Volume I

New Simplified PHYSICS

A Reference Book for Class XI

S.L. Arora

DHANPA

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Preface

This book is an humble attempt at presenting to the students of class XI their complete syllabus prescribed by CBSE in a clear-cut and coordinated manner. The book, based on NCERT pattern, has been carefully designed to make physics learning as easy and straight forward as possible. The present book is the outcome of the author's consistent interaction in the classroom with his students for more than twenty five years. Observing that there is a wide gap between the syllabi of secondary and senior secondary classes, the author has developed the subject matter from the very basic principles of physics. At every stage, student's 'whys' and 'hows' have been anticipated and fully answered so that even the students having weak background of physics and mathematics may easily learn the advanced concepts of physics.

Some of the distinguishing features of the book are :

1. The subject matter has been arranged in a systematic manner strictly in accordance with the CBSE syllabus.
2. The language used in the book is simple, lucid and easily understandable.
3. '*Glimpses*' given in each chapter provide an insight into the entire chapter. These can be used for quick revision also.
4. The book contains a very large number of *illustrative diagrams*. They are neat, explicit, well-labelled and can be easily resketched by the students.
5. A very large number of *solved* and *unsolved numerical problems* on all topics have been given under the appropriate articles for the guidance of the students. It is hoped, doubtlessly, that after a student goes through the solved examples of any topic, sufficient confidence will build up in him to work out unsolved problems of that topic. To further help the students, useful hints or complete solutions of difficult problems have been given at the end of each exercise.
6. '*For Your Knowledge*' boxes given in the book at many places provide additional information to the students which may prove useful for competitive examinations.
7. Another special feature of the book is that it contains a very large number of reasoning based solved problems in the sections '*Very Short Answer Conceptual Problems*' and '*Short Answer Conceptual Problems*'.
8. At the end of each chapter, there is a '*Text Based Exercise*'. Each exercise includes a vast collection of questions asked in previous examination papers of CBSE, ISCE and many other State Boards of India. The exercise has been split into *three* sections : (*a*) *Type A* contains very short answer questions, each carrying 1 mark ; (*b*) *Type B* contains short answer questions, each carrying 2 or 3 marks and (*c*) *Type C* contains long answer questions, each carrying 5 marks. Answers of all these questions have been provided at the end of each section.

I shall feel amply rewarded for the time and labour I have put in during the preparation of this volume, if the students and their teachers find it adequate for their requirements. I need hardly add that suggestions and constructive criticism to improve the book further will be gratefully accepted and incorporated in the next edition of the book.

S.L. ARORA

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Syllabus

Unit		Periods	Marks
I.	Physical World and Measurement	10	23
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UNIT I PHYSICAL WORLD AND MEASUREMENT

Physics-scope and excitement ; nature of physical laws ; Physics, technology and society.

Need for measurement : Units of measurement ; Systems of units ; SI units, Fundamental and derived units, Length, mass and time measurements ; Accuracy and precision of measuring instruments, Errors in measurement ; Significant figures.

Dimensions of physical quantities, dimensional analysis and its applications.

UNIT II KINEMATICS

Frame of reference, Motion in a straight line : Position-time graph, speed and velocity.

Elementary concepts of differentiation and integration for describing motion.

Uniform and non-uniform motion, average speed and instantaneous velocity.

Uniformly accelerated motion, velocity-time, position-time graphs, Relations for uniformly accelerated motion (*graphical treatment*).

Scalar and vector quantities : Position and displacement vectors, general vectors and notation ; Equality of vectors, multiplication of vectors by a real number ; Addition and subtraction of vectors. Relative velocity.

Unit vector, Resolution of a vector in a plane-Rectangular components. Scalar and Vector product of vectors. Motion in a plane, Cases of uniform velocity and uniform acceleration – Projectile motion. Uniform circular motion.

UNIT III LAWS OF MOTION

Intuitive concept of force. Inertia, Newton's first law of motion ; Momentum and Newton's second law of motion, Impulse ; Newton's third law of motion ; Law of conservation of linear momentum and its applications.

Equilibrium of concurrent forces ; Static and Kinetic friction, laws of friction, rolling friction, lubrication.

Uniform circular motion. Dynamics of uniform circular motion ; Centripetal force, examples of circular motion (vehicle on level circular road, vehicle on banked road).

UNIT IV WORK, ENERGY & POWER

Work done by a constant force and a variable force ; Kinetic energy, Work-energy theorem, Power.

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Notion of Potential energy, potential energy of a spring, conservative forces ; conservation of mechanical energy (Kinetic and potential energies), Non-conservative forces ; Motion in a vertical circle ; Elastic and inelastic collisions in one and two dimensions.

UNIT V MOTION OF SYSTEM OF PARTICLES & RIGID BODY

Centre of mass of a two-particle system, momentum conservation and centre of mass motion. Centre of mass of a rigid body ; centre of mass of uniform rod.

Moment of a force, Torque, angular momentum, conservation of angular momentum with some examples.

Equilibrium of rigid bodies, rigid body rotation and equations of rotational motion, comparison of linear and rotational motions. ; Moment of inertia, radius of gyration.

Values of moments of inertia for simple geometrical objects (no derivation). Statement of parallel and perpendicular axes theorems and their applications.

UNIT VI GRAVITATION

Keplar's laws of planetary motion. The universal law of gravitation.

Acceleration due to gravity and its variation with the altitude and depth.

Gravitational potential energy ; gravitational potential. Escape velocity. Orbital velocity of satellite. Geostationary satellites.

UNIT VII PROPERTIES OF BULK MATTER

Elastic behaviour, Stress-Strain relationship, Hooke's law, Young's modulus, bulk modulus, shear modulus of rigidity, poisson's ratio ; elastic energy.

Pressure due to fluid column ; Pascal's law and its applications (hydraulic lift and hydraulic brakes), Effect of gravity on fluid pressure.

Viscosity, Stokes' law, Terminal velocity, Reynold's number, Streamline and turbulent flow. Critical velocity. Bernoulli's theorem and its applications.

Surface energy and surface tension, angle of contact, excess of pressure, applications of surface tension ideas to drops, bubbles and capillary rise.

Heat, temperature, thermal expansion ; Thermal expansion of solids, liquids and gases, anomalous expansion; specific heat capacity ;Cp, Cv - calorimetry ; change of state - latent heat capacity.

Heat transfer - conduction, convection and radiation, Qualitative ideas of Blackbody radiation green house effect, thermal conductivity, Newton's law of cooling, Wein's displacement Law, Stefan's law.

UNIT VIII THERMODYNAMICS

Thermal equilibrium and definition of temperature (zeroth law of thermodynamics), Heat, work and internal energy. First law of thermodynamics. Isothermal and adiabatic processes.

Second law of thermodynamics : reversible and irreversible processes. Heat engines and refrigerators.

UNIT IX BEHAVIOUR OF PERFECT GAS AND KINETIC THEORY

Equation of state of a perfect gas, word done on compressing a gas.

Kinetric theory of gases - assumptions, concept of pressure.. Kinetic energy and temperature ; rms speed of gas molecules ; degrees of freedom, law of equipartition of energy (statement only) and application to specific heat capacities of gases ; concept of mean free path, Avogadro's number.

UNIT X OSCILLATIONS & WAVES

Periodic motion - period, frequency, displacement as a function of time and periodic functions, Simple harmonic motion (S.H.M.) and its equation ; Phase, oscillations of a spring-restoring force and force constant ; Energy in S.H.M. Kinetic and potential energies ; Simple pendulum- derivation of expression for its time period ; Free, forced and damped oscillations (*qualitative ideas only*), resonance.

Wave motion. Transverse and longitudinal waves, speed of wave motion. Displacement relation for a progressive wave ; Principle of superposition of waves, Reflection of waves, standing waves in strings and organ pipes, fundamental mode and harmonics, Beats, Doppler effect.

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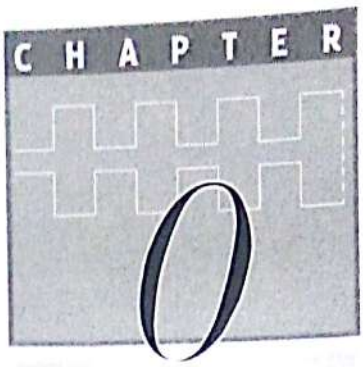

MATHEMATICAL TOOLS

0.1 ALGEBRA

Common Identities

(i) $(a+b)^2 = a^2 + 2ab + b^2 = (a-b)^2 + 4ab$

(ii) $(a-b)^2 = a^2 - 2ab + b^2 = (a+b)^2 - 4ab$

(iii) $a^2 - b^2 = (a+b)(a-b)$

(iv) $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
$= a^3 + b^3 + 3ab(a+b)$

(v) $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
$= a^3 - b^3 - 3ab(a-b)$

(vi) $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
$= (a+b)^3 - 3ab(a+b)$

(vii) $a^3 - b^3 = (a-b)(a^2 + ab + b^3)$
$= (a-b)^3 + 3ab(a-b)$

(viii) $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$

(ix) $(a+b)^2 - (a-b)^2 = 4ab$

(x) $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Quadratic Equation

An equation of second degree is called a quadratic equation. It is of the form :

$$ax^2 + bx + c = 0$$

The roots of a quadratic equation are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

EXAMPLE 1. *Solve the equation :* $6x^2 - 13x + 6 = 0$.

Solution. Here $a = 6, \quad b = -13, \quad c = 6$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{+13 \pm \sqrt{169 - 4 \times 6 \times 6}}{2 \times 6}$$

$$= \frac{13 \pm \sqrt{169 - 144}}{12} = \frac{13 \pm 5}{12} = \frac{18}{12}, \frac{8}{12}$$

or $\quad x = \frac{3}{2}, \frac{2}{3}$.

Binomial Theorem

If n is any integer, positive or negative, or a fraction and x is any real number, then

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$$

where $2! = 2 \times 1, \quad 3! = 3 \times 2 \times 1$

In general, $\quad n! = n(n-1)(n-2)..3 \times 2 \times 1$.

If $|x| << 1$, then $(1+x)^n = 1 + nx$.

EXAMPLE 2. *The acceleration due to gravity at a height h above the surface of the earth (radius = R) is given by*

$$g' = \frac{gR^2}{(R+h)^2}$$

If $h << R$, *then show that :* $g' = g\left(1 - \frac{2h}{R}\right)$.

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Solution. $g' = \dfrac{gR^2}{(R+h)^2}$

$$= \frac{gR^2}{R^2\left(1+\frac{h}{R}\right)^2} = g\left(1+\frac{h}{R}\right)^{-2}$$

Using binomial theorem,

$$g' = g\left[1+(-2)\frac{h}{R}+\frac{(-2)(-3)}{2!}\left(\frac{h}{R}\right)^2+....\right]$$

As $h << R$, h/R is very small, so terms containing higher powers of h/R can be neglected.

$$\therefore \qquad g' = g\left(1-\frac{2h}{R}\right).$$

0.2 ▼ MENSURATION

Important Formulae

Circumference of a circle $= 2\pi r = \pi D$

Area of a circle $= \pi r^2 = \dfrac{\pi D^2}{4}$

Surface area of a sphere $= 4\pi r^2 = \pi D^2$

Volume of a sphere $= \dfrac{4}{3}\pi r^3$

Surface area of a cylinder $= 2\pi r^2 + 2\pi rl = 2\pi r(r+l)$

Volume of a cylinder $= \pi r^2 l$

Curved surface area of a cone $= \pi rl$

Volume of a cone $= \dfrac{1}{3}\pi r^2 h$

Volume of a cube $= (\text{side})^3$

Surface area of a cube $= 6 \times (\text{side})^2$.

0.3 ▼ TRIGONOMETRY

Systems of Measurement of an Angle

(*i*) *Sexagesimal system.* In this system,

1 right angle $= 90°$ (degree)

$1° = 60'$ (minute)

$1' = 60''$ (second)

(*ii*) *Centesimal system.* In this system

1 right angle $= 100^g$ (grade)

$1^g = 100'$ (minute)

$1' = 100''$ (second)

(*iii*) *Circular system.* In this system, the unit of angle is radian.

One radian is the angle subtended at the centre of a circle by an arc whose length is equal to the radius of the circle.

If l is the length of an arc and θ is the angle subtended at the centre of the circle as shown in Fig. 0.1, then

$$\theta = \frac{\text{Arc}}{\text{Radius}} = \frac{l}{r} \text{ radian}$$

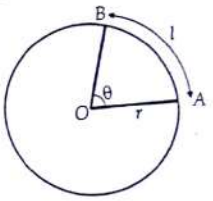

Fig. 0.1

Angle subtended at the centre of the circle is

$$\theta = \frac{\text{Circumference}}{\text{Radius}} = \frac{2\pi r}{r} = 2\pi \text{ radian}$$

π radian $= 180° = 200^g$

1 radian $= 57°916'22'' = 63^g\,63'\,64''$

Trigonometrical Ratios

In right angled ΔOMP, of Fig. 0.2, $\angle OMP = 90°$ and $\angle POM = \theta$.

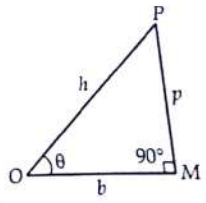

Fig. 0.2

We can define the trigonometric ratios as follows :

$$\text{sine } \theta = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{PM}{OP} = \sin\theta$$

$$\text{cosine } \theta = \frac{\text{base}}{\text{hypotenuse}} = \frac{OM}{OP} = \cos\theta$$

$$\text{tangent } \theta = \frac{\text{perpendicular}}{\text{base}} = \frac{PM}{OM} = \tan\theta$$

$$\text{cosecant } \theta = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{OP}{PM} = \text{cosec } \theta$$

$$\text{secant } \theta = \frac{\text{hypotenuse}}{\text{base}} = \frac{OP}{OM} = \sec\theta$$

$$\text{cotangent } \theta = \frac{\text{base}}{\text{perpendicular}} = \frac{OM}{PM} = \cot\theta$$

Fundamental Trigonometric Relations

1. $\text{cosec } \theta = \dfrac{1}{\sin\theta}$, $\sec\theta = \dfrac{1}{\cos\theta}$, $\cot\theta = \dfrac{1}{\tan\theta}$

2. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

3. $\sin^2\theta + \cos^2\theta = 1$, $1 + \tan^2\theta =$

$1 + \cot^2\theta = \text{cosec}^2\,\theta$

T-ratios of Allied Angles

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

$\sin(90° - \theta) = \cos\theta$

$\cos(90° - \theta) = \sin\theta$

$\tan(90° - \theta) = \cot\theta$

$\sin(90° + \theta) = \cos\theta$

$\cos(90° + \theta) = -\sin\theta$

$\tan(90° + \theta) = -\cot\theta$

$\sin(180° - \theta) = \sin\theta$

$\cos(180° - \theta) = -\cos\theta$

$\tan(180° - \theta) = -\tan\theta$

$\sin(180° + \theta) = -\sin\theta$

$\cos(180° + \theta) = -\cos\theta$

$\tan(180° + \theta) = \tan\theta$

$\sin(270° - \theta) = -\cos$

$\cos(270° - \theta) = -\sin$

$\tan(270° - \theta) = \cot$

$\sin(270° + \theta) = -$

$\cos(270° + \theta) = \sin$

$\tan(270° + \theta) =$

$\sin(360° - \theta) =$

$\cos(360° - \theta)$

$\tan(360° - \theta)$

Values of

angle θ
sin θ
cos
tan

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2. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, $\quad \cot\theta = \dfrac{\cos\theta}{\sin\theta}$

3. $\sin^2\theta + \cos^2\theta = 1$, $\quad 1 + \tan^2\theta = \sec^2\theta$,

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

T-ratios of Allied Angles

$\sin(-\theta) = -\sin\theta$	$\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$
$\cos(-\theta) = \cos\theta$	$\sec(-\theta) = \sec\theta$
$\tan(-\theta) = -\tan\theta$	$\cot\theta = -\cot\theta$
$\sin(90^\circ - \theta) = \cos\theta$	$\operatorname{cosec}(90^\circ - \theta) = \sec\theta$
$\cos(90^\circ - \theta) = \sin\theta$	$\sec(90^\circ - \theta) = \operatorname{cosec}\theta$
$\tan(90^\circ - \theta) = \cot\theta$	$\cot(90^\circ - \theta) = \tan\theta$
$\sin(90^\circ + \theta) = \cos\theta$	$\operatorname{cosec}(90^\circ + \theta) = \sec\theta$
$\cos(90^\circ + \theta) = -\sin\theta$	$\sec(90^\circ + \theta) = -\operatorname{cosec}\theta$
$\tan(90^\circ + \theta) = -\cot\theta$	$\cot(90^\circ + \theta) = -\tan\theta$
$\sin(180^\circ - \theta) = \sin\theta$	$\operatorname{cosec}(180^\circ - \theta) = \operatorname{cosec}\theta$
$\cos(180^\circ - \theta) = -\cos\theta$	$\sec(180^\circ - \theta) = -\sec\theta$
$\tan(180^\circ - \theta) = -\tan\theta$	$\cot(180^\circ - \theta) = -\cot\theta$
$\sin(180^\circ + \theta) = -\sin\theta$	$\operatorname{cosec}(180^\circ + \theta) = -\operatorname{cosec}\theta$
$\cos(180^\circ + \theta) = -\cos\theta$	$\sec(180^\circ + \theta) = -\sec\theta$
$\tan(180^\circ + \theta) = \tan\theta$	$\cot(180^\circ + \theta) = \cot\theta$
$\sin(270^\circ - \theta) = -\cos\theta$	$\operatorname{cosec}(270^\circ - \theta) = -\sec\theta$
$\cos(270^\circ - \theta) = -\sin\theta$	$\sec(270^\circ - \theta) = -\operatorname{cosec}\theta$
$\tan(270^\circ - \theta) = \cot\theta$	$\cot(270^\circ - \theta) = \tan\theta$
$\sin(270^\circ + \theta) = -\cos\theta$	$\operatorname{cosec}(270^\circ + \theta) = -\sec\theta$
$\cos(270^\circ + \theta) = \sin\theta$	$\sec(270^\circ + \theta) = \operatorname{cosec}\theta$
$\tan(270^\circ + \theta) = -\cot\theta$	$\cot(270^\circ + \theta) = -\tan\theta$
$\sin(360^\circ - \theta) = -\sin\theta$	$\operatorname{cosec}(360^\circ - \theta) = -\operatorname{cosec}\theta$
$\cos(360^\circ - \theta) = \cos\theta$	$\sec(360^\circ - \theta) = \sec\theta$
$\tan(360^\circ - \theta) = -\tan\theta$	$\cot(360^\circ - \theta) = -\cot\theta$

Some Important Trigonometrical Formulae

$\sin(A + B) = \sin A \cos B + \cos A \sin B$

$\sin(A - B) = \sin A \cos B - \cos A \sin B$

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A = \frac{2 \tan A}{1 + \tan^2 A}$$

$$\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A$$

$$= 2\cos^2 A - 1 = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$\sin(A + B) + \sin(A - B) = 2 \sin A \cos B$

$\sin(A + B) - \sin(A - B) = 2 \cos A \sin B$

$\cos(A + B) + \cos(A - B) = 2 \cos A \cos B$

$\cos(A + B) - \cos(A - B) = -2 \sin A \sin B$

$$\sin C + \sin D = 2 \sin \frac{C + D}{2} \cos \frac{C - D}{2}$$

$$\sin C - \sin D = 2 \cos \frac{C + D}{2} \sin \frac{C - D}{2}$$

$$\cos C + \cos D = 2 \cos \frac{C + D}{2} \cos \frac{C - D}{2}$$

$$\cos C - \cos D = -2 \sin \frac{C + D}{2} \sin \frac{C - D}{2}$$

Values of Trigonometrical Ratios of Some Standard Angles

angle θ	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	0	1
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0	$-\infty$	0

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0.4 ▼ LOGARITHMS

Definition of Logarithm

The logarithm of any number to a given base is the power to which base must be raised to obtain that number.

For example, $81 = 3^4$, we can say that the logarithm of 81 to the base 3 is equal to 4.

Symbolically, $\log_3 81 = 4$

In general, if $N = a^x$, then $\log_a N = x$

The common logarithm of a number is the power to which 10 must be raised to obtain that number.

As $1000 = 10^3$ $\therefore$ $\log_{10} 1000 = 3$

As $a^0 = 1$ $\therefore$ $\log_a 1 = 0$

As $a^1 = a$ $\therefore$ $\log_a a = 1$

Logarithmic Formulae

Product formula :

$$\log_a mn = \log_a m + \log_a n$$

Quotient formula :

$$\log_a \frac{m}{n} = \log_a m - \log_a n$$

Power formula :

$$\log_a m^n = n \log_a m$$

Base change formula :

$$\log_a m = \log_b m \times \log_a b$$

0.5 ▼ DIFFERENTIAL CALCULUS

Differential Coefficient

Let y be a function of x i.e., $y = f(x)$

Suppose the value of x increases by a small amount Δx. Then the value of y also increases by a small amount, say Δy.

The ratio $\frac{\Delta y}{\Delta x}$ is called the *average rate of change of y with respect to x.*

When Δx approaches zero, the limiting value of $\frac{\Delta y}{\Delta x}$ is called *differential coefficient* or *derivative of y w.r.t. x* and is denoted by $\frac{dy}{dx}$.

Hence $$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

Physically, the *derivative* $\frac{dy}{dx}$ *gives the instantaneous rate of change of function y with respect to variable x.*

Some Important Results on Differentiation

(i) Let c be a constant. Then $\frac{d}{dx}(c) = 0$

(ii) $\frac{d}{dx}(cx) = c \cdot \frac{dy}{dx}$

(iii) $\frac{d}{dx}(x^n) = nx^{n-1}$

(iv) Let $y = u \pm v$, where u and v are functions of x.

Then $\frac{dy}{dx} = \frac{du}{dx} \pm \frac{dv}{dx}$

(v) Product Rule.

Let $y = uv$, then

$$\frac{dy}{dx} = F.F. \frac{d}{dx}(S.F.) + S.F. \frac{d}{dx}(F.F.)$$

$$= u\frac{dv}{dx} + v\frac{du}{dx}$$

(vi) Quotient rule.

Let $y = \frac{u}{v}$, then

$$\frac{dy}{dx} = \frac{\text{Den} \frac{d}{dx}(\text{Num}) - (\text{Num}) \frac{d}{dx}(\text{Den})}{(\text{Den})^2}$$

$$= \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

(vii) Chain rule. Let y be a function of u and u be a function of x. Then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Let $y = u^n$. Then

$$\frac{d}{dx}(u^n) = nu^{n-1} \cdot \frac{du}{dx}$$

(viii) $\frac{d}{dx}(\log_e x) = \frac{1}{x}$

(ix) $\frac{d}{dx}(\log_a x) = \frac{1}{x} \log_e a$

(x) $\frac{d}{dx}(e^x) = e^x$

(xi) $\frac{d}{dx}(a^x) = a^x \log_e a$

(xii) $\frac{d}{dx}(\sin x) = \cos x$

(xiii) $\frac{d}{dx}(\cos x) = -\sin x$

(xiv) $\frac{d}{dx}(\tan x) = \sec^2 x$

(xv) $\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$

(xvi) $\frac{d}{dx}(\sec x) = \sec x \tan x$

(xvii) $\frac{d}{dx}(\text{cosec } x) = -\text{cosec } x \cot x$

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EXAMPLE 3. *Find* $\frac{dy}{dx}$ *for the following functions :*

(i) $y = x^5 + x^3 + 10$ (ii) $y = x + \sqrt{x} + \frac{1}{\sqrt{x}}$

(iii) $y = 5x^4 + 3x^{3/2} + 6x$.

Solution.

(i) $y = x^5 + x^3 + 10$

$$\frac{dy}{dx} = \frac{d}{dx}(x^5) + \frac{d}{dx}(x^3) + \frac{d}{dx}(10)$$

$$= 5x^4 + 3x^2 + 0 = \mathbf{5x^4 + 3x^2}.$$

(ii) $y = x + \sqrt{x} + \frac{1}{\sqrt{x}} = x^1 + x^{1/2} + x^{-1/2}$

$$\frac{dy}{dx} = 1.x^0 + \frac{1}{2}x^{-1/2} - \frac{1}{2}x^{-3/2}$$

$$= 1 + \frac{1}{2\sqrt{x}} - \frac{1}{2x\sqrt{x}}.$$

(iii) $y = 5x^4 + 3x^{3/2} + 6x$

$$\frac{dy}{dx} = 5\frac{d}{dx}(x^4) + 3\frac{d}{dx}(x^{3/2}) + 6\frac{d}{dx}(x)$$

$$= 5 \times 4x^3 + 3 \times \frac{3}{2}x^{1/2} + 6 \times 1$$

$$= \mathbf{20x^3 + \frac{9}{2}\sqrt{x} + 6}.$$

EXAMPLE 4. *Differentiate the following functions :*

(i) $(3x^2 + 7)(6x + 3)$ (ii) $\frac{x^2 + 1}{x - 2}$

(iii) $\sqrt{4x^2 - 7}$.

Solution. (i) Let $y = (3x^2 + 7)(6x + 3)$

Using product rule, we get

$$\frac{dy}{dx} = (3x^2 + 7)\frac{d}{dx}(6x + 3) + (6x + 3)\frac{d}{dx}(3x^2 + 7)$$

$$= (3x^2 + 7)(6 + 0) + (6x + 3)(6x + 0)$$

$$= 18x^2 + 42 + 36x^2 + 18x = \mathbf{54x^2 + 18x + 42}.$$

(ii) $y = \frac{x^2 + 1}{x - 2}$

Using quotient rule, we get

$$\frac{dy}{dx} = \frac{(x-2)\frac{d}{dx}(x^2 + 1) - (x^2 + 1)\frac{d}{dx}(x - 2)}{(x-2)^2}$$

$$= \frac{(x-2)(2x + 0) - (x^2 + 1)(1 - 0)}{(x-2)^2}$$

$$= \frac{2x^2 - 4x - x^2 - 1}{(x-2)^2} = \frac{\mathbf{x^2 - 4x - 1}}{\mathbf{(x-2)^2}}.$$

(iii) $y = \sqrt{4x^2 - 7} = (4x^2 - 7)^{1/2}$

Using chain rule,

$$\frac{dy}{dx} = \frac{1}{2}(4x^2 - 7)^{-1/2}\frac{d}{dx}(4x^2 - 7)$$

$$= \frac{1}{2}(4x^2 - 7)^{-1/2}(8x - 0) = \frac{\mathbf{4x}}{\mathbf{\sqrt{4x^2 - 7}}}.$$

EXAMPLE 5. *Find the differential coefficient of the following functions :*

(i) $\cos(ax^2 + b)$ (ii) $\tan^3 x$ (iii) $\frac{\sin x}{1 + \cos x}$

Solution. (i) Let $y = \cos(ax^2 + b)$

Then $$\frac{dy}{dx} = -\sin(ax^2 + b)\frac{d}{dx}(ax^2 + b)$$

$$= -\sin(ax^2 + b).2ax$$

$$= \mathbf{-2ax \sin(ax^2 + b)}.$$

(ii) Let $y = \tan^3 x = (\tan x)^3$

Then $$\frac{dy}{dx} = 3(\tan x)^2\frac{d}{dx}(\tan x)$$

$$= \mathbf{3\tan^2 x . \sec^2 x}.$$

(iii) Let $y = \frac{\sin x}{1 + \cos x}$

Then

$$\frac{dy}{dx} = \frac{(1 + \cos x)\frac{d}{dx}(\sin x) - \sin x\frac{d}{dx}(1 + \cos x)}{(1 + \cos x)^2}$$

$$= \frac{(1 + \cos x)\cos x - \sin x(0 - \sin x)}{(1 + \cos x)^2}$$

$$= \frac{\cos x + \cos^2 x + \sin^2 x}{(1 + \cos x)^2}$$

$$= \frac{\cos x + 1}{(1 + \cos x)^2} = \frac{\mathbf{1}}{\mathbf{1 + \cos x}}.$$

EXAMPLE 6. *A particle is moving with a uniform acceleration. Its displacement at any instant t is given by* $s = 10t + 4.9t^2$. *What is (i) initial velocity (ii) velocity at* $t = 3$ *second and (iii) the uniform acceleration ?*

Solution. Displacement, $s = 10t + 4.9t^2$

Velocity, $v = \frac{ds}{dt} = 10 + 2 \times 4.9t$

(i) Initial velocity

(*i.e.*, velocity $t = 0$) $= 10 + 0 = \mathbf{10\ ms^{-1}}$.

(ii) Velocity at $t = 3$ second is

$$v = 10 + 2 \times 4.9 \times 3 = \mathbf{39.4\ ms^{-1}}.$$

(iii) Acceleration,

$$a = \frac{dv}{dt} = \frac{d}{dt}(10 + 9.8t) = 0 + 9.8 = \mathbf{9.8\ ms^{-2}}$$

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EXAMPLE 7. *A particle starts rotating from rest according to the formula,*

$$\theta = \frac{3t^3}{20} - \frac{t^2}{3}$$

where θ is in radian and t in second. Find the angular velocity ω and angular acceleration α at the end of 5 seconds.

Solution. Given $\theta = \frac{3t^3}{20} - \frac{t^2}{3}$

Angular velocity,

$$\omega = \frac{d\theta}{dt} = \frac{d}{dt}\left(\frac{3t^3}{20} - \frac{t^2}{3}\right) = \frac{9t^2}{20} - \frac{2t}{3}$$

At $t = 5$ s,

$$\omega = \frac{9 \times 25}{20} - \frac{2 \times 5}{3} = \frac{475}{60} = 7.92 \text{ rad s}^{-1}.$$

Angular acceleration,

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}\left(\frac{9t^2}{20} - \frac{2t}{3}\right) = \frac{18}{20}t - \frac{2}{3}$$

At $t = 5$ s,

$$\alpha = \frac{18 \times 5}{20} - \frac{2}{3} = 3.83 \text{ rad s}^{-2}.$$

EXAMPLE 8. *Show that power is the product of force and velocity.*

Solution. Work = Force × distance

or $W = Fs$

Power = Rate of doing work

$$= \frac{dW}{dt} = \frac{d}{dt}(Fs) = F\frac{ds}{dt} = F \,.\, v.$$

EXAMPLE 9. *A balloon is being filled by air so that its volume V is gradually increasing. Find the rate of increase of volume with radius r when $r = 2$ units.*

Solution. The volume of spherical balloon is

$$V = \frac{4}{3}\pi r^3$$

The rate of increase of volume V *w.r.t.* the radius r is

$$\frac{dV}{dr} = \frac{d}{dr}\left(\frac{4}{3}\pi r^3\right)$$

or $$\frac{dV}{dr} = \frac{4}{3}\pi \,.\, \frac{d}{dr}(r^3) = \frac{4}{3}\pi \,.\, 3r^2 = 4\pi r^2$$

When $r = 2$

$$\frac{dV}{dr} = 4\pi(2)^2 = 16\pi.$$

EXAMPLE 10. *For a particle executing simple harmonic motion, the displacement from the mean position is given by $y = a \sin(\omega t + \phi)$; where a, ω and ϕ are constants. Find the velocity and acceleration of the particle at any instant t.*

Solution. Displacement, $y = a \sin(\omega t + \phi)$

Velocity,

$$v = \frac{dy}{dt} = \frac{d}{dt}[a \sin(\omega t + \phi)]$$

$$= a \cos(\omega t + \phi)\frac{d}{dt}(\omega t + \phi)$$

$$= \omega a \cos(\omega t + \phi)$$

Acceleration,

$$a = \frac{dv}{dt} = \frac{d}{dt}[\omega a \cos(\omega t + \phi)]$$

$$= -\omega a \sin(\omega t + \phi)\frac{d}{dt}(\omega t + \phi)$$

$$= -\omega^2 a \sin(\omega t + \phi).$$

0.6 INTEGRAL CALCULUS

Integration

Integration is the reverse process of differentiation. It is the process of finding a function whose derivative is given. If derivative of function $f(x)$ *w.r.t.* x is $f'(x)$, then integration of $f'(x)$ *w.r.t.* x is $f(x)$. Symbolically, we can say

if $\frac{d}{dx}[f(x)] = f'(x)$, then $\int f'(x)dx = f(x)$.

Some Standard Elementary Integrals

Some standard elementary integrals alongwith their results on differentiation are as follows :

	Differentiation	Integration
1.	$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n\,dx = \frac{x^{n+1}}{(n+1)} + c$, provided $n \neq -1$. Here c is constant of integration.
2.	$\frac{d}{dx}(x) = 1$	$\int dx = x + c$
3.	$\frac{d}{dx}(\log_e x) = \frac{1}{x}$	$\int \frac{dx}{x} = \log_e x + c$
4.	$\frac{d}{dx}(\sin x) = \cos x$	$\int \cos x \,.\, dx = \sin x + c$
5.	$\frac{d}{dx}(\cos x) = -\sin x$	$\int \sin x \,.\, dx = -\cos x + c$
6.	$\frac{d}{dx}(\tan x) = \sec^2 x$	$\int \sec^2 x \,.\, dx = \tan x + c$
7.	$\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$	$\int \text{cosec}^2 x \,.\, dx = -\cot x + c$
8.	$\frac{d}{dx}(\sec x) = \sec x \,.\, \tan x$	$\int \sec x \,.\, \tan x\, dx = \sec x + c$

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	Differentiation	Integration
9.	$\frac{d}{dx}(\text{cosec } x) = -\text{cosec } x \cdot \cot x$	$\int \text{cosec } x \cdot \cot x \, dx = -\text{cosec } x + c$
10.	$\frac{d}{dx}(ax+b)^n = na(ax+b)^{n-1}$	$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$
11.	$\frac{d}{dx}\log_e(ax+b) = \frac{a}{(ax+b)}$	$\int \frac{dx}{(ax+b)} = \frac{1}{a}\log_e(ax+b) + c$
12.	$\frac{d}{dx}(e^x) = e^x$	$\int e^x \cdot dx = e^x + c$
13.	$\frac{d}{dx}(a^x) = a^x \cdot \log_e a$	$\int a^x \cdot dx = \frac{a^x}{\log_e a} = a^x \cdot \log_a e + c$
14.	$y = u \pm v \pm w$; $\frac{dy}{dx} = \frac{du}{dx} \pm \frac{dv}{dx} \pm \frac{dw}{dx}$	$\int (u \pm v \pm w)\, dx = \int u \cdot dx \pm \int v dx \pm \int w dx + c$

Definite integral

When an integral is defined between two definite limits a and b, it is said to be a definite integral. It is given by

$$\int_a^b f(x)dx = [\phi(x)]_a^b = \phi(b) - \phi(a)$$

where $\phi(x)$ is the integral of $f(x)$. Here a and b are the lower and upper limits of integration.

EXAMPLE 11. *Integrate :* $x^2 - \cos x + \frac{1}{x}$.

Solution. $\int \left(x^2 - \cos x + \frac{1}{x}\right) dx$

$$= \int x^2\, dx - \int \cos x\, dx + \int \frac{1}{x} dx$$

$$= \frac{x^3}{3} - \sin x + \log x + c.$$

EXAMPLE 12. *Evaluate* $\int_0^{\pi/6} \sec^2 x dx$.

Solution. $\int_0^{\pi/6} \sec^2 x dx = [\tan x]_0^{\pi/6}$

$$= \tan\frac{\pi}{6} - \tan 0 = \frac{1}{\sqrt{3}} - 0 = \frac{1}{\sqrt{3}}.$$

EXAMPLE 13. *Find the value of* $\int_R^\infty \frac{GMmdx}{x^2}$ *; where G, M and m are constants.*

Solution. $\int_R^\infty \frac{GMm}{x^2} dx = GMm \int_R^\infty x^{-2} dx$

$$= GMm\left[\frac{x^{-1}}{-1}\right]_R^\infty = -GMm\left[\frac{1}{x}\right]_R^\infty$$

$$= -GMm\left[\frac{1}{\infty} - \frac{1}{R}\right]$$

$$= -GMm\left[0 - \frac{1}{R}\right] = \frac{GMm}{R}.$$

EXAMPLE 14. *Find the value of* $\int_0^x F\, dx$ *;*

where $F = kx$.

Solution. $\int_0^x F dx = \int_0^x kx dx$

$$= k\int_0^x x dx = k\left[\frac{x^2}{2}\right]_0^x$$

$$= k\left[\frac{x^2}{2} - 0\right] = \frac{1}{2}kx^2.$$

EXAMPLE 15. *Find the value of* $\int_{V_1}^{V_2} \frac{1}{V} dV$.

Solution. $\int_{V_1}^{V_2} \frac{1}{V} dV = [\log_e V]_{V_1}^{V_2}$

$$= [\log_e V_2 - \log_e V_1]$$

$$= \log_e \frac{V_2}{V_1}.$$

EXAMPLE 16. *Evaluate* $\int_{-l/2}^{+l/2} \frac{M}{l} \cdot x^2 dx$ *; where M and l are constants.*

Solution.

$$\int_{-l/2}^{+l/2} \frac{M}{l} \cdot x^2\, dx = \frac{M}{l}\int_{-l/2}^{+l/2} x^2 dx$$

$$= \frac{M}{l}\left[\frac{x^3}{3}\right]_{-l/2}^{+l/2}$$

$$= \frac{M}{3l}\left[\left(\frac{l}{2}\right)^3 - \left(-\frac{l}{2}\right)^3\right]$$

$$= \frac{M}{3l}\cdot\left[\frac{l^3}{8} + \frac{l^3}{8}\right] = \frac{M}{3l}\cdot\frac{2l^3}{8} = \frac{Ml^2}{12}.$$

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PROBLEMS FOR PRACTICE

1. Find $\frac{dy}{dx}$ for the following functions :

(i) $y = x^3 - 3x^2 + 3x - \frac{2}{5}$ (Ans. $3x^2 - 6x + 3$)

(ii) $y = \frac{(x-1)(x-2)}{\sqrt{x}}$ $\left(\text{Ans. } \frac{3}{2}\sqrt{x} - \frac{3}{2\sqrt{x}} - \frac{1}{x^{3/2}}\right)$

(iii) $y = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$ $\left(\text{Ans. } 1 - \frac{1}{x^2}\right)$

(iv) $y = ax^2 + bx + c$ (Ans. $2ax + b$)

2. Differentiate the following functions :

(i) $(x^2 - 4x + 5)(x^3 - 2)$ (Ans. $5x^4 - 16x^3 + 15x^2 - 4x + 8$)

(ii) $\frac{2x+3}{x^2-5}$ $\left(\text{Ans. } -\frac{2x^2 + 6x + 10}{(x^2-5)^2}\right)$

(iii) $\frac{\sin x + \cos x}{\sin x - \cos x}$ $\left(\text{Ans. } \frac{-2}{(\sin x - \cos x)^2}\right)$

(iv) $(4x^3 - 5x^2 + 1)^4$ [Ans. $8(4x^3 - 5x^2 + 1)^3 (6x^2 - 5x)$]

3. If the motion of a particle is governed by the equation, $s = 2t^3 - 3t^2 + 2t + 2$, find the position, velocity and acceleration of the particle at time $t = 2$ s. (Ans. 10, 14, 18)

4. A particle starts rotating from rest according to the formula,

$$\theta = \frac{t^4}{60} - \frac{t}{4}$$

where θ is in radian and t in second. Find the angular velocity ω and angular acceleration α at time $t = 10$ s. (Ans. 66.5 rad s^{-1}, 20 rad s^{-2})

5. Show that force can be expressed ... mass and acceleration.

[Hint. $F = \frac{dp}{dt} = \frac{d}{dt}(mv) = m\frac{dv}{dt} = ma$]

6. Integrate the following :

(i) $6x + 5x^2 - 2x^3 + \frac{1}{x^2}$ $\left(\text{Ans. } 3x^2 + \frac{5}{3}x^3 - \frac{1}{2}x^4 - \frac{1}{x} + c\right)$

(ii) $\left(x + \frac{1}{x}\right)^3$ $\left(\text{Ans. } \frac{1}{4}x^4 + \frac{3}{2}x^2 + 3\log x - \frac{1}{2x^2} + c\right)$

(iii) $3\,\text{cosec}^2 x - 5x + \sin x$ $\left(\text{Ans. } -3\cot x - \frac{5}{2}x^2 - \cos x + c\right)$

(iv) $3\,\text{cosec}^2 x + 2\sin 3x$ $\left(\text{Ans. } -3\cot x - \frac{2}{3}\cos 3x + c\right)$

7. Evaluate the following integrals :

(i) $\int_1^4 \sqrt{x}\,dx$ $\left(\text{Ans. } \frac{14}{3}\right)$

(ii) $\int_0^4 (t^2 + 1)\,dt$ $\left(\text{Ans. } 25\frac{1}{3}\right)$

(iii) $\int_{-\pi/2}^{\pi/2} \cos x dx$ (Ans. 2)

(iv) $\int_0^{\pi/4} \sec x \tan x\,dx$ (Ans. $\sqrt{2} - 1$)

(v) $\int_2^3 \frac{1}{x}\,dx$ $\left(\text{Ans. } \log\frac{3}{2}\right)$

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CHAPTER

1

PHYSICAL WORLD

1.1 WHAT IS SCIENCE ?

1. *What is science ?*

Science. *It is a systematised and organised knowledge about the various natural phenomena which is obtained by careful experimentation, keen observation and accurate reasoning.* The Sanskrit word *Shastra* and Arabic word *Ilm* also have a similar meaning *i.e.,* organised knowledge.

2. *What are physical and biological sciences ?*

Two types of sciences. The knowledge of science can be divided into two broad categories :

1. ***Biological sciences.*** *The sciences which deal with the behaviour of living things are called biological sciences.* These sciences include Botany, Zoology, Orinthology, Anthropology, Entomology, Forensic Science, etc.

2. ***Physical sciences.*** The two main physical sciences are physics and chemistry. **Chemistry** *is the study of every substance, its structure, its composition and changes in which it takes part.* **Physics** *is the study of the natural world which deals with the concepts of space, time, motion, matter, energy, radiation, etc.* Other physical sciences include Geology, Geography, Astronomy, Astrology, Oceanology, etc.

1.2 SCIENTIFIC ATTITUDE AND SCIENTIFIC METHOD

3. *What is scientific attitude ?*

Scientific attitude. The tremendous growth of science has taken place only due to the use of scientific attitude and scientific method. *The scientific attitude requires a flexible, open-minded approach towards solving problems in which other important points of view are not neglected without any reason.* First a solution is suggested for a problem. This solution is tried. If it works satisfactorily, it is adopted and otherwise it is replaced by a better solution to the same problem.

4. *What is scientific method ? Mention the various steps involved in it.*

Scientific method. *The step by step approach used by a scientist in studying natural phenomena and establishing laws which govern these phenomena is called scientific method.*

Generally, it involves the following steps :

1. Taking a large number of systematic *observations* by doing *controlled experiments.*
2. Studying these observations and making *qualitative* and *quantitative reasoning.*
3. Suggesting *mathematical models* to account for the observed behaviour.
4. *Predicting new phenomena* on the basis of suggested model.
5. Modifying the theory, if necessary, in the light of the fresh evidences.

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1.3 SCIENTIFIC THEORY

5. *What is scientific theory ?*

Scientific theory. *Theory is the name given to a set of limited number of laws in terms of which the behaviour of a physical system can be explained. A good theory should not only explain the already existing phenomenon but it should be able to predict and explain the new phenomenon.*

6. *When does a scientific theory need modification or replacement by a new theory ? Briefly explain giving suitable examples.*

Need for modification of a scientific theory. In science, no theory is final. A theory must be able to explain what may be observed from time to time. Following two types of problems may arise :

1. When any new observation shows some deviation from the existing theory, the theory has to be duly modified. For example, when Johann Kepler examined the extensive data on planetary motion collected by Tycho Brahe, the planetary circular orbits in heliocentric theory (sun at the centre of the solar system) imagined by Nicholas Coperanicus had to be replaced by elliptical orbits to fit the data better.

2. When even the modification of a theory fails to account for the new observations, the theory has to be replaced by a new theory. For example, it was realised in the beginning of twentieth century that Newtonian mechanics could not explain some basic features of atomic phenomena. Also, the wave theory of light failed to explain photoelectric effect. This resulted in the development of an entirely new theory, called Quantum Mechanics, to deal with microscopic phenomena.

7. *Theory and experiments go hand in hand in physics and help each other's progress. Give two suitable examples in support of your answer.*

Or

With the help of suitable examples, briefly explain the interplay between theoretical models and experiments.

Interplay between theoretical models and experiments. The interplay of theory and experiment (or observation) is important for the progress of science.

This is obvious from the following examples :

1. Sometimes a new experiment suggests an alternative theoretical model. For example, alpha particle scattering experiments in 1911 by Ernest Rutherford established the nuclear model of the atom, which then became the basis of quantum theory of hydrogen atom given by Niels Bohr in 1913.

2. Sometimes a theoretical advancement predicts a new experiment. For example, the concept of antiparticle was introduced theoretically by Paul Dirac in 1930 and was confirmed two years later by the experimental discovery of positron (anti-electron) by Carl Anderson.

1.4 WHAT IS PHYSICS ?

8. *What is physics ? What is the origin of the word physics ? What is its Sanskrit equivalent ?*

Physics. The word physics originates from a Greek word which means nature. This word was introduced by ancient scientist *Aristotle* in the year 350 B.C. The Sanskrit equivalent of physics is *bhautika* which refers to the physical world.

Physics is the branch of science that deals with the study of basic laws of nature and their manifestation in various natural phenomena. It is concerned with the interaction of matter with matter or energy. It deals with the various features of the natural world such as space, time, matter, motion, energy, radiation, etc. Physics is the most fundamental of all sciences as it is concerned with the study of various natural phenomena.

1.5 TWO BASIC QUESTS IN PHYSICS

9. *What are the two principal thrusts in physics ? Give suitable examples for each.*

Two basic quests in physics. The two principal thrusts in physics are *unification* and *reductionism*.

1. ***Unification.*** In physics, attempt is made to explain various physical phenomena in terms of just few concepts and laws. We try to see the physical world as manifestation of some universal laws. For example, the same (Newton's) law of gravitation can be used to describe the motion of a body falling towards the earth, motion of the moon around the earth and motion of planets around the sun. Attempts are being made to unify fundamental forces of nature in the persuit of *unification*.

2. ***Reductionism.*** Another attempt made in physics is to explain a macroscopic system in terms of its microscopic constituents. This persuit is called *reductionism*. For example, thermodynamics was developed to explain the macroscopic properties such temperature, internal energy, entropy, etc., of the bulk systems. Later on, these properties were explained in terms of molecules in kinetic theory and statistical mechanics.

1.6 SCOPE OF PHYSICS

Introduction. The scope of physics is very wide. Every event, which occurs around us in our daily life, is governed by one or the other principle of physics.

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One way of getting the idea of the scope of physics is to look at its various sub-disciplines. Another way is look at the vast range of magnitude of the physical quantities it deals with.

10. *Briefly explain the main disciplines and sub-disciplines in physics.*

Branches of physics. Physics has two main domains of study—macroscopic and microscopic. **Classical physics** *deals with macroscopic phenomena* which may be at the laboratory, terrestrial and astronomical scales. It includes branches like mechanics, optics, thermodynamics and electrodynamics. **Quantum mechanics** *deals with microscopic phenomena* at the minute scales of atoms, molecules and nuclei.

Main sub-disciplines in physics :

1. ***Mechanics.*** *It deals with the equilibrium or motion of material bodies at low speeds.* It is based on laws of gravitation. The propulsion of rocket, equilibrium of rod bent under a load, propagation of water waves or sound waves in air, etc., are studied in mechanics.

2. ***Optics.*** *It deals with the nature and propagation of light.* It deals with the formation of images by mirrors and lenses, colours in thin films, etc.

3. ***Thermodynamics.*** *It deals with a macroscopic system in equilibrium* and is concerned with the changes in internal energy, temperature, entropy, etc., of the system through external work and heat. Here we study the efficiency of heat engines and refrigerators.

4. ***Electrodynamics.*** *It deals with electric and magnetic phenomena associated with charged and magnetic bodies.* It is based on laws given by Coulomb, Oersted, Ampere and Faraday, which were later on unified by Maxwell. It deals with problems like motion of current-carrying conductor in a magnetic field, propagation of radiowaves through the atmosphere, etc.

5. ***Quantum mechanics.*** *It deals with the mechanical behaviour of sub-microscopic particles* like atoms and nuclei and their interaction with projectiles like electrons, photons and other elementary particles.

6. ***Relativity.*** *It is theory of invariance in nature.* It deals with the motion of the particles having speeds comparable to the speed of light.

For Your Knowledge

▲ An exciting field of research these days is a new domain, called **mesoscopic physics**, which is intermediate between the microscopic and macroscopic domains and deals with a few tens or hundreds of atoms.

11. *The scope of physics is truly vast. Elaborate.*

The scope of physics is truly vast. We can see in another way that the scope of physics is truly vast. It covers wide ranges of mass, length, time, energy, etc. The mass of an object can vary from 10^{-30} kg (mass of an electron) to 10^{55} kg (mass of the universe). The length of an object can vary from 10^{-14} m (size of a nucleus) to 10^{26} m (size of the universe). Time interval can vary from 10^{-22} s (time taken by light to cross a nuclear distance) to 10^{18} s (life of the sun). The everyday phenomena lie in the middle ranges.

For Your Knowledge

▲ The scope of physics (or science in general) is very wide. It has been quite aptly described by some scientists as follows :

- *Science is a method for describing, creating and understanding human experience.*
  R. Bruce Lindsay
- *Science is the ever unfinished quest to discover all facts, the relationship between things and the laws by which the world runs.* **Gerold Holton**
- *The two processes that of Science and Art are not very different. Both Science and Art, formed in the course of centuries a human language by which we can speak about the more real part of reality.*
  Heisenberg
- *The task of science is both to extend the range of our experience and to reduce it to order.*
  Niels Bohr
- *The most incomprehensible thing about the world is that it is comprehensible.* **Albert Einstein**
- *Nature is pleased with simplicity and effects, not the pump of superfluous causes.* **Isaac Newton**
- *We know very little and yet it is astonishing that we know so much and still more astonishing that so little knowledge can give us so much power.*
  Bertrand Russel

1.7 EXCITEMENT OF PHYSICS

12. *Physics is a science of excitement. How ?*

Excitement of physics. The study of physics is not only educative but also exciting in many ways as follows :

1. Inspite of the wide range and complexity of the physical phenomena, it is quite exciting that these phenomena can be analysed and understood in terms of few universal laws and principles.
2. Some other people get excitement in carrying out new challenging experiments to unfold the mysteries of nature and in verifying or falsifying the existing theories.
3. Applied physicists get great satisfaction when they develop technologies for the welfare of the human beings just by using basic laws of physics.

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13. *What are the reasons behind the large scale progress of physics in the last few centuries ?*

Reasons behind the phenomenal growth of physics in the last few centuries :

1. Precise and accurate measurements are central to the growth of physics because most of laws of nature can be expressed in mathematical forms.
2. The basic laws of physics are universal. The same laws can be applied in widely different domains and conditions.
3. Because of ability of the scientists to separate the important and essential features from the less important ones, it is easier to understand even a highly complex phenomenon.

1.8 PHYSICS IN RELATION TO OTHER SCIENCES

14. *How is the study of physics useful to the study of other sciences ?*

Physics in relation to other sciences. Physics is the most fundamental discipline of all sciences. It has played a key role in the development of all other sciences.

1. ***Physics in relation to mathematics.*** Physics is a quantitative science. Mathematics provides the necessary signs and tools which the physicists use. It has played an important role in the development of theoretical physics. Had Newton not invented calculus, he would not have been able to discover the universal law of gravitation.

2. ***Physics in relation to chemistry.*** In physics, we study the structure of atom, radioactivity, X-ray diffraction, etc. Such studies have enabled chemists to arrange elements in the periodic table on the basis of their atomic numbers. This has further helped to know the nature of valency and chemical bonding and to understand the complex chemical structures.

3. ***Physics in relation to biological sciences.*** The developments in life sciences owe a great deal to physics. (*i*) Optical microscopes are extensively used in the study of biology. (*ii*) With the help of an electron microscope, one can study the structure of cell. (*iii*) The X-rays and neutron diffraction techniques have helped in understanding the structure of nucleic acids, which helped to control vital life processes. (*iv*) Radioisotopes are used in radiation therapy for the cure of deadly diseases like cancer.

4. ***Physics in relation to Astronomy.*** (*i*) Astronomical telescopes are used to study the motion of planets and other heavenly bodies in the sky. (*ii*) Radiotelescopes have helped to discover quasers, pulsars, etc., and can be used to see upto the farthest limits of the universe. (*iii*) Dopler's effect predicted the big-bang theory of the universe.

5. ***Physics in relation to geology.*** Diffraction techniques help to study the crystal structure of various rocks. Radioactivity is used to estimate the age of rocks and fossils.

6. ***Physics in relation to seismology.*** The movement of the earth crust and types of waves so generated help a lot in the study of earthquake and its effect.

7. ***Physics in relation to meteorology.*** By studying variation of pressure with temperature, we can forecast weather.

1.9 PHYSICS IN RELATION TO SOCIETY

15. *The fate of a society is linked to the developments in physics. Explain.*

Physics in relation to society. The fate of a society is closely linked to physics. Whatever is discovered in physics, it immediately affects the society.

For example :

1. The developments of telephone, telegraph, telex enable us to quickly exchange messages between far off places.
2. The discoveries of radio and television have made possible the instantaneous communication with other parts of the world.
3. The launching of satellites into space has revolutionised the concept of communication.
4. The development of alternative sources of energy is of great importance to the mankind.
5. Microelectronics, lasers, computers, superconductivity and nuclear energy have entirely changed the thinking and the living style of the human beings.

The modern science in general and physics in particular has the potentiality to eradicate poverty and hunger from the surface of the earth and to usher in an age of prosperity. On the other hand, a wrong use of discoveries of physics may bring about a disaster. The knowledge of physics can be applied alike, to the creation of heaven on the earth or to the total annihilation. Our wiseness lies in applying physics to solve the pressing problems the society faces and not to annihilate it.

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Table 1.1 Some great physicists and their discoveries

S. No.	Name of Scientist	Country	Discovery
1.	Abdus Salam	America (Pakistan born)	Unification of weak and electromagnetic interactions
2.	Alfred Noble	Sweden	Dynamite
3.	Anderson C.D.	America	Positron
4.	Antony Hewert	England	Pulsars
5.	Appleton, E.	England	Exploration of ionosphere
6.	Ampere	France	Magnetism is due to electric current
7.	Archimedes	Greece	Principle of buoyancy, Principle of the lever
8.	Bardeen, John	America	Transistor, superconductivity
9.	Bequerrel, Henry A.	France	Discovery of spontaneous radioactivity
10.	Bhabha, Homi Jehangir	India	Cosmic ray showers
11.	Bose J.C.	India	Ultrashort radiowaves
12.	Bose, S.N.	India	Bose-Einstein statistics
13.	Bohr, Niels	Denmark	Quantum model of hydrogen atom
14.	Bragg, W.H.	England	Crystal structure by X-rays
15.	Bragg, W.L.	England	Crystal structure by X-rays
16.	Barkla, Charles G.	England	Number of electrons in an atom
17.	Cavendish	England	Experimental determination of 'G'
18.	Cerenkov, P.A.	Russia	Cerenkov radiations
19.	Chadwick, James	England	Neutron
20.	Compton, A.H.	America	Compton effect
21.	Coulomb, Charles A. de	France	Coulomb's law
22.	Curie, Marie Sklodowaska	Poland	Studies on natural radioactivity, Discovery of radium and polonium
23.	de-Broglie, Louis Victor	France	Wave nature of matter
24.	Dirac, Paul	England	Relativistic theory of electron, Quantum statics
25.	Einstein, Albert	Germany	Theory of relativity, mass-energy equivalence, photoelectric effect
26.	Edison, A.	America	Electric bulb, telegraphy
27.	Faraday, Michael	England	Laws of electromagnetic induction, laws of electrolysis, designed electric motor
28.	Fermi, Enrico	Italy	Controlled nuclear fission
29.	Gabor, D.	America	Holography
30.	Galileo, Galilei	Italy	Law of inertia
31.	Grahm Bel	America	Telephone
32.	Hertz, Heinrich Rudolf	Germany	Electromagnetic waves
33.	Hess, V.F.	Austria	Cosmic rays
34.	Huygens, Christiaan	Holland	Wave theory of light
35.	Hubble, Edwin	America	Expanding universe
36.	Heisenberg, Wernes	Germany	Uncertainty principle, Quantum mechanics
37.	Kamerling Onnes	Dutch	Low temperature and liquid helium
38.	Kelvin (William Thomson)	England	Second law of thermodynamics, thermodynamic scale of temperature
39.	Landau, Lev Davidovich	Russia	Theory of condensed matter, Liquid helium
40.	Lawrence, E.O.	America	Cyclotron
41.	Lippman, G.	France	Colour photography
42.	Maxwell, James Cleric	England	Electromagnetic theory, Light as an electromagnetic wave
43.	Michelson, A.A.	America	Precision optical instruments
44.	Marconi	Italy	Wireless telegraphy
45.	Millikan, R.A.	America	Measurement of charge on an electron
46.	Newton, Isaac	England	Law of gravitation, Laws of motion, Reflecting telescope

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S. No.	Name of Scientist	Country	Discovery
47.	Oersted	France	Magnetic effect of current
48.	Planck, Max	Germany	Quantum theory of radiation
49.	Pauli, W.	America	Exclusion principle for filling electrons in orbitals
50.	Raman, C.V.	India	Inelastic scattering of light (Raman effect)
51.	Ramachandran, G.N.	India	Triple helical structure of proteins molecules
52.	Robert Boyle	England	Boyle's law
53.	Robert Hooke	England	Elasticity of solids, balance wheel of a watch
54.	Rutherford, Ernest	England	Nuclear model of atom
55.	Roentgen, W.K.	Germany	X-rays
56.	S. Chandershekhar	America (India born)	Structure and evolution of stars, Chandrashekhar limit
57.	Schrödinger, E.	Germany	Wave mechanics
58.	Saha, M.N.	India	Thermal ionisation
59.	Thomson, J.J.	England	Electron
60.	Thomson, G.P.	England	Diffraction of electrons by crystals
61.	Townes, C.H.	America	Maser, Laser
62.	Van der Walls, J.D.	Dutch	Expansion of gases and liquids
63.	Von Laue, Max	Germany	Diffraction of X-rays in crystals
64.	Volta	Italy	Discovered first battery
65.	Wein, W.	Germany	Laws of radiation of heat
66.	Watt, James	England	Steam Engine
67.	Yukawa, Hedeki	Japan	Meson theory of nuclear forces

1.10 PHYSICS IN RELATION TO TECHNOLOGY

16. *Technological advancements owe a great deal to the developments in physics. Briefly explain.*

Physics in relation to technology. The applications of physics have played a key role in the development of technology. Today we see the applications of physics in every walk of life. Some of the major technologies based on the applications of physics are as follows :

1. Electromagnetic waves are used in radio, television, radar and wireless communication.
2. Newton's concept of gravitation is used in geostationary satellites which help us in forecasting weather and in geophysical survey.
3. X-rays are used in radiotherapy, in detecting fractures or dislocations in bones, in studying crystal systems, in engineering and industry.
4. The study of thermodynamics has helped to design heat engines which have revolutionised the industry.
5. The study of electricity has led to the development of electric appliances like electric motors and generators which are the backbones of industry.
6. Nuclear power stations based on nuclear fission constitute one of the major sources of energy.
7. The concepts of modern electronics find extensive use in telephone exchanges, robots, etc.
8. The discovery of silicon chips has brought a revolution in computer industry.
9. Geothermal energy, *i.e.*, the heat in the depth of the earth is being used these days. The tidal energy in the oceans and solar energy too can be converted into other forms of energy and used.
10. Radioactive isotopes are now being widely used in medicine, agriculture and industry.

These technologies have made our lives comfortable and materially prosperous.

Sometimes physics generates new technology. At other times technology gives rise to new physics. Both have a direct impact on society.

Table 1.2 Some important technologies and their links with physics

Technology	Scientific Principle(s)
Steam engine	Laws of thermodynamics
Nuclear reactor	Controlled nuclear fission
Radio and Television	Generation, propagation and detection of electromagnetic waves
Wireless telegraphy	Propagation of electromagnetic waves
Computers	Digital logic of electronic circuits
Lasers	Light amplification by stimulated emission of radiation (population inversion)
Production of ultra high magnetic fields	Superconductivity
Rocket propulsion	Newton's laws of motion

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Technology	Scientific Principle(s)
Electric generator	Faraday's laws of electromagnetic induction
Hydroelectric power	Conversion of gravitational potential energy into electrical energy
Aeroplane	Bernoulli's principle in fluid dynamics
Particle accelerator / Cyclotron	Motion of charged particles in electromagnetic fields
Sonar	Reflection of ultrasonic waves
Optical fibres	Total internal reflection of light
Non-reflecting coatings	Thin film optical interference
Electron microscope	Wave nature of electrons
Photocell	Photoelectric effect
Fusion test reactor (Tokamak)	Magnetic confinement of plasma
Giant Metrewave Radio Telescope (GMRT)	Detection of cosmic radio waves
Bose-Einstein condensate	Trapping and cooling of atoms by laser beams and magnetic fields

1.11 ▼ FUNDAMENTAL FORCES IN NATURE

17. *Name the four basic forces in nature. Arrange them in the order of their increasing strengths.*

Fundamental forces in nature. In the macroscopic world, we observe several kinds of forces : muscular force, contact forces of support and friction, forces exerted by springs and strings, viscous forces, electric forces, magnetic forces, etc. All these forces between macroscopic objects arise from *two* fundamental forces :

1. Gravitational force 2. Electromagnetic force

In the microscopic world, in addition to the above two forces, two more basic forces are required to account for the various atomic and nuclear processes. These are

1. Strong nuclear force 2. Weak nuclear force

The ratio of the strength of the four fundamental forces in nature is

$$F_G : F_W : F_E : F_S = 1 : 10^{25} : 10^{36} : 10^{38}$$

1.12 ▼ THE GRAVITATIONAL FORCE

18. *What is gravitational force ? Mention its important properties. Give some examples of gravitational force.*

Gravitational force. *It is the force of mutual attraction between two bodies by virtue of their masses.* It is a universal force. Every body attracts every other body of the universe with this force. According to Newton's law of gravitation, the gravitational attraction between two bodies of masses m_1 and m_2 and separated by distance r is given by

$$F = G\frac{m_1 m_2}{r^2}$$

where G is the universal gravitational constant.

Important properties of gravitational force :

1. It is a *universal attractive* force.
2. It is directly proportional to the product of the masses of the two bodies.
3. It obeys *inverse square law.*
4. It is a *long range force* and does not need any intervening medium for its operation.
5. Gravitational force between two bodies does not depend upon the presence of other bodies.
6. It is the *weakest force* known in nature.
7. It is a *central force* (*i.e.*, it acts along the line joining the centres of the two bodies).
8. It is a *conservative force* (*i.e.*, work done in moving a body against the gravitational force is path independent).
9. Gravitational force between two bodies is thought to be caused by an exchange of a particle called *graviton.*

Examples of gravitational force :

1. All bodies fall because of the gravitational force of attraction exerted on them by the earth.
2. Gravitational force governs the motion of the moon and the artificial satellites around the earth ; and the motion of the planets around the sun.
3. Gravitation plays a key role in the formation and evolution of stars, galaxies and galactic clusters.

1.13 ▼ THE ELECTROMAGNETIC FORCE

19. *What is electromagnetic force ? Mention its important properties. Give some examples of the electromagnetic force.*

Electromagnetic force. *The force acting between two electric charges at rest is called electrostatic force.* According to *Coulomb's law*, the magnitude of the electrostatic force F between two point charges q_1 and q_2 separated by distance r in vacuum is given by

$$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r^2}$$

where ε_0 is the permittivity of vacuum. *The force acting between two magnetic poles is called magnetic force.* In fact, electrostatic and magnetic forces are closely interrelated. For example, a moving charge produces a

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magnetic field. Also, a magnetic field exerts a force on a moving charge. This force depends both on the magnitude and direction of the velocity of the electric charge. Thus the electrostatic and magnetic forces are inseparable and are considered as the two facets of a general force known as *electromagnetic force*.

Important properties of electromagnetic force :

1. Electromagnetic force may be *attractive or repulsive*. Like charges repel each other and unlike charges attract each other.
2. It obeys *inverse square law*.
3. It is a *long range force* and does not require any intervening medium for its operation.
4. It is a *central force*.
5. It is a *conservative force*.
6. It is 10^{36} times stronger than the gravitational force.
7. It is caused by the exchange of *photons* (γ) between two charged particles.

Examples of electromagnetic force :

1. When a spring is compressed/elongated, it exerts a force of elasticity due to the net repulsion / attraction between its neighbouring atoms. This net repulsion or attraction is the sum of the electrostatic forces between the electrons and nuclei of the atoms.
2. The Van der Walls' force between two neutral molecules of a gas is net sum of the electrostatic forces between the electrons and nuclei of the two molecules.

1.14 ELECTROMAGNETIC NATURE OF SOME MACROSCOPIC FORCES

Introduction. The macroscopic forces of our daily life such as tension, friction, contact force, spring force, etc., are derived from the fundamental electrostatic force only. All matter consists of charged particles like electrons and protons. The strong electromagnetic force between these particles is responsible for the structure of atoms and molecules, rate of chemical reactions, and the mechanical, thermal and electrical properties of materials.

20. *Discuss the electromagnetic nature of (i) contact force between two bodies (ii) force of friction (iii) elastic force in a spring (iv) Van der Walls' force between gas molecules and (v) force in a taut string/rope.*

(*i*) **Contact force between two bodies.** When we place two bodies in contact with each other, their atoms come close to each other at the surface of contact. Large electromagnetic forces begin to act between the charged constituents of these atoms. Generally, these forces act normal to the surface of contact and are of pushing or repelling nature. For example, a book lying on a table pushes it downwards while the table pushes the book upwards.

(*ii*) **Force of friction.** Sometimes, the electromagnetic contact force between two bodies may have a component acting parallel to the surface of contact. This component is called friction. When bodies are placed with their smooth surfaces in contact, they provide only a small parallel component of contact force and hence friction between them is negligibly small. For example, when we climb up a tree, its rough trunk provides sufficient frictional force parallel to the surface of the tree which helps us cling on to the trunk. It is difficult to climb up a smooth metallic lamp post because it does not provide enough parallel frictional force.

(*iii*) **Elastic force in a spring.** When a spring is elongated (or compressed), it exerts a force of elasticity which arises due to the net attraction (or repulsion) between the neighbouring atoms of the spring. This net attraction (or repulsion) is the (unbalanced) sum of the electrostatic forces between the electrons and nuclei of these atoms.

(*iv*) **Van der Walls' force.** The Van der Walls' force between two neutral molecules of a gas is not a fundamental force but a derived force. It is the net residual force obtained by summing up the fundamental electrostatic forces between the various electrons and nuclei of the two molecules.

(*v*) **Force in a taut string/rope.** Consider a block tied to the lower end of a string suspended from a rigid support. The string is in a state of tension. The electrons and protons of the lower end of the string exert electrostatic forces on the electrons and protons of the block. The resultant of these forces balances the weight of the block. Generally, a string under tension exerts an electromagnetic force of pulling nature on the two bodies attached to its two ends.

21. *Electromagnetic force is enormously stronger than the gravitational force. Give an example from daily life to illustrate it.*

Electromagnetic force is much stronger than the gravitational force. When we hold a book in our hand, the earth exerts a very large gravitational force on the book due to its huge mass. This force is balanced by the normal force exerted by our hand. But the latter force is the net electromagnetic force between the charged constituents of our hand and the book at the surface of contact. Clearly, electromagnetic force is much stronger than the gravitational force.

22. *Although gravitational force is incomparably weaker than the electromagnetic force, yet it governs the large scale motion both on terrestrial and astronomical scales. How ?*

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Gravitational force governs the large scale motion. Mass is only of one type *i.e.*, there are no positive and negative masses. So the gravitational force is always attractive. But electric charges may be positive and negative. Consequently, the electromagnetic forces may be both attractive and repulsive. Between two neutral objects, the gravitational force goes on adding over all pairs of particles of the two objects while the electromagnetic forces being equally attractive and repulsive, add up to zero. That is why the large scale motion in the universe is controlled by the weakest gravitational force.

1.15 THE STRONG NUCLEAR FORCE

23. *What is strong nuclear force ? Mention its important properties. Give some examples of this force.*

Strong nuclear force. *The strong attractive force which binds together the protons and neutrons in a nucleus is called strong nuclear force.* This force cannot be electrostatic force because positively charged protons strongly repel each other at such small separations of the order of 10^{-15} m. Also the gravitational attraction between two protons being much weaker, cannot overcome this electrostatic repulsion. So a new attractive force must be acting between the nucleons (protons and neutrons). This strong nuclear force is strongest of all the fundamental forces, about 100 times stronger than the electromagnetic force.

Important properties of strong nuclear force :

1. It is the strongest interaction known in nature, which is about 10^{38} times stronger than the gravitational force.
2. It is a short range force that operates only over the size of the nucleus ($=10^{-15}$ m).
3. It is basically an attractive force, but becomes repulsive when the distance between the nucleons becomes less than 0.5 fermi (1 fermi $=10^{-15}$ m).
4. It varies inversely with some higher power (>2) of distance.
5. It is a non-central and non-conservative force.
6. It has charge independent character *i.e.*, nuclear forces between proton-proton, proton-neutron and neutron-neutron are almost equally strong.
7. It is caused by the exchange of particles, called Π-mesons.

Examples of nuclear force :

1. Nuclear forces bind together the protons and neutrons in the nuclei. So they are responsible for stability of nuclei and hence of the atoms and ultimately of all matter that exists in the universe.
2. Radioactivity occurs in heavier nuclei because of insufficient nuclear force between their protons and neutrons.
3. The concept of nuclear force is useful in obtaining nuclear energy via the processes of nuclear fission and fusion.

Note *Electrons do not experience the strong nuclear force.*

1.16 THE WEAK NUCLEAR FORCE

24. *What is weak nuclear force ? Explain with the help of an example. Give important properties of weak nuclear force.*

Weak nuclear force. *It is the force that appears only between elementary particles involved in a nuclear process such as the β-decay of a nucleus.* In a β-decay, the nucleus emits an electron and an uncharged particle called neutrino. The electron and neutrino interact with each other through the weak nuclear force. The weak nuclear force is much stronger than the gravitational force, but much weaker than strong nuclear and electromagnetic forces. This is obvious from the fact that the decay of an elementary particle caused by weak nuclear force (*e.g.*, the decay of a pion to a muon and a neutrino) is much slower than the decays caused by strong nuclear or electromagnetic forces.

Important properties of weak nuclear force :

1. Any process involving neutrino and antineutrino is governed by weak nuclear force beacuase these particles can experience only weak interaction and not the strong nuclear interaction.
2. Weak nuclear force is 10^{25} times stronger than the gravitational interaction.
3. It operates only through a range of nuclear size ($\approx 10^{-15}$ m).
4. The messenger particles that transmit the weak force between elementary particles are the massive vector bosons ($W^{\pm}$, Z).

For Your Knowledge

- Neutrino is an elementary particle with zero rest mass and zero charge.
- It will be more correct to say that β^- decay ejects an electron and an antineutrino while β^+ decay ejects a positron (antielectron) and a neutrino.
- **Elementary particles** are grouped as baryons, mesons, leptons. Nucleons (protons and neutrons) and their higher counterparts are called baryons. The pions, kaons, etc., are mesons. Baryons and mesons are together called hadrons which can interact through strong nuclear force. Electrons, neutrinos and their higher mass counterparts are called leptons. Leptons do not experience the strong force, only the weak force is dominant between them.

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1.17 ▼ BASIC MECHANISM OF THE FUNDAMENTAL FORCES

25. *Briefly explain the mechanism that gives rise to the various fundamental forces between elementary particles. Give the names of the exchange particles for each of these forces.*

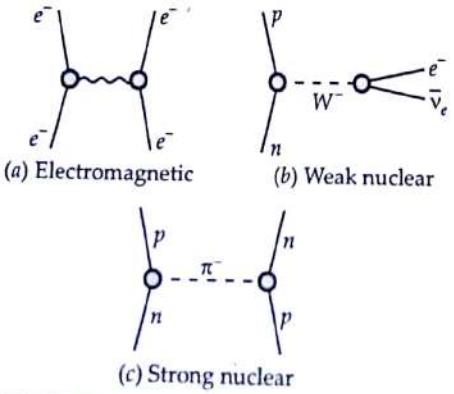
Fundamental forces arise due to exchange of particles. *Each fundamental force between two elementary particles arises from the exchange of its characteristic particles called the quanta of that force or the* **messenger particles**. To understand this mechanism, consider interaction between two persons *A* and *B*. Suppose *A* throws a ball and *B* catches it. By momentum conservation, *A* suffers a recoil when he throws the ball. Similarly, *B* suffers a recoil when he catches the ball. Also the recoil force changes the momentum of each person.

The exchange particles for various fundamental forces are as follows :

1. ***Electromagnetic force*** between two charged particles (like electrons) arises from the exchange of **photons** (γ) between them, as shown in Fig. 1.1(*a*). These photons are emitted by one electron and absorbed by the other within a very short time. As these photons cannot be detected, so they are called **virtual photons.**

2. ***Weak nuclear force*** during the β–decay of a nucleus arises from the exchange of massive particles, called **vector bosons** ($W^{\pm}$, Z) between the elementary particles. As shown in Fig. 1.1(*b*), the charge of vector boson (W^-) is exchanged in the β–decay of a neutron.

3. ***Strong nuclear force*** between two nucleons arises from the exchange of mesons between them. For example, negatively charged pion (π^-) is exchanged in the nuclear force involved in np° pn scattering, as shown in Fig. 1.1(*c*).



(*a*) Electromagnetic (*b*) Weak nuclear

(*c*) Strong nuclear

Fig. 1.1 Origin of fundamental forces due to exchange of particles.

4. ***Gravitational force*** is believed to be caused by the exchange of yet to be detected particles called **gravitons**.

Table 1.3 *Some of the important aspects of fundamental forces in nature*

Fundamental force	Relative strength	Range	Particles on which force acts	Messenger particle
Gravitational force	1	Infinite	All particles	Gravitons
Weak nuclear force	10^{25}	Very short, within nuclear size ($\approx 10^{-15}$ m)	Elementary particles	Vector bosons
Electromagnetic force	10^{36}	Infinite	Charged particles	Photons
Strong nuclear force	10^{38}	Very short, within nuclear size ($\approx 10^{-15}$ m)	Nucleons	Mesons

For Your Knowledge

▲ The strong nuclear force is not a true fundamental force. The nucleons themselves are built of subunits called quarks. The quark-quark force, which binds the nucleons together is now regarded as fundamental force. It is caused by the exchange of massless particles called *gluons*.

1.18 ▼ UNIFICATION OF FORCES

26. *What is the basic quest of modern physicists ? Mention the various significant attempts made towards the unification of forces in a chronical order.*

Basic quest of modern physicists is the unification of forces. The great advacements in physics are the result of unification of different theories and domains. Different attempts made by the distinguished physicists from time to time in persuing the goal of unification of forces are listed in Table 1.4.

Table 1.4 *Some of the milestones towards the unification of forces*

Physicist(s)	Year	Achievement in unification
Issac Newton	1687	Unified celestial and terrestrial domains by explaining both types of phenomena by applying the same laws of motion and law of gravitation.
Hans Cristian Oersted Michael Farady	1820 1830	Showed experimentally that electric and magnetic phenomena are inseparable aspects of a unified domain called electromagnetism.

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Physicist(s)	Year	Achievement in unification
James Clerk Maxwell	1873	Unified electromagnetism and optics by discovering that light is an electromagnetic wave.
Albert Einstein	1905-1916	Asserted the equivalence of mass and energy and uni- fied the motions of space, time and gravitation in his theories of relativity.
Sheldon Glashow, Abdus Salam, Steven Weinberg	1979	Showed that the weak nuclear force and the electromagnetic force are different aspects of a unified electro-weak force
Carlo Rubia, Simon Vander Meer	1984	Experimentally verified the theory of electro-weak force.

1.19 CONSERVATION LAWS

27. *What are conserved quantities and conservation laws ?*

Conservation laws. *In any physical process involving the different forces, some physical quantities remain unchanged with time.* Such quantities are called **conserved quantities**. *The laws which govern the conservation of these quantities are called* **conservation laws**.

In classical physics, we usually deal with the following *four* conservation laws :

1. Law of conservation of energy.
2. Law of conservation of linear momentum.
3. Law of conservation of angular momentum.
4. Law of conservation of charge.

28. *What is an isolated system ?*

Isolated system. *Any system (assembly of particles or bodies) on which no external force acts is called an isolated system.*

1.20 LAW OF CONSERVATION OF ENERGY

29. *State the law of conservation of energy. Give some examples in which this law is obeyed.*

Law of conservation of energy. *This law states that energy can neither be created nor destroyed but it can be changed from one form to another. Equivalently, we can say that the total energy of an isolated system remains constant.*

Examples : (*i*) When a body falls freely, under gravity, its potential energy gradually changes into kinetic energy. But its total mechanical energy (kinetic energy + potential energy) remains constant at any point of its motion.

(*ii*) During the oscillation of a simple pendulum, the energy of the bob changes gradually from kinetic to potential as it moves from mean position to either extreme position. The energy changes from potential to kinetic as the bob moves from either extreme position to the mean position. At all points of its motion, total energy of the bob remains constant.

For Your Knowledge

- When a body moves under a conservative force, its total mechanical energy is conserved. In the presence of a non-conservative forces such as friction or air resistance, mechanical energy is not conserved. It changes into heat, sound, etc.
- Mechanical energy is conserved whether acceleration is constant or variable.
- Inspite of all kinds of violent phenomena occurring in the universe all the time, the total energy of the universe remains constant. In fact, universe is an example of the most ideal isolated system possible.
- According to Albert Einstein, mass and energy are interconvertible. In 1905, he established the mass-energy equivalence. The energy associated with mass m is given by

$$E = mc^2$$

where c is the speed of light in vacuum.

- As mass can be converted into energy, so in law of conservation of energy, we include mass also.

30. *State the law of conservation of linear momentum. Give some examples in which this law is obeyed.*

Law of conservation of linear momentum. *This law states if no external force acts on a system, then its linear momentum remains constant.*

Examples : (*i*) A rifle gives backward kick on firing a bullet. Before firing, both the bullet and the rifle are at rest and initial momentum of the system is zero. As soon as bullet is fired, it moves forward with a large velocity. In order to conserve momentum, the rifle moves backward with such a velocity that the final momentum of the system is zero.

(*ii*) Suppose a radioactive nucleus, initially at rest, decays spontaneously into fragments. To conserve momentum, the heavier and lighter fragments will fly in opposite directions, with the former having a proportionately smaller speed than the latter.

1.21 LAW OF CONSERVATION OF ANGULAR MOMENTUM

31. *State the law of conservation of angular momentum. Give some examples in which this law is obeyed.*

Law of conservation of angular momentum. A body rotating about an axis has a rotational inertia, called moment of inertia. Also, it is associated with a

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momentum, called angular momentum. We shall prove later on that

Angular momentum (L) = Moment of inertia (I) × angular speed (ω)

The law of conservation of angular momentum states that if no external torque acts on a system, then its angular momentum remains constant.

Examples : (*i*) While revolving in its elliptical orbit, when a planet approaches the sun, its moment of inertia about the sun decreases. To conserve the angular momentum, its angular speed increases.

(*ii*) In a Tornado as the air rushes towards the centre, its moment of inertia decreases. To conserve the angular momentum, the angular speed of the air increases.

1.22 LAW OF CONSERVATION OF CHARGE

32. *State the law of conservation of charge. Give some examples in which this law is obeyed.*

Law of conservation of charge. *This law states that the total charge of an isolated system remains constant.* This implies that the electric charges can neither be created nor destroyed, only they can be transferred from one body to another.

Examples : (*i*) When a glass rod is rubbed with silk cloth, both develop charges. It is observed that the positive charge developed on the glass rod has the same magnitude as the negative charge developed on silk cloth. So total charge after rubbing is zero as before rubbing *i.e.*, electric charge is conserved.

(*ii*) Electric charge is conserved during the fission of a $^{235}_{92}U$ nucleus by a neutron.

$$^{1}_{0}n + {}^{235}_{92}U \longrightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3\,{}^{1}_{0}n + \text{Energy}$$

Total charge before fission $(0 + 92)$

= Total charge after fission $(56 + 36 + 3 \times 0)$

For Your Knowledge

- In classical physics, we deal with the four conservation laws of energy, momentum, angular momentum and charge. But in nuclear and particle physics, we also deal with the conservation of quantities like parity, baryon number, strangeness, hypercharge, etc.
- A conservation law is a hypothesis, based on observations and experiments. A conservation law cannot be proved. It can only be verified, or disproved, by experiments.
- Some conservation laws may hold for one fundamental force but not for the other. For example, *parity* is conserved by the strong and electromagnetic forces but not by the weak force. Also, *strangeness* is conserved by the strong force but not by the weak force.

1.23 RELATION BETWEEN CONSERVATION LAWS AND SYMMETRIES OF NATURE

33. *How are the conservation laws related to the symmetries of the nature ?*

Conservation laws are closely related to the symmetries of the nature. *The symmetries of space, time and other types of symmetries have played an important role in developing the modern theories of fundamental forces.*

(*i*) If we perform an experiment at a certain place today and repeat the same experiment after one year at the same place, we obtain exactly the same results. This symmetry of nature with respect to translation or displacement of time is called *homogeneity of time* and it leads to the *law of conservation of energy.*

(*ii*) Laws of nature take the same form everywhere in the universe *i.e.*, there is no preferred location in the universe. This symmetry of the laws of nature with respect to translation in space is called *homogeneity of space* and gives rise to the law of conservation of linear momentum.

(*iii*) *Isotropy of space* (*i.e.*, there is no preferred direction in space) gives rise to the law of conservation of angular momentum.

Very Short Answer Conceptual Problems

Problem 1. Why do we call physics an exact science ? *[Himachal 06C, 08]*

Solution. *It is because of high precision and accuracy with which the physical quantities are measured in physics.*

Problem 2. Why is physics regarded as basic science ?

Solution. *Physics has played a key role in the development of physical sciences, technology, medicine and life science. So many authorities in the other fields of knowledge consider physics to be the basics of all sciences.*

Problem 3. Why was science called natural philosophy in earlier days ?

Solution. This is because in earlier days, scientific knowledge was gained from the direct study of natural phenomena without any experimentation.

Problem 4. Should a scientific discovery which has nothing but dangerous consequences for mankind be made public ?

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Solution. Science is the search for eternal truth. Any discovery good or bad must be made public. A discovery which appears dangerous today may prove useful to the mankind later on. However, a strong public opinion should be built up against the misuse of a dangerous discovery.

Problem 5. Is science not on speaking terms with huminities ? Comment.

Solution. No, science helps in the growth of huminities by preserving old manuscripts and articles of historical values, enriching music, etc.

Problem 6. Is physics more of a philosophy or more of a mathematical science ?

Solution. Physics is more of a philosophy than a mathematical science. In fact, understanding and appreciation of physics without a philosophical outlook is incomplete.

Problem 7. Does imagination play any role in physics ? *[Himachal 07C]*

Solution. Yes, imagination has played an important role in the development of physics. Huygen's principle, Bohr's theory, Maxwell equations, Heisenberg's uncertainty principle, etc. were the imaginations of the scientists which successfully explained the various natural phenomena.

Problem 8. What are the similarities between science and arts ?

Solution. Both science and arts are creative. Both portray realm of experience.

Problem 9. How is science different from technology ?

Solution. Science is the study of natural laws while technology is the practical application of these laws to the daily life problems.

Problem 10. The physicists think at a level far higher than a normal individual. Is it true ?

Solution. Yes, the physicists are always ahead of their times. In order to predict events much in advance, they have to think far higher than normal individual.

Problem 11. What is gravitational force ?

[Himachal 06]

Solution. It is the force of mutual attraction between two bodies by virtue of their masses.

Problem 12. What is electromagnetic force ?

[Himachal 06]

Solution. It is the force due to interaction between two moving charges. It is caused by exchange of photons (γ) between two charged particles.

Problem 13. What is nuclear force ? *[Himachal 06]*

Solution. It is the strongest attractive interaction which binds together the protons and neutrons in a nucleus.

Problem 14. What is weak nuclear force ?

Solution. It is the force that appears only between elementary particles involved in a nuclear process such as β-decay of a nucleus.

Short Answer Conceptual Problems

Problem 1. A theory is not scientific if it is not falsifiable. This means that if a theory is so constructed that it can accommodate any prediction or observation, it is not science. Use this strong argument (and any additional arguments you can think of) to challenge Astrology's claim to being a science.

Solution. Astrology predicts events on the basis of positions of different planets at different times. It predicts the same fate for all children born at the same time which is not found to be true. An astrologer can make any prediction unchallenged. So astrology is not based on scientific laws. It cannot be a science.

Problem 2. Distinguish between classical physics and quantum mechanics.

Solution. Classical physics mainly deals with macroscopic phenomena which may be at the laboratory, terrestrial and astronomical scales. Here the particle size $> 10^{-8}$ m and particle velocity $<< 10^{8}$ m / s. Here we need not consider strong or weak nuclear forces. Gravitational and electromagnetic forces are sufficient to explain these macroscopic phenomena. On the other hand, quantum mechanics deals with microscopic phenomena at the minute scales of atoms, molecules and nuclei. Here the strong and weak nuclear forces become dominant.

Problem 3. Discuss the relation of physics to chemistry. *[Himachal 07]*

Solution. Refer to the answer of Q. 14 on page 1.4.

Problem 4. Discuss the relation of physics with technology. *[Himachal 05, 07, 07C]*

Solution. Refer to answer of Q. 16 on page 1.6.

Problem 5. The fate of society is linked to the developments in physics. Comment.

[Himachal 06C, 07C, 08C]

Solution. Refer to the answer of Q. 15 on page 1.4.

Problem 6. What is the role of physics in your daily life ? *[Himachal 03, 05, 08]*

Solution. Refer to the answer of Q.16 on page 1.6.

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Guidelines to NCERT Exercises

1.1. *Some of the most profound statements on the nature of science have come from Albert Einstein, one of the greatest scientists of all time. What do you think did Einstein mean when he said : "The most incomprehensible thing about the world is that it is comprehensible" ?*

Ans. The physical world is highly complex. It has vast orders of magnitude in respect of mass, length and time. Even then all the physical phenomena can be explained in terms of few basic principles of physics. That is, it is possible to comprehend nature in terms of few basic laws inspite of its vast complexities. This is what Einstein meant to say.

1.2. *Every great physical theory starts as a heresy and ends as a dogma." Give some examples from the history of science of the validity of this incisive remark.*

Ans. Any opinion against the conventional belief is a heresy, while dogma is an established belief. The geocentric theory of Copernicus started as a heresy but it ended as dogma when his theory was explained by Tycho Brahe and Johnes Kepler later on.

1.3. *"Politics is the art of the possible". Similarly, "Science is the art of the soluble". Explain this beautiful aphorism on the nature and practice of science.*

Ans. In politics, everything is possible. The politicians do not follow any principle, discipline or norm. They tend to remain in power by any means, fair or foul. But a scientist observes a phenomenon carefully and patiently, collects data, analyses it and formulates principles, thus solving the mystery of nature.

1.4. *Though India has a large base in science and technology which is fast expanding, it is still a long way from realising its potential of becoming a world leader in science. Name some important factors which in your view have hindered the advancement of science in India.*

Ans. There are many factors which have hindered the advancement of science in India. Some of these are :

1. There is lack of clear cut policies at the political level. There is undue political interference and bureaucratisation in the management of science and technology.
2. Due to excessive population, our country cannot afford to expend sufficient funds on science and technology
3. Talented scientists and technologists are not encouraged to persue research work in the homeland. Instead, they prefer to migrate to developed countries where they get better salaries and research facilities.
4. There is lack of coordination between researchers and industraliasts. Industralists like to borrow technology from advanced countries rather than to use indigeneous technology.

1.5. *No physicist has ever "seen" an atom. Yet, all physicists believe in the existence of atoms. An intelligent but superstitious man advances this analogy to argue that 'ghosts' exist though no one has 'seen' one. How will you refute his argument ?*

Ans. Many phenomena which are based on the assump- tion of existence of atoms have been theoretically predicted and experimentally verified. These phenomena indirectly establish the existence of atoms. But no such phenomenon has been observed which proves the existence of ghosts or which can be explained on the assumption of ghosts.

1.6. *The shells of crabs found around a particular coastal location in Japan seem mostly to resemble the legendary face of a Samurai. Given below are two explanations of this observed fact. Which of these strikes you as a scientific explanation ?*

(*a*) *A tragic sea accident several centuries ago drowned a young Samurai. As a tribute to his bravery, nature through its inscrutable ways immortalised his face by imprinting it on the crab shells in that area.*

(*b*) *After the sea tragedy, fishermen in that area, in a gesture of honour to their dead hero, let free any crab shell caught by them which accidentally had a shape resembling the face of a Samurai. Consequently, the particular shape of the crab shell survived longer and therefore in course of time the shape was genetically propagated. This is an example of evolution by artificial selection.*

Ans. Explanation (*b*) is a scientific explanation to the fact that the shells of crabs resemble the face of Samurai.

1.7. *The industrial revolution in England and Western Europe more than two centuries ago was triggered by some key scientific and technological advances. What were these advances ?*

Ans. Some of the key advances responsible for industrial revolutions were as follows :

1. Invention of steam engine by James Watt in 1769 AD made it possible to drive machines by using steam power.
2. Invention of flying schuttle by John Key and of power-loom by Cartright brought revolution in textile industry.
3. Setting up of blast furnace helped to convert low grade iron into steel.
4. Invention of safety lamp by Humphry Davy helped work safely in mines.

1.8. *It is often said that the world is witnessing now a second industrial revolution, which will transform the society as radically as did the first. List some key contemporary areas of science and technology which are responsible for this revolution.*

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Ans. The key areas of science and technology which may radically transform the present society are as follows :

1. Laser technology which is used for bloodless surgery and for effective automatic control of rockets and satellites.
2. Fabrication of superconductors near the room temperature will help to transmit electric power without any wastage of energy.
3. The extensive use of computers will increase work efficiency.
4. Biotechnology.

1.9. *Write in about 1000 words a fiction piece based on your speculation on the science and technology of the twenty-second century.*

Ans. These days, we go to distant places by means of a car or an aircraft fuelled by petrol. In the twenty-second century, we may plan for a journey to a distant star located hundreds of light years away from the earth by means of a spaceship, without taking care of any fuel needs.

The spaceship is sent into space by firing a rocket engine from a launching pad. As it enters the region of magnetic field in space, it is propelled by electricity generated due to electromagnetic induction. The current induced is fed to the electric motors via superconducting wires. This avoids wastage of electric energy in the form of heat.

Now suppose the spaceship enters a region in space where the temperature is very high. The connecting wires at once lose the superconducting properties. This may cause a panic in the spaceship as no power is available. Another spaceship containing both matter and antimatter in separate chambers may come to its rescue. Hence the spaceship may continue its journey to distant star without bothering about any fuel crisis.

1.10. *Attempt to formulate your 'moral' views on the practice of science. Imagine yourself stumbling upon a discovery, which has great academic interest but is certain to have nothing but dangerous consequences for the human society. How, if at all, will you resolve your dilemma ?*

Ans. Science is the search for eternal truth. It is the moral duty of a scientist to expose the truth. If there is any danger to the mankind from a discovery, he should try to build a public opinion against the misuse of discovery. Moveover, he should develop the means to prevent its misuse. It is equally possible that this discovery may prove to be of immense importance to the mankind later on.

1.11. *Science, like any knowledge, can be put to good or bad use, depending on the user. Given below are some of the applications of science. Formulate your views on whether the particular application is good, bad or something that cannot be so clearly categorised :*

(*a*) *Mass vaccination against small pox to curb and finally eradicate this disease from the population. (This has already been successfully done in India).*

(*b*) *Television for eradication of illiteracy and for mass communication of news and ideas.*

(*c*) *Prenatal sex determination.*

(*d*) *Computers for increase in work efficiency.*

(*e*) *Putting artificial satellites into orbits around the Earth.*

(*f*) *Development of nuclear weapons.*

(*g*) *Development of new and powerful techniques of chemical and biological warfare.*

(*h*) *Purification of water for drinking.*

(*i*) *Plastic surgery.*

(*j*) *Cloning.*

Ans.

(*a*) Good, mass vaccination protects us from this dreaded disease.

(*b*) Good, it educates, entertains and creates awareness amongst the people.

(*c*) It cannot be clearly categorised as it may be misused by people.

(*d*) Good, computers work very fast and with great accuracy.

(*e*) Good, artificial satellites are useful for studying universe, as communication means and in forecasting weather.

(*f*) Bad, nuclear weapons are weapons of mass destruction.

(*g*) Bad, these techniques are the weapons of mass destruction.

(*h*) Good, otherwise the polluted water may cause many diseases.

(*i*) Good, it helps to remove deformations in the body.

(*j*) Good.

1.12. *India has had a long and unbroken tradition of great scholarship – in mathematics, astronomy, linguistics, logic and ethics. Yet, in parallel with this, several superstitious and obscurantistic attitudes and practices flourished in our society and unfortunately continue even today – among many educated people too. How will you use your knowledge of science to develop strategies to counter these attitudes ?*

Ans. Educating the common man is the only way to get rid of superstitious and obscurantistic attitudes and practices flourished in our society. The phenomena which people attribute to evil spirits and gods should be explained scientifically through mass media such as radio, television, newspapers, cinema, etc. The school students must be explained the day to day phenomena occurring around them in an effective scientific manner.

1.13. *Though the law gives women equal status in India, many people hold unscientific views on a woman's innate nature, capacity and intelligence, and in practice give them a secondary status and role. Demolish this view using scientific arguments, and by quoting examples of great women in science and other spheres ; and persuade yourself and others that, given equal opportunity, women are on par with men.*

Ans. The development of human mind depends mainly on the nutrition content of prenatal and postnatal

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diet and the environment around it. The gender factor has no role in the development of human mind. When given equal opportunities, mental development of females will be as rapid as that of males. Madam Marie Curie won the Noble prize twice in science which is a rare gesture. Mrs. Indira Gandhi and Mrs. Margret Thatcher have flourished excellently well in politics.

1.14. *"It is more important to have beauty in the equations of physics than to have them agree with experiments". The great British physicist P. A. M. Dirac held this view. Criticize this statement. Look out for some equations and results in this book which strike you as beautiful.*

Ans. Dirac was his genious best when he pronounced this statement. A physical equation must be simple and hence beautiful. It will automatically agree with the experimental results. For example, consider Einstein mass-energy relationship : $E = mc^2$

This simple equation not only governs energy generation in the sun and the other stars but it is also mainly responsible for energy generation on the earth from the processes of nuclear fission and nuclear fusion.

However, it is not always true. Although some equations of Quantum Mechanics and Theory of Relativity are highly cumbersome and difficult to understand, yet they agree with the experiments.

1.15. *Though the statement quoted above may be disputed, most physicists do have a feeling that the great laws of physics are at once simple and beautiful. Some of the notable physicists, besides Dirac, who have articulated this feeling, are : Einstein, Bohr, Heisenberg, Chandrasekhar and Feynman. You are urged to make special efforts to get access to the general books and writings by these and other great masters of physics. Their writings are truly inspiring.*

Ans. There is no doubt that the great laws of physics are at once simple and beautiful. The Einstein's mass-energy equivalence relation : $E = mc^2$ has tremendous impact on various physical phenomena and on human lives yet it is so simple. Bohr's quantum condition : $E = h\nu$ are $L = nh/2\pi$ and Planck's quantum condition : $L = nh/2\pi$ are quite simple, yet quite important laws of physics. Heisenberg's uncertainty principle : $\Delta x \cdot \Delta p \geq h/2\pi$ is a simple law but changed the thinking of physicists drastically. These are just few examples of the beauty of physics laws which inspired the physicists and opened up new vistas in physics.

1.16. *Textbooks on science may give you a wrong impression that studying science is dry and all too serious and that scientists are absent-minded introverts who never laugh or grin. This image of science and scientists is patently false. Scientists, like any other group of humans, have their share of humorists, and many have led their lives with a great sense of fun and adventure, even as they seriously pursued their scientific work. Two great physicists of this genre are Gamow and Feynman. You will enjoy reading their books.*

Ans. True, the scientists like any other group of humans have their share of humorists, and many have led their lives with a great sense of fun and adventure inspite of pursuing their scientific work seriously. Two such great physicists were Gamow and Feynman.

Text Based Exercises

Type A : Very Short Answer Questions

1 Mark Each

1. What is science ?
2. What is the origin of the word science ?
3. What is meant by the sanskrit word Shastra and Arbic word Ilm ?
4. What is the basic aim of science ?
5. What is the difference between physical and biological sciences ?
6. What is scientific attitude ?
7. What is a theory ?
8. What is physics ?
9. What is the origin of the word physics ?
10. What are the two basic quests in physics ?
11. Name the domain of physics that deals with phenomena intermediate between macroscopic and microscopic domains.
12. Physics has a very limited scope and is only the pastime of a few blessed ones. Is it true ?
13. State one law that holds good in all natural processes.
14. Some exciting and educative physical phenomena and experience lead to the development of a theory. Give two such examples.
15. Name two Indian born physicists who have been awarded Noble Price in physics. **[Chandigarh 02]**
16. Name the scientist who replaced circular orbits by elliptical orbits in the heliocentric theory of the sun.
17. Name the scientist who discovered X –rays.
18. Who discovered electrons ?
19. Name the scientist who first proposed the wave theory of light.
20. Name the famous scientist who discovered the law of gravitation.
21. Name the physicist who first proposed the nuclear model of the atom.
22. Who gave the quantum model of hydorgen atom ?
23. Name the scientists responsible for the development of Quantum Mechanics.

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24. Name the physicist who first proposed the concept of antiparticle.
25. Which physicist first confirmed the existence of positrons experimentally ?
26. Name the physicist who first measured the charge on an electron experimentally.
27. Name the physicist associated with wave- particle duality.
28. Who first discovered neutrons ?
29. Name the scientist who received Nobel Prize twice in physics.
30. Name the physicist who first gave the exchange theory of nuclear forces.
31. Name the scientist who was first awarded two Nobel Prizes.
32. What were the important contributions of Madame Marie Curie ?
33. Who first discovered radioactivity ?
34. Who first gave theory of expanding universe ?
35. Mention some important contributions of Albert Einstein to physics.
36. Name the Indian physicist, who was first awarded the Nobel Prize.
37. What was the important discovery of C.V. Raman ?
38. What was the major contribution of Indian physicist S.N. Bose ?
39. With which field work was the famous Indian physicist H.J. Bhabha associated ?
40. Name the Indian physicist associated with the triple helical structure of proteins.
41. Mention the major areas of modern scientific research in chemical and biological sciences that are bound to affect the human society of the twenty-first century.
42. On which scientific principle does an aeroplane work ?
43. On which scientific principle calculators and computers are based ?
44. Which technology of physics has triggered the computer revolution in the last three decades of the twentieth century ?
45. What does the word LASER stand for ? On what basic principle does it work ?
46. Name the phenomenon used in the production of ultra high magnetic fields.
47. Name the philosopher who said about science "we know very little yet it is astonishing that we know so much and still astonishing that so little knowledge (of science) can give us so much power".
48. Name the four fundamental forces in nature.
49. Name the force responsible for the stability of nuclei and having range of 10^{-15} m.
[Central Schools 13]
50. Arrange the weak-nuclear force, electromagnetic force and gravitational force in the decreasing order of the strengths.
51. Give the ratio of the strengths of the four fundamental forces in nature.
52. What is the nature of the intermolecular Van der Walls' forces ?
53. How many times is the strong nuclear force stronger than the electromagnetic force ?
54. Is strong nuclear force a true fundamental force ?
55. Which force mainly governs the structure of atoms and molecules ?
56. Which fundamental force governs the large scale motion in the universe ?
57. What is meant by charge independent character of strong nuclear force ?
58. Which class of elementary particles experiences the weak nuclear force and not the strong nuclear force ?
59. Among which type of elementary particles does the electromagnetic force act ?
60. Name the class of elementary particldes on which strong nuclear force acts.
61. Which particle initiates the electromagnetic force between two electrons ?
62. What are the exchange particles for the operation of (*i*) strong nuclear force and (*ii*) weak nuclear force ?
63. How much is the range of gravitational force ? What are the messenger particles for this force ?
64. Name the physicists who first unified the electric and magnetic phenomena.
65. Name the unified domain of weak nuclear force and electromagnetic force.
66. Name the physicists who first predicted the existence of electro-weak force.
67. What are the conserved quantities in nature ? Give two examples of such quantities.
68. State the law of conservation of momentum. From which symmetry principle is this law obtained ?
69. Which symmetry laws lead to (*i*) law of conservation of energy and (*ii*) law of conservation of angular momentum ?
70. Some conservation laws are true for one fundamental force but not for other. Give two such examples.

Answers

1. Science is a systematised knowledge about the various natural phenomena gained by the man through his careful experimentation, keen observation and accurate reasoning.
2. The word science originated from the Latin verb *scientia* which means *to know*.
3. Both of these words stand for organised knowledge.

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4. The basic aim of science is to search for truth. It tends to analyse the natural phenomena occurring around us.
5. Physical sciences deal with the properties and behaviour of non-living matter while biological sciences deal with living things.
6. Scientific attitude requires a flexible open-minded approach towards solving problems in which other important points of view are not neglected.
7. Theory is the name given to a minium number of laws in terms of which the behaviour of a physical system can be explained.
8. Physics is the study of the basic laws of nature and their manifestations in various natural phenomena.
9. The word physics originates from a Greek word meaning nature.
10. (*i*) Unification and (*ii*) Reductionism.
11. Meso-scopic physics which deals with a few tens or hundreds of atoms.
12. No, physics has a very wide scope. We find application of physics in every walk of life.
13. One such law is the Newton's law of gravitation. It states that every body in the universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.
14. (*i*) The fall of an apple from a tree and motion of the moon led Newton to his famous law of gravitation.
    (*ii*) The spherical shape of the liquid drops led to the concept of surface tension.
15. (*i*) C.V. Raman & (*ii*) Subramanium Chandrasekhar.
16. Johann Kepler.
17. W.K. Roentgen.
18. J.J. Thomson.
19. Christiaan Huygens.
20. Isaac Newton.
21. Earnest Rutherford.
22. Neils Bohr in 1913.
23. Heisenberg and Schrödinger.
24. Paul Dirac in 1930.
25. Carl Anderson in 1932.
26. R.A. Millikan.
27. de Broglie.
28. James Chadwick in 1932.
29. John Bardeen.
30. Hideki Yukava of Japan.
31. Madame Marie Skladowak Curie, for physics in 1903 and for chemistry in 1911.
32. (*i*) Discoveries of radium and polonium.
    (*ii*) Studies on natural radioactivity.
33. A.H. Becquerrel in 1896.
34. Edwin Hubble of U.S.A.
35. (*i*) Mass-energy equivalence (*ii*) Photoectric effect and (*iii*) Theory of relativity.
36. C.V. Raman.
37. Inelastic scattery of light by molecules.
38. Quantum statistics (Bose-Einstein statistics).
39. Cascade process in cosmic radiation.
40. G.N. Ramchandran.
41. Genetic engineering, biotechnology, new chemical materials, etc.
42. Bernoulli's principle in fluid dynamics.
43. Digital logic of electronic circuits.
44. Technology of silicon chip.
45. The word LASER stands for light amplification by stimulated emission of radiation. It works on the principle of population inversion.
46. Superconductivity.
47. Bertrand Russel.
48. (*i*) Gravitational force
    (*ii*) Electromagnetic force
    (*iii*) Strong nuclear force and
    (*iv*) Weak nuclear force.
49. Strong nuclear force.
50. Electromagnetic force > Weak nuclear force > Gravitational force.
51. $F_G : F_W : F_E : F_N = 1 : 10^{25} : 10^{36} : 10^{38}$.
52. Intermolecular Van der Walls' forces are electromagnetic in nature.
53. 100 times.
54. No, strong nuclear force is now thought to be a force derived from quark-quark force.
55. Electromagnetic force.
56. Gravitational force.
57. Nuclear forces between a proton and proton, a neutron and a neutron, and a proton and a neutron are nearly equally strong. This indicates that the strong nuclear force does not depend on the charge of the nucleons.
58. Leptons.
59. Electromagnetic force acts on all electrically charged particles.
60. Hadrons.
61. Photon (γ).
62. (*i*) Mesons
    (*ii*) Vector bosons.
63. The range of gravitational force is infinite. Gravitons are thought to be the exchange particles for this force.
64. Oersted and Faraday.
65. Electro-weak force.

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66. Sheldon Glashow, Abdus Salam, Steven Weinberg in 1979.

67. The physical quantities that remain unchanged in a process are called conserved quantities. For example, energy, linear momentum, etc.

68. Law of conservation of linear momentum states that if no external force acts on a system then, its total linear momentum remains constant. This law follows from the homogeneity of space.

69. (*i*) Homogeneity of time leads to the law of conservation of energy.
(*ii*) Isotropy of space leads to the law of conservation of angular momentum.

70. (*i*) Parity is conserved by the strong and electromagnetic forces but not by the weak force.
(*ii*) Strangeness is conserved by the strong force but not by the weak force.

Type B : Short Answer Questions — 2 or 3 Marks Each

1. What is a scientific method ? Mention the various steps involved in a scientific method.
2. Mathematics is an important tool in the development of physics. Comment on this statement.
3. Mention four important discoveries of physics which have revolutionised modern chemistry.
4. Name some advancements in physics which have played an important role in the development of medical sciences.
5. Name *two* advancements made in technology on the basis of physics. **[Himachal 06C]**
6. Theory and experiments go hand in hand in physics and help each other's progress. Give two suitable examples in support of this view.
7. The scope of physics is truly vast. Comment.
8. Physics is a science of excitement. How ?
9. Give reasons for the tremendous progress in physics in the last few centuries.
10. Discuss the relation of physics with other sciences. **[Himachal 02, 04]**
11. Discuss the relation of physics with mathematics and chemistry. **[Himachal 03]**
12. What is physics ? Discuss the relation of physics with technology. **[Himachal 03, 04, 05]**
13. Discuss the scope and excitement of physics. **[Himachal 04]**
14. Explain the electromagnetic nature of the contact forces between two bodies.
15. Briefly discuss the nature of the force of friction.
16. Explain how does the elastic force in a spring arise.
17. Briefly explain the origin of Van der Walls' force between two neutral molecules.
18. Taking a suitable example from daily life, show that electromagnetic force is enormously stronger than the gravitational force.
19. Although gravitational force is the weakest of the four fundamental forces, yet it governs the large scale motion in the universe. Explain how.
20. What is a strong nuclear force ? Give its important properties. Give an example of nuclear force.
21. What is a weak nuclear force ? Give its important properties. Give an example of a weak nuclear force.
22. Name the *four* fundamental forces in nature. Out of the four which one is (*a*) strongest, and (*b*) weakest ? **[Delhi 03]**
23. Write the names of the *four* fundamental forces in nature. Give any *two* main characteristics of each type. **[Delhi 03C]**
24. How do we classify elementary particles as baryons, mesons and leptons ? Amongst which of these particles is the strong nuclear force dominant and amongst which is the weak nuclear force dominant ?
25. State the law of conservation of energy. Give a suitable example for it.
26. State the law of conservation of linear momentum. Give an example for it.
27. State the law of conservation of angular momentum. Give an example for it.
28. Explain the law of conservation of charge with the help of a suitable example.
29. How are the conservation laws related with the symmetries of nature ?
30. Match the scientist's name against discovery :

A. Scientist	B. Discovery
1. Faraday	1. Law of gravitation
2. Rutherford	2. Quantum model of hydrogen atom
3. Chadwick	3. Unification of light and electromagnetism
4. Bohr	4. Theory of Relativity
5. Newton	5. Inelastic scattering of light by molecules
6. Maxwell	6. Unification of weak and strong electromagnetic interaction
7. Salam	7. Law of electromagnetic induction
8. Einstein	8. Expansion of the Universe
9. Raman	9. Neutron
10. Hubble	10. Nuclear model of the atom

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31. Match the scientist in column A against the country of origin in column B :

A. Scientist	B. Country of origin
1. Michelson	1. Denmark
2. Newton	2. U.S.A.
3. Landau	3. Italy
4. Bhabha	4. France
5. Bohr	5. India
6. Archimedes	6. Germany
7. Galileo	7. U.S.S.R.
8. Curie	8. Britain
9. Heisenberg	9. Japan
10. Yukawa	10. Greece

32. Match the technology in column A to its related scientific principle(s) in column B :

A. Technology	B. Scientific principle(s)
1. Steam engine	1. Propagation of electromagnetic waves
2. Nuclear reactor	2. Newton's laws of motion
3. Radio and T.V.	3. Superconductivity
4. Computers	4. Role of DNA in heredity
5. Lasers	5. Thermodynamics
6. Production of ultra high magnetic fields	6. Faraday's law of induction
7. Rocket propulsion	7. Conversion of gravitational potential energy into electrical energy
8. Genetic engineering	8. Motion of charged particles in electromagnetic fields
9. Electric generator	9. Fission of uranium by slow neutrons
10. Hydroelectric power	10. Amplification by population inversion
11. Aeroplane	11. Digital logic of electronic circuits
12. Particle accelerators	12. Bernoulli's principle in fluid dynamics

Answers

1. Refer answer to Q. 4 on page 1.1.
2. Refer answer to Q. 14 on page 1.4.
3. (*i*) X-ray diffraction techniques (*ii*) Study of radioactivity (*iii*) Discovery of isotopes by mass spectrographs (*iv*) Nuclear magnetic resonance.
4. (*i*) Electron microscope in studying structure of a cell.
   (*ii*) X-ray and neutron diffraction techniques in studying the structure of nucleic acids.
   (*iii*) Radioisotopes in radiation therapy for curing skin cancer.
   (*iv*) Ultrasonics in diagonosing the human body.
5. Refer answer to Q. 16 on page 1.6.
6. (*i*) The alpha scattering experiments of Rutherford gave the nuclear model of the atom.
   (*ii*) Dirac's theoretical work introduced the concept of antiparticle, which was later confirmed experimentally by Anderson's discovery of positron.
7. Refer answer to Q. 11 on page 1.3.
8. Refer answer to Q. 12 on page 1.3.
9. Refer answer to Q. 13 on page 1.4.
10. Refer answer to Q. 14 on page 1.4.
11. Refer answer to Q. 14 on page 1.4.
12. Refer answer to Q. 16 on page 1.6.
13. Refer answer to Q. 11 & Q. 12 on page 1.3.
14. Refer answer to Q. 20(*i*) on page 1.8.
15. Refer answer to Q. 20(*ii*) on page 1.8.
16. Refer answer to Q. 20(*iii*) on page 1.8.
17. Refer answer to Q. 20(*iv*) on page 1.8.
18. Refer answer to Q. 21 on page 1.8.
19. Refer answer to Q. 22 on page 1.8.
20. Refer answer to Q. 23 on page 1.9.
21. Refer answer to Q. 24 on page 1.9.
22. Refer answer to Q. 17 on page 1.7.
23. Refer answer to Q. 18 and Q. 19 on page 1.7 and Q. 23 and Q. 24 on page 1.9.
24. Refer to 'For your knowledge' box on page 1.9.
25. Refer answer to Q. 29 on page 1.11.
26. Refer answer to Q. 30 on page 1.11.
27. Refer answer to Q. 31 on page 1.12.
28. Refer answer to Q. 32 on page 1.12.
29. Refer answer to Q. 33 on page 1.12.
30. (1 – 7) ; (2 – 10) ; (3 – 9) ; (4 – 2) ; (5 – 1) ; (6 – 3) ; (7 – 6) ; (8 – 4) ; (9 – 5) ; (10 – 8).
31. (1 – 2) ; (2 – 8) ; (3 – 7) ; (4 – 5) ; (5 – 1) ; (6 – 10) ; (7 – 3) ; (8 – 4) ; (9 – 6) ; (10 – 9).
32. (1 – 5) ; (2 – 9) ; (3 – 1) ; (4 – 11) ; (5 – 10) ; (6 – 3) ; (7 – 2) ; (8 – 4) ; (9 – 6) ; (10 – 7) ; (11 – 12) ; (12 – 8).

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Competition Section

Physical World

GLIMPSES

1. **Science.** Science is the systematised and organised knowledge about the various natural phenomena which is obtained by careful experimentation, keen observation, and accurate reasoning. It has *two* main branches, (*a*) physical sciences, (*b*) biological sciences.
   (*i*) **Physical sciences.** These sciences deal with the properties and behaviour of non-living things.
   (*i*) **Biological sciences.** These sciences deal with the behaviour of living things.
2. **Physics.** The word physics comes from a Greek word which means nature. Physics is the study of the natural laws and their manifestation in the natural phenomena. It deals with the concepts of space, time, motion, matter, energy, radiation, etc.
3. **Chemistry.** Chemistry is the study of every substance, its structure, its composition and changes in which it takes part.
4. **Scientific attitude.** Scientific attitude requires a flexible, open-minded approach towards solving problems in which other important points of view are not neglected.
5. **Scientific method.** The step by step approach used by a scientist in studying natural phenomena and establishing laws which govern these phenomena, is called scientific method. It involves the steps like (*i*) experimentation and observation, (*ii*) formation of hypothesis, (*iii*) verification of hypothesis and (*iv*) theoretical predictions.
6. **Scientific theory.** Theory is the name given to a set of a limited number of laws in terms of which the behaviour of a physical system can be explained.
7. **Physics is an exact science.** It is because of high precision and accuracy obtainable in the measurement of physical quantities.
8. **Physics is a basic science.** Because of its important role played in the development of life science, medicine, technology and industry, physics is considered as the most basic of all sciences.
9. **Physics and mathematics.** Mathematics has proved to be the most important tool in the development of physics.
10. **Mechanics.** It deals with the slow motion or equilibrium of material bodies.
11. **Optics.** It deals with the nature and propagation of light.
12. **Thermodynamics.** It deals with the change in internal energy, temperature, entropy etc., of the macroscopic systems through external work and heat.
13. **Electrodynamics.** It deals with electric and magnetic phenomena associated with charged and magnetic bodies.
14. **Quantum mechanics.** It deals with the mechanical behaviour of atoms, molecules and nuclei.
15. **Relativity.** It deals with the particles having speeds comparable to the speed of light. It is the theory of invariance in nature.
16. **Physics, technology and society.** Sometimes, physics generates new technology. At other times technology gives rise to new physics. Both have a direct impact on society.
17. **Scope of physics.** The scope of physics is truly vast. It covers a wide range of magnitudes of physical quantities such as mass, length, time, energy, etc.
18. **Basic quests in physics.** The *two* basic quests in modern physics are : (*i*) unification and (*ii*) reductionism. These have resulted in great advances in physics.

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19. **Fundamental forces in nature.** There are four fundamental forces which govern both macroscopic and microscopic phenomena. These are : (*i*) gravitational force, (*ii*) electromagnetic force, (*iii*) strong nuclear force and (*iv*) weak nuclear force.

The relative strengths of these forces are

$$F_G : F_W : F_E : F_S = 1 : 10^{25} : 10^{36} : 10^{38}.$$

20. **Gravitational force.** It is the force of mutual attraction between two bodies by virtue of their masses. It obeys Newton's law of gravitation. It is caused by the exchange particles called *gravitons*.

21. **Electromagnetic force.** It is the force due to interaction between two moving charges. It is caused by exchange of photons (γ) between two charged particles.

22. **Strong nuclear force.** It is the strongest attractive interaction which binds together the protons and neutrons in a nucleus. Electrons do not experience this force.

23. **Weak nuclear force.** It is the force that appears only between elementary particles involved in a nuclear process such as the β –decay of a nucleus. It is caused by exchange of vector bosons.

24. **Isolated system.** Any system on which no external force acts is called isolated system. Universe is the most ideal isolated system possible.

25. **Law of conservation of energy.** It states that the total energy of an isolated system remains constant. It follows from the nature's symmetry called homogeneity of time.

26. **Law of conservation of linear momentum.** It states that if no external force acts on a system, then its total linear momentum remains constant. It follows from nature's symmetry called homogeneity of space.

27. **Law of conservation of angular momentum.** It states that if no external torque acts on a system, then its angular momentum remains constant. It follows from nature's symmetry called isotropy of space.

28. **Law of conservation of charge.** It states that the total charge of an isolated system remains constant.

Multiple Choice Questions

1. A supposition that is put forward as a probable solution to a natural phenomenon is called a

(*a*) hypothesis (*b*) theory
(*c*) law (*d*) relief.

2. A thought experiment in physics is one

(*a*) which is theoretically possible but experimentally not feasible.
(*b*) which is neither theoretically nor experimentally feasible.
(*c*) which is performed by non-physicist.
(*d*) none of the above.

3. "Nature is pleased with simplicity and effects not the pump of superfluous causes". It has been said by the great scientist

(*a*) Einstein (*b*) Dirac
(*c*) Russel (*d*) Newton.

4. The book titled 'Principia' was written by

(*a*) Maxwell (*b*) Einstein
(*c*) Newton (*d*) Galileo. [AFMC 01]

5. "It is more important to have beauty in the equations of physics than to have them agree with experiments". The scientist who made this statement is

(a) Dirac (*b*) Maxwell
(*c*) Faraday (*d*) Raman.

6. "The most incomprehensible thing about the world is that it is comprehensible." This statement was made by the scientist

(*a*) Aristotle (*b*) Newton
(*c*) Galileo (*d*) Einstein.

7. The country which awards the prestigeous Nobel prize is

(*a*) U.S.A. (*b*) U.K.
(*c*) Sweden (*d*) France.

8. An Indian scientist who won Nobel prize for physics is

(a) Sir J.C. Bose (*b*) H.J. Bhaba
(*c*) M.N. Saha (*d*) Sir C.V. Raman. [AFMC 01]

9. The man who has won Nobel Prize twice in physics is

(*a*) Einstein (*b*) Bardeen
(*c*) Heisenberg (*d*) Faraday. [AFMC 01]

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10. Madam Marie Curie won Nobel Prize twice which were in the field of

(a) Physics and chemistry

(b) Chemistry only

(c) Physics only

(d) Biology only.

11. Prof. Albert Einstein got noble prize in physics for his work on

(a) special theory of relativity

(b) general theory of relativity

(c) photoelectric effect

(d) theory of specific heats. [CBSE 91]

12. The value of universal gravitational constant 'G' was first experimentally determined by

(a) Newton (b) Galileo

(c) Kelvin (d) Cavendish.

13. Charge on an electron was first determined by :

(a) Millikan (b) Bohr

(c) Thomson (d) Rutherford.

14. J. D. Van der Walls discovered equation of gases and liquids. He was a

(a) English scientist (b) French scientist

(c) Dutch scientist (d) German scientist.

15. X-rays were discovered by

(a) Coolidge (b) Roentgen

(c) Maxwell (d) Fermi.

16. The working of an aeroplane is based on

(a) Newton's third law of motion

(b) Bernoulli's principle

(c) Newton's law of gravitation

(d) Law of conservation of momentum.

17. Chandershekher, an America based Indian scientist was awarded Nobel Prize in physics on the subject concerning

(a) Geophysics (b) Astronomy

(c) Superconductivity

(d) Laser Technology.

18. Radioactivity was discovered by

(a) Chadwick (b) Rutherford

(c) Becquerel (d) Roentgen.

19. Abdus Salam, an America based Pakistani physicist won Nobel Prize in the field of

(a) Inelastic scattering of light by molecules

(b) Unification of weak and electromagnetic forces

(c) Superconductivity

(d) Laser technology.

20. Neutrons were discovered by

(a) Fermi (b) Yukawa

(c) Dirac (d) Chadwick.

21. Louis de-Broglie is credited for his work on

(a) Theory of relativity

(b) Electromagnetic theory

(c) Matter waves

(d) Law of distribution of velocities.

22. The country to which de Broglie belongs is

(a) Germany (b) England

(c) France (d) America.

23. Lightning was discovered by [AFMC 96]

(a) Faraday (b) Franklin

(c) Ohm (d) Edison.

24. The discoverer of loudness and intensity of sound is

(a) Newton (b) Bell

(c) Leplace (d) Edison.

25. The man who is known as the Father of Experimental Physics is

(a) Newton (b) Albert Einstein

(c) Galileo (d) Rutherford.

26. The total number of discoveries made by Michael Faraday is

(a) 16 (b) 41

(c) 5 (d) 16401.

27. The person who has been awarded the title of the Father of Physics of 20th century is

(a) Madame Curie (b) Sir C.V. Raman

(c) Neils Bohar (d) Albert Einstein.

28. It has been postulated that there may be some particles moving with speed greater than the speed of light. Such particles are known as

(a) Electrons (b) Neutrons

(c) Nucleons (d) Tachyons. [AFMC 01]

29. The book "Pisces of Physics" has been written by

(a) Einstein (b) Newton

(c) Archimedes

(d) Galileo. [AFMC 01]

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30. Which of the following is wrongly matched ?
(a) Barometer-Pressure
(b) Lactometer-Milk
(c) Coulomb's law-charges
(d) Humidity-Calorimeter [AFMC 01]

31. C.V. Raman got Nobel Prize for his experiment on
(a) dispersion of light
(b) reflection of light
(c) deflection of light
(d) scattering of light. [AFMC 01]

32. The idea of calculus was given by
(a) Newton
(b) Einstein
(c) Marconi
(d) Planck. [AFMC 03]

33. Which of the following principles is being used in Sonar Technology ?
(a) Reflection of ultrasonic waves
(b) Newton's laws of motion
(c) Reflection of electromagnetic waves
(d) Laws of thermodynamics [DPMT 2011]

Answers

1. (a)	2. (b)	3. (d)	4. (c)	5. (a)	6. (d)
7. (c)	8. (d)	9. (b)	10. (a)	11. (c)	12. (d)
13. (a)	14. (c)	15. (b)	16. (b)	17. (b)	18. (c)
19. (b)	20. (d)	21. (c)	22. (c)	23. (b)	24. (b)
25. (c)	26. (d)	27. (d)	28. (d)	29. (a)	30. (d)
31. (d)	32. (a)	33. (a)			

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CHAPTER 2

UNITS AND MEASUREMENTS

2.1 NEED FOR THE MEASUREMENT

1. *Explain the need for measurement in physics.*

Need for the measurement. Physics is an exact science which gives an accurate knowledge about the nature and the natural phenomena. It expresses various natural phenomena in terms of the relationships among the quantities involved. The exactness or accuracy of these relationships depends upon the measurements we make. The accuracy of the measurements, in turn, depends on the accuracy of the measuring instruments and techniques. The recent progress in science and technology has been possible only due to the development of high precision instruments. One can measure mass as small as that of an electron ($\sim 10^{-30}$ kg) and as large as that of the universe ($\sim 10^{55}$ kg).

Prof. *William S. Franklin* emphasised the importance of measurement in the following words :

"The most important thing for a young man to acquire from his first course in physics is an appreciation for precise details."

Stressing upon the importance of measurement, *Lord Kelvin*, a great English physicist of 19th century, once remarked that

"When you can measure what you are speaking about and express it in numbers, you know something about it, but when you cannot measure it and express it in numbers, your knowledge is of a meagre and unsatisfactory kind, but you have scarcely, in your thoughts, advanced to the stage of science whatever the matter may be."

2.2 PHYSICAL QUANTITIES : FUNDAMENTAL AND DERIVED

2. *What are physical quantities ? Distinguish between fundamental and derived quantities.*

Physical quantities. *All those quantities which can be measured directly or indirectly and in terms of which the laws of physics can be expressed are called physical quantities.* For example, length, mass, temperature, speed, force, electric current, etc. The physical quantities are the building blocks of physics in terms of which the basic laws of physics can be expressed in mathematical forms.

Physical quantities are of *two* types – fundamental and derived.

(*i*) ***Fundamental quantities.*** *The physical quantities which can be treated as independent of other physical quantities and are not usually defined in terms of other physical quantities are called fundamental quantities.* To

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give a consistent and unambiguous description to all physical quantities, we need a minimum of **seven fundamental** or **base quantities**. These are mass, length, time, electric current, temperature, luminous intensity and amount of substance.

(*ii*) ***Derived quantities.*** *The physical quantities whose defining operations are based on other physical quantities are called derived quantities.* All physical quantities other than the seven base quantities are derived quantities. For example, velocity, acceleration, force, momentum, etc.

2.3 THE MEASURING PROCESS

3. *What is meant by the term measurement of a physical quantity ? How is the result of measurement of a physical quantity expressed ?*

Measurement. The measurement of a physical quantity is the process of comparing this quantity with a standard amount of the physical quantity of the same kind, called its unit.

To express the measurement of a physical quantity, we need to know *two* things :

(*i*) The *unit* in which the quantity is measured.

(*ii*) The *numerical value* or the *magnitude* of the quantity *i.e.*, the number of times that unit is contained in the given physical quantity.

$\therefore$ Measure of a physical quantity

= Numerical value of the physical quantity × size of the unit

or $Q = nu$

Let length of a room = 5 m = 500 cm.

Clearly, the smaller the size of the unit, the larger is the numerical value associated with the physical quantity.

Thus the numerical value (n) is inversely proportional to the size (u) of the unit.

$$n \propto \frac{1}{u} \quad \text{or} \quad nu = \text{constant}.$$

. If n_1 and n_2 are numerical values for a physical quantity Q corresponding to the units u_1 and u_2, then

$$Q = n_1 u_1 = n_2 u_2 \qquad ...(2.1)$$

2.4 PHYSICAL UNIT AND ITS DESIRABLE CHARACTERISTICS

4. *What is a physical unit ? Write the essential requirements that a physical unit/standard must meet.*

Physical unit. *The standard amount of a physical quantity chosen to measure the physical quantity of the same kind is called a physical unit.*

Desirable characteristics of a physical unit :

1. It should be *well-defined.*
2. It should be of *convenient size, i.e.,* neither too small nor too large in comparison with the measurable physical quantity.
3. It should not change with time.
4. It should be easily *reproducible.*
5. It should be *imperishable* or *indestructible.*
6. It should not be affected by the change in physical conditions such as pressure, temperature, etc.
7. It should be *internationally acceptable.*
8. It should be easily *accessible.*

2.5 FUNDAMENTAL AND DERIVED UNITS

5. *Although the number of physical quantities which we measure is very large, yet we do not need a very large number of units for this measurement. Why ?*

This is possible because the various physical quantities are related to each other and so their units can be expressed in terms of just seven basic or fundamental units.

6. *What are fundamental and derived units ? Give some examples.*

Fundamental units. *The physical units which can neither be derived from one another, nor they can be further resolved into more simpler units are called fundamental units.* The units of fundamental quantities such as mass, length, etc. are fundamental units.

Derived units. *All the other physical units which can be expressed in terms of the fundamental units are called derived units.* Let us consider the unit of speed.

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$$

$$\therefore \text{Unit of speed} = \frac{\text{Unit of distance}}{\text{Unit of time}}$$

$$= \frac{\text{metre}}{\text{second}} = \text{ms}^{-1}$$

Thus the unit of speed (ms^{-1}) is a derived unit as it has been expressed in the fundamental units of length and time.

2.6 SYSTEMS OF UNITS

7. *What is a system of units ? Mention the various types of systems of units.*

System of units. *A complete set of units which is used to measure all kinds of fundamental and derived quantities is called a system of units.*

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Some of the commonly used systems of units are as follows :

(*i*) ***cgs system.*** It was set up in France. It is based on *centimetre, gram* and *second* as the fundamental units of length, mass and time respectively.

(*ii*) ***fps system.*** It is a British system based on *foot, pound* and *second* as the fundamental units of length, mass and time respectively.

(*iii*) ***mks system.*** It is also a French system based on *metre, kilogram* and *second* as the fundamental units of length, mass and time respectively.

(*iv*) ***SI : The international system of units.*** SI is the abbreviation for *"Systeme Internationale d' Unites"*, which is French equivalent for *international system of units*. It is a modernised and extended form of the metric systems like cgs and mks systems. This system was adopted by eleventh *General Conference of Weights and Measures in 1960*. It covers all branches of science and technology. It is based on the following seven basic units and two supplementary units.

Table 2.1 *Basic SI quantities and units*

S. No.	Basic physical quantity	Basic unit	Symbol
1.	Length	metre	m
2.	Mass	kilogram	kg
3.	Time	second	s
4.	Temperature	kelvin	K
5.	Electric current	ampere	A
6.	Luminous intensity	candela	cd
7.	Quantity of matter	mole	mol

Table 2.2 *Supplementary SI units*

S. No.	Supplementary quantity	Basic unit	Symbol
1.	Plane angle	radian	rad
2.	Solid angle	steradian	sr

For Your Knowledge

- The cgs, mks and SI are *metric* or *decimal systems* of units. This is because the multiples and sub-multiples of their basic units are related to the practical units by powers of 10.
- The fps system is not a metric system. This system is not in much use these days.

2.7 DEFINITIONS OF BASIC AND SUPPLEMENTARY SI UNITS

8. *Name and define all the basic and supplementary units of SI.*

The *seven* **basic SI units** are defined as follows :

(*i*) **Metre (m).** It is the SI unit of length. *One metre is defined as the length of the path travelled by light in vacuum during a time interval of 1/299, 792, 458 of a second.*

(*ii*) **Kilogram (kg).** It is the SI unit of mass. *One kilogram is the mass of prototype cylinder of platinum-iridium alloy (whose height is equal to its diameter)* preserved at the International Bureau of Weights and Measures, at Sevres, near Paris.

(*iii*) **Second (s).** It is the SI unit of time. *One second is the duration of 9, 192, 631, 770 periods of the radiation corresponding to the transition between two hyperfine levels of the ground state of the cesium – 133 atom.*

(*iv*) **Ampere (A).** It is the SI unit of electric current. *One ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to* 2×10^{-7} *newton per metre of length.*

(*v*) **Kelvin (K).** It is the SI unit of temperature. *One kelvin is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.* The triple point of water is the temperature at which ice, water and water vapour co-exist.

(*vi*) **Candela (cd).** It is the SI unit of luminous intensity. *One candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency* 540×10^{12} *hertz and* that has a radiant intensity of 1/683 watt per steradian in that direction.

(*vii*) **Mole (mol).** *One mole is that amount of a substance which contains as many elementary entities as there are atoms in 0.012 kg of carbon-12 isotope.* The entities may be atoms, molecules, ions etc.

The *two* **supplementary SI units** are defined as follows :

(*a*) **Radian (rad).** *It is defined as the plane angle subtended at the centre of a circle by an arc equal in length to the radius of the circle.*

$$\theta \text{ (in radians)} = \frac{\text{Arc}}{\text{Radius}} = \frac{l}{r}$$

(*b*) **Steradian (sr).** *It is defined as the solid angle subtended at the centre of a sphere by a surface of the sphere equal in area to that of a square, having each side equal to the radius of the sphere.*

$$\Omega \text{ (in steradian)} = \frac{\text{Surface area}}{\text{Radius}^2}$$

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For Your Knowledge

A HISTORICAL VIEW OF BASIC SI UNITS

metre (m)

- In 1791, the Paris Academy of Sciences defined metre as *one ten millionth part of the distance measured along the meridian from the north pole to the equator.*
- In 1889, the General Conference of Weights and Measures defined metre *as the distance between two lines marked on a platinum-iridium rod preserved at a constant temperature of 273.16 K at 1 bar pressure in the* International Bureau of Weights and Measures at Sevres, near Paris.

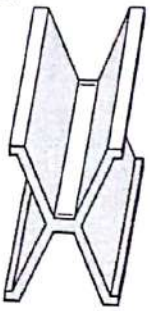

Fig. 2.1 Prototype metre.

- In 1960, the standard metre was defined in terms of wavelength of light. *One metre is defined as the distance which contains 1650, 763.73 wavelengths of certain orange-red radiation (of wavelength 6057.8021 Å) emitted by krypton-86 source kept at the temperature of triple point of nitrogen.*

kilogram (kg)

- In 1791 in France, originally kilogram was defined as the mass of one cubic decimetre (or 1 litre) of water at 4°C. Water has maximum density at this temperature.
- In 1889, the General Conference of Weights and Measures defined kilogram as the mass of platinum-iridium cylinder preserved at Sevres.

second (s)

- The Paris Academy of Sciences defined second as the time taken by a simple pendulum of one metre length to swing from one extreme position to the other.
- A *solar day* is defined as the time interval between the noons of two successive days. A *mean solar day* is the average of the solar days in one complete year. The $\frac{1}{24 \times 60 \times 60}$ or $\frac{1}{86,400}$ part of a mean solar day is called *mean solar second.*
- In 1956, the second was defined in terms of the tropical year 1900, the duration of which was taken as 365.242 mean solar days. So second was defined as the fraction $\frac{1}{365.242 \times 86,400}$ or $\frac{1}{31,556,925.9747}$ of the tropical year 1900 and was called ephimeris second.

2.8 COHERENT SYSTEM OF UNITS

9. *What is a coherent system of units ? Give examples.*

Coherent system. *It is a system of units based on a certain set of fundamental units from which all derived units can be obtained by simple multiplication or division without introducing any numerical factor.* For example, mks system is a coherent system of units in mechanics. All derived units in mechanics such as those of area, volume, density, acceleration, force, etc., can be obtained by the multiplication or division of the fundamental units of mass, length and time. SI is a coherent system of units for all branches of physics.

2.9 ADVANTAGES OF SI

10. *State the advantages of SI over other systems of units.*

Advantages of SI over other systems of units :

(*i*) ***SI is a coherent system of units.*** All derived units can be obtained by simple multiplication or division of fundamental units without introducing any numerical factor.

(*ii*) ***SI is a rational system of units.*** It uses only one unit for a given physical quantity. For example, all forms of energy are measured in joule. On the other hand, in mks system, the mechanical energy is measured in joule, heat energy in calorie and electrical energy in watt hour.

(*iii*) ***SI is a metric system.*** The multiples and sub-multiples of SI units can be expressed as powers of 10.

(*iv*) ***SI is an absolute system of units.*** It does not use gravitational units. The use of '*g*' is not required.

(*v*) ***SI is an internationally accepted system of units.***

2.10 GUIDELINES FOR WRITING SI UNITS IN SYMBOLS

11. *State the rules that are followed in writing SI units in symbolic form.*

Rules for writing SI units in symbolic form :

(*i*) Small letters are used for symbols of units.

(*ii*) Symbols are not followed by a full stop.

(*iii*) The initial letter of a symbol is capital only when the unit is named after a scientist.

(*iv*) The full name of a unit always begins with a small letter even if it has been named after a scientist.

(*v*) Symbols do not take plural form.

2.11 ABBREVIATIONS IN POWERS OF TEN

12. *Make a list of various prefixes used for powers of 10. Give some examples.*

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Abbreviations in powers of ten. When the magnitudes of the physical quantities are very large or very small, it is convenient to express them in the multiples or submultiples of the SI units. The various prefixed used for powers of 10 are listed in table 2.3.

Table 2.3 *Prefixes for powers of ten*

Multiple	Prefix	Symbol	Submultiple	Prefix	Symbol
10^{1}	deca	da	10^{-1}	deci	d
10^{2}	hecto	h	10^{-2}	centi	c
10^{3}	kilo	k	10^{-3}	milli	m
10^{6}	mega	M	10^{-6}	micro	μ
10^{9}	giga	G	10^{-9}	nano	n
10^{12}	tera	T	10^{-12}	pico	p
10^{15}	peta	P	10^{-15}	femto	f
10^{18}	exa	E	10^{-18}	atto	a

Examples :

1 megaohm = 1 MΩ = 10^{6} Ω

1 kilometre = 1 km = 10^{3} m

1 decagram = 1 da g = 10 g

1 centimetre = 1 cm = 10^{-2} m

1 milliampere or 1 mA = 10^{-3} A

1 microvolt or 1 μV = 10^{-6} V

1 nanosecond or 1 ns = 10^{-9} s

1 picofarad or 1 pF = 10^{-12} F.

2.12 SOME COMMON PRACTICAL UNITS

13. *Make a list of some commonly used practical units. How these units are related to SI units ?*

A. Practical units for measuring small distances :

(*i*) ***Fermi.*** It is the small practical unit of distance used for measuring nuclear sizes. It is also called femtometre.

1 fermi = 1 fm = 10^{-15} m

The radius of a proton is 1.2 fermi.

(*ii*) ***Angstrom.*** It is used to express wavelength of light.

1 angstrom = 1 Å = 10^{-10} m = 10^{-8} cm

(*iii*) ***Nanometre.*** It is also used for expressing wavelength of light.

1 nanometre = 1 nm = 10^{-9} m

(*iv*) ***Micron.*** It is the unit of distance defined as micrometre.

1 micron = 1 μm = 10^{-6} m

B. Practical units used for measuring large distances :

(*i*) ***Light year.*** *It is the distance travelled by light in vacuum in one year.*

1 light year = Speed of light in vacuum × 1 year

$= 3\times 10^{8}\ ms^{-1}\times 365.25\times 24\times 60\times 60\ s$

∴ **1 light year = 1 ly = 9.467×10^{15} m**

Light year is used in astronomy to measure distances of nearby stars. *Alpha centauri,* the nearest star outside the solar system is 4.3 light years away from the earth.

(*ii*) ***Astronomical unit.*** *It is defined as the mean distance of the earth from the sun.* It is used in astronomy to measure distances of planets.

1 astronomical unit = 1 AU = 1.496×10^{11} m

(*iii*) ***Parsec (parallactic second).*** It is the largest practical unit of distance used in astronomy. *It is defined as the distance at which an arc of length 1 astronomical unit subtends an angle of 1 second of arc.*

As $\theta = \frac{l}{r}$ ∴ $r = \frac{l}{\theta}$

$$1\ \text{parsec} = \frac{1\ \text{AU}}{1''} = \frac{1.496\times 10^{11}\ \text{m}}{\frac{1}{3600}\times\frac{\pi}{180}\ \text{rad}} = 3.08\times 10^{16}\ \text{m}$$

1 parsec = 3.08×10^{16} m = 3.26 ly

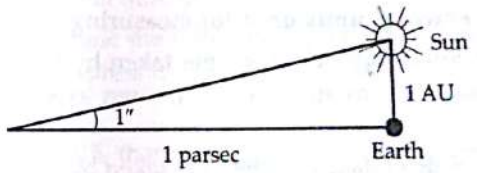

Fig. 2.2 Parsec

Relations between astronomical unit, light year and parsec

$1\ \text{AU} = 1.5\times 10^{11}$ m

$1\ \text{ly} = 9.46\times 10^{15}$ m

$1\ \text{parsec} = 3.08\times 10^{16}$ m

$$\frac{1\ \text{ly}}{1\ \text{AU}} = \frac{9.46\times 10^{15}}{1.5\times 10^{11}} = 6.3\times 10^{4}$$

∴ **1 ly = 6.3×10^{4} AU**

Also $\frac{1\ \text{parsec}}{1\ \text{ly}} = \frac{3.08\times 10^{16}}{9.46\times 10^{15}} = 3.26$

∴ **1 parsec = 3.26 ly**

Clearly, **1 parsec > 1 ly > 1 AU**

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C. Practical units for measuring areas :

(*i*) ***Barn.*** It is used for very small areas, such as nuclear cross-sections.

$$1 \text{ barn} = 10^{-28} \text{ m}^2$$

(*ii*) ***Acre.*** It is used measuring large areas.

$$1 \text{ acre} = 4047 \text{ m}^2$$

(*iii*) ***Hectare.*** It is also used for measuring large areas.

$$1 \text{ hectare} = 10^4 \text{ m}^2$$

D. Practical units used for measuring large masses :

1 tonne or 1 metric ton = 1000 kg

1 quintal = 100 kg

1 slug = 14.57 kg

1 pound = 1 lb = 0.4536 kg

1 Chandra Shekher limit = 1 CSL = 1.4 times the mass of the sun.

CSL is the largest practical unit of mass.

E. Practical unit used for measuring very small masses :

Atomic mass unit. It is defined as $\frac{1}{12}$th of the mass of one $^{12}_{6}C$ atom.

1 atomic mass unit = 1 amu = 1 u = 1.66×10^{-27} kg

The mass of a proton or a neutron is of the order of one amu.

F. Practical units used for measuring time :

(*i*) ***Solar day.*** It is the time taken by the earth to complete one rotation about its own axis w.r.t. the sun.

(*ii*) ***Sedrial day.*** It is the time taken by the earth to complete one rotation about its own axis w.r.t. a distant star.

(*iii*) ***Solar year.*** It is the time taken by the earth to complete one revolution around the sun in its orbit.

1 solar year = 365.25 average solar days

= 366.25 sedrial days

(*iv*) ***Tropical year.*** The year in which there is total solar eclipse is called tropical year.

(*v*) ***Leap year.*** The year which is divisible by 4 and in which the month of February has 29 days is called a leap year.

(*vi*) ***Lunar month.*** It is the time taken by the moon to complete one revolution around the earth in its orbit.

1 lunar month = 27.3 days

(*vii*) ***Shake.*** It is the smallest practical unit of time.

1 shake = 10^{-8} s

G. Practical units used for measuring pressu

1 bar = 1 atmospheric pressure

= 10^5 Nm^{-2} = 10^5 pascal (Pa)

1 millibar = 10^2 Pa

1 torr = 1 mm of Hg column

1 atmospheric pressure

= 1 bar = 760 mm of Hg column

= 760 torr.

Examples based on Simple Conversion of Units

CONVERSIONS USED

$$1 \text{ kg m}^{-3} = 10^{-3} \text{ g cm}^{-3}$$
$$1 \text{ g cm}^{-3} = 10^3 \text{ kg m}^{-3}$$
$$1 \text{ N} = 10^5 \text{ dyn}$$
$$1 \text{ amu} = 1.66 \times 10^{-27} \text{ kg}$$
$$1 \text{ AU} = 1.496 \times 10^{11} \text{ m}$$
$$1 \text{ ly} = 9.46 \times 10^{15} \text{ m}$$
$$1 \text{ parsec} = 3.08 \times 10^{16} \text{ m}$$
$$1 \text{ Å} = 10^{-10} \text{ m} = 0.1 \text{ nm}$$

EXAMPLE 1. How many light years are there in one metre ? *[Himachal 06C*

Solution. $1 \text{ ly} = 9.46 \times 10^{15} \text{ m}$

or $9.46 \times 10^{15} \text{ m} = 1 \text{ ly}$

$$\therefore 1 \text{ m} = \frac{1}{9.46 \times 10^{15}} \text{ ly} = 1.057 \times 10^{-16} \text{ ly}.$$

EXAMPLE 2. What is the number of electrons that would weigh 1 kg ? Mass of an electron is 9.11×10^{-31} *kg.*

Solution. Mass of each electron = 9.11×10^{-31} kg

Total mass = 1 kg

$\therefore$ Number of electrons

$$= \frac{\text{Total mass}}{\text{Mass of each electron}} = \frac{1 \text{ kg}}{9.11 \times 10^{-31} \text{ kg}}$$

$$= 1.1 \times 10^{30}.$$

EXAMPLE 3. The radius of gold nucleus is 41.3 fermi. Express its volume in m^3.

Solution. Here $r = 413$ fermi $= 41.3 \times 10^{-15}$ m

Volume, $V = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (41.3 \times 10^{-15} \text{ m})^3$

$$= 2.95 \times 10^{-40} \text{ m}^3.$$

EXAMPLE 4. Convert an acceleration of 2 km h^{-2} *into cm* s^{-2}.

Solution. $a = 2 \frac{\text{km}}{\text{h}^2} = \frac{2 \times 10^5 \text{ cm}}{\left(\frac{1}{3600} \text{ s}\right)^2} = 0.0154 \text{ cm s}^{-2}.$

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EXAMPLE 5. The Young's modulus of steel is $1.9 \times 10^{11}\ Nm^{-2}$. Express it in dyn cm^{-2}.

Solution. $Y = 1.9 \times 10^{11}\ \frac{N}{m^2}$

$$= 1.9 \times 10^{11} \times \frac{10^5\ \text{dyn}}{(100\ \text{cm})^2}$$

$$= \mathbf{1.9 \times 10^{12}\ dyn\ cm^{-2}}.$$

EXAMPLE 6. The density of a material is 0.8 g cm^{-3}. Express it in SI units.

Solution. As $1\ g\ cm^{-3} = 10^3\ kg\ m^{-3}$

$$\therefore\ 0.8\ g\,cm^{-3} = 0.8 \times 10^3\ kg\ m^{-3} = \mathbf{800\ kg\ m^{-3}}.$$

EXAMPLE 7. How many amu would make up 1 kg ?

Solution. As $1\ amu = 1.66 \times 10^{-27}\ kg$

$$\therefore 1\ kg = \frac{1}{1.66 \times 10^{-27}}\ amu = \mathbf{6.024 \times 10^{26}\ amu}.$$

EXAMPLE 8. Express the average distance of the earth from the sun in (i) light year and (ii) parsec.

Solution. Average distance of the earth from the sun,

$$r = 1\ AU = 1.496 \times 10^{11}\ m$$

(*i*) As 1 light year $= 9.46 \times 10^{15}$ m

$$\therefore\ r = \frac{1.496 \times 10^{11}}{9.46 \times 10^{15}}\ ly = \mathbf{1.58 \times 10^{-5}\ ly}.$$

(*ii*) As 1 parsec $= 3.08 \times 10^{16}$ m

$$\therefore\ r = \frac{1.496 \times 10^{11}}{3.08 \times 10^{16}}\ parsec = \mathbf{4.86 \times 10^{-6}\ parsec}.$$

PROBLEMS FOR PRACTICE

1. Calculate the number of astronomical units in one metre. (**Ans.** 6.67×10^{-12} AU)
2. How many parsec are there in one metre ? (**Ans.** 3.25×10^{-17} parsec)
3. How many parsec are there in one light year ? (**Ans.** 0.3067 parsec)
4. The density of air is $1.293\ kg\ m^{-3}$. Express it in cgs units. (**Ans.** $0.001293\ g\ cm^{-3}$)
5. Convert an acceleration of $10\ ms^{-2}$ in $km\ h^{-2}$. (**Ans.** $1.29 \times 10^5\ kmh^{-2}$)
6. The mass of a proton is 1.67×10^{-27} kg. How many protons would make 1 g ? (**Ans.** 5.99×10^{23})

HINTS

1. As $1\ AU = 1.5 \times 10^{11}$ m

$$\therefore\ 1\ m = \frac{1}{1.5 \times 10^{11}}\ AU = \mathbf{6.67 \times 10^{-12}\ AU}.$$

2. 1 parsec $= 3.08 \times 10^{16}$ m

$$\therefore\ 1\ m = \frac{1}{3.08 \times 10^{16}}\ parsec$$

$$= \mathbf{3.25 \times 10^{-17}\ parsec}.$$

3. As 1 parsec $= 3.26$ light year

$$\therefore\ 1\ \text{light year} = \frac{1}{3.26}\ \text{parsec} = \mathbf{0.3067\ parsec}.$$

4. Density $= 1.293\ kg\ m^{-3} = 1.293 \times 10^{-3}\ g\ cm^{-3}$

$$= \mathbf{0.001293\ g\ cm^{-3}}.$$

5. Acceleration $= 10\ \frac{m}{s^2} = \frac{10 \times 10^{-3}\ km}{\left(\frac{1}{3600}\ h\right)^2}$

$$= (3600)^2 \times 10^{-2}\ km\ h^{-2} = \mathbf{1.29 \times 10^5\ km\ h^{-2}}.$$

6. No. of protons $= \frac{\text{Total mass}}{\text{Mass of each proton}}$

$$= \frac{10^{-3}\ kg}{1.67 \times 10^{-27}\ kg} = \mathbf{5.99 \times 10^{23}}.$$

2.13 ORDER OF MAGNITUDE

14. *Define order of magnitude of a physical quantity. How is it determined ? Illustrate it with same examples.*

Order of magnitude. *The order of magnitude of a physical quantity is that power of 10 which is closest to its magnitude.* It gives an idea about how big or how small a given physical quantity is.

To determine the order of magnitude of a number N, we first express it as

$$N = n \times 10^x.$$

If $0.5 < n \leq 5$, then x will be the order of magnitude of N.

Few examples are illustrated in table 2.4.

Table 2.4 *Order of magnitude*

Measure number N	Expressed in neartest power of 10	Order of magnitude
8	0.8×10^1	1
49	4.9×10^1	1
52	0.52×10^2	2
555	0.555×10^3	3
999	0.999×10^3	3
1001	1.001×10^3	3
753000	0.753×10^6	6
0.135	1.35×10^{-1}	−1
0.05	5×10^{-2}	−2
0.99	0.99×10^0	0

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Examples based on Order of Magnitude

Formulae Used

To determine the order of magnitude of a number N, we express it as

$$N = n \times 10^x.$$

If $0.5 < n \leq 5$, then x will be the order of magnitude of N.

Conversions Used

$1\text{ AU} = 1.496 \times 10^{11}$ m, 1 parsec $= 3.08 \times 10^{16}$ m

Constant Used

Speed of light in vacuum, $c = 3 \times 10^8$ ms^{-1}

Example 9. Write the order of magnitude of the following measurements :

(i) 25,710,000 m (ii) 0.00000521 kg.

Solution. We first express each number in terms of nearest power of 10.

(i) $25{,}710{,}000\text{ m} = 2.571 \times 10^7$ m

As $0.5 < 2.571 < 5$

$\therefore$ Order of magnitude = **7.**

(ii) $0.00000521\text{ kg} = 0.521 \times 10^{-5}$ kg

As $0.5 < 0.521 < 5$

$\therefore$ Order of magnitude = **– 5.**

Example 10. Express 1 light year in terms of metres. What is its order of magnitude ?

Solution. One light year is the distance travelled by light in one year.

Speed of light, $c = 3 \times 10^8$ ms^{-1}

$t = 1$ year $= 365.25$ days $= 365.25 \times 24 \times 60 \times 60$ s

$\therefore$ 1 light year $= ct$

$= 3 \times 10^8 \times 365.25 \times 24 \times 60 \times 60$ m

$= \mathbf{9.467 \times 10^{15}\text{ m} = 0.9467 \times 10^{16}\text{ m}}$.

Order of magnitude of light year = **16.**

Problems for Practice

1. Write the order of magnitude of the following :
   (i) 8 (ii) 49 (iii) 52 (iv) 999
   (v) 1001 (vi) 753000 (vii) 0.05 (viii) 0.99.
   (Ans. 1, 1, 2, 3, 3, 6, – 2, 0**)**
2. What is one astronomical unit ? Express it in metres. Write its order of magnitude.
   (Ans. Order of magnitude = 11**)**
3. What is the order of magnitude of seconds in a day ?
   (Ans. 5**)**

Hints

1. (i) $8 = 0.8 \times 10^1$ (ii) $49 = 4.9 \times 10^1$
   (iii) $52 = 0.52 \times 10^2$ (iv) $999 = 0.999 \times 10^3$
   (v) $1001 = 1.001 \times 10^3$ (vi) $753000 = 0.753 \times 10^6$
   (vii) $0.05 = 5 \times 10^{-2}$ (viii) $0.99 = 0.99 \times 10^0$
2. Astronomical unit is the mean distance of the earth from the sun. $1\text{ AU} = 1.496 \times 10^{11}$ m
   $\therefore$ Order of magnitude = 11.
3. $1\text{ day} = 24 \times 60 \times 60 = 86400\text{ s} = 0.864 \times 10^5$ s
   $\therefore$ Order of magnitude = 5.

2.14 MEASUREMENT OF LENGTH BY DIRECT METHODS

15. *Define Length. Name the device used for measuring directly the lengths (i) from 10^{-3} m to 10^2 m (ii) to an accuracy of 10^{-4} m and (ii) to an accuracy of 10^{-5} m.*

Length. *Length may be defined as the distance of separation between two points in space.*

(i) Metre scale.
(ii) Vernier callipers.
(iii) Screw gauge or spherometer.

2.15 INDIRECT METHODS FOR MEASURING LARGE DISTANCES

16. *Describe an indirect method to measure the height of an accessible tree or a tower.*

Triangulation method for the height of an accessible object. Let $AB = h$ be the height of the tree or the tower to be measured. Let C be the point of observation at distance x from B. Place a sextant at C and measure the angle of elevation,

$\angle AOB = \theta$.

From right ΔABC, we have

$$\tan\theta = \frac{AB}{CB} = \frac{h}{x}$$

or height, $h = x \tan\theta$

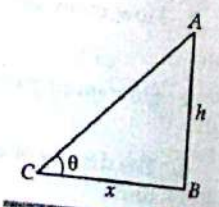

Fig. 2.3 Height of a tree.

Knowing the distance x, the height h can be determined.

17. *Describe a method to measure the height of an inaccessible object like a mountain.*

Triangulation method for the height of an inaccessible object. Let $AB = h$ be the height of the mountain to be measured. By using a sextant, we first measure the angle of elevation of its peak from a point C on the ground. Let it be θ_1.

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Move the sextant to another position D such that $CD = d$. Again measure the angle of elevation, $\angle ADB = \theta_2$.

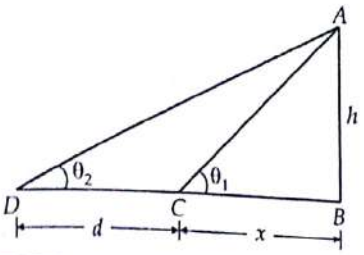

Fig. 2.4 Height of a mountain.

In rt. ΔABC, $\cot\theta_1 = \dfrac{CB}{AB} = \dfrac{x}{h}$

In rt. ΔABD, $\cot\theta_2 = \dfrac{DB}{AB} = \dfrac{d+x}{h}$

$\therefore \quad \cot\theta_2 - \cot\theta_1 = \dfrac{d+x}{h} - \dfrac{x}{h} = \dfrac{d}{h}$

or $$h = \frac{d}{\cot\theta_2 - \cot\theta_1}.$$

Knowing d, the height h can be determined.

18. *What is meant by parallax and parallactic angle ? How can we find the distance of the moon (or any planet) by parallax method ?*

Parallax. *Parallax is the apparent shift in the position of an object with respect to another when we shift our eye sidewise.* The closer object always appears to move in the direction opposite to that of our eye.

To understand parallax, hold a pen O at distance S from the eyes. Look at the pen first by the left eye L (closing the right eye) and then by the right eye R (closing the left eye). The position of the pen appears to change with respect to the background. This is called *parallax*. The distance between the two points of observation is called *basis*. In this case, the distance ($LR = b$) between the two eyes is the basis. $\angle LOR = \theta$ is called *parallax angle* or *parallactic angle.*

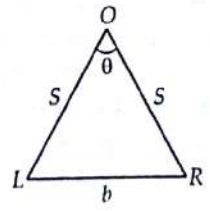

Fig. 2.5 Parallax

Distance of the moon or any planet. To measure the distance S of the moon or a far away planet P, we observe it simultaneously from two different positions (observatories) A and B on the earth, separated by a large distance $AB = b$. We select a distant star S' whose position and direction can be taken approximately same from A and B.

Now $\angle PAS' = \phi_1$ and $\angle PBS' = \phi_2$ are measured from the two observatories at the same time. As $b << S$, so we can take AB as an arc of length b.

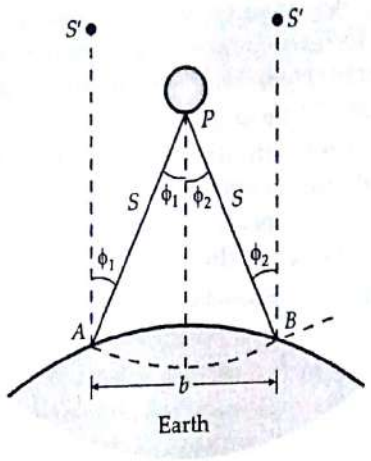

Fig. 2.6 Distance of a planet.

Now $$\theta = \frac{\text{Arc}}{\text{Radius}} = \frac{b}{S}$$

$\therefore \quad S = \dfrac{b}{\theta}$.

where $\theta = \angle APB = \phi_1 + \phi_2$, is the parallactic angle.

19. *Describe the parallax method for the determination of the distance of a nearby star from the earth.*

Distance of a nearby star by parallax method.

As shown in Fig. 2.7, suppose N is the nearby star whose distance d from the earth is to be found. F is a far off star whose direction and position is fixed for all positions of the earth in its orbital motion. When the earth is at position A, the parallax angle between distant star F and nearby star N is determined. Let it be θ_1. After six months, the earth is at diametrically opposite position B. The parallax angle, $\angle NBF = \theta_2$ is measured.

Total parallax angle subtended by N on the earth's orbital diameter AB is

$$\theta = \theta_1 + \theta_2$$

As $$\theta = \frac{\text{Arc}}{\text{Radius}}$$

$\therefore \quad \theta = \dfrac{AB}{d}$

or $$d = \frac{AB}{\theta}.$$

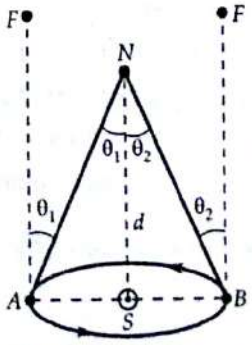

Fig. 2.7 Distance of a nearby star.

The parallax method is useful for measuring distances of the stars which are less than 100 light years away from the earth.

20. *Briefly describe a method for the determination of distance of a far away star.*

Intensity method. This is a spectroscopic method based on **inverse square law of intensity.** According

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to this law, *the intensity of illumination at a point is inversely proportional to the square of the distance from the source of light*. Here we assume that the intrinsic brightness of all the stars is same. We compare the intensity I_1 of the faint image of a far away star taken on a photographic plate with the intensity I_2 of the bright image of a nearby star. Let r_1 and r_2 be the respective distances of these two stars.

From inverse square law of intensity,

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} \quad \text{or} \quad r_1 = r_2\left[\frac{I_2}{I_1}\right]^{1/2}$$

knowing the distance r_2 of the nearby star, the distance r_1 of the far away star can be determined. This method is useful for measuring distances of stars which are more than 100 light years away from the earth.

21. *What are inferior and superior planets ? Give the names of these planets.*

Inferior planets. *The planets which are closer to the sun than the earth are called inferior planets.* Mercury and Venus are the inferior planets.

Superior planets. *The planets which are farther from the sun than the earth are called superior planets.* Jupiter, Saturn, Uranus, Neptune and Pluto are the superior planets.

22. *Describe a method for determining the distance of an inferior planet.*

Copernicus method. Copernicus assumed circular orbits for the planets. The angle formed at the earth between the earth-planet direction and earth-sun direction is called the *planet's elongation*. As shown in Fig. 2.8, let

R_{ps} = distance of the planet from the sun
R_{pe} = distance of the planet from the earth
R_{es} = distance of the earth from the sun
ε = planet's elongation

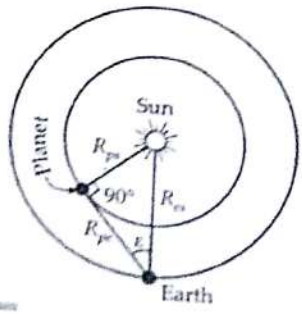

Fig. 2.8 Copernicus method.

When the elongation attains its maximum value and the planet appears farthest from the sun, the angle subtended by the sun and the earth at the planet is 90°. Then from the right angle triangle shown in the figure, we find that

$$\frac{R_{ps}}{R_{es}} = \sin\varepsilon$$

Hence the distance of the planet from the sun is

$$R_{ps} = \sin\varepsilon \,.\, R_{es} = \sin\varepsilon \,.\, \text{AU}$$

where R_{es} is the average distance of the earth from the sun and is called *astronomical unit* (AU).

23. *How can we determine the distance of a superior planet from the earth ?*

Distance of a superior planet. By knowing the distance of any planet from the sun, we can determine the distance of any superior planet. For this purpose we use **Kepler's third law of planetary motion.** This law states *the square of the period (T) of revolution of a planet round the sun is proportional to cube of the semi-major axia (a) of the orbit i.e.,*

$$T^2 \propto a^3$$

If T_1 and T_2 are periods of revolution of two planets, and a_1 and a_2 are their respective semi-major axes, then

$$\frac{a_2^3}{a_1^3} = \frac{T_2^2}{T_1^2} \quad \text{or} \quad a_2 = a_1\left(\frac{T_2}{T_1}\right)^{2/3}$$

Knowing the values of a_1, T_1 and T_2, we can find out a_2.

24. *Describe a method to measure the diameter of the moon or any planet.*

Diameter of the moon. Let $AB = D$ be the diameter of the moon (or planet) which is to be measured by an observer O on the earth. A telescope is focussed on the moon and the angle AOB subtended by it on the point O of the earth is found.

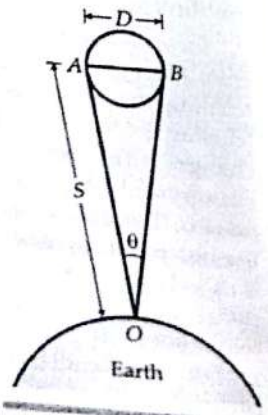

Fig. 2.9 Diameter of the moon.

$$\text{As } \theta = \frac{\text{Arc}}{\text{Radius}} = \frac{AB}{OB} = \frac{D}{S}$$

or $D = S\theta$

Linear diameter = Distance × angular diameter

Knowing S and θ, D can be determined.

25. *Describe a method to measure distance of a hill.*

Reflection or echo method. To find the distance of a hill, a gun is fired towards the hill. The time interval between the instant of firing the gun and the instant of

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hearing the echo of the gun shot is noted. During this time interval, the sound first travels towards the hill and then back from the hill to the place of firing. If v is the speed of sound, S is the distance of the hill and t is the total time taken, then

$$2S = v \times t \quad \text{or} \quad S = \frac{v \times t}{2}$$

26. *Describe a reflection method for measuring the distance of the moon.*

Laser method. The word laser stands for *light amplification by stimulated emission of radiation*. Laser is a source of very intense, highly monochromatic (of one wavelength) and highly directional beam of light. A laser beam is sent towards the moon and its reflected pulse is received. If t is the time elapsed between the instants the laser beam is sent and received back, then the distance of the moon from the earth is given by

$$S = \frac{c \times t}{2}$$

where $c = 3 \times 10^8$ ms^{-1}, is the speed of light.

27. *Briefly describe a reflection method for estimating the distance of a nearby planet from the earth.*

Radar method. The word RADAR stands for *radio detection and ranging*. A radar can be used to measure accurately the distance of a nearby planet (such as Venus). Here radiowaves are sent from a transmitter which after reflection from the planet are detected by the receiver. By measuring the time interval (t) between the instants the radiowaves are sent and received, the distance of the planet can be determined as

$$S = \frac{c \times t}{2}$$

where c is the speed of the radiowave. This method can also be used to determine the distance of an aeroplane.

28. *How is a sonar used in finding the depth of the sea-bed ?*

Sonar method. The word SONAR stands for *sound navigation and ranging*. On a sonar, ultrasonic waves (sound waves having frequencies greater than 20,000 Hz) are transmitted through the ocean. They are reflected by the submerged rock or submarines and received by the receiver. By measuring the time delay t of the receipt of the echo, the distance S of the submerged rock or submarine can be determined as

$$S = \frac{v \times t}{2}$$

where v is the speed of the ultrasonic waves in water.

Examples based on Indirect Methods for Long Distances

FORMULAE USED

1. *Reflection or echo method*

$$S = \frac{c \times t}{2} \quad \text{or} \quad \frac{v \times t}{2}$$

2. *Triangulation method*

(i) Height of an accessible object

$$h = x \tan \theta$$

where x is the distance of observation point from the foot of the object.

(ii) Height of an inaccessible object

$$h = \frac{d}{\cot \theta_2 - \cot \theta_1}$$

where d is the distance between the two observation points.

3. *Parallax method. The distance of an astronomical object*

$$S = \frac{\text{Basis}}{\text{Parallactic angle}} = \frac{b}{\theta}$$

4. Size of an astronomical object,

Linear diameter = Distance × angular diameter

or $D = S \times \theta$

5. *Copernicus method*

(i) The distance of a planet from the sun,

$$r_{ps} = \sin \varepsilon \,.\, r_{es} = \sin \varepsilon \text{ AU}$$

(ii) The distance of a planet from the earth,

$$r_{pe} = \cos \varepsilon \,.\, r_{es} = \cos \varepsilon \text{ AU}$$

6. *From Kepler's law of periods*

$$\frac{a_2^3}{a_1^3} = \frac{T_2^2}{T_1^2} \quad \text{or} \quad a_2 = \left[\frac{T_2}{T_1}\right]^{2/3} . a_1$$

UNITS USED

Distances S, x, d, h, a, b and D are in metre, angle θ in radian, velocities v and c in ms^{-1}

CONVERSIONS USED

$$1'' = 4.85 \times 10^{-6} \text{ rad}$$

EXAMPLE 11. *Calculate the angle of (a) 1° (degree) (b) 1′ (minute of arc or arcmin) and (c) 1″ (second of arc or arc second) in radians. Use 360° = 2π rad, 1° = 60′ and 1′ = 60″.* [NCERT]

Solution. (a) As $360° = 2\pi$ rad

$$\therefore \quad 1° = \frac{2\pi}{360} \text{ rad} = \mathbf{1.745 \times 10^{-2} \text{ rad.}}$$

(b) $1° = 60' = 1.745 \times 10^{-2}$ rad

$$\therefore \quad 1' = 2.908 \times 10^{-4} \approx \mathbf{2.91 \times 10^{-4} \text{ rad}}$$

(c) $1' = 60'' = 2.908 \times 10^{-4}$ rad.

$$\therefore \quad 1'' = 4.847 \times 10^{-4} \text{ rad} \approx \mathbf{4.85 \times 10^{-6} \text{ rad.}}$$

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Example 12. The shadow of a tower standing on a level plane is found to be 50 m longer when sun's altitude is 30° than when it is 60°. Find the height of the tower.

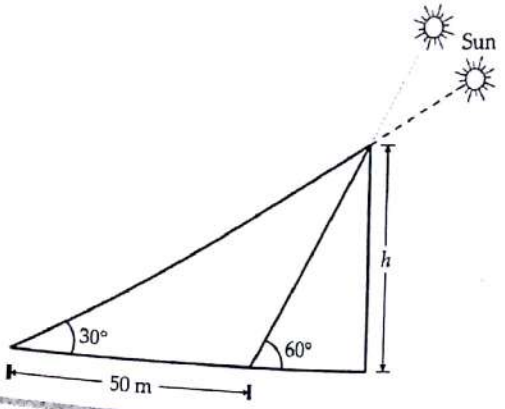

Fig. 2.10

Solution. Here $d = 50$ m, $\theta_1 = 60°$, $\theta_2 = 30°$.

$$h = \frac{d}{\cot\theta_2 - \cot\theta_1} = \frac{50}{\cot 30° - \cot 60°}$$

or
$$h = \frac{50}{\sqrt{3} - 1/\sqrt{3}} = \frac{50\sqrt{3}}{3-1} = 25\sqrt{3}$$

$$= 25 \times 1.732 = \mathbf{43.3\ m.}$$

Example 13. A man wishes to estimate the distance of a nearby tower from him. He stands at a point A in front of the tower C and spots a very distant object O in line with AC. He then walks perpendicular to AC upto B, a distance of 100 m, and looks at O and C again. Since O is very distant, the direction BO is practically the same as AO ; but he finds the line of sight of C shifted from the original line of sight by an angle $\theta = 40°$ (θ is known as 'parallax'). Estimate the distance of the tower C from his original position A. **[NCERT]**

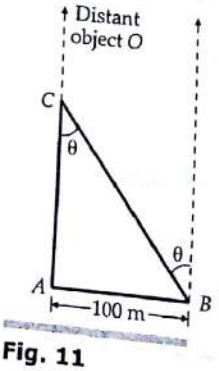

Fig. 11

Solution. Here parallax angle, $\theta = 40°$

Now $\tan\theta = \dfrac{AB}{AC}$

$\therefore$ $$AC = \frac{AB}{\tan\theta} = \frac{100\text{ m}}{\tan 40°} = \frac{100\text{ m}}{0.8391} = \mathbf{119\ m.}$$

Example 14. The moon is observed from two diametrically opposite points A and B on the earth. The angle θ subtended at the moon by the two directions of observation is 1°54'. Given the diameter of the earth to be 1.276×10^7 m, compute the distance of the moon from the earth. **[NCERT]**

Solution. Here parallactic angle,

$\theta = 1°54' = 114' = (144 \times 60)''$

$= 114 \times 60 \times 4.85 \times 10^{-6}$ rad $= 3.32 \times 10^{-2}$ rad

Basis, $b = AB = 1.276 \times 10^7$ m

The distance of the moon from the earth,

$$S = \frac{b}{\theta} = \frac{1.276 \times 10^7}{3.32 \times 10^{-2}} = \mathbf{3.84 \times 10^8\ m.}$$

Example 15. The angular diameter of the sun is 1920". If the distance of the sun from the earth is 1.5×10^{11} m, what is the linear diameter of the sun ? **[NCERT]**

Solution. Distance of the sun from the earth,

$S = 1.5 \times 10^{11}$ m

Angular diameter of the sun,

$\theta = 1920'' = 1920 \times 4.85 \times 10^{-6}$ rad

Linear diameter of the sun,

$D = S \times \theta = 1.5 \times 10^{11} \times 1920 \times 4.85 \times 10^{-6}$

$= \mathbf{1.4 \times 10^9\ m.}$

Example 16. Assuming that the orbit of the planet Mercury around the sun to be a circle, Copernicus determined the orbital radius to be 0.38 AU. From this, determine the angle of the maximum elongation for Mercury and its distance from the earth where the elongation is maximum.

Solution. Here $r_{ps} = 0.38$ AU,

$r_{es} = 1$ AU [By Definition]

The angle of maximum elongation ε for a planet is given by

$$\sin\varepsilon = \frac{r_{ps}}{r_{es}} = \frac{0.38}{1} = 0.38$$

$$\varepsilon = \sin^{-1}(0.38) = \mathbf{22.3°}$$

Distance of mercury from the earth is

$r_{pe} = \cos\varepsilon\ .\ \text{AU} = \cos(22.3°) \times 1.496 \times 10^{11}$ m

$= 0.9252 \times 1.496 \times 10^{11}$ m

$= 1.384 \times 10^{11}$ m $= \mathbf{1.384 \times 10^8\ km.}$

Example 17. In the case of Venus, the angle of maximum elongation is found to be approximately 47°. Determine the distance between Venus and the Sun r_{vs} and the distance between Venus and the Earth.

Solution. Here angle of maximum elongation, $\varepsilon = 47°$

Distance between Venus and the sun is

$r_{vs} = \sin\varepsilon\ .\ \text{AU} = \sin 47° \times 1$ AU

$= 0.73 \times 1.496 \times 10^{11}$ m $= \mathbf{1.09 \times 10^{11}\ m.}$

Distance between Venus and the earth is

$r_{ve} = \cos\varepsilon\ .\ \text{AU} = \cos 47° \times 1$ AU

$= 0.68 \times 1.496 \times 10^{11}$ m $= \mathbf{1.02 \times 10^{11}\ m.}$

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EXAMPLE 18. Suppose there existed a planet that went around the sun twice as fast as the earth. What would be its orbital size as compared to that of the earth ?

Solution. Let the period of revolution of the earth $= T_e$

As the planet goes round the sun twice as fast as the earth, so its period of revolution is

$$T_p = \frac{1}{2} T_e$$

Orbital size of the earth, $a_e = 1$ AU

Orbital size of the planet, $a_p = ?$

From Kepler's law of periods,

$$\frac{T_p^2}{T_e^2} = \frac{a_p^3}{a_e^3}$$

$$\therefore \quad a_p = \left[\frac{T_p}{T_e}\right]^{2/3} \times a_e = \left[\frac{T_e/2}{T_e}\right]^{1/3} \times 1 \text{ AU}$$

$$= (0.5)^{2/3} \text{ AU} = \mathbf{0.63 \text{ AU}}.$$

PROBLEMS FOR PRACTICE

1. In a submarine fitted with a SONAR, the time interval between the generation of an ultrasonic wave and the receipt of its echo is 200 s. What is the distance of the enemy submarine ? The speed of sound in water is 1.450 km s^{-1}. **(Ans.** 145 km)
2. A radar signal is beamed towards a planet and its echo is received 7 minutes later. If the distance between the planet and the earth is 6.3×10^{10} m, calculate the speed of the signal. **(Ans.** 3×10^8 ms^{-1})
3. A rock under water is 1595 m deep. Find the time in which an ultrasonic signal returns after reflection from the rock. Given speed of ultrasonic waves in water = 1450 ms^{-1}. **(Ans.** 2.2 s)
4. The angle of elevation of the top of a hill is 30° from a point on the ground. On walking 1 km towards the hill, angle is found to be 45°. Calculate the height of the hill. **(Ans.** 1.366 km)
5. The moon subtends an angle of 57 minutes at the base-line equal to the radius of the earth. What is the distance of the moon from the earth ? Radius of the earth = 6.4×10^6 m. **(Ans.** 3.86×10^8 **m)**
6. The parallax of a heavenly body measured from two points diametrically opposite on equator of earth is 1.0 minute. If the radius of the earth is 6400 km, find the distance of the heavenly body from the centre of the earth in AU.

   Given 1 AU = 1.5×10^{11} m. **(Ans.** 0.293 AU)
7. Assuming that a planet goes round the sun in a circular orbit of radius 0.5 AU, determine the angle of maximum elongation for the planet and its distance from the earth when elongation is maximum. **(Ans.** 30°, 0.866 AU)
8. Find the period of revolution of planet mars about the sun compared with that of the earth about it. The mean distance of the Mars from the Sun is 1.52 AU. **(Ans.** 1.87 year)

HINTS

1. $S = \frac{v \times t}{2} = \frac{1.450 \times 200}{2} = \mathbf{145 \text{ km}}.$
3. $t = \frac{2S}{v} = \frac{2 \times 1595}{1450} = \mathbf{2.2 \text{ s}}.$
4. $h = \frac{d}{\cot\theta_2 - \cot\theta_1} = \frac{1 \text{ km}}{\cot 30° - \cot 45°} = \mathbf{1.366 \text{ km}}.$
5. Here parallactic angle,

   $$\theta = 57' = \frac{57°}{60} = \frac{57}{60} \times \frac{\pi}{180} \text{ rad}$$

   Basis, b = Radius of earth = 6.4×10^6 m

   Distance of the moon from the earth,

   $$S = \frac{b}{\theta} = \frac{6.4 \times 10^6 \times 60 \times 180}{57 \times \pi} = \mathbf{3.86 \times 10^8 \text{ m}}.$$
6. $\theta = 1.0 \text{ min} = \frac{1°}{60} = \frac{1}{60} \times \frac{\pi}{180} \text{ rad}$

   $$S = \frac{b}{\theta} = \frac{2 \times 6400 \times 180 \times 60}{\pi}$$

   $$= 4.4 \times 10^7 \text{ km} = 4.4 \times 10^{10} \text{ m}$$

   $$= \frac{4.4 \times 10^{10}}{1.5 \times 10^{11}} \text{ AU} = \mathbf{0.293 \text{ AU}}.$$
7. Here $r_{ps} = 0.5$ AU, $r_{es} = 1$ AU

   $$\therefore \quad \sin\varepsilon = \frac{r_{ps}}{r_{es}} = \frac{0.5}{1} = \sin 30°, \quad \therefore \quad \varepsilon = 30°.$$

   Also, $r_{pe} = \cos\varepsilon \,.\, \text{AU} = \cos 30° \text{ AU} = \mathbf{0.866 \text{ AU}}.$
8. For the mars : $a_m = 1.52$ AU

   For the earth : $a_e = 1$ AU, $T_e = 1$ year

   $$T_m = \left[\frac{a_m}{a_e}\right]^{3/2} \times T_e = \left[\frac{1.52}{1}\right]^{3/2} \times 1 = \mathbf{1.87 \text{ year}}.$$

2.16 INDIRECT METHODS FOR MEASURING SMALL DISTANCES

29. *Explain how will you determine the radius of an atom by Avogadro's hypothesis.*

Atomic radius by Avogadro's hypothesis. Atoms are spherical in shape. So when a large number of atoms are packed together, some empty spaces are left between them. According to *Avogadro's hypothesis, the actual volume occupied by the atoms in one gram of a substance is two-third of the volume of one gram of the substance.*

Let M be the molecular mass of a substance. Then M grams of the substance will contain N (Avogadro number) of atoms.

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$\therefore$ Number of atoms in 1 gram $= \frac{N}{M}$

If r be the radius of each atom, then volume of atoms in one gram $= \frac{N}{M} \cdot \frac{4}{3}\pi r^3$.

Let V be the actual volume occupied by molecules in 1 gram of the substance. Then by

Avogadro's hypothesis, $\frac{N}{M} \cdot \frac{4}{3}\pi r^3 = \frac{2}{3} V$

If ρ is the density of the substance, then

$$\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{1}{V} \quad \text{or} \quad V = \frac{1}{\rho}$$

$$\therefore \frac{N}{M} \cdot \frac{4}{3}\pi r^3 = \frac{2}{3} \cdot \frac{1}{\rho} \quad \text{or} \quad r = \left[\frac{M}{2\pi N\rho}\right]^{1/3}$$

Thus, the radius of an atom of the substance can be determined.

30. *Describe a method for measuring the molecular size of oleic acid.*

Size of molecule of oleic acid. Oleic acid is a soapy liquid with large molecular size. We dissolve 1 cm^3 of oleic acid in 20 cm^3 of alcohol and then redissolve 1 cm^3 of this solution in 20 cm^3 of alcohol. Then the concentration of oleic acid is $1/400$ cm^3 in 1 cm^3 of alcohol. We then determine the approximate volume of each drop (V cm^3). Now pour n drops of the solution on the surface of water taken in a broad vessel. We stretch the film carefully. As the alcohol evaporates, a very thin film of oleic acid is left on water surface. We measure the area A of the film using a graph paper.

Volume of n drops of the solution $= nV$ cm^3

Amount of oleic acid in this solution $= \frac{nV}{400}$ cm^3

Thickness of the oil film,

$$t = \frac{\text{Volume of the film}}{\text{Area of the film}} = \frac{nV}{400\ A}\ \text{cm}$$

Assuming that the film has one molecular thickness, then t will be approximately the size or diameter of a molecule of oleic acid. The value of t is found to be of the order of 10^{-9} m.

Table 2.5 *Orders of Magnitude of length*

S. No.	Size of object or distance	Length (m)
1.	Radius of a proton	10^{-15}
2.	Size of atomic nucleus	10^{-14}
3.	Size of hydrogen atom	10^{-10}
4.	Size of typical virus	10^{-8}
5.	Wavelength of light	10^{-7}
6.	Size of red blood corpuscle	10^{-5}
7.	Thickness of a paper	10^{-4}
8.	Height of a person	10^{0}
9.	Height of Mount Everest	10^{4}
10.	Radius of the earth	10^{7}
11.	Distance of the moon from the earth	10^{8}
12.	Radius of the sun	10^{9}
13.	Distance of the sun from the earth	10^{11}
14.	Distance of Pluto from the earth	10^{13}
15.	Size of Milkway	10^{21}
16.	Distance to Andromeda galaxy	10^{22}
17.	Distance to the boundary of observable universe	10^{26}

Examples based on Indirect Methods for Small Distances

Formulae Used

1. Molar volume = Volume of 1 mole of a gas at S.T.P. $= 22.4$ L
2. Volume of a sphere $= \frac{4}{3}\pi r^3$
3. Thickness of an oil film $= \frac{\text{Volume of oil drop}}{\text{Area of the film}}$

Units Used

Volume V is in m^3 or litre, radius r in metre.

Constant Used

Avogadro's number, $N = 6.023 \times 10^{23}$

Conversions Used

$1\ \text{Å} = 10^{-10}$ m, $1\text{L} = 10^{-3}\ m^3$

Example **19.** *The radius of a muonic hydrogen atom is 2.5×10^{-13} m. What is the total atomic volume in m^3 of a mole of such hydrogen atoms ?*

Solution. Radius, $r = 2.5 \times 10^{-13}$ m.

Volume of one atom $= \frac{4}{3}\pi r^3$

Number of atoms in 1 mole $= 6.023 \times 10^{23}$

Volume of 1 mole of H-atoms

$$= N \times \frac{4}{3}\pi r^3$$

$$= 6.023 \times 10^{23} \times \frac{4}{3} \times 3.14 \times (2.5 \times 10^{-13})^3$$

$$= 3.94 \times 10^{-14}\ m^3.$$

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EXAMPLE 20. A drop of olive oil of radius 0.25 mm spreads into a circular film of radius 10 cm on the water surface. Estimate the molecular size of olive oil.

Solution. Thickness of oil film

$$= \frac{\text{Volume of oil drop}}{\text{Area of the film}}$$

$$= \frac{\frac{4}{3}\pi \times (0.025)^3 \text{ cm}^3}{\pi \times (10)^2 \text{ cm}^2} = \frac{4}{3} \times (25)^3 \times 10^{-11} \text{ cm}$$

$$= 2.08 \times 10^{-7} \text{ cm}$$

Assuming that the film has one molecular thickness, then molecular size of olive oil

$$= \mathbf{2.08 \times 10^{-7} \text{ cm}}.$$

✕ PROBLEM FOR PRACTICE

1. A drop of olive oil of radius 0.30 mm spreads into a rectangular film of 30 cm × 15 cm on the water surface. Estimate the molecular size of olive oil.
(**Ans.** 7.5×10^{-7} cm)

Examples based on Magnification of Sizes

FORMULAE USED

1. Linear magnification $= \frac{\text{Final size}}{\text{Initial size}} = \frac{\text{Size of image}}{\text{Size of object}}$
2. Linear magnification $= \sqrt{\text{Areal magnification}}$

UNITS USED

Magnification has no units.

EXAMPLE 21. If the size of a nucleus ($\simeq 10^{-15}$ m) is scaled upto the tip of a sharp pin ($\simeq 10^{-5}$ m), what roughly is the size of the atom ? [NCERT]

Solution. Magnification in size

$$= \frac{\text{Size of the tip of sharp pin}}{\text{Size of nucleus}} = \frac{10^{-5}}{10^{-15}} = 10^{10}$$

Actual size of an atom $= 1 \text{ Å} = 10^{-10}$ m, which is scaled up by a factor 10^{10}.

$\therefore$ Apparent size of atom $= 10^{-10} \times 10^{10} = \mathbf{1 m}$.

Thus a nucleus in an atom is as small in size as the tip of a sharp pin placed at the centre of a sphere of radius about one metre.

Example 22. A 35 mm wide slide with a 24 mm × 36 mm picture is projected on a screen placed 12 cm from the slide. The image of the slide picture on the screen measures 1.0 m × 1.5 m. What is the linear magnification of the projector-screen arrangement ?

Solution. Areal magnification

$$= \frac{1.0 \text{ m} \times 1.5 \text{ m}}{24 \times 10^{-3} \text{ m} \times 36 \times 10^{-3} \text{ m}}$$

$$= 1736$$

Linear magnification $= \sqrt{1736} = \mathbf{41.67}$.

✕ PROBLEMS FOR PRACTICE

1. If the size of an atom ($\simeq 1$ Å) were enlarged to the tip of a sharp pin ($\simeq 10^{-5}$ m), how large would the height of Mount Everest be ? (**Ans.** 10^9 m)
2. If the size of an atom ($\simeq 1$ Å) were enlarged to the size of the earth ($\simeq 10^7$ m), how large would its nucleus be ? Take size of nucleus $= 10^{-15}$ m.
(**Ans.** 100 m)
3. If the universe were shrunk to the size of the earth, how large would the earth be on this scale ?
(**Ans.** 10^{-11} m *i.e.* size of the atom)

✕ HINTS

1. Magnification $= \frac{10^{-5} \text{ m}}{10^{-10} \text{ m}} = 10^5$

Apparent height of Mount Everest
= Actual height × magnification
$= 10^4 \text{ m} \times 10^5 = \mathbf{10^9 \text{ m}}$.

2. Magnification $= \frac{\text{Size of earth}}{\text{Size of atom}} = \frac{10^7 \text{ m}}{10^{-10} \text{ m}} = 10^{17}$

Apparent size of nucleus
= Actual size × magnification
$= 10^{-15} \times 10^{17} = \mathbf{100 \text{ m}}$.

3. Magnification $= \frac{\text{Size of earth}}{\text{Size of universe}} = \frac{10^7 \text{ m}}{10^{25} \text{ m}} = 10^{-18}$

Apparent size of earth $= 10^7 \text{ m} \times 10^{-18} = \mathbf{10^{-11} \text{ m}}$.

2.17 ▼ MASS AND WEIGHT

31. ***Define the terms mass and weight. Give their SI units.***

Mass. *The mass of a body is the quantity of matter contained in it.* It is a basic property of matter. It does not depend on the temperature, pressure or location of the body in space. The SI unit of mass is *kilogram* (kg).

Weight. *The weight of a body is the force with which a body is pulled towards the centre of the earth.* It is equal to the product of the mass (m) of the body and the acceleration due to gravity (g) of the earth on body.

Thus $W = mg$

As the value of 'g' changes from place to place, so the weight of a body is different at different places.

The SI unit of weight is *newton* (N).

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32. *Distinguish between mass and weight.*

Mass	Weight
1. Mass is the measure of inertia.	Weight is the measure of gravity
2. It is a scalar quantity.	It is a vector quantity.
3. It is a constant quantity.	It varies from place to place.
4. It cannot be zero for a body.	Weight of a body is zero at the centre of the earth.
5. It is an essential property of material bodies.	It is not an essential property.
6. It is not affected by the presence of other bodies.	It is affected by the presence of other bodies.
7. Its units are gram, kilogram, etc.	Its units are dyne, newton, etc.

2.18 INERTIAL AND GRAVITATIONAL MASSES

33. *Distinguish between inertial mass and gravitational mass.*

Inertial mass. *The mass of a body which determines its inertia in translatory motion is called its inertial mass.* It is defined by Newton's second law of motion and is equal to the ratio of the external force applied on the body to the acceleration produced in it. By Newton's second law,

$$F = m_i a \qquad \text{or} \qquad m_i = \frac{F}{a}$$

Here m_i is the inertial mass of the body which can be measured by using an inertial balance.

Gravitational mass. *The mass of a body which determines the gravitational pull acting upon it due to the earth is called its gravitational mass.* It is defined by Newton's law of gravitation. According to this law, the force of gravitation of the earth on a body of mass m_g is given by

$$F = \frac{GMm_g}{R^2} \qquad \text{or} \qquad m_g = \frac{FR^2}{GM}$$

Here m_g is the gravitational mass of the body which can be measured by using a physical balance.

34. *Briefly explain how can we measure the inertial mass of a body.*

Measurement of inertial mass. The inertial mass of a body is measured by using a device called *inertial balance*. As shown in Fig. 2.12, it consists of a long strip of metal.

One end of the strip is clamped to a table. The other end of the strip carries a pan in which body whose inertial mass is to be measured is kept. When the strip vibrates horizontally, its inertia comes into play and not the force of gravity of the earth. It is found that *period of vibration T is directly proportional to the square root of the inertial mass m of the body.*

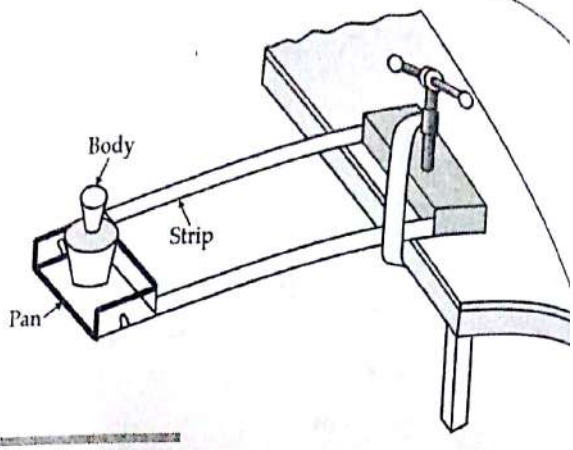

Fig. 2.12 Inertial balance.

Thus $T \propto \sqrt{m}$ or $T^2 \propto m$

Let m_1 and m_2 be the inertial masses of two objec and T_1 and T_2 be their corresponding periods vibration, then

$$\frac{m_2}{m_1} = \frac{T_2^2}{T_1^2} \qquad \text{or} \qquad m_2 = m_1 \cdot \frac{T_2^2}{T_1^2}$$

If m_1 is a standard mass, then unknown mass m_2 ca be determined.

35. *How is gravitational mass of a body measured b a common (or physical) balance ?*

Gravitational mass by a physical balance. A physical balance works on the *principle of moments* i.e., when an object is balanced under the action of several forces acting in the same plane, the sum of the clockwise moments is equal to the sum of anticlockwise moments. The essential parts of a physical balance are shown in Fig. 2.13.

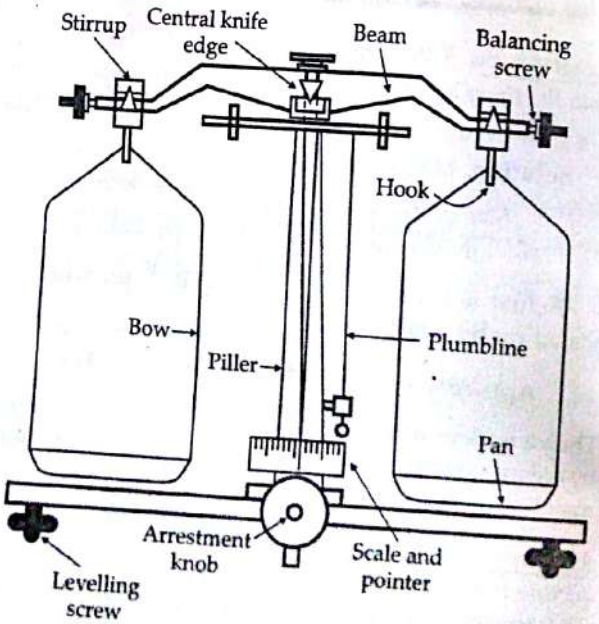

Fig. 2.13 Physical balance.

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The object to be weighed is placed in the left pan and the standard weights are placed in the right pan. The weights are adjusted till the beam becomes horizontal. In this condition, the gravitational force on the object is equal to the gravitational force on standard weights. Hence the gravitational mass of the object is equal to that of the standard weights (cancelling the effect of g).

36. *How can a spring balance be used to measure the gravitational mass of a body ?*

Gravitational mass by a spring balance. In a spring balance, the gravitational force on a body stretches the spring and we measure the elongation of the spring. The elongation depends on gravitational force which, in turn, is proportional to the gravitational mass of the body. Thus the elongation of the spring gives a measure of the gravitational mass. For this we first calibrate the spring balance by using standard masses and measuring the elongations. The calibration of spring balance and the mass measurement should be done at same place because the value of g varies from place to place.

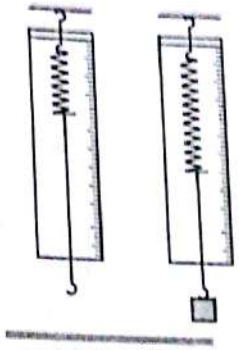

Fig. 2.14 Spring balance.

Table 2.6 Orders of magnitude of mass

S.No.	Object	Mass (kg)
1.	Electron	10^{-30}
2.	Proton	10^{-27}
3.	Uranium atom	10^{-25}
4.	Cell	10^{-10}
5.	Dust particle	10^{-9}
6.	Rain drop	10^{-6}
7.	Mosquito	10^{-5}
8.	Grape	10^{-3}
9.	Man	10^{2}
10.	Automobile	10^{3}
11.	Ship	10^{5}
12.	Moon	10^{23}
13.	Earth	10^{25}
14.	Milkyway	10^{41}
15.	Observable universe	10^{55}

Examples based on Mass Density

Formulae Used

1. Density $= \dfrac{\text{Mass}}{\text{Volume}}$ or $\rho = \dfrac{M}{V}$
2. Volume of a sphere, $V = \dfrac{4}{3}\pi r^3$

Units Used

Radius r is in metre, volume V in m^3 and density ρ in kg m^{-3}.

Example 23. Consider a white dwarf and a neutron star each of one solar mass. The radius of the white dwarf is same as that of the earth (= 6400 km) and the radius of the neutron star is 10 km. Determine the densities of the two types of the stars. Take mass of the sun = 2.0×10^{30} kg.

Solution. *For white dwarf :* $M = 2 \times 10^{30}$ kg, $r = 6400\text{ km} = 64 \times 10^5$ m.

Density of white dwarf

$$= \frac{M}{\frac{4}{3}\pi r^3} = \frac{2 \times 10^{30}}{\frac{4}{3}\pi \times (64 \times 10^5)^3} = 1.822 \times 10^9 \text{ kg m}^{-3}$$

For neutron star : $M = 2 \times 10^{30}$ kg, $r = 10\text{ km} = 10^4$ m

Density of neutron star

$$= \frac{2 \times 10^{30}}{\frac{4}{3}\pi \times (10^4)^3} = 4.77 \times 10^{17} \text{ kgm}^{-3}.$$

Example 24. Assume that the mass of a nucleus is given by $M = A\,m_p$, where A is the mass number and radius of a nucleus $r = r_0\,A^{1/3}$, where $r_0 = 1.2\ f$. Estimate the density of nuclear matter in kg m^{-3}. Given $m_p = 1.67 \times 10^{-27}$ kg.

Solution. Here $m_p = 1.67 \times 10^{-27}$ kg,

$$r_0 = 1.2\ f = 1.2 \times 10^{-15} \text{ m}$$

$$\text{Nuclear density} = \frac{Am_p}{\frac{4}{3}\pi r^3} = \frac{Am_p}{\frac{4}{3}\pi (r_0 A^{1/3})^3} = \frac{3m_p}{4\pi r_0^3}$$

$$= \frac{3 \times 1.67 \times 10^{-27}}{4 \times 3.14 \times (1.2 \times 10^{-15})^3} \text{ kg m}^{-3}$$

$$= 2.3 \times 10^{17} \text{ kg m}^{-3}.$$

Problem For Practice

1. A neutron star has a density equal to that of nuclear matter ($= 2.8 \times 10^{17}$ kg m^{-3}). Assume the star to be spherical, find the radius of the neutron star whose mass is 4.0×10^{10} kg (twice the mass of the sun).

(Ans. 15 km)

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2.19 MEASUREMENT OF TIME

37. *What type of phenomena can be used as a time standard ? Give some examples. Which one do you select as the most suitable standard for measuring time ?*

Measurement of time. According to Einstein, *"Time is simply what a clock reads. Any phenomenon that repeats itself after equal intervals of time can be used as a time standard"*. Examples of such a phenomenon are

1. Beating of human heart.
2. Oscillations of a pendulum.
3. Rotation of the earth about its own-axis.
4. Revolution of the earth around the sun.
5. Vibrations of a quartz crystal in quartz wristwatch.
6. Period of vibration of cesium – 133 atom.

Of the above examples, the period of vibration of cesium – 133 atom serves as the most accurate standard of time.

38. *What are the advantages of defining second in terms of period of radiation from cesium – 133 atom ?*

Advantages of defining second in terms of period of radiation from cesium – 133 atom :

1. Cesium atomic clock has high accuracy of 1 part in 10^{13}.
2. It can be easily reproduced in any good laboratory.
3. It is not affected by physical conditions of pressure, temperature, etc.
4. It is imperishable.

39. *Give a brief account of the various techniques used for measuring small and large time intervals.*

Some techniques used for measuring time intervals over different ranges :

1. ***Electrical oscillators.*** In India, a.c. is supplied at a frequency of 50 Hz. A motor running synchronously with this a.c. can be used to provide a time scale.

2. ***Electronic oscillators.*** A junction transistor can be used to produce oscillations of very high frequency which can be used to measure small time intervals.

3. ***Quartz clocks.*** A quartz crystal shows piezo-electric effect. If fluctuating pressure is applied across a pair of its parallel faces, an oscillatory emf is developed across another pair of perpendicular faces and vice versa. The oscillations produced can be used to measure time intervals.

4. ***Atomic clocks.*** These clocks are based on periodic vibrations taking place within the atoms. The first cesium atomic clock was set up in 1964.

5. ***Decay of elementary particles.*** The life of many elementary particles varies from 10^{-16} to 10^{-24} s. By making use of their decay times, very small time intervals can be measured.

6. ***Radioactive dating.*** This technique is used to measure long time intervals by finding the ratio of the number of radioactive atoms that have undergone decay to the number of atoms left undecayed. Carbon dating is used to estimate the age of fossils, whereas uranium dating is used to estimate the age of the rocks.

Table 2.7 *Orders of magnitude of time intervals*

S. No.	Event	Time (s)
1.	Life of most unstable particle	10^{-24}
2.	Time taken by light to cross a nuclear distance	10^{-22}
3.	Time period of electron in hydrogen atom	10^{-16}
4.	Period of visible light waves	10^{-15}
5.	Life time of an excited state of atom	10^{-8}
6.	Period of radio wave	10^{-6}
7.	Period of sound wave	10^{-3}
8.	Wink of eye	10^{-1}
9.	Time between two successive heart beats	10^{0}
10.	Travel time for light from moon to earth	10^{0}
11.	Travel time for light from sun to earth	10^{2}
12.	Time period of a satellite	10^{4}
13.	Period of rotation of the earth (one day)	10^{5}
14.	Period of revolution of the earth (1 year)	10^{7}
15.	Travel time for light from nearest star	10^{8}
16.	Average human life-span	10^{9}
17.	Age of the universe	10^{17}

For Your Knowledge

- In India, the National Physical Laboratory (New Delhi) has the responsibility of maintenance and improvement of physical standards of length, mass, time, etc.
- The iodine stabilized helium neon laser has been used to realize the latest definition of standard metre.
- Italian physicist Galileo was the first to set up pendulum clocks.
- The cesium atomic clocks have an high accuracy of 1 part in 10^{13}. They lose or gain not more than 3 μs in one year.
- Table 2.5 shows that the ratio of the largest and shortest lengths of objects in the universe is about 10^{41}.
- Table 2.7 shows that the ratio of the largest and the shortest time intervals associated with the events in the universe is also about 10^{41}.
- Table 2.6 shows that the ratio of the largest and the smallest masses of the objects in the universe is about $(10^{41})^2$.

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Examples based on Measurement of Time

FORMULAE USED

$$\text{Fractional error in time} = \frac{\text{Difference in time}}{\text{Time internal}} = \frac{\Delta t}{t}$$

UNITS USED

Times t and Δt are in second.

EXAMPLE 25. The average life of an Indian is 56 years. Find the number of times the human heart beats in the life of an Indian, if the heart beats once in 0.8 s.

Solution. Average life of an Indian = 56 years

$$= 56 \times 365.25 \times 24 \times 60 \times 60 \text{ s}$$

Period of heart beat = 0.8 s

Total number of heart beats in 56 years

$$= \frac{56 \times 365.25 \times 24 \times 60 \times 60}{0.8} = 2.2 \times 10^9 \text{ times.}$$

EXAMPLE 26. The mean life of an elementary particle pion is 2×10^{-7} nanosecond. The age of the universe is about 4×10^9 years. Identify a physically meaning time interval that is approximately half way between these two on a logarithmic scale.

Solution. Mean life of pion,

$$t_1 = 2 \times 10^{-7} \text{ ns} = 2 \times 10^{-7} \times 10^{-9} \text{ s}$$
$$= 2 \times 10^{-16} \text{ s} \simeq 10^{-16} \text{ s}$$

Mean life of universe,

$$t_2 = 4 \times 10^9 \text{ years} = 4 \times 10^9 \times 3 \times 10^7 \text{ s}$$
$$= 1.2 \times 10^{17} \text{ s} \simeq 10^{17} \text{ s}$$

Let t be the time interval that is half way between t_1 and t_2 on the logarithmic scale. Then

$$\log t = \frac{1}{2}[\log t_1 + \log t_2]$$
$$= \frac{1}{2}[\log 10^{-16} + \log 10^{17}]$$
$$= \frac{1}{2}[-16 + 17] = 0.5 \simeq 1$$

$$\therefore \quad t = 10 \text{ s}$$

which is the time taken by an athlete to run a 100 m track.

PROBLEMS FOR PRACTICE

1. Find the number of seconds in 1 year. Express them in order of magnitude. (**Ans.** 3.158×10^7 s, 10^7)
2. Find the number of times the human heart beats in the life of 60 years of a man, assuming that the heart beats once in 0.8 s. (**Ans.** 2.3668×10^9 s)
3. Two atomic clocks allowed to run for average life of an Indian (say, 70 years) differ by 0.2 s only. Calculate the accuracy of standard atomic clock in measuring a time interval of 1s. (**Ans.** 1 s in 10^{10} s)
4. Age of the universe is about 10^{10} years whereas the mankind has existed for 10^6 years. For how many seconds would the man have existed if age of universe were 1 day ? (**Ans.** 8.64 s)

HINTS

3. Here t = 70 years = $70 \times 365 + 17$

   [17 days added due to 17 leap years in 70 years]

   or $t = 25567 \text{ days} = 25567 \times 86400 \text{ s}$

   $\Delta t = 0.2 \text{ s}$

   Fractional error in time

   $$= \frac{\Delta t}{t} = \frac{0.2}{25567 \times 86400} = 0.904 \times 10^{-10} \simeq 10^{-10}.$$

   So the accuracy shown by atomic clock is 10^{-10} part in 1 s or 1 s in 10^{10} s.

4. Magnification in time

   $$= \frac{\text{Age of mankind}}{\text{Age of universe}} = \frac{10^6}{10^{10}} = 10^{-4}$$

   Apparent age of mankind

   $= 10^{-4} \times 1 \text{ day} = 10^{-4} \times 86400 \text{ s} = \mathbf{8.64 \text{ s}}$.

2.20 DIMENSIONS OF A PHYSICAL QUANTITY

40. *What do you mean by seven dimensions of the world ?*

Seven dimensions of the world. All the derived physical quantities can be expressed in terms of some combination of the seven fundamental or base quantities. We call these fundamental quantities as the *seven dimensions of the world*, which are denoted with square brackets [ ].

Dimension of length = [L]
Dimension of mass = [M]
Dimension of time = [T]
Dimension of electric current = [A]
Dimension of thermodynamic temperature = [K]
Dimension of luminous intensity = [cd]
Dimension of amount of substance = [mol]

41. *What do you mean by dimensions of a physical quantity ? Explain with the help of an example.*

Dimensions of a physical quantity. *The dimensions of a physical quantity are the powers (or exponents) to which the fundamental quantities must be raised to represent that quantity completely.*

For example,

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{\text{Mass}}{\text{Length} \times \text{breadth} \times \text{height}}$$

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∴ Dimensions of density

$$= \frac{[M]}{[L][L][L]} = [ML^{-3}] = [M^1L^{-3}T^0]$$

Hence the dimensions of density are '1' in mass, '–3' in length and '0' in time.

2.21 DIMENSIONAL FORMULAE AND DIMENSIONAL EQUATIONS

42. *What is meant by dimensional formula and dimensional equation ? Give examples.*

Dimensional formula. *The expression which shows how and which of the fundamental quantities represent the dimensions of a physical quantity is called the dimensional formula of the given physical quantity.*

Examples : The dimensional formula of the volume is $[M^0L^3T^0]$ and that of momentum is $[MLT^{-1}]$

Dimensional equation. *The equation obtained by equating a physical quantity with its dimensional formula is called the dimensional equation of the given physical quantity.*

Examples : The dimensional equation of force is

$$[\text{Force}] = [MLT^{-2}]$$

The dimensional equation for pressure is

$$[\text{Pressure}] = [ML^{-1}T^{-2}]$$

Dimensional formuale and SI units of some physical quantities are given in table 2.8.

Table 2.8 Dimensional Formulae and SI units of some physical quantities

S. No.	Physical Quantity	Relation with other quantities	Dimensional formula	SI unit
		A. Mechanical Quantities		
1.	Area	Length × breadth	$L \times L = L^2 = [M^0L^2T^0]$	m^2
2.	Volume	Length × breadth × height	$L \times L \times L = [M^0L^3T^0]$	m^3
3.	Density	$\frac{\text{Mass}}{\text{Volume}}$	$\frac{M}{L^3} = [ML^{-3}T^0]$	$kg\ m^{-3}$
4.	Speed or velocity	$\frac{\text{Distance}}{\text{Time}}$	$\frac{L}{T} = [M^0LT^{-1}]$	ms^{-1}
5.	Acceleration	$\frac{\text{Change in velocity}}{\text{Time}}$	$\frac{LT^{-1}}{T} = LT^{-2} = [M^0LT^{-2}]$	ms^{-2}
6.	Momentum	Mass × velocity	$M \times LT^{-1} = [MLT^{-1}]$	$kg\ ms^{-1}$
7.	Force	Mass × acceleration	$M \times LT^{-2} = [MLT^{-2}]$	N
8.	Work	Force × distance	$MLT^{-2} \times L = [ML^2T^{-2}]$	J
9.	Energy	Amount of work	$[ML^2T^{-2}]$	J
10.	Power	$\frac{\text{Work}}{\text{Time}}$	$\frac{ML^2T^{-2}}{T} = [ML^2T^{-3}]$	W
11.	Pressure	$\frac{\text{Force}}{\text{Area}}$	$\frac{ML^1T^{-2}}{L^2} = [ML^{-1}T^{-2}]$	Pa or Nm^{-2}
12.	Moment of force or torque	Force × $\perp_r$ distance	$MLT^{-2} \times L = [ML^2T^{-2}]$	Nm
13.	Gravitational constant 'G'	$\frac{\text{Force} \times (\text{distance})^2}{\text{Mass} \times \text{mass}}$	$\frac{MLT^{-2}L^2}{M \times M} = [M^{-1}L^3T^{-2}]$	Nm^2kg^{-2}
14.	Impulse of a force	Force × time	$MLT^{-2} \times T = [MLT^{-1}]$	Ns
15.	Stress	$\frac{\text{Force}}{\text{Area}}$	$\frac{MLT^{-2}}{L^2} = [ML^{-1}T^{-2}]$	Nm^{-2}
16.	Strain	$\frac{\text{Change in dimension}}{\text{Original dimension}}$	$[M^0L^0T^0]$ (dimensionless)	–
17.	Coefficient of elasticity	$\frac{\text{Stress}}{\text{Strain}}$	$\frac{ML^{-1}T^{-2}}{1} = [ML^{-1}T^{-2}]$	Nm^{-2}

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S. No.	Physical Quantity	Relation with other quantities	Dimensional formula	SI unit
18.	Surface tension	$\frac{\text{Force}}{\text{Length}}$	$\frac{MLT^{-2}}{L} = MT^{-2} = [ML^0T^{-2}]$	Nm^{-1}
19.	Surface energy	$\frac{\text{Work}}{\text{Area}}$	$\frac{ML^2T^{-2}}{L^2} = MT^{-2} = [ML^0T^{-2}]$	Jm^{-2}
20.	Coefficient of viscosity	$\frac{\text{Force} \times \text{distance}}{\text{Area} \times \text{velocity}}$	$\frac{MLT^{-2} \times L}{L^2 \times LT^{-1}} = [ML^{-1}T^{-1}]$	Nm^{-2} s or Pa s
21.	Angle	$\frac{\text{Arc}}{\text{Radius}}$	$\frac{L}{L} = 1 = [M^0L^0T^0]$ (dimensionless)	rad
22.	Angular velocity	$\frac{\text{Angle}}{\text{Time}}$	$\frac{1}{T} = T^{-1} = [M^0L^0T^{-1}]$	rad s^{-1}
23.	Angular acceleration	$\frac{\text{Angular velocity}}{\text{Time}}$	$\frac{T^{-1}}{T} = T^{-2} = [M^0L^0T^{-2}]$	rad s^{-2}
24.	Moment of inertia	Mass × (distance)^2	$ML^2 = [ML^2T^0]$	kg m^2
25.	Radius of gyration	Distance	$L = [M^0LT^0]$	m
26.	Angular momentum	Mass × velocity × radius	$M \times LT^{-1} \times L = [ML^2T^{-1}]$	kg m^2s^{-1}
27.	T-ratios ($\sin\theta$, $\cos\theta$, $\tan\theta$)	$\frac{\text{Length}}{\text{Length}}$	$\frac{L}{L} = 1 = [M^0L^0T^0]$ (dimensionless)	—
28.	Time period	Time	$T = [M^0L^0T]$	s
29.	Frequency	$\frac{1}{\text{Time period}}$	$\frac{1}{T} = T^{-1} = [M^0L^0T^{-1}]$	s^{-1} or Hz
30.	Planck's constant 'h'	$\frac{E}{\nu} = \frac{\text{Energy}}{\text{Frequency}}$	$\frac{ML^2T^{-2}}{T^{-1}} = [ML^2T^{-1}]$	Js
31.	Relative density	$\frac{\text{Density of substance}}{\text{Density of water at 4°C}}$	$\frac{ML^{-3}}{ML^{-3}} = 1 = [M^0L^0T^0]$ (dimensionless)	—
32.	Velocity gradient	$\frac{\text{Velocity}}{\text{Distance}}$	$\frac{LT^{-1}}{L} = T^{-1} = [M^0L^0T^{-1}]$	s^{-1}
33.	Pressure gradient	$\frac{\text{Pressure}}{\text{Distance}}$	$\frac{ML^{-1}T^{-2}}{L} = [ML^{-2}T^{-2}]$	Pa m^{-1}
34.	Force constant	$\frac{\text{Force}}{\text{Displacement}}$	$\frac{MLT^{-2}}{L} = MT^{-2} = [ML^0T^{-2}]$	Nm^{-1}
	B. Thermal Quantities			
35.	Heat or enthalpy	Energy	$[ML^2T^{-2}]$	J
36.	Specific heat	$\frac{\text{Heat}}{\text{Mass} \times \text{Temperature}}$	$\frac{ML^2T^{-2}}{M \cdot K} = [M^0L^2T^{-2}K^{-1}]$	J kg^{-1} K^{-1}
37.	Latent heat	$\frac{\text{Heat}}{\text{Mass}}$	$\frac{ML^2T^{-2}}{M} = [M^0L^2T^{-2}]$	J kg^{-1}
38.	Thermal conductivity	$\frac{\text{Heat} \times \text{distance}}{\text{Area} \times \text{temp.} \times \text{time}}$	$\frac{ML^2T^{-2} \cdot L}{L^2 \cdot K \cdot T} = [MLT^{-3}K^{-1}]$	$Js^{-1}m^{-1}K^{-1}$
39.	Entropy	$\frac{\text{Heat}}{\text{Temperature}}$	$\frac{ML^2T^{-2}}{K} = [ML^2T^{-2}K^{-1}]$	JK^{-1}
40.	Universal gas constant	$\frac{PV}{nT}$	$\frac{ML^{-1}T^{-2}L^3}{\text{mol} \cdot K} = [ML^2T^{-2}K^{-1}\ \text{mol}^{-1}]$	J mol^{-1} K^{-1}

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S. No.	Physical Quantity	Relation with other quantities	Dimensional formula	SI unit
41.	Boltzmann's constant	$\frac{\text{Energy}}{\text{Temperature}}$	$\frac{ML^2T^{-2}}{K} = [ML^2T^{-2}K^{-1}]$	JK^{-1}
42.	Stefan's constant	$\frac{\text{Energy}}{\text{Area} \times \text{time} \times (\text{temp.})^4}$	$\frac{ML^2T^{-2}}{L^2 \cdot T \cdot K^4} = [ML^0T^{-3}K^{-4}]$	$Js^{-1}m^{-2}K^{-4}$
43.	Solar constant	$\frac{\text{Energy}}{\text{Area} \times \text{time}}$	$\frac{ML^2T^{-2}}{L^2 \cdot T} = [ML^0T^{-3}]$	$Js^{-1}m^{-2}$
44.	Mechanical equivalent of heat	$J = \frac{W}{H}$	$\frac{ML^2T^{-2}}{ML^2T^{-2}} = [M^0L^0T^0]$ (dimensionless)	—
	C. Electrical Quantities			
45.	Electric charge	Time × Current	T. A = $[M^0L^0TA]$	C (coulomb)
46.	Electrical potential	$\frac{\text{Work}}{\text{Charge}}$	$\frac{ML^2T^{-2}}{TA} = [ML^2T^{-3}A^{-1}]$	V (volt)
47.	Resistance	$\frac{\text{Potential difference}}{\text{Current}}$	$\frac{ML^2T^{-3}A^{-1}}{A} = [ML^2T^{-3}A^{-2}]$	Ω (ohm)
48.	Capacitance	$\frac{\text{Charge}}{\text{Potential difference}}$	$\frac{TA}{ML^2T^{-3}A^{-1}} = [M^{-1}L^{-2}T^4A^2]$	F (farad)
49.	Inductance	$\frac{\text{EMF}}{\text{Current / time}}$	$\frac{ML^2T^{-3}A^{-1}}{AT^{-1}} = [ML^2T^{-2}A^{-2}]$	H (henry)
50.	Permittivity of free space	$\varepsilon_0 = \frac{q_1 q_2}{Fr^2}$	$\frac{AT \cdot AT}{MLT^{-2}L^2} = [M^{-1}L^{-3}T^4A^2]$	$A^2C^2N^{-1}m^{-2}$
51.	Relative permittivity or dielectric constant	ε_r or $\kappa = \frac{\varepsilon_0}{\varepsilon}$	a pure ratio = $[M^0L^0T^0]$ (dimensionless)	—
52.	Intensity of electric field	$E = \frac{F}{q} = \frac{\text{Force}}{\text{Charge}}$	$\frac{MLT^{-2}}{AT} = [MLT^{-3}A^{-1}]$	NC^{-1} or Vm^{-1}
53.	Conductance	$C = \frac{1}{R}$	$\frac{1}{ML^2T^{-3}A^{-2}} = [M^{-1}L^{-2}T^3A^2]$	Ω^{-1} or mho
54.	Specific resistance or resistivity	$\rho = \frac{RA}{l}$	$\frac{ML^2T^{-3}A^{-2} \cdot L^2}{L} = [ML^3T^{-3}A^{-2}]$	Ωm
55.	Specific conductance or conductivity	$\sigma = \frac{1}{\rho}$	$[M^{-1}L^{-3}T^3A^2]$	$\Omega^{-1}m^{-1}$
56.	Electric dipole moment	$q \times 2l$	$AT \cdot L = [M^0LTA]$	Cm
	D. Magnetic Quantities			
57.	Magnetic field	$B = \frac{F}{qv \sin 0}$	$\frac{MLT^{-2}}{AT \cdot LT^{-1} \cdot 1} = [ML^0T^{-2}A^{-1}]$	T (tesla)
58.	Magnetic flux	$\phi = BA$	$MT^{-2}A^{-1} \cdot L^2 = [ML^2T^{-2}A^{-1}]$	Wb (weber)
59.	Permeability of free space	$\mu_0 = \frac{4\pi r \cdot F}{I_1 I_2 l}$	$\frac{L \cdot MLT^{-2}}{A^2 \cdot L} = [MLT^{-2}A^{-2}]$	
60.	Magnetic moment	Current × area	$A \cdot L^2 = [M^0L^2T^0A]$	Am^2
61.	Pole strength	$\frac{\text{Magnetic moment}}{\text{Magnetic length}}$	$\frac{AL^2}{L} = [M^0LT^0A]$	Am

Examples ba

Derivation of Dimens

Concept Used

The dimensional formul can be obtained by defin quantities, whose dime known.

Example 27. Name the physic formulae are as follows :

(i) ML^2T^{-2} (ii)

(iv) $ML^{-1}T^{-1}$ (v)

Solution. (i) ML^2T

(ii) $ML^2T^{-3} = \frac{ML^2}{}$

(iii) $MT^{-2} = \frac{M}{}$

$= S$

(iv) $ML^{-1}T^{-1} =$

(v) $ML^{-1}T$

Example 28. following phys

(i) Gravit

(ii) Powe

(iv) Coef

(vi) Plan

Soluti

$\therefore$

(ii

$\therefore$

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Examples based on Derivation of Dimensional Formulae

CONCEPT USED

The dimensional formula of a physical quantity can be obtained by defining its relation with other quantities, whose dimensions in M, L and T are known.

EXAMPLE 27. Name the physical quantities whose dimensional formulae are as follows :

(i) ML^2T^{-2} *(ii)* ML^2T^{-3} *(iii)* MT^{-2}
(iv) $ML^{-1}T^{-1}$ *(v)* $ML^{-1}T^{-2}$.

Solution. (i) $ML^2T^{-2} = MLT^{-2}.L$

= Force × distance = Work.

(ii) $ML^2T^{-3} = \frac{ML^2T^{-2}}{T} = \frac{\text{Work}}{\text{Time}}$ = Power.

(iii) $MT^{-2} = \frac{MLT^{-2}}{L} = \frac{\text{Force}}{\text{Length}}$

= Surface tension or force constant.

(iv) $ML^{-1}T^{-1} = \frac{MLT^{-2}}{L^2T^{-1}} = \frac{\text{Force}}{\text{Area} \times \text{velocity gradient}}$

= Coefficient of viscosity.

(v) $ML^{-1}T^{-2} = \frac{MLT^{-2}}{L^2} = \frac{\text{Force}}{\text{Area}}$

= Pressure or stress.

EXAMPLE 28. Deduce the dimensional formulae for the following physical quantities :

(i) Gravitational constant **[Delhi 13]**
(ii) Power *(iii) Young's modulus*
(iv) Coefficient of viscosity *(v) Surface tension*
(vi) Planck's constant.

Solution. (i) According to Newton's law of gravitation,

$$F = G\frac{m_1 m_2}{r^2}$$

$$\therefore \quad [G] = \frac{[F][r^2]}{[m_1][m_2]} = \frac{MLT^{-2}.L^2}{MM} = [M^{-1}L^3T^{-2}].$$

(ii) $\text{Power} = \frac{\text{Work}}{\text{Time}} = \frac{\text{Force} \times \text{Distance}}{\text{Time}}$

$$\therefore \quad [\text{Power}] = \frac{MLT^{-2}.L}{T} = [ML^2T^{-3}].$$

(iii) Young's modulus,

$$Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}} = \frac{F/A}{\Delta l/l} = \frac{F}{A}.\frac{l}{\Delta l}$$

$$\therefore \quad [Y] = \frac{MLT^{-2}.L}{L^2.L} = [ML^{-1}T^{-2}].$$

(iv) Coefficient of viscosity,

$$\eta = \frac{\text{Force}}{\text{Area} \times \text{velocity gradient}} = \frac{\text{Force}}{\text{Area}} \times \frac{\text{Distance}}{\text{Velocity}}$$

$$[\eta] = \frac{MLT^{-2}.L}{L^2.LT^{-1}} = [ML^{-1}T^{-1}].$$

(v) Surface tension,

$$\sigma = \frac{\text{Force}}{\text{Length}}$$

$$[\sigma] = \frac{MLT^{-2}}{L} = [MT^{-2}].$$

(vi) Planck's constant,

$$h = \frac{\text{Energy}}{\text{Frequency}}$$

$$[h] = \frac{ML^2T^{-2}}{T^{-1}} = [ML^2T^{-1}].$$

EXAMPLE 29. Deduce the dimensional formulae of the following physical quantities :

(i) Heat *(ii) Specific heat* *(iii) Latent heat*
(iv) Gas constant
(v) Boltzmann's constant
(vi) Coefficient of thermal conductivity
(vii) Mechanical equivalent of heat.

Solution. (i) Heat = Energy

$$\therefore \quad [\text{Heat}] = [ML^2T^{-2}].$$

(ii) $\text{Specific heat} = \frac{\text{Heat}}{\text{Mass} \times \text{temperature}}$

$$\therefore \quad [\text{Specific heat}] = \frac{ML^2T^{-2}}{M.K} = [L^2T^{-2}K^{-1}].$$

(iii) $\text{Latent heat} = \frac{\text{Heat}}{\text{Mass}}$

$$\therefore \quad [\text{Latent heat}] = \frac{ML^2T^{-2}}{M} = [L^2T^{-2}].$$

(iv) $PV = nRT$

or $R = \frac{PV}{nT} = \frac{\text{Force} \times \text{volume}}{\text{mol} \times \text{area} \times \text{temp.}}$

$$\therefore \quad [R] = \frac{MLT^{-2}.L^3}{\text{mol}.L^2.K} = [ML^2T^{-2}K^{-1}\text{mol}^{-1}].$$

(v) Boltzmann's constant,

$$k = \frac{\text{Heat}}{\text{Temperature}}$$

$$[k] = \frac{ML^2T^{-2}}{K} = [ML^2T^{-2}K^{-1}].$$

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(vi) Coefficient of thermal conductivity,

$$x = \frac{\text{Heat} \times \text{distance}}{\text{Area} \times \text{time} \times \text{temperature difference}}$$

$$[x] = \frac{ML^2T^{-2}.L}{L^2.T.K} = [MLT^{-3}K^{-1}]$$

(vii) Joule's mechanical equivalent of heat,

$$J = \frac{\text{Work}}{\text{Heat}}$$

$$[J] = \frac{ML^2T^{-2}}{ML^2T^{-2}} = 1 = [M^0L^0T^0].$$

EXAMPLE 30. Find the dimensional formulae of (i) charge (ii) potential (iii) resistance (iv) capacitance.

Solution. (i) Charge, q = Current × time

$$\therefore \quad [q] = [AT].$$

(ii) Potential,

$$V = \frac{\text{Work}}{\text{Charge}}$$

$$[V] = \frac{ML^2T^{-2}}{AT} = [ML^2A^{-1}T^{-3}]$$

(iii) Resistance,

$$R = \frac{\text{Potential difference}}{\text{Current}}$$

$$= \frac{ML^2A^{-1}T^{-3}}{A} = [ML^2A^{-2}T^{-3}].$$

(iv) Capacitance,

$$C = \frac{\text{Charge}}{\text{Potential}}$$

$$[C] = \frac{AT}{ML^2A^{-1}T^{-3}} = [M^{-1}L^{-2}A^2T^4]$$

EXAMPLE 31. Taking velocity, time and force as the fundamental quantities, find the dimensions of mass.

Solution. Force = Mass × Acceleration

$$= \text{Mass} \times \frac{\text{Velocity}}{\text{Time}}$$

or

$$\text{Mass} = \frac{\text{Force} \times \text{Time}}{\text{Velocity}}$$

$$\therefore \quad [\text{Mass}] = [FTV^{-1}]$$

EXAMPLE 32. If density ρ, acceleration due to gravity g and frequency ν are the basic quantities, find the dimensions of force.

Solution. We have $\rho = ML^{-3}$, $g = LT^{-2}$, $\nu = T^{-1}$

Solving for M, L and T in terms of ρ, g and ν, we get

$$M = \rho g^3 \nu^{-6}, \quad L = g\nu^{-2}, \quad T = \nu^{-1}$$

$$\therefore \quad [\text{Force}] = MLT^{-2} = \rho g^3 \nu^{-6} . g\nu^{-2} . \nu^2$$

$$= [\rho g^4 \nu^{-6}].$$

EXAMPLE 33. If the velocity of light c, acceleration due to gravity g and atmospheric pressure p are the fundamental quantities, find the dimensions of length.

Solution. We have,

$$c = LT^{-1}, \quad g = LT^{-2}, \quad p = ML^{-1}T^{-2}$$

$$[\text{Length}] = L = \frac{L^2T^{-2}}{LT^{-2}} = \frac{(LT^{-1})^2}{LT^{-2}} = \left[\frac{c^2}{g}\right].$$

EXAMPLE 34. The number of particles crossing a unit area perpendicular to X-axis in unit time is given by

$$n = -D\frac{n_2 - n_1}{x_2 - x_1}$$

where n_1 and n_2 are number of particles per unit volume for the values of x meant to be x_1 and x_2. Find the dimensions of the diffusion constant D.

Solution. As $n = -D\dfrac{n_2 - n_1}{x_2 - x_1}$

$$\therefore \quad D = \frac{n(x_2 - x_1)}{(n_2 - n_1)} \qquad \text{(numerically)}$$

Now n = number of particles per unit area per second,

$$\therefore \quad [n] = L^{-2}T^{-1}$$

$n_2 - n_1$ = number of particles per unit volume

$$\therefore \quad [n_2 - n_1] = L^{-3}$$

$x_2 - x_1$ = position

$$\therefore \quad [x_2 - x_1] = L$$

Hence $[D] = \dfrac{L^{-2}T^{-1}.L}{L^{-3}} = [L^2T^{-1}]$.

PROBLEMS FOR PRACTICE

1. Deduce dimensional formulae for (*i*) angle (*ii*) angular velocity (*iii*) angular acceleration (*iv*) torque (*v*) angular momentum and (*vi*) moment of inertia.

   [Ans. (*i*) Dimensionless (*ii*) T^{-1} (*iii*) T^{-2} (*iv*) ML^2T^{-2} (*v*) ML^2T^{-1} (*vi*) ML^2**]**

2. Obtain dimensions of (*i*) impulse (*ii*) power (*iii*) surface energy (*iv*) coefficient of viscosity (*v*) bulk modulus (*vi*) force constant.

   [Ans. (*i*) MLT^{-1} (*ii*) ML^2T^{-3} (*iii*) ML^0T^{-2} (*iv*) $ML^{-1}T^{-1}$**]** (*v*) $ML^{-1}T^{-2}$ (*vi*) ML^0T^{-2}**]**

3. By the use of dimensions, show that energy per unit volume is equal to the pressure.

4. Show that angular momentum has the same physical units as the Planck's constant h which is given by the relation $E = h\nu$.

5. If force (F), length (L) and time (T) are chosen as the fundamental quantities, then what would be the dimensional formula for density ? **(Ans.** $FL^{-4}T^{-1}$**)**

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6. Calculate the dimensions of force and impulse taking velocity, density and frequency as basic quantities. (Ans. $\rho v^4 \nu^{-2}$, $\rho v^4 \nu^{-3}$)

7. Find the dimensions of linear momentum and surface tension in terms of velocity v, density ρ and frequency ν as fundamental quantities. (Ans. $\rho v^4 \nu^{-3}$, $\rho v^3 \nu^{-1}$)

8. In the expression $P = El^2 m^{-5} G^{-2}$; E, m, l and G denote energy, mass angular momentum and gravitational constant, respectively. Show that P is a dimensionless quantity. **(Exemplar Problem)**

HINTS

3. Energy per unit volume

$$= \frac{\text{Energy}}{\text{Volume}} = \frac{ML^2 . T^{-2}}{L^3} = \frac{MLT^{-2}}{L^2}$$

$$= \frac{\text{Force}}{\text{Area}} = \text{Pressure.}$$

4. Angular momentum,

$$L = mvr$$

$$[L] = MLT^{-1} . L = ML^2T^{-1}$$

As $E = h\nu$

$$\therefore \quad h = \frac{E}{\nu}$$

$$[h] = \frac{ML^2 T^{-2}}{T^{-1}} = ML^2 T^{-1} = [L]$$

6. $v = LT^{-1}$, $\rho = ML^{-3}$, $\nu = T^{-1}$

Solving for M, L and T in terms of ν, ρ and v, we get

$$T = \nu^{-1}, \quad L = v\nu^{-1}, \quad M = \rho v^3 \nu^{-3}$$

$$[\text{Force}] = MLT^{-2} = \rho v^3 \nu^{-3} . v\nu^{-1} . \nu^2 = \rho v^4 \nu^{-2}$$

$$[\text{Impulse}] = \text{Force} \times \text{time} = \rho v^4 \nu^{-2} . \nu^{-1} = \rho v^4 \nu^{-3}$$

7. Using the dimensions of M, L and T as obtained in Problem 6, we get

$$[p] = [mv] = MLT^{-1}$$

$$= \rho v^3 \nu^{-3} . v\nu^{-1} . \nu = \rho v^4 \nu^{-3}$$

$$\text{Surface tension} = \frac{\text{Force}}{\text{Length}}$$

$$= \frac{MLT^{-2}}{L} = MT^{-2} = \rho v^3 \nu^{-3} \nu^2 = \rho v^3 \nu^{-1}.$$

8. $[E] = ML^2 T^{-2}$, $[M] = M$, $[l] = ML^2 T^{-1}$,

$[G] = M^{-1}L^3T^{-2}$

$$\therefore \left[\frac{El^2}{M^5 G^2}\right] = \frac{ML^2T^{-2}[ML^2T^{-1}]^2}{M^5[M^{-1}L^3T^{-2}]^2}$$

$$= \frac{M^3L^6T^{-4}}{M^3L^6T^{-4}} = [M^0L^0T^0]$$

2.22 DIFFERENT TYPES OF VARIABLES AND CONSTANTS

43. *How can we classify variables and constants on the basis of dimensions ? Give examples of each type.*

Different types of variables and constants. On the basis of dimensions, we can classify quantities into *four* categories :

1. ***Dimensional variables.*** *The physical quantities which possess dimensions and have variable values are called dimensional variables.*

Examples. Area, volume, velocity, force, etc.

2. ***Dimensionless variables.*** *The physical quantities which have no dimensions but have variable values are called dimensionless variables.*

Examples. Angle, specific gravity, strain, etc.

3. ***Dimensional constants.*** *The physical quantities which possess dimensions and have constant values are called dimensional constants.*

Examples. Gravitational constant, Planck's constant, etc.

4. ***Dimensionless constants.*** *The constant quantities having no dimensions are called dimensionless constants.*

Examples. π, e, etc.

2.23 APPLICATIONS OF DIMENSIONAL ANALYSIS

44. *Mention some applications of dimensional analysis.*

Applications of dimensional analysis. *The method of studying a physical phenomenon on the basis of dimensions is called dimensional analysis.* Following are the *three* main uses of dimensional analysis :

1. To convert a physical quantity from one system of units to another.
2. To check the correctness of a given physical relation.
3. To derive a relationship between different physical quantities.

2.24 CONVERSION OF ONE SYSTEM OF UNITS TO ANOTHER

45. *How can a physical quantity be converted from one system of units to another ? Explain it with the help of a suitable example.*

To convert a physical quantity from one system of units to another. It is based on the fact that *the magnitude of a physical quantity remains the same, whatever may be the system of units.* If u_1 and u_2 are the units of measurement of a physical quantity Q and n_1 and n_2 are the corresponding numerical values, then

$$Q = n_1u_1 = n_2u_2$$

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Let M_1, L_1 and T_1 be the sizes of fundamental units of mass, length and time in one system ; and M_2, L_2, T_2 be corresponding units in another system. If the dimensional formula of quantity Q be $M^a L^b T^c$, then

$$n_1 = M_1^a L_1^b T_1^c$$

and $$n_2 = M_2^a L_2^b T_2^c$$

$$\therefore \quad n_1 [M_1^a L_1^b T_1^c] = n_2 [M_2^a L_2^b T_2^c]$$

or $$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c.$$

This equation can be used to find the numerical value in the second or new system of units.

Example. Let us convert one joule into erg.

Joule is SI unit of energy and erg is the CGS unit of energy. Dimensional formula of energy is ML^2T^{-2}.

$\therefore \quad a = 1, \ b = 2, \ c = -2.$

SI	CGS
$M_1 = 1\text{ kg} = 1000\text{ g}$	$M_2 = 1\text{ g}$
$L_1 = 1\text{ m} = 100\text{ cm}$	$L_2 = 1\text{ cm}$
$T_1 = 1\text{ s}$	$T_2 = 1\text{ s}$
$n_1 = 1$ (joule)	$n_2 = ?$ (erg)

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

$$= 1\left[\frac{1000}{1}\right]^1 \left[\frac{100}{1}\right]^2 \left[\frac{1}{1}\right]^{-2}$$

$$= 1 \times 10^3 \times 10^4 = 10^7$$

$\therefore$ **1 joule = 10^7 erg.**

For Your Knowledge

▲ The above conversion technique is applicable to only absolute systems of units. The gravitational or other practical units must be first converted into absolute units before using the above technique.

Examples based on Conversion of Units from one System to another

FORMULAE USED

1. $n_1 u_1 = n_2 u_2$
2. $n_1 [M_1^a L_1^b T_1^c] = n_2 [M_2^a L_2^b T_2^c]$
3. $n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$

UNITS USED

The SI units of mass, length and time are kg, m and s and the corresponding CGS units are g, cm and s.

EXAMPLE 35. The value G in CGS system is 6.67×10^{-8} dyne cm^2 g^{-2}. Calculate the value in SI units.

Solution. As $F = G\frac{m_1 m_2}{r^2}$

$$\therefore \quad G = \frac{Fr^2}{m_1 m_2}$$

$$[G] = \frac{MLT^{-2} \cdot L^2}{MM} = M^{-1}L^3T^{-2}$$

$\therefore \quad a = -1, \ b = 3, \ c = -2$

CGS units	SI units
$n_1 = 6.67 \times 10^{-8}$	$n_2 = ?$
$M_1 = 1\,g$	$M_2 = 1\text{ kg} = 1000\text{ g}$
$L_1 = 1\text{ cm}$	$L_2 = 1\text{ m} = 100\text{ cm}$
$T_1 = 1s$	$T_2 = 1\text{ s}$

$$\therefore \quad n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

$$= 6.67 \times 10^{-8} \left[\frac{1}{1000}\right]^{-1} \left[\frac{1}{100}\right]^3 \left[\frac{1}{1}\right]^{-2}$$

$$= 6.67 \times 10^{-11}$$

Hence in SI units,

$$G = 6.67 \times 10^{-11} \text{Nm}^2\text{kg}^{-2}.$$

EXAMPLE 36. Find the value of 60 J per min on a system that has 100 g, 100 cm and 1 min as the base units.

Solution. $P = 60\,\frac{\text{joule}}{\text{min}} = \frac{60\text{ joule}}{60\text{ s}} = 1\text{ watt}$

which is the SI unit of power.

Now $[\text{Power}] = ML^2T^{-3}$

$\therefore \quad a = 1, \ b = 2, \ c = -3$

SI	New System
$n_1 = 1$	$n_2 = ?$
$M_1 = 1\text{ kg} = 1000\text{ g}$	$M_2 = 100\text{ g}$
$L_1 = 1\text{ m} = 100\text{ cm}$	$L_2 = 100\text{ cm}$
$T_1 = 1\text{ s}$	$T_2 = 1\text{ min} = 60\text{ s}$

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

$$= 1\left[\frac{1000}{100}\right]^1 \left[\frac{100}{100}\right]^2 \left[\frac{1}{60}\right]^{-3}$$

$$= 2.16 \times 10^6$$

$\therefore$ **60 J min^{-1} = 2.16×10^6 new units of power.**

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EXAMPLE 37. *In CGS system, the value of Stefan's constant is* $\sigma = 5.67 \times 10^{-5}$ *erg* s^{-1} cm^{-2} K^{-4}. *Find its value in SI units. Given* $1\ J = 10^7$ *erg.*

Solution. In CGS system, $\sigma = 5.67 \times 10^{-5}$ erg $s^{-1}cm^{-2}K^{-4}$. The SI unit of work is joule. We have, $1\ erg = 10^{-7}$ J and $1\ cm = 10^{-2}$ m

$\therefore$ The value of Stefan's constant in SI units is

$$\sigma = 5.67 \times 10^{-5}[10^{-7}\ J]\ s^{-1}\ [10^{-2}\ m]^{-2}\ K^{-4}$$
$$= 5.67 \times 10^{-5} \times 10^{-7} \times 10^{4}\ Js^{-1}m^{-2}\ K^{-4}$$
$$= \mathbf{5.67 \times 10^{-8}\ Js^{-1}m^{-2}K^{-4}}.$$

EXAMPLE 38. *If the unit of force is 1 kN, unit of length 1 km and the unit of time is 100 s, what will be the unit of mass ?*

Solution. $M = \dfrac{MLT^{-2}.T^2}{L} = \dfrac{FT^2}{L}$

$$= \frac{1000\ N \times 10^4\ s^2}{1000\ m} = 10^4\ \frac{Ns^2}{m}$$
$$= 10^4\ \frac{kg\ m\ s^{-2}.\ s^2}{m} = \mathbf{10^4\ kg.}$$

PROBLEMS FOR PRACTICE

1. Convert one dyne into newton. **[Himachal 09]** (**Ans.** 10^{-5} newton)
2. If the value of universal gravitational constant in SI is $6.6 \times 10^{-11}\ Nm^2kg^{-2}$, then find its value in CGS system. **[Himachal 09]** (**Ans.** 6.6×10^{-8} dyne cm^2g^{-2})
3. The density of mercury is 13.6 g cm^{-3} in CGS system. Find its value in SI units. (**Ans.** 13.6×10^3 kg m^{-3})
4. The surface tension of water is 72 dyne cm^{-1}. Express it in SI units. (**Ans.** 0.072 Nm^{-1})
5. An electric bulb has a power of 500 W. Express it in CGS units. (**Ans.** 5×10^9 erg s^{-1})
6. If the value of atmospheric pressure is 10^6 dyne cm^{-2}, find its value in SI units. (**Ans.** 10^5 Nm^{-2})
7. In SI units, the value of Stefan's constant is $\sigma = 5.67 \times 10^{-8}\ Js^{-1}\ m^{-2}\ K^{-4}$. Find its value in CGS system. (**Ans.** 5.67×10^{-5} erg s^{-1} cm^{-2} K^{-4})
8. Find the value of 100 J on a system which has 20 cm, 250 g and half minute as fundamental units of length, mass and time. (**Ans.** 9×10^6 new units)
9. If the units of force, energy and velocity are 20 N, 200 J and 5 ms^{-1}, find the units of length, mass and time. (**Ans.** 10 m, 8 kg, 2 s)
10. When 1 m, 1 kg and 1 min are taken as the fundamental units, the magnitude of the force is 36 units. What will be the value of this force in CGS system ? (**Ans.** 10^3 dyne)

HINTS

3. [Density] $= M^1L^{-3}$

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^1\left[\frac{L_1}{L_2}\right]^{-3}$$
$$= 13.6\left[\frac{1g}{1kg}\right]^1\left[\frac{1cm}{1m}\right]^{-3}$$
$$= 13.6\left[\frac{1g}{1000\,g}\right]\left[\frac{1cm}{100\,cm}\right]^{-3} = 13.6 \times 10^3$$

$\therefore$ $13.6\ g\ cm^{-3} = \mathbf{13.6 \times 10^3\ kgm^{-3}}$.

4. [Surface Tension] $= M^1T^{-2}$

$$n_2 = 72\left[\frac{1g}{1kg}\right]^1\left[\frac{1s}{1s}\right]^{-2}$$
$$= 72\left[\frac{1g}{1000\,g}\right]^1\left[\frac{1s}{1s}\right]^{-2} = 0.072$$

$\therefore$ $72\ dyne\ cm^{-1} = \mathbf{0.072\ Nm^{-1}}$.

6. [Pressure] $= M^1L^{-1}T^{-2}$

$$n_2 = 10^6\left[\frac{1g}{1kg}\right]^1\left[\frac{1cm}{1m}\right]^{-1}\left[\frac{1s}{1s}\right]^{-2} = 10^5$$

$\therefore$ $10^6\ dyne\ cm^{-2} = \mathbf{10^5\ Nm^{-2}}$.

7. $\sigma = 5.67 \times 10^{-8}\ Js^{-1}m^{-2}K^{-4}$

$$= 5.67 \times 10^{-8} \times 10^7\ erg\ s^{-1}\ (100\,cm)^{-2}\ K^{-4}$$
$$= \mathbf{5.67 \times 10^{-5}\ erg\ s^{-1}\ cm^{-2}\ K^{-4}}.$$

8. [Work] $= ML^2\ T^{-2}$

$$n_2 = 100\left[\frac{1kg}{250\,g}\right]^1\left[\frac{1m}{20\,cm}\right]^2\left[\frac{1s}{0.5\,min}\right]^{-2}$$
$$= 100\left[\frac{1000\,g}{250\,g}\right]^1\left[\frac{100\,cm}{20\,cm}\right]^2\left[\frac{1s}{30\,s}\right]^{-2}$$
$$= 100 \times 4 \times 25 \times 30 \times 30 = \mathbf{9 \times 10^6\ new\ units.}$$

9. (*i*) $MLT^{-2} = 20\ N$ (*ii*) $ML^2\ T^{-2} = 200\ J$

(*iii*) $LT^{-1} = 5\ ms^{-1}$

Dividing (*ii*) by (*i*), $L = \dfrac{200}{20} = \mathbf{10\ m}$

Putting the value of L in (*iii*),

$$10\ T^{-1} = 5 \text{ or } T = \mathbf{2\ s}$$

From (*i*),

$M \times 10 \times (2)^{-2} = 20$ or $M = \mathbf{8\ kg.}$

10. $n_2 = 36\left[\frac{1kg}{1g}\right]^1\left[\frac{1m}{1cm}\right]^1\left[\frac{1min}{1s}\right]^{-2}$

$$= 36\left[\frac{1000\,g}{1g}\right]\left[\frac{100\,cm}{1cm}\right]^1\left[\frac{60\,s}{1s}\right]^{-2} = \mathbf{10^3\ dyne.}$$

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2.25 CHECKING THE DIMENSIONAL CONSISTENCY OF EQUATIONS

46. *State the principle of homogeneity of dimensions. What is its basis ?*

Principle of homogeneity of dimensions. According to this principle, *a physical equation will be dimensionally correct if the dimensions of all the terms occurring on both sides of the equation are the same.* This principle is based on the fact that only the physical quantities of the same kind can be added, subtracted or compared. Thus, velocity can be added to velocity but not to force.

47. *How can we check the dimensional correctness of a physical equation ? Explain it with a suitable example.*

To check the dimensional correctness of a physical equation. For this purpose we make use of the principle of homogeneity of dimensions. If the dimensions of all the terms on the two sides of the equation are same, then the equation is dimensionally correct.

Example. Let us check the dimensional accuracy of the equation of motion,

$$s = ut + \frac{1}{2}at^2$$

Dimensions of different terms are

$$[s] = [L]$$

$$[ut] = [LT^{-1}][T] = [L]$$

$$\left[\frac{1}{2}at^2\right] = [LT^{-2}][T^2] = [L]$$

As all the terms on both sides of the equations have the same dimensions, so the given equation is dimensionally correct.

For Your Knowledge

A dimensionally correct equation need not be actually a correct equation, but a dimensionally inconsistent equation must be wrong. The equation of motion : $s = ut + at^2$ is dimensionally correct but numerically it is wrong.

Examples based on Dimensional Correctness of Physical Relations

CONCEPT USED

By the principle of homogeneity of dimensions, a physical relation will be dimensionally correct if the dimensions of all the terms in the equation are the same.

EXAMPLE 39. The distance x travelled by a body in time t which starts from the position x_0 with initial velocity v_0 and has uniform acceleration a, is given by $x = x_0 + v_0t + \frac{1}{2}at^2$. Check the dimensionally consistency of this equation.

Solution. The dimensions of the various terms are

$$[x] = [L]$$

$$[x_0] = [L]$$

$$[v_0t] = [LT^{-1}][T] = [L]$$

$$[\tfrac{1}{2}at^2] = [LT^{-2}][T^2] = [L]$$

Since the dimensions of all the terms are same, hence the given equation is dimensionally correct.

EXAMPLE 40. Check whether the following equation is dimensionally correct.

$$\frac{1}{2}mv^2 = mgh. \qquad \text{[NCERT, Himachal 06]}$$

Solution. $[\frac{1}{2}mv^2] = [M][LT^{-1}]^2 = [ML^2T^{-2}]$

$$[mgh] = [M][LT^{-2}][L] = [ML^2T^{-2}]$$

$\therefore$ Dimensions of LHS = Dimensions of RHS

Hence the given equation is dimensionally correct.

EXAMPLE 41. Check the correctness of the equation,

$$FS = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2$$

where F is the force acting on a body of mass m and S is the distance moved by the body when its velocity changes from u to v. [Delhi 08]

Solution.

$$[FS] = MLT^{-2} \cdot L = ML^2T^{-2}$$

$$[\tfrac{1}{2}mv^2] = M[LT^{-1}]^2 = ML^2T^{-2}$$

$$[\tfrac{1}{2}mu^2] = M[LT^{-1}]^2 = ML^2T^{-2}$$

Since dimensions of all the terms in the given equation are same, hence the given equation is dimensionally correct.

EXAMPLE 42. Check the correctness of the relation $\tau = I\alpha$, where τ is the torque acting on a body, I is the moment of inertia and α is angular acceleration. [Punjab 1990]

Solution. Given $\tau = I\alpha$

As torque, τ = Force × distance

$\therefore$ $[\tau] = MLT^{-2} \cdot L = ML^2T^{-2}$

Moment of inertia,

I = Mass × distance^2

$\therefore$ $[I] = ML^2$

Angular acceleration,

$$\alpha = \frac{\text{Angle}}{(\text{Time})^2}$$

$\therefore$ $$[\alpha] = \frac{1}{T^2} = T^{-2}$$

$$[I\alpha] = ML^2T^{-2}$$

$\therefore$ Dimensions of LHS = Dimensions of RHS

Hence the given equation is dimensionally correct.

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EXAMPLE 43. Check the dimensional consistency of the following equations :

(i) de-Broglie wavelength, $\lambda = \frac{h}{mv}$

(ii) Escape velocity, $v = \sqrt{\frac{2GM}{R}}$. **[Himachal 05]**

Solution. (i) Given $\lambda = \frac{h}{mv}$

As wavelength is a distance,

$\therefore \quad [\lambda] = L$

Also $\left[\frac{h}{mv}\right] = \frac{\text{Planck's constant}}{\text{Mass} \times \text{velocity}} = \frac{ML^2T^{-1}}{M \times LT^{-1}} = L$

$\because$ Dimensions of LHS = Dimensions of RHS

Hence the given equation is dimensionally consistent.

(ii) Given $v = \sqrt{\frac{2GM}{R}}$

$[v] = LT^{-1}$

$$\left[\frac{2GM}{R}\right]^{\frac{1}{2}} = \left[\frac{M^{-1}L^3T^{-2} \cdot M}{L}\right]^{\frac{1}{2}} = [L^2T^{-2}]^{\frac{1}{2}} = LT^{-1}$$

$\because$ Dimensions of LHS = Dimensions of RHS

Hence the equation is dimensionally correct.

EXAMPLE 44. Check by the method of dimensions whether the following equations are correct :

(i) $E = mc^2$ *(ii)* $T = 2\pi\sqrt{\frac{l}{g}}$ **[Delhi 98]**

(iii) $v = \sqrt{\frac{P}{\rho}}$, *where* v = *velocity of sound,* P = *pressure and* ρ = *density of medium.*

(iv) $\nu = \frac{1}{2l}\sqrt{\frac{T}{m}}$, *where* ν = *frequency of vibration,* l = *length of the string,* T = *tension in the string and* m = *mass per unit length.*

Solution.

(i) [L.H.S] = $[E]$ = Energy = ML^2T^{-2}

[R.H.S] = $[mc^2] = M[LT^{-1}]^2 = ML^2T^{-2}$

Hence the relation is correct.

(ii) $[RHS] = \sqrt{\frac{[l]}{[g]}} = \left[\frac{L}{LT^{-2}}\right]^{1/2} = T$

$[LHS] = [T] = T$.

Hence the relation is correct.

(iii) $[RHS] = \sqrt{\frac{[P]}{[\rho]}} = \left[\frac{ML^{-1}T^{-2}}{ML^{-3}}\right]^{1/2} = LT^{-1}$

$[LHS] = [v] = LT^{-1}$.

Hence the relation is correct.

(iv) $RHS = \frac{1}{[l]}\sqrt{\frac{[T]}{[m]}} = \frac{1}{L}\left[\frac{MLT^{-2}}{ML^{-1}}\right]^{1/2}$

$= \frac{1}{L} \cdot LT^{-1} = T^{-1}$

$[LHS] = [\nu] = T^{-1}$.

Hence the relation is correct.

EXAMPLE 45. By the method of dimensions, test the accuracy of the equation : $\delta = \frac{mgl^3}{4bd^3Y}$

where δ *is the depression produced in the middle of a bar of length* l, *breadth* b *and depth* d, *when it is loaded in the middle with mass* m. Y *is the Young's modulus of the material of the bar.*

Solution. [LHS] = $[\delta]$ = depression = L

$$[RHS] = \left[\frac{mgl^3}{4bd^3Y}\right]$$

$$= \frac{\text{Mass} \times \text{acceleration} \times \text{length}^3}{4 \times \text{breadth} \times \text{depth}^3 \times \text{Young's modulus}}$$

$$= \frac{M \cdot LT^{-2} \cdot L^3}{L \cdot L^3 \cdot ML^{-1}T^{-2}} = L$$

$\therefore$ [LHS] = [RHS]. Hence the relation is correct.

EXAMPLE 46. Find the dimensions of a/b in the equation : $F = a\sqrt{x} + bt^2$, *where F is force, x is distance and t is time.*

Solution. $[a\sqrt{x}] = [F]$

$\therefore \quad [a] = \frac{[F]}{[\sqrt{x}]} = \frac{MLT^{-2}}{L^{1/2}} = ML^{1/2}T^{-2}$

$[bt^2] = [F]$

$\therefore \quad [b] = \frac{[F]}{[t^2]} = \frac{MLT^{-2}}{T^2} = MLT^{-4}$

$[a/b] = \frac{ML^{1/2}T^{-2}}{MLT^{-4}} = L^{-1/2}T^2$.

EXAMPLE 47. Find the dimensions of $a \times b$ *in the relation :* $P = \frac{b - x^2}{at}$; *where P is power, x is distance and t is time.*

Solution. $[b] = [x^2] = L^2 \quad \therefore \quad [P] = \frac{L^2}{[at]}$

or $\quad [a] = \frac{L^2}{[P][t]} = \frac{L^2}{ML^2T^{-3} \cdot T} = M^{-1}T^2$

Hence $[a \times b] = M^{-1}L^2T^2$.

EXAMPLE 48. The Vander Wall's equation for a gas is

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

Determine the dimensions of a and b. Hence write the SI units of a and b. **[Himachal 06C ; Central Schools 14]**

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Solution. Since dimensionally similar quantities can be added or subtracted, therefore,

$$[P] = \left[\frac{a}{V^2}\right]$$

or $$[a] = [PV^2] = [ML^{-1}T^{-2}][L^3]^2 = ML^5T^{-2}$$

Also $$[b] = [V] = L^3.$$

The SI unit of a is $kg\ m^5 s^{-2}$ and that of b is m^3.

EXAMPLE 49. When white light travels through glass, the refractive index of glass (μ = velocity of light in air/velocity of light in glass) is found to vary with wavelength as $\mu = A + \frac{B}{\lambda^2}$. Using the principle of homogeneity of dimensions, find the SI units in which the constants A and B must be expressed.

Solution. Here $\mu = \dfrac{\text{Velocity of light in air}}{\text{Velocity of light in glass}}$

= a dimensionless number

$\therefore$ $[A] = [\mu]$ = a dimensionless number

As $$\left[\frac{B}{\lambda^2}\right] = [\mu]$$

$\therefore$ $$[B] = [\mu][\lambda^2] = 1 . L^2 = L^2$$

Hence A, being dimensionless, has no units and SI unit of B is m^2.

EXAMPLE 50. In the equation : $y = a \sin(\omega t - kx)$, t and x stand for time and distance respectively. Obtain the dimensional formula for ω and k.

Solution. An angle is a dimensionless quantity,

$\therefore$ $[\omega t] = 1$ or $[\omega] = \dfrac{1}{[t]} = \dfrac{1}{T} = T^{-1}$.

$[kx] = 1$ or $[k] = \dfrac{1}{[x]} = \dfrac{1}{L} = L^{-1}$.

EXAMPLE 51. Rule out or accept the following formulae for kinetic energy on the basis of dimensional arguments :

(i) $\frac{3}{16}mv^2$ *(ii)* $\frac{1}{2}mv^2 + ma$ **[Central Schools 08]**

Solution. $K.E. = \frac{1}{2}mv^2$

Dimensions of $K.E. = [ML^2T^{-2}]$

(i) Dimensions of $\frac{3}{16}mv^2 = [ML^2T^{-2}]$

This formula is dimensionally acceptable for K.E.

(ii) $\left[\frac{1}{2}mv^2\right] + [ma] = [ML^2T^{-2}] + [MLT^{-2}]$

This formula cannot be accepted for K.E. as it is dimensionally inhomogeneous.

PROBLEMS FOR PRACTICE

1. Test the dimensional consistency of the following equations : (i) $v = u + at$ (ii) $s = ut + \frac{1}{2}at^2$ (iii) $v^2 - u^2 = 2as$ **[Himachal 06]**
(Ans. All relations are dimensionally correct)

2. The viscous force 'F' acting on a small sphere of radius 'r' moving with velocity v through a liquid is given by $F = 6\pi\eta rv$. Calculate the dimensions of η, the coefficient of viscosity. (Ans. $ML^{-1}T^{-1}$)

3. The distance covered by a particle in time t is given by $x = a + bt + ct^2 + dt^3$; find the dimensions of a, b, c and d. (Ans. L, LT^{-1}, LT^{-2}, LT^{-3})

4. The critical velocity of the flow of a liquid through a pipe of radius r is given by $v_c = \dfrac{K\eta}{r\rho}$ where ρ is the density and η is the coefficient of viscosity of the liquid. Check if this relation is dimensionally correct. (Ans. Correct)

5. The rate of flow (V) of a liquid flowing through a pipe of radius r and a pressure gradient (P/l) is given by Poiseuille's equation : $V = \dfrac{\pi}{8}\dfrac{Pr^4}{\eta l}$

Check the dimensional consistency of this equation.
(Ans. Correct)

6. Test if the following equation is dimensionally correct :

$$h = \frac{2S\cos\theta}{r\rho g}$$

where h = height, S = surface tension, ρ = density, r = radius, and g = acceleration due to gravity.
(Ans. Correct)

7. Find the dimensions of the quantity v in the equation, $v = \dfrac{\pi p(a^2 - x^2)}{2\eta l}$, where a is the radius and l is the length of the tube in which the fluid of coefficient of viscosity η is flowing, x is the distance from the axis of the tube and p is the pressure difference.
(Ans. LT^{-1})

8. Find the dimensions of the quantity q from the expression. $T = 2\pi\sqrt{\dfrac{ml^3}{3Yq}}$, where T is the time period of a bar of length l, mass m and Young's modulus Y.
(Ans. L^4)

9. An artificial satellite of mass m is revolving in a circular orbit around a planet of M and radius R. If the radius of the orbit of the satellite be r.
Justify by the method of dimensions that the time period of the satellite is given by : $T = \dfrac{2\pi}{R}\sqrt{\dfrac{r^3}{g}}$.

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10. Find the dimensions of $(a \times b)$ in the equation : $E = \frac{b - x^2}{at}$; where E is energy, x is distance and t is time. **(Ans.** $M^{-1} L^2 T$**)**

11. Find the dimensions of (a/b) in the equation :
$$P = \frac{a - t^2}{bx}$$
where P is pressure, x is distance and t is time. **(Ans.** MT^{-2}**)**

12. Time period of an oscillating drop of radius r, density ρ and surface tension S is : $t = K\sqrt{\frac{\rho r^3}{S}}$
Check the correctness of the relation. **[Himachal 04] (Ans.** correct**)**

13. Out of formulae (*i*) $y = a\sin 2\pi t / T$ and (*ii*) $y = a\sin vt$ for the displacement y of particle undergoing a certain periodic motion, rule out the wrong formula on dimensional grounds. (where a = maximum displacement of the particle, v = speed of the particle, T = time period of motion). **[Delhj 09]**

HINTS

1. (*i*) $[v] = LT^{-1}$, $[u] = LT^{-1}$,
$[at] = LT^{-2}.T = LT^{-1}$
(*ii*) $[s] = L$, $[ut] = LT^{-1}.T = L$,
$\left[\frac{1}{2}at^2\right] = LT^{-2}.T^2 = L$
(*iii*) $[v^2] = [LT^{-1}]^2 = L^2T^{-2}$,
$[u^2] = [LT^{-1}]^2 = L^2T^{-2}$
$[2aS] = LT^{-2}.L = L^2T^{-2}$

4. $[v_c]$ = critical velocity $= LT^{-1}$
$$\left[\frac{K\eta}{r\rho}\right] = \frac{1.ML^{-1}T^{-1}}{L.ML^{-3}} = LT^{-1}$$
Hence the relation is correct.

5. $[V]$ = Rate of flow of liquid
= Volume / Time $= L^3T^{-1}$
$$\left[\frac{\pi}{8}\frac{Pr^4}{\eta l}\right] = \frac{ML^{-1}T^{-2}.L^4}{ML^{-1}T^{-1}.L} = L^3T^{-1}$$
Hence the relation is correct.

6. $[h] = L$
$$\left[\frac{2S\cos\theta}{r\rho g}\right] = \frac{MT^{-2}.1}{L.ML^{-3}.LT^{-2}} = L$$
Hence the relation is correct.

7. $[v] = \frac{[ML^{-1}T^{-2}].[L^2 - L^2]}{[ML^{-1}T^{-1}].[L]} = \frac{ML^{-1}T^{-2}.L^2}{MT^{-1}} = LT^{-1}$.

8. $[q] = \left[\frac{4\pi^2 ml^3}{3YT^2}\right] = \frac{ML^3}{ML^{-1}T^{-2}.T^2} = L^4$.

9. $[T] = T$
$$\left[\frac{2\pi}{R}\sqrt{\frac{r^3}{g}}\right] = \frac{1}{L}\left[\frac{L^3}{LT^{-2}}\right]^{1/2} = T.$$
Hence the relation is correct.

11. $[a] = [t^2] = T^2 \quad \therefore [P] = \frac{T^2}{[bx]}$
or $[b] = \frac{T^2}{[P][x]} = \frac{T^2}{ML^{-1}T^{-2}.L} = M^{-1}T^4$
$$[a/b] = \frac{T^2}{M^{-1}T^4} = MT^{-2}.$$

12. $[RHS] = \left[K\sqrt{\frac{\rho r^3}{S}}\right] = \left[\frac{[ML^{-3}][L^3]}{[MT^{-2}]}\right]^{1/2} = [T] = [LHS]$

13. Refer to the answer of NCERT Exercise 2.14.

2.26 DEDUCING RELATION AMONG THE PHYSICAL QUANTITIES

48. *How can the method of dimensions be used to deduce a relation among the physical quantities ? Explain it with the help of a suitable example.*

To derive the relationship among physical quantities. By making use of the homogeneity of dimensions, we can derive an expression for a physical quantity if we know the various factors on which it depends.

Example. Let us derive an expression for the centripetal force F acting on a particle of mass m moving with velocity v in a circle of radius r.

Let $F \propto m^a v^b r^c$ or $F = K m^a v^b r^c$ ...(1)

where K is a dimensionless constant. Writing the dimensions of various quantities in equation (1), we get

$$[MLT^{-2}] = 1[M]^a[LT^{-1}]^b[L]^c$$

or $M^1L^1T^{-2} = M^aL^{b+c}T^{-b}$.

Comparing the dimensions of similar quantities on both sides, we get

$$a = 1$$
$$b + c = 1 \quad \text{or} \quad c = 1 - b = 1 - 2 = -1$$
$$-2 = -b \quad \text{or} \quad b = 2$$

From equation (1), we get

$$F = K mv^2 r^{-1} = K\frac{mv^2}{r}.$$

This is the required expression for the centripetal force.

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Examples based on Deriving Relationship between Physical Quantities

Concept Used

By using the principle of homogeneity of dimensions, the form of expression for a given physical quantity can be obtained if we know the factors upon which that physical quantity depends.

Example 52. Consider a simple pendulum, having a bob attached to a string, that oscillates under the action of the force of gravity. Suppose that the period of oscillation of the simple pendulum depends on (i) mass m of the bob (ii) length l of the pendulum and (iii) acceleration due to gravity g at the place. Derive the expression for its time period using method of dimensions. [NCERT]

Solution. Let us assume that $T \propto m^a\ l^b\ g^c$

or $T = Km^a\ l^b\ g^c$ ...(i)

where K is a dimensionless constant.

The dimensions of various quantities are

$[T] = T,\ [m] = M,\ [l] = L,\ [g] = LT^{-2}$

Substituting these dimensions in eqn. (i), we get

$T = [M]^a\ [L]^b\ [LT^{-2}]^c$

or $M^0L^0T^1 = M^aL^{b+c}T^{-2c}$

Equating the exponents of M, L and T on both sides, we get

$a = 0,\quad b + c = 0,\quad -2c = 1$

On solving, $a = 0,\quad b = \frac{1}{2},\quad c = -\frac{1}{2}$

$\therefore\quad T = Km^0l^{1/2}g^{-1/2} = K\sqrt{\frac{l}{g}}$

From experiments, $K = 2\pi$.

Therefore $T = 2\pi\sqrt{\frac{l}{g}}$.

Example 53. The velocity 'v' of water waves depends on the wavelength 'λ', density of water 'ρ' and the acceleration due to gravity 'g'. Deduce by the method of dimensions the relationship between these quantities. [Central Schools 08]

Solution. Let $v = K\lambda^a\rho^b g^c$ ...(i)

where K = a dimensionless constant.

Dimensions of the various quantities are

$[v] = LT^{-1},\ [\lambda] = L,\ [\rho] = ML^{-3},\ [g] = LT^{-2}$

Substituting these dimensions in equation (i), we get

$[LT^{-1}] = [L]^a\ [ML^{-3}]^b\ [LT^{-2}]^c$

$M^0L^1T^{-1} = M^bL^{a-3b+c}T^{-2c}$

Equating the powers of M, L and T on both sides,

$b = 0,\quad a - 3b + c = 1,\quad -2c = -1$

On solving, $a = \frac{1}{2},\quad b = 0,\quad c = \frac{1}{2}$

$\therefore\quad v = K\lambda^{\frac{1}{2}}\rho^0 g^{\frac{1}{2}} = K\sqrt{\lambda g}$.

Example 54. Assuming that the mass M of the large stone that can be moved by a flowing river depends upon the velocity, 'v' the density of water and on 'g', the acceleration due to gravity. Show that M varies with the sixth power of the velocity of flow.

Solution. Let $M = Kv^a\ \rho^b\ g^c$

where K = a dimensionless constant.

Dimensions of the various quantities are :

$[M] = M,\ [v] = LT^{-1},\ [\rho] = ML^{-3},\ [g] = LT^{-2}$

Substituting these dimensions in equation (i), we get

$[M] = [LT^{-1}]^a\ [ML^{-3}]^b\ [LT^{-2}]^c$

$M^1L^0T^0 = M^b\ L^{a-3b+c}\ T^{-a-2c}$

Equating the powers of M, L and T, we get

$b = 1,\quad a - 3b + c = 0,\quad -a - 2c = 0$

On solving, $a = 6,\quad b = 1,\quad c = -3$

$\therefore\quad M = Kv^6\ \rho^1\ g^{-3}$

Hence $M \propto v^6$.

Example 55. The velocity of sound waves 'v' through a medium may be assumed to depend on :

(i) the density of the medium 'd' and

(ii) the modulus of elasticity 'E'.

Deduce by the method of dimensions the formula for the velocity of sound. Take dimensional constant $K = 1$.

Solution. Let the velocity of sound waves be given by

$v = K\ d^a\ E^b$ ...(i)

where K = a dimensionless constant.

Dimensions of the various quantities are

$[v] = LT^{-1},\quad [d] = ML^{-3}$

$[E] = \frac{\text{stress}}{\text{strain}} = \frac{\text{force}}{\text{area} \times \text{strain}} = \frac{MLT^{-2}}{L^2.1} = ML^{-1}T^{-2}$

Substituting these dimensions in equation (i), we get

$[LT^{-1}] = [ML^{-3}]^a\ [ML^{-1}T^{-2}]^b$

or $M^0L^1T^{-1} = M^{a+b}\ L^{-3a-b}\ T^{-2b}$

Equating the dimensions of M, L and T, we get

$a + b = 0,\quad -3a - b = 1,\quad -2b = -1$

On solving, $a = -\frac{1}{2},\quad b = \frac{1}{2}$

$\therefore\quad v = Kd^{-1/2}E^{1/2}$ or $v = \sqrt{\frac{E}{d}}$. $[\because\ K = 1]$

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EXAMPLE 56. *The frequency 'ν' of vibration of a stretched string depends upon :*

(i) its length l,

(ii) its mass per unit length 'm' and

(iii) the tension T in the string.

Obtain dimensionally an expression for frequency ν. [Delhi 2002]

Solution. Let the frequency of vibration of the string be given by

$$\nu = K l^a m^b T^c \quad ...(i)$$

where K = a dimensionless constant.

Dimensions of the various quantities are :

$[\nu] = T^{-1}$, $[l] = L$, $[T] = \text{Force} = MLT^{-2}$

$$[m] = \frac{\text{mass}}{\text{length}} = \frac{M}{L} = ML^{-1},$$

Substituting these dimensions in equation (*i*), we get

$$[T^{-1}] = [L]^a [ML^{-1}]^b [MLT^{-2}]^c$$

or $$M^0L^0T^{-1} = M^{b+c} L^{a-b+c} T^{-2c}$$

Equating the dimensions of M, L and T, we get

$$b + c = 0, \quad a - b + c = 0, \quad -2c = -1$$

On solving, $$a = -1, \quad b = -\frac{1}{2}, \quad c = \frac{1}{2}.$$

$$\therefore \quad \nu = Kl^{-1} m^{-1/2} T^{1/2}$$

or $$\nu = \frac{K}{l}\sqrt{\frac{T}{m}}.$$

EXAMPLE 57. *A planet moves around the sun in nearly circular orbit. Its period of revolution 'T' depends upon :*

(i) radius 'r' of orbit (ii) mass 'M' of the sun and

(iii) the gravitational constant G.

Show dimensionally that $T^2 \propto r^3$.

Solution. Let $$T = Kr^a M^b G^c \quad ...(i)$$

where K = a dimensionless constant.

Dimensions of the various quantities are :

$$[T] = T, \ [r] = L, \ [M] = M$$

$$[G] = \frac{Fr^2}{m_1 m_2} = \frac{MLT^{-2} \cdot L^2}{MM} = M^{-1}L^3T^{-2}.$$

Substituting these dimensions in equation (*i*), we get

$$[T] = [L]^a [M]^b [M^{-1}L^3T^{-2}]^c$$

$$M^0L^0T^1 = M^{b-c} L^{a+3c} T^{-2c}$$

Equating the dimensions of M, L and T, we get

$$b - c = 0, \quad a + 3c = 0, \quad -2c = 1$$

On solving,

$$a = \frac{3}{2}, \quad b = -\frac{1}{2}, \quad c = -\frac{1}{2}$$

$$\therefore \quad T = Kr^{3/2} M^{-1/2} G^{-1/2}$$

or $$T^2 = \frac{K^2 r^3}{MG}$$

$$\therefore \quad T^2 \propto r^3.$$

EXAMPLE 58. *Reynold number* N_R *(a dimensionless quantity) determines the condition of laminar flow of a viscous liquid through a pipe.* N_R *is a function of the density of the liquid 'ρ', its average speed 'v' and coefficient of viscosity 'η'. Given that* N_R *is also directly proportional to 'D' (the diameter of the pipe), show from dimensional considerations that* $N_R \propto \frac{\rho v D}{\eta}$.

The unit of η in SI system is $kg\ m^{-1}\ s^{-1}$.

Solution. Let $$N_R = K\rho^a v^b \eta^c D \quad ...(i)$$

where K = a dimensionless constant.

Dimensions of various quantities are

$$[N_R] = 1 = M^0 L^0 T^0$$

$$[\rho] = ML^{-3}, \ [v] = LT^{-1},$$

$$[\eta] = ML^{-1}T^{-1}, \ [D] = L$$

Substituting these dimensions in equation (*i*), we get

$$[M^0L^0T^0] = [ML^{-3}]^a [LT^{-1}]^b [ML^{-1}T^{-1}]^c [L]$$

$$M^0L^0T^0 = M^{a+c} L^{-3a+b-c+1} T^{-b-c}$$

Equating the powers of M, L and T, we get

$$a + c = 0, \quad -3a + b - c + 1 = 0, \quad -b - c = 0$$

On solving, $a = 1$, $b = 1$, $c = -1$.

$$\therefore \quad N_R = K\rho^1 v^1 \eta^{-1} D = K\frac{\rho v D}{\eta}$$

or $$N_R \propto \frac{\rho v D}{\eta}.$$

EXAMPLE 59. *Derive by the method of dimensions, an expression for the volume of a liquid flowing out per second through a narrow pipe. Assume that the rate of flow of liquid depends on*

(i) the coefficient of viscosity 'η' of the liquid

(ii) the radius 'r' of the pipe and

(iii) the pressure gradient $\left(\frac{p}{l}\right)$ *along the pipe.*

Take $K = \frac{\pi}{8}$.

Solution. Let volume flowing out per second through the pipe be given by

$$V = K\eta^a r^b \left(\frac{p}{l}\right)^c \quad ...(i)$$

where K = a dimensionless constant.

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Dimensions of the various quantities are

$$[V] = \frac{\text{volume}}{\text{time}} = \frac{L^3}{T} = L^3T^{-1}$$

$$[\eta] = ML^{-1}T^{-1},\ [r] = L$$

$$\left[\frac{p}{l}\right] = \frac{\text{pressure}}{\text{length}} = \frac{\text{force}}{\text{area} \times \text{length}}$$

$$= \frac{MLT^{-2}}{L^2 . L} = ML^{-2}T^{-2}$$

Substituting these dimensions in equation (*i*), we get

$$[L^3T^{-1}] = [ML^{-1}T^{-1}]^a\ [L]^b\ [ML^{-2}T^{-2}]^c$$

or $$M^0L^3T^{-1} = M^{a+c}\ L^{-a+b-2c}\ T^{-a-2c}$$

Equating the powers of M, L and T, we get

$$a + c = 0,\ -a + b - 2c = 3,\ -a - 2c = -1$$

On solving,

$$a = -1,\ b = 4,\ c = 1$$

$$\therefore \quad V = K\eta^{-1} r^4 \left(\frac{p}{l}\right)^1$$

or $$V = \frac{\pi r^4 p}{8\eta l}$$ [Poiseuille's equation]

EXAMPLE 60. *The period of vibration of a tuning fork depends on the length l of its prong, density d and Young's modulus Y of its material. Deduce an expression for the period of vibration on the basis of dimensions.*

Solution. Let $$T = K l^a\ d^b\ Y^c$$ ...(*i*)

where K = a dimensionless constant.

Dimensions of various quantities are

$$[T] = T,\ [l] = L,\ [d] = ML^{-3},\ [Y] = ML^{-1}T^{-2}$$

Substituting these dimensions in equation (*i*), we get

$$T = [L]^a\ [ML^{-3}]^b\ [ML^{-1}T^{-2}]^c$$

or $$M^0L^0T^1 = M^{b+c}\ L^{a-3b-c}\ T^{-2c}$$

Equating the powers of M, L and T, we get

$$b + c = 0,\ a - 3b - c = 0,\ -2c = 1$$

On solving,

$$a = 1,\ b = \frac{1}{2},\ c = -\frac{1}{2}$$

$$\therefore \quad T = K l^1 d^{1/2} Y^{-1/2} = Kl\sqrt{\frac{d}{Y}}.$$

EXAMPLE 61. *The frequency ν of an oscillating drop may depend upon radius r of the drop, density ρ of the liquid and surface tension S of the liquid. Establish an expression for ν dimensionally.* **[Delhi 13]**

Solution. Let $$\nu = K r^a\ \rho^b\ S^c$$ ...(*i*)

where K = a dimensionless constant.

Dimensions of various quantities are

$$[\nu] = T^{-1},\ [r] = L,\ [\rho] = ML^{-3},\ [S] = MT^{-2}$$

Substituting these dimensions in equation (i), we get

$$T^{-1} = [L]^a\ [ML^{-3}]^b\ [MT^{-2}]^c$$

or $$M^0L^0T^{-1} = M^{b+c}\ L^{a-3b}\ T^{-2c}$$

Equating the powers of M, L and T on both sides,

$$b + c = 0,\ a - 3b = 0,\ -2c = -1$$

On solving,

$$a = -\frac{3}{2},\ b = -\frac{1}{2},\ c = \frac{1}{2}$$

$$\therefore \quad \nu = Kr^{-3/2}\rho^{-1/2} S^{1/2} = K\sqrt{\frac{S}{\rho r^3}}.$$

EXAMPLE 62. *The escape velocity v of a body depends upon (i) the acceleration due to gravity of the planet and (ii) the radius of the planet R. Establish dimensionally the relationship between v, g and R.* **[Himachal 06 ; Delhi 05]**

Solution. Let $$v = Kg^a\ R^b$$

where $K = a$ dimensionless constant.

Putting the dimensions,

$$LT^{-1} = [LT^{-2}]^a [L]^b = L^{a+b}T^{-2a}$$

Equating the powers of L and T,

$$a + b = 1,\ -2a = -1$$

$$\therefore \quad a = \frac{1}{2},\ b = \frac{1}{2}$$

Hence $$v = K g^{1/2}\ R^{1/2} = K\sqrt{gR}.$$

EXAMPLE 63. *A large fluid star oscillates in shape under the influence of its own gravitational field. Using dimensional analysis, find the expression for period of oscillation (T) in terms of radius of star (R), mean density of fluid (ρ) and universal gravitational constant G.* **[Chandigarh 04]**

Solution. Let $$T = KR^a\rho^b G^c$$

where K is a dimensionless constant.

Putting the dimensions,

$$T^1 = [L]^a [ML^{-3}]^b [M^{-1}L^3T^{-2}]^c$$

or $$M^0L^0T^1 = M^{b-c}L^{-3b+3c+a}T^{-2c}$$

Equating the powers of M, L and T, we get

$$b - c = 0,\ -3b + 3c + a = 0,\ -2c = 1$$

On solving,

$$a = 0,\ b = -\frac{1}{2},\ c = -\frac{1}{2}$$

$$\therefore \quad T = KR^0\rho^{-1/2}G^{-1/2} = K\frac{1}{\sqrt{\rho G}}.$$

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PROBLEMS FOR PRACTICE

1. The wavelength λ associated with a moving electron depends on its mass m, its velocity v and Planck's constant h. Prove dimensionally that $\lambda \propto \frac{h}{mv}$.
[Himachal 04 ; Chandigarh 08]

2. Obtain an expression for the centripetal force F acting on a particle of mass m moving with velocity v in a circle of radius r. Take dimensionless constant $K = 1$
[Himachal 2000]
$\left(\text{Ans. } F = \frac{mv^2}{r}\right)$

3. The orbital velocity v of a satellite may depend on its mass m, the distance r from the centre of the earth and acceleration due to gravity g. Obtain an expression for its orbital velocity. (Ans. $v = K\sqrt{rg}$)

4. A small spherical ball of radius r falls with velocity v through a liquid having coefficient of viscosity η. Find the viscous drag F on the ball assuming it depends on η, r and v. Take $K = 6\pi$.
(Ans. $F = 6\pi\eta rv$)

5. The velocity of a freely falling body is a function of the distance fallen through (h) and acceleration due to gravity g. Show by the method of dimensions that $v = K\sqrt{gh}$.

6. Using the method of dimensions, derive an expression for the energy of a body executing SHM ; assuming this energy depends upon its mass m, frequency ν and amplitude of vibration r. [Himachal 06]
(Ans. $E = Km\nu^2 r^2$)

7. A body of mass m hung at one end of the spring executes SHM. Prove that the relation $T = 2\pi m/k$ is incorrect, where k is the force constant of the spring. Also derive the correct relation. (Ans. $T = K\sqrt{m/k}$)

8. Assuming that the critical velocity v_c of a viscous liquid flowing through a capillary tube depends only upon the radius r of the tube, density ρ and the coefficient of viscosity η of the liquid, find the expression for critical velocity. $\left(\text{Ans. } v_c = \frac{K\eta}{r\rho}\right)$

9. By the method of dimensions, obtain an expression for the surface tension S of a liquid rising in a capillary tube. Assume that the surface tension depends upon (*i*) mass m of the liquid (*ii*) pressure p of the liquid and (*iii*) radius r of the capillary tube. Take $K = 1/2$ (Ans. $S = pr/2$)

10. The depth x to which a bullet penetrates a human body depends upon (*i*) coefficient of elasticity η and (*ii*) kinetic energy E_k. By the method of dimensions, show that : $x \propto \left[\frac{E_k}{\eta}\right]^{1/3}$.

11. A U-tube of uniform cross-section contains mercury upto a height h in either limb. The mercury in one limb is depressed a little and then released. Obtain an expression for the time period of oscillation assuming that T depends on h, ρ and g.
$\left(\text{Ans. } T = K\sqrt{\frac{h}{g}}\right)$

12. The critical angular velocity ω_c of a cylinder inside another cylinder containing a liquid at which its turbulence occurs depends on viscosity η, density ρ and the distance d between the walls of the cylinder. Find an expression for ω_c.
$\left(\text{Ans. } \omega_c = \frac{K\eta}{\rho d^2}\right)$

13. A body of mass m is moving in a circle of radius r with angular velocity ω. Find expression for centripetal force acting on it by the method of dimensions. [Himachal 03, 09C] (Ans. $F = Km r\omega^2$)

14. Consider a simple pendulum. The period of oscillation of the simple pendulum depends on its length 'l' and acceleration due to gravity 'g'. Derive the expression for its period of oscillation by the method of dimensions. [Himachal 06, 06C, 07]

HINTS

1. Let $\lambda = Km^a v^b h^c$, then
$M^0L^1T^0 = [M]^a[LT^{-1}]^b[ML^2T^{-1}]^c$
$= M^{a+c}L^{b+2c}T^{-b-c}$
$\therefore \quad a + c = 0, \quad b + 2c = 1, \quad -b - c = 0$
On solving, $a = -1, \quad b = -1, \quad c = 1$
$\therefore \quad \lambda = k\frac{h}{mv}$ or $\lambda \propto \frac{h}{mv}$.

4. Let $F = K\eta^a r^b v^c$, then
$M^1L^1T^{-2} = [ML^{-1}T^{-1}]^a[L]^b[LT^{-1}]^c$
$= M^aL^{-a+b+c}T^{-a-c}$
$\therefore \quad a = 1, \quad -a + b + c = 1, \quad -a - c = -2$
On solving, $a = b = c = 1$
Hence $F = K\eta rv = 6\pi\eta rv$. (Stoke's law)

6. Let $E = Km^a v^b r^c$, then
$M^1L^2T^{-2} = [M]^a[T^{-1}]^b[L]^c = M^aL^cT^{-b}$
$\therefore \quad a = 1, \ b = 2, \ c = 2$ Hence $E = Kmv^2r^2$.

7. $[LHS] = [T] = T$
$[RHS] = \left[\frac{2\pi m}{k}\right] = \frac{M}{MT^{-2}} = T^2$
$\left[\because [k] = \frac{\text{Force}}{\text{Length}} = \frac{MLT^{-2}}{L} = MT^{-2}\right]$
$[LHS] \neq [RHS]$. Hence the relation is incorrect.

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To find the correct relation, let $T = K m^a k^b$, then

$$T^1 = [M]^a [MT^{-2}]^b = M^{a+b} T^{-2b}$$

$\therefore \quad a + b = 0, \quad -2b = 1$

On solving, $a = \frac{1}{2}, \quad b = -\frac{1}{2}$

Hence $T = K m^{1/2} k^{-1/2} = K\sqrt{\frac{m}{k}}$.

8. Let $v_c = K r^a \rho^b \eta^c$, then

$$LT^{-1} = [L]^a [ML^{-3}]^b [ML^{-1} T^{-1}]^c$$

$$= M^{b+c} L^{a-3b-c} T^{-c}$$

$\therefore \quad b + c = 0, \quad a - 3b - c = 1, \quad -c = -1$

On solving,

$$a = -1, \quad b = -1, \quad c = 1$$

$\therefore \quad v_c = K r^{-1} \rho^{-1} \eta^1 = \frac{K\eta}{r\rho}$.

10. Let $x = K\eta^a [F_k]^b$, then

$$L^1 = [ML^{-1} T^{-2}]^a [ML^2 T^{-2}]^b$$

$$= M^{a+b} L^{-a+2b} T^{-2a-2b}$$

$\therefore \quad a + b = 0, \quad -a + 2b = 1, \quad -2a - 2b = 0$

On solving, $\quad a = -\frac{1}{3}, \quad b = \frac{1}{3}$

Hence $x = K\eta^{-1/3} F_k^{1/3} = K\left[\frac{F_k}{\eta}\right]^{1/3}$.

12. Let $\omega_c = K\eta^a \rho^b d^c$, then

$$T^{-1} = [ML^{-1} T^{-1}]^a [ML^{-3}]^b [L]^c$$

$$= M^{a+b} L^{-a-3b+c} T^{-a}$$

$\therefore \quad a + b = 0, \quad -a - 3b + c = 0, \quad -a = -1$

On solving,

$$a = 1, \quad b = -1, \quad c = -2$$

Hence $\omega_c = K\eta^1 \rho^{-1} d^{-2} = \frac{K\eta}{\rho d^2}$.

2.27 LIMITATIONS OF DIMENSIONAL ANALYSIS

49. *Mention the limitations of the method of dimensional analysis.*

Limitations of the method of dimensions :

1. The method does not give any information about the dimensionless constant K.
2. It fails when a physical quantity depends on more than three physical quantities.
3. It fails when a physical quantity (*e.g.*, $s = ut + \frac{1}{2} at^2$) is the sum or difference of two or more quantities.
4. It fails to derive relationships which involve trigonometric, logarithmic or exponential functions.
5. Sometimes, it is difficult to identify the factors on which the physical quantity depends. The method becomes more complicated when dimensional constants like G, h, etc. are involved.

2.28 SIGNIFICANT FIGURES

50. *What is meant by significant figures in a measured quantity ?*

Significant figures. *The significant figures are normally those digits in a measured quantity which are known reliably or about which we have confidence in our measurement plus one additional digit that is uncertain.* The larger the number of significant figures in a measurement, the higher is the accuracy of the measurement. Suppose the time period of a simple pendulum is 1.62s. This digits 1 and 6 are reliable and certain, while the digit 2 is uncertain. So the time period has three significant figures. Again, suppose the length of an object is measured as 273. 6 cm. It has four significant figures. The digits 2, 7 and 3 are reliable while the digit 6 is uncertain.

51. *State the rules for counting the number of significant figures in a measured quantity.*

Rules for determining the number of significant figures :

(*i*) *All non-zero digits are significant.* So 13.75 has four significant figures.

(*ii*) *All zeros between two non-zero digits are significant.* Thus 100.05 km has five significant figures.

(*iii*) *All zeros to the right of a non-zero digit but to the left of an understood decimal point are not significant.* For example, 86400 has three significant figures. But *such zeros are significant if they come from a measurement.* For example, 86400 s has five significant figures.

(*iv*) *All zeros to the right of a non-zero digit but to the left of a decimal point are significant.* For example, 648700. has six significant figure.

(*v*) *All zeros to the right of a decimal point are significant.* So 161 cm, 161.0 cm and 161.00 cm have three, four and five significant figures respectively.

(*vi*) *All zeros to the right of a decimal point but to the left of a non-zero digit are not significant.* So 0.161 cm and 0.0161 cm, both have three significant figures. Moreover, zero conventionally placed to the left of the decimal point is not significant.

(*vii*) *The number of significant figures does not depend on the system of units.* So 16.4 cm, 0.164 m and 0.000164 km, all have three significant figures.

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For Your Knowledge

▲ **In scientific notation**, a number is expressed in the power of 10 as $a \times 10^b$, where a is the number between 1 and 10, and b is any positive or negative exponent of 10. The decimal point is written after the first digit. Suppose the length of a rod is reported as 3.500 m. In scientific notation, it can be expressed in different units as

$$3500 \text{ m} = 3.500 \times 10^2 \text{ cm} = 3.500 \times 10^3 \text{ mm} = 3.500 \times 10^{-3} \text{ km}.$$

▲ We retain only those zeros in the base number which are the result of a measurement. Now *the power of 10 is not relevant to the determination of significant figures.* Each of the above numbers has four significant figures.

52. *State the rules for rounding off a measurement.*

Rules for rounding off a measurement :

(*i*) If the digit to be dropped is smaller than 5, then the preceding digit is left unchanged.

(*ii*) If the digit to be dropped is greater than 5, then the preceding digit is increased by 1.

(*iii*) If the digit to be dropped is 5 followed by non-zero digits, then the preceding digit is increased by 1.

(*iv*) If the digit to be dropped is 5, then the preceding digit is left unchanged if it is even.

(*v*) If the digit to be dropped is 5, then the preceding digit is increased by 1 if it is odd.

53. *State the rules for determining the significant figures in the sum, difference, product and quotient of two numbers.*

Arithmetic operations with significant figures. The result of an arithmetic operation involving measured values of quantities cannot be more accurate than the measured values themselves. So certain rules have to be followed while doing arithmetic operations with significant figures so as to ensure that the precision of the final result is consistent with the precision of the original measured values.

1. ***Significant figures in the sum or difference of two numbers.*** *In addition or subtraction, the final result should be reported to the same number of decimal places as that of the original number with minimum number of decimal places.*

2. ***Significant figures in the product or quotient of two numbers.*** *In multiplication or division, the final result should be reported to the same number of significant figures as that of the original number with minimum number of significant figures.*

Examples based on Significant Figures

Formulae Used

1. **Rules for rounding off a measurement.** Refer to Q. 52.
2. **Rules for counting significant figures.** Refer to Q. 51.
3. **Significant figures in the sum or difference of two numbers.** The final result should be reported to the same number of decimal places as that of the number with minimum number of decimal places.
4. **Significant figures in the product or quotient of two numbers.** The final result should be reported to the same number of significant figures as that of the number with minimum number of significant figures.

Example 64. State the number of significant figures in the following : (i) 2.000 m (ii) 5100 kg (iii) 0.050 cm.

Solution. (*i*) Four : 2, 0, 0, 0 (*ii*) Four : 5, 1, 0, 0
(*iii*) Two : 5, 0.

Example 65. Round off the following numbers as indicated :

(*i*) 18.35 *upto 3 digits* (*ii*) 143.45 *upto 4 digits*
(*iii*) 18967 *upto 3 digits* (*iv*) 12.653 *upto 3 digits*
(*v*) 248337 *upto 3 digits* (*vi*) 321.135 *upto 5 digits*
(*vii*) 101.55×10^6 *upto 4 digits*
(*viii*) 31.325×10^{-5} *upto 4 digits.*

Solution. (*i*) 18.4 (*ii*) 143.4 (*iii*) 19000
(*iv*) 12.7 (*v*) 248000 (*vi*) 321.14
(*vii*) 101.6×10^6 (*viii*) 31.32×10^{-5}.

Example 66. Add 7.21, 12.141 and 0.0028, and express the result to an appropriate number of significant figures.

Solution.

$$\begin{array}{rl} & 7.21 \\ & +12.141 \\ & +0.0028 \\ \hline \text{Sum} & =19.3538 \end{array}$$

Corrected sum = 19.35 [Rounded off upto 2nd decimal place]

Here 7.21 has minimum number of decimal places (two), so result is rounded off upto second place of decimal point.

Example 67. Subtract 4.27153 from 6.807 and express the result to an appropriate number of significant figures.

Solution.

$$\begin{array}{rl} & 6.807 \\ & -4.27153 \\ \hline \text{Difference} & =2.53547 \end{array}$$

Corrected difference = 2.535 [Rounded off upto 3rd decimal place]

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Here 6.807 has the lesser number of decimal places (three), so difference is rounded off to 3rd place of decimal point.

EXAMPLE 68. *Subtract 2.5×10^{-6} from 4.0×10^{-4} with due regard to significant figures.*

Solution. Le $x = 2.5 \times 10^{-6} = 0.0000025$ (2 significant figures)

$y = 4.0 \times 10^{-4} = 0.00040$ (2 significant figures)

$\therefore \quad y - x = 0.00040 - 0.0000025 = 0.0003975$

$= 3.975 \times 10^{-4} = \mathbf{4.0 \times 10^{-4}}$

[Rounded off upto 2 significant figures]

EXAMPLE 69. *Solve the following and express the result to an appropriate number of significant figures :*

(i) Add 6.2 g, 4.33 g and 17.456 g.

(ii) Subtract 63.54 kg from 187.2 kg

(iii) $75.5 \times 125.2 \times 0.51$.

(iv) $\dfrac{2.13 \times 24.78}{458.2}$ *(v) $\dfrac{2.51 \times 10^{-4} \times 1.81 \times 10^{7}}{0.4463}$*

Solution.

(i) $6.2\ g + 4.33\ g + 17.456\ g = 27986 = \mathbf{28.0\ g}$.

[Rounded off to first decimal place]

(ii) $187.2\ kg - 63.54\ kg = 123.66\ kg = \mathbf{123.7\ kg}$.

[Rounded off to first decimal place]

(iii) $75.5 \times 125.2 \times 0.51 = 4820.826 = \mathbf{4800}$.

[Rounded off upto two significant figures]

(iv) $\dfrac{2.13 \times 24.78}{458.2} = 0.115193 = \mathbf{0.115}$.

[Rounded off to 3 significant figures]

(v) $\dfrac{2.51 \times 10^{-4} \times 1.81 \times 10^{7}}{0.4463} = 10.1795 \times 10^{3}$

$= \mathbf{10.2 \times 10^{3}}$.

[Rounded off to 3 significant figures]

EXAMPLE 70. *Each side of a cube is measured to be 7.203 m. What are the total surface area and the volume of the cube to appropriate significant figures ?* **[NCERT]**

Solution. Side of the cube = 7.203 m

Total surface area

$= 6 \times \text{side}^2 = 6 \times (7.203)^2\ m$

$= 311.299254\ m^2 = 3113\ m^2$

[Rounded off to 4 significant figures]

Volume $= \text{side}^3 = (7.203)^3 = 373.714754\ m^3$

$= \mathbf{373.7\ m^3}$.

[Rounded off to 4 significant figures]

EXAMPLE 71. *The radius of a sphere is 1.41 cm. Express its volume to an appropriate number of significant figures.*

Solution. Radius of the sphere, $r = 1.41$ cm (3 significant figures)

Volume of the sphere $= \dfrac{4}{3}\pi r^3$

$= \dfrac{4}{3} \times 3.14 \times (1.41)^3\ cm^3 = 11.736\ cm^3$

$= \mathbf{11.7\ cm^3}$.

[Rounded off upto 3 significant figures]

EXAMPLE 72. *The length and the radius of a cylinder measured with slide callipers are found to be 4.54 cm and 1.75 cm respectively. Calculate the volume of the cylinder.*

Solution. Length of cylinder, $h = 4.54$ cm (3 significant figures)

Radius of cylinder, $r = 1.75$ cm (3 significant figures)

Volume of cylinder

$= \pi r^2 h = 3.14 \times (1.75)^2 \times 4.54\ cm^3$

$= 43.657775\ cm^3 = \mathbf{43.7\ cm^3}$.

[Rounded off upto 3 significant figures]

EXAMPLE 73. *The mass and radius of the earth are 5.975×10^{24} kg and 6.37×10^{6} m respectively. Calculate the average earth's density to correct significant figures. Take $\pi = 3.142$.*

Solution. Radius of earth, $R = 6.37 \times 10^{6}$ m (3 significant figures)

Mass of earth, $M = 5.975 \times 10^{24}$ kg (4 significant figures)

Average density

$= \dfrac{M}{V} = \dfrac{M}{\frac{4}{3}\pi R^3} = \dfrac{5.975 \times 10^{24}}{\frac{4}{3} \times 3.142 \times (6.37 \times 10^{6})^3}$

$= 0.005517 \times 10^{6}\ kgm^{-3}$

$= \mathbf{5.52 \times 10^{3}\ kg\ m^{-3}}$.

[Rounded off upto 3 significant figures]

EXAMPLE 74. *5.74 g of a substance occupies 1.2 cm^3. Express its density keeping significant figures in view.* **[NCERT]**

Solution. Density

$= \dfrac{\text{Mass}}{\text{Volume}} = \dfrac{5.74\ g}{1.2\ cm^3} = 4.783\ gcm^{-3}$

$= \mathbf{4.8\ g\ cm^{-3}}$.

[Rounded of upto 2 significant figures]

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PROBLEMS FOR PRACTICE

1. State the number of significant figures in the following measurements :
   (*i*) 0.009 m^2 (*ii*) 5.049 Nm^{-2} (*iii*) 0.1890 g cm^{-3}
   (*iv*) 1.90×10^{11} kg (*v*) 0.020800 m (*vi*) 5.308 J.
   [**Ans.** (*i*) 1 (*ii*) 4 (*iii*) 4 (*iv*) 3 (*v*) 5 (*vi*) 4]
2. Subtract 2.5×10^4 from 3.9×10^5 with due regard to significant figures. (**Ans.** 3.7×10^5)
3. Round off the following numbers as indicated :
   (*i*) 15.654 upto 3 digits (*ii*) 15.75 upto 3 digits
   (*iii*) 15.654 upto 4 digits (*iv*) 15.65 upto 3 digits
   (*v*) 142667 upto 5 digits (*vi*) 5.996×10^5 upto 3 digits.
   (*vii*) 0.7995 upto 1 digit
   (*viii*) 2.5946×10^{-4} upto 2 digits.
   [**Ans.** (*i*) 15.7 (*ii*) 15.8 (*iii*) 15.65 (*iv*) 15.6 (*v*) 142670 (*vi*) 6.00×10^5 (*vii*) 0.8 (*viii*) 2.6×10^{-4}]
4. A jeweller puts a diamond in a box weighing 1.2 kg. Find the total weight of the box and diamond with due regard to significant figures, if the weight of diamond is 5.42 g. (**Ans.** 1.2 kg)
5. The diameter of a circle is 1.06 m. Calculate the area to an appropriate number of significant figures. Take $\pi = 3.14$. (**Ans.** 0.882 m^2)
6. The radius of a solid sphere is measured as 11.24 cm. What is the surface area of the sphere to appropriate significant figures ? **[Delhi 02]**
   (**Ans.** 1588 cm^2)
7. The mass of a body is 275.32 g and its volume is 36.41 cm^3. Express its density upto appropriate significant figures. (**Ans.** 7.562 gcm^{-3})
8. 9.74 g of a substance occupies 1.2 cm^3. Express its density by keeping the significant figures in view. **[Central Schools 09]**
   (**Ans.** 8.1 gcm^{-3})

2.29 ACCURACY AND PRECISION

54. *Distinguish between the terms precision and accuracy of a measurement.*

Every measurement is limited by the *reliability of the measuring instrument and skill of the person making the measurement.* If we repeat a particular measurement, we usually do not get precisely the same result as each result is subject to some experimental error. This imperfection in measurement can be described in two ways :

1. ***Accuracy.*** *It refers to the closeness of a measurement to the true value of the physical quantity.* It indicates the relative freedom from errors. As we reduce the errors, the measurement becomes more accurate.

2. ***Precision.*** *It refers to the resolution or the limit to which the quantity is measured.* Precision is determined by the least count of the measuring instrument. *The smaller the least count, greater is the precision.* If we repeat a particular measurement of a quantity a number of times, then the precision refers to the closeness of the set of values so obtained.

We can illustrate the difference between accuracy and precision with the help of an example. Suppose three students are asked to find the mass of a piece of metal whose mass is known to be 0.520 g. They obtain the data given in Table 2.9.

Table 2.9 *Data to illustrate accuracy and precision*

Student	Measurement 1	Measurement 2	Measurement 3	Average mass
A	0.52 g	0.51 g	0.50 g	0.51 g
B	0.516 g	0.515 g	0.514 g	0.515 g
C	0.521 g	0.520 g	0.520 g	0.520 g

The data obtained by the student *A* are neither very precise nor accurate, the individual values differ widely and also the average value is not accurate. The data for student *B* are more precise, as they vary slightly from one another but the average mass is not accurate. The data for student *C* are both precise and accurate. The resolution for *A* is 0.01 g and that for *B* or *C* is 0.001 g.

2.30 ERRORS IN A MEASUREMENT

55. *What do you mean by error in a measurement ? Briefly explain the different types of errors and their causes. How can these errors be minimised ?*

Error in a measurement. Every measurement is done with the help of some instrument. While making the measurement, some uncertainty gets introduced in the measurement. As a result, the measured value is always somewhat different from the actual or true value. *The error in a measurement is equal to the difference between the true value and the measured value of the quantity.*

Error = True value – Measured value

An error gives an indication of the limits within which the true value may lie. Every measurement has an error. Every calculated value which is based on measured values has an error.

Different types of errors :

1. Constant errors. *The errors which affect each observation by the same amount are called constant errors.* Such errors are due to the faulty calibration of the scale

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of the measuring instrument. Such errors can be eliminated by measuring the same physical quantity by a number of different methods, apparatus or technique.

2. Systematic errors. *The errors which tend to occur in one direction, either positive or negative, are called systematic errors.* We can eliminate such errors once we know the rule which governs them. These errors may be of the following types :

(*i*) ***Instrumental errors.*** *These errors occur due to the inbuilt defect of the measuring instrument.* For example, wearing off the metre scale at one end, zero error in a vernier callipers (zero of the vernier scale may not coincide with the zero of main scale), etc. This error can be detected by measuring a physical quantity with two different instruments of the same type or by measuring the same physical quantity by two different methods.

(*ii*) ***Imperfections in experimental technique.*** *These errors are due to the limitations of the experimental arrangement.* For example, error due to radiation loss in calorimetric experiments, error due to buoyancy of air when we weigh a body in air. Such errors cannot be eliminated altogether but necessary corrections can be applied for them.

(*iii*) ***Personal errors.*** *These errors arise due to individual's bias, lack of proper setting of apparatus or individual's carelessness in taking observations without observing proper precautions,* etc. For example, when an observer (by habit) holds his head towards right, while reading a scale, he introduces some error due to *parallax.* Such errors can be minimised if measurements are repeated by different persons or removing the personal bias as far as possible.

(*iv*) ***Errors due to external causes.*** *These errors arise due to the change in external conditions* like pressure, temperature, wind, etc. For example, the expansion of a scale due to the increase in temperature. Such errors can be easily detected and necessary corrections may be made accordingly. These errors can also be minimised by controlling the external conditions during the experimentation.

3. Random errors. *The errors which occur irregularly and at random, in magnitude and direction, are called random errors.* Such errors occur by chance and arise due to slight variation in the attentiveness of the observer while taking the readings or because of slight variations in the experimental conditions. For example, if a person repeats the observation a number a times, he may get different readings everytime. Random errors have almost equal chances for both positive and negative errors. Hence the *arithmetic mean* of a large number of observations can be taken as the true value of the measured quantity.

4. Least count error. *This error is due to the limitations imposed by the least count of the measuring instrument.* It is an uncertainty associated with the resolution of the measuring instrument. *The smallest division on the scale of the measuring instrument is called its least count.* For example, a metre scale has a least count of 1 mm, i.e., readings are good only upto this value. The error in reading will be half of this value *i.e.*, ± 0.5 mm $= \pm 0.05$ cm.

5. Gross errors or mistakes. These errors are due to either carelessness of the person or due to improper adjustment of the apparatus. No corrections can be applied for gross errors.

For Your Knowledge

- Least count errors are random errors but within a limited size ; they occur with both random and systematic errors.
- The accuracy of measurement is related to the systematic errors but its precision is related to the random errors, which include least count error also.

2.31 ABSOLUTE ERROR, RELATIVE ERROR AND PERCENTAGE ERROR

56. *How is random error eliminated ? What do you mean by (i) absolute error (ii) mean absolute error (iii) relative error and (iv) percentage error ?*

Elimination of error. *Normal* or *Gaussian law* of random errors shows that the probability of occurrence of positive and negative errors is same, so random error can be minimised by repeating measurements a large number of times. Then the *arithmetic mean of all measurements can be taken as the true value of the measured quantity.*

If $a_1, a_2, a_3, \ldots a_n$ be the n measured values of a physical quantity, then its true value, $\bar{a}$ is given by the arithmetic mean,

$$\bar{a} \text{ or } a_{mean} = \frac{a_1 + a_2 + a_3 + \ldots + a_n}{n} = \frac{1}{n} \sum_{i=1}^{n} a_i .$$

(*i*) ***Absolute error.*** *The magnitude of the difference between the true value of the quantity measured and the individual measured value is called absolute error.*

If we take arithmetic mean $\bar{a}$ as the true value, then the absolute errors in the individual measured values will be

$$\Delta a_1 = \bar{a} - a_1 \qquad \Delta a_2 = \bar{a} - a_2$$
$$\Delta a_3 = \bar{a} - a_3 \qquad \Delta a_n = \bar{a} - a_n$$

(*ii*) ***Mean or final absolute error.*** *The arithmetic mean of the positive magnitudes of all the absolute errors is called mean absolute error.* It is given by

$$\Delta \bar{a} = \frac{|\Delta a_1| + |\Delta a_2| + \ldots + |\Delta a_n|}{n} = \frac{1}{n} \sum_{i=1}^{n} |\Delta a_i|$$

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Thus the final result of the measure of a physical quantity can be expressed as $a = \bar{a} \pm \Delta\bar{a}$.

Clearly, any measured value of a will be such that

$$\bar{a} - \Delta\bar{a} \le a \le \bar{a} + \Delta\bar{a}$$

(iii) ***Relative error.*** *The ratio of the mean absolute error to the true value of the measured quantity is called relative error.*

$$\text{Relative error, } \delta a = \frac{\Delta\bar{a}}{\bar{a}}$$

(iv) ***Percentage error.*** *The relative error expressed in percent is called percentage error.*

$$\text{Percentage error} = \frac{\Delta\bar{a}}{\bar{a}} \times 100\%$$

For Your Knowledge

- The unit of absolute error is same as that of the quantity being measured.
- It is the relative error or the percentage error and not the absolute error which truly indicates the accuracy of a measurement.

Examples based on Errors in Measurements

FORMULAE USED

1. *True value.* If $a_1, a_2, a_3, \ldots\ldots a_n$ are the readings of an experiment, then true value of the quantity is given by the arithmetic mean,
$$\bar{a} = \frac{a_1 + a_2 + a_3 + \ldots\ldots + a_n}{n} = \frac{1}{n}\sum_{i=1}^{n} a_i.$$
2. Absolute error = True value – Measured value
or $\Delta a_i = \bar{a} - a_i$
3. Final absolute error
= Arithmetic mean of absolute errors
$$\Delta\bar{a} = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + \ldots. + |\Delta a_n|}{n} = \frac{1}{n}\sum_{i=1}^{n} |\Delta a_i|$$
4. Relative error or fractional error
$$= \frac{\text{Final absolute error}}{\text{True value}} \quad \text{or} \quad \delta a = \frac{\Delta\bar{a}}{\bar{a}}$$
5. Percentage error $= \frac{\Delta\bar{a}}{\bar{a}} \times 100\%$

EXAMPLE 75. The length of a rod as measured in an experiment was found to be 2.48 m, 2.46 m, 2.49 m, 2.50 m, and 2.48 m. Find the average length, the absolute error in each observation and the percentage error.

Solution. Average length

$$= \frac{2.48 + 2.46 + 2.49 + 2.50 + 2.48}{5} = \frac{12.41}{5} \text{ m}$$

$= 2.482 \text{ m} = 2.48 \text{ m}$

[Rounded off to 2 places of decimal point]

The absolute errors in the different measurements are

$$\Delta L_1 = 2.48 - 2.48 = 0.00 \text{ m}$$
$$\Delta L_2 = 2.48 - 2.46 = 0.02 \text{ m}$$
$$\Delta L_3 = 2.48 - 2.49 = -0.01 \text{ m}$$
$$\Delta L_4 = 2.48 - 2.50 = -0.02 \text{ m}$$
$$\Delta L_5 = 2.48 - 2.48 = 0.00 \text{ m}$$

$$\text{The absolute error} = \frac{\Sigma|\Delta L|}{5}$$

$$= \frac{0.00 + 0.02 + 0.01 + 0.02 + 0.00}{5} = \frac{0.05}{5} = 0.01 \text{ m}$$

$\therefore$ Correct length $= \mathbf{2.48 \pm 0.01\ m}$

$$\text{Percentage error} = \frac{0.01}{2.48} \times 100 = \mathbf{0.40\%}.$$

EXAMPLE 76. In successive measurements, the readings of the period of oscillation of a simple pendulum were found to be 2.63 s, 2.56 s, 2.42 s, 2.71 s and 2.80 s in an experiment. Calculate (i) mean value of the period of oscillation (ii) absolute error in each measurement (iii) mean absolute error (iv) relative error (v) percentage error and (vi) express the result in proper form. **[NCERT]**

Solution. (i) Mean period of oscillation,

$$\bar{T} = \frac{2.63 + 2.56 + 2.42 + 2.71 + 2.80}{5}$$

$$= \frac{13.12}{5} = 2.624 \text{ s}$$

$= 2.62$ s [Rounded off to 2nd decimal place]

(ii) Absolute errors in different measurements are

$$\Delta T_1 = 2.62 - 2.63 = -0.01 \text{ s}$$
$$\Delta T_2 = 2.62 - 2.56 = 0.06 \text{ s}$$
$$\Delta T_3 = 2.62 - 2.42 = 0.20 \text{ s}$$
$$\Delta T_4 = 2.62 - 2.71 = -0.09 \text{ s}$$
$$\Delta T_5 = 2.62 - 2.80 = -0.18 \text{ s}$$

(iii) Mean absolute error $= \frac{\Sigma|\Delta T_i|}{n}$

$$\Delta\bar{T} = \frac{0.01 + 0.06 + 0.20 + 0.09 + 0.18}{5}$$

$$= \frac{0.54}{5} = 0.108 \text{ s} = \mathbf{0.11\ s}.$$

[Rounded off to 2nd decimal place]

(iv) Relative error,

$$\delta T = \frac{\Delta\bar{T}}{\bar{T}} = \frac{0.11}{2.62} = 0.04198 = \mathbf{0.04}.$$

(v) Percentage error in $T = 0.04 \times 100 = \mathbf{4\%}$.

(vi) In terms of absolute error, $T = \mathbf{(2.62 \pm 0.11)\ s}$.

In terms of percentage error, $T = \mathbf{(2.62 \pm 4\%)\ s}$.

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EXAMPLE 77. *In an experiment, refractive index of glass was observed to be 1.45, 1.56, 1.54, 1.44, 1.54 and 1.53. Calculate (i) Mean value of refractive index ; (ii) Mean absolute error ; (iii) Fractional error ; (iv) Percentage error. Express the result in terms of absolute error and percentage error.* [Chandigarh 03]

Solution. (i) Mean value of refractive index,

$$\bar{\mu} = \frac{1.45+1.56+1.54+1.44+1.54+1.53}{6} = 1.51.$$

(ii) Absolute errors in different measurements are

$$\Delta\mu_1 = 1.51 - 1.45 = 0.06$$
$$\Delta\mu_2 = 1.51 - 1.56 = -0.05$$
$$\Delta\mu_3 = 1.51 - 1.54 = -0.03$$
$$\Delta\mu_4 = 1.51 - 1.44 = 0.07$$
$$\Delta\mu_5 = 1.51 - 1.54 = -0.03$$
$$\Delta\mu_6 = 1.51 - 1.53 = -0.02$$

Mean absolute error,

$$\Delta\bar{\mu} = \frac{\Sigma|\Delta\mu_i|}{n} = \frac{0.06+0.05+0.03+0.07+0.03+0.02}{6}$$
$$= 0.26/6 = 0.0433 \approx 0.04.$$

(iii) Fractional error,

$$\delta\mu = \frac{\Delta\bar{\mu}}{\bar{\mu}} = \frac{0.04}{1.51} = 0.02649 = 0.03.$$

(iv) Percentage error $= \frac{\Delta\bar{\mu}}{\bar{\mu}} \times 100 = 3\%$

In terms of absolute error, $\mu = 1.51 \pm 0.04$

In terms of percentage error, $\mu = 1.51 \pm 3\%$.

PROBLEMS FOR PRACTICE

1. The diameter of a wire as measured by a screw gauge was found to be 0.026 cm, 0.028 cm, 0.029 cm, 0.027 cm, 0.024 cm and 0.027 cm. Calculate (*i*) mean value of the diameter (*ii*) mean absolute error (*iii*) relative error (*iv*) percentage error. Also express the result in terms of absolute error and percentage error. [**Ans.** (*i*) 0.027 cm (*ii*) 0.001 cm (*iii*) 0.037 (*iv*) 3.7%; (0.027 ± 0.001) cm, (0.027 ± 3.7%) cm]

2. The refractive index of water as measured by the relation $\mu = \frac{\text{Real depth}}{\text{Apparent depth}}$ was found to have the values 1.29, 1.33, 1.34, 1.35, 1.32, 1.36, 1.30, 1.33. Calculate (*i*) mean value of μ (*ii*) mean value of absolute error (*iii*) relative error (*iv*) percentage error.

[**Ans.** (*i*) 1.33 (*ii*) 0.02 (*iii*) 0.015 (*iv*) 1.5%]

3. In an experiment to measure focal length of a concave mirror, the value of focal length in successive observations turns out to be 17.3 cm, 17.8 cm, 18.3 cm, 18.2 cm, 17.9 cm and 18.0 cm. Calculate the mean absolute error and percentage error. Express the result in a proper way.

[**Ans.** 0.25 cm, 1.4%, (17.9 ± 0.25) cm]

2.32 COMBINATION OF ERRORS

57. *How can we estimate the error in the (i) sum (ii) difference (iii) product (iv) quotient (v) power of different measured quantities. Deduce the general rule for evaluating the error in a combined calculation.*

Propagation or combination of errors. An experiment involves several measurements and the final result is arrived at using different physical relations. The error in the final result depends on the individual measurements as well as the mathematical operations done to get the final result. Following rules are used to evaluate *maximum permissible* error in a measurement.

(i) *Error in the sum of two quantities.* Let ΔA and ΔB be the absolute errors in the two quantities A and B respectively. Then

Measured value of $A = A \pm \Delta A$

Measured value of $B = B \pm \Delta B$

Consider the sum, $Z = A + B$

The error ΔZ in Z is then given by

$$Z \pm \Delta Z = (A \pm \Delta A) + (B \pm \Delta B)$$
$$= (A + B) \pm (\Delta A + \Delta B)$$
$$= Z \pm (\Delta A + \Delta B)$$

or $$\Delta Z = \Delta A + \Delta B$$

Hence the rule. *The maximum possible error in the sum of two quantities is equal to the sum of the absolute errors in the individual quantities.*

(ii) *Error in the difference of two quantities.* Consider the difference,

$$Z = A - B$$

The error ΔZ in Z is given by

$$Z \pm \Delta Z = (A \pm \Delta A) - (B \pm \Delta B)$$
$$= (A - B) \pm \Delta A \mp \Delta B$$
$$= Z \pm \Delta A \mp \Delta B$$

For error ΔZ to be maximum, ΔA and ΔB must have the same sign, therefore

$$\Delta Z = \Delta A + \Delta B$$

Hence the rule. *The maximum error in the difference of two quantities is equal to the sum of the absolute errors in the individual quantities.*

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(iii) Error in the product of two quantities. Consider the product,

$$Z = AB$$

The error ΔZ in Z is given by

$$Z \pm \Delta Z = (A \pm \Delta A)(B \pm \Delta B)$$

$$= AB \pm A\Delta B \pm B\Delta A \pm \Delta A . \Delta B$$

Dividing L.H.S. by Z and R.H.S. by AB, we get

$$1 \pm \frac{\Delta Z}{Z} = 1 \pm \frac{\Delta B}{B} \pm \frac{\Delta A}{A} \pm \frac{\Delta A}{A} \cdot \frac{\Delta B}{B}$$

As $\frac{\Delta A}{A}$ and $\frac{\Delta B}{B}$ are small quantities, their product term can be neglected. The maximum fractional error in Z is

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

Hence the rule. *The maximum fractional error in the product of two quantities is equal to the sum of the fractional errors in the individual quantities.*

(iv) **Error in the division or quotient.** Consider the quotient,

$$Z = \frac{A}{B}$$

The error ΔZ in Z is given by

$$Z \pm \Delta Z = \frac{A \pm \Delta A}{B \pm \Delta B} = \frac{A\left(1 \pm \frac{\Delta A}{A}\right)}{B\left(1 \pm \frac{\Delta B}{B}\right)}$$

$$= \frac{A}{B}\left(1 \pm \frac{\Delta A}{A}\right)\left(1 \pm \frac{\Delta B}{B}\right)^{-1}$$

or $$Z \pm \Delta Z = Z\left(1 \pm \frac{\Delta A}{A}\right)\left(1 \mp \frac{\Delta B}{B}\right)$$

$[\because (1 + x)^n = 1 + nx$, when $x << 1]$

Dividing both sides by Z, we get

$$1 \pm \frac{\Delta Z}{Z} = \left(1 \pm \frac{\Delta A}{A}\right)\left(1 \mp \frac{\Delta B}{B}\right)$$

$$= 1 \pm \frac{\Delta A}{A} \mp \frac{\Delta B}{B} \pm \frac{\Delta A}{A} \cdot \frac{\Delta B}{B}$$

As the terms $\frac{\Delta A}{A}$ and $\frac{\Delta B}{B}$ are small, their product term can be neglected. The maximum fractional error in Z is given by

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

Hence the rule. *The maximum fractional error in the quotient of two quantities is equal to the sum of their individual fractional errors.*

(v) Error in the power of a quantity. Consider the n^{th} power of A,

$$Z = A^n$$

The error ΔZ in Z is given by

$$Z \pm \Delta Z = (A \pm \Delta A)^n = A^n\left(1 \pm \frac{\Delta A}{A}\right)^n$$

$$= Z\left(1 \pm n\frac{\Delta A}{A}\right)$$

$[\because (1 + x)^n = 1 + nx$, when $x << 1]$

Dividing both sides by Z, we get

$$1 \pm \frac{\Delta Z}{Z} = 1 \pm n\frac{\Delta A}{A}$$

or $$\frac{\Delta Z}{Z} = n . \frac{\Delta A}{A}$$

Hence the rule. *The fractional error in the n^{th} power of a quantity is n times the fractional error in that quantity.*

General rule. If $Z = \frac{A^p B^q}{C^r}$, then maximum fractional error in Z is given by

$$\frac{\Delta Z}{Z} = p\frac{\Delta A}{A} + q\frac{\Delta B}{B} + r\frac{\Delta C}{C}$$

The percentage error in Z is given by

$$\frac{\Delta Z}{Z} \times 100 = p\frac{\Delta A}{A} \times 100 + q\frac{\Delta B}{B} \times 100 + r\frac{\Delta C}{C} \times 100$$

Derivation of general rule using differentiation. We have

$$Z = \frac{A^p B^q}{C^r}$$

Taking logarithms, we get

$$\log Z = p \log A + q \log B - r \log C$$

On differentiating both sides, we get

$$\frac{dZ}{Z} = p\frac{dA}{A} + q\frac{dB}{B} - r\frac{dC}{C}$$

Writing the above equation in terms of fractional errors,

$$\pm \frac{\Delta Z}{Z} = \pm p\frac{\Delta A}{A} \pm q\frac{\Delta B}{B} \pm \frac{\Delta C}{C}$$

The maximum permissible error in Z is given by

$$\frac{\Delta Z}{Z} = p\frac{\Delta A}{A} + q\frac{\Delta B}{B} + r\frac{\Delta C}{C}$$

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For Your Knowledge

- In all mathematical operations, the errors are of additive nature.
- When a quantity appears with a power n greater than one in an expression, its error contribution to the final result increases n times. So quantities with higher power in the expression should be measured with maximum accuracy.
- If a quantity appears with a power less than one in an expression, then its error contribution in the final result is reduced.

Examples based on Combination of Errors

Formulae Used

1. If $Z = A + B$, then the maximum possible error in Z,
$$\Delta Z = \Delta A + \Delta B$$
2. If $Z = A - B$, then the maximum possible error in Z,
$$\Delta Z = \Delta A + \Delta B$$
3. If $Z = AB$, then the maximum fractional error in Z
$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$
4. If $Z = A / B$, then the maximum fractional error in Z
$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$
5. If $Z = A^n$, then the maximum fractional error in Z,
$$\frac{\Delta Z}{Z} = n.\frac{\Delta A}{A}$$
6. If $Z = \frac{A^p B^q}{C^r}$, then the maximum fractional error in Z,
$$\frac{\Delta Z}{Z} = p\frac{\Delta A}{A} + q\frac{\Delta B}{B} + r\frac{\Delta C}{C}$$

The percentage error in Z
$$\frac{\Delta Z}{Z} \times 100 = p\frac{\Delta A}{A} \times 100 + q\frac{\Delta B}{B} \times 100 + r\frac{\Delta C}{C} \times 100$$

Units Used

The maximum possible error has the same units as the quantity itself but fractional error has no units.

Example 78. *Two resistances $R_1 = 100 \pm 3\,\Omega$ and $R_2 = 200 \pm 4\,\Omega$ are connected in series. What is their equivalent resistance ?* [NCERT]

Solution. Equivalent resistance,
$$R = R_1 + R_2 = (100 \pm 3) + (200 \pm 4)$$
$$= (100 + 200) \pm (3 + 4) = \mathbf{(300 \pm 7)\,\Omega.}$$

Example 79. *Two different masses are determined as (23.7 ± 0.5) g and (17.6 ± 0.3) g. What is the sum of their masses?*

Solution. Sum of the masses
$$= (23.7 \pm 0.5) + (17.6 \pm 0.3)$$
$$= (23.7 + 17.6) \pm (0.5 + 0.3) = \mathbf{(41.3 \pm 0.8)\ g.}$$

Example 80. *The initial and final temperatures of a bath are $(18 \pm 0.5)^\circ C$ and $(40 \pm 0.3)^\circ C$. What is the rise in temperature of the bath ?*

Solution. Rise in temperature
$$= \text{Final temperature} - \text{Initial temperature}$$
$$= (40 \pm 0.3) - (18 \pm 0.5) = (40 - 18) \pm (0.3 + 0.5)$$
$$= \mathbf{(22 \pm 0.8)^\circ C.}$$

Example 81. *The resistance $R = \frac{V}{I}$, where $V = 100 \pm 5$ V and $I = 10 \pm 0.2$ A. Find the percentage error in R.* [NCERT]

Solution. The percentage error in V is 5% and in I is 2%.

The total percentage error in R is given by
$$\frac{\Delta R}{R} \times 100 = \frac{\Delta V}{V} \times 100 + \frac{\Delta I}{I} \times 100$$
$$= 5\% + 2\% = 7\%.$$

Example 82. *If the errors involved in the measurement of a side and mass of a cube are 3% and 4% respectively, what is the maximum permissible error in the density of the material ?*

Solution. Density, $\rho = \frac{M}{V} = \frac{M}{L^3}$

The percentage error in density is given by
$$\frac{\Delta\rho}{\rho} \times 100 = \frac{\Delta M}{M} \times 100 + 3 \times \frac{\Delta L}{L} \times 100$$
$$= 4\% + 3 \times 3\% = \mathbf{13\%.}$$

Example 83. *The error in the measurement of radius of a sphere is 2%. What would be the error in the volume of the sphere ?*

Solution. Given : $\frac{\Delta r}{r} \times 100 = 2\%$

Volume of sphere, $V = \frac{4}{3}\pi r^3$

Percent error in volume,
$$\frac{\Delta V}{V} \times 100 = 3\frac{\Delta r}{r} \times 100 = 3 \times 2 = \mathbf{6\%.}$$

Example 84. *The percentage errors in the measurement of mass and speed are 2% and 3% respectively. How much will be the maximum error in the estimate of kinetic energy obtained by measuring mass and speed ?* [NCERT]

Solution. Given :
$$\frac{\Delta m}{m} \times 100 = 2\% \quad \text{and} \quad \frac{\Delta v}{v} \times 100 = 3\%$$

Kinetic energy, $K = \frac{1}{2}mv^2$

Percent error in K.E.,
$$\frac{\Delta K}{K} \times 100 = \frac{\Delta m}{m} \times 100 + 2\frac{\Delta v}{v} \times 100$$
$$= 2\% + 2 \times 3\% = \mathbf{8\%.}$$

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EXAMPLE 85. The length, breadth and height of a rectangular block of wood were measured to be :

$l = 12.13 \pm 0.02$ *cm* ; $b = 8.16 \pm 0.01$ *cm*;

$h = 3.46 \pm 0.01$ *cm.*

Determine the percentage error in the volume of the block.

Solution. Volume of block, $V = lbh$

The percentage error in the volume is given by

$$\frac{\Delta V}{V} \times 100 = \left(\frac{\Delta l}{l} + \frac{\Delta b}{b} + \frac{\Delta h}{h}\right) \times 100$$

$$= \left(\frac{0.02}{12.13} + \frac{0.01}{8.16} + \frac{0.01}{3.46}\right) \times 100$$

$$= \frac{200}{1213} + \frac{100}{816} + \frac{100}{346}$$

$$= 0.1649 + 0.1225 + 0.2890 = \mathbf{0.58\%}.$$

[Rounded off to 2 significant figures]

EXAMPLE 86. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{L/g}$. *Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of 1 s resolution. What is the accuracy in the determination of g ?* [NCERT]

Solution. As $T = 2\pi\sqrt{\frac{L}{g}}$

or $$T^2 = 4\pi^2 \frac{L}{g} \quad \text{or} \quad g = \frac{4\pi^2 L}{T^2}$$

$$\therefore \frac{\Delta g}{g} \times 100 = \frac{\Delta L}{L} \times 100 + 2 \times \frac{\Delta T}{T} \times 100$$

Now $L = 20.0$ cm, $\Delta L = 1$ mm $= 0.1$ cm, T for 100 oscillations $= 90$ s, $\Delta T = 1$ s

$$\therefore \frac{\Delta g}{g} \times 100 = \frac{0.1}{20.0} \times 100 + 2 \times \frac{1}{90} \times 100$$

$$= 0.5 + 2.22 = 2.72\% = \mathbf{3\%}.$$

EXAMPLE 87. Find the relative error in Z, if

$$Z = A^4 B^{1/3} / CD^{3/2}.$$ [NCERT]

Solution. The relative error in Z is

$$\frac{\Delta Z}{Z} = 4 \times \frac{\Delta A}{A} + \frac{1}{3} \times \frac{\Delta B}{B} + \frac{\Delta C}{C} + \frac{3}{2} \frac{\Delta D}{D}.$$

EXAMPLE 88. A physical quantity X is given by $X = \frac{a^2 b^3}{c\sqrt{d}}$.

If the percentage errors of measurement in a, b, c and d are 4%, 2%, 3% and 1% respectively, then calculate the percentage error in X. [Delhi 13]

Solution. Given : $X = \frac{a^2 b^3}{c\sqrt{d}}$

The percentage error in X is given by

$$100 \times \frac{\Delta X}{X} = 2\frac{\Delta a}{a} \times 100 + 3\frac{\Delta b}{b} \times 100 + \frac{\Delta c}{c} \times 100 + \frac{1}{2}\frac{\Delta d}{d} \times 100$$

$$= 2 \times 4\% + 3 \times 2\% + 3\% + \tfrac{1}{2} \times 1\% = \mathbf{17.5\%}.$$

EXAMPLE 89. For the estimation of Young's modulus :

$$Y = \frac{4\,Mg}{\pi d^2} \cdot \frac{L}{l}$$

for the specimen of a wire, following observations were recorded : $L = 2.890$, $M = 3.00$, $d = 0.082$, $g = 9.81$, $l = 0.087$. *Calculate the maximum percentage error in the value of Y and mention which physical quantity causes maximum error.*

Solution. Given $Y = \frac{4\,Mg}{\pi d^2} \cdot \frac{L}{l}$

As 4 and π are constants and a standard value of 'g' is taken, so the percentage error in Y will be

$$\frac{\Delta Y}{Y} \times 100 = \left(\frac{\Delta M}{M} + \frac{\Delta L}{L} + 2\frac{\Delta d}{d} + \frac{\Delta l}{l}\right) \times 100$$

It is obvious from the given data that

$$\Delta M = 0.01,\ \Delta L = 0.001,\ \Delta d = 0.001,\ \Delta l = 0.001$$

$$\therefore \frac{\Delta Y}{Y} \times 100 = \left(\frac{0.01}{3.00} + \frac{0.001}{2.890} + \frac{2 \times 0.001}{0.082} + \frac{0.001}{0.087}\right) \times 100$$

$$= (0.0033 + 0.0003 + 0.0244 + 0.0115) \times 100$$

$$= 0.0395 \times 100 = \mathbf{3.95\%}.$$

Diameter of the wire causes maximum error in the value of Y.

EXAMPLE 90. The specific resistance σ *of a thin wire of radius r cm, resistance* $R\,\Omega$ *and length L cm is given by*

$$\sigma = \frac{\pi r^2 R}{L}$$

If $r = 0.26 \pm 0.02$ *cm,* $R = 32 \pm 1\,\Omega$

and $L = 78 \pm 0.01$ *cm, find the percentage error in* σ.

Solution. The percentage error in specific resistance σ is given by

$$\frac{\Delta\sigma}{\sigma} \times 100 = \left(2\frac{\Delta r}{r} + \frac{\Delta R}{R} + \frac{\Delta L}{L}\right) \times 100$$

$$= \left[\frac{2 \times 0.02}{0.26} + \frac{1}{32} + \frac{0.01}{78}\right] \times 100$$

$$= [0.15 + 0.03 + 0.0001] \times 100$$

$$= 0.1801 \times 100 = 0.18 \times 100 = \mathbf{18\%}.$$

EXAMPLE 91. If two resistors of resistances $R_1 = (4 \pm 0.5)\,\Omega$ *and* $R_2 = (16 \pm 0.5)\,\Omega$ *are connected (i) in series and (ii) in parallel; find the equivalent resistance in each case with limits of percentage error.*

Solution. (*i*) The equivalent resistance of the series combination,

$$R = R_1 + R_2 = (4 \pm 0.5)\Omega + (16 \pm 0.5)\Omega$$

$$= \mathbf{(20 \pm 1)\Omega} \text{ or } \mathbf{20 \pm 5\%}$$

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(ii) the equivalent resistance of parallel combination

$$R' = \frac{R_1 R_2}{R_1 + R_2} = \frac{4 \times 16}{4 + 16} = 3.2\ \Omega$$

From $\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2}$, we get

$$\frac{\Delta R'}{R'^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$$

$$\therefore \quad \Delta R' = (R'^2)\frac{\Delta R_1}{R_1^2} + (R'^2)\frac{\Delta R_2}{R_2^2}$$

$$= \left(\frac{3.2}{4}\right)^2 \times 0.5 + \left(\frac{3.2}{16}\right)^2 \times 0.5$$

$$= 0.34\ \Omega$$

Here $R' = (3.2 \pm 0.34)\ \Omega = 3.2\ \Omega \pm 10.625\%$

PROBLEMS FOR PRACTICE

1. If $A = (12.0 \pm 0.1)$ cm and $B = (8.5 \pm 0.5)$ cm, find (i) $A + B$ and (ii) $A - B$ **[Delhi 04]**
[Ans. (i) 20.5 ± 0.6 cm (ii) 3.5 ± 0.6 cm]

2. The temperatures of two bodies measured by a thermometer are $t_1 = 20°C \pm 0.5°C$ and $t_2 = 50°C \pm 0.5°C$. Calculate the temperature difference and the error therein. **[NCERT ; Central Schools 12]**
(Ans. $30°C \pm 1°C$)

3. The lengths of two rods are 15.2 ± 0.2 cm and 10.7 ± 0.2 cm. Find difference in lengths of the two rods with the limits of error. **[Himachal 07]**
(Ans. 4.5 ± 0.4 cm)

4. The length and breadth of a rectangle are (5.7 ± 0.1) cm and (3.4 ± 0.2) cm. Calculate area of the rectangle with error limits. **[Chandigarh 04]**
(Ans. $19.4 \pm 1.5\ cm^2$)

5. If displacement of a body, $s = (200 \pm 5)$ meters and time taken by it $t = (20 \pm 0.2)$ seconds, then find the percentage error in the calculation of velocity.
[Delhi 05] **(Ans. 3.5%)**

6. If the length and time period of an oscillating pendulum have errors of 1% and 2% respectively, what is the error in the estimate of g ?
[Central Schools 04] **(Ans. 5%)**

7. If $l_1 = (10.0 \pm 0.1)$ cm and $l_2 = (9.0 \pm 0.1)$ cm, find their sum, difference and error in each.
[Ans. (19.0 ± 0.2) cm, 1% ; (1.0 ± 0.2) cm, 20%]

8. The length and breadth of a field are 22.4 cm and 15.8 cm respectively and have been measured to an accuracy of 0.2 cm. Find the percentage error in the area of the field. **[Himachal 07]** **(Ans. 2.16%)**

9. The relative density of a material is found by weighing the body first in air and then in water. If the weight in air is (10.0 ± 0.1) N and weight in water is (5.0 ± 0.1) N, what would be the maximum percentage error in relative density ? **(Ans. 5%)**

10. The voltage across a lamp is (6.0 ± 0.1) volt and the current passing through it is (4.0 ± 0.2) ampere. Find the power consumed. **[Ans. (24.0 ± 1.6) watt]**

11. The radius of a sphere is measured as (2.1 ± 0.5) cm. Calculate its surface area with error limits.
[Ans. $(55.4 \pm 26.4)\ cm^2$]

12. The radius of a sphere is 5.3 ± 0.1 cm. Calculate the percentage error in its volume. **[Himachal 06]**
(Ans. 5.7 %)

13. The measure of the diameter of a cylinder is (1.60 ± 0.01) cm and its length is (5.0 ± 0.1) cm. Calculate the percentage error in its volume.
(Ans. 3.25%)

14. The measured mass and volume of a body are 2.00 g and 5.0 cm^3 respectively. With possible errors of 0.01 g and 0.1 cm^3, what would be the percent error in density ?
(Ans. 2.5%)

15. A body travels uniformly a distance of (13.8 ± 0.2) m in a time (4.0 ± 0.3) s. Calculate its velocity with error limits. What is percentage error in velocity ?
[Ans. $(3.5 \pm 0.3)\ ms^{-1}$, 9%]

16. Find the percentage error in Z, if $Z = \frac{A^{1/3}B^4}{CD^{2/3}}$.
[Delhi 08]
[Ans. % error in $Z = (\frac{1}{3}\frac{\Delta A}{A} + 4\frac{\Delta B}{B} + \frac{\Delta C}{C} + \frac{2}{3}\frac{\Delta D}{D})100$]

17. A physical quantity X is related to three observables a, b, c as $X = \sqrt{a}\, b^2 / c^2$. The errors of measurement in a, b and c are 2%, 1% and 3% respectively. What is the percentage error in the quantity X ?
(Ans. 9%)

18. A physical quantity Q is given by $Q = \frac{A^2 B^{3/2}}{C^{1/4} D^{1/2}}$

The percentage errors in A, B, C and D are 1%, 2%, 4% and 2% respectively. Find the percentage error in Q.
[Central Schools 05] **(Ans. 22%)**

19. Two resistors of resistances $R_1 = 100 \pm 3$ ohm and $R_2 = 200 \pm 4$ ohm are connected (a) in series, (b) in parallel. Find the equivalent resistance of the (a) series combination, (b) parallel combination. Use for (a) the relation $R = R_1 + R_2$ and for (b) $\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2}$ and $\frac{\Delta R'}{R'^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$. **[NCERT]**
(Ans. $66.7 \pm 1.8\ \Omega$)

HINTS

2. $t' = t_2 -$
$t' = 30°$

4. Here l
Area

ΔA

$\therefore A$

5. As

6. A

7.

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HINTS

2. $t' = t_2 - t_1 = (50°C \pm 0.5°C) - (20°C \pm 0.5°C)$
$t' = 30°C \pm 1°C$.

4. Here $l = 5.7 \pm 0.1$ cm and $b = 3.4 \pm 0.2$ cm
Area $= lb = 5.7 \times 3.4 = 19.38\ cm^2 = 19.4\ cm^2$
$$\Delta A = \left(\frac{\Delta l}{l} + \frac{\Delta b}{b}\right) lb = \left(\frac{0.1}{5.7} + \frac{0.2}{3.4}\right) \times 19.4$$
$$= (0.1017 + 0.059) \times 19.4 = 1.47 = 1.5$$
$\therefore A = \mathbf{(19.4 \pm 1.5)\ cm^2}$.

5. As $v = s/t$
$$\therefore \frac{\Delta v}{v} \times 100 = \frac{\Delta s}{s} \times 100 + \frac{\Delta t}{t} \times 100$$
$$= 2.5\% + 1\% = 3.5\%.$$

6. As $g = \dfrac{4\pi^2 l}{T^2}$
$$\therefore \frac{\Delta g}{g} \times 100 = \frac{\Delta l}{l} \times 100 + 2\frac{\Delta T}{T} = 1 + 2 \times 2 = 5\%.$$

7. Sum $= (10.0 + 9.0) \pm (0.1 + 0.1) = \mathbf{(19.0 \pm 0.2)}$.
Percentage error $= \dfrac{0.2}{19.0} \times 100 = \mathbf{1\%}$
Difference $= (10.0 - 9.0) \pm (0.1 + 0.1) = \mathbf{(1.0 \pm 0.2)\ cm.}$
Percentage error $= \dfrac{0.2}{1.0} \times 100 = \mathbf{20\%}.$

8. Area, $A = l \times b$
$$\% \text{ Error} = \frac{\Delta A}{A} \times 100 = \frac{\Delta l}{l} \times 100 + \frac{\Delta b}{b}$$
$$= \frac{0.2}{22.4} \times 100 + \frac{0.2}{15.8} \times 100$$
$$= 0.89 + 1.27 = \mathbf{2.16\%}$$

9. Relative density $= \dfrac{\text{Weight in air}}{\text{Loss of weight in water}}$
Loss of weight in water
$= (10.0 - 5.0) \pm (0.1 + 0.1) = 5.0 \pm 0.2$ N
Percentage error in relative density
= Percentage error in weight in air
+ Percentage error in loss of weight in water
$$= \left(\frac{0.1}{10.0} + \frac{0.2}{5.0}\right) \times 100 = \mathbf{5\%}.$$

10. Power consumed, $P = VI = 6.0 \times 4.0 = 24.0$ W
$$\therefore \frac{\Delta P}{P} = \frac{\Delta V}{V} + \frac{\Delta I}{I} = \frac{0.1}{6.0} + \frac{0.2}{4.0} = \frac{1}{15}$$
$$\therefore \Delta P = \frac{1}{15} \times P = \frac{1}{15} \times 24.0 = 1.6\ \text{W}$$
Power consumed $= \mathbf{(24.0 \pm 1.6)\ W.}$

11. Surface area of a sphere, $A = 4\pi r^2$
$$\therefore \frac{\Delta A}{A} = 2\frac{\Delta r}{r}.$$

13. Volume of a cylinder, $V = \pi r^2 h = \dfrac{\pi d^2 h}{4}$
$$\% \text{ Error}, \frac{\Delta V}{V} \times 100 = \left[2\frac{\Delta d}{d} + \frac{\Delta h}{h}\right] \times 100$$
$$= \left[2 \times \frac{0.01}{1.60} + \frac{0.1}{5.0}\right] \times 100 = [0.0125 + 0.02] \times 100$$
$$= 0.0325 \times 100 = \mathbf{3.25\%}.$$

14. $$\frac{\Delta \rho}{\rho} = \left[\frac{\Delta m}{m} + \frac{\Delta V}{V}\right] \times 100$$
$$= \left[\frac{0.01}{2.00} + \frac{0.1}{5.0}\right] \times 100 = \mathbf{2.5\%}.$$

15. Given : $s = (13.8 \pm 0.2)$ m, $t = (4.0 \pm 0.3)$ s
Velocity, $v = \dfrac{s}{t} = \dfrac{13.8}{4.0} = 3.45\ \text{ms}^{-1} = 3.4\ \text{ms}^{-1}$
[Rounded off to first place of decimal]
$$\frac{\Delta v}{v} = \frac{\Delta s}{s} + \frac{\Delta t}{t} = \frac{0.2}{13.8} + \frac{0.3}{4.0} = \frac{0.8 + 4.14}{13.8 \times 4.0}$$
$$= \frac{4.94}{13.8 \times 4.0} = 0.0895$$
$$\Delta v = 0.0895 \times v = 0.0895 \times 3.45 = 0.3087 = \pm 0.3$$
$\therefore$ Velocity $= \mathbf{(3.4 \pm 0.3)\ ms^{-1}}$.
$\therefore$ Error, $\dfrac{\Delta v}{v} \times 100 = 0.0895 \times 100 = 8.95\% = \mathbf{9\%}.$

18. $$\frac{\Delta Q}{Q} \times 100 = 2\frac{\Delta A}{A} \times 100 + \frac{3}{2}\frac{\Delta B}{B} \times 100 + 4\frac{\Delta C}{C} \times 100 + \frac{1}{2}\frac{\Delta D}{D} \times 100$$
$$= 2 \times 1 + \frac{3}{2} \times 2 + 4 \times 4 + \frac{1}{2} \times 2$$
$$= 2 + 3 + 16 + 1 = \mathbf{22\%}.$$

19. (*a*) The equivalent resistance of series combination,
$$R = R_1 + R_2 = (100 \pm 3)\,\Omega + (200 \pm 4)\,\Omega$$
$$= \mathbf{300 \pm 7\,\Omega}.$$
(*b*) The equivalent resistance of parallel combination,
$$R' = \frac{R_1 R_2}{R_1 + R_2} = \frac{200}{3} = 66.7\,\Omega$$
From $\dfrac{1}{R'} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$, we get
$$\frac{\Delta R'}{R'^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$$
$$\therefore \Delta R' = (R'^2)\frac{\Delta R_1}{R_1^2} + (R'^2)\frac{\Delta R_2}{R_2^2}$$
$$= \left(\frac{66.7}{100}\right)^2 3 + \left(\frac{66.7}{200}\right)^2 4 = 1.8$$
Hence $R' = \mathbf{66.7 \pm 1.8\,\Omega}$.

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2.33 RULES FOR DETERMINING THE UNCERTAINTY OF NUMBERS IN ARITHMATIC OPERATIONS

58. *State and illustrate the rules for determining the uncertainty of a number in arithmatic operations.*

Rules for determining the uncertainty of a number in arithmatic operations :

1. *The uncertainty in a measured value is equal to half the least count of the measuring instrument.*

Illustration. Suppose the measured values of the length and breadth of a rectangular sheet are 16.2 cm and 10.1 cm respectively. Then its true length and breadth may be expressed as

$$l = 16.2 \pm \frac{1}{2} \times 0.1 = 16.2 \pm 0.05 \text{ cm}$$

or $$l = 16.2 \text{ cm} \pm \frac{0.05}{16.2} \times 100 = 16.2 \text{ cm} \pm 0.3\%.$$

and $$b = 10.1 \pm \frac{1}{2} \times 0.1 = 10.1 \pm 0.05 \text{ cm}.$$

or $$b = 10.1 \text{ cm} \pm \frac{0.05}{10.1} \times 100 = 10.1 \text{ cm} \pm 0.5\%.$$

2. *When two or more measured values are multiplied, the percentage uncertainty of the final result is equal to the square root of the sum of the squares of the percentage uncertainties in the original values.*

Illustration. The true area of the above rectangular sheet can be expressed as

$$lb = 16.2 \times 10.1 \text{ cm}^2 \pm \sqrt{(0.3\%)^2 + (0.5\%)^2}$$

$$= 163.62 \text{ cm}^2 \pm 0.6\%$$

or $$lb = 163.62 \pm \frac{163.62 \times 0.6}{100} = 163.62 \pm 1.0 \text{ cm}^2.$$

3. *If a set of experimental data is specified to n significant figures, then the result obtained by combining this data will also be valid upto n significant figures. But sometimes the number of significant figures get reduced in subtraction.*

Illustration. $12.9 \text{ g} - 7.06 \text{ g} = 5.84 \text{ g} = 5.8 \text{ g}$, the corrected difference on being rounded off to first place of decimal. Both 2.9 g and 7.06 g have three significant figures each but their difference contains two significant figures.

4. *The fractional errors of the value of a number specified to n significant figures depends not only on n but also on the original number.*

Illustration. Accuracy in the measurement of mass $1.02 \text{ g} = \pm 0.005 \text{ g}$

Fractional error in 1.02 g

$$= \pm \frac{0.005}{1.02} \times 100 = \pm 0.5\%.$$

Accuracy in the measurement of mass 9.89 g

$$= \pm 0.005 \text{ g}$$

Fractional error in 9.89 g

$$= \pm \frac{0.005}{9.89} \times 100 = \pm 0.05\%.$$

Both 1.02 g and 9.89 g contain three significant figures each. Both the numbers have same accuracy but different fractional errors.

5. *The results in the intermediate steps of a multi-step calculation should be calculated to one extra significant figure in each measurement than the number of digits in the least precise measurement. This avoids building up of errors in the process of rounding off the numbers.*

Illustration. Reciprocal of 9.58 after being rounded off to 3 significant figures = 0.104.

But the reciprocal of 0.104 calculated to 3 significant figures = 9.62.

However, if we take 1/9.58 = 0.1044, then find the reciprocal to three significant figures, we get back the original number 9.58.

Very Short Answer Conceptual Problems

Problem 1. What is the necessity of selecting some units as fundamental units ?

Solution. The number of physical units required to be measured is very large. If a separate unit is defined for each of them, then it will become very difficult to remember all of them as they will be quite unrelated to each other.

Problem 2. How is SI a coherent system of units ?

Solution.In SI, all derived units can be obtained by multiplying and dividing the basic and supplementary units and no numerical factors are required to be introduced. So SI is a coherent system of units.

Problem 3. In defining the standard of length, (the prototype metre), we have to specify the temperature at which the measurement should be made. Are we justified in calling length a fundamental quantity, if another physical quantity (temperature) has to be specified in choosing a standard ?

Solution. Yes, the choice of length as a fundamental quantity is justified. The modern definition of metre in terms of wavelength of light radiation is not affected by temperature.

Problem 4. Do Å and AU stand for the same unit of length ? *[Himachal 03, 05C, 08]*

Solution. No. 1 Å(angstrom) = 10^{-10} m

1 AU (astronomical unit) = 1496×10^{11} m.

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Problem 5. Why is it convenient to express the distances of stars in terms of light year rather than in metre or kilometre ?

Solution. One light year

$$= 9.46 \times 10^{15} \text{ m} = 9.46 \times 10^{12} \text{ km}.$$

As the distances of stars are extra-ordinarily large, so it is convenient to express them in light year rather than in metre or kilometre.

Problem 6. Comment on the statement : "To define a physical quantity for which no method of measurement is given or known has no meaning."

Solution. The given statement is not correct. A physical quantity, if it is called so, must have a physical meaning. If it cannot be measured by any direct method, these must be some indirect method for its measurement. Entropy is one such physical quantity.

Problem 7. Is the measure of an angle dependent upon the unit of length ?

Solution. $\theta \text{ (radian)} = \dfrac{\text{Arc}}{\text{Radius}}$

As an angle is the ratio of the length of an arc and the radius *i.e.*, it is the ratio of two lengths, so the measure of an angle doe not depend upon the unit of length.

Problem 8. What is meant by angular diameter of the moon ? What is its value ?

Solution. The angle subtended by the two diametrically opposite ends of the moon at a point on the earth is called angular diameter of the moon. Its value is about 0.5°.

Problem-9. For a given base line, which will show a greater parallax—a distant star or a nearby star ?

Solution. Parallactic angle, $\theta = \dfrac{\text{base line}}{\text{distance of star}} = \dfrac{b}{S}$

Thus for a given base line b, parallax of a star is inversely proportional to its distance S. Hence the nearby star will show a greater parallax.

Problem 10. Why is parallax method not useful for measuring the distances of stars more than 100 light years away ?

Solution. For a star more than 100 light years away, the parallax angle is so small that it cannot be measured accurately.

Problem 11. What is the difference between mN, Nm and nm ? *[Himachal 03]*

Solution. 1 mN = 1 millinewton = 10^{-3} newton (unit of force).

1 Nm = 1 Newton metre (unit of work)

1 nm = 1 nanometre = 10^{-9} metre (unit of distance).

Problem 12. Do all physical quantities have dimensions ? If no, name three physical quantities which are dimensionless.

Solution. No, all physical quantities do not have dimensions. The physical quantities like angle, strain and relative density are dimensionless.

Problem 13. If 'slap' times speed equals power, what will be the dimensional equation for 'slap' ?

Solution. As slap × speed = power

$$\therefore \text{ Slap} = \frac{\text{Power}}{\text{Speed}} = \frac{ML^2T^{-3}}{LT^{-1}} = MLT^{-2}.$$

Problem 14. Write the dimensions and SI unit of linear momentum.

Solution. Linear momentum = Mass × velocity

$$\therefore \quad [\text{Linear momentum}] = [M][LT^{-1}] = [MLT^{-1}]$$

SI unit of linear momentum = kgms^{-1}.

Problem 15. What is the basis of the principle of homogeneity of dimensions ?

Solution. The principle of homogeneity of dimensions is based on the fact that only the physical quantities of the same kind can be added, subtracted or compared.

Problem 16. If $x = a + bt + ct^2$, where x is in metre and t in second; then what is the unit of c ?

[Himachal 05C]

Solution. $[x] = [ct^2]$ or $[c] = \left[\dfrac{x}{t^2}\right]$

$\therefore$ Unit of $c = \text{ms}^{-2}$.

Problem 17. What are the dimensions of a and b in the relation : $F = a + bx$, where F is force and x is distance ?

Solution. $[a] = [F] = MLT^{-2}$

$$[b] = \left[\frac{F}{x}\right] = \frac{MLT^{-2}}{L} = MT^{-2}.$$

Problem 18. Name two physical quantities having the dimensions $[ML^2T^{-2}]$.

Solution. Work and torque.

Problem 19. Write three physical quantities having dimensions $[ML^{-1}T^{-2}]$.

Solution. Pressure, stress and Young's modulus of elasticity.

Problem 20. If the units of force and length each are doubled, then how many times would the unit of energy be affected ?

Solution. Energy = Work done = Force × length

So when the units of force and length each are doubled, the unit of energy will increase four times.

Problem 21. The velocity v of a particle depends on time t as : $v = At^2 + Bt + C$

where v is in m/s and t in second. What are the units of A, B and C ?

Solution. Unit of $A = \dfrac{\text{Unit of } v}{\text{Unit of } t^2} = \dfrac{\text{ms}^{-1}}{\text{s}^2} = \text{ms}^{-3}$

$$\text{Unit of } B = \frac{\text{Unit of } v}{\text{Unit of } t} = \frac{\text{ms}^{-1}}{\text{s}} = \text{ms}^{-2}.$$

Unit of C = Unit of $v = \text{ms}^{-1}$.

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Problem 22. Can a quantity have dimensions but still has no units ?

Solution. No, a quantity having dimensions must have some units of its measurement.

Problem 23. Can a quantity have different dimensions in different systems of units ?

Solution. No, a quantity has same dimensions in all system of units.

Problem 24. Can a quantity have units but still be dimensionless ?

Solution. Yes. For example, a plane angle has no dimensions but has unit like radian for its measurement.

Problem 25. Does the magnitude of a physical quantity depend on the system of units chosen ?

Solution. No. The magnitude of physical quantity remains same in all systems of units.

Problem 26. Justify L + L = L and L – L = L.

Solution. When we add a length to another length, we get length only so L + L = L is justified.

When we subtract a length from another length, again we get length. This justifies L – L = L.

Problem 27. Can there be a physical quantity that has no units and no dimensions ?

Solution. Yes, strain is a physical quantity that has no units and no dimensions.

Problem 28. Can an instrument be called precise without being accurate ? Can it be accurate without being precise ?

Solution. Yes, an instrument can be precise without being accurate but the measurements cannot be accurate without being precise.

Problem 29. Which of the following length measurements is (*i*) most precise and (*ii*) least precise ? Give reason (*i*) $l = 5$ cm (*ii*) $l = 5.00$ cm (*iii*) 5.000 cm (*iv*) 5.0000 cm (*v*) 5.00000 cm.

Solution. (*i*) The last measurement is most precise because it has been taken with an instrument whose least count is equal to 0.00001 cm.

(*ii*) The first measurement is least precise because it has been taken with a device having least count equal to 1 cm.

Problem 30. Which of the following readings is the most accurate :

(*i*) 5000 m (*ii*) 5×10^2 m (*iii*) 5×10^3 m ?

Solution. (*i*) 5000 m is most accurate.

Problem 31. Which quantity in a given formula should be measured most accurately ?

Solution. The quantity which has maximum power (say, n) in the formula should be measured more accurately because an error in measurement is multiplied n times in the final result.

Problem 32. Which of the following length measured is most accurate and why ? *[Central Schools 14]*

(*a*) 500.0 cm (*b*) 0.0005 cm (*c*) 6.00 cm

Solution. The measurement (*b*) 0.0005 cm is most accurate because it is correct upto fourth decimal place.

Short Answer Conceptual Problems

Problem 1. Why do we treat length, mass and time as basic or fundamental quantities in mechanics ?

Solution. In mechanics, the quantities like length, mass and time are taken as fundamental quantities because :

(*i*) these quantities represent basic scientific notations,

(*ii*) there is no other quantity simpler to them,

(*iii*) length, mass and time cannot be expressed in terms of one another, and

(*iv*) all other physical quantities in mechanics can be expressed in terms of these quantities.

Problem 2. SI is a rational system of units while mks system is not so. Justify.

Solution. SI assigns only one unit to a particular physical quantity so it is a rational system. For example, all types of energies are measured in joule in SI. But in mks system, mechanical energy is measured in joule, heat energy in calorie and electrical energy in watt-hour. So mks system is not a rational system of units.

Problem 3. Why it became necessary to redefine metre on atomic standard ?

Solution. It became necessary to redefine metre on atomic standard because the prototype metre offered the following difficulties :

(*i*) It is difficult to preserve a metre bar.

(*ii*) It is difficult to produce replicas of metre bar for their use in different countries.

(*iii*) The techniques used for producing replicas are not of very high accuracy.

Problem 4. What are the advantages of defining metre in terms of the wavelength of light radiation ?

Solution. The advantages of defining metre in terms of wavelength of light radiation are as follows :

(*i*) It can be easily reproduced anywhere and at any time.

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(*ii*) It is invariant in time and space.

(*iii*) It is unaffected by environmental conditions like temperature and pressure.

(*iv*) It has an high accuracy of 1 part in 10^9.

Problem 5. Give reasons why is platinum iridium alloy used in making prototype metre and kilogram.

Solution. The reasons for making standard kilogram and metre from platinum-iridium alloy are as follows :

(*i*) The alloy is least affected by temperature variations.

(*ii*) It is non-corrosive and so does not wear out easily.

(*iii*) It is quite hard.

(*iv*) It does not change with time.

Problem 6. Suggest a distance corresponding to each of the following order of length :

(*i*) 10^7 m (*ii*) 10^4 m (*iii*) 10^2 m (*iv*) 10^{-3} m

(*v*) 10^{-6} m (*vi*) 10^{-14} m.

Solution. (*i*) 10^7 m = Radius of the earth

(*ii*) 10^4 m = Height of Mount Everest

(*iii*) 10^2 m = Length of Hockey Field

(*iv*) 10^{-3} m = Thickness of a card-board

(*v*) 10^{-6} m = Mean free path of air molecule

(*vi*) 10^{-14} m = Size of atomic nucleus.

Problem 7. Suggest an indirect method for measuring the height of a tree on a sunny day.

Solution. Let AB be the height of a tree, as shown in Fig. 2.15 and BC be its shadow cast by the sun.

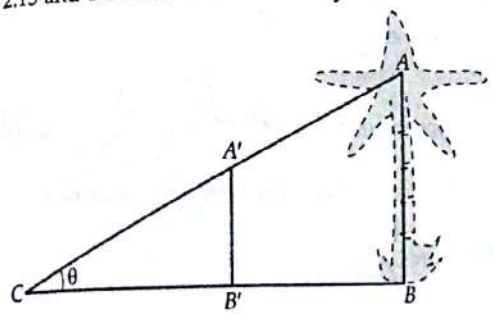

Fig. 2.15

Let $\angle ACB = \theta$.

Clearly, $\tan\theta = \dfrac{AB}{BC}$

Take a rod $A'B'$ and fix it at such a point that the tip of its shadow coincides with the point C. Then

$$\tan\theta = \frac{A'B'}{B'C}.$$

Hence $$\frac{AB}{BC} = \frac{A'B'}{B'C}.$$

$\therefore$ Height of tree, $AB = \dfrac{A'B'}{B'C} \times BC$.

Problem 8. What is the basic principle of alpha particle scattering method for estimating the size of the nucleus ? *[Pubjab 90]*

Solution. Both the α-particle and nucleus are positively charged. When an α-particle approaches a nucleus, its kinetic energy gradually changes into potential energy due to repulsive forces. At the distance of closest approach r_0, the entire kinetic energy changes into potential energy. This concept can be used to calculate r_0, which gives the order of the size of the nucleus.

Problem 9. If the velocity of light is taken as the unit of velocity and one year as the unit of time, what must be the unit of length ? What is it called ?

Solution. Unit of length = unit of velocity × unit of time

$$= 3\times10^8\ \text{ms}^{-1} \times 1\ \text{year}$$
$$= 3\times10^8\ \text{ms}^{-1} \times 365\times24\times60\times60\ \text{s}$$
$$= 9.45\times10^{15}\ \text{ms}^{-1}$$
$$= \textbf{1 light year.}$$

Problem 10. What is common between bar and torr ?

Solution. Both bar and torr are the units of pressure.

1 bar = 1 atmospheric pressure

= 760 mm of Hg column

1 torr = 1 mm of Hg column

$\therefore$ 1 bar = 760 torr.

Problem 11. Distinguish between accuracy and precision.

Solution. By accuracy of a measurement we mean that the measured value of a physical quantity is as close to the true value as possible. On the other hand, a measurement is said to be precise, if same value of the quantity is obtained in each of the various measurements carried out with the given apparatus.

Problem 12. Which of the following measurements is most accurate and which is most precise :

(*i*) 4.00 mm (*ii*) 4.00 cm

(*iii*) 4.00 m (*iv*) 40.00 m ?

Solution.

Length (l)	*Least Count* (Δl)	*Relative Error* ($\Delta l / l$)
(*i*) 4.00 mm	0.01 mm	2.5×10^{-3}
(*ii*) 4.00 cm	0.01 cm	2.5×10^{-3}
(*iii*) 4.00 m	0.01 m	2.5×10^{-3}
(*iv*) 40.00 m	0.01 m	2.5×10^{-4}

Since the relative error in the measurement of 40.00 m is minimum, so this measurement is most accurate. The measurement 4.00 mm is most precise because it is measured with an instrument of minimum least count.

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Problem 13. Two clocks are being tested against a standard clock located in a national laboratory. At 12:00:00 noon by the standard clock, the readings of the two clocks are :

Day	Clock 1	Clock 2
Monday	12:00:05	10:15:06
Tuesday	12:01:15	10:14:59
Wednesday	11:59:08	10:15:18
Thursday	12:01:50	10:15:07
Friday	11:59:15	10:14:53
Saturday	12:01:30	10:15:24
Sunday	12:01:19	10:15:11

If you are doing an experiment that requires precision time interval measurements, which of the two clocks will you prefer ? *[NCERT]*

Solution. The range of variation over the seven days of observations is 162 s for clock 1, and 31 s for clock 2. The average reading of clock 1 is much closer to the standard time than the average reading of clock 2. The important point is that a clock's zero error is not as significant for precision work as its variation, because a 'zero-error' can always be easily corrected. Hence clock 2 is to be preferred to clock 1.

Problem 14. For the determination of 'g' using a simple pendulum, measurements of l and T are required. Error in the measurement of which of these will have larger effect on the value of 'g' thus obtained and why ? What is done to minimize this error ?

[Delhi 04]

Solution. Time period of a simple pendulum,

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \therefore \quad g = \frac{4\pi^2 l}{T^2}$$

Clearly, the error in the measurement of time period T has larger effect on the value of g than the error in the measurement of length l.

Reasons (*i*) T is very small.

(*ii*) In contrast to l, T^2 appears in the formula for g.

To minimise the error, time period for a large number of oscillations is measured.

Problem. 15. Magnitude of force F experienced by a certain object moving with speed v is given by $F = Kv^2$, where K is a constant. Find the dimensions of K.

[Delhi 04]

Solution. $[K] = \dfrac{[F]}{[v^2]} = \dfrac{[MLT^{-2}]}{[LT^{-1}]^2} = [ML^{-1}]$.

Problem. 16. The SI unit of energy is $J = kgm^2s^{-2}$, that of speed v is ms^{-1} and acceleration a is ms^{-2}. Which of the formulae for kinetic energy (K) given below can you rule out on the basis of dimensional arguments (m stands for the mass of the body) :

(*a*) $K = m^2v^3$ (*b*) $K = (1/2)mv^2$

(*c*) $K = ma$ (*d*) $K = (3/16)mv^2$

(*e*) $K = (1/2)mv^2 + ma$

Solution. As SI unit of energy, $J = kgm^2s^{-2}$ [NCERT, so

$[\text{Energy}] = [ML^2T^{-2}]$

(*a*) $[m^2v^2] = [M^2][LT^{-1}]^2 = [M^2L^2T^{-2}]$

(*b*) $[1/2\ mv^2] = [M][LT^{-1}]^2 = [ML^2T^{-2}]$

(*c*) $[ma] = [M][LT^{-2}] = [MLT^{-2}]$

(*d*) $[3/16\ mv^2] = [M][LT^{-1}]^2 = [ML^2T^{-2}]$

(*e*) The quantities $(1/2)\ mv^2$ and ma have different dimensions and hence cannot be added.

Since the kinetic energy K has the dimensions of $[ML^2T^{-2}]$, formulas (*a*), (*c*) and (*e*) are clearly ruled out.

Dimensional analysis cannot tell which of the two, (*b*) or (*d*), is the correct formula. From the actual definition of kinetic energy, only (*b*) is the correct formula for kinetic energy.

Problem. 17. Using the principle of homogeneity of dimensions, find which of the following is correct:

(*i*) $T^2 = 4\pi^2r^2$ (*ii*) $T^2 = \dfrac{4\pi^2r^3}{G}$ (*iii*) $T^2 = \dfrac{4\pi^2r^3}{GM}$

where T is time period, G is gravitational constant, M is mass and r is radius of orbit. *[Central Schools 05]*

Solution. (*i*) $T^2 = 4\pi^2r^2$

Dimensionally, $M^0L^0T^2 = M^0L^2T^0$

As LHS ≠ RHS, the formula is *incorrect.*

(*ii*) $T^2 = \dfrac{4\pi^2r^3}{G}$

Dimensionally, $M^0L^0T^2 = \dfrac{L^3}{M^{-1}L^3T^{-2}} = M^1L^0T^2$

As LHS ≠ RHS, the formula is *incorrect.*

(*iii*) $T^2 = \dfrac{4\pi^2r^3}{GM}$

Dimensionally, $M^0L^0T^2 = \dfrac{L^3}{M^{-1}L^3T^{-2}M} = M^0L^0T^2$

As LHS = RHS, the formula is *correct.*

Problem 18. The mean value of period of oscillation of a simple pendulum in an experiment is 2.825 s. The arithmatic mean of all the absolute errors is 0.11 s. Round off the period of simple pendulum to appropriate number of significant figures. Give reasons.

[Central School 08]

Solution. The absolute error 0.11 s has only two significant figures.

∴ Period of simple pendulum = 2.9 s

[Rounded off upto 2 significant figures]

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HOTS

Problems on Higher Order Thinking Skills

Problem 1. *If nth division of main scale coincides with (n + 1)th division of vernier scale, find the least count of the vernier. Given one main scale division is equal to 'a' units.* **[IIT Mains 03]**

Solution. $(n+1)$ divisions of vernier scale

$= n$ divisions of main scale

$$\therefore \quad 1 \text{ V.S.D.} = \frac{n}{n+1} \text{ M.S.D.}$$

$$\text{Least count} = 1 \text{ M.S.D.} - 1 \text{ V.S.D.}$$

$$= 1 \text{ M.S.D.} - \frac{n}{n+1} \text{ M.S.D.}$$

$$= \frac{1}{n+1} \text{ M.S.D.}$$

$$= \frac{1}{n+1} \times a \text{ units} = \frac{a}{n+1} \textbf{ units.}$$

Problem 2. *If the velocity of light c, the constant of gravitation G and Planck's constant h be chosen as fundamental units, find the dimensions of mass, length and time in terms of c, G and h.* **[IIT 92]**

Solution. We have,

$$[c] = LT^{-1}, \quad [G] = M^{-1}L^3T^{-2}, \quad [h] = ML^2T^{-1}$$

$$\therefore \quad \frac{[h][c]}{[G]} = \frac{ML^2T^{-1}.\,LT^{-1}}{M^{-1}L^3T^{-2}} = M^2$$

Hence $[M] = h^{1/2}c^{1/2}G^{-1/2}$.

Again, $\dfrac{[h]}{[c]} = \dfrac{ML^2T^{-1}}{LT^{-1}} = ML$

$$\therefore \quad [L] = \frac{h}{c[M]} = \frac{h}{ch^{1/2}c^{1/2}G^{-1/2}}$$

$$= h^{1/2}c^{-3/2}G^{1/2}$$

As $[c] = LT^{-1}$

$$\therefore \quad [T] = \frac{[L]}{c} = \frac{h^{1/2}c^{-3/2}G^{1/2}}{c} = h^{1/2}c^{-5/2}G^{1/2}.$$

Problem 3. *The velocity of a body which has fallen freely under gravity varies as $g^p h^q$, where g is the acceleration due to gravity at the place and h is the height through which the body has fallen. Determine the values of p and q.* **[NCERT 83]**

Solution. Let $v = K g^p h^q$,

where K = a dimensionless constant.

Putting the dimensions of various quantities, we get

$$LT^{-1} = [LT^{-2}]^p [L]^q$$

or $\quad L^1T^{-1} = L^{p+q}T^{-2p}$

Equating the powers of L and T on both sides, we get :

$$p + q = 1, \quad -2p = -1$$

On solving, $p = \dfrac{1}{2}$, $q = \dfrac{1}{2}$.

Problem 4. *A gas bubble, from an explosion under water, oscillates with a period T proportional to $p^a d^b E^c$, where p is the static pressure, d is the density of water and E is the total energy of the explosion. Find the values of a, b and c.* **[IIT 81 ; MNREC 90]**

Solution. Let $T = K p^a d^b E^c$,

where K = a dimensionless constant.

Putting the dimensions of various quantities,

$$T = [ML^{-1}T^{-2}]^a [ML^{-3}]^b [ML^2T^{-2}]^c$$

or $\quad M^0L^0T = M^{a+b+c}L^{-a-3b+2c}T^{-2a-2c}$

Equating the powers of M, L and T on both sides, we get

$$a + b + c = 0, \quad -a - 3b + 2c = 0, \quad -2a - 2c = 1$$

On solving, $a = -\dfrac{5}{6}$, $b = \dfrac{1}{2}$, $c = \dfrac{1}{3}$.

Problem 5. *A small steel ball of radius r is allowed to fall under gravity through a column of a viscous liquid of coefficient of viscosity η. After some time the velocity of the body attains a constant value v_T. The terminal velocity depends upon (i) the weight of the ball mg (ii) the coefficient of viscosity η and (iii) the radius of the ball r. By the method of dimensions, determine the relation expressing terminal velocity.* **[Chandigarh 07]**

Solution. Let $v_T = K(mg)^a \eta^b r^c$,

where K = a dimensionless constant.

Putting the dimensions of various quantities,

$$LT^{-1} = [MLT^{-2}]^a [ML^{-1}T^{-1}]^b [L]^c$$

or $\quad M^0L^1T^{-1} = M^{a+b}L^{a-b+c}T^{-2a-b}$

Equating the powers of M, L and T on both sides, we get

$$a + b = 0, \quad a - b + c = 1, \quad -2a - b = -1$$

On solving, $a = 1$, $b = -1$, $c = -1$

$$\therefore \quad v_T = K(mg)^1\eta^{-1}r^{-1} \quad \text{or} \quad v_T \propto \frac{mg}{\eta r}.$$

Problem 6. *Derive dimensionally the relation :*

$$S = ut + \frac{1}{2}at^2.$$

Solution. Let $S = Ku^x a^y t^z$,

where K = a dimensionless constant.

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Putting dimensions of various quantities,

$$L = [LT^{-1}]^x [LT^{-2}]^y [T]^z$$

or $$L^1T^0 = L^{x+y}T^{-x-2y+z}$$

Equating the powers of M, L and T, we get

$$x + y = 1$$

$$-x - 2y + z = 0$$

These two equations cannot be solved for three unknowns x, y and z. The problem is split into two parts.

(*i*) Suppose the body has no acceleration. Then

$$S = K_1 u^x t^z$$

$$\therefore \quad L = [LT^{-1}]^x [T]^z = L^x T^{-x+z}$$

Equating the powers of L and T, $x = 1$, $-x + z = 0$

On solving, $x = 1$, $z = 1$

Hence $S = K_1 ut$

(*ii*) Suppose the body has no initial velocity. Then

$$S = K_2 a^y t^z$$

$$\therefore \quad L = [LT^{-2}]^y [T]^z = L^y T^{-2y+z}$$

Equating the powers of L and T, $y = 1$, $-2y + z = 0$

On solving, $y = 1$, $z = 2$

Hence $S = K_2 at^2$

(*iii*) Suppose the body has both acceleration and initial velocity. Then

$$S = K_1 ut + K_2 at^2$$

It is found that $K_1 = 1$ and $K_2 = 1/2$. Therefore,

$$S = ut + \frac{1}{2} at^2.$$

Problem 7. *The specific heats of a gas are measured as* $C_p = (12.28 \pm 0.2)$ *units and* $C_v = (3.97 \pm 0.3)$ *units. Find the value of real gas constant R and percentage error in it.*

Solution. Gas constant,

$$R = C_p - C_v$$

$$= (12.28 \pm 0.2) - (3.97 \pm 0.3)$$

$$= \mathbf{(8.31 \pm 0.5)\ units}$$

% Error in R,

$$\frac{\Delta R}{R} \times 100 = \left[\frac{\Delta C_p + \Delta C_v}{C_p - C_v}\right] \times 100$$

$$= \frac{0.5}{8.31} \times 100 = \mathbf{6.016\%}.$$

Problem 8. *The heat dissipated in a resistance can be determined from the relation :*

$$H = \frac{I^2 Rt}{42}\ cal$$

If the maximum errors in the measurement of current, resistance and time are 2%, 1% and 1% respectively, what would be the maximum error in the dissipated heat ?

Solution. Given : $H = \dfrac{I^2 Rt}{4.2}$

% Error, $$\frac{\Delta H}{H} \times 100 = \left(2\frac{\Delta I}{I} + \frac{\Delta R}{R} + \frac{\Delta t}{t}\right) \times 100$$

$$= 2 \times 2 + 1 + 1 = \mathbf{6\%}.$$

Guidelines to NCERT Exercises

2.1. *Fill in the blanks :*

(*i*) *The volume of a cube of side 1 cm is equal to ..... m^3.*

(*ii*) *The surface area of a solid cylinder of radius 2 cm and height 10 cm is equal to ..... $(mm)^2$.*

(*iii*) *A vehicle moving with a speed of 18 km/h covers .....m in 1 s.*

(*iv*) *The relative density of lead is 11.3. Its density is ..... gcm^{-3} or ..... kgm^{-3}.*

Ans.

(*i*) $V = l^3 = (1\,\text{cm})^3 = (10^{-2}\,\text{m})^3 = \mathbf{10^{-6}\,m^{-3}}$.

(*ii*) $r = 2\,\text{cm} = 20\,\text{mm}$, $h = 10\,\text{cm} = 100\,\text{mm}$

$S = 2\pi r(r + h) = 2 \times 3.14 \times 20(20 + 100)$

$= \mathbf{15072\ mm^2}$.

(*iii*) $v = 18\,\dfrac{\text{km}}{\text{h}} = \dfrac{18 \times 1000\,\text{m}}{60 \times 60\,\text{s}} = \mathbf{5\ ms^{-1}}$.

(*iv*) Density = Relative density × density of water at 4°C

$= 11.3 \times 1\,\text{gcm}^{-3} = \mathbf{11.3\ gcm^{-3}}$

$= 11.3 \times 10^3\ \text{kg m}^{-3} = \mathbf{11300\ kgm^{-3}}$.

2.2. *Fill in the blanks by suitable conversion of units :*

(*i*) $1\,kg\,m^2s^{-2} = ...g\,cm^2s^{-2}$

(*ii*) $1\,m = ...light\ year$ (*iii*) $3\,ms^{-2} = ...kmh^{-2}$

(*iv*) $G = 6.67 \times 10^{-11}\ Nm^2kg^{-2} = ...cm^3s^{-2}g^{-1}$

Ans. (*i*) $1\,\text{kg m}^2\text{s}^{-2} = 1\,(10^3\,\text{g})\,(10^2\,\text{cm})^2\,\text{s}^{-2}$

$= \mathbf{10^7\,g\,cm^2s^{-2}}$.

(*ii*) As 1 light year $= 9.46 \times 10^{15}$ m

$\therefore \quad 1\,\text{m} = \dfrac{1}{9.46 \times 10^{15}}$ light year

$= 1.053 \times 10^{-16}$ light year $\simeq \mathbf{10^{-16}}$ **light year.**

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(iii) $3\ ms^{-2} = 3(10^{-3}\ km)\left(\frac{1}{60\times 60}\ h\right)^{-2}$

$= 3\times 10^{-3}\times 3600\times 3600\ kmh^{-2}$

$= 3.888\times 10^{4}\ kmh^{-2} \simeq \mathbf{3.9\times 10^{4}\ kmh^{-2}}$.

(iv) $G = 6.67\times 10^{-11}\ Nm^2kg^{-2}$

$= 6.67\times 10^{-11}\ kg\ m\ s^{-2}.\ m^2\ kg^{-2}$

$= 6.67\times 10^{-11}\ m^3s^{-2}kg^{-1}$

$= 6.67\times 10^{-11}\ (10^2cm)^3s^{-2}(1000\ g)^{-1}$

$= \mathbf{6.67\times 10^{-8}\ cm^3s^{-2}g^{-1}}$.

2.3. *A calorie is a unit of heat energy and it equals about 4.2 J, where $1\ J = 1\ kg\ m^2\ s^{-2}$. Suppose we employ a system of units in which the unit of mass equals α kg, the unit of length equals β m, the unit of time is γ s. Show that a calorie has a magnitude $4.2\ \alpha^{-1}\beta^{-2}\gamma^2$ in terms of the new units.*

Ans. As 1calorie = 4.2 J, where $1\ J = 1\ kg\ m^2s^{-2}$

Clearly, $[Energy] = ML^2T^{-2}$

$\therefore\ a = 1,\ b = 2,\ c = -2$

SI	*New System*
$n_1 = 4.2$	$n_2 = ?$
$M_1 = 1\ kg$	$M_2 = \alpha\ kg$
$L_1 = 1\ m$	$L_2 = \beta\ m$
$T_1 = 1\ s$	$T_2 = \gamma\ s$

$$\therefore\ n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

$$= 4.2\left[\frac{1\ kg}{\alpha\ kg}\right]^1\left[\frac{1\ m}{\beta\ m}\right]^2\left[\frac{1\ s}{\gamma\ s}\right]^{-2}$$

$$= 4.2\ \alpha^{-1}\beta^{-2}\gamma^2$$

Hence 1 calorie = 4.2 J = $\mathbf{4.2\alpha^{-1}\beta^{-2}\gamma^2}$ **new units of energy.**

2.4. (i) *Explain the statement clearly : To call a dimensional quantity 'large' or 'small' is meaningless without specifying a standard for comparison.*

(ii) *In view of this, reframe the following statements, wherever necessary :*

(a) *Atoms are very small objects*

(b) *A jet plane moves with great speed*

(c) *The mass of jupiter is very large*

(d) *The air inside this room contains a large number of molecules*

(e) *A proton is much more massive than an electron*

(f) *The speed of sound is much smaller than the speed of light.*

Ans. (i) The given statement is correct. Measurement is basically a comparison process. Without specifying a standard of comparison, it is not possible to get an exact idea about the magnitude of a dimensional quantity. For example, the statement that the mass of the earth is very large, is meaningless. To correct it, we can say that the mass of the earth is large in comparison to any object lying on its surface.

(ii) (a) The size of an atom is much smaller than even the sharp tip of a pin.

(b) A jet planet moves with a speed greater than that of a superfast train.

(c) The mass of jupiter is very large compared to that of the earth.

(d) The air inside this room contains more number of molecules than in one mole of air.

(e) This is a correct statement.

(f) This is a correct statement.

2.5. *A new unit of length is chosen such that the speed of light in vacuum is unity. What is the distance between the sun and the earth in terms of the new unit if light takes 8 min and 20 s to cover this distance ?*

Ans. Speed of light

= 1 new unit of length / s

Time = 8 min 20 s = 8 × 60 + 20 = 500 s

Distance between the earth and the sun

= Speed of light × time = 1 × 500

= **500 new units of length.**

2.6. *Which of the following is the most precise device for measuring length :*

(a) *a vernier calliper with 20 divisions on the sliding scale,*

(b) *a screw gauge of pitch 1 mm and 100 divisions on the circular scale*

(c) *an optical instrument that can measure length to within a wavelength of visible light ?*

Ans. The device that has minimum least count will be more precise for measuring length.

(a) Least count of vernier callipers

$$= 1\ MSD - 1\ VSD = 1\ MSD - \frac{19}{20}\ MSD = \frac{1}{20}\ MSD$$

$$= \frac{1}{20}\times 1\ mm = \frac{1}{200}\ cm = 0.005\ cm.$$

(b) Least count of screw gauge

$$= \frac{\text{Pitch}}{\text{No. of divisions on circular scale}}$$

$$= \frac{1.0\ mm}{100} = \frac{1}{1000}\ cm = 0.001\ cm$$

(c) Least count of optical instrument

= Wavelength of visible (red) light

$= 6000\ Å = 6000\times 10^{-8}\ cm$

$= 0.00006\ cm.$

Hence the most precise device for measuring length is the given optical instrument.

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2.7. *A student measures the thickness of a human hair by looking at it through a microscope of magnification 100. He makes 20 observations and finds that the average width of the hair in the field of view of the microscope is 3.5 mm. What is the estimate on the thickness of hair ?*

Ans. Average thickness of hair as observed through microscope = 3.5 mm

Magnification produced by the microscope = 100

Actual thickness of hair

$$= \frac{\text{Observed thickness}}{\text{Magnification}} = \frac{3.5}{100} = \mathbf{0.035\ mm.}$$

2.8. *Answer the following :*

(a) You are given a thread and a metre scale. How will you estimate the diameter of the thread ?

(b) A screw gauge has a pitch of 1.0 mm and 200 divisions on the circular scale. Do you think it is possible to increase the accuracy of the screw gauge arbitrarily by increasing the number of divisions on the circular scale ? **[Delhi 13]**

(c) The mean diameter of a thin brass rod is to be measured by vernier calipers. Why is a set of 100 measurements of the diameter expected to yield a more reliable estimate than a set of 5 measurements only ?

Ans. (*a*) The thread is wound on the metre scale so that its turns are as close as possible. Thickness 'l' of the thread coil is measured and the number of turns 'n' of the thread coil is counted.

$$\therefore \quad \text{Thickness of thread} = \frac{l}{n}\ \text{cm}.$$

(*b*) Least count of a screw gauge

$$= \frac{\text{Pitch}}{\text{Number of divisions on circular scale}}$$

Theoretically, it appears that the least count can be decreased by increasing the number of divisions on the circular scale. Practically, it may not be possible to take the reading precisely due to low resolution of human eye.

(*c*) Larger the number of readings, closer is the arithmetic mean to the true value and hence smaller the random error. Hence result with a set of 100 measurements is more reliable than that with a set of 5 measurements.

2.9. *The photograph of a house occupies an area of 1.75 cm^2 on a 35 mm slide. The slide is projected on to a screen and the area of the house on the screen is 1.55 m^2. What is the linear magnification of the projector-screen arrangement ?*

Ans. Size of object = $1.75\ \text{cm}^2 = 1.75 \times 10^{-4}\ \text{m}^2$

Size of image = $155\ \text{m}^2$

Areal magnification

$$= \frac{\text{Size of image}}{\text{Size of object}} = \frac{1.55}{1.75 \times 10^{-4}} = 8857$$

Linear magnification

$$= \sqrt{\text{Areal magnification}} = \sqrt{8857} = \mathbf{94.1.}$$

2.10. *State the number of significant figures in the following :*

(i) $0.007\ m^2$ *(ii)* $2.64 \times 10^{24}\ kg$

(iii) $0.2370\ g\ cm^{-3}$ *(iv)* $6.320\ J$

(v) $6.032\ Nm^{-2}$ *(vi)* $0.0006032\ m^2$

Ans. (*i*) One : 7 (*ii*) Three : 2, 6, 4

(*iii*) Four : 2, 3, 7, 0 (*iv*) Four : 6, 3, 2, 0

(*v*) Four : 6, 0, 3, 2 (*vi*) Four 6, 0, 3, 2

2.11. *The length, breadth and thickness of a rectangular sheet of metal are 4.234 m, 1.005 m and 2.01 cm, respectively. Give the area and volume of the sheet to correct significant figures.*

Ans. Here $l = 4.234$ m, $b = 1.005$ m,

$h = 2.01$ cm $= 0.0201$ m

Area of sheet $= 2(lb + bh + hl)$

$= 2\,(4.234 \times 1.005 + 1.005 \times 0.0201 + 0.0201 \times 4.234)\ \text{m}^2$

$= 2\,(4.25517 + 0.0202005 + 0.0851034)\ \text{m}^2$

$= 2 \times 4.3604739\ \text{m}^2 = 8.7209478\ \text{m}^2$

$= 8.72\ \text{m}^2$ [Rounded off upto 3 significant figures]

Volume of the sheet

$= lbh = 4.234 \times 1.005 \times 0.0201\ \text{m}^3$

$= 0.0855289\ \text{m}^3 = \mathbf{0.0855\ m^3}$.

[Rounded off upto three significant figures]

2.12. *The mass of a box measured by a grocer's balance is 2.3 kg. Two gold pieces of masses 20.15 g and 20.17 g are added to the box. What is (a) the total mass of the box and (b) the difference in the masses of the pieces to correct significant figures ?*

Ans. (*a*) Total mass of the box

$= 2.3\ \text{kg} + 0.02015\ \text{kg} + 0.02017\ \text{kg}$

$= 2.34032\ \text{kg} = \mathbf{2.3\ kg.}$

The result has been rounded off to first place of decimal because mass (2.3 kg) of box has digits upto this place of decimal.

(*b*) Difference in masses of 2 gold pieces

$= 20.17\ \text{g} - 20.15\ \text{g} = \mathbf{0.02\ g.}$

2.13. *A physical quantity P is related to four observations : a, b, c and d as follows :* $P = a^3b^2/\sqrt{cd}$

The percentage errors of measurement in a, b, c and d are 1%, 3%, 4% and 2% respectively. What is the percentage error in the quantity P ? If the value of P calculated using the above relation turns out to be 3.763 , to what value should you round off the result ? **[Central Schools 12 ; Delhi 09]**

Ans. Given : $P = \dfrac{a^3 b^2}{\sqrt{c}\,d}$

The percentage error in the quantity P is given by

$$100 \times \frac{\Delta P}{P} = 3 \times 100 . \frac{\Delta a}{a} + 2 \times 100 . \frac{\Delta b}{b} + \frac{1}{2} \times 100 . \frac{\Delta c}{c} + 100 \times \frac{\Delta d}{d}$$

$$= 3 \times 1\% + 2 \times 3\% + \frac{1}{2} \times 4\% + 2\% = \mathbf{13\%.}$$

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Since 13% = 0.13, so there are two significant figures in the percentage error. Hence P should also be rounded off to 2 significant figures.

$$P = 3.763 = 3.8.$$

∴

2.14. *A book with many printing errors contains four different formulae for the displacement y of a particle undergoing a certain periodic motion :*

(i) $y = a \sin \frac{2\pi t}{T}$, (ii) $y = a \sin vt$,

(iii) $y = \left(\frac{a}{T}\right) \sin \frac{t}{a}$

(iv) $y = \left(\frac{a}{\sqrt{2}}\right)\left(\sin \frac{2\pi t}{T} + \cos \frac{2\pi t}{T}\right)$

(a = maximum displacement of the particle, v = speed of the particle, T = time-period of motion). Rule out the wrong formula on dimensional grounds.

Ans. Dimensions of LHS in all the cases (*i*) to (*iv*) = L

Dimensions of RHS in different cases are as follows :

(i) $\left[a \sin \frac{2\pi t}{T}\right] = L \cdot \sin \frac{T}{T} = L$

(Angle $\frac{2\pi t}{T}$ is dimensionless)

This relation is dimensionally correct.

(ii) $[a \sin vt] = L \sin (LT^{-1} T) = L \sin (L)$

(Angle is not dimensionless)

This relation is dimensionally wrong.

(iii) $\left[\left(\frac{a}{T}\right) \sin \frac{t}{a}\right] = \frac{L}{T} \sin \frac{T}{L}$

(Angle is not dimensionless)

This relation is dimensionally wrong.

(iv) $\left[\frac{a}{\sqrt{2}}\left(\sin \frac{2\pi t}{T} + \cos \frac{2\pi t}{T}\right)\right] = L\left[\sin \frac{T}{T} + \cos \frac{T}{T}\right]$

(Angle is dimensionless)

This relation is dimensionally correct.

Hence formulae (*ii*) and (*iii*) are dimensionally wrong.

2.15. *A famous relation in physics relates 'moving mass' m to the 'rest mass' m_0 of a particle in terms of its speed v and the speed of light c. (This relation first arose as a consequence of special relatively due to Albert Einstein). A boy recalls the relation almost correctly but forgets where to put the constant c. He writes :* $m = \frac{m_0}{(1 - v^2)^{1/2}}$

Guess where to put the missing c. **[Central Schools 14]**

Ans. Since quantities of similar nature can only be added or subtracted, v^2 cannot be subtracted from dimensionless constant 1. It should be divided by c^2 so as to make it dimensionless. Hence the corrected relation is

$$m = \frac{m_0}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}}.$$

2.16. *The radius of a hydrogen atom is about 0.5 Å. What is the total atomic volume in m^3 of a mole of hydrogen atoms ?*

Ans. Radius of a hydrogen atom,

$$r = 0.5 \text{ Å} = 0.5 \times 10^{-10} \text{ m}$$

Volume of one atom $= \frac{4}{3} \pi r^3$

No. of atoms in 1 mole $= 6.023 \times 10^{23}$

Volume of 1 mole of H-atoms $= N \times \frac{4}{3} \pi r^3$

$$= 6.023 \times 10^{23} \times \frac{4}{3} \times 3.14 \times (0.5 \times 10^{-10})^3$$

$$= 3.154 \times 10^{-7} \text{ m}^3 \simeq \mathbf{3 \times 10^{-7} \text{ m}^3}.$$

2.17. *One mole of an ideal gas at S.T.P. occupies 22.4 L. What is the ratio of molar volume to the atomic volume of a mole of hydrogen ? Why is this ratio so large ? Take the radius of hydrogen molecule to be 1 Å.*

Ans. Radius of a hydrogen molecule

$$= 1 \text{ Å} = 10^{-10} \text{ m}$$

Atomic volume of a mole of hydrogen

= Avogadro's no.
× Volume of a hydrogen molecule

$$= 6.023 \times 10^{23} \times \frac{4}{3} \times 3.14 \times (10^{-10})^3$$

$$= 25.2 \times 10^{-7} \text{ m}^3$$

Molar volume $= 22.4 \text{ L} = 22.4 \times 10^{-3} \text{ m}^3$

$$\therefore \quad \frac{\text{Molar volume}}{\text{Atomic volume}} = \frac{22.4 \times 10^{-3}}{25.2 \times 10^{-7}} = 0.89 \times 10^4 = \mathbf{10^4}.$$

This ratio is large because the actual size of the gas molecules is negligibly small in comparison with the intermolecular separation.

2.18. *Explain why on looking through the window of a fast moving train, the nearby trees and electric poles etc. appear to run in direction opposite to that of motion of the train, while far off houses, hilltops, Moon, stars etc. appear stationary.*

Ans. The line joining the object and the eye is called the line of slight. The direction of the line of sight of the nearby objects like trees, poles etc. ; changes very rapidly due to fast motion of the train and accordingly they appear to be moving opposite to the direction of motion of the train. But the line of sight of a distant object almost does not change its direction due to its extremely large distance from the eye. Hence the distant objects like hilltops, moon, stars etc. appear stationary.

2.19. *A parsec is a convenient unit of length on the astronomical scale. It is the distance of an object that will show a parallax of 1" (second) of arc from opposite ends of a baseline equal to the distance from the earth to the sun. How much is parsec in terms of metres ?*

Ans. One parsec is the distance at which an arc of length 1 AU makes an angle of 1 second of an arc.

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As $\theta(\text{rad}) = \dfrac{\text{Arc}}{\text{Radius}} = \dfrac{l}{r}$ $\quad\therefore\quad r = \dfrac{l}{\theta}$.

Here $\quad l = 1\text{ AU} = 1.496 \times 10^{11}\text{ m}$

$$\theta = 1\text{ s of arc} = \frac{\pi}{60 \times 60 \times 180}\text{ rad}$$

$$= 4.85 \times 10^{-6}\text{ rad}$$

$$\therefore \quad 1\text{ parsec} = r = \frac{1496 \times 10^{11}}{4.85 \times 10^{-6}} = 3.08 \times 10^{16}\text{ m}.$$

Order of magnitude of parsec = 16.

2.20. *The nearest star (Alpha Centauri) to our solar system is 4.29 light years away. How much is this distance in terms of parsec ? How much parallax would this star show when viewed from two locations of the earth six months apart in its orbit around the sun ?*

Ans. As $\quad 1\text{ light year} = 9.46 \times 10^{15}\text{ m},$

$$1\text{ parsec} = 3.08 \times 10^{16}\text{ m}$$

$\therefore$ Distance of Alpha Centauri from the earth,

$$S = 4.29\text{ light years} = 4.29 \times 9.46 \times 10^{15}\text{ m}$$

$$= \frac{4.29 \times 9.46 \times 10^{15}}{3.08 \times 10^{16}}\text{ parsec} = 1.32\text{ parsec}$$

In an orbit around the sun, the distance between the two locations of the earth six months apart,

b = Diameter of the earth's orbit = 2 AU

Parallax of the star,

$$\theta = \frac{\text{Arc}}{\text{Radius}} = \frac{b}{S} = \frac{2\text{ AU}}{1.32\text{ parsec}} = \mathbf{1.515\text{ s of arc.}}$$

2.21. *Precise measurements of physical quantities are a need of science. For example, to ascertain the speed of an aircraft, one must have an accurate method to find its positions at closely separated instants of time. This was the actual motivation behind the discovery of radar in World War II. Think of different examples in modern science where precise measurements of length, time, mass etc. are needed. Also, wherever you can, give a quantitative idea of the precision needed.*

Ans. Some of the examples of modern science, where precise measurements play an important role, are as follows :

1. Electron microscope uses an electron beam of wavelength 0.2 Å to study very minute objects like viruses, microbes and the crystal structure of solids.
2. The successful launching of artificial satellites has been made possible only due to the precise technique available for accurate measurement of time-intervals.
3. The precision with which the distances are measured in Michelson-Morley Interferometer helped in discarding the idea of hypothetical medium ether and in developing the Theory of Relativity by Einstein.

2.22. *Just as precise measurements are necessary in science, it is equally important to be able to make rough estiamtes of quantities using rudimentary ideas and common observations. Think of ways by which you can estimate the following (where an estimate is difficult to obtain, try to get an upper bound on the quantity) :*

(a) the total mass of rain-bearing clouds over India during the Monsoon

(b) the mass of an elephant

(c) the wind speed during a storm

(d) the number of strands of hair on your head

(e) the number of air molecules in your classroom.

Ans. (*a*) The average rainfall during the Monsoon in India is about 100 cm or 1 m.

Total surface area of India

$$= 3.3 \times 10^6\text{ km}^2 = 3.3 \times 10^6 \times (10^3\text{ m})^2$$

$$= 3.3 \times 10^{12}\text{ m}^2.$$

Volume of rain water,

$$V = Ah = 3.3 \times 10^{12}\text{ m}^2 \times 1\text{ m} = 3.3 \times 10^{12}\text{ m}^3$$

Density of water, $\rho = 10^3\text{ kg m}^{-3}$

Hence total mass of rain-bearing clouds over India,

$$m = V\rho = 3.3 \times 10^{12} \times 10^3 = \mathbf{3.3 \times 10^{15}\text{ kg}}.$$

(*b*) To estimate the mass of an elephant, consider a boat of base area A in a river. Let x_1 be the depth of the boat inside water. Now move the elephant into the boat. Again measure the depth x_2 of the boat inside water.

Volume of water displaced by elephant

$$V = A(x_2 - x_1)$$

According to Archimedes' principle, mass of the elephant is

m = Mass of water displaced by the elephant

$$= V\rho = A(x_2 - x_1)\rho$$

Mass of an elephant is about 10^3 kg.

(*c*) The wind speed during a storm can be measured by floating a gas filled balloon in air. When there is no wind storm, suppose the balloon is at vertical height $OA = h$, from the ground. Due to the wind storm, suppose the balloon moves to position B in a small time interval t, as shown in Fig. 2.16.

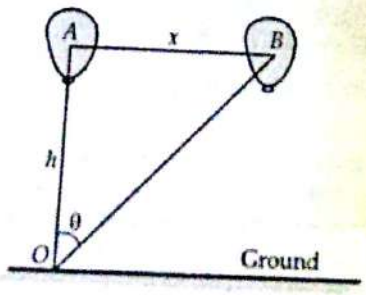

Fig. 2.16

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If θ is the angle of drift of the balloon, then from right angled ΔOAB,

$$\tan\theta = \frac{AB}{OA} = \frac{x}{h} \qquad [AB = x, \text{ say}]$$

or $$x = h\tan\theta$$

Hence the wind speed during the storm,

$$v = \frac{x}{t} = \frac{h\tan\theta}{t}.$$

(*d*) First we count the strands of hair of 1 cm^2 area of the head. Then by multiplying it by the total area of the head, we can estimate the total number of stands of hair on the head. Its order of magnitude may be as large as 10^8.

(*e*) The dimensions of a typical classroom are

$$8\text{ m} \times 6\text{ m} \times 4\text{ m}$$

Volume of the class room $= 8\times6\times4 = 192\,m^3$

Now one mole of air molecules occupy a volume of 22.4 litres or $22.4\times10^{-3}\ m^3$.

$\therefore$ Number of molecules in $22.4\times10^{-3}\ m^3$

$$= 6.023\times10^{23}$$

Number of molecules in the classroom

$$= \frac{6.023\times10^{23}}{22.4\times10^{-3}}\times192 = 516\times10^{26}\text{ molecules}$$

$$\approx \mathbf{10^{28}\ molecules.}$$

2.23. *The sun is a hot plasma (ionized matter) with its inner core at a temperature exceeding 10^7 K, and its outer surface at a temperature of about 6000 K. At these high temperatures no substance remains in a solid or liquid phase. In what range do you expect the mass density of the sun to be ? In the range of densities of solids and liquids or gases ? Check if your guess is correct from the following data : mass of the sun $= 2.0\times10^{30}$ kg, radius of the sun $= 7.0\times10^8$ m.*

Ans. Mass of the sun, $M = 2.0\times10^{30}$ kg

Radius of the sun, $R = 7.0\times10^8$ m

Volume of the sun,

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi\times(7.0\times10^8)^3$$

$$= 1.437\times10^{27}\ m^3$$

Density of the sun,

$$\rho = \frac{M}{V} = \frac{2.0\times10^{30}}{1.437\times10^{27}}$$

$$= 1391.8\text{ kg m}^{-3} \approx \mathbf{1.4\times10^3\ kg\ m^{-3}}.$$

The density of the sun is in the range of the densities of the solids and liquids but not gases. The high density is due to the invard gravitational attraction on the outer layers due to the inward layers of the sun.

2.24. *When the planet Jupiter is at a distance of 824.7 million kilometres from the earth, its angular diameter is measured to be 35.72 s of arc. Calculate the diameter of Jupiter.*

Ans. Distance of Jupiter from the earth,

$$S = 824.7\times10^6\text{ km}$$

Angular diameter of Jupiter,

$$\theta = 35.72'' = \left(\frac{35.72}{60\times60}\right)^\circ = \frac{35.72}{3600}\times\frac{\pi}{180}\text{ rad}$$

Diameter of Jupiter,

$$D = S\times\theta = 824.7\times10^6\times\frac{35.72}{3600}\times\frac{\pi}{180}$$

$$= \mathbf{1.428\times10^5\ km.}$$

2.25. *A man walking briskly in rain with speed v must slant his umbrella forward making an angle θ with the vertical. A student derives the following relation between θ and v : $\tan\theta = v$ and checks that the relation has a correct limit : as $v \to 0$, $\theta \to 0$, as expected. (We are assuming there is no strong wind and that the rain falls vertically for a stationary man). Do you think this relation can be correct ? If not, guess at the correct relation.*

Ans. Since trigonometric functions are dimensionless,

$\therefore$ $[\tan\theta] = 1$

But $[v] = LT^{-1}$

$\therefore$ Dimensions of LHS $\neq$ Dimensions of RHS

Hence the given relation is dimensionally wrong.

This relation can be corrected by dividing RHS by the speed '*u*' of the rainfall. So the corrected relation is

$$\tan\theta = \frac{v}{u}.$$

2.26. *It is claimed that two cesium clocks, if allowed to run for 100 years, free from any disturbance, may differ by only about 0.02 s. What does this imply for the accuracy of the standard cesium clock in measuring a time-interval of 1 s ?*

Ans. Here $\Delta t = 0.02$ s,

$$t = 100\text{ years} = 100\times365.25\times86{,}400\text{ s}$$

Fractional error

$$= \frac{\Delta t}{t} = \frac{0.02}{100\times365.25\times86400} = 0.63\times10^{-11}$$

So there is an accuracy of 10^{-11} part in 1s or 1s in 10^{11} s.

2.27. *Estimate the average density of a sodium atom assuming its radius to be about 2.5 Å. Compare it with the density of sodium in crystalline phase : 970 kg m^{-3}. Are the two densities of the same order of magnitude ? If so, why ?*

Ans. Radius of a sodium atom,

$$r = 2.5\text{ Å} = 2.5\times10^{-10}\text{ m}$$

Volume of a sodium atom,

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\times(2.5\times10^{-10})^3$$

$$= 65.42\times10^{-30}\text{ kg}$$

Mass of a sodium atom

$$= \frac{\text{Mass number}}{\text{Avogadro's number}} = \frac{23}{6.02\times10^{23}}\text{ g}$$

$$= 3.82\times10^{-23}\text{ g} = 3.82\times10^{-26}\text{ kg}$$

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Average density of sodium atom

$$= \frac{\text{Mass}}{\text{Volume}} = \frac{3.82 \times 10^{-26}}{65.42 \times 10^{-30}}$$

$$= 0.58 \times 10^{3} \text{ kg m}^{-3}$$

Density of sodium in crystalline phase

$$= 970 \text{ kg m}^{-3} = 0.970 \times 10^{3} \text{ kg m}^{-3}$$

Hence the average mass density of sodium atom and the density of crystalline sodium are of the same order of magnitude (10^3). This is because sodium atoms in crystalline phase are closely packed.

2.28. *The unit of length convenient on the nuclear scale is a fermi : 1 $f = 10^{-15}$ m. Nuclear sizes obey roughly the following empirical relation $r = r_0 A^{1/3}$; where r is the radius of the nucleus, A its mass number and r_0 is a constant equal to about 1.2 f. Show that the rule implies that nuclear mass density is nearly constant for different nuclei. Estimate the mass density of sodium nucleus. Compare it with the average mass density of a sodium atom obtained in Exercise 2.27.*

Ans. Radius of a nucleus, $r = r_0 A^{1/3}$

Mass of a nucleus $= \dfrac{\text{Mass number}}{\text{Avogadro's number}} = \dfrac{A}{N_A}$

Nuclear mass density $= \dfrac{\text{Mass of a nucleus}}{\text{Volume of a nucleus}}$

or $$\rho = \frac{A}{N_A \cdot \frac{4}{3}\pi r^3} = \frac{A}{N_A \cdot \frac{4}{3}\pi (r_0 A^{1/3})^3} = \frac{3}{4\pi N_A r_0^3}$$

As ρ is independent of A, so nuclear mass density is same for different nuclei.

For a kilomole,

$$N_A = 6.02 \times 10^{26}, \; r_0 = 1.2 \; f = 1.2 \times 10^{-15} \text{ m}$$

$$\therefore \quad \rho = \frac{3}{4\pi \times 6.02 \times 10^{26} \times (1.2 \times 10^{-15})^3}$$

$$= \mathbf{2.3 \times 10^{17} \text{ kg m}^{-3}}$$

Density of sodium nucleus should also be

$$= 2.3 \times 10^{17} \text{ kg m}^{-3}$$

From Exercise 2.27, density of sodium atom

$$= 0.58 \times 10^{3} \text{ kg m}^{-3}$$

$$\therefore \quad \frac{\text{Nuclear mass density}}{\text{Atomic mass density}} = \frac{2.3 \times 10^{17}}{0.58 \times 10^{3}} = 3.96 \times 10^{14}.$$

Nuclear density is typically 10^{15} times atomic density of matter.

2.29. *A laser is a source of very intense, monochromatic, and unidirectional beam of light. These properties of a laser light can be exploited to measure long distances. The distance of the Moon from the Earth has been already determined very precisely using a laser as a source of light. A laser light beamed at the moon takes 2.56 s to return after reflection at the moon's surface. How much is the radius of the lunar orbit around the earth ?*

Ans. Here $t = 2.56$ s, $c = 3 \times 10^8 \text{ ms}^{-1}$

Radius of the lunar orbit around the earth

= Distance of the moon from the earth

$$= \frac{c \times t}{2} = \frac{3 \times 10^8 \times 2.56}{2} = \mathbf{3.84 \times 10^8 \text{ m}}.$$

2.30. *A SONAR (Sound Navigation and Ranging) uses ultrasonic waves to detect and locate objects under water. In a submarine equipped with a SONAR, the time delay between generation of a probe wave and the reception of its echo after reflection from an enemy submarine is found to be 77 s. What is the distance of the enemy submarine ?*

(Speed of sound in water = 1450 ms^{-1}).

Ans. Here $t = 77$ s, $c = 1450 \text{ ms}^{-1}$

Distance of enemy submarine

$$= \frac{c \times t}{2} = \frac{1450 \times 77}{2} = \mathbf{55825 \text{ m}}.$$

2.31. *The farthest objects (known as quasers) in our universe are so distant that light emitted by them takes billion of years to reach the earth. What is the distance in km of a quaser from which light takes 3.0 billion years to reach us ?*

Ans. Here $t = 3.0$ billion years

$$= 3.0 \times 10^9 \times 365.25 \times 24 \times 60 \times 60 \text{ s}$$

Speed of light, $c = 3 \times 10^5 \text{ kms}^{-1}$

Distance of quaser

$$= ct = 3 \times 10^5 \times 3.0 \times 10^9 \times 365.25 \times 24 \times 60 \times 60$$

$$= \mathbf{2.84 \times 10^{22} \text{ km}}.$$

2.32. *It is a well known fact that during a solar eclipse the disc of the moon almost completely covers the disc of the sun. From this fact and from the information that sun's angular distance α is measured to be 1920'', determine the approximate diameter of the moon. Given earth-moon distance = 3.8452×10^8 m.*

Ans. During total solar eclipse, the disc of the moon completely covers the disc of the sun, so the angular diameters of both the sun and the moon must be equal.

∴ Angular diameter of the moon,

θ = Angular diameter of the sun

$$= 1920'' = 1920 \times 4.85 \times 10^{-6} \text{ rad}$$

$$[\because \; 1'' = 4.85 \times 10^{-6} \text{ rad}]$$

Earth-moon distance, $S = 3.8452 \times 10^8$ m

Diameter of the moon,

$$D = \theta \times S = 1920 \times 4.85 \times 10^{-6} \times 3.8452 \times 10^8$$

$$= 3.581 \times 10^6 \text{ m} = \mathbf{3581 \text{ km}}.$$

2.33. *A great physicist of this century (P.A.M. Dirac) loved playing with numerical values of Fundamental constants of nature. This led him to an interesting observation. Dirac found that from the basic constants of atomic physics (c, e, mass of electron, mass of proton) and the gravitational constant G, he could arrive at a number with the dimension of time. Further, it was a very large number, its magnitude being close to the present estimate on the age of the universe (≈ 15 billion years).*

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From the table of fundamental constants in this book, try to see if you too can construct this number (or any other interesting number you can think of). If its coincidence with the age of the universe were significant, what would this imply for the constancy of fundamental constant ?

Ans. Using basic constants such as speed of light (c), charge on electron (e), mass of electron (m_e), mass of proton (m_p) and gravitational constant (G), we can construct the quantity,

$$\tau = \left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \times \frac{1}{m_p m_e^2 c^3 G}$$

$$\text{Now } \left[\frac{e^2}{4\pi\varepsilon_0}\right] = \left[\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}r^2\right] = [Fr^2]$$

$$= [MLT^{-2}.L^2] = [ML^3T^{-2}]$$

$$\therefore \quad [\tau] = \frac{[ML^3T^{-2}]^2}{[M][M]^2[LT^{-1}]^3[M^{-1}L^3T^{-2}]} = [T]$$

Clearly, the quantity τ has the dimensions of time.

Put $G = 6.67\times10^{-11}$ Nm^2 kg^{-2}, $c = 3\times10^8$ ms^{-1}, $e = 1.6\times10^{-19}$ C, $m_e = 9.1\times10^{-31}$ kg, $m_p = 1.67\times10^{-27}$ kg and $\frac{1}{4\pi\varepsilon_0} = 9\times10^9$ Nm^{2}C^{-2}

$$\therefore \quad \tau = \frac{[9\times10^9\times(1.6\times10^{-19})^2]^2}{1.67\times10^{-27}\times(9.1\times10^{-31})^2\times(3\times10^8)^3\times6.67\times10^{-11}}$$

$$= 2.13\times10^{16} \text{ s}$$

$$= \frac{2.13\times10^{16}}{3.156\times10^7} \text{ years} = 0.667\times10^9 \text{ years.}$$

$$= 0.667 \text{ billion years.}$$

This time is slightly less than the age of the universe ($\approx$ 15 billion years). It implies that the values of the basic constants of physics should change with time because the age of the universe increases with time.

Text Based Exercises

Type A : Very Short Answer Questions — 1 Mark Each

1. What is a physical unit ? **[Himachal 01]**
2. Write the relationship between numerical value of a quantity and the size of the unit.
3. Does the magnitude of a physical quantity change with the change in the system of units.
4. Define length.
5. Define a prototype metre.
6. Define standard metre in terms of the wavelength of light.
7. What is the accuracy of the metre defined in terms of wavelength of light radiation ?
8. Define standard metre in terms of velocity of light.
9. Why was metre redefined in terms of velocity of light in 1983 ?
10. Is light year a unit of time ?
11. Define light year and express it in metres. **[Himachal 2K]**
12. Define one astronomical unit. **[Himachal 06C]**
13. Name the unit used to express the distances of stars.
14. Define parsec and express it in metres. **[Himachal 02, 05]**
15. Express parsec in light years.
16. Arrange in ascending order-astronomical unit, parsec and light year.
17. Name two commonly units used to express wavelength of light.
18. Express one micron in metre. **[Delhi 10]**
19. Name the unit used to measure size of a nucleus and express it in metre.
20. Express nanometre in terms of metre and angstrom units.
21. Express the wavelength of yellow light (5893 Å) in terms of nm.
22. How many nanometres are there in one metre ? **[Himachal 07C, 08C]**
23. How many angstrom are there in one metre ? **[Himachal 07C, 08C]**
24. How many fermi are there in one metre ? **[Himachal 07C]**
25. How many light years are there in one metre ? **[Himachal 06C]**
26. Write the full name of the technique used in locating (*a*) an under-water obstacle (*b*) position of an aeroplane in space. **[Chandigarh 07]**
27. How much far away is the nearest star alpha centuri from us ?
28. What is the order of size of our galaxy ?
29. Give the order of mean free path of an air molecule.
30. What is the shortest distance measured indirectly so far ?
31. What is the estimated radius of the universe ?
32. Suggest a distance corresponding to 10^7 m.
33. Name the unit used for measuring nuclear cross-sections.
34. Name the device used for measuring directly the lengths (*i*) from 10^{-3} m to 10^2 m (*ii*) to an accuracy of 10^{-4} m (*iii*) to an accuracy of 10^{-5} m.

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35. Name the device that can be used to measure the number of wavelengths of light in a given distance.
36. What does the word RADAR stand for ?
37. What does the word SONAR stand for ?
38. What does the word LASER stand for ?
39. What is a laser ?
40. Express light year in metres. What is its order of magnitude ?
41. What types of waves are used in a SONAR ?
42. Define international standard of mass.
43. Define atomic mass unit. Express it in kg.
44. Are the inertial and gravitational masses of an object different from one another ?
45. It is said that common balance compares masses and spring balance measures weight. If a body is 2 kg in common balance, it is 2 kg in spring balance, does it mean that mass is equal to weight ?
46. How many times larger is kg than a mg ? **[Himachal 03, 06C]**
47. How many metric tons are there in a teragram ?
48. What is the order of mass of an electron ?
49. What is the order of mass of universe ?
50. What is the smallest mass measured indirectly so far ?
51. Define second in terms of casium-133 vibrations.
52. Human heart is an in-built clock. Comment.
53. Which is the most accurate atomic clock ?
54. Name the unit used for measuring very small time intervals. How is it related to second ?
55. Which technique is used for measuring age of fossils, rocks, etc.
56. How many times is millisecond larger than a microsecond ?
57. Are there more microseconds in a second than the number of seconds in a year ?
58. What is the order of age of the earth ?
59. Give the order of average life of a human being.
60. What is the shortest time interval measured indirectly so far ?
61. Name the SI unit of (*i*) temperature and (*ii*) electric current.
62. Name and define the SI unit of luminous intensity.
63. Define one mole.
64. What is leap year ?
65. Name the year in which there is total solar eclipse.
66. The density of wood is 0.5 cm^{-3}. Write this value in SI units.
67. The radius of the sun is 696 000 000 m. Express it in scientific notation (in powers of 10).
68. Express 0.00000538 in powers of 10.
69. Define dimensions of a physical quantity.
70. State the principle of homogeneity of dimensions.
71. Which of the following has the same dimensions as 'Planck's constant' : Torque, gravitational constant and angular momentum ?
72. What are the dimensions of rate of flow ?
73. Give names of a scalar quantity and a vector quantity which have same dimensions.
74. What is the difference between the measurements 4.0 cm and 4.000 cm ?
75. What importance is attached to the final zeros in a number without any decimal point ?
76. If $f = x^3$, then relative error in f would be how many times the relative error in x ?
77. A junior research fellow takes 100 careful readings in an experiment. If he repeats the same experiment by taking 500 readings, then by what factor will the probable error be reduced ?
78. Give two examples of non-dimensional variables.
79. Write the names of three dimensional constants.
80. Write two examples of non-dimensional constants.
81. If g is the acceleration due to gravity and λ is wavelength, then which physical quantity does $\sqrt{\lambda g}$ represent ?
82. Write the dimensional formula of wavelength and frequency of a wave. **[Delhi 09]**
83. If $x = a + bt^2$, where x is in metres and t in seconds, find the units of a and b. **[Himachal 08]**
84. If $x = at + bt^2$, where x is in metres and t in hours (hr), what will be the units of 'a' and 'b' ? **[Central Schools 08 ; Delhi 10]**
85. State the number of significant figures in
(*a*) 0.007 m (*b*) 2.67×10^{-24} kg **[Central Schools 14]**
86. How can a systematic error be eliminated ?
87. How can we eliminate constant error ?
88. Name the SI unit used to express the amount of substance. **[Central Schools 04]**
89. Write the number of significant figures in each of the following measurements :
(*a*) 1.67×10^{-27} kg ; (*b*) 0.270 cm **[Delhi 04]**
90. Add 8.2 and 10.163 and round off the sum to two significant figures. **[Delhi 06]**
91. Give approximate ratio of 1 AU and 1 light year. **[Delhi 01]**
92. How many kilograms are there in 1 amu ? **[Himachal 2K]**
93. What do you understand by absolute error ? **[Himachal 04]**

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Answers

1. The standard amount of a physical quantity chosen to measure the physical quantity of the same kind is called a physical unit.

2. The numerical value (n) of a physical quantity is inversely proportional to the size of the unit (u) *i.e.*,

$$n \propto \frac{1}{u} \quad \text{or} \quad nu = \text{constant}.$$

3. No, the magnitude of a physical quantity does not change with the change in the system of units.

4. Length may be defined as distance of separation between two points in space.

5. In 1899, the general conference of weights and measures defined metre as the distance between two transverse lines marked on a platinum-iridium rod preserved at a constant temperature of 273.16 K at 1 bar pressure in the International Bureau of Weights and Measures at Sevres, near Paris in France.

6. Standard metre is defined as the distance which contains 1650,763.73 wavelengths of certain orange-red radiation emitted by Kr-86 atom in its transition between $2p_{10}$ and $2d_5$ levels.

7. Standard metre defined in terms of the wavelength of light radiation has an accuracy of 1 part in 10^9.

8. The velocity of light in vacuum, $c = 299, 792, 458\ ms^{-1}$. So one metre is defined as the length of the path travelled by light in vacuum in 1/299, 792, 458 of a second.

9. Because of the tremendous accuracy (1 part in 10^{13}) achieved in the measurement of time, metre has been redefined in terms of velocity of light since 1983.

10. No, light year is a unit of distance.

11. One light year is defined as the distance travelled by light in one year.

$$1 \text{ light year} = 9.46 \times 10^{15} \text{ m}$$

12. One astronomical unit (AU) is the mean distance of the earth from the sun.

$$1 \text{ AU} = 1.496 \times 10^{11} \text{ m}.$$

13. Parallactic second (parsec).

14. One parsec is defined as the distance at which an arc of length one astronomical unit subtends an angle of one second of an arc.

$$1 \text{ parsec} = 3.08 \times 10^{16} \text{ m}.$$

15. 1 parsec = 3.26 light years.

16. 1 AU < 1 light year < 1 parsec.

17. nm and Å.

18. One micron is one millionth part of a metre.

$$1 \text{ micron} = 1\mu = 10^{-6} \text{ m}.$$

19. The unit used to measure nuclear size is fermi.

$$1 \text{ fermi} = 10^{-15} \text{ m}.$$

20. 1 nanometre or $1 \text{ nm} = 10^{-9} \text{ m} = 10 \text{ Å}$.

21. As $10 \text{ Å} = 1 \text{ nm}$

$$\therefore 5893 \text{ Å} = \frac{5893}{10} = 589.3 \text{ nm}.$$

22. 1 nanometre (nm) $= 10^{-9}$ m $\quad \therefore 1 \text{ m} = 10^9$ nm.

23. 1 angstrom (Å) $= 10^{-10}$ m $\quad \therefore 1 \text{ m} = 10^{10}$ Å

24. 1 fermi (f) $= 10^{-15}$ m $\quad \therefore 1 \text{ m} = 10^{15}\ f$

25. 1 light year (ly) $= 9.46 \times 10^{15}$ m

$$\therefore 1 \text{ m} = \frac{1}{9.46 \times 10^{15}} \text{ ly} = \mathbf{1.057 \times 10^{-16}\ ly}.$$

26. (*a*) Sound navigation and ranging (SONAR)
(*b*) Radio navigation and ranging (RADAR)

27. Alpha centauri is 4.26 light years away from us.

28. The order of size of our galaxy is 10^{20} m.

29. The order of mean free path of an air molecule is 10^{-6} m.

30. The shortest distance measured indirectly so far is the radius of a proton ($\approx 10^{-15}$ m).

31. The estimated radius of the universe is 10^{25} m.

32. Radius of the earth is the order of 10^7 m.

33. Barn is used to measure nuclear cross-section.

$$1 \text{ barn} = 10^{-28} \text{ m}^2.$$

34. (*i*) Metre-scale (*ii*) Vernier callipers (*iii*) Screw gange or spherometer.

35. Optical interferometer.

36. The word RADAR stands for *radio detection and ranging*.

37. The word SONAR stands for *sound navigation and ranging*.

38. The word LASER stands for *light amplification by stimulated emission of radiation*.

39. A laser a source of very intense, highly monochromatic and highly directional beam of light.

40. 1 light year $= 9.46 \times 10^{15}$ m $= 0.946 \times 10^{16}$ m.
$\therefore$ Order of magnitude of light year $= 10^{16}$ m.

41. Ultrasonic waves.

42. The international standard of mass is kilogram (kg). One kilogram is the mass of a platinum iridium (90% Pt and 10% Ir) cylinder of diameter equal to its height preserved in the International Bureau of Weight and Measures at Sevres.

43. One atomic mass unit is the mass of $\frac{1}{12}$th of the mass of an atom of carbon-12 isotope.

$$1 \text{ amu} = 1.66 \times 10^{-27} \text{ kg}$$

44. No, the inertial and gravitational masses of an object are equivalent.

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45. No, mass and weight are two different physical quantities. If mass is 2 kg in common balance, it is 2 kilogram force (kg f) in spring balance.

46. $1\ kg = 10^6\ mg$

47. $1\ Teragram = 10^{12}\ g = 10^9\ kg = 10^6$ metric tons.

48. Mass of an electron $= 10^{-30}$ kg

49. Mass of universe $= 10^{55}$ kg

50. The smallest mass measured so far is the mass of an electron ($\approx 10^{-30}$ kg).

51. One second is defined as the duration of 9192, 631, 770 vibrations corresponding to the transition between two hyperfine levels of cesium-133 atom in the ground state.

52. Yes, because human heart beats at a regular rate.

53. Cesium atomic clock.

54. Shake is the unit used for measuring very small time-intervals.

$$1\ shake = 10^{-8}\ second$$

55. Radioactive dating.

56. 1 millisecond $= 10^3$ microsecond.

57. No. As 1 microsecond $= 10^{-6}$ s, therefore

1 second $= 10^6$ microseconds

Also, no. of seconds in a year

$$= 365 \times 24 \times 60 \times 60 = 31.536 \times 10^6.$$

Clearly, the number of microseconds in a second is smaller than the number of seconds in a year.

58. Age of earth $= 10^{17}$ second.

59. Average life of a human being ≈ 100 years $= 10^9$ s.

60. The shortest time interval measured so far is the life-span of most unstable nucleus ($\approx 10^{-24}$ s).

61. (*i*) The SI unit of temperature is kelvin (K).

(*ii*) The SI unit of electric current is ampere (A).

62. The SI unit of luminous intensity is candela (cd). One candela is defined as the luminous intensity, in a given direction, of a source that emits a mochromatic radiation of frequency 540×10^{12} hertz and has a radiant intensity in that direction of 1/ 683 watt per steradian.

63. One mole is that amount of a substance which contains as many elementary entities as there are atoms in 12 g of carbon-12 isotope. These entities may be atoms, molecules, ions, etc.

64. Leap year is the year which is divisible by four. The February month of a leap year has 29 days.

65. Tropical year.

66. $\text{Density} = 0.5\,\frac{g}{cm^3} = \frac{0.5 \times 10^{-3}\ kg}{10^{-6}\ m^3} = 500\ kg\ m^{-3}$.

67. Radius of the sun = 696000 000 m

$$= 6.96 \times 10^8\ m.$$

68. $0.00000538 = 5.38 \times 10^{-6}$.

69. The dimensions of a physical quantity are the powers to which the fundamental quantities must be raised to represent that quantity completely.

70. The principle of homogeneity of dimensions states that a physical equation will be dimensionally correct if the dimensions of all the terms occurring on both sides of the equation are the same.

71. Angular momentum.

72. $[\text{Rate of flow}] = \frac{[\text{Volume}]}{[\text{Time}]} = \frac{[L^3]}{[T]} = [L^3T^{-1}]$.

73. Speed is a scalar quantity and velocity is a vector quantity. Both have the same dimensions of $[LT^{-1}]$.

74. (*i*) 4.0 cm has two significant figures while 4.000 cm has four significant figures.

(*ii*) 4.0 cm is correct upto first decimal place while 4.000 cm is correct upto third place of decimal.

75. All such zeros are not significant. For example 86,400 has three significant figures.

76. When $f = x^3$, $\frac{\Delta f}{f} = 3 \times \frac{\Delta x}{x}$

$\therefore$ Relative error in f

= Three times the relative error in x.

77. The probable error will be reduced by a factor of 1/5.

78. Specific gravity and strain.

79. Gravitational constant, Planck's constant and Boltzmann's constant.

80. π, e, all trigonometric functions.

81. $\sqrt{\lambda g} = \sqrt{L \cdot LT^{-2}} = LT^{-1}$

$\therefore$ $\sqrt{\lambda g}$ represents velocity.

82. $[\lambda] = [M^0L^1T^0]$ and $[v] = [M^0L^0T^{-1}]$

83. Unit of a = unit of x = metre

Unit of b = unit of $(x/t^2) = m/s^2$.

84. Unit of a = unit of $(x/t) = m/h$

Unit of b = unit of $(x/t^2) = m/h^2$.

85. (*a*) Three : 0, 0, 7 (*b*) Three : 2, 6, 7

86. The errors which tend to occur in one direction, positive or negative, are called systematic errors. Such an error can be eliminated by detecting the source of error and the rule governing this error.

87. The errors which affect each observation by the same amount are called constant errors. Such an error can be eliminated measuring the same physical quantity by number of different methods, apparatus or technique.

88. Mole

89. (*a*) Three : 1, 6, 7
(*b*) Three : 2, 7, 0

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90. $\begin{array}{r} 8.2 \\ +\ 10.163 \\ \hline \end{array}$

Sum = 18.363

Corrected sum = 18

[Rounded off upto 2 significant figures]

91. $\frac{1\text{ AU}}{1\text{ light year}} = \frac{1.496 \times 10^{11}\text{ m}}{9.46 \times 10^{15}\text{ m}} \approx 10^{-5}$.

92. $1\text{ amu} = 1.66 \times 10^{-27}\text{ kg}$.

93. Refer answer to Q.56 on page 2.40.

Type B : Short Answer Questions

2 or 3 Marks Each

1. Define the following : (*i*) Light year (*ii*) Parsecond (*iii*) Astronomical unit. **[Himachal 05]**
2. Explain the triangulation method to measure the height of an inaccessible object. **[Himachal 07]**
3. Distinguish between mass and weight. **[Himachal 01, 04]**
4. Distinguish between inertial mass, gravitational mass and weight of a body.
5. Which technique is used for measuring large time intervals ?
6. Define dimensional formula. Give uses of dimensional analysis. Write down the limitations of dimensional analysis. **[Himachal 05]**
7. In what way is the knowledge of the dimensions of a physical quantity useful ?
8. How is a dimensional formula different from a differential equation ?
9. Distinguish between the dimensions and unit of a physical quantity.
10. Differentiate between dimensional and non-dimensional variables.
11. Distinguish between dimensional and non-dimensional constants.
12. Name any three physical quantities having the same dimensions and also give their dimensions. **[Central Schools 07]**
13. What are the dimensional formulae of the following : (*i*) pressure (*ii*) power (*iii*) density (*iv*) angle ? **[Himachal 01]**
14. Write the dimensional formulae of the following physical quantities : (*i*) work (*ii*) angular velocity (*iii*) pressure (*iv*) Planck's constant. **[Himachal 05]**
15. Mention the various sources of occurrence of errors, while taking measurements.
16. State the errors other than random and instrumental errors.
17. Show that the maximum error in the sum of the two quantities is equal to the sum of the absolute errors in the two individual quantities.
18. Show that the absolute error in the difference of two quantities is equal to the sum of the absolute errors in the individual quantities.
19. Show that the maximum fractional error in the product of two quantities is equal to the sum of the fractional errors in the individual quantities.
20. Show that the maximum error in the quotient of two quantities is equal to the sum of their individual fractional errors.
21. Show that the fractional error in the nth power of a quantity is equal to n times the fractional error in the quantity itself. State the general rule for evaluating the error in a combined calculation.
22. What is meant by the term "measurement of a physical quantity ? How is the result of measurement of a physical quantity expressed ? **[Delhi 03C]**
23. State the principle of homogeneity of dimensions. Test the dimensional homogeneity of the following equation : $h = h_0 + v_0 t + \frac{1}{2} g t^2$ **[Delhi 03]**
24. Define least count error. What is the value of least count error associated with the scale in your geometry box ? **[Central Schools 03]**
25. Describe the parallax method to find the distance of an inferior planet from earth. **[Delhi 06]**
26. What is meant by RADAR and SONAR ? How are long distances measured using these techniques ? **[Central Schools 04]**
27. If $x = 2at - 5bt^2$, where x is in metres and t in seconds, find the dimensions of a and b. **[Central Schools 13]**

Answers

1. Refer answer to Q.13 on page 2.5.
2. Refer answer to Q.17 on page 2.8.
3. Refer answer to Q.32 on page 2.16.
4. Inertial mass of a body is the measure of its inertia in translatory motion. It is equal to the force required to produce unit acceleration in the body.

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The mass of a body which determines the gravitational pull acting upon it due to the earth is called its gravitational mass. It is equal to force experienced by the body in a gravitational field of unit intensity.

Weight of a body is the force with which a body is attracted towards the centre of the earth.

5. The technique of radioactive dating is used to measure long time intervals by finding the ratio of the number of radioactive atoms that have undergone decay to the number of atoms left undecayed.

6. Refer answer to Q. 44 on page 2.25 and Q.49 on page 2.36.

7. The dimensions of a physical quantity represent the powers of the fundamental units, on which the given physical quantity depends. So we can express its units in terms of the fundamental units. Suppose the dimensions of a physical quantity are 1 in mass, 1 in length and – 2 in time, then its SI unit is $kgms^{-2}$.

8. Dimensional formula is an expression which shows how and which of the fundamental units of mass, length and time occur in the derived unit of a physical quantity. The equation obtained by equating a physical quantity to its dimensional formula is called its dimensional equation.

9. Dimensions of a physical quantity are the powers to which the fundamental units must be raised to represent the unit of the given physical quantity. The unit is compact mathematical expression involving fundamental units. The unit may also be given some name. *e.g.*, $[F] = [MLT^{-2}]$ and unit of force = $kg\ ms^{-2}$ or newton.

10. The quantities which have dimensions but do not possess a constant value are called dimensional variables *e.g.*, velocity, force etc. On the other hand, the quantities which have neither dimensions nor they have a constant value are called non-dimensional variables *e.g.*, relative density, strain, etc.

11. The quantities which have dimensions as well as a constant value are called dimensional constants *e.g.*, Planck's constant, Boltzmann's constant. On the other hand, the quantities which have no dimensions but a constant value are called non-dimensional constants *e.g.*, π, $\sin\theta$, $\cos\theta$ etc.

12. Work, energy and torque. Each quantity has the dimensions $[ML^2T^{-2}]$.

13. (*i*) $[ML^{-1}T^{-2}]$ (*ii*) $[ML^2T^{-3}]$
(*iii*) $[ML^{-3}T^0]$ (*iv*) dimensionless.

14. (*i*) $[ML^2T^{-2}]$ (*ii*) $[M^0L^0T^{-1}]$
(*iii*) $[ML^{-1}T^{-2}]$ (*iv*) $[ML^2T^{-1}]$

15. While taking measurements, the sources of occurrence of errors may be as follows :

(*i*) *Instrumental errors* due to faulty graduations, zero error, etc.

(*ii*) *Personal errors* due to personal peculiarities of the experimenter.

(*iii*) *Error due to imperfection* in experimental arrangement, such as loss of heat due to radiation in calorimetry.

(*iv*) *Errors due to external causes* such as expansion of scale due to rise in temperature.

16. The errors other than random and instrumental errors are (*i*) absolute error (*ii*) relative error (*iii*) percentage error and (*iv*) gross error. The gross error may be due to incorrect reading of the instrument or due to incorrect recording of the reading.

17. Refer answer to Q. 57 on page 2.42.

18. Refer answer to Q. 57 on page 2.42.

19. Refer answer to Q. 57 on page 2.42.

20. Refer answer to Q. 57 on page 2.42.

21. Refer answer to Q. 57 on page 2.42.

22. Refer answer to Q. 3 on page 2.2.

23. For principle of homogeneity of dimensions, refer answer to Very Short Answer Q. 70 on page 2.64.

$$[h] = L, \quad [h_0] = L$$

$$[u_0 t] = LT^{-1} \cdot T = L$$

$$\left[\frac{1}{2} gt^2\right] = LT^{-2} \cdot T^2 = L$$

As all the terms have the same dimensions, so the given equation is dimensionally homogeneous.

24. Refer answer to Q. 55 on page 2.39.

25. Refer answer to Q. 22 on page 2.10.

26. Refer answer to Q. 27 and Q. 28 on page 2.11.

27. $[a] = [LT^{-1}]$, $[b] = [LT^{-2}]$.

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Competition Section

Units and Measurements

GLIMPSES

1. **Physical quantities.** All those quantities which can be measured directly or indirectly and in terms of which the laws of physics can be expressed are called *physical quantities*.

2. (*i*) **Physical unit.** The standard amount of a physical quantity chosen to measure the physical quantity of the same kind is called a *physical unit*.

   Measure of a physical quantity = Numerical value of the quantity × size of the unit = nu

   (*ii*) **Relationship between the numerical value and the size of the unit.** The numerical value (n) of a physical quantity is inversely proportional to the size of the unit.

   $$n \propto \frac{1}{u} \quad \text{or} \quad nu = \text{constant} \quad \text{or} \quad n_1 u_1 = n_2 u_2 .$$

3. **Fundamental quantities.** The physical quantities which can be treated as independent of other physical quantities and are not usually defined in terms of other physical quantities are called *fundamental quantities*. The seven fundamental quantities are mass, length, time, electric current, temperature, luminous intensity and amount of substance.

4. **Derived quantities.** The physical quantities whose defining operations are based on other physical quantities are called derived quantities.

5. **Fundamental units.** The physical units which can neither be derived from one another, nor they can be further resolved into more simpler units are called *fundamental units*. The units of length, mass, time, etc. are fundamental units.

6. **Derived units.** All other units which can be expressed in terms of fundamental units are called *derived units*.

7. **System of units.** A complete set of units which is used for measuring all kinds of fundamental and derived quantities is called a *system of units*.

   (*i*) ***The CGS or the metric system.*** In this system the fundamental units of length, mass and time are centimetre, gram and second respectively.

   (*ii*) ***The FPS or the British system.*** In this system the fundamental units of length, mass and time are foot, pound and second respectively.

   (*iii*) ***The MKS system.*** In this system the fundamental units of length, mass and time are metre, kilogram and second respectively.

   (*iv*) ***The SI.*** SI is the abbreviation for *Systeme Internationale d'Unites*, which is the French equivalent for *international system of units*. In this system the fundamental units of length, mass, time, electric current, temperature, luminous intensity and amount of substance are metre, kilogram, second, ampere, kelvin, candela and mole, respectively.

8. **The seven basic SI units.**

   (*i*) ***Metre (m).*** One metre is defined as the length of the path travelled by light in vacuum during a time interval of 1/299,792,458 of a second (1983).

   (*ii*) ***Kilogram (kg).*** It is the mass of a platinum-iridium cylinder (90% Pt and 10% Ir) whose height is equal to its diameter (each = 3.9 cm) preserved at International Bureau of Weights and Measures at Sevres (1889).

   (*iii*) ***Second (s).*** One second is defined as the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom (1967).

   (*iv*) ***Ampere (A).*** One ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre

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apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length (1948).

(v) ***Kelvin (K).*** One kelvin is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water (1967).

(vi) ***Candela (cd).*** Candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian (1979).

(vii) ***Mole (mol).*** One mole is the amount of substance of a system, which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon-12 (1971).

9. **The two supplementary SI units.**

(i) ***Radian (rad).*** It is defined as the plane angle subtended at the centre of a circle by an arc equal in length to the radius of the circle.

$$\theta \text{ (in radians)} = \frac{\text{Arc}}{\text{Radius}} = \frac{l}{r}$$

(ii) ***Steradian (sr).*** It is the solid angle subtended at the centre of a sphere by a surface of the sphere equal in area to that of a square, having each side equal to the radius of the sphere.

$$\Omega \text{(in steradian)} = \frac{\text{Surface area}}{\text{Radius}^2}$$

10. **Coherent system.** A coherent system of units is a system based on a certain set of fundamental units from which all derived units are obtained by multiplication or division without introducing any numerical factor.

11. **Advantages of SI.** It is a metric, rational, coherent and internationally acceptable system of units which covers all branches of physics.

12. **Some Practical Units.**

(i) 1 fermi = 10^{-15} m

(ii) 1 angstrom (Å) = 10^{-10} m

(iii) 1 nanometre (nm) = 10^{-9} m

(iv) 1 micron (μm) = 10^{-6} m

(v) 1 light year (ly) = 9.46×10^{15} m

(vi) 1 astronomical unit (AU) = 1.496×10^{11} m

(vii) 1 parallactic second (parsec) = 3.08×10^{16} m

(viii) 1 barn = 10^{-28} m^2

(ix) 1 tonne = 1000 kg

(x) 1 quintal = 100 kg

(xi) 1 slug = 14.57 kg

(xii) 1 atomic mass unit (amu) = 1.66×10^{-27} kg

(xiii) 1 solar year = 365.25 average solar days
= 366.25 sidereal days

(xiv) 1 lunar month = 27.3 days

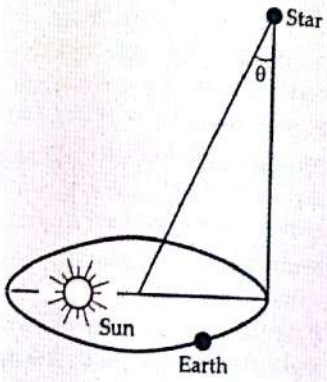
(xv) 1 shake = 10^{-8} s

(xvi) 1 Torr = 1 mm of Hg

(xvii) 1 bar = 1 atmospheric pressure
= 760 mm of Hg = 10^5 Nm^{-2}.

13. **Order of magnitude.** The order of magnitude of a physical quantity is that power of ten which is closest to its magnitude. To determine the order of magnitude of a number N, we express it as $N = n \times 10^x$. If $0.5 < n \leq 5$, then x will be the order of magnitude of N.

14. **Annual parallax.** It is the angle (θ) at which the semi-major axis of the earth's orbit perpendicular to the star's direction is seen from the star.



15. **Indirect methods for measuring long distances.**

(i) ***Reflection or echo method.*** The distance (S) of a hill can be determined by noting the time interval t of sending a sound wave towards the hill and receiving back its echo.

$$S = \frac{c \times t}{2},$$

where c = Speed of sound waves

The same principle is used in radars and sonars.

(ii) ***Triangulation method.*** The height (h) of an accessible object like a tower or a tree can be found by measuring the angle θ subtended by the object at the observation point. If x is the distance of the observation point from the foot of the object, then

$$h = x \tan \theta$$

To find the height of an inaccessible object like a hill, its angles of elevation θ_1 and θ_2 are measured at two points separated by distance d.

$$h = \frac{d}{\cot \theta_2 - \cot \theta_1}.$$

(iii) ***Parallax method.*** By measuring the angle θ (called parallax angle or parallactic angle) subtended

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by the astronomical object at two locations on the earth separated by a large distance b (called basis), the distance S of the object can be determined.

$$\theta = \frac{\text{Arc}}{\text{Radius}} = \frac{b}{S} \quad \text{or} \quad S = \frac{\text{Basis}}{\text{Parallactic angle}} = \frac{b}{\theta}$$

(iii) ***Size of an astronomical object.*** The angle θ (called angular diameter) subtended by the diameter of the planet is measured at a point on the surface of the earth. Knowing the distance S of the planet from the earth, its diameter D can be determined.

$$\theta = \frac{\text{Arc}}{\text{Radius}} = \frac{D}{S} \quad \text{or} \quad D = S \times \theta$$

Linear diameter = Distance × Angular diameter.

(iv) ***Copernicus method.*** This method is used to determine the distance of an inferior planet from the sun. The angle formed at the earth between the earth-planet direction and the earth-sun direction is called the planet's elongation (ε).

Distance of inferior planet from the sun,

$$r_{ps} = \sin\varepsilon \,.\, r_{es} = \sin\varepsilon \text{ AU}$$

Distance of inferior planet from the earth,

$$r_{pe} = \cos\varepsilon \,.\, r_{es} = \cos\varepsilon \text{ AU}.$$

(vi) ***By applying Kepler's third law.*** If T_1 and T_2 be the periods of revolution of two planets and a_1 and a_2 be their semi-major axes, then

$$\frac{a_2^3}{a_1^3} = \frac{T_2^2}{T_1^2}$$

or $$a_2 = \left[\frac{T_2}{T_1}\right]^{2/3} . a_1$$

16. **Molar Volume.** One mole of every gas occupies a volume of 22.4 litres at S.T.P. This is called molar volume of the gas. It contains an Avogadro's number ($N = 6.023 \times 10^{23}$) of molecules.

17. **Indirect methods for measuring small distances.**

(i) ***By using Avogadro's hypothesis.*** According to Avogadro's hypothesis, the actual volume occupied by the atoms in one gram of a substance is two-third of the volume of one gram of the substance.

Radius of atom,

$$r = \left[\frac{M}{2\pi N \rho}\right]^{1/3}$$

where N, M and ρ are the Avogadro's number, molecular weight and density of the substance.

(ii) ***Molecular size of oleic acid.*** A large and very thin film of oil is formed on water surface and its area is measured.

Thickness of oil film,

$$t = \frac{\text{Volume of the film}}{\text{Area of the film}}$$

Assuming that the film is of one molecular thickness, then t is approximately the size of oil molecule.

19. **Mass.** The mass of a body is the quantity of matter contained in it. Its SI unit is kg.

20. **Weight.** The weight of a body is the force with which a body is attracted towards the centre of the earth. $W = mg$.

21. **Inertial mass.** The mass of a body which determines its inertia in translatory motion is called its inertial mass. It is measured from Newton's second law of motion.

$$m_i = \frac{F}{a}$$

22. **Gravitational mass.** The mass of a body which determines the gravitational pull upon it due to the earth is called its gravitational mass. It is measured by using Newton's law of gravitation.

$$m_g = \frac{FR^2}{GM}$$

23. **Dimensions of the derived quantity.** These are the powers to which the fundamental units of mass, length and time must be raised in order to represent a derived quantity completely.

24. **Dimensional formula.** It is an expression which shows how and which of the fundamental units of mass, length and time occur in the derived unit of a physical quantity.

25. **Dimensional equation.** The equation which expresses a physical quantity in terms of the fundamental units of mass, length and time, is called dimensional equation.

26. **Principle of homogeneity of dimensions.** This principle states that a physical equation will be dimensionally correct if the dimensions of all the terms occurring on both sides of the equation are the same.

27. **Uses of dimensional equations :**

(i) To convert a physical quantity from one system of units to another.

Suppose a physical quantity has dimensional formula $M^a L^b T^c$. Let n_1 and n_2 be its numerical values when the units are u_1 and u_2. Then

$$n_1 u_1 = n_2 u_2$$

$$n_1 [M_1^a \, L_1^b \, T_1^c] = n_2 [M_2^a \, L_2^b \, T_2^c]$$

$$\therefore \; n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

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(ii) To check the correctness of a given physical relation.

(iii) To derive a relationship between different physical quantities.

28. **Significant figures.** The significant figures are normally those digits in a measured quantity which are known reliably plus one additional digit that is uncertain.

29. **Rules for counting the number of significant figures in a measured quantity :**

(i) All non-zero digits are significant.

(ii) All zeros between two non-zero digits are significant.

(iii) All zeros to the right of a non-zero digit but to left of an understood decimal point are not significant. But such zeros are significant if they come from a measurement.

(iv) All zeros to the right of a non-zero digit but to the left of a decimal point are significant. For example, 648700. has six significant figures.

(v) All zeros to the right of a decimal point are significant.

(vi) All zeros to the right of a decimal point but to the left of a non-zero digit are not significant. Single zero conventionally placed to the left of the decimal point is not significant.

(vii) The number of significant figures does not depend on the system of units.

30. **Significant figures in the sum or difference of two numbers.** In addition or subtraction the result should be reported to the same number of decimal places as that of the number with minimum number of decimal places.

31. **Significant figures in the product or quotient of two numbers.** In multiplication or division, the result should be reported to the same number of significant figures as that of the number with minimum of significant figures.

32. **Rules for rounding off a measurement :**

(i) If the digit to be dropped is less than 5, then the preceding digit is left unchanged.

(ii) If the digit to be dropped is greater than 5, then the preceding digit is increased by 1.

(iii) If the digit to be dropped is 5 followed by non-zero digits, then the preceding digit is increased by 1.

(iv) If the digit to be dropped is 5, then the preceding digit is left unchanged if it is even.

(v) If the digit to be dropped is 5, then the preceding digit is increased by 1 if it is odd.

33. **Accuracy and precision of a measurement.** Accuracy refers to the closeness of a measurement to the true value of the physical quantity. Precision refers to the resolution or the limit to which the quantity is measured.

34. **Error in a measurement.** It is the difference between the measured value and the true value of a physical quantity. It gives an indication of the limits within which the true value may lie.

35. **Elimination of random error.** The errors which occur irregularly and at random, in magnitude and direction, are called random errors. To eliminate random error, a large number of readings are taken and their arithmetic mean is taken as the true value.

$$\bar{a} = \frac{a_1 + a_2 + a_3 + \ldots + a_n}{n} = \frac{1}{n}\sum_{i=1}^{n} a_i$$

36. **Absolute error.** The magnitude of the difference between the true value and the measured value is called absolute error.

Such errors are given by

$$\Delta a_1 = \bar{a} - a_1 \qquad \Delta a_2 = \bar{a} - a_2$$

$$\Delta a_3 = \bar{a} - a_3 \qquad \Delta a_n = \bar{a} - a_n$$

37. **Mean absolute error.** The arithmetic mean of the positive magnitudes of all the absolute errors is called mean absolute error

$$\Delta\bar{a} = \frac{|\Delta a_1| + |\Delta a_2| + \ldots + |\Delta a_n|}{n} = \frac{1}{n}\sum_{i=1}^{n} |\Delta a_i|$$

38. **Relative error.** It is the ratio of the mean absolute error to the true value.

$$\delta a = \frac{\Delta\bar{a}}{\bar{a}}$$

39. **Percentage error.** The relative error expressed in percent is called percentage error.

$$\text{Percentage error} = \frac{\Delta\bar{a}}{\bar{a}} \times 100\%$$

40. **Error combination in a sum or a difference.** When two quantities are added or subtracted, the absolute error in the final result is the sum of the absolute errors associated with the individual quantities.

$$\Delta Z = \Delta A + \Delta B$$

41. **Error combination in a product or a quotient.** When two quantities are multiplied or divided, the fractional error in the final result is the sum of the fractional errors of the two quantities.

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

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42. **Error due to the power of a measured quantity.** The fractional error in the nth power of a quantity is equal to n times the fractional error in the quantity itself.

If $Z = A^n$, then $\frac{\Delta Z}{Z} = n\frac{\Delta A}{A}$

43. **General rule.** If $Z = \frac{A^p B^q}{C^r}$, then the maximum fractional or relative error in Z will be

$$\frac{\Delta Z}{Z} = p\frac{\Delta A}{A} + q\frac{\Delta B}{B} + r\frac{\Delta C}{C}$$

$$\% \text{ Error in } Z = \frac{\Delta Z}{Z} \times 100$$

$$= p\frac{\Delta A}{A} \times 100 + q\frac{\Delta B}{B} \times 100 + r\frac{\Delta C}{C} \times 100$$

44. **Propagation or combination of errors.** It is summarised in the adjacent table.

Operation	Formula Z	Absolute error ΔZ	Relative error $\Delta Z / Z$	Percentage error $100\Delta Z/Z$
Sum	$A + B$	$\Delta A + \Delta B$	$\frac{\Delta A + \Delta B}{A + B}$	$\frac{\Delta A + \Delta B}{A + B} \times 100$
Difference	$A - B$	$\Delta A + \Delta B$	$\frac{\Delta A + \Delta B}{A - B}$	$\frac{\Delta A + \Delta B}{A - B} \times 100$
Multiplication	$A \times B$	$A\Delta B + B\Delta A$	$\frac{\Delta A}{A} + \frac{\Delta B}{B}$	$\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right) \times 100$
Division	$\frac{A}{B}$	$\frac{B\Delta A + A\Delta B}{B^2}$	$\frac{\Delta A}{A} + \frac{\Delta B}{B}$	$\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right) \times 100$
Power	A^n	$n A^{n-1}\Delta A$	$n\frac{\Delta A}{A}$	$n\frac{\Delta A}{A} \times 100$
Root	$A^{1/n}$	$\frac{1}{n} A^{1/n - 1}\Delta A$	$\frac{1}{n}\frac{\Delta A}{A}$	$\frac{1}{n}\frac{\Delta A}{A} \times 100$

IIT Entrance Exam

MULTIPLE CHOICE QUESTIONS WITH ONE CORRECT ANSWER

1. The equation of state for a real gas is given by

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

The dimensions of the constant a are

(a) $[ML^5T^{-2}]$ (b) $[M^{-1}L^5T^2]$

(c) $[ML^{-5}T^{-1}]$ (d) $[ML^5T^{-1}]$

[IIT 97 ; CBSE PMT 96]

2. Pressure depends on distance as $P = \frac{\alpha}{\beta}\exp\left(\frac{-\alpha z}{k\theta}\right)$, where α, β are constants, z is distance, k is Boltzmann's constant and θ is temperature. The dimensions of β are

(a) $[M^0L^0T^0]$ (b) $[M^{-1}L^{-1}T^{-1}]$

(c) $[M^0L^2T^0]$ (d) $[M^{-1}L^{-1}T^2]$ [IIT 04]

3. The dimensions of $\left(\frac{1}{2}\right)\varepsilon_0 E^2$ are

(ε_0 : permittivity of free space, E : electric field)

(a) $[MLT^{-1}]$ (b) $[ML^2T^{-2}]$

(c) $[ML^{-1}T^{-2}]$ (d) $[ML^2T^{-1}]$ [IIT 2K]

4. In the formula : $X = 3YZ^2$, X and Z have dimensions of capacitance and magnetic induction respectively. What are the dimensions of Y in MKSQ system ?

(a) $[M^{-3}L^{-1}T^3Q^4]$ (b) $[M^{-3}L^{-2}T^4Q^4]$

(c) $[M^{-2}L^{-2}T^4Q^4]$ (d) $[M^{-3}L^{-2}T^4Q^1]$ [IIT 95]

5. A quantity X is given by $\varepsilon_0 L\frac{\Delta V}{\Delta t}$ where ε_0 is the permittivity of the free space, L is a length, ΔV is a potential difference and Δt is a time interval. The dimensional formula for X is the same as that of

(a) resistance (b) charge

(c) voltage (d) current [IIT 01]

6. Which of the following sets have different dimensions ?

(a) Pressure, Young's modulus, stress

(b) Emf, potential difference, electric potential

(c) Heat, work done, energy

(d) Dipole moment, electric flux, electric field

[IIT 05]

7. A cube has a side of length 1.2×10^{-2} m. Calculate its volume.

(a) $1.7 \times 10^{-6}\,m^3$ (b) $1.73 \times 10^{-6}\,m^3$

(c) $1.0 \times 10^{-6}\,m^3$ (d) $1.732 \times 10^{-6}\,m^3$ [IIT 03]

8. A student performs an experiment for determination of $g\left(=\frac{4\pi^2 l}{T^2}\right)$. The error in length l is Δl and in time T is ΔT and n is number of times the reading is taken. The measurement of g is most accurate for

	Δl	ΔT	n
(a)	5 mm	0.2 sec	10
(b)	5 mm	0.2 sec	20
(c)	5 mm	0.1 sec	10
(d)	1 mm	0.1 sec	50

[IIT 06]

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9. A student performs an experiment to determine the Young's modulus of a wire, exactly 2 m long, by Searle's method. In a particular reading, the student measures the extension in the length of the wire to be 0.8 mm with an uncertainty of ± 0.05 mm at a load of exactly 1.0 kg. The student also measures the diameter of the wire to be 0.4 mm with an uncertainty of ± 0.01 mm. Take $g = 9.8$ m/s^2 (exact). The Young's modulus obtained from the reading is

(*a*) $(2.0 \pm 0.3) \times 10^{11}$ N/m^2

(*a*) $(2.0 \pm 0.2) \times 10^{11}$ N/m^2

(*c*) $(2.0 \pm 0.1) \times 10^{11}$ N/m^2

(*d*) $(2.0 \pm 0.05) \times 10^{11}$ N/m^2 [IIT 07]

10. A wire has a mass 0.3 ± 0.003 g, radius 0.5 ± 0.005 mm and length 6 ± 0.06 cm. The maximum percentage error in the measurement of its density is

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) 4 [IIT 04]

11. Students I, II and III perform an experiment for measuring the acceleration due to gravity (g) using a simple pendulum. They use different lengths of the pendulum and/or record time for different number of oscillations. The observations are shown in the table.

Least count for length = 0.1 cm

Least count for time = 0.1 s

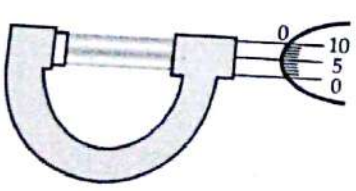
Student	Length of the pendulum (cm)	No. of oscillations (n)	Total time for (n) oscillation (s)	Time period (s)
I	64.0	8	128.0	16.0
II	64.0	4	64.0	16.0
III	20.0	4	36.0	9.0

If E_I, E_{II} and E_{III} are the percentage errors in g *i.e.*, $\left(\frac{\Delta g}{g} \times 100\right)$ for students I, II and III respectively,

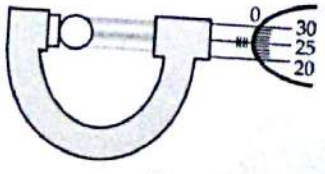
(*a*) $E_I = 0$ (*b*) E_I is maximum

(*c*) $E_I = E_{II}$ (*d*) E_{II} is maximum [IIT 08]

12. In a screw gauge, the zero of main scale coincides with fifth division of circular scale in figure (*i*). The circular divisions of screw gauge are 50. It moves 0.5 mm on main scale in one rotation. The diameter of the ball in figure (*ii*) is



(*i*)



(*ii*)

(*a*) 2.25 mm (*b*) 2.20 mm

(*c*) 1.20 mm (*d*) 1.25 mm [IIT 06]

13. A Vernier calipers has 1 mm marks on the main scale. It has 20 equal divisions on the Vernier scale which match with 16 main scale divisions. For this Vernier calipers, the least count is

(*a*) 0.02 mm (*b*) 0.05 mm

(*c*) 0.1 mm (*d*) 0.2 mm [IIT 2010]

14. The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge, whose pitch is 0.5 mm and there are 50 divisions on the circular scale. The reading on the main scale is 2.5 mm and that on the circular scale is 20 divisions. If the measured mass of the ball has a relative error of 2%, the relative percentage error in the density is

(*a*) 0.9% (*b*) 2.4%

(*c*) 3.1% (*d*) 4.2% [IIT 2011]

Multiple Choice Questions with One or More than One Correct Answer

15. The dimensions of the quantities in one (or more) of the following pairs are the same. Identify the pair(s)

(*a*) Torque and work

(*b*) Angular momentum and work

(*c*) Energy and Young's modulus

(*d*) Light year and wavelength [IIT 86]

16. The pairs of physical quantities that have the same dimensions are

(*a*) Reynold number and coefficient of friction

(*b*) Curie and frequency of a light wave

(*c*) Latent heat and gravitational potential

(*d*) Planck's constant and torque [IIT 95]

17. The dimensions of length are expressed as $G^x c^y h^z$ where G, c and h are the universal gravitational constant, speed of light and Planck's constant respectively, then

(*a*) $x = (1/2), y = (1/2)$

(*b*) $x = (1/2), z = (1/2)$

(*c*) $y = (-3/2), z = (1/2)$

(*d*) $y = (1/2), z = (3/2)$ [IIT 92]

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18. The SI unit of inductance, the henry can be written as

(a) weber/ampere (b) volt-sec/amp

(c) joule/(ampere)^2 (d) ohm-second [IIT 98]

19. If L, C, R represent inductance, capacitance and resistance respectively, the combinations having dimensions of frequency are

(a) $\frac{1}{\sqrt{CL}}$ (b) $\frac{L}{C}$

(c) $\frac{R}{L}$ (d) $\frac{R}{C}$ [IIT 84]

20. Which of the following combinations have the dimensions of time ? L, C, R represent inductance, capacitance and resistance respectively.

(a) RC (b) $\sqrt{LC}$

(c) R/L (d) C/L [IIT 86]

21. Let $[\varepsilon_0]$ denote the permittivity of the vacuum, and $[\mu_0]$ denote the permeability of the vacuum. If M = mass, L = length, T = time and I = electrical current, then the dimensional formulae of ε_0 and μ_0 are

(a) $[\varepsilon_0] = M^{-1}L^{-3}T^2I$ (b) $[\varepsilon_0] = M^{-1}L^{-3}T^4I^2$

(c) $[\mu_0] = MLT^{-2}I^{-2}$ (d) $[\mu_0] = ML^2T^{-1}I$ [IIT 98]

22. A student uses a simple pendulum of exactly 1 m length to determine g, the acceleration due to gravity. He uses a stop watch with the least count of 1 second for this and records 40 seconds for 20 oscillations. For this observation, which of the following statement(s) is (are) true ?

(a) Error ΔT in measuring T, the time period, is 0.05 second

(b) Error ΔT in measuring T, the time period, is 1 second

(c) Percentage error in the determination of g is 5%

(d) Percentage error in the determination of g is 2.5% [IIT 2010]

MATRIX-MATCH TYPE

23. Match the physical quantities given in column I with dimensions expressed in terms of mass (M), length (L), time (T), and charge (Q) given in column II. [IIT 83]

I	II
(a) Angular momentum	(p) ML^2T^{-2}
(b) Torque	(q) ML^2T^{-1}
(c) Inductance	(r) $M^{-1}L^{-2}T^2Q^2$
(d) Latent heat	(s) ML^2Q^{-2}
(e) Capacitance	(t) $ML^3T^{-1}Q^{-2}$
(f) Resistivity	(u) L^2T^{-2}

24. Column I gives three physical quantities. Select the appropriate units for the choices given in column II. Some of the physical quantities may have more than one choice correct. [IIT 90]

I	II
(a) Capacitance	(p) ohm-second
(b) Inductance	(q) coulomb^2-joule^{-1}
(c) Magnetic Induction	(r) coulomb(volt)$^{-1}$
	(s) newton (ampere m)$^{-1}$
	(t) volt-second (ampere)$^{-1}$

25. Some physical quantities are given in column I and some possible SI units in which these quantities may be expressed are given in column II. Match the physical quantities in column I with the units in column II. [IIT 07]

I	II
(a) GM_eM_s ; G = universalgravitation constant, M_e = mass of earth, M_s = mass of sun	(p) (volt) (coulomb) (metre)
(b) $\frac{3RT}{M}$; R = universal gas constant, T = absolute temperature, M = molar mass	(q) (kilogram) (metre)^3 (second)$^{-2}$
(c) $\frac{F^2}{q^2B^2}$; F = force, q = charge, B = magnetic field	(r) (metre)^2 (second)$^{-2}$
(d) $\frac{GM_e}{R_e}$; G = universal gravitational constant, M_e = mass of earth, R_e = radius of earth	(s) (farad) (volt)^2 (kg)$^{-1}$

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Answers and Explanations

1. (c) As quantities with similar dimensions can be added only, so

$$\left[\frac{a}{V^2}\right] = [P]$$

or $$[a] = [P][V^2] = [ML^{-1}T^{-2}][L^3]^2$$

$$= [ML^5T^{-2}]$$

2. (c) Here k = Boltzmann's constant

$$\therefore \quad [k] = \frac{[\text{joule}]}{[\text{kelvin}]} = [ML^2T^{-2}\theta^{-1}]$$

As an exponential factor is dimensionless,

$$\therefore \quad \left[\frac{\alpha z}{k\theta}\right] = [M^0L^0T^0]$$

or $$[\alpha] = \left[\frac{k\theta}{z}\right] = \frac{ML^2T^{-2}\theta^{-1}\cdot\theta}{L} = [MLT^{-2}]$$

$$\beta = \frac{\alpha}{P}\exp\left(\frac{\alpha z}{k\theta}\right)$$

$$\therefore \quad [\beta] = \frac{[\alpha]}{[P]} \times [M^0L^0T^0] = \frac{[MLT^{-2}]}{[ML^{-1}T^{-2}]} = [L^2]$$

3. (c) $\frac{1}{2}\varepsilon_0 E^2 = \frac{\text{Energy}}{\text{Volume}}$

$$\therefore \quad \left[\frac{1}{2}\varepsilon_0 E^2\right] = \frac{[ML^2T^{-2}]}{[L^3]} = [ML^{-1}T^{-2}]$$

4. (b) $[X] = [C] = [M^{-1}L^{-2}T^2Q^2]$

$$[Z] = [B] = [MT^{-1}Q^{-1}]$$

$$\therefore \quad [Y] = \frac{[X]}{[Z^2]} = \frac{[M^{-1}L^{-2}T^2Q^2]}{[MT^{-1}Q^{-1}]^2}$$

$$= [M^{-3}L^{-2}T^4Q^4]$$

5. (d) $[\varepsilon_0 L] = [\text{capacitance}]$

$$\therefore \quad \varepsilon_0 L\frac{\Delta V}{\Delta t} = \frac{C\Delta V}{\Delta t} = \frac{\Delta q}{\Delta t} = \frac{\text{charge}}{\text{time}} = \text{current}.$$

6. (d) Dipole moment = Charge × distance

Electric flux = Electric field × area

∴ Dipole moment, electric flux and electric field have different dimensions.

7. (a) Volume = $(\text{side})^3 = (1.2 \times 10^{-2}\text{m})^3$

$$= 1.728 \times 10^{-6}\text{m}^3 = 1.7 \times 10^{-6}\text{ m}^3$$

As the side has 2 significant figures only, so the expressed volume must have 2 significant figures only.

8. (d) Given $g = \frac{4\pi^2 l}{T^2}$

$$\therefore \quad \frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\frac{\Delta T}{T}$$

Δl is minimum in option (d),

ΔT is minimum in options (c) and (d),

n is maximum in option (d)

Hence error will be minimum or the measurement of g will be most accurate in option (d).

9. (b) Young's modulus,

$$Y = \frac{F}{A}\cdot\frac{L}{l} = \frac{4Mg}{\pi d^2}\cdot\frac{L}{l}$$

$$\therefore \quad Y = \frac{4 \times 1.0 \times 9.8 \times 2}{\pi(0.4 \times 10^{-3})^2 \times 0.8 \times 10^{-3}} = 2.0 \times 10^{11}\ \text{Nm}^{-2}$$

As the values of L, M and g are exactly given, so

$$\frac{\Delta Y}{Y} = 2\frac{\Delta d}{d} + \frac{\Delta l}{l}$$

or $$\Delta Y = \left(\frac{2 \times 0.01}{0.4} + \frac{0.05}{0.8}\right) \times 2 \times 10^{11}\ \text{Nm}^{-2}$$

$$= 0.2 \times 10^{11}\ \text{Nm}^{-2}$$

∴ Measured value of $Y = (2.0 \pm 0.2) \times 10^{11}\ \text{Nm}^{-2}$.

10. (d) $\text{Density} = \frac{\text{Mass}}{\text{Volume}}$

$$\rho = \frac{m}{\pi r^2 l}$$

The maximum percentage error in density,

$$\frac{\Delta\rho}{\rho} \times 100 = \left[\frac{\Delta m}{m} + 2\frac{\Delta r}{r} + \frac{\Delta l}{l}\right] \times 100$$

$$= \left[\frac{0.003}{0.3} + \frac{2 \times 0.005}{0.5} + \frac{0.06}{6}\right] \times 100$$

$$= \left[\frac{1}{100} + \frac{2}{100} + \frac{1}{100}\right] \times 100 = 4.$$

11. (b) $g = 4\pi^2\left(\frac{l}{T^2}\right)$

$$\frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\frac{\Delta T}{T}$$

$$\Rightarrow \quad E = \frac{\Delta l}{l} + 2\frac{\Delta t}{t},$$

greater the value of t, lesser the error.

Hence, fractional error in the 1st observation is minimum.

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12. (c) Least count of screw gauge

$$= \frac{\text{Pitch}}{\text{No. of circular divisions}} = \frac{0.5\,\text{mm}}{50}$$

$$= 0.01 \text{ mm}$$

Initial reading $= 0 + 5 \times 001 = 0.05$ mm

Final reading $= 2 \times 0.5 + 25 \times 001 = 1.25$ mm

Diameter of ball $= 125 - 005 = \mathbf{1.20\ mm}$.

13. (d) $20\,\text{VSD} = 16\,\text{MSD}$

$$1\,\text{VSD} = \frac{16}{20}\text{MSD} = \frac{4}{5}\text{MSD}$$

$$\text{LC} = 1\,\text{MSD} - 1\,\text{VSD} = 1\,\text{MSD} - \frac{4}{5}\text{MSD}$$

$$= \frac{1}{5}\text{MSD} = \frac{1}{5} \times 1\,\text{mm} = \mathbf{0.2\ mm}$$

14. (c) Least count of screw gauge

$$= \frac{0.5\,\text{mm}}{50} = \frac{1}{100}\,\text{mm} = 0.01 \text{ mm}$$

$\therefore$ Diameter D

$$= \text{MSR} + \text{CSR} \times \text{LC}$$

$$= 2.5 + 20 \times 0.01 = 2.70 \text{ mm}$$

Now, $$\text{density} = \frac{m}{\frac{4}{3}\pi\left(\frac{D}{2}\right)^3}$$

$\therefore$ % error in density

$$= \frac{\Delta m}{m} \times 100 + 3\frac{\Delta D}{D} \times 100$$

$$= 2\% + 3 \times \frac{0.01}{2.70} \times 100 = \mathbf{3.1\%}$$

15. (a), (d)

(a) Torque and work both have the dimensions $[ML^2T^{-2}]$.

(d) Lightyear and wavelength both have the dimensions [L].

16. (a), (b), (c)

(a) Reynold's number and coefficient of friction both are dimensionless quantities.

(b) $$\text{Curie} = \frac{\text{Number of atoms}}{\text{Time}} = [T^{-1}]$$

$$\text{Frequency} = \frac{\text{No. of vibrations}}{\text{Time}} = [T^{-1}]$$

(c) [Latent heat] = [Gravitational potential]

$$= \frac{\text{Energy}}{\text{Mass}}$$

17. (b) and (c).

Here $L = G^x c^y h^z$.

$$[L] = [M^{-1}L^3T^{-2}]^x [LT^{-1}]^y [ML^2T^{-1}]^z$$

$$M^0L^1T^0 = M^{-x+z} L^{3x+y+2z} T^{-2x-y-z}$$

$\therefore$ $\quad -x + z = 0,\ 3x + y + 2z = 1,\ -2x - y - z = 0$

Hence $x = \frac{1}{2}$, $y = -\frac{3}{2}$, $z = \frac{1}{2}$.

18. (a), (b), (c), (d)

(a) $L = \frac{\text{flux}}{\text{current}}$ $\quad \therefore\ 1\,\text{H} = \frac{1\,\text{Wb}}{1\,\text{A}}$

(b) $L = \frac{\varepsilon}{dI/dT}$ $\quad \therefore\ 1\,\text{H} = \frac{1\,\text{V}}{\text{A/s}} = 1\,\text{Vs A}^{-1}$

(c) $U = \frac{1}{2}LI^2$ or $L = \frac{2U}{I^2}$ $\quad \therefore 1\,\text{H} = \frac{1\,\text{J}}{1\,\text{A}^2}$

(d) $1\,\text{H} = \frac{1\,\text{V.s}}{1\,\text{A}} = 1\,\Omega\,\text{s}$.

19. (a) and (c)

$$[R] = [ML^2T^{-3}A^{-2}]$$

$$[L] = [ML^2T^{-2}A^{-2}]$$

$$[C] = [M^{-1}L^{-2}T^4A^2]$$

$\therefore$ $\quad \frac{1}{\sqrt{LC}} = \frac{1}{[T]} = [\text{frequency}]$

and $\quad \left[\frac{R}{L}\right] = \frac{1}{[T]} = [\text{frequency}]$

20. (a) and (b)

$$[RC] = [ML^2T^{-3}A^{-2}][M^{-1}L^{-2}T^4A^2] = [T]$$

$$[LC] = [ML^2T^{-2}A^{-2}][M^{-1}L^{-2}T^4A^2] = [T^2]$$

$\therefore$ $\quad [\sqrt{LC}] = [T]$

21. (c) $\quad F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$ or $\varepsilon_0 = \frac{1}{4\pi F}\frac{q_1q_2}{r^2}$

$\therefore$ $\quad [\varepsilon_0] = \frac{[IT][IT]}{[MLT^{-2}][L^2]} = [M^{-1}L^{-3}T^4I^2]$

Also, $\quad \frac{1}{\mu_0\varepsilon_0} = c^2$

or $\quad [\mu_0] = \frac{1}{[\varepsilon_0][c^2]} = \frac{1}{[M^{-1}L^{-3}T^4I^2][LT^{-1}]^2}$

$$= \mathbf{[MLT^{-2}I^{-2}]}.$$

22. (a), (c) Relative error in measurement of time

$$= \frac{1\,\text{second}}{40\,\text{second}} = \frac{1}{40}$$

Time period $= \frac{40}{20} = 2$ second

$\therefore$ Error in measurement of time period

$$= 2 \times \frac{1}{40} = \frac{1}{20}\,\text{second} = 0.05 \text{ second}$$

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Now, $g \propto \frac{1}{T^2}$ [For constant l]

$\Rightarrow \quad \frac{\Delta g}{g} = \frac{2\Delta T}{T} = \frac{2 \times 1}{40} = \frac{1}{20}$

$\Rightarrow \quad \frac{\Delta g}{g} \times 100 = 5\%$

23.

(a) [Angular momentum] = $[ML^2T^{-1}]$

(d) [Latent heat] = $[L^2T^{-2}]$

(b) [Torque] = $[ML^2T^{-2}]$

(e) [Capacitance] = $[M^{-1}L^{-2}T^2Q^2]$

(c) [Inductance] = $[ML^2Q^{-2}]$

(f) [Resistivity] = $[ML^3T^{-1}Q^{-2}]$

$\therefore (a) \to q$; $(b) \to p$; $(c) \to s$;

$(d) \to u$; $(e) \to r$; $(f) \to t$.

24. (a) Capacitance, $C = \frac{q}{V}$ = coulomb volt^{-1}

or $\quad C = \frac{q}{W/q} = \frac{q^2}{W}$ = coulomb^2 joule^{-1}

(b) Inductance, $L = \frac{\mathcal{E}}{dI/dt} = \frac{\text{volt}}{\text{ampere/sec}}$

= volt sec ampere^{-1} = ohm sec.

(c) Magnetic induction, $B = \frac{F}{Il} = \frac{\text{newton}}{\text{ampere metre}}$

= newton (ampere metre)$^{-1}$

$\therefore (a) \to q, r$; $(b) \to p, t$; $(c) \to s$.

25. $(a) \to p, q$; $(b) \to r, s$;

$(c) \to r, s$; $(d) \to r, s$.

AIEEE

1. Which of the following represents the correct dimensions of the coefficient of viscosity ?

(a) $[ML^{-1}T^{-2}]$ (b) $[ML^{-2}T^{-2}]$

(c) $[ML^{-1}T^{-1}]$ (d) $[MLT^{-1}]$ [AIEEE 04]

2. Dimensions of $\frac{1}{\mu_0 \varepsilon_0}$, where symbols have their usual meanings, are :

(a) $[L^{-1}T]$ (b) $[L^2T^2]$

(c) $[L^2T^{-2}]$ (d) $[LT^{-1}]$ [AIEEE 03]

3. Identify the pair whose dimensions are equal.

(a) torque and work (b) stress and energy

(c) force and stress (d) force and work [AIEEE 02]

4. Out of the following pairs, which one does not have identical dimensions

(a) Moment of inertia and moment of a force

(b) Work and torque

(c) Angular momentum and Planck's constant

(d) Impulse and momentum [AIEEE 05]

5. The physical quantities not having the same dimensions are

(a) torque and work

(b) momentum and Planck's constant

(c) stress and Young's modulus

(d) speed and $1/\sqrt{\mu_0 \varepsilon_0}$ [AIEEE 03]

6. Which of the following units denotes the dimensions ML^2Q^{-2}, where Q denotes the electric charge?

(a) henry (H) (b) weber (Wb)

(c) Wb m^{-2} (d) Hm^{-2} [AIEEE 04]

7. The dimensions of magnetic field in M, L, T and C (Coulomb) are given as

(a) $[MLT^{-1}C^{-1}]$ (b) $[MT^2C^{-2}]$

(c) $[MT^{-1}C^{-1}]$ (d) $[MT^{-2}C^{-1}]$ [AIEEE 06]

8. Two full turns of the circular scale of a screw gauge cover a distance of 1 mm on its main scale. The total number of divisions on the circular scale is 50. Further, it is found that the screw gauge has a zero error of – 0.03 mm. While measuring the diameter of a thin wire, a student notes the main scale reading of 3 mm and the number of circular scale divisions in line with the main scale is 35. The diameter of the wire is

(a) 3.32 mm (b) 3.73 mm

(c) 3.67 mm (d) 3.38 mm [AIEEE 08]

9. In an experiment the angles are required to be measured using an instrument. 29 divisions of the main scale exactly coincide with the 30 divisions of the vernier scale. If the smallest division of the main scale is half-a-degree (=0.5°), then the least count of the instrument is

(a) one minute (b) half minute

(c) one degree (d) half degree [AIEEE 09]

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10. The respective number of significant figures for the numbers 23.023, 0.0003 and 2.1×10^{-3} are
(a) 4, 4, 2 (b) 5, 1, 2
(c) 5, 1, 5 (d) 5, 5, 2 [AIEEE 2010]

11. A screw gauge gives the following reading when used to measure the diameter of a wire.
Main scale reading : 0 mm
Circular scale reading : 52 divisions
Given that 1 mm on main scale corresponds to 100 divisions of the circular scale.
The diameter of wire from the above data is
(a) 0.52 cm
(b) 0.052 cm
(c) 0.026 cm
(d) 0.005 cm [AIEEE 2011]

Answers and Explanations

1. (c) $\eta = \dfrac{\text{Force}}{\text{Area}} \times \dfrac{\text{Distance}}{\text{Velocity}}$

$$[\eta] = \frac{[MLT^{-2}]}{[L^2]} \cdot \frac{[L]}{[LT^{-1}]} = [ML^{-1}T^{-1}].$$

2. (c) $\dfrac{1}{\mu_0\varepsilon_0} = c^2$, c = speed of light

$$\therefore \quad \left[\frac{1}{\mu_0\varepsilon_0}\right] = [LT^{-1}]^2 = [L^2T^{-2}]$$

3. (a) [Torque] = [Work] = $[ML^2T^{-2}]$

4. (a) [Moment of inertia] = $[ML^2T^0]$

[Moment of force] = Force × distance
= $[ML^2T^{-2}]$

5. (b) [Momentum] = $[MLT^{-1}]$

[Planck's constant] = $[ML^2T^{-1}]$.

6. (a) $\text{Inductance} = \dfrac{\text{EMF}}{\text{Current / time}}$

$$= \frac{\text{Work}}{\text{Charge}} \cdot \frac{\text{Time}}{\text{Current}}$$

$$[\text{Inductance}] = \frac{[ML^2T^{-2}]}{[Q]} \cdot \frac{[T]}{[Q/T]}$$

$$[\text{Henry}] = [ML^2Q^{-2}]$$

Henry is the SI unit of inductance.

7. (c) $F = IlB$ or $B = \dfrac{F}{Il}$

$$\therefore \quad [B] = \frac{[MLT^{-2}]}{[CT^{-1}][L]} = [MT^{-1}C^{-1}].$$

8. (d) Pitch of screw gauge $= \dfrac{1}{2}$ mm $= 0.5$ mm

$$\text{Least count} = \frac{0.5\,\text{mm}}{50} = 0.01 \text{ mm}$$

Observed diameter $= 3\,\text{mm} + 35 \times 0.01\,\text{mm}$
$= 3.35$ mm

Corrected diameter
= Observed diameter − Zero error
$= 3.35 - (-0.03) = \mathbf{3.38\ mm}$.

9. (a) 29 MSD = 30 VSD

$$1 \text{ VSD} = \frac{29}{30} \text{ MSD}$$

Least count = 1 MSD − 1 VSD

$$= \left(1 - \frac{29}{30}\right)\text{MSD} = \frac{1}{30}\text{MSD}$$

$$= \frac{1}{30}\left(\frac{1}{2}\right)^0 = \left(\frac{1}{60}\right)^0 = \mathbf{1\ minute}.$$

10. (b) $23.023 \rightarrow 5$ (2, 3, 0, 2, 3) ;
$0.0003 \rightarrow 1$ (3) ; $2.1\times10^{-3} \rightarrow 2$ (2, 1).

11. (b) $d = \text{MSR} + \text{CSR}$

$$= 0 + 52 \times \frac{1}{100} = 0.52 \text{ mm} = \mathbf{0.052\ cm}$$

DCE and G.G.S. Indraprastha University Engineering Entrance Exam

(Similar Questions)

1. Out of the following four dimensional quantities, which one qualifies to be called a dimensional constant ?
(a) Acceleration due to gravity
(b) Surface tension of water
(c) Weight of a standard kilogram mass
(d) The velocity of light in vacuum. [IPUEE 06]

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2. The dimensions of torque are
(a) $[ML^2T^2]$ (b) $[ML^2T^{-2}]$
(c) $[M^2L^2T^{-2}]$ (d) $[MLT^{-1}]$ [DCE 00]

3. $[ML^2T^{-2}]$ are dimensions of
(a) force (b) moment of force
(c) momentum (d) power [DCE 04]

4. The dimensions of Planck's constant are
(a) $[M^2L^2T^{-2}]$ (b) $[MLT^{-2}]$
(c) $[ML^2T^{-2}]$ (d) $[ML^2T^{-1}]$ [DPM 01 ; DCE 07]

5. Dimensions of bulk modulus are
(a) $[M^{-1}LT^{-2}]$ (b) $[ML^{-1}T^{-2}]$
(c) $[ML^{-2}T^{-2}]$ (d) $[M^2L^2T^{-1}]$ [DCE 07]

6. The unit of a in van der Waal's gas equation is
(a) atm $L^{-2}mol^2$ (b) atm L^2per mol
(c) atm $L^{-1}mol^{-2}$ (d) atm L^2mol^{-2} [DCE 97]

7. In the relation, $y = r\sin(\omega t + kx)$, the dimensional formula for kx or ωt is same as
(a) r/ω (b) r/y
(c) $\omega t/r$ (d) $yr/\omega t$ [DCE 03]

8. Which of the following quantities can be written in SI units in $kgm^2A^{-2}s^{-3}$?
(a) Resistance (b) Inductance
(c) Capacitance (d) Magnetic flux [DCE 07]

9. The time period T of a small drop of liquid (due to surface tension) depends on density ρ, radius r and surface tension S. The relation is
(a) $T \propto \left(\frac{\rho r^3}{S}\right)^{1/2}$ (b) $T \propto \rho rS$
(c) $T \propto \frac{\rho r}{S}$ (d) $T \propto \frac{S}{\rho r}$

10. Which one of the following pairs of quantities has the same dimension ?
(a) force and work done
(b) momentum and impulse
(c) pressure and force
(d) surface tension and stress. [DCE 05]

11. If $L = 2.331$ cm, $B = 2.1$ cm, then $L + B$ = ?
(a) 4.431 cm (b) 4.43 cm
(c) 4.4 cm (d) 4 cm [DCE 03]

12. If error in radius is 3%, what is error in volume of sphere ?
(a) 3% (b) 27%
(c) 9% (d) 6% [DCE 06]

13. In an experiment, on the measurement of g, using a simple pendulum, the time period was measured with an accuracy of 0.2% while the length was measured with an accuracy of 0.5%. The percentage accuracy in the value of g thus obtained is
(a) 0.7% (b) 0.1%
(c) 0.25% (d) 0.9% [DCE 07]

Answers and Explanations

1. (d) The velocity of light in vacuum ($c = 3 \times 10^8$ m/s) is a dimensional constant.

2. (b) [Torque] = [Force] [Distance]

$$= [MLT^{-2}][L] = [ML^2T^{-2}]$$

3. (b) [Moment of force] = [Torque] = $[ML^2T^{-2}]$

4. (d) $[h] = \frac{\text{Energy}}{\text{Frequency}} = \frac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T^{-1}]$

5. (b) $[\text{Bulk modulus}] = \frac{\text{Stress}}{\text{Strain}} = \frac{[ML^{-1}T^{-2}]}{1}$

$$= [ML^{-1}T^{-2}]$$

6. (b) Van der Wall's equation is

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

$$\therefore \quad [P] = \left[\frac{a}{V^2}\right] \quad \text{or} \quad [a] = [P][V^2]$$

Unit of a = atm L^2 per mole.

7. (b) ωt and kx both are dimensionless. Out of the given options, only r/y is dimensionless.

8. (a) $[\text{Resistance}] = [ML^2T^{-3}A^{-2}]$

$\therefore$ Unit of resistance = kg $m^2A^{-2}s^{-3}$.

9. (a) Let $T = K\rho^a r^b S^c$

$$\therefore \quad [T] = [ML^{-3}]^a[L]^b[MT^{-2}]^c$$

or $\quad M^0L^0T^1 = M^{a+c}L^{-3a+b}T^{-2c}$

$$\therefore \quad a + c = 0, \; -3a + b = 0, \; -2c = 1$$

On solving, $a = \frac{1}{2}$, $b = \frac{3}{2}$, $c = -\frac{1}{2}$.

Hence $\quad T \propto \left(\frac{\rho r^3}{S}\right)^{1/2}$.

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10. (*b*) [Change in momentum] = [Impulse]
$= [MLT^{-1}]$.

11. (*c*) $L + B = 2.331 + 2.1 = 4.431 = 4.4$ cm
$\because$ B has 2 significant figures.
$\therefore$ $L + B$ must have only 2 significant figures.

12. (*c*) $V = \frac{4}{3}\pi r^3$

$\therefore \frac{\Delta V}{V} \times 100 = 3 \times \frac{\Delta r}{r} \times 100 = 3 \times 3 = 9\%$.

13. (*d*) As $T = 2\pi\sqrt{\frac{l}{g}}$

$\therefore \quad g = 4\pi^2 \frac{l}{T^2}$

$\frac{\Delta g}{g} \times 100 = \frac{\Delta l}{l} \times 100 + 2 \times \frac{\Delta T}{T} \times 100$

$= 0.5\% + 2 \times 0.2\% = 0.9\%$.

AIIMS Entrance Exam

1. How many wavelengths of Kr^{86} are there in one metre ?
(*a*) 15,53,164.13 (*b*) 16,50,763.73
(*c*) 23,48,123.73 (*d*) 6,52,189.63 [AIIMS 94]

2. One nanometre is equal to
(*a*) 10^9 mm (*b*) 10^{-6} cm
(*c*) 10^{-7} cm (*d*) 10^{-9} m [AIIMS 94]

3. Light year is the unit of
(*a*) time (*b*) distance
(*c*) velocity (*d*) intensity of light [AIIMS 96]

4. Parsec is the unit of
(*a*) time (*b*) distance
(*c*) frequency (*d*) angular momentum [AIIMS 96]

5. Length cannot be measured by
(*a*) fermi (*b*) debye
(*c*) micron (*d*) light year [AIIMS 02]

6. The difference in the length of a mean solar day and a sidereal day is about
(*a*) 1 minute (*b*) 4 minutes
(*c*) 15 minutes (*d*) 56 minutes [AIIMS 03]

7. Gravitational mass is proportional to gravitational
(*a*) field (*b*) force
(*c*) intensity (*d*) all of these [AIIMS 08]

8. SONAR emits which of the following waves ?
(*a*) radio (*b*) light
(*c*) ultrasound (*d*) none of these [AIIMS 99]

9. The dimension of torque is
(*a*) $[MLT^{-2}]$ (*b*) $[ML^2T^{-2}]$
(*c*) $[ML^{-1}T^{-1}]$ (*d*) $[ML^3T^{-3}]$ [AIIMS 02]

10. What is the dimensional formula of gravitational constant G ?
(*a*) $[M^{-1}L^3T^{-2}]$ (*b*) $[M^{-2}L^3T^{-2}]$
(*c*) $[M^{-1}L^2T^{-2}]$ (*d*) $[M^{-1}L^3T^{-1}]$ [AIIMS 00]

11. The dimension of angular velocity is
(*a*) $[MLT^{-2}]$ (*b*) $[M^2L^0T^{-1}]$
(*c*) $[M^0L^0T^{-1}]$ (*d*) $[ML^2T^{-2}]$ [AIIMS 08]

12. Which of the following physical quantity has the dimension of $[ML^2T^{-3}]$?
(*a*) work (*b*) power
(*c*) pressure (*d*) impulse [AIIMS 94]

13. The dimension of the modulus of rigidity, is
(*a*) $[MLT^{-2}]$ (*b*) $[ML^{-1}T^{-2}]$
(*c*) $[ML^{-2}T^{-2}]$ (*d*) $[ML^{-1}T^{-1}]$ [AIIMS 94]

14. The dimension of Planck's constant is
(*a*) $[ML^2T^{-1}]$ (*b*) $[ML^{-3}T^{-1}]$
(*c*) $[ML^{-2}T^{-1}]$ (*d*) $[M^0L^{-1}T^{-3}]$ [AIIMS 97]

15. Dimensions $[ML^{-1}T^{-1}]$ are related to
(*a*) work (*b*) torque
(*c*) energy (*d*) coefficient of viscosity [AIIMS 99]

16. Which of the following is a dimensionless quantity ?
(*a*) Strain (*b*) Stress
(*c*) Specific heat (*d*) Quantity of heat [AIIMS 94]

17. Which of the following pair does not have similar dimensions ?
(*a*) stress and pressure
(*b*) angle and strin
(*c*) tension and surface tension
(*d*) Planck's constant and angular momentum [AIIMS 01, 07]

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18. Dimensions of electrical resistance are
(a) $[ML^2T^{-3}A^{-1}]$ (b) $[ML^2T^{-3}A^{-2}]$
(c) $[ML^3T^3A^{-2}]$ (d) $[ML^{-1}T^3A^2]$ [AIIMS 94]

19. The magnetic moment has the dimensions of
(a) [LA] (b) $[L^2A]$
(c) $[LT^{-1}A]$ (d) $[L^2T^{-1}A]$ [AIIMS 06]

20. Which of the following pairs does not have same dimensions ?
(a) impulse and momentum
(b) moment of inertia and moment of force
(c) angular momentum and Planck's constant
(d) work and torque [AIIMS 2010]

21. If the energy, $E = G^p h^q c^r$, where G is the universal gravitational constant, h is the Planck's constant and c is the velocity of light, then the values of p, q and r are, respectively
(a) $-1/2, 1/2$ and $5/2$
(b) $1/2, -1/2$ and $-5/2$
(c) $-1/2, 1/2$ and $3/2$
(d) $1/2, -1/2$ and $-3/2$ [AIIMS 2010]

Answers and Explanations

1. (b) One metre contains 1650763.73 wavelengths of certain orange radiation of Kr^{86}.

2. (d) 1 nm $= 10^{-9}$ m.

3. (b) One light year is the *distance* travelled by light in one year.

4. (b) One parse is the *distance* at which an arc of length 1 AU subtends an angle of 1 second of arc.

5. (b) Debye is the unit of electric dipole moment not of length or distance.

6. (b) The sidereal day is about 4 minutes (more precisely, 3 minutes 56 seconds) shorter than a mean solar day (of 24 hours).

7. (b) Gravitational mass is proportional to gravitational force.

8. (c) SONAR emtis ultrasound.

9. (b) [Torque] $= [ML^2T^{-2}]$.

10. (a) $[G] = \frac{Fr^2}{m_1 m_2} = \frac{[MLT^{-2}][L^2]}{[M][M]} = [M^{-1}L^3T^{-2}]$.

11. (c) $[\omega] = \frac{[\theta]}{[t]} = \frac{1}{T} = [M^0L^0T^{-1}]$

12. (b) $[\text{Power}] = \frac{\text{work}}{\text{time}} = \frac{[ML^2T^{-2}]}{[T]}$

$= [ML^2T^{-3}]$.

13. (b) $[\text{Modulus of rigidity}] = \frac{\text{Stress}}{\text{Strain}} = \frac{[ML^{-1}T^{-2}]}{1}$

$= [ML^{-1}T^{-2}]$.

14. (a) $[h] = \frac{\text{Energy}}{\text{Frequency}} = \frac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T^{-1}]$

15. (d) [Coefficient of viscosity] $= [ML^{-1}T^{-1}]$

16. (a) Strain is a dimensionless quantity.

17. (a) [Tension] = [Force] $= [MLT^{-2}]$

$$[\text{Surface tension}] = \frac{[\text{Force}]}{[\text{Length}]} = \frac{[MLT^{-2}]}{[L]} = [MT^{-2}]$$

18. (b) $\text{Resistance} = \frac{\text{P.D.}}{\text{Current}} = \frac{\text{Work}}{\text{Charge} \times \text{Current}}$

$$[\text{Resistance}] = \frac{[ML^2T^{-2}]}{[TA][A]} = [ML^2T^{-3}A^{-2}].$$

19. (b) [Magnetic moment] = [Current] [Area]

$= [A][L^2] = [L^2A]$.

20. (b) Moment of inertia,

$[I] = \text{Mass} \times \text{distance}^2 = [ML^2]$

Moment of force,

$[\tau] = \text{Force} \times \text{distance}$

$= ML^{-2}T \cdot L = [ML^2T^{-2}]$

$\therefore \quad [I] \neq [\tau]$.

21. (a) Given $E = G^p h^q c^r$

$\therefore \quad [M^1L^2T^{-2}] = [M^{-1}L^3T^{-2}]^p [ML^2T^{-1}]^q [LT^{-1}]^r$

or $M^1L^2T^{-2} = M^{-p+q} L^{3p+2q+r} T^{-2p-q-r}$

Equating the powers of M, L and T, we get

$$-p + q = 1$$
$$3p + 2q + r = 2$$
$$-2p - q - r = -2$$

On solving, we get

$$p = -\frac{1}{2}, \quad q = \frac{1}{2}, \quad r = \frac{5}{2}$$

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CBSE PMT Prelims and Final Exams

1. Which of the following has the dimensions of pressure ?
(a) $[ML^{-2}]$ (b) $[ML^{-1}T^{-2}]$
(c) $[ML^{-2}T^{-2}]$ (d) $[M^{-1}L^{-1}]$
[CBSE PMT 90, 94]

2. The dimensional formula of torque is
(a) $[ML^2T^{-2}]$ (b) $[ML^2T^{-1}]$
(c) $[ML^{-1}T^{-2}]$ (d) $[ML^{-2}T^{-2}]$ [CBSE PMT 89]

3. The dimensions of universal gravitational constant are
(a) $[M^{-1}L^3T^{-2}]$ (b) $[ML^2T^{-1}]$
(c) $[M^{-2}L^3T^{-2}]$ (d) $[M^{-2}L^2T^{-1}]$
[CBSE PMT 92, 04]

4. The dimensions of impulse are equal to that of
(a) pressure (b) linear momentum
(c) force (d) angular momentum
[CBSE PMT 89]

5. According to Newton, the viscous force acting between liquid layers of area A and velocity gradient $\Delta v/\Delta x$ is given by

$$F = -\eta A\frac{\Delta v}{\Delta x},$$

where η is constant called coefficient of viscosity. The dimensional formula of η is
(a) $[ML^{-2}T^{-2}]$ (b) $[M^0L^0T^0]$
(c) $[ML^2T^{-2}]$ (d) $[ML^{-1}T^{-1}]$ [CBSE PMT 90]

6. Turpentine oil is flowing through a tube of length l and radius r. The pressure difference between the two ends of the tube is P. The viscosity of oil is given by $\eta = \frac{P(r^2 - x^2)}{4vl}$, where v is the velocity of oil at a distance x from the axis of the tube. The dimensions of η are
(a) $[M^0L^0T^0]$ (b) $[MLT^{-1}]$
(c) $[ML^2T^{-2}]$ (d) $[ML^{-1}T^{-1}]$ [CBSE PMT 93]

7. The dimensional formula of angular momentum is
(a) $[ML^2T^{-2}]$ (b) $[ML^{-2}T^{-1}]$
(c) $[MLT^{-1}]$ (d) $[ML^2T^{-1}]$ [CBSE PMT 88]

8. The dimensions of Planck's constant equals to that of
(a) energy (b) momentum
(c) angular momentum (d) power [CBSE PMT 01]

9. The ratio of the dimensions of Planck constant and that of moment of inertia is the dimensions of
(a) time (b) frequency
(c) angular momentum (d) velocity [CBSE PMT 05]

10. Dimensions of resistance in an electrical circuit, in terms of dimension of mass M, of length L, of time T and of current I, would be
(a) $[ML^2T^{-2}]$ (b) $[ML^2T^{-1}I^{-1}]$
(c) $[ML^2T^{-3}I^{-2}]$ (d) $[ML^2T^{-3}I^{-1}]$
[CBSE PMT 07]

11. Dimensional formula of self inductance is
(a) $[MLT^{-2}A^{-2}]$ (b) $[ML^2T^{-1}A^{-2}]$
(c) $[ML^2T^{-2}A^{-2}]$ (d) $[ML^2T^{-2}A^{-1}]$
[CBSE PMT 89]

12. If C and R denote capacitance and resistance, the dimensional formula of CR is
(a) $[M^0L^0T^1]$ (b) $[M^0L^0T^0]$
(c) $[M^0L^0T^{-1}]$
(d) not expressible in terms of MLT [CBSE PMT 89]

13. The dimension of RC is
(a) square of time (b) square of inverse time
(c) time (d) inverse time.
[CBSE PMT 95]

14. Which of the following dimensions will be the same as that of time ?
(a) $\frac{L}{R}$ (b) $\frac{C}{L}$
(c) LC (d) $\frac{R}{L}$.
[CBSE PMT 96]

15. The unit of permittivity of free space ε_0 is
(a) coulomb/newton-metre
(b) newton-metre^2/coulomb^2
(c) coulomb^2/newton-metre^2
(d) coulomb^2/(newton-metre)^2 [CBSE PMT 04]

16. The dimensional formula of magnetic flux is
(a) $[M^0L^{-2}T^{-2}A^{-2}]$ (b) $[ML^0T^{-2}A^{-2}]$
(c) $[ML^2T^{-2}A^{-1}]$ (d) $ML^2T^{-1}A^3$
[CBSE PMT 99]

17. The dimensional formula of permeability of free space μ_0 is
(a) $[MLT^{-2}A^{-2}]$ (b) $[M^0L^1T]$
(c) $[M^0L^2T^{-1}A^2]$ (d) none of these.
[CBSE PMT 91]

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18. Which pairs do not have equal dimensions ?
(a) Energy and torque (b) Force and impulse
(c) Angular momentum and Planck's constant
(d) Elastic modulus and pressure. [CBSE PMT 2K]

19. Of the following quantities, which one has dimensions different from the remaining three ?
(a) Energy per unit volume
(b) Force per unit area
(c) Product of voltage and charge per unit volume
(d) Angular momentum. [CBSE PMT 89]

20. If $x = at + bt^2$, where x is the distance travelled by the body in kilometres while t is the time in seconds, then the unit of b is
(a) km/s (b) kms
(c) km/s^2 (d) kms^2 [CBSE PMT 89]

21. An equation is given here, $\left(P + \frac{a}{V^2}\right) = b\frac{\theta}{V}$, where P = pressure, V = volume and θ = absolute temperature. If a and b are constants, the dimensions of a will be
(a) $[ML^{-5}T^{-1}]$ (b) $[ML^5T^1]$
(c) $[ML^5T^{-2}]$ (d) $[M^{-1}L^5T^2]$ [CBSE PMT 96]

22. The velocity v of a particle at time t is given by $v = at + \frac{b}{t+c}$, where a, b and c are constants. The dimensions of a, b and c are
(a) [L], [LT] and $[LT^{-2}]$
(b) $[LT^{-2}]$, [L] and [T]
(c) $[L^2]$, [T] and $[LT^{-2}]$
(d) $[LT^{-2}]$, [LT] and [L]. [CBSE PMT 06]

23. The time dependence of a physical quantity p is given by $p = p_0 \exp(-\alpha t^2)$, where α is a constant and t is the time. The constant α
(a) is dimensionless (b) has dimensions $[T^{-2}]$
(c) has dimensions $[T^2]$
(d) has dimensions of p. [CBSE PMT 93]

24. Which of the following five physical parameters have the same dimensions ?

1. energy density	2. refractive index
3. dielectric constant	4. Young's modulus
5. magnetic field	

(a) 2 and 4 (b) 3 and 5
(c) 1 and 4 (d) 1 and 5 [CBSE PMT 08]

25. The frequency of vibration f of a mass m suspended from a spring of spring constant k is given by a relation $f = am^x k^y$, where a is a dimensionless constant. The values of x and y are
(a) $x = \frac{1}{2}, y = \frac{1}{2}$ (b) $x = -\frac{1}{2}, y = -\frac{1}{2}$
(c) $x = \frac{1}{2}, y = -\frac{1}{2}$ (d) $x = -\frac{1}{2}, y = \frac{1}{2}$ [CBSE PMT 90]

26. P represents radiation pressure, c represents speed of light and S represents radiation energy striking per unit area per sec. The non-zero integers x, y, z such that $P^x S^y c^z$ is dimensionless, are
(a) $x = 1, y = 1, z = 1$ (b) $x = -1, y = 1, z = 1$
(c) $x = 1, y = -1, z = 1$ (d) $x = 1, y = 1, z = -1$ [CBSE PMT 92]

27. If the dimensions of a physical quantity are given by $M^aL^bT^c$, then the physical quantity will be
(a) Velocity if $a = 1, b = 0, c = -1$
(b) Acceleration if $a = 1, b = 1, c = -2$
(c) Force if $a = 0, b = -1, c = -2$
(d) Pressure if $a = 1, b = -1, c = -2$. [CBSE PMT 09]

28. The density of a material in CGS system of units is 4 g / cm^2. In a system of units in which unit of length is 10 cm and unit of mass is 100 g, the value of density of material will be
(a) 400 (b) 0.04
(c) 0.4 (d) 40 [CBSE Final 2011]

29. Which of the following is a dimensional constant ?
(a) relative density (b) gravitational constant
(c) refractive index (d) poisson ratio. [CBSE PMT 95]

30. Percentage errors in the measurement of mass and speed are 2% and 3% respectively. The error in the estimate of kinetic energy obtained by measuring mass and speed will be
(a) 8% (b) 2%
(c) 12% (d) 10% [CBSE PMT 95]

31. The density of a cube is measured by measuring its mass and length of its sides. If the maximum errors in the measurement of mass and lengths are 3% and 2% respectively, the maximum error in the measurement of density would be
(a) 12% (b) 14%
(c) 7% (d) 9% [CBSE PMT 89]

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32. A certain body weighs 22.42 g and has a measured volume of 4.7 cc. The possible errors in the measurement of mass and volume are 0.01 g and 0.1 cc. Then maximum error in the density will be

(a) 22% (b) 2%

(c) 0.2% (d) 0.02% [CBSE PMT 91]

33. If the error in the measurement of radius of a sphere is 2%, then the error in the determination of volume of the sphere will be

(a) 4% (b) 6%

(c) 8% (d) 2% [CBSE PMT 08]

Answers and Explanations

1. (b) [Pressure] = $[ML^{-1}T^{-2}]$

Refer to point 11 of Table 2.8 on page 2.20.

2. (a) [Torque] = $[ML^2T^{-2}]$

Refer to point 12 of Table 2.8 on page 2.20.

3. (a) $[G] = [M^{-1}L^3T^{-2}]$

Refer to point 13 of Table 2.8 on page 2.20.

4. (b) [Impulse] = $[MLT^{-1}]$

Refer to point 14 of Table 2.8 on page 2.20.

5. (d) $[\eta] = \left[\frac{F\Delta x}{A\Delta v}\right] = \frac{[MLT^{-2}][L]}{[L^2][LT^{-1}]}$

$= [ML^{-1}T^{-1}]$

6. (d) $[\eta] = \frac{P(r^2 - x^2)}{4vl} = \frac{[ML^{-1}T^{-2}][L^2]}{[LT^{-1}][L]}$

$= [ML^{-1}T^{-1}]$

7. (d) $[L] = [mvr] = [M][LT^{-1}][L]$

$= [ML^2T^{-1}]$.

8. (c) $[h] = \frac{\text{Energy}}{\text{Frequency}}$

$= \frac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T^{-1}]$.

9. (b) $\left[\frac{h}{l}\right] = \frac{E}{vl} = \frac{[ML^2T^{-2}]}{[T^{-1}][ML^2]}$

$= [T^{-1}] = [\text{Frequency}]$

10. (c) $[R] = \frac{V}{I} = \frac{W}{ql} = \frac{[ML^2T^{-2}]}{[IT][I]} = [ML^2T^{-3}I^{-2}]$

11. (c) [Self inductance] = $[ML^2T^{-2}A^{-2}]$

Refer to point 49 of Table 2.8 on page 2.22.

12. (a) $[CR] = \frac{q}{V}.\frac{V}{I} = \frac{q}{I} = \frac{[IT]}{[I]} = [M^0L^0T^1]$

13. (c) Dimension of RC is same as that of time, as proved in the above problem.

14. (a) $\left[\frac{L}{R}\right] = \frac{[ML^2T^{-2}A^{-2}]}{[ML^2T^{-3}A^{-2}]} = [T]$.

15. (c) From Coulomb's law,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$$

or $$\varepsilon_0 = \frac{q_1q_2}{4\pi Fr^2}$$

SI unit of $\varepsilon_0 = \frac{C.C}{Nm^2} = C^2/Nm^2$

16. (c) $[\phi] = BA = \frac{F}{qv\sin\theta}A$

$= \frac{[MLT^{-2}][L^2]}{[AT][LT^{-1}].1}$

$= [ML^2T^{-2}A^{-1}]$

17. (a) $[\mu_0] = \frac{4\pi rF}{I_1I_2l} = \frac{L.MLT^{-2}}{A^2.L}$

$= [MLT^{-2}A^{-2}]$

18. (b) [Force] = $[MLT^{-2}]$, [Impulse] = $[MLT^{-1}]$

19. (d) $\left[\frac{E}{V}\right] = \frac{ML^2T^{-2}}{L^3} = [ML^{-1}T^{-2}]$

$\left[\frac{F}{A}\right] = \frac{MLT^{-2}}{L^2} = [ML^{-1}T^{-2}]$

$\left[\text{voltage}\frac{q}{V}\right] = \frac{ML^2T^{-3}A^{-1}\cdot AT}{L^3} = [ML^{-1}T^{-2}]$

$[L] = [ML^2T^{-1}]$

Dimensions of angular momentum L are different from remaining three.

20. (c) Unit of b = unit of $\frac{x}{t^2} = km/s^2$

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21. (c) $\left[\frac{a}{V^2}\right] = [P]$

$\therefore \quad [a] = [P][V^2] = [ML^{-1}T^{-2}][L^3]^2$

$= [ML^5T^{-2}]$

22. (b) $V = at + \frac{b}{t+c}$

As c is added to t, so

$[c] = [T]$,

$[at] = [v]$

$\therefore \quad [a] = \frac{[v]}{[t]} = \frac{LT^{-1}}{T} = [LT^{-2}]$

$\frac{[b]}{[T]} = [v]$

or $[b] = LT^{-1}.T = [L]$

23. (b) Given $p = p_0 e^{-\alpha x^2}$

αx^2 must be dimensionless

$\therefore \quad [\alpha] = \frac{1}{[x^2]} = \frac{1}{T^2} = [T^{-2}]$

24. (c) Energy density

$= \frac{\text{Energy}}{\text{Volume}} = \frac{[ML^2T^{-2}]}{[L^3]}$

$= [ML^{-1}T^{-2}]$

Young's modulus,

$= \frac{F}{A}.\frac{L}{l} = \frac{MLT^{-2}}{L^2}.\frac{L}{L} = [ML^{-1}T^{-2}]$.

25. (d) Given : $f = am^x k^y$

Putting the dimensions of various quantities,

$T^{-1} = 1.[M]^x [MT^{-2}]^y \quad \left[\because k = \frac{\text{Force}}{\text{Distance}}\right]$

$M^0T^{-1} = M^{x+y}T^{-2y}$

$x + y = 0$ and $-2y = -1$

$\therefore \quad y = \frac{1}{2}$ and $x = -\frac{1}{2}$

26. (c) $[P] = \frac{\text{Force}}{\text{Area}} = \frac{MLT^{-2}}{L^2} = [ML^{-1}T^{-2}]$

$[S] = \frac{\text{Energy}}{\text{Area} \times \text{Time}}$

$= \frac{ML^2T^{-2}}{L^2.T} = [MT^{-3}]$

$[c] = [LT^{-1}]$

Dimensionless constant,

$k = P^x S^y c^z$

$\therefore \quad M^0L^0T^0 = [ML^{-1}T^{-2}]^x [MT^{-3}]^y [LT^{-1}]^z$

$M^0L^0T^0 = M^{x+y} L^{-x+z} T^{-2x-3y-z}$

$\therefore \quad x + y = 0, \; -x + z = 0, \; -2x - 3y - z = 0$

On solving, $x = 1, \; y = -1, \; z = 1$

27. (d) Pressure, $[P] = \frac{[F]}{[A]} = \frac{MLT^{-2}}{L^2}$

$= [M^1L^{-1}T^{-2}]$

$\therefore a = 1, \; b = -1, \; c = -2$.

28. (d) $n_2 = n_1\left[\frac{M_1}{M_2}\right]\left[\frac{L_1}{L_2}\right]^{-3}$

$= 4\left[\frac{1}{100}\right]\left[\frac{1}{10}\right]^{-3}$

$= 40$

29. (b) Only G has dimensions of $[M^{-1}L^3T^{-2}]$. Remaining three quantities are pure ratios.

30. (a) $K = \frac{1}{2}mv^2$

$\therefore \quad \frac{\Delta K}{K} \times 100 = \frac{\Delta m}{m} \times 100 + 2\frac{\Delta v}{v} \times 100$

$= 2\% + 2 \times 3\%$

$= 8\%$.

31. (d) $\rho = \frac{M}{V} = \frac{M}{L^3}$

$\therefore \quad \frac{\Delta \rho}{\rho} \times 100 = \frac{\Delta M}{M} \times 100 + 3\frac{\Delta L}{L} \times 100$

$= 3\% + 3 \times 2\% = 9\%$.

32. (b) $\rho = \frac{M}{V}$

$\therefore \quad \frac{\Delta \rho}{\rho} \times 100 = \frac{\Delta M}{M} \times 100 + \frac{\Delta V}{V} \times 100$

$= \left(\frac{0.01}{22.42} + \frac{0.1}{4.7}\right) \times 100 = 2\%$.

33. (b) Here $V = \frac{4}{3}\pi r^3$

$\therefore \quad \frac{\Delta V}{V} \times 100 = 3\frac{\Delta r}{r} \times 100$

$= 3 \times 2\% = 6\%$.

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Delhi PMT and VMMC Entrance Exam

(Similar Questions)

1. Light year is used to meaure
(a) distance between stars
(b) distance between atoms
(c) stationary charge
(d) none of these. [DPMT 07]

2. Which of the following is true for the solid angle ?
(a) $\delta\omega = \frac{\delta A\cos\theta}{r^2}$ (b) $d\omega = \frac{\delta A\cos^2\theta}{r^2}$
(c) $\delta\omega = \frac{\delta A\cos\theta}{r^3}$ (d) $\delta\omega = \frac{\delta A\cos^2\theta}{r^3}$ [DPMT 99]

3. Dimensions of coeffcient of viscosity are
(a) $[ML^{-1}T^{-1}]$ (b) $[ML^{-3}T^{-4}]$
(c) $[ML^{-1}T^{-2}]$ (d) $[MT^2]$ [DPMT 99, 04]

4. The dimensions of surface tension are
(a) Nm (b) Nm^2
(c) Nm^{-1} (d) N-s. [DPMT 01]

5. Dimensions of Hubble's constant are
(a) $[T^{-1}]$ (b) $[MLT^4]$
(c) $[M^0L^0T^{-2}]$ (d) $[MLT^1]$ [DPMT 97]

6. The units of Planck's constant are
(a) J/s (b) Js^2
(c) Js (d) Js^{-2} [DPMT]

7. Which of the following have same dimensions ?
(a) torque and force
(b) torque and potential energy
(c) potential energy and force
(d) Planck's constant and momentum [DPMT 98]

8. Force F is given by
$$F = at + bt^2,$$
where t is time. What are the dimensions of a and b ?
(a) $[MLT^{-3}]$ and $[MLT^{-4}]$
(b) $[MLT^{-1}]$ and $[MLT^0]$
(c) $[MLT^{-3}]$ and $[MLT^4]$
(d) $[MLT^{-4}]$ and $[MLT^1]$ [DPMT 93]

9. The dimensional formula of the constant a in van der Waal's gas equation
$$\left(P + \frac{a}{V^2}\right)(V - b) = RT \text{ is}$$
(a) $[ML^3T^2]$ (b) $[ML^2T^{-1}]$
(c) $[ML^5T^{-2}]$ (d) $[ML^5T^{-3}]$ [DPMT 92]

10. The equation $\left(P + \frac{a}{V^2}\right)(V - b) =$ Constant. The units of a are
(a) dyne × cm^5 (b) dyne × cm^4
(c) dyne cm^{-3} (d) dyne cm^{-2}. [DPMT 06]

11. In a system of units, the units of mass, length and time are 1 quintal, 1 km and 1 h respectively. In this system 1 N force will be equal to
(a) 1 new unit (b) 129.6 new units
(c) 125.7 new units
(d) 10^3 new units.

12. What is the dimension of surface tension ?
(a) $[ML^1T^0]$ (b) $[ML^1T^{-1}]$
(c) $[ML^0T^{-2}]$ (d) $[M^1L^1T^{-2}]$ [DPMT 2011]

13. The significant figures of the number 6.0023 are
(a) 1 (b) 5
(c) 4 (d) 2 [DPMT 95]

14. The angle subtended by a coin of radius 1 cm held at a distance of 80 cm from your eyes is
(a) 1.43° (b) 0.72°
(c) 0.0125° (d) 0.025° [DPMT 09]

Answers and Explanations

1. (a) Light year is the *distance* travelled by light is one year.

2. (a) Solid angle is normal projection of area divided by square of radius of the curved surface
$$\delta\omega = \frac{\delta A.\cos\theta}{r^2}$$

3. (a) [coefficient of viscosity] = $[ML^{-1}T^{-1}]$.

4. (c) Surface tension = $\frac{\text{Force}}{\text{Length}}$

∴ SI unit of surface tension = Nm^{-1}.

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5. According to Hubble's law, speed of recession of a galaxy,

$$v = Hr$$

$$\therefore \quad [H] = \frac{[v]}{[r]} = \frac{[LT^{-1}]}{[L]} = [T^{-1}]$$

6. $$h = \frac{\text{Energy}}{\text{Frequency}}$$

Unit of $h = \frac{J}{s^{-1}} = Js.$

7. [Torque] = [Potential energy]

$$= [ML^2T^{-2}]$$

8. $$[a] = \frac{[F]}{[t]} = \frac{[MLT^{-2}]}{[T]} = [MLT^{-3}]$$

$$[b] = \frac{[F]}{[t]^2} = \frac{[MLT^{-2}]}{[T]^2}$$

$$= [MLT^{-4}]$$

9. $$[P] = \left[\frac{a}{V^2}\right]$$

$$\therefore \quad [a] = [P][V^2] = [ML^{-1}T^{-2}][L^3]^2$$

$$= [ML^5T^{-2}]$$

10. Unit of a = Unit of $P \times$ Unit of V^2

$$= \text{dyne cm}^{-2} \times (\text{cm}^3)^2$$

$$= \textbf{dyne cm}^4.$$

11. [Force] = $[MLT^{-2}]$

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^1\left[\frac{L_1}{L_2}\right]^1\left[\frac{T_1}{T_2}\right]^{-2}$$

$$= 1\left[\frac{1\text{ kg}}{1\text{ quintal}}\right]^1\left[\frac{1\text{m}}{1\text{km}}\right]^1\left[\frac{1\text{s}}{1\text{h}}\right]^{-2}$$

$$= 1\left[\frac{1}{100}\right]\left[\frac{1}{1000}\right]\left[\frac{1}{3600}\right]^{-2}$$

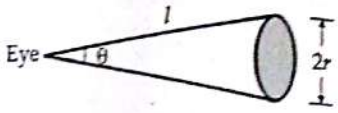
$$= \frac{3600 \times 3600}{100 \times 1000} = \textbf{129.6 new units.}$$

12. Surface tension

$$= \frac{\text{Force}}{\text{Length}} = \frac{MLT^{-2}}{L} = [ML^0T^{-2}]$$

13. The number of significant figures in 6.0023 is five (6, 0, 0, 2, 3).

14. Here $l = 80$ cm, $2r = 2$ cm



$$\theta = \frac{\text{Arc}}{\text{Radius}} = \frac{2r}{l} = \frac{2}{80} = \frac{1}{40}\text{ rad}$$

$$= \left(\frac{180}{\pi} \times \frac{1}{40}\right)^\circ = \textbf{1.43°}.$$

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MOTION IN A STRAIGHT LINE

3.1 MECHANICS

1. *What is mechanics ? What are its various sub-branches ?*

Mechanics. Mechanics is the branch of physics that *deals with the conditions of rest or motion of the material objects* around us. It is one of the oldest branches of physics.

Sub-branches of mechanics :

(i) **Statics.** It is the branch of mechanics that *deals with the study of objects at rest or in equilibrium,* even when they are under the action of several forces. The measurement of time is not essential in statics.

(ii) ***Kinematics.*** It is the branch of mechanics that *deals with the study of motion of objects without considering the cause of motion.* Here measurement of time is essential. The word *kinematics* comes from Greek word *kinema* which means motion.

(iii) ***Dynamics.*** It is the branch of mechanics that *deals with the study of motion of objects taking into consideration the cause of their motion.* The word *dynamics* comes from a Greek word *dynamics* which means power. Dynamics is concerned with the forces which cause motion. Newton contributed a lot to dynamics by postulating his famous laws of motion.

3.2 REST AND MOTION

2. *Define the terms rest and motion. Show that rest and motion are relative terms.*

Rest. *An object is said to be at rest if it does not change its position w.r.t. its surroundings with the passage of time e.g.,* a book lying on a table.

Motion. *An object is said to be in motion if it changes its position w.r.t. its surroundings with the passage of time e.g.,* a train moving on rails.

Rest and motion are relative terms. A passenger sitting in a moving train is at rest with respect to his fellow passengers but he is in motion with respect to the objects outside the train. Thus an object may be at rest w.r.t. one object and at the same time it may be in motion relative to another object. Hence rest and motion are relative terms.

3. *Can a body exist in a state of absolute rest or of absolute motion ? Explain.*

Absolute rest and motion are unknown. In order to know whether the position of an object changes with time or not, a point *absolutely fixed in space has to be chosen as reference point.* But no such point is known in the universe. The earth revolves around the sun, the entire solar system travels through our own galaxy, the milkyway and clusters of galaxies move with respect to other clusters. So *no object in the universe is in a state of absolute rest.*

As no object in the universe is at absolute rest, so the absolute motion cannot be realised. Only relative rest and relative motion can be realised.

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3.3 ▼ CONCEPT OF A POINT OBJECT

4. *What is meant by a point object ? Give suitable examples.*

Point object. *If the position of an object changes by distances much greater than its own size in a resonable duration of time, then the object may be regarded as a point object.* When a point object moves, its rotational and vibrational motions may be ignored.

Examples :

(*i*) Earth can be regarded as a point object for studying its motion around the sun.

(*ii*) A train under a journey of several hundred kilometres can be regarded as a point object.

3.4 ▼ MOTION IN ONE, TWO AND THREE DIMENSIONS

5. *What do you mean by motion in one, two and three dimensions ? Give examples of each type.*

One dimensional motion. *The motion of an object is said to be one dimensional if only one of the three coordinates specifying the position of the object changes with time.* Here the object moves along a straight line. This motion is also called **rectilinear** or **linear motion**. As shown in Fig. 3.1, only the x-coordinate changes from x_1 to x_2 when the particle moves from P_1 to P_2 along a *straight line* path.

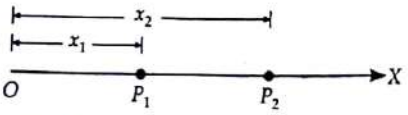

Fig. 3.1 One dimensional motion.

Examples of one dimensional motion :

(*i*) Motion of a train along a straight track.

(*ii*) Motion of a freely falling body.

Two dimensional motion. *The motion of an object is said to be two dimensional if only two of the three coordinates specifying its position change with time.* Here an object moves along a plane. As shown in Fig. 3.2, the coordinates (x_1, y_1) change to (x_2, y_2) as the particle moves from P_1 to P_2 in a *plane*.

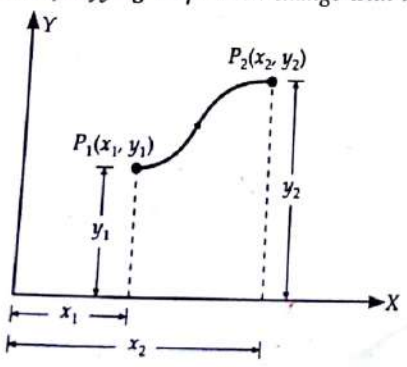

Fig. 3.2 Two dimensional motion.

Examples of two dimensional motion :

(*i*) Motion of planets around the sun.

(*ii*) A car moving along a zig-zag path on a level road.

Three dimensional motion. *The motion of an object is said to be three dimensional if all the three coordinates specifying its position change with time.* Here an object moves in space. As shown in Fig. 3.3, the coordinates (x_1, y_1, z_1) change to (x_2, y_2, z_2) as the particle moves from P_1 to P_2 in *space*.

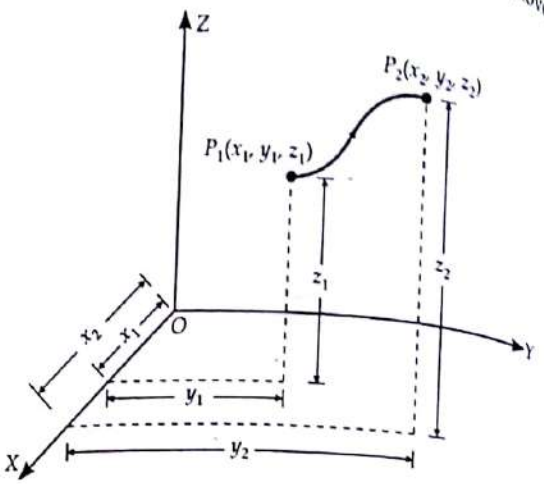

Fig. 3.3 Three dimensional motion.

Examples of three dimensional motion :

(*i*) A kite flying on a windy day.

(*ii*) Motion of an aeroplane in space.

3.5 ▼ DISTANCE AND DISPLACEMENT

6. *Distinguish between the terms distance and displacement.*

Distance or path length. *It is the length of the actual path traversed by a body between its initial and final positions.* As shown in Fig. 3.4, suppose a body moves from position A to B through C. Then

$$\text{Distance covered} = AC + CB$$

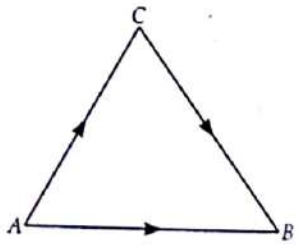

Fig. 3.4

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Distance is a scalar quantity because it has only magnitude and no direction. Distance covered is always positive or zero.

The SI unit of distance is metre (m).

The CGS unit of distance is centimetre (cm).

Displacement. *The displacement of an object is the change in the position of an object in a fixed direction.* It is the shortest (or the straight line) path measured in the direction from initial point to the final point. As displacement has both magnitude and direction, so it is a vector quantity.

In Fig. 3.4, displacement = $\vec{AB}$.

Displacement may be positive, negative or zero.

The SI unit of displacement is metre (m).

The CGS unit of displacement is centimetre (cm).

1. Mention some important characteristics of displacement. Give illustrations to support these characteristics.

Characteristics of Displacement :

(i) Displacement has the units of length.

(ii) The displacement of an object can be positive, negative or zero.

When an object moves towards right in time t_1 to t_2, its displacement is positive as shown in Fig. 3.5.

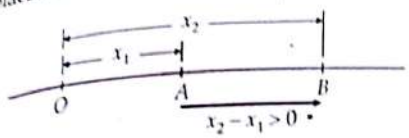

Fig. 3.5 Positive displacement.

When an object moves to the left in time t_1 to t_2, its displacement is negative as shown in Fig. 3.6.

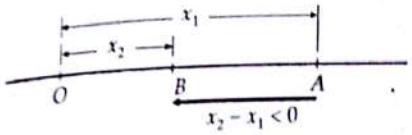

Fig. 3.6 Negative displacement.

When an object remains stationary, or it moves first towards right and then an equal distance towards left, its displacement is zero as shown in Fig. 3.7.

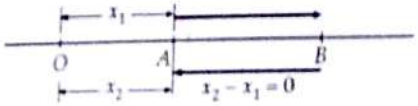

Fig. 3.7 Zero displacement.

(iii) Displacement is not dependent on the choice of the origin O of the position coordinates. As shown in Fig. 3.8, the shift of the origin from O to O' on the line l, produces equal changes in the position coordinates at times t_1 and t_2, so that the difference in the new position coordinates still remains the same. Hence the displacement AB remains unaffected.

Clearly,

$$x_2 - x_1 = AB = x_2' - x_1'$$

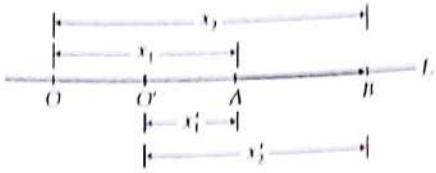

Fig. 3.8 Shifting origin causes no change in displacement.

(iv) The actual distance travelled by an object in a given time interval is greater than or equal to the magnitude of the displacement. The first situation is shown in Fig. 3.9 and second one in Fig. 3.10.

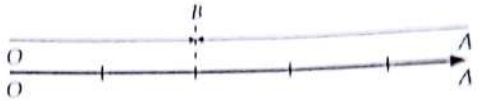

Fig. 3.9 Magnitude of displacement $\vec{OB}$ (2 units) < Actual distance (OA + AB) travelled (8 units).

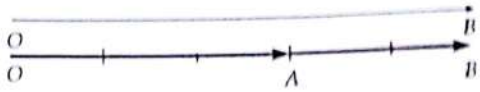

Fig. 3.10. Magnitude of displacement $\vec{OB}$ = Actual distance travelled (OA + AB) = 5 units each.

(v) The displacement of an object between two points is the unique path that takes the body from its initial to final position.

(vi) The displacement of an object between two positions does not give any information regarding the shape of the actual path followed by the object between these two positions.

(vii) The magnitude of the displacement of an object between two positions gives the shortest distance between these positions.

(viii) Displacement is a vector quantity. Displacement of an object between two given positions is independent of the actual path followed by the object in moving from one position to another.

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3.6 SPEED

8. *Define the term speed. Is it a scalar or vector quantity ? Give its units and dimensions.*

Speed. *The rate of change of position of an object with time in any direction is called its speed.* It is equal to the distance travelled by the object per unit time.

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$$

Speed has only magnitude and no direction, so it is a *scalar quantity*. Also the distance travelled by an object is either positive or zero, so the speed may be positive or zero but never negative.

The SI unit of speed is ms^{-1}.

The CGS unit of speed is cms^{-1}.

The dimensional formula of speed is $[M^0L^1T^{-1}]$.

9. *Define : (i) uniform speed (ii) variable speed (iii) average speed and (iv) instantaneous speed.*

Different Types of Speed :

(*i*) ***Uniform speed.*** An object is said to be moving with uniform speed, if it covers equal distances in equal intervals of time, however small these time intervals may be.

(*ii*) ***Variable speed.*** An object is said to be moving with variable speed if it covers unequal distances in equal intervals of time.

(*iii*) ***Average speed.*** For an object moving with variable speed, the average speed is the total distance travelled by the object divided by the total time taken to cover that distance.

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

(*iv*) ***Instantaneous speed.*** *The speed of an object at any particular instant of time or at a particular point of its path is called the instantaneous speed of the object.* Suppose an object covers distance Δx in a small time interval Δt around the instant t, then its average speed is $\Delta x / \Delta t$. The limiting value of this average speed when the time interval Δt approaches zero, gives the instantaneous speed at the instant t. Thus

Instantaneous speed,

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Here $\frac{dx}{dt}$ is the first order derivative of distance x with respect to time t.

The **speedometer** of an automobile indicates its instantaneous speed at any instant.

For Your Knowledge

Average Speed in Different Situations

▲ **A body covering different distances with different speeds.** Suppose a body covers distances $s_1, s_2, s_3, \ldots$ with speeds $v_1, v_2, v_3, \ldots$ respectively, then its average speed will be

$$v_{av} = \frac{\text{Total distance travelled}}{\text{Total time taken}} = \frac{s}{t}$$

$$= \frac{s_1 + s_2 + s_3 + \ldots}{t_1 + t_2 + t_3 + \ldots}$$

or $$v_{av} = \frac{s_1 + s_2 + s_3 + \ldots}{\left(\frac{s_1}{v_1} + \frac{s_2}{v_2} + \frac{s_3}{v_3} + \ldots\right)}$$

Special case If $s_1 = s_2 = s$ *i.e.*, the body covers equal distances with different speeds, then

$$v_{av} = \frac{2s}{s\left(\frac{1}{v_1} + \frac{1}{v_2}\right)} = \frac{2\,v_1 v_2}{v_1 + v_2}$$

Clearly, *the average speed is the harmonic mean of the individual speeds.*

▲ **A body moving with different speeds in different time intervals.** Suppose a body travels with speeds $v_1, v_2, v_3, \ldots$ in time intervals $t_1, t_2, t_3, \ldots$ respectively, then

Total distance travelled $= v_1t_1 + v_2t_2 + v_3t_3 + \ldots$

Total time taken $= t_1 + t_2 + t_3 + \ldots$

$$\therefore \quad v_{av} = \frac{v_1t_1 + v_2t_2 + v_3t_3 + \ldots}{t_1 + t_2 + t_3 + \ldots}$$

Special case If $t_1 = t_2 = t_3 = \ldots = t_n = t$ (say), then

$$v_{av} = \frac{(v_1 + v_2 + v_3 + \ldots + v_n)\,t}{nt} = \frac{v_1 + v_2 + v_3 + \ldots + v_n}{n}$$

Clearly, *average speed is the arithmetic mean of the individual speeds.*

3.7 VELOCITY

10. *Define the term velocity. Is it a scalar or vector quantity ? Give its units and dimensions.*

Velocity. *The rate of change of position of an object with time in a given direction is called its velocity.* It can also be defined as the speed of an object in a given direction. It is equal to the displacement covered per unit time.

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time}}$$

As velocity has both magnitude and direction, it is a vector quantity. Velocity can be positive, zero or negative depending on the displacement is positive, zero or negative.

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The SI unit of velocity is ms^{-1}.

The CGS unit of velocity is cms^{-1}.

The dimensional formula for the velocity is $[M^0L^1T^{-1}]$.

11. *Define (i) uniform velocity (ii) variable velocity (iii) average velocity and (iv) instantaneous velocity.*

Different Types of Velocities :

(i) ***Uniform velocity.*** *A body is said to be moving with uniform velocity if it covers equal displacements in equal intervals of time, however small these time intervals may be.*

(ii) ***Variable velocity.*** A body is said to be moving with variable velocity if either its speed changes or direction of motion changes or both change with time.

(iii) ***Average velocity.*** *For an object moving with variable velocity, average velocity is defined as the ratio of its total displacement to the total time interval in which that displacement occurs.*

$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time}}$$

If x_1 and x_2 are the positions of an object at times t_1 and t_2, then the average velocity from time t_1 to t_2 is given by

$$v_{av} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

(iv) ***Instantaneous velocity.*** *The velocity of an object at a particular instant of time or at a particular point of its path is called its instantaneous velocity.* It is equal to the limiting value of the average velocity of the object in a small time interval taken around that instant, when the time interval approaches zero. Thus

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t} = \frac{d\vec{x}}{dt}$$

Thus instantaneous velocity of an object is equal to the first order derivative of its displacement with respect to time.

12. *When is the average speed of an object equal to the magnitude of its average velocity ? Give reason also.*

We know that

$$\text{Average speed} = \frac{\text{Total path length}}{\text{Time interval}}$$

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time interval}}$$

When an object moves along a straight line and in the same direction, its total path length is equal to the magnitude of its displacement. Consequently, its average speed is equal to the magnitude of its average velocity.

3.8 UNIFORM AND NON-UNIFORM MOTIONS

13. *What is meant by uniform motion ?*

Uniform motion. *An object is said to be in uniform motion if it covers equal distances in equal intervals of time, however small these time intervals may be, in the same fixed direction.* So if an object in uniform motion covers 1200 m in 1 minute in a given direction, it indicates that it covers 600 m in every 30 s, 300 m in every 15 s, 100 m in every 5 s, and so on.

14. *Obtain the formulae for the position of an object moving with uniform velocity v (i) at any time t in terms of its position at $t = 0$ and (ii) at any time t′ in terms of its position at another time t.*

Kinematic formulae for uniform motion. Consider an object moving with uniform velocity v along the positive direction of x-axis. Suppose it occupies positions x_0, x and x' at times $t = 0$, t and t' respectively, as shown in Fig. 3.11.

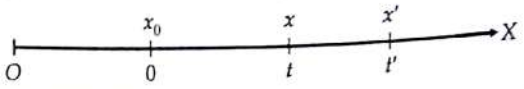

Fig. 3.11 Uniform motion.

Velocity of the object in the time interval 0 to t is

$$v = \frac{\text{Displacement}}{\text{Time}} = \frac{x - x_0}{t - 0}$$

or $$x = x_0 + vt \quad \ldots(1)$$

This equation gives position of the particle at instant t.

Similarly, velocity of the object in the time interval t to t' is

$$v = \frac{x' - x}{t' - t} \quad \text{or} \quad x' = x + v(t' - t)$$

This equation gives position of the object at instant t' in terms of position x at time t.

From equation (1), we have : $x - x_0 = vt$

But $x - x_0$ = displacement in time interval t

$= s$ (say)

$\therefore \quad s = vt$

This equation can be used to determine the distance travelled by an object in uniform motion.

15. *Give some important features of uniform motion.*

Some Important Features of Uniform Motion :

(i) The velocity in uniform motion does not depend on the choice of origin.

(ii) The velocity in uniform motion does not depend on the choice of the time interval $(t_2 - t_1)$.

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(iii) For uniform motion along a straight line in the same direction, the magnitude of the displacement is equal to the actual distance covered by the object.

(iv) The velocity is positive if the object is moving towards the right of the origin and negative if the object is moving towards the left of the origin.

(v) For an object in uniform motion, no force is required to maintain its motion.

(vi) In uniform motion, the instantaneous velocity is equal to the average velocity at all times because velocity remains constant at each instant or at each point of the path.

16. *What is non-uniform motion ?*

Non-uniform motion. *A body is said to be in non-uniform motion if its velocity changes with time.* Here either the speed of the body or its direction of motion or both change with time. For example, when a vehicle starts moving from rest, its velocity increases for sometime, then its velocity may become constant for some time and finally slow down and come to rest again. The velocity of the vehicle is different at different instants and so it has non-uniform motion.

Examples based on Calculation of Distance Covered, Displacement, Average Speed and Average Velocity

FORMULAE USED

1. Distance covered = Length of actual path traversed by the body
2. Displacement = Vector drawn from initial to final position of the body
3. Average speed $= \dfrac{\text{Total distance travelled}}{\text{Total time taken}}$

UNITS USED

Distance and displacement are in metre (or km), average speed and average velocity in ms^{-1} (or $km\ h^{-1}$) and time in second (or hour).

EXAMPLE 1. In Fig. 3.12, a particle moves along a circular path of radius r. It starts from point A and moves anticlockwise. Find the distance travelled by the particle as it (i) moves from A to B (ii) moves from A to C (iii) moves A to D (iv) completes one revolution. Also find the magnitude of displacement in each case.

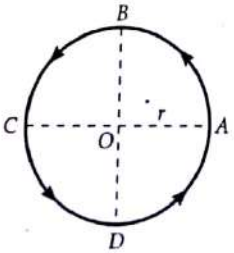

Fig. 3.12

Solution. *(i) From A to B.*

Distance covered $= \frac{1}{4} \times 2\pi r = \frac{1}{2}\pi r.$

Displacement $= |\vec{AB}| = \sqrt{OA^2 + OB^2}$

$= \sqrt{r^2 + r^2} = \sqrt{2}r.$

(ii) From A to C.

Distance covered $= \frac{1}{2} \times 2\pi r = \pi r.$

Displacement $= |\vec{AC}| = 2r.$

(iii) From A to D.

Distance covered $= \frac{3}{4} \times 2\pi r = \frac{3}{2}\pi r.$

Displacement $= |\vec{AD}| = \sqrt{r^2 + r^2} = \sqrt{2}\,r$

(iv) From A to A. Distance covered $= 2\pi r$

As the final position coincides with the initial position, displacement covered = zero.

EXAMPLE 2. A car is moving along X-axis. As shown in Fig. 3.13, it moves from 0 to P in 18 s and returns from P to Q in 6 s. What are the average velocity and average speed of the car in going from (i) from O to P and (ii) from O to P and back to Q ? [NCERT]

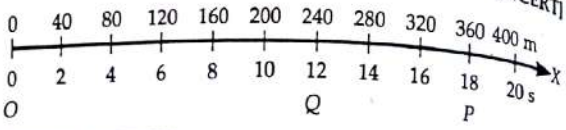

Fig. 3.13

Solution. *(i) From O to P.*

Average velocity $= \dfrac{\text{Displacement}}{\text{Time interval}}$

$= \dfrac{+360\,\text{m}}{18\,\text{s}} = +\,\mathbf{20\ ms^{-1}}.$

Average speed $= \dfrac{\text{Path length}}{\text{Time interval}} = \dfrac{360\ \text{m}}{18\ \text{s}} = \mathbf{20\ ms^{-1}}.$

(ii) From O to P and back to Q.

Average velocity $= \dfrac{\text{Displacement}}{\text{Time interval}} = \dfrac{OQ}{18 + 6}$

$= \dfrac{+240\,\text{m}}{24\,\text{s}} = \mathbf{10\ ms^{-1}}.$

Average speed $= \dfrac{\text{Path length}}{\text{Time interval}} = \dfrac{OP + PQ}{18 + 6}$

$= \dfrac{(360 + 120)}{24\,\text{s}}\,\text{m} = \mathbf{20\ ms^{-1}}.$

EXAMPLE 3. A body travels from A to B at 40 ms^{-1} and from B to A at 60 ms^{-1}. Calculate the average speed and average velocity. [Himachal 07]

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Solution. Total time taken by the body to travel from A to B and then from B to A,

$$t_1 + t_2 = \frac{AB}{40} + \frac{BA}{60} = AB\left(\frac{1}{40} + \frac{1}{60}\right) = \frac{AB}{24} \text{ s.}$$

Total distance covered

$$= AB + BA = 2AB$$

$$\therefore \text{ Average speed} = \frac{2AB}{t_1 + t_2} = \frac{2AB}{AB/24} = \mathbf{48\ ms^{-1}}.$$

As the body comes back to its initial position A, its net displacement is zero.

$$\therefore \text{ Average velocity} = \frac{\text{Displacement}}{\text{Time}} = \frac{0}{AB/24} = \mathbf{0}.$$

EXAMPLE 4. On a 60 km track, a train travels the first 30 km with a uniform speed of 30 km h^{-1}. How fast must the train travel the next 30 km so as to average 40 km h^{-1} for the entire trip ?

Solution. Here $s_1 = s_2 = s = 30$ km, $v_1 = 30$ km h^{-1}, $v_{av} = 40$ km h^{-1}, $v_2 = ?$

$$\text{But } v_{av} = \frac{s_1 + s_2}{t_1 + t_2} = \frac{s + s}{\frac{s}{v_1} + \frac{s}{v_2}} = \frac{2s}{s\left(\frac{1}{v_1} + \frac{1}{v_2}\right)} = \frac{2v_1v_2}{v_1 + v_2}$$

$$\therefore \quad 40 = \frac{2 \times 30 \times v_2}{30 + v_2} \quad \text{or} \quad 30 + v_2 = \frac{3}{2}v_2$$

or $$v_2 = \mathbf{60\ km\ h^{-1}}.$$

EXAMPLE 5. A body covers one-third of its journey with speed 'u', next one-third with speed 'v' and the last one-third with speed 'w'. Calculate the average speed of the body during the entire journey.

Solution. Let total distance $= 3x$. Then,

$$\text{Total time taken} = \frac{x}{u} + \frac{x}{v} + \frac{x}{w} = x\left(\frac{vw + uw + uv}{uvw}\right)$$

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

$$= \frac{3x}{x\left(\frac{vw + uw + uv}{uvw}\right)}$$

$$= \frac{3uvw}{uv + vw + uw}.$$

EXAMPLE 6. A body travelling along a straight line traversed one-half of the total distance with a velocity v_0. The remaining part of the distance was covered with a velocity v_1, for half the time and with velocity v_2 for the other half of time. Find the mean velocity averaged over the whole time of motion.

Solution. Let total distance $= x$

Let the time taken to cover first one-half distance $= t_1$.

Then $$t_1 = \frac{x/2}{v_0} = \frac{x}{2v_0}$$

Let the time taken for next $x/2$ distance $= t_2$.

Then $$\frac{x}{2} = v_1 \cdot \frac{t_2}{2} + v_2 \cdot \frac{t_2}{2} = \frac{(v_1 + v_2)\,t_2}{2}$$

or $$t_2 = \frac{x}{v_1 + v_2}$$

Total time taken

$$= t_1 + t_2 = \frac{x}{2v_0} + \frac{x}{v_1 + v_2}$$

$$= \frac{x(v_1 + v_2 + 2v_0)}{2v_0(v_1 + v_2)}$$

Average velocity

$$= \frac{\text{Total distance}}{\text{Total time}} = \frac{x}{\frac{x(v_1 + v_2 + 2v_0)}{2v_0(v_1 + v_2)}}$$

$$= \frac{2v_0(v_1 + v_2)}{v_1 + v_2 + 2v_0}.$$

PROBLEMS FOR PRACTICE

1. A cyclist moving on a circular track of radius 100 m completes one revolution in 4 minutes. What is his (*i*) average speed (*ii*) average velocity in one full revolution ? [**Ans.** (*i*) 50 π metre/minute (*ii*) 0]

2. A body travels the first half of the total distance with velocity v_1 and the second half with velocity v_2. Calculate the average velocity. $\left(\textbf{Ans.}\ \frac{2v_1v_2}{v_1 + v_2}\right)$

3. A car covers the first half of the distance between two places at a speed of 40 kmh^{-1} and the second half at 60 kmh^{-1}. What is the average speed of the car ? **[Central Schools 07]**
(**Ans.** 48 kmh^{-1})

4. A train moves with a speed of 30 kmh^{-1} in the first 15 minutes, with another speed of 40 kmh^{-1} the next 15 minutes, and then with a speed of 60 kmh^{-1} in the last 30 minutes. Calculate the average speed of the train for this journey. (**Ans.** 47.5 kmh^{-1})

5. A body travels a distance s_1 with velocity v_1 and distance s_2 with velocity v_2 in the same direction. Calculate the average velocity of the body.
$\left[\textbf{Ans.}\ \frac{(s_1 + s_2)\,v_1\,v_2}{s_1\,v_2 + s_2\,v_1}\right]$

6. A car travels along a straight line for the first half time with speed 50 kmh^{-1} and the second half time with speed 60 kmh^{-1}. Find the average speed of the car. (**Ans.** 55 kmh^{-1})

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HINTS

1. Distance travelled in one revolution
$= 2\pi r = 2\pi \times 100$ m
Time taken = 4 minute
Displacement in 4 minute = 0
(i) Average speed $= \frac{\text{Distance}}{\text{Time}} = \frac{2\pi \times 100 \text{ m}}{4 \text{ min}}$
$= 50\pi$ **metre/ minute.**
(ii) Average velocity $= \frac{\text{Displacement}}{\text{Time}} = 0$.

2. Let total distance = 2x. Then,
Total time taken $= \frac{x}{v_1} + \frac{x}{v_2} = x\left(\frac{v_1 + v_2}{v_1 v_2}\right)$
$\therefore$ Average speed $= \frac{2x}{x\left(\frac{v_1 + v_2}{v_1 v_2}\right)} = \frac{2v_1 v_2}{v_1 + v_2}$.

3. $v_{av} = \frac{2v_1 v_2}{v_1 + v_2} = \frac{2 \times 40 \times 60}{40 + 60} =$ **48 km h^{-1}.**

4. Total distance covered
$= 30 \times \frac{15}{60} + 40 \times \frac{15}{60} + 60 \times \frac{30}{60} = 47.5$ km
Total time taken = 15 min + 15 min + 30 min
= 60 min = 1 h
Average speed $= \frac{47.5}{1} =$ **47.5 km h^{-1}.**

5. Total distance covered $= s_1 + s_2$
Total time taken
$= t_1 + t_2 = \frac{s_1}{v_1} + \frac{s_2}{v_2} = \frac{s_1 v_2 + s_2 v_1}{v_1 v_2}$
Average velocity
$= \frac{s_1 + s_2}{\frac{s_1 v_2 + s_2 v_1}{v_1 v_2}} = \frac{(s_1 + s_2) v_1 v_2}{s_1 v_2 + s_2 v_1}$.

6. Average speed $= \frac{50 \times \frac{t}{2} + 60 \times \frac{t}{2}}{t} =$ **55 kmh^{-1}.**

3.9 ACCELERATION

17. *Define the term acceleration. Is it a scalar or a vector quantity ? Give its units and dimensions.*

Acceleration. *The rate of change of velocity of an object with time is called its acceleration.* It tells how fast the velocity of an object changes with time.

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time taken}}$$

Acceleration is a vector quantity. It has the same direction as that of the change in velocity.

The SI unit of acceleration is ms^{-2}.

The CGS unit of acceleration is cm s^{-2}.

The dimensional formula of acceleration is $[M^0L^1T^{-2}]$.

18. *Define (i) uniform acceleration (ii) variable acceleration (iii) average acceleration and (iv) instantaneous acceleration.*

Different Types of Acceleration :

(i) ***Uniform acceleration.*** *The acceleration of an object is said to be uniform acceleration if its velocity changes by equal amounts in equal intervals of time, however small these time intervals may be.*

(ii) ***Variable acceleration.*** *The acceleration of an object is said to be variable acceleration if its velocity changes by unequal amounts in equal intervals of time.*

(iii) ***Average acceleration.*** For an object moving with variable velocity, the *average acceleration is defined as the ratio of the total change in velocity of the object to the total time interval taken.* Suppose v_1 and v_2 are the velocities of an object at times t_1 and t_2 respectively, then its average acceleration is

$$a_{av} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

(iv) ***Instantaneous acceleration.*** *The acceleration of an object at a given instant of time or at a given point of its motion, is called its instantaneous acceleration.* It is equal to the limiting value of the average acceleration of the object in small time interval around that instant, when the time interval approaches zero. Thus

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

As $v = \frac{dx}{dt}$

$\therefore$ $a = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$

Thus, acceleration is the first order derivative of velocity and second order derivative of position with respect to time.

19. *What are positive and negative accelerations ? Give examples.*

Positive acceleration. *If the velocity of an object increases with time, its acceleration is positive.* When a bus leaves a bus-stop, its acceleration is positive.

Negative acceleration. *If the velocity of an object decreases with time, its acceleration is negative.* Negative acceleration is also called **retardation** or **deceleration**. When a bus slows down on approaching a bus-stop, its acceleration is negative.

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Examples based on Instantaneous Velocity and Instantaneous Acceleration

FORMULAE USED

1. $v_{av} = \frac{s_2 - s_1}{t_2 - t_1} = \frac{\Delta s}{\Delta t}$
2. $v = \underset{\Delta t \to 0}{\text{Lim}} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$
3. $a_{av} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$
4. $a = \underset{\Delta t \to 0}{\text{Lim}} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2s}{dt^2}$

UNITS USED

All velocities are in ms^{-1} and accelerations in ms^{-2}

EXAMPLE 7. The position of an object moving along x-axis is given by $x = a + bt^2$, where $a = 8.5$ m, $b = 2.5\ ms^{-2}$ and t is measured in seconds. What is its velocity at $t = 0$ s and $t = 2s$? What is the average velocity between $t = 2s$ and $t = 4s$? **[NCERT ; Himachal 08]**

Solution. Given $x = a + bt^2$

Instantaneous velocity,

$$v = \frac{dx}{dt} = \frac{d}{dt}(a + bt^2) = 0 + b \times 2t = 2bt$$

At $t = 0$, $v = 0$.

At $t = 2s$, $v = 2 \times 2.5 \times 2 = \mathbf{10\ ms^{-1}}$.

At $t = 2s$, $x = a + 4b$

At $t = 4s$, $x = a + 16b$

Average velocity

$$= \frac{x_2 - x_1}{t_2 - t_1} = \frac{(a + 16b) - (a + 4b)}{4 - 2} = 6b$$

$$= 6 \times 2.5 = \mathbf{15.0\ ms^{-1}}.$$

Example 8. The displacement (in metre) of a particle moving along x-axis is given by $x = 18t + 5t^2$. Calculate :

(i) the instantaneous velocity at $t = 2$ s,

(ii) average velocity between $t = 2$ s and $t = 3$s,

(iii) instantaneous acceleration. **[Central Schools 05]**

Solution. Given : $x = 18t + 5t^2$

(i) Velocity, $v = \frac{dx}{dt} = \frac{d}{dt}(18t + 5t^2) = 18 + 10t$

At $t = 2$ s, the instantaneous velocity

$$= 18 + 10 \times 2 = \mathbf{38\ ms^{-1}}.$$

(ii) Displacement at $t = 2$ s is

$$x_1 = 18 \times 2 + 5 \times 2^2 = 56 \text{ m}$$

Displacement at $t = 3$ s is

$$x_2 = 18 \times 3 + 5 \times 3^2 = 54 + 45 = 99 \text{ m}$$

Average velocity,

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{99 - 56}{3 - 2} = \mathbf{43\ ms^{-1}}.$$

(iii) Acceleration,

$$a = \frac{dv}{dt} = \frac{d}{dt}(18 + 10t) = \mathbf{10\ ms^{-2}}.$$

EXAMPLE 9. The displacement x of a particle varies with time t as $x = 4t^2 - 15t + 25$.

Find the position, velocity and acceleration of the particle at $t = 0$. When will the velocity of the particle become zero ? Can we call the motion of the particle as one with uniform acceleration ?

Solution. Position, $x = 4t^2 - 15t + 25$

Velocity, $v = \frac{dx}{dt} = 8t - 15$

Acceleration, $a = \frac{dv}{dt} = 8$

At time $t = 0$, we have

$$x = \mathbf{25\ m}, \quad v = \mathbf{-15\ ms^{-1}}, \quad a = \mathbf{8\ ms^{-2}}.$$

Velocity will become zero, when $8t - 15 = 0$

or $$t = \frac{15}{8} = \mathbf{1.875\ s}.$$

Yes, the particle has a uniform acceleration because a does not depend on time t.

EXAMPLE 10. The velocity of a particle is given by the equation, $v = 2t^2 + 5\ cms^{-1}$. Find (i) the change in velocity of the particle during the time interval between $t_1 = 2$ s and $t_2 = 4$ s (ii) the average acceleration during the same interval and (iii) the instantaneous acceleration at $t_2 = 4$ s.

Solution. Given $v = 2t^2 + 5\ \text{cms}^{-1}$

(i) When $t_1 = 2s$, $v_1 = 2\ (2)^2 + 5 = 13\ \text{cms}^{-1}$

When $t_2 = 4$ s, $v_2 = 2\ (4)^2 + 5 = 37\ \text{cms}^{-1}$

Change in velocity

$$= v_2 - v_1 = 37 - 13 = \mathbf{24\ cms^{-1}}.$$

(ii) Average acceleration,

$$a_{av} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{37 - 13}{4 - 2} = \mathbf{12\ cms^{-2}}.$$

(iii) Instantaneous acceleration,

$$a = \frac{dv}{dt} = \frac{d}{dt}(2t^2 + 5) = 4t$$

At $t = 4$ s, $a = 4 \times 4 = \mathbf{16\ cms^{-2}}$.

EXAMPLE 11. The distance x of a particle moving in one dimension, under the action of a constant force is related to time t by the equation, $t = \sqrt{x} + 3$, where x is in metres and t in seconds. Find the displacement of the particle when its velocity is zero. **[IIT]**

Solution. Given : $t = \sqrt{x} + 3$ or $\sqrt{x} = t - 3$

or $$x = (t - 3)^2 = t^2 - 6t + 9$$

Velocity, $v = \frac{dx}{dt} = 2t - 6$

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Velocity becomes zero when $2t-6=0$ or $t=3$ s

$\therefore$ At $t=3$s, $x=(t-3)^2=(3-3)^2=0$

Hence displacement of the particle is zero when its velocity is zero.

EXAMPLE 12. The acceleration of a particle in ms^{-2} is given by $a=3t^2+2t+2$, where time t is in second. If the particle starts with a velocity $v=2\ ms^{-1}$ at $t=0$, then find the velocity at the end of 2s. **[Delhi 13]**

Solution. As $a=\frac{dv}{dt}=3t^2+2t+2$

or $dv=(3t^2+2t+2)\,dt$

Integrating both sides, we get

$$\int dv=\int (3t^2+2t+2)\,dt$$

or $$v=3\frac{t^3}{3}+2\frac{t^2}{2}+2t+C=t^3+t^2+2t+C$$

At $t=0$, $v=2\,ms^{-1}$, therefore

$$2=0+C \quad \text{or} \quad C=2\ ms^{-1}$$

$$v=t^3+t^2+2t+2$$

At $t=2s$, $v=8+4+4+2=18\ ms^{-1}$.

PROBLEMS FOR PRACTICE

1. The displacement x of a particle at time t along a straight line is given by $x=\alpha-\beta t+\gamma t^2$. Find the acceleration of the particle. **(Ans. 2γ)**
2. The displacement x of a particle is dependent on time t according to the relation : $x=3-5t+2t^2$. If t is measured in seconds and x in metres, find its *(i)* velocity at $t=2$s and *(ii)* acceleration at $t=4$s.
   [Central Schools 13] [Ans. (i) $3ms^{-1}$ (ii) $4ms^{-2}$]
3. The displacement x of a particle along X-axis is given by $x=3+8t+7t^2$. Obtain its velocity and acceleration at $t=2$s. **(Ans. $36\ ms^{-1}$, $14\ ms^{-2}$)**
4. The distance traversed by a particle moving along a straight line is given by $x=180t+50t^2$ metre. Find :
   (i) the initial velocity of the particle
   (ii) the velocity at the end of 4s and
   (iii) the acceleration of the particle.
   [Ans. (i) $180\ ms^{-1}$ (ii) $580\ ms^{-1}$ (iii) $100\ ms^{-2}$]

3.10 KINEMATIC EQUATIONS FOR UNIFORMLY ACCELERATED MOTION

20. Derive the following equations of motion for an object moving with constant acceleration along a straight line :

(i) $v=v_0+at$

(ii) $x=x_0+v_0t+\frac{1}{2}at^2$ or $s=v_0t+\frac{1}{2}at^2$

(iii) $v^2=v_0^2+2a(x-x_0)=v_0^2+2as$

(iv) $s_{nth}=v_0+\frac{a}{2}(2n-1)$.

Equations for uniformly accelerated motion along a straight line. Consider an object moving with constant acceleration along positive direction of X-axis.

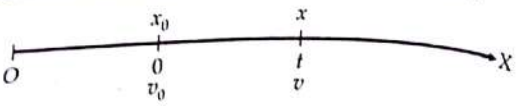

Fig. 3.14 Uniformly accelerated motion.

As shown in Fig. 3.14, let

x_0 = position of the object at instant $t=0$

x = position of the object at instant t

v_0 = velocity of the object at instant $t=0$

v = velocity of the object at instant t

(i) Velocity after a certain time. By definition,

$$\text{Acceleration}=\frac{\text{Change in velocity}}{\text{Time taken}}$$

$$a=\frac{v-v_0}{t-0}$$

or $$at=v-v_0$$

or $$v=v_0+at \qquad ...(1)$$

This is the required velocity-time relation.

(ii) Distance covered in a certain time. Average velocity of the object in time interval 0 to t is given by

$$v_{av}=\frac{\text{Displacement}}{\text{Time}}=\frac{x-x_0}{t-0}$$

or $$x-x_0=v_{av}\times t$$

But $$v_{av}=\frac{\text{Initial velocity}+\text{Final velocity}}{2}$$

$$=\frac{v_0+v}{2}$$

$\therefore$ $$x-x_0=\frac{v_0+v}{2}\times t=\frac{2v_0+at}{2}\times t \qquad [\because v=v_0+at]$$

$$=v_0t+\frac{1}{2}at^2$$

or $$x=x_0+v_0t+\frac{1}{2}at^2 \qquad ...(2)$$

Also, $x-x_0$ = distance travelled in time t = s, say

$\therefore$ $$s=v_0t+\frac{1}{2}at^2 \qquad ...(3)$$

Equation (2) is the required *position-time relation* and equation (3) is the required *distance-time relation*.

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(iii) Velocity at a certain position. We know that

$$v = v_0 + at \qquad ...(4)$$

$$v - v_0 = at$$

Also, Average velocity × time = Displacement

$$\frac{v + v_0}{2} \times t = x - x_0$$

$$v + v_0 = \frac{2}{t}(x - x_0) \qquad ...(5)$$

Multiplying equations (4) and (5), we get

$$(v + v_0)(v - v_0) = \frac{2}{t}(x - x_0)\, at$$

$$v^2 - v_0^2 = 2a(x - x_0)$$

$$v^2 = v_0^2 + 2a(x - x_0) \qquad ...(6)$$

But $x - x_0 = s$, the distance travelled in time t,

therefore

$$v^2 = v_0^2 + 2as \qquad ...(7)$$

Equation (6) is the required *velocity-position relation* and equation (7) is the required *velocity-distance relation.*

(iv) Distance covered in nth second. As shown in Fig. 3.15, the distance travelled in nth second can be obtained by subtracting the distance travelled in first $(n-1)$ seconds from the distance travelled in first n seconds.

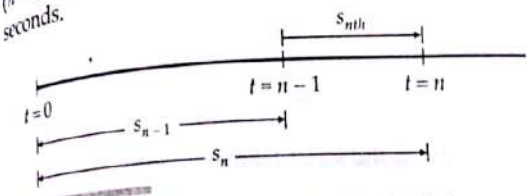

Fig. 3.15 Distance travelled in nth second.

Distance travelled in first t seconds is given by

$$s_t = v_0 t + \frac{1}{2} at^2$$

∴ Distance travelled in first $(n-1)$ seconds is

$$s_{n-1} = v_0(n-1) + \frac{1}{2} a(n-1)^2$$

Distance travelled in first n seconds is

$$s_n = v_0 n + \frac{1}{2} an^2$$

Hence the distance travelled in nth second is

$$s_{nth} = s_n - s_{n-1}$$

$$= (v_0 n + \frac{1}{2} an^2) - [v_0(n-1) + \frac{1}{2} a(n-1)^2]$$

$$= (v_0 n + \frac{1}{2} an^2) - [v_0 n - v_0 + \frac{1}{2} a(n^2 - 2n + 1)]$$

$$= v_0 n + \frac{1}{2} an^2 - v_0 n + v_0 - \frac{1}{2} an^2 + an - \frac{a}{2}$$

or $$s_{nth} = v_0 + \frac{a}{2}(2n - 1).$$

3.11 EQUATIONS OF MOTION BY CALCULUS METHOD

21. *Deduce the following equations for uniformly accelerated motion by using integration technique :*

(i) $v = u + at$ (ii) $s = ut + \frac{1}{2} at^2$

(iii) $v^2 - u^2 = 2as$ (iv) $s_{nth} = u + \frac{a}{2}(2n - 1)$.

First equation of motion. Acceleration is defined as

$$a = \frac{dv}{dt}$$

or $$dv = a\,dt \qquad ...(1)$$

When time = 0, velocity = u (say)

When time = t, velocity = v (say)

Integrating equation (1) within the above limits of time and velocity, we get

$$\int_u^v dv = \int_0^t a\,dt$$

or $$[v]_u^v = a\int_0^t dt = a[t]_0^t$$

or $$v - u = a(t - 0)$$

or $$v = u + at \qquad ...(2)$$

Second equation of motion. Velocity is defined as

$$v = \frac{ds}{dt}$$

or $$ds = vdt = (u + at)\,dt \qquad ...(3)$$

When time = 0, distance travelled = 0

When time = t, distance travelled = s (say)

Integrating equation (3) within the above limits of time and distance, we get

$$\int_0^s ds = \int_0^t (u + at)\,dt = u\int_0^t dt + a\int_0^t t\,dt$$

or $$[s]_0^s = u[t]_0^t + a\left[\frac{t^2}{2}\right]_0^t$$

or $$s - 0 = u(t - 0) + a\left[\frac{t^2}{2} - 0\right]$$

or $$s = ut + \frac{1}{2} at^2 \qquad ...(4)$$

Third equation of motion. By the definitions of acceleration and velocity,

$$a = \frac{dv}{dt} = \frac{dv}{ds} \times \frac{ds}{dt} = \frac{dv}{ds} \times v$$

or $$ads = vdv \qquad ...(5)$$

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When time = 0, velocity = u, distance travelled = 0
When time = t, velocity = v, distance travelled = s (say)

Integrating equation (5) within the above limits of velocity and distance, we get

$$\int_0^s a\,ds = \int_u^v v\,dv$$

or $$a\int_0^s ds = \int_u^v v\,dv$$

or $$a\,[s]_0^s = \left[\frac{v^2}{2}\right]_u^v$$

or $$a\,[s-0] = \frac{v^2}{2} - \frac{u^2}{2}$$

or $$2as = v^2 - u^2$$

or $$v^2 - u^2 = 2as \quad ...(6)$$

Fourth equation of motion. By definition of velocity,

$$v = \frac{ds}{dt}$$

or $$ds = vdt = (u + at)\,dt \quad ...(7)$$

When time = $(n-1)$ second, distance travelled = s_{n-1} (say)

When time = n second, distance travelled = s_n (say)

Integrating equation (7) within the above limits of time and distance, we get

$$\int_{s_{n-1}}^{s_n} ds = \int_{n-1}^{n} (u + at)\,dt$$

or $$[s]_{s_{n-1}}^{s_n} = u\int_{n-1}^{n} dt + a\int_{n-1}^{n} t\,dt$$

or $$s_n - s_{n-1} = u\,[t]_{n-1}^{n} + a\left[\frac{t^2}{2}\right]_{n-1}^{n}$$

$$= u\,[n-(n-1)] + \frac{a}{2}[n^2 - (n-1)^2]$$

$$= u + \frac{a}{2}[n^2 - (n^2 - 2n + 1)]$$

or $$s_{nth} = u + \frac{a}{2}(2n-1) \quad ...(8)$$

where $s_{nth} = s_n - s_{n-1}$ = distance travelled in n^{th} second.

Examples based on Motion with Uniform Acceleration

Formulae Used

1. Equations of motion in conventional form,
   (i) $v = u + at$
   (ii) $s = ut + \frac{1}{2}at^2$
   (iii) $v^2 - u^2 = 2as$ or $v^2 = u^2 + 2as$
   (iv) $s_{nth} = u + \frac{a}{2}(2n-1)$
2. Equations of motion in Cartesian form,
   (i) $v(t) = v(0) + at$
   (ii) $v(t') = v(t) + a(t'-t)$
   (iii) $x(t) = x(0) + v(0)t + \frac{1}{2}at^2$
   (iv) $x(t') = x(t) + v(t)(t'-t) + \frac{1}{2}a(t'-t)^2$
   (v) $[v(t')]^2 - [v(t)^2] = 2a\,[x(t') - x(t)]$.

Units Used

Displacement is in metre, velocity in ms^{-1}, acceleration in ms^{-2} and time in second.

Example 13. A jet plane starts from rest with an acceleration of 3 ms^{-2} and makes a run for 35 s before taking off. What is the minimum length of the runway and what is the velocity of the jet at take off ?

Solution. Here $u = 0$, $a = 3\ ms^{-2}$, $t = 35\,s$

Minimum length of the runway is given by

$$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}\times 3 \times (35)^2 = \mathbf{1837.5\ m.}$$

Velocity of the jet at take off is

$$v = u + at = 0 + 3 \times 35 = \mathbf{105\ ms^{-1}.}$$

Example 14. An electron travelling with a speed of $5 \times 10^3\ ms^{-1}$ passes through an electric field with an acceleration of $10^{12}\ ms^{-2}$. (i) How long will it take for the electron to double its speed ? (ii) What will be the distance covered by the electron in this time ?

Solution. Here $u = 5 \times 10^3\,ms^{-1}$,

$v = 2 \times 5 \times 10^3\ ms^{-1}$, $a = 10^{12}\ ms^{-2}$

(i) Time, $$t = \frac{v-u}{a} = \frac{2\times 5\times 10^3 - 5\times 10^3}{10^{12}}$$

$$= \frac{5\times 10^3}{10^{12}} = \mathbf{5\times 10^{-9}\ s.}$$

(ii) Distance,

$$s = ut + \frac{1}{2}at^2$$

$$= 5\times 10^3 \times 5\times 10^{-9} + \frac{1}{2}\times 10^{12}\times (5\times 10^{-9})^2$$

$$= (25 + 12.5)\times 10^{-6} = \mathbf{3.75\times 10^{-5}\ m.}$$

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EXAMPLE 15. A driver takes 0.20 s to apply the brakes after he sees a need for it. This is called the reaction time of the driver. If he is driving car at a speed of 54 kmh^{-1} and the brakes cause a deceleration of 6.0 ms^{-2}, find the distance travelled by the car after he sees the need to put the brakes.

Solution. During the time of reaction, the car continues to move with uniform speed of 54 kmh^{-1} or 15 ms^{-1}.

∴ Distance covered during 0.20 s = 15 × 0.20 = 3.0 m

For motion with deceleration :

$$u = 15 \text{ ms}^{-1}, \quad v = 0, a = -6.0 \text{ ms}^{-2}$$

As $v^2 - u^2 = 2as$

$$\therefore \quad 0^2 - 15^2 = 2 \times (-6.0)\, s$$

$$s = \frac{225}{12} = 18.75 \text{ m}$$

or

Total distance travelled = 3.0 + 18.75 = **21.75 m.**

EXAMPLE 16. On a foggy day two drivers spot each other when they are just 80 metres apart. They are travelling at 72 kmh^{-1} and 60 kmh^{-1}, respectively. Both of them applied brakes retarding their cars at the rate of 5 ms^{-2}. Determine whether they avert collision or not.

Solution. *For the first car :*

$$u = 72 \text{ kmh}^{-1} = 20 \text{ ms}^{-1}, \quad v = 0, a = -5 \text{ ms}^{-2}$$

As $v^2 - u^2 = 2as$

$$0^2 - 20^2 = 2(-5)s_1$$

∴ Distance covered by first car, $s_1 = 40$ m

For the second car :

$$u = 60 \text{ kmh}^{-1} = \frac{60 \times 5}{18}$$

$$= \frac{50}{3} \text{ ms}^{-1}, \quad v = 0, a = -5 \text{ ms}^{-2}$$

As $v^2 - u^2 = 2as$

$$\therefore \quad 0^2 - \left(\frac{50}{3}\right)^2 = 2(-5)s_2$$

Distance covered by second car,

$$s_2 = \frac{2500}{9 \times 10} = 27.78 \text{ m}$$

Total distance covered by the two cars

$$= s_1 + s_2 = 40 + 27.78 = 67.78 \text{ m}$$

As this distance is less than the initial distance (=80 m) between the two cars, *so the collision will be averted.*

EXAMPLE 17. A hundred metre sprinter increases her speed from rest uniformly at the rate of 1 ms^{-2} upto three quaters of the total run and covers the last quater with uniform speed. How much time does she take to cover the first half and the second half of the run ?

Solution. The situation is shown in Fig. 3.16.

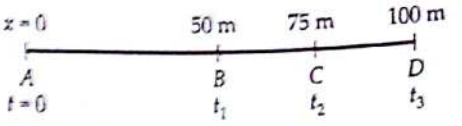

Fig. 3.16

For the motion from A to B :

$$u = 0, \quad a = 1 \text{ ms}^{-2}, \quad s = 50 \text{ m}$$

As $s = ut + \frac{1}{2}at^2$

$$\therefore \quad 50 = 0 + \frac{1}{2} \times 1 \times t_1^2$$

or $t_1^2 = 100$

∴ Time taken for first half run $AB = t_1 = \mathbf{10\ s.}$

For the motion from A to C :

$$u = 0, \quad a = 1 \text{ ms}^{-2}, \quad s = 75 \text{ m}$$

As $s = ut + \frac{1}{2}at^2$

$$\therefore \quad 75 = 0 + \frac{1}{2} \times 1 \times t_2^2$$

or $t_2 = \sqrt{150} = 12.2$ s

Time taken to run $BC = 12.2 - 10 = 2.2$ s

Again for part AC, $u = 0, a = 1 \text{ ms}^{-2}, \ t = 12.2$ s

$$\therefore \quad v = u + at = 0 + 1 \times 12.2 = \mathbf{12.2\ ms^{-1}}$$

For motion from C to D : This motion is uniform with speed equal to 12.2 ms^{-1}.

$$\therefore \text{ Time taken} = \frac{25 \text{ m}}{12.2 \text{ ms}^{-1}} = 2.04 \text{ s}$$

Total time taken for 2nd half run BD

$$= 2.2 + 2.04 = \mathbf{4.24\ s.}$$

EXAMPLE 18. A motor car starts from rest and accelerates uniformly for 10 s to a velocity of 20 ms^{-1}. It then runs at a constant speed and is finally brought to rest in 40 m with a constant acceleration. Total distance covered is 640 m. Find the value of acceleration, retardation and total time taken.

Solution. Let s_1, s_2 and s_3 be distances covered in the three parts of the motion.

For first part of the motion :

$$u = 0, \quad t = 10 \text{ s}, \quad v = 20 \text{ ms}^{-1}$$

As $v = u + at \quad \therefore \ 20 = 0 + a \times 10$

Acceleration, $a = 2 \text{ ms}^{-2}$

Distance, $s_1 = ut + \frac{1}{2}at^2$

$$= 0 \times 10 + \frac{1}{2} \times 2 \times (10)^2 = 100 \text{ m}$$

For second part of the motion : We have

$$s_1 = 100 \text{ m}, \quad s_3 = 40 \text{ m}$$

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As $s_1 + s_2 + s_3 = 640$

$\therefore \quad 100 + s_2 + 40 = 640$

or $s_2 = 500$ m

This distance is covered with a uniform speed of 20 ms^{-1}.

$\therefore$ Time taken $= \frac{500}{20} = 25$ s

For third part of the motion :

$u = 20\ ms^{-1}$, $v = 0$, $s = s_3 = 40$ m

As $v^2 - u^2 = 2as$

$\therefore \quad 0^2 - 20^2 = 2 \times a \times 40$

or $a = -\frac{400}{80} = -5\ ms^{-2}$

$\therefore$ Retardation $= 5\ ms^{-2}$

Time taken, $t = \frac{v-u}{a} = \frac{0-20}{-5} = 4$ s

Total time taken $= 10 + 25 + 4 = 39$ s.

EXAMPLE 19. An athlete runs a distance of 1500 m in the following manner. (i) Starting from rest, he accelerates himself uniformly at 2 ms^{-2} till he covers a distance of 900 m. (ii) He, then runs the remaining distance of 600 m at the uniform speed developed. Calculate the time taken by the athlete to cover the two parts of the distance covered. Also find the time, when he is at the centre of the track.

Solution. The situation is shown in Fig. 3.17.

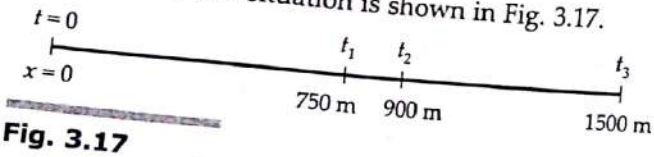

Fig. 3.17

For motion between times $t = 0$ and t_2 :

$u = 0$, $a = 2\ ms^{-2}$, $s = 900$ m, $v = v_2$ (say)

As $v^2 - u^2 = 2as$

$\therefore \quad v_2^2 - 0^2 = 2 \times 2 \times 900$

or $v_2 = 60\ ms^{-1}$

Time, $t_2 = \frac{v-u}{a} = \frac{60-0}{2} = 30$ s.

For motion between times t_2 and t_3 : This motion occurs at a constant speed of 60 ms^{-1}.

$\therefore$ Time taken, $t_3 - t_2 = \frac{1500-900}{60} = 10$ s.

For motion between time $t = 0$ and t_1 :

$u = 0$, $s = 750$ m, $a = 2\ ms^{-2}$, $t = t_1$ (say)

As $s = ut + \frac{1}{2}at^2 \quad \therefore \quad 750 = 0 \times t_1 + \frac{1}{2} \times 2 \times t_1^2$

or $t_1 = \sqrt{750} = 27.4$ s.

EXAMPLE 20. A man is $s = 9$ m behind the door of a train when it starts moving with acceleration $a = 2\ ms^{-2}$. The man runs at full speed. How far does he have to run and after what time does he get into the train ? What is his full speed ?

Solution. Let v be the full speed of the man. He gets into the train when his velocity becomes equal to the velocity of the train.

$\therefore \quad v = 0 + at$

Also, the man's acceleration is $-a$ relative to train and his initial velocity is v relative to train, so

$$s = vt - \frac{1}{2}at^2 = at \,.\, t - \frac{1}{2}at^2 = \frac{1}{2}at^2$$

or $t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 9}{2}} = 3$ s

$$s = vt = at \,.\, t = at^2 = a \,.\, \frac{2s}{a} = 2s = 2 \times 9 = 18 \text{ m}$$

and $v = at = 2 \times 3 = 6\ ms^{-1}$.

EXAMPLE 21. A car accelerates from rest at a constant rate α for some time, after which it decelerates at a constant rate β to come to rest. If the total time elapsed is t second, then calculate :

(i) the maximum velocity attained by the car, and

(ii) the total distance travelled by the car in terms of α, β and t.

Solution. (*i*) Let the car accelerate for time t_1 and v be the maximum velocity so attained in time t_1.

As $v = u + at$

$\therefore \quad v = 0 + \alpha t_1$

or $t_1 = \frac{v}{\alpha}$ ...(1)

Now, starting with the maximum velocity v, the car decelerates at constant rate β and comes to rest in time $(t - t_1)$. Therefore,

$0 = v - \beta(t - t_1)$ [$\because\ v = u + at$]

or $t - t_1 = \frac{v}{\beta}$ ...(2)

Adding equations (1) and (2), we get

$$t = \frac{v}{\alpha} + \frac{v}{\beta} = v\left(\frac{\alpha + \beta}{\alpha\beta}\right)$$

or $v = \frac{\alpha\beta\, t}{\alpha + \beta}$ ...(3)

This gives the maximum velocity attained by the car.

(*ii*) Distance covered by the car in time t_1 is

$$x_1 = 0 + \frac{1}{2}\alpha\, t_1^2 = \frac{1}{2}\alpha \,.\, \frac{v^2}{\alpha^2} = \frac{v^2}{2\alpha} \quad \text{[using (1)]}$$

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Distance covered by the car in time $(t - t_1)$ is

$$x_2 = v(t - t_1) - \frac{1}{2}\beta(t - t_1)^2$$

$$= v \cdot \frac{v}{\beta} - \frac{1}{2}\beta \cdot \frac{v^2}{\beta^2} = \frac{v^2}{2\beta} \quad \text{[using (2)]}$$

Total distance travelled by the car is

$$x = x_1 + x_2 = \frac{v^2}{2\alpha} + \frac{v^2}{2\beta} = \frac{v^2}{2}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$$

$$= \frac{\alpha^2\beta^2 t^2}{2(\alpha+\beta)^2} \cdot \frac{(\alpha+\beta)}{\alpha\beta} \quad \text{[using (3)]}$$

$$x = \frac{\alpha\beta t^2}{2(\alpha+\beta)}.$$

EXAMPLE 22. A body covers 12 m in 2nd second and 20 m in 4th second. How much distance will it cover in 4 seconds after the 5th second ? **[Chandigarh 03]**

Solution. Here $s_{2nd} = 12$ m, $s_{4th} = 20$ m

As $s_{n\,th} = u + \frac{a}{2}(2n - 1)$

$$\therefore \quad s_{2nd} = u + \frac{a}{2}(2 \times 2 - 1) = 12$$

or $$u + \frac{3}{2}a = 12 \qquad ...(i)$$

Also $$s_{4th} = u + \frac{a}{2}(2 \times 4 - 1) = 20$$

or $$u + \frac{7a}{2} = 20 \qquad ...(ii)$$

Subtracting (*i*) from (*ii*),

$$2a = 8 \quad \text{or} \quad a = 4 \text{ ms}^{-2}$$

From (*i*),

$$u + \frac{3}{2} \times 4 = 12 \quad \text{or} \quad u = 12 - 6 = 6 \text{ ms}^{-1}$$

Distance covered in 4 seconds after 5th second

$$= s_9 - s_5 \qquad [\text{Here } s_t = ut + \tfrac{1}{2}at^2]$$

$$= (6 \times 9 + \frac{1}{2} \times 4 \times 9^2) - (6 \times 5 + \frac{1}{2} \times 4 \times 5^2)$$

$$= 216 - 80 = \mathbf{136\ m}.$$

EXAMPLE 23. If x, y, z be the distances described by a particle during the pth, qth and rth second respectively, prove that : $(q - r)x + (r - p)y + (p - q)z = 0$

Solution. As $s_{nth} = u + \frac{a}{2}(2n - 1)$

$$\therefore \quad x = u + \frac{a}{2}(2p - 1) \qquad ...(1)$$

$$y = u + \frac{a}{2}(2q - 1) \qquad ...(2)$$

$$z = u + \frac{a}{2}(2r - 1) \qquad ...(3)$$

Subtracting (3) from (2),

$$y - z = \frac{a}{2}(2q - 2r) \quad \text{or} \quad q - r = \frac{y - z}{a}$$

Subtracting (1) from (3),

$$z - x = \frac{a}{2}(2r - 2p) \quad \text{or} \quad r - p = \frac{z - x}{a}$$

Subtracting (2) from (1),

$$x - y = \frac{a}{2}(2p - 2q) \quad \text{or} \quad p - q = \frac{x - y}{a}$$

Hence $(q - r)x + (r - p)y + (p - q)z$

$$= \frac{(y - z)x}{a} + \frac{(z - x)y}{a} + \frac{(x - y)z}{a}$$

$$= \frac{(xy - xz) + (yz - xy) + (xz - yz)}{a} = \mathbf{0}.$$

EXAMPLE 24. Two buses A and B are at positions 50 m and 100 m from the origin at time t = 0. They start moving in the same direction simultaneously with uniform velocity of 10 ms^{-1} and 5 ms^{-1}. Determine the time and position at which A overtakes B.

Solution. Here we use equation of motion for constant velocity in Cartesian form.

Given $x_1(0) = 50$ m, $x_2(0) = 100$ m,

$v_1 = 10 \text{ ms}^{-1}$, $v_2 = 5 \text{ ms}^{-1}$

The positions of the two buses at any instant t are

$$x_1(t) = x_1(0) + v_1 t = 50 + 10t$$

$$x_2(t) = x_2(0) + v_2 t = 100 + 5t$$

When A overtakes B,

$$x_1(t) = x_2(t)$$

$$50 + 10t = 100 + 5t$$

or $$5t = 50$$

$$t = \mathbf{10\ s}$$

$$x_1(10) = x_2(10) = \mathbf{150\ m}.$$

Thus A overtakes B at a position of 150 m from the origin at time $t = 10$ s.

EXAMPLE 25. An object is moving along +ve x-axis with a uniform acceleration of 4 ms^{-2}. At time $t = 0$, $x = 5$ m and $v = 3$ ms^{-1}.

(a) What will be the velocity and position of the object at time $t = 2$ s ?

(b) What will be the position of the object when it has a velocity of 5 ms^{-1} ?

Solution. Here we use equations of motion for constant acceleration in Cartesian form.

$$x(0) = 5 \text{ m}, \quad v(0) = 3 \text{ ms}^{-1}, \quad a = 4 \text{ ms}^{-2}$$

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(i) Velocity of the object at time $t = 2$ s is given by

$$v(t) = v(0) + at$$

$$v(2) = 3 + 4 \times 2 = 11 \text{ ms}^{-1}$$

Position of the object at time $t = 2$ s is given by

$$x(t) = x(0) + v(0)t + \frac{1}{2}at^2$$

$$x(2) = 5 + 3 \times 2 + \frac{1}{2} \times 4 \times (2)^2 = 19 \text{ m}.$$

(ii) The position $x(t)$ of the body when its velocity is 5 ms^{-1} is given by

$$2a[x(t) - x(0)] = v(t)^2 - v(0)^2$$

$$2 \times 4[x(t) - 5] = 5^2 - 3^2$$

or $$x(t) - 5 = \frac{16}{8}$$

or $$x(t) = 2 + 5 = 7 \text{ m}.$$

PROBLEMS FOR PRACTICE

1. A race car accelerates on a straight road from rest to a speed of 180 kmh^{-1} in 25 s. Assuming uniform acceleration of the car throughout, find the distance covered in this time. [Delhi 02]

(Ans. 625 m)

2. A bullet travelling with a velocity of 16 ms^{-1} penetrates a tree trunk and comes to rest in 0.4 m. Find the time taken during the retardation.

(Ans. 0.05 s)

3. A car moving along a straight highway with a speed of 72 kmh^{-1} is brought to a stop within a distance of 100 m. What is the retardation of the car and how long does it take for the car to stop ?

[Delhi 95, 05C]

(Ans. 2 ms^{-2}, 10 s)

4. On turning a corner a car driver driving at 36 kmh^{-1}, finds a child on the road 55 m ahead. He immediately applies brakes, so as to stop within 5 m of the child. Calculate the retardation produced and the time taken by the car to stop. (Ans. 1 ms^{-2}, 10 s)

5. The reaction time for an automobile driver is 0.6 s. If the automobile can be decelerated at 5 ms^{-2}, calculate the total distance travelled in coming to stop from an initial velocity of 30 kmh^{-1}, after a signal is observed. (Ans. 11.94 m)

6. A car starts from rest and accelerates uniformly for 10 s to a velocity of 8 ms^{-1}. It then runs at a constant velocity and is finally brought to rest in 64 m with a constant retardation. The total distance covered by the car is 584 m. Find the value of acceleration, retardation and total time taken.

(Ans. 0.8 ms^{-2}, 0.5 ms^{-2}, 86 s)

7. Two trains, one travelling at 72 kmh^{-1} and other at 90 kmh^{-1} are heading towards one another along a straight level track. When they are 1.0 km apart, both the drivers simultaneously see the other's train and apply brakes which retard each train at the rate of 1.0 ms^{-2}. Determine whether the trains would collide or not. (Ans. No)

8. A burglar's car had a start with an acceleration of 2 ms^{-2}. A police vigilant party came after 5 seconds and continued to chase the burglar's car with a uniform velocity of 20 ms^{-1}. Find the time in which the police van overtakes the burglar's car. (Ans. 5 s)

9. A ball rolls down an inclined track 2 m long in 4 s. Find (i) acceleration (ii) time taken to cover the second metre of the track and (iii) speed of the ball at the bottom of the track.

[Ans. (i) 0.25 ms^{-2} (ii) 1.17 s (iii) 1 ms^{-1}]

10. A bus starts from rest with a constant acceleration of 5 ms^{-2}. At the same time a car travelling with a constant velocity of 50 ms^{-1} overtakes and passes the bus. (i) Find at what distance will the bus overtake the car ? (ii) How fast will the bus be travelling then ? [Ans. (i) 1000 m (ii) 100 ms^{-1}]

11. A body starting from rest accelerates uniformly at the rate of 10 cms^{-2} and retards uniformly at the rate of 20 cms^{-2}. Find the least time in which it can complete the journey of 5 km if the maximum velocity attained by the body is 72 kmh^{-1}.

(Ans. 400 s)

12. A body covers a distance of 20 m in the 7th second and 24 m in the 9th second. How much shall it cover in 15th s ? (Ans. 36 m)

13. A body covers a distance of 4 m is 3rd second and 12 m in 5th second. If the motion is uniformly accelerated, how far will it travel in the next 3 seconds ? (Ans. 60 m)

14. An object is moving with uniform acceleration. Its velocity after 5 seconds is 25 ms^{-1} and after 8 seconds, it is 34 ms^{-1}. Find the distance travelled by the object in 12th second. (Ans. 44.5 m)

HINTS

1. Here $u = 0$, $t = 25$ s,

$$v = 180 \text{ kmh}^{-1} = \frac{180 \times 5}{18} = 50 \text{ ms}^{-1}$$

Acceleration,

$$a = \frac{v - u}{t} = \frac{50 - 0}{25} = 2 \text{ ms}^{-2}$$

Distance,

$$s = ut + \frac{1}{2}at^2 = 0 \times 25 + \frac{1}{2} \times 2 \times (25)^2$$

$$= 625 \text{ m}.$$

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Here $u = 16\ \text{ms}^{-1}, v = 0,\ s = 0.4\ \text{m}, t = ?$

As $v^2 - u^2 = 2as$

$0^2 - 16^2 = 2a \times 0.4$

$a = -\dfrac{16 \times 16}{2 \times 0.4} = -320\ \text{ms}^{-2}$

or $t = \dfrac{v-u}{a} = \dfrac{0-16}{-320} = \mathbf{0.05\ s.}$

Time,

Distance covered in $0.6\ \text{s} = 30 \times \dfrac{5}{18} \times 0.6 = 5.0\ \text{m}$

During deceleration,

$v^2 - u^2 = 2as$

$0^2 - \left(\dfrac{25}{3}\right)^2 = 2 \times (-5)\, s$

or $s = 6.94\ \text{m}$

or Total distance travelled $= 5.0 + 6.94 = \mathbf{11.94\ m.}$

8. Suppose the police van overtakes the burglar's car in time t second after it starts chasing.

Distance covered by police van in t seconds $= 20t$

Distance covered by burglar's car in $(t + 5)$ seconds,

$s = u(t+5) + \frac{1}{2} a (t+5)^2$

$= 0(t+5) + \frac{1}{2} \times 2 (t+5)^2 = (t+5)^2$

$20t = (t+5)^2$

Now $t^2 - 10t + 25 = 0$

or $(t-5)^2 = 0$

or $t = \mathbf{5\ s.}$

$\therefore$

9. (i) Here $s = 2\ \text{m},\ t = 4\ \text{s},\ u = 0$

As $s = ut + \frac{1}{2} at^2$

$\therefore\ 2 = 0 + \frac{1}{2} \times a \times 4^2$

or $a = \mathbf{0.25\ ms^{-2}.}$

(ii) Time taken to cover first one metre of the track is given by

$s = ut + \frac{1}{2} at^2$

$1 = 0 + \frac{1}{2} \times 0.25 \times t^2 \quad \Rightarrow \quad t^2 = 8$

$\therefore\ t = 2\sqrt{2} = 2 \times 1.414 = 2.828 = 2.83\ \text{s}$

Hence time taken to cover the second metre

$= 4 - 2.83 = \mathbf{1.17\ s.}$

(iii) Speed at the bottom,

$v = u + at = 0 + 0.25 \times 4 = \mathbf{1\ ms^{-1}.}$

10. (i) Suppose the bus overtakes the car after covering distance s.

When the two meet, time taken t is same.

For bus, $s = ut + \frac{1}{2} at^2 = 0 + \frac{1}{2} \times 5t^2$

For car, $s = 50t$

$\therefore\ \dfrac{5}{2} t^2 = 50t$ or $t = 20\ \text{s}$

Hence $s = 50t = 50 \times 20 = \mathbf{1000\ m.}$

(ii) $v^2 = u^2 + 2as = 0 + 2 \times 5 \times 1000 = 10{,}000$

or $v = \mathbf{100\ ms^{-1}.}$

11. (i) *For motion with uniform acceleration :*

$u = 0,\ v = 72\ \text{kmh}^{-1} = 20\ \text{ms}^{-1},$

$a = 10\ \text{cms}^{-2} = 0.1\ \text{ms}^{-2},\ t = t_1 = ?,\ s = ?$

As $v = u + at$

$\therefore\ 20 = 0 + 0.1 \times t_1$ or $t_1 = 200\ \text{s}$

Also, $v^2 = u^2 + 2as$

$\therefore\ 20^2 = 0^2 + 2 \times 0.1 \times s$ or $s = 2000\ \text{m} = 2\ \text{km}$

(ii) *For motion with uniform retardation :*

$u = 20\ \text{ms}^{-1},\ v = 0,$

$a = -20\ \text{cms}^{-2} = -0.2\ \text{ms}^{-2},\ t = t_2 = ?\ \ s = ?$

As $v = u + at$

$\therefore\ 0 = 20 - 0.2 \times t_2$ or $t_2 = 100\ \text{s}$

Again, $v^2 = u^2 + 2as$

$\therefore\ 0 = 20^2 + 2 \times (-0.2)\, s$

or $s = 1000\ \text{m} = 1\ \text{km}$

Remaining part of journey

$= 5 - (2 + 1) = 2\ \text{km} = 2000\ \text{m}$

This journey occurs at the uniform maximum velocity of $20\ \text{ms}^{-1}$.

(iii) *For motion with uniform velocity :*

$u = 20\ \text{ms}^{-1},\ s = 2000\ \text{m},\ t = t_3 = ?$

Time, $t_3 = \dfrac{s}{u} = \dfrac{2000}{20} = 100\ \text{s}$

Total time $= t_1 + t_2 + t_3 = 200 + 100 + 100 = \mathbf{400\ s.}$

12. Here $s_{7th} = u + \dfrac{a}{2}(2 \times 7 - 1) = 20$

or $u + \dfrac{13}{2} a = 20\ \text{m}$ ...(i)

$s_{9th} = u + \dfrac{a}{2}(2 \times 9 - 1) = 24\ \text{m}$

or $u + \dfrac{17}{2} a = 24$ ...(ii)

Subtracting (i) from (ii), we get

$2a = 4$ or $a = 2\ \text{ms}^{-2}$

From (i),

$u + \dfrac{13}{2} \times 2 = 20$ or $u = 7\ \text{ms}^{-1}.$

Hence,

$s_{15th} = 7 + \dfrac{2}{2}(2 \times 15 - 1) = 7 + 29 = \mathbf{36\ m.}$

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13. $s_{3rd} = u + \frac{a}{2}(2 \times 3 - 1) = 4$

or $u + \frac{5}{2}a = 4$

$s_{5th} = u + \frac{a}{2}(2 \times 5 - 1) = 12$

or $u + \frac{9}{2}a = 12$

On solving,

$u = -6\ \text{ms}^{-1}$, $a = 4\ \text{ms}^{-2}$

Distance travelled in next 3 seconds

$$= s_8 - s_5 = \left[-6 \times 8 + \frac{1}{2} \times 4 \times (8)^2\right] - \left[-6 \times 5 + \frac{1}{2} \times 4 \times (5)^2\right]$$

$= 80 - 20 = \mathbf{60\ m}$.

14. As $v = u + at$

$\therefore$ In first case, $25 = u + 5a$

In second case, $34 = u + 8a$

On solving,

$u = 10\ \text{ms}^{-1}$, $a = 3\ \text{ms}^{-2}$

$s_{12th} = 10 + \frac{3}{2}(2 \times 12 - 1)$

$= 10 + 34.5 = \mathbf{44.5\ m}$.

3.12 MOTION UNDER GRAVITY

22. *Discuss the motion under free fall. Write the equations of motion for a freely falling body.*

Free fall. A body released near the surface of the earth is accelerated downward under the influence of force of gravity. In the absence of air resistance, all bodies fall with the same acceleration near the surface of the earth. This motion of body falling towards the earth from a small height is called *free fall*. The acceleration with which a body falls is called acceleration due to gravity and is denoted by g.

Near the surface of the earth, $g = 9.8\ \text{ms}^{-2}$

Equations of motion for a freely falling body. For a freely falling body, the following equations of motion hold good :

$$(i)\ v = u + gt \quad (ii)\ s = ut + \frac{1}{2}gt^2 \quad (iii)\ v^2 - u^2 = 2gs$$

When a body falls freely under the action of gravity, it velocity increases and the value of g is taken positive.

When a body is thrown vertically upward, its velocity decreases and the value of g is taken negative.

Examples based on Motion under Gravity

FORMULAE USED

1. For a freely falling body, the equations of motion are

   $(i)\ v = u + gt$ $(ii)\ s = ut + \frac{1}{2}gt^2$ $(iii)\ v^2 - u^2 = 2gs$

2. For a body falling freely under the action of gravity, g is taken *positive*.
3. For a body thrown vertically upward, g is taken *negative*.
4. When a body is just dropped, $u = 0$
5. For a body thrown vertically up with initial velocity u,

   (i) Maximum height reached, $h = \frac{u^2}{2g}$

   (ii) Time of ascent = Time of descent $= \frac{u}{g}$

   (iii) Total time of flight $= \frac{2u}{g}$

   (iv) Velocity of fall at the point of projection $= u$

   (v) Velocity attained by a body dropped from height h, $v = \sqrt{2gh}$.

UNITS USED

Velocities u, v are in ms^{-1}, acceleration due to gravity g is in ms^{-2}, distances s and h are in metres.

EXAMPLE 26. A ball thrown vertically upwards with a speed of 19.6 ms^{-1} from the top of a tower returns to the earth in 6 s. Find the height of the tower.

Solution. Here : $u = 19.6\ \text{ms}^{-1}$, $g = -9.8\ \text{ms}^{-2}$, $t = 6\ \text{s}$

Net displacement,

$$s = -h$$

Negative sign is taken because displacement is in the opposite direction of initial velocity.

As $s = ut + \frac{1}{2}gt^2$

$\therefore -h = 19.6 \times 6 + \frac{1}{2} \times (-9.8) \times 6^2$

$= 117.6 - 176.4 = -58.8$

$h = \mathbf{58.8\ m}$.

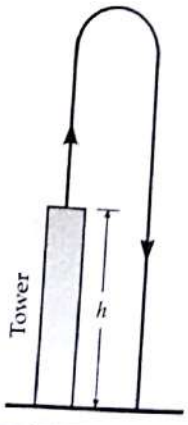

Fig. 3.18

EXAMPLE 27. A ball is thrown vertically upwards with a velocity of 20 ms^{-1} from the top of a multistoreyed building. The height of the point from where the ball is thrown is 25 m

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...the ground. (i) How high will the ball rise ? (ii) How ...s will it be before the ball hits the ground ? (iii) Trace the ...ectory of this ball. [Central Schools 08, NCERT]

Solution. (i) Here : $u = +20\ \text{ms}^{-1}$, $g = -10\,\text{ms}^{-2}$

At the highest point, $v = 0$

Suppose the ball rises to the height h from the point of projection.

As $v^2 - u^2 = 2gs$

$0^2 - 20^2 = 2 \times (-10) \times h$

$h = +20$ m.

(ii) Net displacement, $s = -25$ m

Negative sign is taken because displacement is in the opposite direction of initial velocity.

As $s = ut + \frac{1}{2} gt^2$

$-25 = 20\,t + \frac{1}{2} \times (-10) \times t^2$

$5t^2 - 20\,t - 25 = 0$

or $t^2 - 4t - 5 = 0$

or $(t+1)(t-5) = 0$

or As $t \neq -1$, so $t = 5$ s.

(iii) The trajectory of the ball will be a vertical line.

$y = ut + \frac{1}{2} gt^2$, $x = 0$, $y \neq 0$.

EXAMPLE 28. A ball thrown up is caught by the thrower after 4 s. How high did it go and with what velocity was it thrown ? How far was it below the highest point 3 s after it was thrown ?

Solution. As time of ascent = time of descent

∴ Time taken by the ball to reach the highest point = 2 s

For upward motion of the ball :

$u = ?$, $v = 0$, $t = 2$s, $g = -9.8\ \text{ms}^{-2}$

As $v = u + gt$

$0 = u - 9.8 \times 2$

$u = \mathbf{19.6\ ms^{-1}}$.

or Maximum height attained by the ball is given by

$s = ut + \frac{1}{2} gt^2$

$= 19.6 \times 2 + \frac{1}{2} \times (-9.8) \times 2^2$

$= \mathbf{19.6\ m}$.

Distance of the ball from the highest point 3 s after it was thrown

= Distance covered by the ball in 1s from the highest point

$= 0 \times 1 + \frac{1}{2} \times 9.8 \times (1)^2$

$= \mathbf{4.9\ m}$.

EXAMPLE 29. A balloon is ascending at the rate of 9.8 ms^{-1} at a height of 39.2 m above the ground when a food packet is dropped from the balloon. After how much time and with what velocity does it reach the ground ? Take $g = 9.8\ ms^{-2}$.

Solution. Initially the food packet shares the upward velocity of the balloon, so

$u = 9.8\ \text{ms}^{-1}$, $g = -9.8\ \text{ms}^{-2}$, $s = -39.2$ m

Here s is taken negative because it is in the opposite direction of initial velocity.

Using, $s = ut + \frac{1}{2} gt^2$, we get

$-39.2 = 98\,t - \frac{1}{2} \times 9.8\,t^2$

or $4.9\,t^2 - 9.8\,t - 39.2 = 0$ or $t^2 - 2t - 8 = 0$

or $(t-4)(t+2) = 0$ or $t = 4$ s or -2 s

As time is never negative, so $t = 4$ s.

Velocity with which the food packet reaches the ground is

$v = u + gt = 9.8 - 9.8 \times 4 = \mathbf{-29.4\ ms^{-1}}$.

Negative sign shows that the velocity is directed vertically downwards.

EXAMPLE 30. A food packet is released from a helicopter which is rising steadily at 2 ms^{-1}. After two seconds (i) What is the velocity of the packet ? (ii) How far is it below the helicopter ? Take $g = 9.8\ ms^{-2}$.

Solution. Here $u = 2\ \text{ms}^{-1}$,

$g = -9.8\ \text{ms}^{-2}$, $t = 2$ s

(i) $v = u + gt = 2 - 9.8 \times 2 = \mathbf{-17.6\ ms^{-1}}$.

Negative sign shows that the velocity is directed vertically downwards.

(ii) Distance covered by the food packet in 2 s,

$s = ut + \frac{1}{2} at^2 = 2 \times 2 - \frac{1}{2} \times 9.8 \times 2^2$

$= 4 - 19.6 = -15.6$ m

Thus the food packet falls through a distance of 15.6 m in 2 s but in the mean time the helicopter rises up through a distance

$= 2\ \text{ms}^{-1} \times 2\ \text{s} = 4$ m

∴ After 2 s, the distance of the food packet from the helicopter

$= 15.6 + 4 = \mathbf{19.6\ m}$.

EXAMPLE 31. A parachutist bails out from an aeroplane and after dropping through a distance of 40 m, he opens the parachute and decelerates at 2 ms^{-2}. If he reaches the ground with a speed of 2 ms^{-1}, how long is he in the air ? At what height did he bail out from the plane ? [AIEEE 05]

Solution. *When the parachutist falls freely :*

$u = 0$, $g = 9.8\ \text{ms}^{-2}$, $s = 40$ m, $t = ?$, $v = ?$

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As $s = ut + \frac{1}{2}gt^2$

$\therefore \quad 40 = 0 - \frac{1}{2} \times 9.8 \times t^2$

or $\quad t = \sqrt{\frac{80}{9.8}} = \frac{20}{7}\ s = 2.86\ s$

Also, $\quad v = u + gt = 0 + 9.8 \times \frac{20}{7} = 28\ ms^{-1}$

When the parachutist decelerates uniformly :

$u = 28\ ms^{-1},\ a = -2\ ms^{-2},\ v = 2\ ms^{-1}$

Time taken,

$$t = \frac{v-u}{a} = \frac{2-28}{-2} = 13\ s$$

Distance, $s = ut + \frac{1}{2}at^2 = 28 \times 13 - \frac{1}{2} \times 2 \times (13)^2$

$= 364 - 169 = 195\ m$

Total time of parachutist in air

$= 2.86 + 13 = \mathbf{15.86\ s}$

Height at which parachutist bails out

$= 40 + 195 = \mathbf{235\ m.}$

EXAMPLE 32. Two balls are thrown simultaneously, A vertically upwards with a speed of 20 ms^{-1} from the ground, and B vertically downwards from a height of 40 m with the same speed and along the same line of motion. At what points do the two balls collide ? Take $g = 9.8\ ms^{-2}$.

Solution. Suppose the two balls meet at a height of x metre from the ground after time t s from the start.

For upward motion of ball A :

$u = 20\ ms^{-1},\ g = -9.8\ ms^{-2}$

$s = ut + \frac{1}{2}gt^2$

$x = 20\,t - \frac{1}{2} \times 9.8t^2$

$= 20\,t - 4.9\,t^2 \quad ...(i)$

For downward motion of ball B :

$40 - x = 20 \times t + \frac{1}{2} \times 9.8\,t^2$

$= 20t + 4.9\,t^2 \quad ...(ii)$

Adding (*i*) and (*ii*),

$40 = 40\,t \quad$ or $\quad t = \mathbf{1\ s}$

From (*i*), $\quad x = 20 \times 1 - 4.9 \times (1)^2 = \mathbf{15.1\ m}$

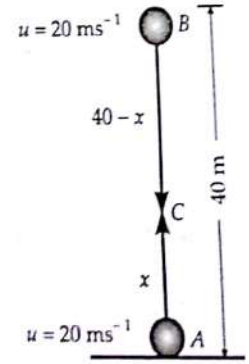

Fig. 3.19

Hence the two balls will collide after 1 s at a height of 15.1 m from the ground.

EXAMPLE 33. A tennis ball is dropped on to the floor from a height of 4 m. It rebounds to a height of 3 m. If the ball was in contact with the floor for 0.01 s, what was its average acceleration during contact ?

Solution. *For downward motion of the ball.* Suppose the ball falls from height h_1 and strikes the floor with velocity v_1. Then

$u = 0, \quad v = v_1, \quad a = +g, \quad s = h_1$

As $\quad v^2 - u^2 = 2as$

$\therefore \quad v_1^2 - 0^2 = 2gh_1 \quad$ or $\quad v_1 = \sqrt{2gh_1}$

For upward motion of the ball. Suppose the ball gets rebounded from the floor with velocity v_2 and rises to height h_2. Then

$u = v_2, \quad v = 0, \quad a = -g, \quad s = h_2$

As $\quad v^2 - u^2 = 2as$

$\therefore \quad 0^2 - v_2^2 = 2 \times (-g)\,h_2 \quad$ or $\quad v_2 = \sqrt{2gh_2}$

Average acceleration,

$$a = \frac{\Delta v}{\Delta t} = \frac{v_2 - (-v_1)}{\Delta t}$$

$$= \frac{v_2 + v_1}{\Delta t} = \frac{\sqrt{2gh_2} + \sqrt{2gh_1}}{\Delta t}$$

But $\quad h_1 = 4m,\ h_2 = 3m,\ \Delta t = 0.01\,s,\ g = 9.8\ ms^{-2}$

$\therefore \quad a = \dfrac{\sqrt{2 \times 9.8 \times 3} + \sqrt{2 \times 9.8 \times 4}}{0.01} = \mathbf{1652\ ms^{-2}}.$

EXAMPLE 34. A stone falls from a cliff and travels 24.5 m in the last second before it reaches the ground at the foot of the cliff. Find the height of the cliff.

Solution. Here $u = 0,\ g = 9.8\ ms^{-2}$

Let nth second be the last second. Then

$$s_{nth} = u + \frac{g}{2}(2n-1)$$

or $\quad 24.5 = 0 + \frac{9.8}{2}(2n-1)$

or $\quad 2n - 1 = \frac{24.5}{4.9} = 5$

or $\quad n = 3 \quad$ or $\quad t = 3s$

Distance, $\quad s = ut + \frac{1}{2}gt^2 = 0 + \frac{1}{2} \times 9.8 \times 3^2 = \mathbf{44.1\ m}$

Hence the height of cliff is 44.1 m.

PROBLEMS FOR PRACTICE

1. A stone is thrown vertically upwards with a velocity of 4.9 ms^{-1}. Calculate (*i*) the maximum height reached (*ii*) the time taken to reach the maximum height (*iii*) the velocity with which it returns to the ground and (*iv*) the time taken to reach the ground.

   [**Ans.** (*i*) 1.225 m (*ii*) 0.5 s (*iii*) 4.9 ms^{-1} (*iv*) 1 s]

2. A stone thrown upwards from the top of a tower 85 m high, reaches the ground in 5 s. Find (*i*) the

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greatest height above the ground (*ii*) the velocity with which it reaches the ground and (*iii*) the time taken to reach the maximum height. Take $g = 10\ ms^{-2}$. [**Ans.** (*i*) 88.2 m (*ii*) 42 ms^{-1} (*iii*) 0.8 s]

3. From the top of a multi-storeyed building, 39.2 m tall, a boy projects a stone vertically upwards with an initial velocity of 9.8 ms^{-1} such that it finally drops to the ground. (*i*) When will the stone reach the ground ? (*ii*) When will it pass through the point of projection ? (*iii*) What will be its velocity before striking the ground ? Take $g = 9.8\ ms^{-2}$.

(**Ans.** 4s, 2s, 29.4 ms^{-1})

4. A rocket is fired vertically from the ground with a resultant vertical acceleration of 10 ms^{-2}. The fuel is finished in 1 minute and it continues to move up. What is the maximum height reached ?

(**Ans.** 36.4 km)

5. A balloon is ascending at the rate of 14 ms^{-1} at a height of 98 m above the ground when the food packet is dropped from the balloon. After how much time and with what velocity does it reach the ground ? Take $g = 9.8\ ms^{-2}$.

(**Ans.** 6.12 s, 45.98 ms^{-1})

6. A stone is dropped from a balloon rising upwards with a velocity of 16 ms^{-1}. The stone reaches the ground in 4 s. Calculate the height of the balloon when the stone was dropped. (**Ans.** 14.4 m)

7. From the top of a tower 100 m in height a ball is dropped and at the same time another ball is projected vertically upwards from the ground with velocity of 25 ms^{-1}. Find when and where the two balls will meet. Take $g = 9.8\ ms^{-2}$. **[Delhi 05]**

(**Ans.** 78.4 from top, 4 s)

8. A body is dropped from rest at a height of 150 m, and simultaneously, another body is dropped from rest from a point 100 m above the ground. What is their difference in height after they have fallen (*i*) 2 s (*ii*) 3 s ? How does the difference in height vary with time ? Take $g = 10\ ms^{-2}$.

(**Ans.** 50 m, difference in height remains constant at 50 m)

9. A body falling freely under gravity passes two points 30 m apart in 1s. Find from what point above the upper point it began to fall ? Take $g = 9.8\ ms^{-2}$.

(**Ans.** 32.1 m)

10. Four balls are dropped from the top of a tower at intervals of one-one second. The first ball reaches the ground after 4 s of dropping. What are the distances between first and second, second and third, third and fourth balls at this instant ?

(**Ans.** 34.3 m, 24.5 m, 14.7 m)

✠ HINTS

2. Net displacement, $s = -85$ m, $t = 5$ s, $g = -10\ ms^{-2}$

As $s = ut + \frac{1}{2}gt^2$

$\therefore \quad -85 = u \times 5 - \frac{1}{2} \times 10 \times 25$

or $u = 8\ ms^{-1}$.

(*i*) Height covered in upward motion is given by

$v^2 = u^2 + 2gs$

or $0 = 8^2 + 2 \times (-10)\, s$

or $s = 3.2$ m

$\therefore$ The greatest height above the ground

$= 85 + 3.2 = 88.2$.

(*ii*) *For the downward motion* :

$u = 0, \quad g = +10\ ms^{-2}, \quad s = 88.2$ m

$v^2 = u^2 + 2gs = 0 + 2 \times 10 \times 88.2 = 1764$

$\therefore$ Velocity on reaching the ground, $v = \mathbf{42\ ms^{-1}}$.

(*iii*) Let t be the time taken to reach the maximum height.

For upward motion,

$u = 8\ ms^{-1}, \quad v = 0, \quad g = -10\ ms^{-2}$

$\therefore \quad t = \dfrac{v-u}{g} = \dfrac{0-8}{-10} = \mathbf{0.8\ s.}$

4. Height covered in 1 min,

$s_1 = ut + \frac{1}{2}gt^2 = 0 + \frac{1}{2} \times 10 \times (60)^2 = 18000$ m

Velocity attained after 1 min,

$v = u + at = 0 + 10 \times 60 = 600\ ms^{-1}$

After the fuel is finished, $u = 600\ ms^{-1}, \quad v = 0,$

$v^2 - u^2 = 2gs$

or $0 - (600)^2 = 2 \times (-9.8) \times s_2$

or $s_2 = \dfrac{(600)^2}{2 \times 9.8} = 18367.3$ m

Maximum height reached

$= s_1 + s_2 = 36367.3$ m $= \mathbf{36.4\ km.}$

6. Here $u = 16\ ms^{-1}, \ g = -9.8\ ms^{-2}, \ t = 4$ s, $\ s = -h$

$s = ut + \frac{1}{2}gt^2$

$-h = 16 \times 4 - \frac{1}{2} \times 9.8 \times 16 = -14.4 \quad$ or $\quad h = \mathbf{14.4\ m.}$

8. *For downward motion of the first body* :

$u = 0, \quad t = 2$ s, $\quad g = 10\ ms^{-2}$

$s = ut + \frac{1}{2}gt^2 = 0 \times 2 + \frac{1}{2} \times 10 \times 4 = 20$ m

Height from the ground, $h_1 = 150 - 20 = 130$ m

For downward motion of the second body :

$u = 0, \quad t = 2$ s, $\quad g = 10\ ms^{-2}$.

$s = 0 \times 2 + \frac{1}{2} \times 10 \times 4 = 20$ m

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Height from the ground, $h_2 = 100 - 20 = 80$ m

Difference in heights $= h_1 - h_2 = 130 - 80 = 50$ m

Similarly at $t = 3$ s, difference in heights = 50 m

Hence the difference in heights does not depend on time so long as both the bodies are in air.

9. Suppose the body passes the upper point at t second and lower point at $(t + 1)$ s, then

$$s_2 - s_1 = \frac{1}{2} g (t + 1)^2 - \frac{1}{2} gt^2 = \frac{1}{2} g (2t + 1)$$

or $\quad 30 \text{ m} = \frac{1}{2} \times 9.8 (2t + 1)$

or $\quad t = 2.56$ s

$\therefore \quad s_1 = \frac{1}{2} gt^2 = \frac{1}{2} \times 9.8 \times (2.56)^2 = \mathbf{32.1 \text{ m}}.$

10. Here we need to find the distances travelled by the first ball in 4 s, by the second ball in 3 s, by the third ball in 2 s and by the fourth ball in 1 s.

$$h_1 = \frac{1}{2} g (4)^2 = 8 \times 9.8 = 78.4 \text{ m};$$

$$h_2 = \frac{1}{2} g (3)^2 = \frac{9}{2} \times 9.8 = 44.1 \text{ m};$$

$$h_3 = \frac{1}{2} g (2)^2 = 2 \times 9.8 = 19.6 \text{ m};$$

$$h_4 = \frac{1}{2} g (1)^2 = \frac{9.8}{2} = 4.9 \text{ m}.$$

$\therefore$ Distance between first and second balls is

$$h_1 - h_2 = 78.4 - 44.1 = \mathbf{34.3 \text{ m}}$$

Distance between second and third balls is

$$h_2 - h_3 = 44.1 - 19.6 = \mathbf{24.5 \text{ m}}$$

Distance between third and fourth balls is

$$h_3 - h_4 = 19.6 - 4.9 = \mathbf{14.7 \text{ m}}.$$

3.13 POSITION-TIME GRAPHS

23. *Draw the position-time graph for a stationary object.*

Position-time graph for a stationary object. The position of a stationary object does not change with time. The object remains at a constant distance $x = x_0$ from the origin at all times. So the position-time $(x - t)$ graph for a stationary object is a straight line parallel to the time-axis, as shown in Fig. 3.20.

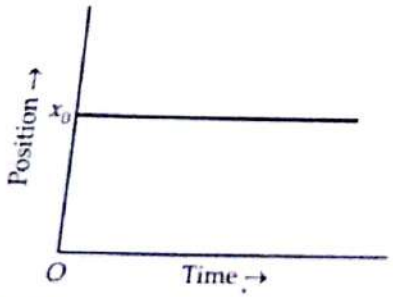

Fig. 3.20 Position-time graph for a stationary object.

24. *Draw the position-time graph for an object in uniform motion. Show that the slope of the position-time graph gives the velocity of the object.*

Position-time graph for uniform motion. An object in uniform motion covers equal distances in equal intervals of time. So the *position-time graph for an object in uniform motion along a straight line path is a straight line inclined to the time-axis*, as shown in Fig. 3.21.

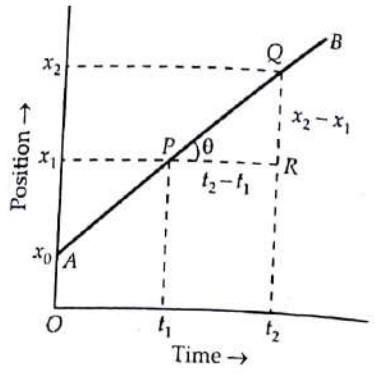

Fig. 3.21. Position-time graph for uniform motion.

Slope of position-time graph AB

$$= \tan \theta = \frac{QR}{PR} = \frac{x_2 - x_1}{t_2 - t_1}$$

$$= \frac{\text{Displacement}}{\text{Time}} = \text{Velocity } (v).$$

Hence the slope of the position-time graph gives velocity of the object.

25. *Draw the position-time graph for uniformly accelerated motion. What does its slope give ?*

Position-time graph for uniformly accelerated motion. The position-time relation for uniformly accelerated motion along a straight line is

$$x = x_0 + v_0 t + \frac{1}{2} at^2$$

Clearly, $x \propto t^2$ *i.e.*, x is a quadratic function of t. So the *position-time graph for uniformly accelerated motion is a parabola*, as shown in Fig. 3.22.

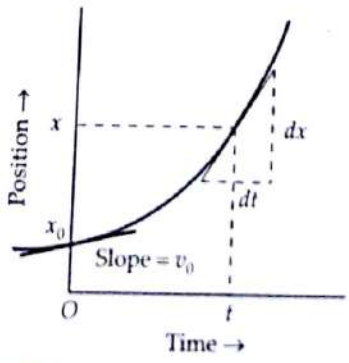

Fig. 3.22 Position-time graph for uniform acceleration.

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slope of position-time graph

$= \frac{\text{small change in vertical coordinate}}{\text{small change in horizontal coordinate}}$

$= \frac{dx}{dt}$ = velocity at instant t

Thus the slope of the position-time graph gives the instantaneous velocity of the object. Moreover, the slope of the x-t graph at time $t = 0$ gives the initial velocity v_0 of the object.

3.14 VELOCITY-TIME GRAPHS

26. *Draw the velocity-time graph for an object in uniform motion. Show that the area under the velocity-time graph gives the displacement of the object in the given time interval.*

Velocity-time graph for uniform motion. When an object has uniform motion, it moves with uniform velocity v in the same fixed direction. So *the velocity-time graph for uniform motion is a straight line parallel to the time-axis*, as shown in Fig. 3.23

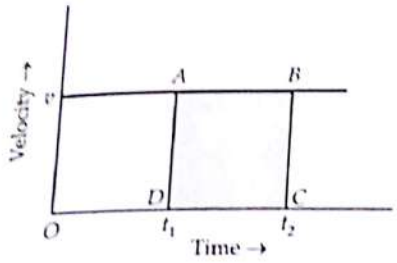

Fig. 3.23 Velocity-time graph for uniform motion.

Area under the velocity-time graph between times t_1 and t_2

= Area of rectangle $ABCD$

$= AD \times DC = v(t_2 - t_1)$

= Velocity × time

= Displacement

Hence *the area under the velocity-time graph gives the displacement of the object in the given time interval.*

27. *Draw the velocity-time graph for uniformly accelerated motion. Show that slope of the velocity-time graph gives acceleration of the object.*

Velocity-time graph for uniformly accelerated motion. When a body moves with a uniform acceleration, its velocity changes by equal amounts in equal intervals of time. So the *velocity-time graph for a uniformly accelerated motion is a straight line inclined to the time-axis*, as shown in Fig. 3.24.

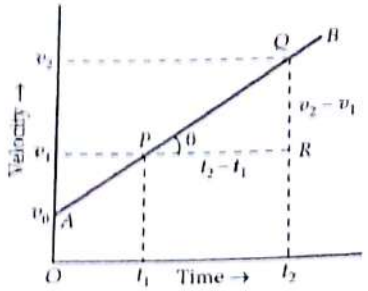

Fig. 3.24 Velocity-time graph for uniform acceleration.

Slope of velocity-time graph AB

$= \tan\theta = \frac{QR}{PR} = \frac{v_2 - v_1}{t_2 - t_1}$

$= \frac{\text{Change in velocity}}{\text{Time interval}}$

= Acceleration (a)

Hence the *slope of the velocity-time graph gives the acceleration of the object.*

28. *Show that the area under the velocity-time graph of an object moving with constant acceleration in a straight line in certain time interval is equal to the distance covered by the object in that interval.*

Distance covered as area under the velocity-time graph. In Fig. 3.25, the straight line AB is the velocity-time graph of an object moving along a straight line path with uniform acceleration a. Let its velocities be v_0 and v at times 0 and t respectively.

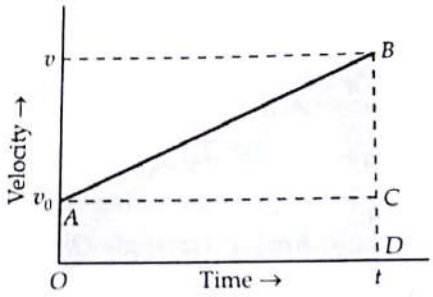

Fig. 3.25. Area under velocity-time graph.

Area under the velocity-time graph AB

= Area of trapezium $OABD$

$= \frac{1}{2}(OA + BD) \times OD = \frac{1}{2}(v_0 + v) \times (t - 0)$

= Average velocity × time interval

= Distance travelled in time t

Hence *the area under the velocity-time graph gives the distance travelled by the object in the given time interval.*

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3.15 DERIVATION OF EQUATIONS OF MOTION BY GRAPHICAL METHOD

29. *Derive the following equations of motion for uniformly accelerated motion from velocity-time graph :*

(i) $v = u + at$ *(ii)* $s = ut + \frac{1}{2}at^2$ *(iii)* $v^2 - u^2 = 2as$.

Equations of motion by graphical method. Consider an object moving along a straight line path with initial velocity u and uniform acceleration a. Suppose it travels distance s in time t. As shown in Fig. 3.26, its velocity-time graph is straight line. Here $OA = ED = u$, $OC = EB = v$ and $OE = t = AD$.

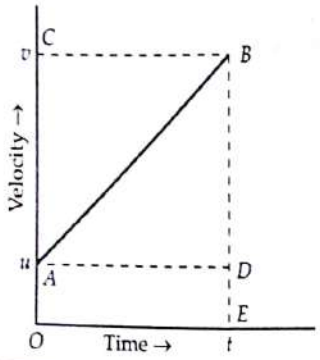

Fig. 3.26 Velocity-time graph for uniform acceleration.

(i) We know that,

Acceleration = Slope of velocity-time graph AB

or $$a = \frac{DB}{AD} = \frac{DB}{OE} = \frac{EB - ED}{OE} = \frac{v - u}{t}$$

or $$v - u = at \quad \text{or} \quad v = u + at$$

This proves the *first equation of motion.*

(ii) From part *(i)*, we have

$$a = \frac{DB}{AD} = \frac{DB}{t} \quad \text{or} \quad DB = at$$

Distance travelled by the object in time t is

s = Area of the trapezium $OABE$

= Area of rectangle $OADE$ + Area of triangle ADB

$$= OA \times OE + \frac{1}{2} DB \times AD$$

$$= ut + \frac{1}{2} at \times t \quad \text{or} \quad s = ut + \frac{1}{2}at^2$$

This proves the *second equation of motion.*

(iii) Distance travelled by object in time t is

s = Area of trapezium $OABE$.

$$= \frac{1}{2}(EB + OA) \times OE = \frac{1}{2}(EB + ED) \times OE$$

Acceleration,

a = Slope of velocity-time graph AB

or $$a = \frac{DB}{AD} = \frac{EB - ED}{OE} \quad \text{or} \quad OE = \frac{EB - ED}{a}$$

$$\therefore \quad s = \frac{1}{2}(EB + ED) \times \frac{(EB - ED)}{a}$$

$$= \frac{1}{2a}(EB^2 - ED^2) = \frac{1}{2a}(v^2 - u^2)$$

or $$v^2 - u^2 = 2as$$

This proves the *third equation of motion.*

For Your Knowledge

- A straight line graph has a *single slope*. So if the displacement-time graph is a straight line, it represents a constant velocity. If the velocity-time graph is a straight line, it represents a constant acceleration.
- A curved graph has *multiple slopes*. In Fig. 3.27, as the displacement-time graph bends upwards with the passage of time, the value of θ increases, slope (= tan θ) of the curve increases, consequently the velocity increases and hence the motion is accelerated.

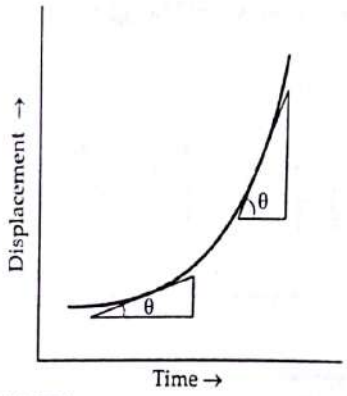

Fig. 3.27 Displacement-time graph for accelerated motion.

- In Fig. 3.28, as the displacement-time graph bends downwards with the passage of time, the value of θ decreases, the slope of the curve decreases, consequently the velocity decreases and hence the motion is decelerated.

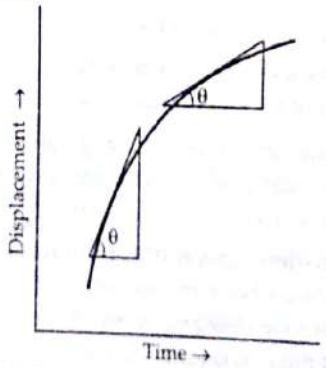

Fig. 3.28 Displacement-time graph for decelerated motion.

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3.16 ANALYSING NATURE OF MOTION FROM VARIOUS GRAPHS

30. *Discuss the nature of motion from the given distance-time graphs.*

Different Types of Distance-Time Graphs :

(*i*) For a body at rest, the distance-time graph is a straight line *AB*, as shown in Fig. 3.29. As the slope of *AB* is zero, so speed of the body is zero.

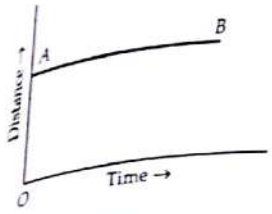

Fig. 3.29

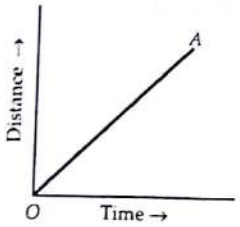

Fig. 3.30

(*ii*) For a body moving with uniform speed, the distance-time graph is a straight line inclined to the time-axis, as shown in Fig. 3.30. As the graph passes through *O*, so distance travelled at $t = 0$ is also zero.

(*iii*) The distance-time graph in Fig. 3.31 represents accelerated (speeding up) motion, because the slope of the graph is increasing with time.

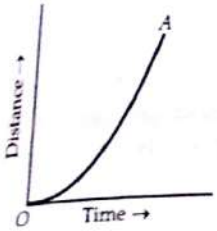

Fig. 3.31

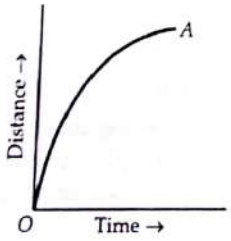

Fig. 3.32

(*iv*) The distance-time graph in Fig. 3.32 represents decelerated (speeding down) motion, because slope of the graph is decreasing with time.

(*v*) In Fig. 3.33, distance-time graph is a straight line parallel to distance-axis. It represents infinite speed which is not possible.

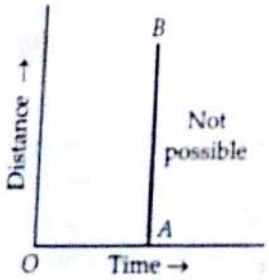

Fig. 3.33

(*vi*) The distance covered by a body cannot decrease with the increase of time. So the distance-time graph of the type shown in Fig. 3.34 is not possible.

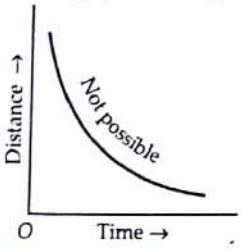

Fig. 3.34

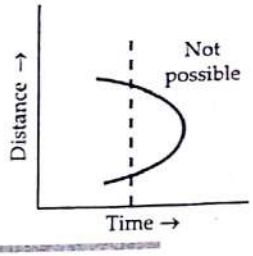

Fig. 3.35

(*vii*) The distance-time graph shown in Fig. 3.35 is not possible because it represents two different positions of the body at the same instant which is not possible.

31. *Discuss the nature of the motion from the given displacement-time graphs.*

Different Types of Displacement-Time Graphs :

(*i*) For a stationary body, the displacement-time graph is a straight line *AB* parallel to time-axis. The zero slope of line *AB* indicates zero velocity.

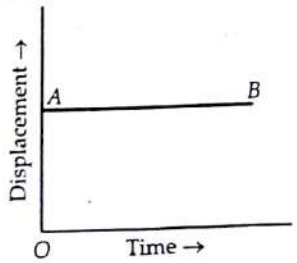

Fig. 3.36

(*ii*) In Fig. 3.37, the displacement-time graph is a straight line *OA* inclined to time-axis. It has a single slope. So it represents a constant velocity and hence zero acceleration.

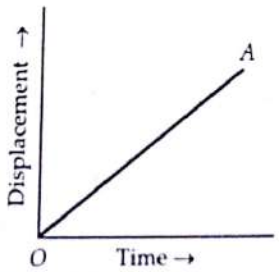

Fig. 3.37

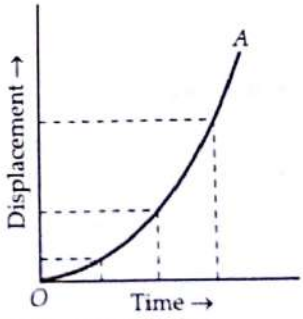

Fig. 3.38

(*iii*) In Fig. 3.38, greater displacements are taking place in equal intervals of time. So the displacement-time curve *OA* represents an increasing velocity or an accelerated motion. For constant acceleration, the displacement-time graph is a parabola bending upwards.

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(*iv*) In Fig. 3.39, decreasing displacements are taking place in equal intervals of time. So the displacement-time curve OA represents a decreasing velocity or deceleration. For uniform deceleration, the displacement-time graph is a parabola bending downwards.

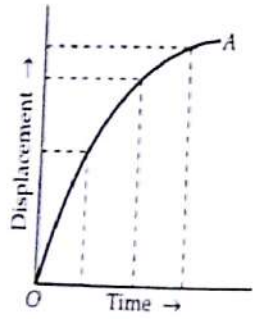

Fig. 3.39

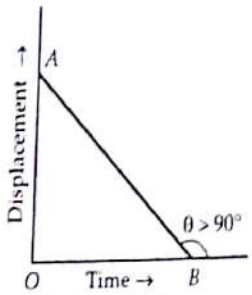

Fig. 3.40

(*v*) In Fig. 3.40, displacement is decreasing uniformly with time and becomes zero after a certain time. Displacement-time graph has a negative slope ($\theta > 90°$ and $\tan\theta < 0$) and represents a uniform negative velocity. It indicates that the body is returning back to its original position with a uniform velocity.

(*vi*) In Fig. 3.41, the displacement of the body becomes negative after time t and then increases in magnitude with time. It indicates that the body returning from position A, moves past the original position B and then moves towards C with a uniform velocity.

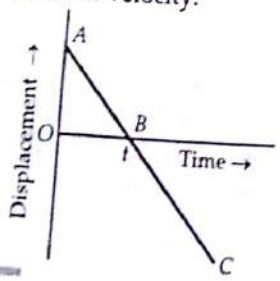

Fig. 3.41

32. *Discuss the nature of the motion from the given velocity-time graphs.*

Different Types of Velocity-Time Graphs :

(*i*) For a body moving with a uniform velocity, the v-t graph is a straight line parallel to the time-axis as shown in Fig. 3.42. The zero slope of line AB indicates zero acceleration.

(*ii*) When a body starts from rest and moves with uniform acceleration, its v-t graph is straight line OA inclined to the time-axis and passing through the origin O, as shown in Fig. 3.43. Greater is the slope of the v-t graph, greater will be the acceleration.

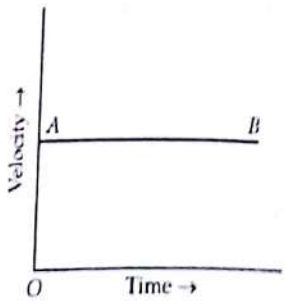

Fig. 3.42

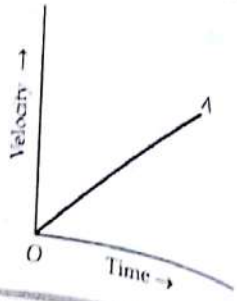

Fig. 3.43

(*iii*) In Fig. 3.44, the straight line v-t graph does not pass through origin O. The body has an initial velocity u ($= OA$) and then it moves with a uniform acceleration.

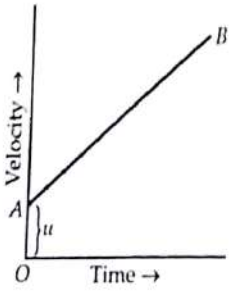

Fig. 3.44

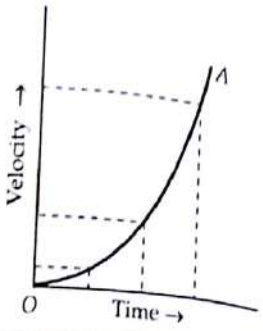

Fig. 3.45

(*iv*) In Fig. 3.45, greater changes in velocity are taking place in equal intervals of time. So the v-t graph bending upwards represents an increasing acceleration.

(*v*) In Fig. 3.46, smaller changes in velocity are taking place in equal intervals of time. So the v-t graph bending downwards represents a decreasing acceleration.

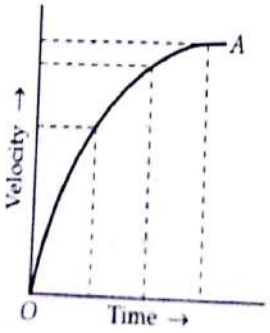

Fig. 3.46

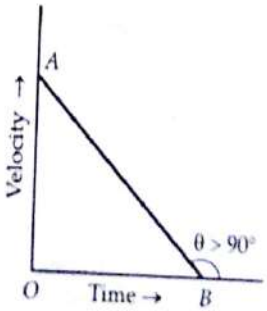

Fig. 3.47

(*vi*) In Fig. 3.47, the body starts with an initial velocity u. The velocity decreases uniformly with time, becoming zero after some time. As $\theta > 90°$, the graph has a negative slope. The v-t graph represents uniform negative acceleration.

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(iii) In Fig. 3.48, the v-t graph represents a body projected upwards with an initial velocity u. The velocity decreases with time (negative uniform acceleration), becoming zero after certain time t. Then the velocity becomes negative and increases in magnitude, showing body is returning to original position with positive uniform acceleration.

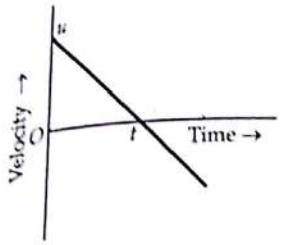

Fig. 3.48

(iv) The area between the velocity-time graph and the time-axis gives the displacement. In Fig. 3.49, the v-t graph represents variable acceleration.

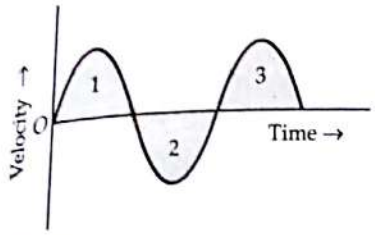

Fig. 3.49

Displacement covered = Area 1 – Area 2 + Area 3

Distance covered = Area 1 + Area 2 + Area 3.

34. *Discuss the nature of motion from the given speed-time graphs.*

Different Types of Speed-Time Graphs :

(i) For a body projected upwards, the speed-time graph is of the type shown in Fig. 3.50. When the body moves, its speed decreases uniformly, becoming zero at the highest point. As the body moves down, its speed increases uniformly. It returns with the same speed with which it was thrown up.

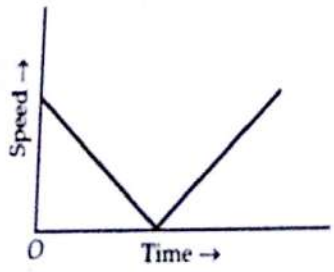

Fig. 3.50

(ii) For a ball dropped on the ground from a certain height, the speed-time graph is of the type shown in Fig. 3.51. As the ball falls, its speed increases. As the ball bounces back, its speed decreases uniformly and becomes zero at the highest point.

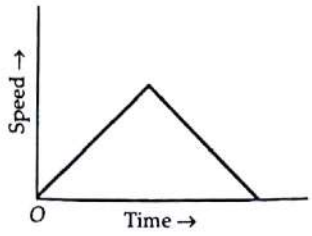

Fig. 3.51

34. *Discuss the nature of motion from the given acceleration-time graphs.*

Different Types of Acceleration-Time Graphs :

(i) For a body moving with constant acceleration, the acceleration-time graph is a straight line AB parallel to the time-axis, as shown in Fig. 3.52.

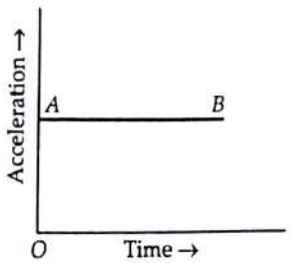

Fig. 3.52

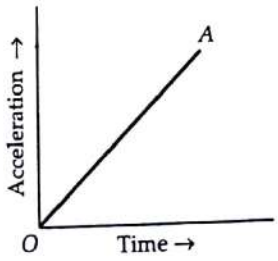

Fig. 3.53

(ii) When the acceleration of a body increases uniformly with time, its a-t graph is a straight line OA inclined to the time-axis as shown in Fig. 3.53.

(iii) For a body moving with variable acceleration, the a-t graph is a curve. The area between the a-t graph and the time-axis gives the change in velocity, as shown in Fig. 3.54.

Change in velocity = Area 1 – Area 2 + Area 3

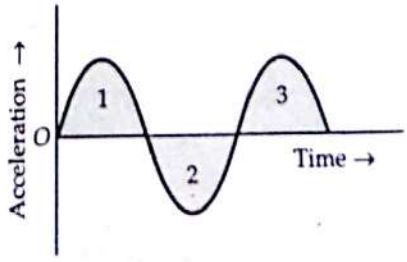

Fig. 3.54

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Examples based on Position-Time and Velocity-Time Graphs

FORMULAE USED

1. Slope of position-time (x-t) graph gives velocity.
$$v = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}.$$
2. Slope of velocity-time (v-t) graph gives acceleration.
$$a = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$
3. Distance travelled = Area between the (v-t) graph and time-axis.
4. Change in velocity = Area between the (a-t) graph and time-axis.

Example 35. From the top of a tower, a ball is dropped to fall freely under gravity and at the same time, another ball is thrown up with a velocity of 50 ms^{-1}. Plot the position-time graph for the motion of the two balls during the time interval $t = 0$ to $t = 5$ s. Take $g = 10\ ms^{-2}$.

Solution. We can find the distances covered by the two balls at different instants of time by using the formula,

$$s = ut + \frac{1}{2}gt^2$$

Time	For the ball dropped down $u = 0,\ g = +10\ ms^{-2}$	For the ball thrown up $u = 50\ ms^{-1},\ g = -10\ ms^{-2}$
	$s = 5t^2$	$s = 50t - 5t^2$
$t = 1$ s	$s = 5$ m	$s = 45$ m
$t = 2$ s	$s = 20$ m	$s = 80$ m
$t = 3$ s	$s = 45$ m	$s = 105$ m
$t = 4$ s	$s = 80$ m	$s = 120$ m
$t = 5$ s	$s = 125$ m	$s = 125$ m

The position-time graphs for the motion of the two balls are as shown in Fig. 3.55.

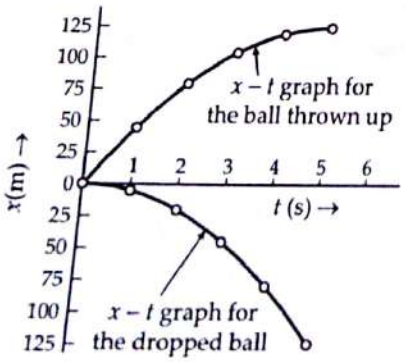

Fig. 3.55

Example 36. Fig. 3.56 shows the distance-time graphs of two trains, which start moving simultaneously in the same direction. From the graphs, find :

(i) How much ahead of A is B when the motion starts?

(ii) What is the speed of B ?

(iii) When and where will A catch B ?

(iv) What is the difference between the speeds of A and B?

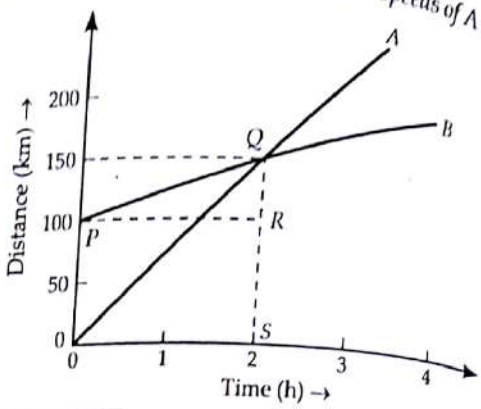

Fig. 3.56

Solution. *(i)* B is ahead of A by the distance $OP = 100$ **km,** when the motion starts.

(ii) Speed of $B = \frac{QR}{PR} = \frac{150 - 100}{2 - 0} = 25\ \text{kmh}^{-1}$.

(iii) Since the two graphs intersect at point Q, so A will catch B after 2 hours and at a distance of 150 km from the origin.

(iv) Speed of $A = \frac{QS}{OS} = \frac{150 - 0}{2 - 0} = 75\ \text{kmh}^{-1}$

$\therefore$ Difference in speeds $= 75 - 25 = 50\ \text{kmh}^{-1}$.

Example 37. The speed-time graph of a particle moving along a fixed direction is shown in figure. Find :

(i) distance travelled by the particle between 0 sec to 10 sec

(ii) average speed between this interval

(iii) the time when the speed was minimum

(iv) the time when speed was maximum. **[Delhi 06]**

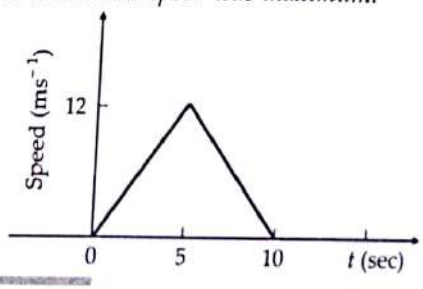

Fig. 3.57

Solution. *(i)* Distance travelled by the particle between 0 sec and 10 sec is

s = Area under speed-time graph

$= \frac{1}{2}(10 - 0)(12 - 0) = 60$ **m.**

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(*d*) Average speed $= \dfrac{\text{Total distance travelled}}{\text{Time taken}}$

$= \dfrac{60}{10} = 6 \text{ ms}^{-1}$.

(*e*) Speed is minimum at $t = 0$ s and $t = 10$ s.

(*f*) Speed is maximum at $t = 5$ s.

Example 38. A body starting from rest accelerates uniformly along a straight line at the rate of 10 ms^{-2} for 5 seconds. It moves for 2 second with uniform velocity of 50 ms^{-1}. Then it retards uniformly and comes to rest in 3 s. Draw velocity-time graph of the body and find the total distance travelled by the body. **[Central Schools 04]**

Solution. The velocity-time graph is shown below.

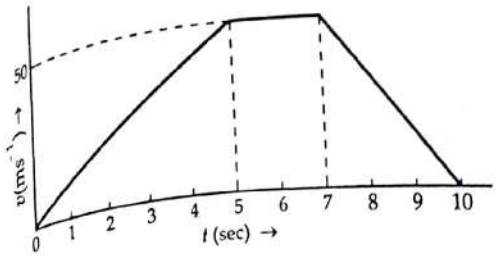

Fig. 3.58

Distance travelled by the body

= Area under v-t graph

$= \dfrac{1}{2}(2 + 10) \times 50 = \mathbf{300 \text{ m}}$.

Example 39. A train moves from one station to another in two hours' time. Its speed-time graph during the motion is shown in Fig. 3.59. (i) Determine the maximum acceleration during the journey (ii) Also calculate the distance covered during the time interval from 0.75 hour to 1 hour.

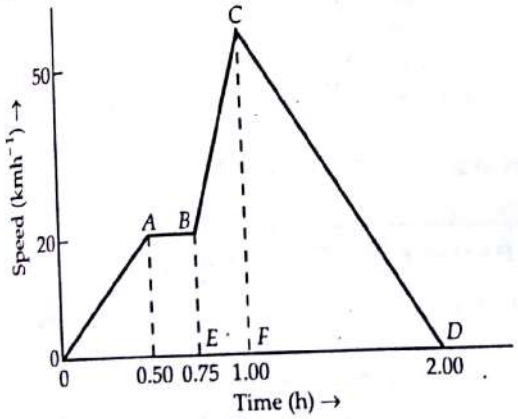

Fig. 3.59

Solution. (*i*) As the part BC of the speed-time graph has maximum slope, so acceleration is maximum during this interval.

Maximum acceleration

= Slope of BC

$= \dfrac{(50 - 20) \text{ kmh}^{-1}}{(1.00 - 0.75) \text{ h}} = \dfrac{30}{0.25} = \mathbf{120 \text{ kmh}^{-2}}$.

(*ii*) Distance covered from 0.75 h to 1.00 h

= Area of trapezium $BEFC$

$= \dfrac{1}{2}[BE + CF] \times EF$

$= \dfrac{1}{2}[20 + 50] \times (1.00 - 0.75) \text{ km} = \mathbf{8.75 \text{ km}}$.

Example 40. A ball is thrown upward with an initial velocity of 100 ms^{-1}. After how much time will it return ? Draw velocity-time graph for the ball and find from the graph (i) the maximum height attained by the ball and (ii) height of the ball after 15 s. Take $g = 10$ ms^{-2}.

Solution. Here $u = 100 \text{ ms}^{-1}$, $g = -10 \text{ ms}^{-2}$

At highest point, $v = 0$

As $v = u + gt$ $\quad \therefore \quad 0 = 100 - 10 \times t$

$\therefore$ Time taken to reach highest point,

$$t = \frac{100}{10} = 10 \text{ s}$$

The ball will return to the ground at $t = 20$ s.

Velocities of the ball at different instants of time will be as follows :

At $t = 0$, $\quad v = 100 - 10 \times 0 = 100 \text{ ms}^{-1}$

At $t = 5$ s, $\quad v = 100 - 10 \times 5 = 50 \text{ ms}^{-1}$

At $t = 10$ s, $\quad v = 100 - 10 \times 10 = 0$

At $t = 15$ s, $\quad v = 100 - 10 \times 15 = -50 \text{ ms}^{-1}$

At $t = 20$ s, $\quad v = 100 - 10 \times 20 = -100 \text{ ms}^{-1}$

The velocity–time graph will be as shown in Fig. 3.60.

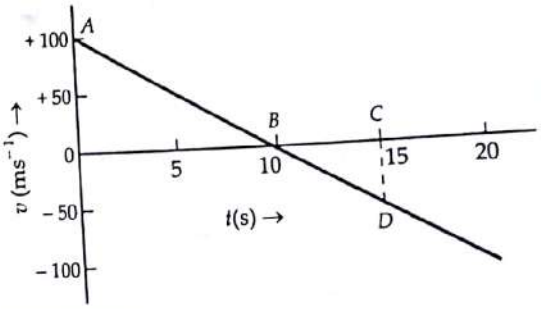

Fig. 3.60

(*i*) Maximum height attained by the ball

= Area of ΔAOB

$= \dfrac{1}{2} \times 10 \text{ s} \times 100 \text{ ms}^{-1} = \mathbf{500 \text{ m}}$.

(*ii*) Height attained after 15 s

= Area of ΔAOB + Area of ΔBCD

$= 500 + \dfrac{1}{2}(15 - 10) \times (-50) = 500 - 125$

$= \mathbf{375 \text{ m}}$.

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EXAMPLE 41. The velocity-time graph for a vehicle is shown in Fig. 3.61. Draw acceleration-time graph from it.

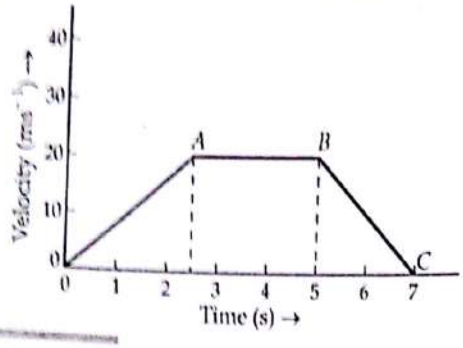

Fig. 3.61

Solution. In time 0 to 2.5 s, acceleration of the vehicle,

$$a = \text{Slope of } OA = \frac{20-0}{2.5-0} = 8 \text{ ms}^{-2}$$

In time 2.5 to 5 s, velocity of the vehicle is uniform. So acceleration = 0.

In time 5 to 7 s, acceleration of the vehicle,

$$a = \text{Slope of } BC = \frac{0-20}{7-5} = -10 \text{ ms}^{-2}$$

Hence the acceleration-time graph will be as shown in Fig. 3.62.

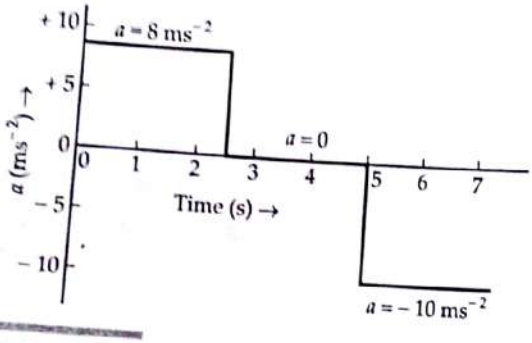

Fig. 3.62

EXAMPLE 42. The velocity of a train increases at a constant rate α from 0 to v and then remains constant for some time interval and then finally decreases to zero at a constant rate β. If the total distance covered by the particle be x, then show that the total time taken will be

$$t = \frac{x}{v} + \frac{v}{2}\left[\frac{1}{\alpha} + \frac{1}{\beta}\right].$$

Solution. The given motion can be represented by the velocity-time graph $OABC$ shown in Fig. 3.63. The portion OA represents the motion with constant acceleration α, the straight line AB parallel to time-axis represents the motion with uniform velocity v and the line BC represents the motion with constant acceleration β.

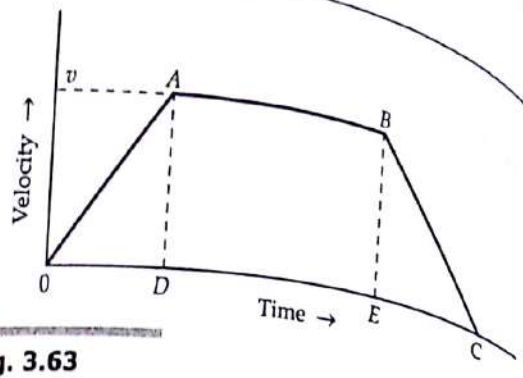

Fig. 3.63

Acceleration, α = slope of $OA = \frac{AD}{OD} = \frac{v}{OD}$

$\therefore \quad OD = \frac{v}{\alpha}$

Acceleration, β = slope of $BC = \frac{BE}{EC} = \frac{v}{EC}$

$\therefore \quad EC = \frac{v}{\beta}$

Now, total distance covered

= Area under the graph $OABC$

or $\quad x = \frac{1}{2}[AB + OC] \times AD$

$= \frac{1}{2}[DE + OD + DE + EC] \times AD \quad [\because AB = DE]$

$= \frac{1}{2}\left[2\,DE + \frac{v}{\alpha} + \frac{v}{\beta}\right] \times v$

or $\quad \frac{2x}{v} = 2\,DE + \frac{v}{\alpha} + \frac{v}{\beta}$

or $\quad DE = \frac{1}{2}\left[\frac{2x}{v} - \frac{v}{\alpha} - \frac{v}{\beta}\right]$

Now, total time taken is given by

$t = OD + DE + EC$

$= \frac{v}{\alpha} + \frac{1}{2}\left[\frac{2x}{v} - \frac{v}{\alpha} - \frac{v}{\beta}\right] + \frac{v}{\beta}$

or $\quad t = \frac{x}{v} + \frac{v}{2}\left[\frac{1}{\alpha} + \frac{1}{\beta}\right].$

PROBLEMS FOR PRACTICE

1. Figure 3.64 shows the position-time graphs of three cars *A*, *B* and *C*. On the basis of the graphs, answer the following questions :
   (*i*) Which car has the highest speed and which the lowest ?
   (*ii*) Are the three cars ever at the same point on the road ?
   (*iii*) When *A* passes *C*, where is *B* ?

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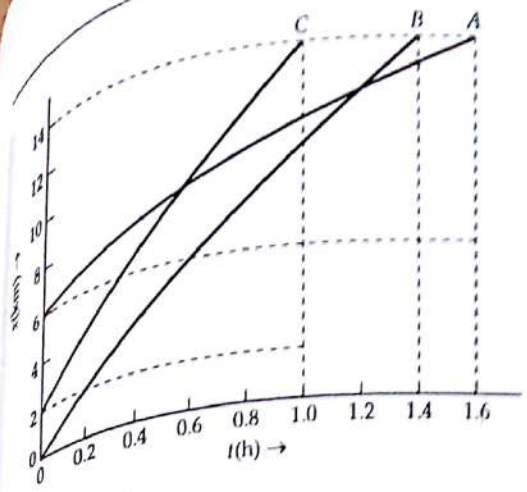

Fig. 3.64

(iv) How far did car A travel between the time it passed cars B and C ?

(v) What is the relative velocity of car C with respect to car A ?

(vi) What is the relative velocity of car B with respect to car C ?

[**Ans.** (*i*) C has the highest speed and A has the lowest speed (*ii*) No (*iii*) 6 km from the origin (*iv*) 3 km (*v*) 7 kmh^{-1} (*vi*) – 2 kmh^{-1}]

2. An insect crawling up a wall crawls 5 cm upwards in the first minute but then slides 3 cm downwards in the next minute. It again crawls up 5 cm upwards in the third minute but again slides 3 cm downwards in the fourth minute. How long will the insect take to reach a crevice in the wall at a height of 24 cm from its starting point ? How does the position-time graph of the insect look like ?

(**Ans.** 21 min)

3. A driver of a car travelling at 52 km h^{-1} applies the brakes and decelerates uniformly. The car stops in 5 seconds. Another driver going at 34 km h^{-1} applies his brakes slower and stops after 10 seconds. On the same graph paper, plot the speed versus time graph for two cars. Which of the two cars travelled farther after the brakes were applied ?

(**Ans.** Second car travelled farther)

4. A motor car, starting from rest, moves with uniform acceleration and attains a velocity of 8 ms^{-1} in 8 s. It then moves with uniform velocity and finally brought to rest in 32 m under uniform retardation. The total distance covered by the car is 464 m. Find (*i*) the acceleration (*ii*) the retardation and (*iii*) the total time taken. [**Ans.** (*i*) 1 ms^{-2} (*ii*) 1 ms^{-2} (*iii*) 66 s]

5. Starting from rest a car accelerates uniformly with 3 ms^{-2} for 5 s and then moves with uniform velocity. Draw the distance-time graph of the motion of the car upto t = 7s.

6. The velocity-time graph of an object moving along a straight line is as shown in Fig. 3.65. Find the net distance covered by the object in time interval between t = 0 to t = 10 s. Also find the displacement in time 0 to 10 s. (**Ans.** 100 m, 60 m)

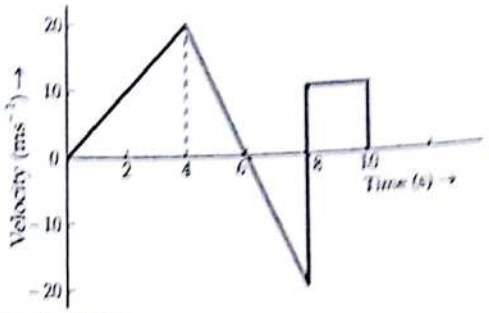

Fig. 3.65

7. As soon as a car just starts from rest in a certain direction, a scooter moving with a uniform speed overtakes the car. Their velocity-time graphs are shown in Fig. 3.66. Calculate (*i*) the difference

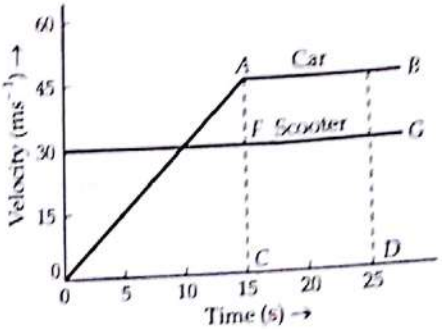

Fig. 3.66

between the distances travelled by the car and the scooter in 15 s (*ii*) the time when the car will catch up the scooter and (*iii*) the distance of car and scooter from the starting point at that instant.

[**Ans.** (*i*) 112.5 m (*ii*) 22.5 s (*iii*) 675 m]

8. The velocity-time graph of an object moving along a straight line is as shown below :

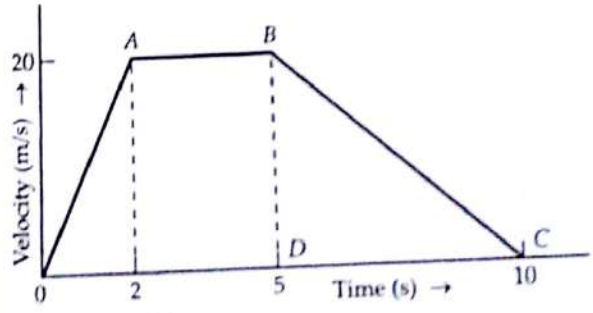

Fig. 3.67

Calculate the distance covered by object between :

(*i*) t = 0 to t = 5 s (*ii*) t = 0 to t = 10 s.

[**Chandigarh 08**] [**Ans.** (*i*) 80 m (*ii*) 130 m]

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HINTS

1. (*i*) Slope of line *C* is highest and that of *A* is lowest.

(*ii*) The lines do not meet at one point at any time.

(*iii*) *A* passes *C* at 0.6 h, at this time *B* is at 6 km from the origin.

(*v*) v_A = Slope of line $A = \dfrac{14-6}{1.6-0} = 5 \text{ km h}^{-1}$

v_C = Slope of line $C = \dfrac{14-2}{1.0-0} = 12 \text{ km h}^{-1}$

Relative velocity of *C* w.r.t. *A*

$= v_C - v_A = 12 - 5 = 7 \text{ km h}^{-1}$.

(*vi*) v_B = Slope of line $B = \dfrac{14-0}{1.4-0} = 10 \text{ km h}^{-1}$

Relative velocity of *B* w.r.t. *C*

$= v_B - v_C = 10 - 12 = -2 \text{ km s}^{-1}$.

2. Net distance crawled upwards in 2 min time

$= 5 - 3 = 2$ cm

∴ Insect will crawl $2 \times 10 = 20$ cm upward in $2 \times 10 = 20$ min.

In 21st min, insects covers 5 cm upward and reaches the crevice located at 25 cm.

∴ Total time taken = 20 + 1 = 21 min.

The positions of insect at intervals of 1 min each will be

t (min)	*x* (cm)	*t* (min)	*x* (cm)
0	0	11	15
1	5	12	12
2	2	13	17
3	7	14	14
4	4	15	19
5	9	16	16
6	6	17	21
7	11	18	18
8	8	19	23
9	13	20	20
10	10	21	25

Position-time (*x*-*t*) graph can be drawn with the help of the above table.

3. Fig. 3.68 shows the speed-time graphs for the two cars.

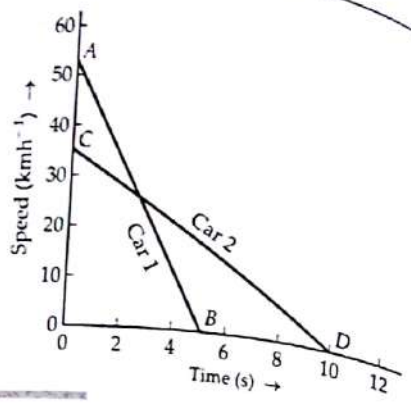

Fig. 3.68

Distance-travelled by car 1,

s_1 = Area of ΔAOB

$= \frac{1}{2} OA \times OB = \frac{1}{2} \times 52 \text{ km h}^{-1} \times 5 \text{ s}$

$= \frac{1}{2} \times 52 \times \frac{5}{18} \text{ ms}^{-1} \times 5 \text{ s} = 36.1 \text{ m}.$

Distance travelled by car 2,

s_2 = Area of ΔCOD

$= \frac{1}{2} OC \times OD = \frac{1}{2} \times 34 \times \frac{5}{18} \text{ ms}^{-1} \times 10 \text{ s}$

$= 47.2 \text{ m}.$

Hence second car travelled farther than first after the brakes were applied.

4. Fig. 3.69 shows the velocity-times graph for the motion of the motor car.

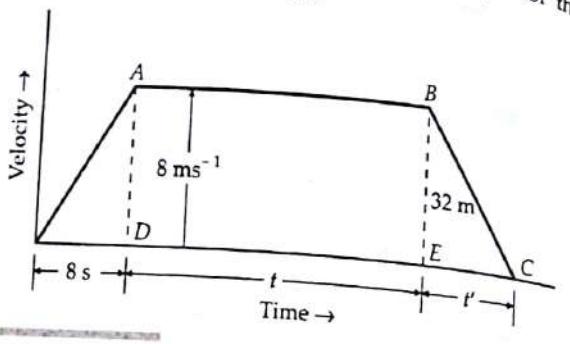

Fig. 3.69

Distance covered by the motor car,

Area of ΔAOD + Area of rect. *ADEB*

+ Area of $\Delta BEC = 464$ m

$\frac{1}{2} \times 8 \times 8 + 8 \times t + 32 = 464$

or $t = 50$ s.

Suppose the motor car retards for time t'.

Area of $\Delta BEC = \frac{1}{2} \times 8 \times t' = 32$

or $t' = 8$ s

∴ Total time taken $= 8 + t + t' = 8 + 50 + 8 = 66$ s.

Acceleration = Slope of $OA = \dfrac{AD}{OD} = \dfrac{8}{8} = 1 \text{ ms}^{-2}$.

Retardation = Slope of $BC = \dfrac{BE}{EC} = \dfrac{8}{8} = 1 \text{ ms}^{-2}$.

5. Distance covered by car in first 5 s is given by

$s = ut + \frac{1}{2} at^2 = 0 + \frac{1}{2} \times 3 \times t^2 = \frac{3}{2} t^2$

At $t = 5$ s, $v = u + at = 0 + 3 \times 5 = 15 \text{ ms}^{-1}$

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After $t = 5$ s, car covers a distance 15 m in each second upto $t = 7$ s.

Hence positions of the car at different instants of time will be

t (s)	0	1	2	3	4	5	6	7
x (m)	0	1.5	6.0	13.5	24.0	37.5	52.5	67.5

Fig. 3.70 shows the distance-time (x-t) graph for the car. For accelerated motion (0 to 5 s), the graph is a parabola (OA) and for uniform velocity (5 s to 7 s), the graph is straight line AB.

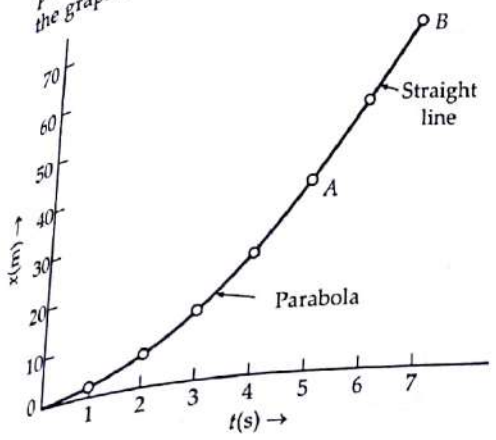

Fig. 3.70

6. Total distance covered in 10 s

$$= \frac{1}{2} \times 6 \times 20 + \frac{1}{2} \times 2 \times 20 + 2 \times 10 = \mathbf{100\ m}.$$

Displacement is positive from 0 to 6 s, negative from 6 to 8 s and positive from 8 to 10 s.

∴ Total displacement in 10 s

$$= \frac{1}{2} \times 6 \times 20 - \frac{1}{2} \times 2 \times 20 + 2 \times 10 = \mathbf{60\ m}.$$

7. (*i*) Distance travelled by the car in 15 s

$$= \text{Area of } \Delta OAC$$

$$= \frac{1}{2} \times OC \times AC = \frac{1}{2} \times 15 \times 45 = 337.5 \text{ m}$$

Distance travelled by the scooter in 15 s

$$= \text{Area of rect. } OEFC$$

$$= 15 \times 30 = 450 \text{ m}$$

Difference in the distances travelled

$$= 450 - 337.5 = \mathbf{112.5\ m}.$$

(*ii*) After $t = 15$ s, relative velocity of the car w.r.t. the scooter

$$= 45 - 30 = 15 \text{ ms}^{-1}$$

∴ Time taken in covering a difference of 112.5 m

$$= \frac{112.5 \text{ m}}{15 \text{ ms}^{-1}} = 7.5 \text{ s}$$

∴ Time after which car will catch up the scooter

$$= 15 + 7.5 = \mathbf{22.5\ s}.$$

(*iii*) Distance travelled by the scooter in 22.5 s

$$= 30 \text{ ms}^{-1} \times 22.5 \text{ s} = \mathbf{675\ m}.$$

So the car catches the scooter when both are at 675 m from the starting point.

8. (*i*) Distance covered between $t = 0$ to $t = 5$ s

$$= \text{Area } OABDO = \frac{1}{2}(3 + 5) \times 20 = \mathbf{80\ m}.$$

(*ii*) Distance covered between $t = 0$ to $t = 10$ s

$$= \text{Area } OABCO = \frac{1}{2}(3 + 10) \times 20 = \mathbf{130\ m}.$$

3.17 RELATIVE VELOCITY

35. *Define relative velocity. Deduce an expression for relative velocity of one object with respect to another in terms of their velocities relative to the earth.*

Relative velocity. *The relative velocity of an object 2 with respect to object 1, when both are in motion, is the time rate of change of position of object 2 with respect to that of object 1.*

Expression for relative velocity. As shown in Fig. 3.71, consider the objects 1 and 2 moving along the same direction with constant velocities v_1 and v_2 (relative to the earth) respectively.

Fig. 3.71 Relative velocity.

Suppose the position coordinates of the two objects are $x_1(0)$ and $x_2(0)$ at time $t = 0$. At time $t = t$, their position coordinates will be

$$x_1(t) = x_1(0) + v_1 t \quad \ldots(1)$$

$$x_2(t) = x_2(0) + v_2 t \quad \ldots(2)$$

Subtracting (1) from (2), we find that

$$x_2(t) - x_1(t) = x_2(0) - x_1(0) + (v_2 - v_1)\, t$$

or $[x_2(t) - x_2(0)] - [x_1(t) - x_1(0)] = (v_2 - v_1)\, t$

or Displacement of object 2 in time t

– Displacement of object 1 in time $t = (v_2 - v_1)\, t$

or Relative displacement of object 2 w.r.t. object 1 in time $t = (v_2 - v_1)\, t$

or $$\frac{\text{Relative displacement of object 2 w.r.t. object 1}}{\text{Time } t} = v_2 - v_1$$

or Relative velocity of object 2 w.r.t. object 1,

$$v_{21} = v_2 - v_1$$

Similarly, relative velocity of object 1 w.r.t. object 2,

$$v_{12} = v_1 - v_2$$

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3.18 RELATIVE VELOCITY IN TERMS OF POSITION-TIME GRAPHS

36. *Draw and discuss the position-time graphs of two objects moving along a straight line, when their relative velocity is (i) zero (ii) positive and (iii) negative.*

Analysis of relative velocity in terms of position-time graphs. The relative velocity, $v_{21} = v_2 - v_1$ may be zero, positive or negative.

Case 1. ***When the two objects move with same velocity in the same direction.*** That is $v_1 = v_2$ and relative velocity, $v_2 - v_1 = 0$

Then, $x_2(t) - x_1(t) = x_2(0) - x_1(0)$

Thus the two objects remain a constant distance apart throughout their motion. Their position-time graphs are parallel straight lines as shown in Fig. 3.72. Here the initial positions of the objects are $x_1(0) = 10$ m and $x_2(0) = 30$ m. Velocities are $v_1 = v_2 = 5\ \text{ms}^{-1}$. The two objects remain 20 m apart at all instants.

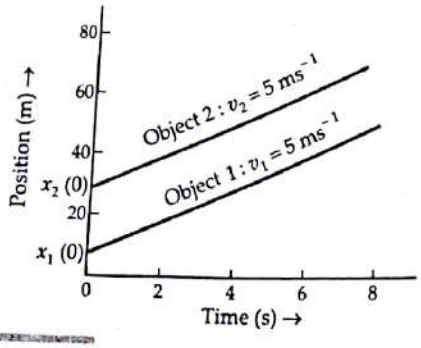

Fig. 3.72 Position-time graphs for $v_1 = v_2$.

Case 2. ***When $v_2 > v_1$ or relative velocity $(v_2 - v_1)$ is positive.*** The relative separation $x_2(t) - x_1(t)$ increases by the amount $(v_2 - v_1)$ after every second. So the position-time graphs gradually move apart, as shown in Fig. 3.73.

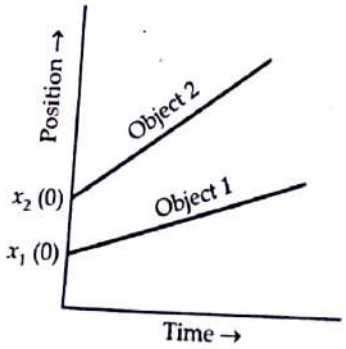

Fig. 3.73 Position-time graphs for $v_2 > v_1$.

Case 3. ***When $v_2 < v_1$ or relative velocity $(v_2 - v_1)$ is negative.*** Initially the object 2 is ahead of object 1 and $x_2(t) - x_1(t)$ is positive. The relative separation $x_2(t) - x_1(t)$ first decreases till the two objects meet at the position $x_1(t) = x_2(t)$. Then the separation $x_2(t) - x_1(t)$ becomes negative. The object 1 overtakes the object 2 and the relative separation between them again begins to increase.

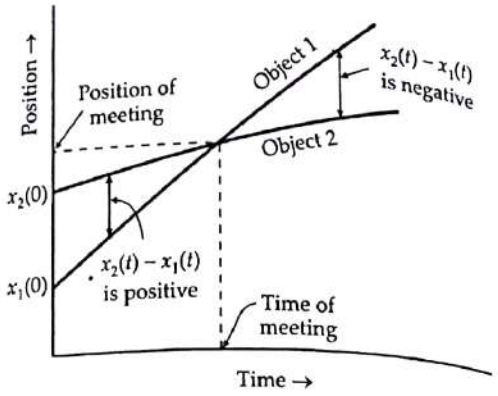

Fig. 3.74 Position-time graphs for $v_2 < v_1$.

3.19 DETERMINATION OF RELATIVE VELOCITY

37. *Two bodies A and B are moving with velocities v_A and v_B, making an angle θ with each other. Determine the relative velocity of A w.r.t. B. What will be the relative velocity (i) when the two bodies move in same direction and (ii) when the two bodies move in opposite directions ?*

Rule to determine relative velocity. The relative velocity of a body A with respect to another body B when both are in motion can be obtained by adding to the velocity of A, a velocity equal and opposite to that of B. Thus

$$v_{AB} = v_A + (-v_B)$$

This law can be applied even to the bodies moving in directions inclined to each other.

Consider two bodies A and B moving with velocities v_A and v_B respectively, making an angle θ with each other as shown in Fig. 3.75(*a*).

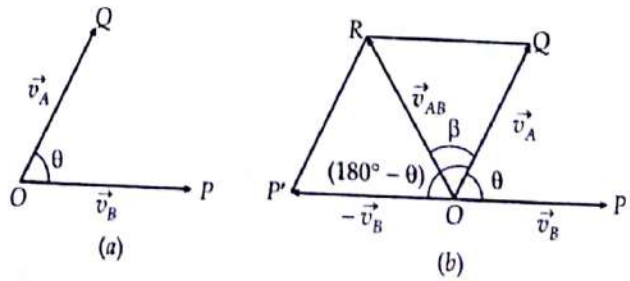

Fig. 3.75 Determination of relative velocity.

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To find the relative velocity v_{AB} of the body A with respect to B, draw $OP' = -v_B$ as shown in Fig. 3.75(b). Now we add v_A and $(-v_B)$ which make an angle $(180° - \theta)$ with each other. The relative velocity of A with respect to B is given by the diagonal OR of the parallelogram OQRP'.

The magnitude of the relative velocity v_{AB} is

$$v_{AB} = \sqrt{v_A^2 + v_B^2 + 2\, v_A\, v_B \cos(180° - \theta)}$$

$$= \sqrt{v_A^2 + v_B^2 - 2\, v_A\, v_B \cos\theta}$$

Suppose the relative velocity v_{AB} makes angle β with v_A. Then

$$\tan\beta = \frac{v_B \sin(180° - \theta)}{v_A + v_B \cos(180° - \theta)}$$

$$= \frac{v_B \sin\theta}{v_A - v_B \cos\theta}$$

or

$$\beta = \tan^{-1}\left(\frac{v_B \sin\theta}{v_A - v_B \cos\theta}\right)$$

This gives the direction of the relative velocity v_{AB}.

Special cases :

(i) **When both the bodies are moving along parallel straight lines in the same direction.** We have $\theta = 0°$.

$$\therefore \quad v_{AB} = \sqrt{v_A^2 + v_B^2 - 2\, v_A\, v_B \cos 0°}$$

$$= \sqrt{v_A^2 + v_B^2 - 2\, v_A\, v_B}$$

$$= \sqrt{(v_A - v_B)^2} = v_A - v_B.$$

Thus the relative velocity of A with respect to B is equal to the difference between the magnitudes of their velocities.

(ii) **When the two bodies are moving along parallel straight lines in the opposite directions.** We have $\theta = 180°$.

$$\therefore \quad v_{AB} = \sqrt{v_A^2 + v_B^2 - 2 v_A\, v_B \cos 180°}$$

$$= \sqrt{v_A^2 + v_B^2 + 2 v_A\, v_B}$$

$$= \sqrt{(v_A + v_B)^2} = v_A + v_B$$

Thus the relative velocity of body A w.r.t. body B is equal to the sum of the magnitudes of their velocities. That is why when two fast trains cross each other in opposite directions, each appears to go very fast relative to the other.

Examples based on Relative Velocity

FORMULAE USED

1. Relative velocity of object A w.r.t. object B,
$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$
2. Relative velocity of object B w.r.t. object A,
$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$$
where $\vec{v}_A$ and $\vec{v}_B$ are the velocities w.r.t. the ground
3. When the objects A and B move in the same direction,
$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$
4. When the object B moves in the opposite direction of A,
$$v_{AB} = v_A + v_B$$

UNITS USED

All velocities are in ms^{-1} or kmh^{-1}.

EXAMPLE 43. A car A moving at 10 ms^{-1} on a straight road, is ahead of car B moving in the same direction at 6 ms^{-1}. Find the velocity of A relative to B and vice versa.

Solution. Here $v_A = 10\ ms^{-1}$, $v_B = 6\ ms^{-1}$

Velocity of A relative to B,

$$v_{AB} = v_A - v_B = 10 - 6 = 4\ ms^{-1}$$

Positive velocity indicates that the driver of car B sees car A moving ahead from him at the rate of 4 ms^{-1}.

Velocity of B relative to A

$$v_{BA} = v_B - v_A = 6 - 10 = -4\ ms^{-1}$$

Negative velocity indicates that the driver of car A (when looks back) sees the car B lagging behind at the rate of 4 ms^{-1}.

EXAMPLE 44. Two parallel rail tracks run north south. Train A moves north with a speed of 54 kmh^{-1} and train B moves south with a speed of 90 kmh^{-1}. What is the

(i) relative velocity of B with respect to A ?

(ii) relative velocity of ground with respect to B ?

(iii) velocity of a monkey running on the roof of the train A against its motion (with a velocity of 18 km h^{-1} with respect to the train A) as observed by a man standing on the ground ? **[NCERT ; Delhi 05C]**

Solution. Taking south to north direction as the positive direction of x-axis, we have

$$v_A = +54\ \text{km/h} = \frac{54 \times 1000}{3600}\ ms^{-1} = 15\ ms^{-1}$$

$$v_B = -90\ \text{km/h} = \frac{-90 \times 1000}{3600}\ ms^{-1} = -25\ ms^{-1}$$

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(*i*) Relative velocity of *B* with respect to *A*

$$= v_B - v_A = -25 - 15 = -40 \text{ ms}^{-1}.$$

So to an observer in train *A*, the train *B* appears to move with a speed of 40 ms^{-1} from north to south.

(*ii*) Relative velocity of ground with respect to *B*

$$= 0 - v_B = 0 + 25 = 25 \text{ ms}^{-1}.$$

So to an observer in train *B*, the earth appears to move with a speed of 25 ms^{-1} from south to north.

(*iii*) Let velocity of monkey with respect to ground

$$= v_M$$

∴ Relative velocity of monkey with respect to train *A*

$$= v_M - v_A = -18 \text{ km h}^{-1} = -5 \text{ ms}^{-1}$$

or $$v_M = v_A - 5 = 15 - 5 = 10 \text{ ms}^{-1}.$$

EXAMPLE 45. Two trains 120 m and 80 m in length are running in opposite directions with velocities 42 kmh^{-1} and 30 kmh^{-1}. In what time they will completely cross each other ?

Solution. Relative velocity of one train w.r.t. the other

$$= 42 - (-30) = 72 \text{ kmh}^{-1} = 20 \text{ ms}^{-1}$$

Total distance to be travelled by each train to cross other train

$$= 120 + 80 = 200 \text{ m}$$

Time taken by each train to cross other train

$$= \frac{200}{20} = \mathbf{10\ s}.$$

EXAMPLE 46. The speed of a motor launch with respect to still water = 7 ms^{-1} and the speed of stream is $u = 3$ ms^{-1}. When the launch began travelling upstream, a float was dropped from it. The launch travelled 4.2 km upstream, turned about and caught up with the float. How long is it before the launch reaches the float ?

Solution. *For upstream motion of launch :*

Relative velocity = $7 - 3 = 4$ ms^{-1}

Distance moved = 4.2 km = 4200 m

Time taken, $t_1 = \frac{4200}{4} = 1050$ s

For downstream motion of launch :

Distance moved downstream by float in 1050 s

$$= 3 \times 1050 = 3150 \text{ m}$$

Distance between float and launch turned about

$$= 4200 + 3150 = 7350 \text{ m}$$

This distance is to be covered by launch with its own velocity (7 ms^{-1}) because stream velocity is being shared by both.

∴ Time taken, $t_2 = \frac{7350}{7} = 1050$ s

Total time taken, $t = t_1 + t_2 = 1050 + 1050$

$$= 2100 \text{ s} = \mathbf{35\ min}.$$

PROBLEMS FOR PRACTICE

1. A jet airplane travelling at the speed of 450 kmh^{-1} ejects the burnt gases at the speed of 1200 kmh^{-1} relative to the jet airplane. Find the speed of the burnt gases *w.r.t.* a stationary observer on earth. (**Ans.** 750 kmh^{-1})
2. Two cars *A* and *B* are moving with velocities of 60 kmh^{-1} and 45 kmh^{-1} respectively. Calculate the relative velocity of *A w.r.t. B*, if (*i*) both cars are travelling eastwards and (*ii*) car *A* is travelling eastwards and car *B* is travelling westwards. [**Ans.** (*i*) 15 kmh^{-1} eastwards (*ii*) 105 km h^{-1} eastwards]
3. An open car is moving on a road with a speed of 100 kmh^{-1}. A man sitting in the car fires a bullet from the gun in the opposite direction. If the speed of the bullet is 250 kmh^{-1} relative to the car, then find its (bullet's) speed with respect to an observer on the ground. (**Ans.** 150 kmh^{-1})
4. A car *A* is moving with a speed of 60 kmh^{-1} and car *B* is moving with a speed of 75 kmh^{-1}, along parallel straight paths, starting from the same point. What is the position of car *A w.r.t. B* after 20 minutes ? (**Ans.** 5 km behind)
5. Two buses start simultaneously towards each other from towns *A* and *B* which are 480 km apart. The first bus takes 8 hours to travel from *A* to *B* while the second bus takes 12 hours to travel from *B* to *A*. Determine when and where the buses will meet. (**Ans.** 4.8 h, 288 km from *A*)
6. Two trains *A* and *B*, each of length 100 m, are running on parallel tracks. One overtakes the other in 20 s and one crosses the other in 10 s. Calculate the velocities of each train. (**Ans.** 15 ms^{-1}, 5 ms^{-1})
7. A man swims in a river with and against water at the rate of 15 kmh^{-1} and 5 kmh^{-1}. Find the man's speed in still water and the speed of the river. (**Ans.** 10 kmh^{-1}, 5 kmh^{-1})
8. A motorboat covers the distance between the two spots on the river in 8 h and 12 h downstream and upstream respectively. Find the time required by the boat to cover this distance in still water. (**Ans.** 9.6 h)
9. A car *A* is travelling on a straight level road with a speed of 60 kmh^{-1}. It is followed by another car *B* which is moving with a speed of 70 kmh^{-1}. When the distance between them is 2.5 km, the car *B* is given a deceleration of 20 kmh^{-2}. After what distance and time will the car *B* catch up with car *A* ? (**Ans.** 32.5 km, 0.5 h)

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HINTS

4. Relative speed of A w.r.t. B
$= 60 - 75 = -15\ \text{kmh}^{-1}$

Distance of A from B after 20 min
$= -15 \times \frac{20}{60} = -5$ **km.**

5. Speed of first bus $= \frac{480}{8} = 60\ \text{kmh}^{-1}$

Speed of second bus $= \frac{480}{12} = 40\ \text{kmh}^{-1}$

Suppose the two buses meet after time t. Then
$60t + 40t = 480$
$t = \mathbf{4.8\ h}$
or
Distance from $A = 60 \times 4.8 = \mathbf{288\ km.}$

6. Let u and v be the velocities of trains A and B respectively.
During overtaking, relative velocity of A w.r.t. B
$= u - v$
During crossing, relative velocity of A w.r.t B
$= u + v$
Total distance to be covered by A during overtaking or crossing
$= 100 + 100 = 200$ m
$\therefore \quad \frac{200}{u - v} = 20$ and $\frac{200}{u + v} = 10$
or $\quad u - v = 10$ and $u + v = 20$
On solving, $u = \mathbf{15\ ms^{-1}}$, $v = \mathbf{5\ ms^{-1}}$.

7. Let u be the speed of man in still water and v be the speed of river. Then
$u + v = 15\ \text{kmh}^{-1}$ and $u - v = 5\ \text{kmh}^{-1}$
On solving,
$u = \mathbf{10\ kmh^{-1}}$, $v = \mathbf{5\ kmh^{-1}}$.

8. Let u and v be the velocity of boat in still water and velocity of river respectively. If x is the distance between the two spots, then
$u + v = \frac{x}{8}$ *(for upstream)*
$u - v = \frac{x}{12}$ *(for downstream)*
On adding,
$2u = \frac{20}{96}x$ or $u = \frac{10}{96}x$
Time required by boat in still water
$= \frac{x}{u} = \frac{x}{10x/96} = \mathbf{9.6\ h.}$

9. Relative velocity of car B w.r.t. A
$= 70 - 60 = 10\ \text{kmh}^{-1}$
$\therefore$ For car B,
$u = 10\ \text{kmh}^{-1}$, $s = 2.5$ km, $a = -20\ \text{kmh}^{-1}$
As $\quad s = ut + \frac{1}{2}at^2$
$\therefore \quad 2.5 = 10t - \frac{1}{2} \times 20 \times t^2$ or $t = 0.5$ h
Actual distance travelled by car B during this time,
$s = ut + \frac{1}{2}at^2 = 70 \times 0.5 - \frac{1}{2} \times 20 \times (0.5)^2$
$= 35 - 2.5 = \mathbf{32.5\ km.}$

Very Short Answer Conceptual Problems

Problem 1. When does a cyclist appear to be stationary with respect to another moving cyclist ?

Solution. When both the cyclists are moving in the same direction with the same velocity parallel to each other.

Problem 2. Can the earth be regarded as point object when it is describing its yearly journey around the sun ?

Solution. Yes, because size of the earth is much smaller than the distance from the sun.

Problem 3. Can the displacement be greater than the distance travelled by an object ? Give reason.

Solution. No, the displacement of an object can be either equal to or less than the distance travelled by the object. This is because displacement is shortest distance between the initial and final positions of the object while distance travelled is the length of the actual path traversed by the object.

Problem 4. Can the speed of a body be negative ?

Solution. No, because the speed of an object is the distance travelled per unit time and distance travelled is never negative.

Problem 5. Can a body have a constant speed and still have a varying velocity ? *[Himachal 06C]*

Solution. Yes. A particle in uniform circular motion has a constant speed but varying velocity because of the change in its direction of motion at every point.

Problem 6. Can a body have a constant velocity and still have a varying speed ? *[Himachal 04, 06]*

Solution. No, a body cannot have a constant velocity while having a varying speed. Whenever the speed changes, velocity also changes.

Problem 7. Can a particle in one-dimensional motion have zero speed and a non-zero velocity ?

Solution. No. If the speed is zero, the velocity will be necessarily zero.

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Problem 8. Can a body have zero velocity and still be accelerating ? *[Delhi 05C]*

Solution. Yes. A body thrown vertically upwards has zero velocity at its highest point but has acceleration equal to the acceleration due to gravity.

Problem 9. Can an object have an eastward velocity while experiencing a westward acceleration ?

Solution. Yes. A pendulum oscillating in east-west direction will have eastward velocity and westward acceleration in half cycle of its oscillation.

Problem 10. Can the direction of velocity of an object change, when acceleration is constant ?

Solution. Yes. For an object thrown vertically upwards, the direction of velocity changes during its rise and fall. But acceleration acts always downwards and remains constant.

Problem 11. Is it possible for a body to be accelerated without speeding up or slowing down ? If so, give an example. *[Himachal 01, 04]*

Solution. Yes. An object in uniform circular motion is accelerating but its speed neither decreases nor increases.

Problem 12. Under what condition is the average velocity equal to the instantaneous velocity ?

Solution. When a body moves with a constant velocity, its average velocity over any time interval is same as is instantaneous velocity.

Problem 13. Why is the speed, in general, greater than the magnitude of the velocity ?

Solution. Because of the change in the direction of motion, the length of the path traversed by a body is generally greater than the magnitude of its displacement. So speed is generally greater than the magnitude of velocity.

Problem 14. Is the direction of acceleration same as the direction of velocity ?

Solution. Not necessarily. If velocity increases, acceleration acts in the direction of velocity and if velocity decreases, then acceleration acts in the opposite direction of velocity.

Problem 15. Can we use the equations of kinematics to find the height attained by a body projected upwards with any velocity ?

Solution. No. The equations of kinematics are applicable only so long as the acceleration is uniform. The acceleration due to gravity is uniform only near the surface of the earth.

Problem 16. Two balls of different masses (one lighter and other heavier) are thrown vertically upward with same initial speed. Which one will rise to the greater height ? *[I.I.T.]*

Solution. Both the balls will rise to the same heights. It is because, for a body moving with given initial velocity and acceleration, the distance covered by the body does not depend on the mass of the body.

Problem 17. Two balls of different masses (one lighter and other heavier) are thrown vertically upwards with the same speed. Which one will pass through the point of projection in their downward direction with the greater speed ? *[I.I.T.]*

Solution. In case of motion under gravity, the speed with which a body returns back is always equal to the speed with which it is thrown up. Since expression for final speed does not involve mass, both the balls will acquire the same speed.

Problem 18. Can the relative velocity of two bodies be greater than the absolute velocity of either body ?

Solution. Yes. When two bodies move in opposite directions, the relative velocity of each is greater than the individual velocity of either body.

Problem 19. A car travelling with a velocity of 50 km h^{-1} on a straight road is ahead of a motor-cycle travelling with a speed of 75 kmh^{-1}. How would the relative velocity be altered if motor cycle is ahead of car ?

Solution. The relative velocity will remain same as it does not depend on the position of the two bodies.

Problem 20. Even when rain is falling vertically downwards, the front screen of a moving car gets wet while the back screen remains dry. Why ?

Solution. This is because the rain strikes the car in the direction of relative velocity of rain with respect to car.

Problem 21. Is it possible that the brakes of a car are so perfect that the car stops instantaneously. If not, why ?

Solution. No, it is not possible. In order to make velocity zero in an infinitesimally small interval of time, the car needs an infinite declaration which is not possible.

Problem 22. When an observer is standing on earth, the trees and houses appear stationary to him. However, when he is sitting in a moving train, all these objects appear to move in backward direction. Why ?

Solution. For the stationary observer, the relative velocity of trees and houses is zero. For the observer sitting in the moving train, the relative velocity of houses and trees is negative. So these objects appear to move in backward direction.

Problem 23. The displacement of a body is given to be proportional to the cube of time elapsed. What is the nature of the acceleration of the body ?

Solution. Given : $s \propto t^3$ or $s = kt^3$

$$\therefore \quad \text{Velocity} = \frac{ds}{dt} = 3kt^2$$

$$\text{Acceleration} = \frac{dv}{dt} = 6kt \quad i.e., \quad \text{acceleration} \propto t.$$

Clearly, the acceleration increases uniformly with time.

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Problem 24. The average velocity of a particle is equal to its instantaneous velocity. What is the nature of its displacement-time graph ?

Solution. When the average velocity is equal to the instantaneous velocity, the particle has a uniform motion. So its displacement-time graph is a straight line inclined to the time-axis.

Problem 25. Can the position-time graph have a negative slope ?

Solution. Yes, when the velocity of the object is negative.

Problem 26. What is the nature of the displacement-time curve of a body moving with constant velocity ?

Solution. Parabola.

Problem 27. Can the direction of motion of a body change if its velocity is changing at uniform rate ?

Solution. Yes. This can happen in a retarded motion. A body thrown up moves under constant retardation and from the highest point it begins to fall downwards.

Problem 28. Draw position-time graphs for two objects having zero relative velocity. *[Central Schools 14]*

Solution. As the relative velocity is zero, the two bodies A and B have equal velocities. Hence their position-time graphs are parallel straight lines, equally inclined to the time-axis as shown in Fig. 3.76.

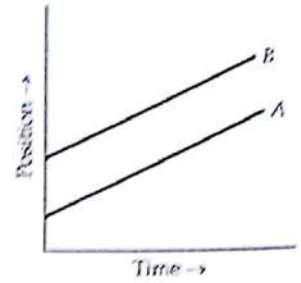

Fig. 3.76

Problem 29. Is it possible to have a constant rate of change of velocity when velocity changes both in magnitude and direction ? If yes, give one example.

[Central Schools 08]

Solution. Yes, in projectile motion, a body has uniform acceleration in the downward direction while its velocity changes both in magnitude and direction at every point of its trajectory.

Short Answer Conceptual Problems

Problem 1. Can a body be at rest as well as in motion at the same time ? Explain. *[Himachal 06]*

Or

Rest and motion are relative terms. Explain.

[Himachal 09]

Solution. Yes, object may be at rest relative to one object and at the same time it may be in motion relative to another object. For example, a passenger sitting in a moving train is at rest with respect to his fellow passengers but he is in motion with respect to the objects outside the train. Rest and motion are relative terms.

Problem 2. If the displacement of a body is zero, is the distance covered by it necessarily zero ? Comment with suitable illustration.

Solution. No, it is not necessary that the distance covered by a body is zero when its displacement is zero. Consider a particle moving along a circular track of radius r. After the particle completes one revolution, its displacement is zero while the distance covered is $2\pi r$.

Problem 3. State in the following cases, whether the motion is one, two or three dimensional :

(*i*) a kite flying on a windy day,

(*ii*) a speeding car on a long straight high way,

(*iii*) a carrom coin rebounding from the side of the board,

(*iv*) an insect crawling on a globe, and

(*v*) a planet revolving around its star ?

Solution.

(*i*) three-dimensional, (*ii*) one-dimensional,
(*iii*) two-dimensional, (*iv*) two-dimensional,
(*v*) two-dimensional.

Problem 4. Which of the two-velocity or acceleration, gives the direction of motion of the body. Justify your answer by an example.

Solution. It is the velocity and not the acceleration, which gives the direction of motion of a body. When a body is projected upward, both its direction of motion and velocity are in upward direction but the acceleration is in the downward direction.

Problem 5. Two straight lines drawn on the same displacement-time graph make angles 30° and 60° with time-axis respectively, as shown in Fig. 3.77. Which line represents greater velocity ? What is the ratio of the two velocities ? *[Delhi 05, 08]*

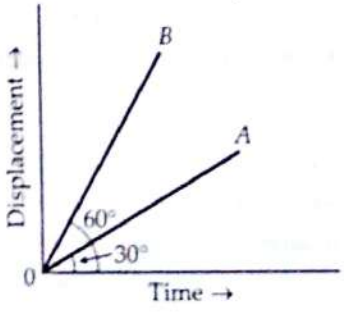

Fig. 3.77

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Solution. Slope of displacement-time graph
= Velocity of the object

As slope of line OB > slope of line OA

∴ The line making angle of $60°$ with time-axis represents greater velocity.

Ratio of the two velocities

$$= \frac{v_A}{v_B} = \frac{\tan 30°}{\tan 60°}$$

$$= \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3} = 1:3.$$

Problem 6. If in case of a motion, displacement is directly proportional to the square of the time elapsed, what do you think about its acceleration *i.e.*, constant or variable ? Explain why. *[Chandigarh 02]*

Solution. Given $x \propto t^2$

or $x = ct^2$,

where c is a constant

Velocity, $v = \frac{dx}{dt} = c \times 2t$

Acceleration, $a = \frac{dv}{dt} = 2c$ = a constant

Hence the object is moving with uniform acceleration.

Problem 7. Show that the average velocity of an object over an interval of time is either smaller than or equal to the average speed of the object over the same interval.

Solution. We know that

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time interval}}$$

$$\text{Average speed} = \frac{\text{Total path length}}{\text{Time interval}}.$$

As the magnitude of the displacement of an object can be equal to or less than the total length of the path traversed by the body in given time, so average velocity of an object over an interval of time is either equal to or smaller than the average speed of the object over the same interval.

Problem 8. An object is in uniform motion along a straight line. What will be position-time graph for the motion of the object if

(*a*) x_0 = +ve, v = +ve

(*b*) x_0 = +ve, v = −ve

(*c*) x_0 = +ve, v = −ve

(*d*) both x_0 and v are negative ?

The letters x_0 and v represent position of the object at $t = 0$ and uniform velocity of the object respectively.

Solution. The position of the object moving with a uniform velocity v and time t is given by $x = x_0 + vt$

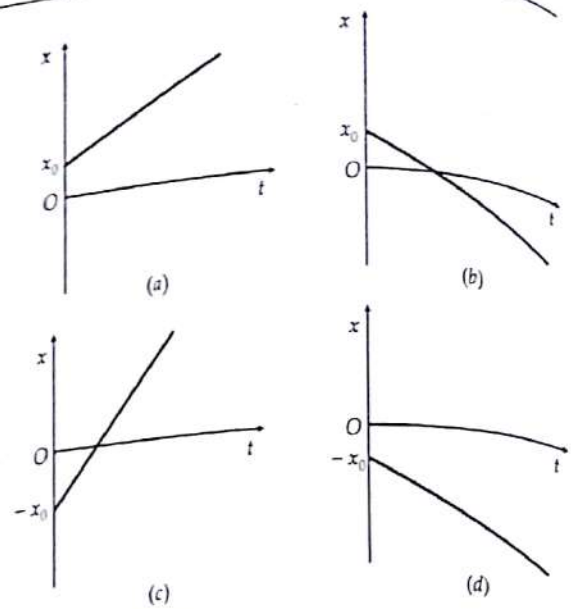

Fig. 3.78

(*a*) If x_0 = + ve and v = + ve, position-time graph will be as shown in Fig. 3.78(*a*).

(*b*) If x_0 = + ve and v = − ve, the position-time graph will be as shown in Fig. 3.78(*b*).

(*c*) If x_0 = − ve and v = + ve, the position-time graph will be as shown in Fig. 3.78(*c*).

(*d*) If both x_0 and v are negative, the position-time graph will be as shown in Fig. 3.78(*d*).

Problem 9. An object has uniformly accelerated motion. The object always slows down before the time, when its velocity becomes zero. Prove this statement graphically, when (*a*) both u and a are positive (*b*) u = − ve and a = + ve (*c*) u = + ve and a = −ve and (*d*) both u and a are negative.

Solution.

(*a*) **When both u and a are +ve :** In such a case, the v-t graph will be as shown in Fig. 3.79(*a*). At the time corresponding to point A, the velocity becomes zero. It can be seen that before this time, the velocity is negative but its magnitude decreases with time till it becomes zero at A.

(*b*) **When u is − ve and a is + ve :** In this case, graph will be shown in Fig. 3.92(*b*). At the time corresponding to point A, the velocity becomes zero. It can be seen that before this time the velocity is − ve but its magnitude decreases with time till it becomes zero at A.

(*c*) **When u + ve and a is − ve :** In such a case, graph between v and t will be as shown in Fig. 3.79(*c*). Again at A, velocity is zero. The velocity decreases before the time corresponding to point A.

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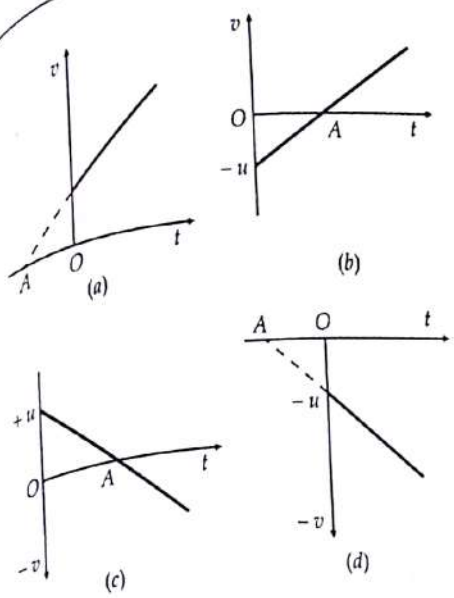

Fig. 3.79

(*d*) **When both u and a are – ve :** In this case, v-t graph will be as shown in Fig. 3.79(*d*). If we produce graph backwards, it meets the time-axis at point *A*. Before this time, velocity is +ve and decreases till it becomes zero at point *A*.

Problem 10. The distance covered by an object between times t_1 and t_2 is given by the area under the v-t graph between t_1 and t_2. Prove this statement for an object moving with negative acceleration and having a positive velocity at time t_1 and a negative velocity at time t_2

Solution. The velocity-time graph for the given motion of the object is as shown in Fig. 3.80. Let v_1 be the velocity at time t_1 and $-v_2$ at time t_2.

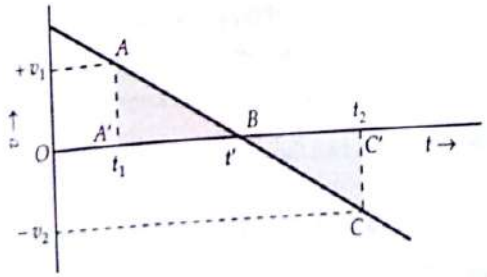

Fig. 3.80

Area under v-t graph between times t_1 and t_2

$$= \text{Area } AA'B + \text{Area } CC'B$$

$$= \frac{1}{2} A'B \times AA' + \frac{1}{2} BC' \times CC'$$

$$= \frac{1}{2}(t' - t_1)(v_1) + \frac{1}{2}(t_2 - t')(-v_2)$$

$$= \frac{1}{2} v_1 (t' - t_1) - \frac{1}{2} v_2 (t_2 - t').$$

Between times t_1 and t', the acceleration of the object is

$$a = \text{Slope of } v\text{-}t \text{ graph } AB = \frac{0 - v_1}{t' - t_1}$$

$$\therefore \quad t' - t_1 = -\frac{v_1}{a}$$

Between times t' and t_2, the acceleration of the object is

$$a = \text{slope of } v\text{-}t \text{ graph } BC = \frac{-v_2 - 0}{t_2 - t'}$$

$$\therefore \quad t_2 - t' = -\frac{v_2}{a}$$

Hence area under v-t graph between times t_1 and t_2

$$= \frac{1}{2} v_1 \left(-\frac{v_1}{a}\right) - \frac{1}{2} v_2 \left(-\frac{v_2}{a}\right)$$

$$= \frac{v_2^2 - v_1^2}{2a} = \frac{2as}{2a} = s$$

= Distance covered by the object between times t_1 and t_2.

Problem 11. Distinguish between distance and displacement. *[Himachal 04, 06, 09C]*

Solution.

Distance	*Displacement*
1. Distance is the length of the actual path traversed by a body, irrespective of its motion.	Displacement is the shortest distance between the initial and final positions of body in a given direction.
2. Distance between two points may be same or different for different paths chosen.	Displacement between two given points is always same.
3. It is a scalar quantity.	It is a vector quantity.
4. Distance covered may be positive or zero.	Displacement covered may be positive, negative or zero.

Problem 12. Distinguish between speed and velocity. *[Himachal 06C, 08, 09C]*

Solution.

Speed	*Velocity*
1. It is the distance travelled by a body per unit time in any direction.	It is the distance travelled by a body per unit time in a fixed direction.
2. It is a scalar quantity.	It is a vector quantity.
3. Speed may be positive or zero but never negative.	Velocity may be positive, negative or zero.

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HOTS

Problems on Higher Order Thinking Skills

Problem 1. *The velocity-time relation of an electron starting from rest is given by $v = kt$, where $k = 2\ ms^{-2}$. Calculate the distance traversed in 3 s.*

Solution. Velocity, $v = kt$

Acceleration, $a = \frac{dv}{dt} = \frac{d}{dt}(kt) = k = 2\ ms^{-2}$

Distance traversed in 3 s,

$$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 2 \times (3)^2 = 9\ \text{m}.$$

Problem 2. *The acceleration experienced by a boat after the engine is cut off, is given by $\frac{dv}{dt} = -kv^3$, where k is a constant. If v_0 is the magnitude of the velocity at cut off, find the magnitude of the velocity at time t after the cut off.* **[CBSE 94]**

Solution. Given : $\frac{dv}{dt} = -kv^3$ or $v^{-3}\,dv = -k\,dt$

Integrating within the conditions of motion,

$$\int_{v_0}^{v} v^{-3}dv = -k\int_0^t dt$$

or $$\left[\frac{v^{-2}}{-2}\right]_{v_0}^{v} = -k\,[t]_0^t$$

or $$-\frac{1}{2}\left[\frac{1}{v^2} - \frac{1}{v_0^2}\right] = -k\,[t-0]$$

or $$\frac{1}{v^2} - \frac{1}{v_0^2} = 2\,kt$$

or $$\frac{1}{v^2} = \frac{1}{v_0^2} + 2kt = \frac{1 + 2kt\,v_0^2}{v_0^2}$$

or $$v = \frac{v_0}{\sqrt{1 + 2kt\,v_0^2}}.$$

Problem 3. *If a body moving with uniform acceleration in a straight line describes successive equal distances in time intervals t_1, t_2 and t_3, then show that*

$$\frac{1}{t_1} - \frac{1}{t_2} + \frac{1}{t_3} = \frac{3}{t_1 + t_2 + t_3}.$$

Solution. As shown in Fig. 3.81, let the three successive equal distances be represented by AB, BC and CD.

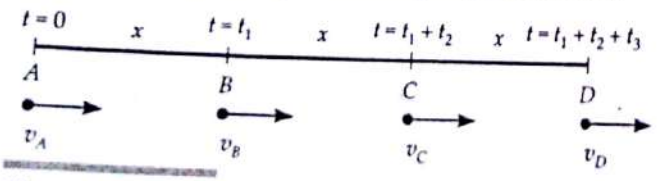

Fig. 3.81

Let each distance be x m. Let v_A, v_B, v_C and v_D be the velocities at points A, B, C and D respectively.

Average velocity between A and $B = \frac{v_A + v_B}{2}$

$$\therefore \quad \frac{v_A + v_B}{2} \times t_1 = x$$

or $$v_A + v_B = \frac{2x}{t_1}$$

Similarly, $v_B + v_C = \frac{2x}{t_2}$ and $v_C + v_D = \frac{2x}{t_3}$

Average velocity between A and $D = \frac{v_A + v_D}{2}$

$$\therefore \quad \frac{v_A + v_D}{2}(t_1 + t_2 + t_3) = x + x + x$$

or $$v_A + v_D = \frac{6x}{t_1 + t_2 + t_3}$$

Hence $v_A + v_D = (v_A + v_B) - (v_B + v_C) + (v_C + v_D)$

or $$\frac{6x}{t_1 + t_2 + t_3} = \frac{2x}{t_1} - \frac{2x}{t_2} + \frac{2x}{t_3}$$

or $$\frac{3}{t_1 + t_2 + t_3} = \frac{1}{t_1} - \frac{1}{t_2} + \frac{1}{t_3}.$$

Problem 4. *In a car race, car A takes time t less than car B and passes the finishing point with a velocity v more than the velocity with which car B passes the point. Assuming that the cars start from rest and travel with constant accelerations a_1 and a_2, show that $v = t\sqrt{a_1 a_2}$.*

Solution. Let s be the distance covered by each car. Let the times taken by the two cars to complete the journey be t_1 and t_2, and their velocities at the finishing point be v_1 and v_2 respectively. According to the problem,

$$v_1 - v_2 = v \quad \text{and} \quad t_2 - t_1 = t$$

When $u = 0$, $s = \frac{0 + v}{2} \times t = \frac{v}{2} \cdot t$

$$\therefore \quad s = \frac{v_1 t_1}{2} = \frac{v_2 t_2}{2}$$

Also $$s = \frac{1}{2}a_1 t_1^2 = \frac{1}{2}a_2 t_2^2$$

Hence $$\frac{v}{t} = \frac{v_1 - v_2}{t_2 - t_1} = \frac{\frac{2s}{t_1} - \frac{2s}{t_2}}{t_2 - t_1} = \frac{2s(t_2 - t_1)}{t_1 t_2 (t_2 - t_1)} = \frac{2s}{t_1 t_2}$$

$$= \sqrt{\frac{4s^2}{t_1^2 t_2^2}} = \sqrt{\frac{2s}{t_1^2} \cdot \frac{2s}{t_2^2}} = \sqrt{a_1 a_2}$$

or $$v = t\sqrt{a_1 a_2}.$$

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Problem 5. *The driver of a train moving at a speed v_1 observes another train at a distance d ahead of him on the same track moving in the same direction with a slower speed v_2. He applies the brakes and gives his train constant deceleration α. Show that if $d > \frac{(v_1 - v_2)^2}{2\alpha}$, there will be no collision and if $d < \frac{(v_1 - v_2)^2}{2\alpha}$, there will be collision.*

Solution. There will be no collision if the driver of the train moving at a speed v_1, reduces the speed of his train to v_2 before the two trains meet *i.e.*, the relative velocity of the two trains must be reduced to zero.

Initial relative velocity $= v_1 - v_2$

If s is the distance covered by first train before its relative velocity becomes zero, then

$$v^2 = u^2 + 2as \quad \text{or} \quad 0 = (v_1 - v_2)^2 - 2\alpha s$$

or $$s = \frac{(v_1 - v_2)^2}{2\alpha}$$

For no collision, $d > s$ or $d > \frac{(v_1 - v_2)^2}{2\alpha}$.

Problem 6. *A juggler maintains four balls in motion, making each in turn rise to a height of 20 m from his hand. With what velocity does he project them and where will the other three balls be at the instant when the fourth one is just leaving the hand ? Take $g = 10\ ms^{-2}$.*

Solution. *For upward motion of a ball :*

$v = 0,\ a = -10\ ms^{-2},\ s = 20\ m,\ u = ?,\ t = ?$

As $v^2 - u^2 = 2as$ $\quad\therefore\quad 0 - u^2 = -2 \times 10 \times 20$

or $$u = \mathbf{20\ ms^{-1}}$$

Also, $v = u + at$ $\quad\therefore\quad 0 = 20 - 10t$

or $$t = 2\ s$$

So the ball returns to the hand of the juggler after 4 s.

To maintain proper distance, the balls must be thrown up at an interval of $\frac{4}{4} = 1$ s.

When the fourth ball is in hand, the third ball has travelled for 1 s, second for 2 s and first for 3 s.

(i) For third ball, $s = ut + \frac{1}{2}at^2$

$$= 20 \times 1 - \tfrac{1}{2} \times 10 \times (1)^2 = \mathbf{15\ m}$$

Third ball will be 15 m above the ground *going upward.*

(ii) For second ball, $s = 20 \times 2 - \frac{1}{2} \times 10\,(2)^2 = \mathbf{20\ m}$

Second ball will be 20 m above the ground and will be *at rest.*

(iii) For first ball, $s = 20 \times 3 - \frac{1}{2} \times 10 \times (3)^2 = \mathbf{15\ m}$

First ball will be 15 m above the ground *going downward.*

Problem 7. *A bullet fired into a fixed target loses half of its velocity after penetrating 3 cm. How much further will it penetrate before coming to rest assuming that it faces constant resistance to motion ?* **[AIEEE 05]**

Solution. ***In first case.*** If u is initial velocity, then $v = u/2, s = 3$ cm

As $v^2 - u^2 = 2as$

$\therefore\quad (u/2)^2 - u^2 = 2as \quad$ or $\quad a = -u^2/8$

In second case : $v = 0,\ a = -u^2/8$

Initial velocity $= u/2$

$\therefore\quad 0^2 - (u/2)^2 = 2(-u^2/8)\,s$

or $$s = \mathbf{1\ cm}$$

Thus the bullet will penetrate a further distance of 1 cm before coming to rest.

Problem 8. *A car, starting from rest, accelerates at the rate f through a distance s, then continues at constant speed for some time t and then decelerates at the rate $f/2$ to come to rest. If the total distance is 5 s, then prove that $s = \frac{1}{2}ft^2$.* **[AIEEE 05]**

Solution. ***For accelerated motion.*** $u = 0,\ a = f,\ s = s$

As $v^2 - u^2 = 2as$

$\therefore\quad v_1^2 - 0^2 = 2fs \quad$ or $\quad v_1 = \sqrt{2fs}$

For uniform motion : $u = v_1 = \sqrt{2fs},\ t = t$

Distance travelled, $s_2 = ut = \sqrt{2fs}\,t$

For decelerated motion : $u = \sqrt{2fs},\ a = -f/2,\ v = 0$

As $v^2 - u^2 = 2as$,

$\therefore\quad 0^2 - (\sqrt{2fs})^2 = 2 \times (-f/2)\,s_3$

Distance travelled, $s_3 = 2s$

Given $s + s_2 + s_3 = 5s$

or $s + \sqrt{2fs}\,t + 2s = 5s$

or $\sqrt{2fs}\,t = 2s$

or $$s = \tfrac{1}{2}ft^2.$$

Problem 9. *A car moving with a speed of 50 kmh^{-1} can be stopped by brakes after atleast 6 m. What will be the minimum stopping distance, if the same car is moving at a speed of 100 kmh^{-1} ?* **[AIEEE 03 ; Delhi 10]**

Solution. ***In first case*** :

$u = 50\ km\ h^{-1} = 50 \times \frac{5}{18} = \frac{125}{9}\ ms^{-1}\ ;\ v = 0,\ s = 6$ m

As $v^2 - u^2 = 2as$

$\therefore\quad 0^2 - \left(\frac{125}{9}\right)^2 = 2a \times 6$

or $$a = -\frac{125 \times 125}{81 \times 2 \times 6} \approx -16\ ms^{-2}$$

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In second case :

$$u = 100 \text{ kmh}^{-1} = \frac{250}{9} \text{ ms}^{-1}$$

$$v = 0, \quad a = -16 \text{ ms}^{-2}$$

$$\therefore \quad 0^2 - \left(\frac{250}{9}\right)^2 = 2 \times (-16) \times s$$

or $$s = \frac{250 \times 250}{81 \times 32} = 24.1 \text{ m.}$$

Problem 10. *The relation between t and distance x is $t = ax^2 + bx$ where a and b are constants. Express the instantaneous acceleration in terms of instantaneous velocity.* **[AIEEE 05]**

Solution. Given : $t = ax^2 + bx$

$$\therefore \quad \frac{dt}{dx} = 2ax + b$$

Velocity, $$v = \frac{dx}{dt} = (2ax + b)^{-1}$$

$$\text{Acceleration} = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = v \frac{dv}{dx} \qquad \left[\because \frac{dx}{dt} = v\right]$$

$$= v \frac{d}{dx}(2ax + b)^{-1}$$

$$= v(-1)(2ax + b)^{-2} \cdot 2a$$

$$= -2a(2ax + b)^{-3} = -2av^3.$$

Problem 11. *A ball is released from the top of a tower of height h metres. It takes T seconds to reach the ground. What is the position of the ball in T / 3 seconds ?* **[AIEEE 04]**

Solution. As $s = ut + \frac{1}{2}gt^2$

$$\therefore \quad h = 0 \times T + \frac{1}{2} g T^2$$

or $$T = \sqrt{\frac{2h}{g}}$$

Distance covered in time $T/3$,

$$h' = 0 \times \frac{T}{3} + \frac{1}{2} g \left(\frac{T}{3}\right)^2$$

$$= \frac{gT^2}{18} = \frac{g}{18} \times \frac{2h}{g} = \frac{h}{9}$$

Position of the ball after time $T/3$,

$$= h - \frac{h}{9} = \frac{8h}{9}, \text{ above the ground.}$$

Problem 12. *Points P, Q and R are in a vertical line such that PQ = QR. A ball at P is allowed to fall freely. What is the ratio of the times of descent through PQ and QR ?* **[Central Schools 08]**

Solution. Let t_1 and t_2 be the times of descent through PQ and QR respectively.

Let $PQ = QR = h$.

Then $h = \frac{1}{2} g t_1^2$ and $2h = \frac{1}{2} g (t_1 + t_2)^2$

$$\therefore \quad \frac{1}{2} = \frac{t_1^2}{(t_1 + t_2)^2} \quad \text{or} \quad \frac{1}{\sqrt{2}} = \frac{t_1}{t_1 + t_2}$$

Hence $t_1 : t_2 = 1 : (\sqrt{2} - 1)$.

Guidelines to NCERT Exercises

3.1. *In which of the following examples of motion can the body be considered approximately a point object :*

(i) a railway carriage moving without jerks between two stations.

(ii) a monkey sitting on the top of a man cycling smoothly on a circular track.

(iii) a spinning cricket ball that turns sharply on hitting the ground, and

(iv) tumbling beaker that has slipped off the edge of a table ?

Ans.

(i) The carriage can be considered as the point object because the distance between two stations is much larger than the size of the carriage.

(ii) The monkey can be considered as a point object because its size is much smaller than the distance covered by the cyclist.

(iii) The spinning ball cannot be considered as point object because its size is quite appreciable as compared to the distance through which it turns on hitting the ground.

(iv) The tumbling beaker slipping off the edge of a table cannot be considered a point object because its size is not negligibly smaller than the height of the table.

3.2. *The position-time (x-t) graphs for two children A and B returning from their school O to their homes P and Q respectively are shown in Fig. 3.82. Choose the correct entries in the brackets below :*

(a) A / B lives closer to the school than B / A.

(b) A / B starts from the school earlier than B / A.

(c) A / B walks faster than B / A.

(d) A and B reach home at the (same/different) time.

(e) A / B overtakes B / A on the road (once/twice.)

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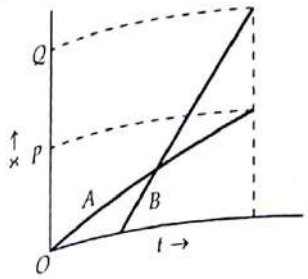

Fig. 3.82

Ans. (*a*) As $OP < OQ$, **A lives closer to the school than B.**

(*b*) When $x = 0, t = 0$ for A; while t has some finite value when B starts moving. So **A starts from the school earlier than B**

(*c*) Speed = Slope of x - t graph.

Slope of x - t graph for B > Slope of x - t graph for A.

$\therefore$ B walks faster than A

(*d*) Corresponding to the positions P and Q time t is same on time-axis.

$\therefore$ A and B reach home at the same time.

(*e*) The x - t graphs for A and B intersect each other only once. Since B starts from the school afterwards, so **B overtakes A on the road once.**

3.3. A woman starts from her home at 9.00 A.M. walks with a speed of 5 kmh^{-1} on a straight road upto her office 2.5 km away, stays at the office upto 5 P.M. and returns home by an auto with a speed of 25 kmh^{-1}. Choose suitable scales and plot the x-t graph of her motion.

Ans. ***For the journey from home to office :***

The time at which woman leaves for her office is 9 A.M.

As she travels with a speed of 5 kmh^{-1} and the distance of office is 2.5 km, hence time taken by her to reach office,

$$t = \frac{\text{distance}}{\text{speed}} = \frac{2.5 \text{ km}}{5 \text{ kmh}^{-1}} = \frac{1}{2} \text{ h}$$

Hence the time at which she reaches office is 9.30 A.M.

Between 9.30 A.M. to 5.00 P.M., she stays in her office i.e. at a distance of 2.5 km from her home.

For the return journey from office to home :

The time at which she leaves her office = 5 P.M.

Now she travels a distance of 2.5 km with a speed of 25 kmh^{-1}, hence the time taken

$$t' = \frac{2.5 \text{ km}}{25 \text{ kmh}^{-1}} = \frac{1}{10} \text{ h} = 6 \text{ minutes}$$

The time at which she reaches her home = 5.06 P.M.

The x-t graph of the woman's motion is shown in Fig. 3.83. Here 9 A.M. is regarded as the origin for time-axis and home as the origin for the position-axis.

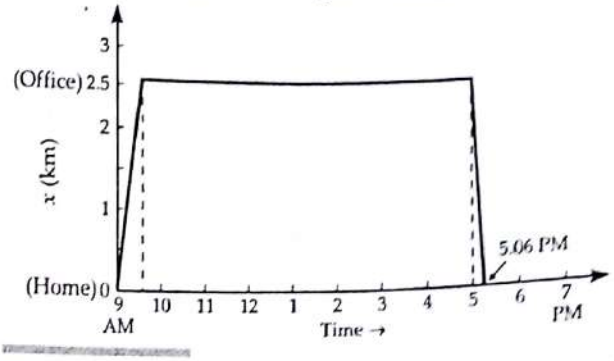

Fig. 3.83

3.4. *A drunkard walking in a narrow lane takes 5 steps forward and 3 steps backward, followed again by 5 steps forward and 3 steps backward, and so on. Each step is 1 m long and requires 1 s. Plot the x-t graph of his motion. Determine graphically and otherwise how long the drunkard takes to fall in a pit 13 m away from the start.*

Ans. (*a*) **Graphical method.** Taking the starting point as origin, the positions of the drunkard at various instants of time are given in the following table.

t (s)	0	5	8	13	16	21	24	29	32	37
x (m)	0	5	2	7	4	9	6	11	8	13

The position-time (x-t) graph for the motion of the drunkard is shown in Fig. 3.84. As is obvious from graph that the drunkard would take 37 s to fall in a pit 13 m away from the starting point.

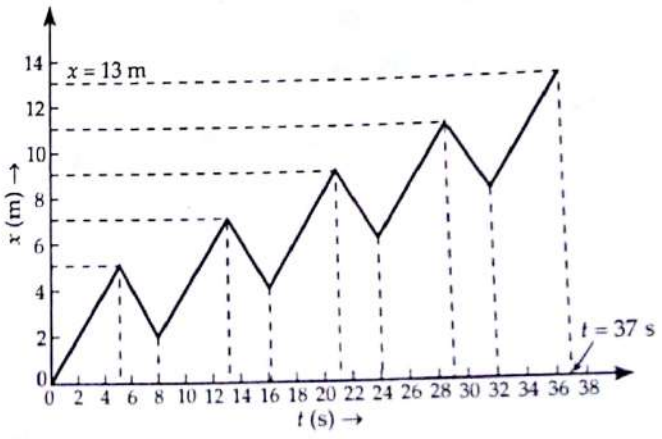

Fig. 3.84

(*b*) **Analytical method.** In each forward motion of 5 steps and backward motion of 3 steps, net distance covered = 5 – 3 = 2 m and time taken = 5 + 3 = 8 s.

$\therefore$ Time required to cover a distance of 8 m

$$= \frac{8}{2} \times 8 = 32 \text{ s}$$

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Remaining distance of the pit = 13 – 8 = 5 m

In next 5 s, as he moves 5 steps forward, he falls into the pit.

∴ Total time taken = 32 + 5 = **37 s.**

3.5. *A jet airplane travelling at the speed of 500 kmh^{-1} ejects its products of combustion at the speed of 1500 kmh^{-1} relative to the jet plane. What is the speed of the latter with respect to an observer on the ground ?*

Ans. Here speed of jet airplane,

$$v_1 = 500 \text{ kmh}^{-1}$$

Let v_2 be the speed of products w.r.t. the ground. Suppose the direction of motion of the jet plane is positive. Then the relative velocity of products w.r.t. jet plane is

$$v_2 - v_1 = -1500$$

or $$v_2 = v_1 - 1500 = 500 - 1500 = -1000 \text{ kmh}^{-1}$$

Negative sign shows that the direction of products of combustion is opposite to that of the jet plane.

∴ Speed of products of combustion w.r.t. ground

$$= \mathbf{1000 \text{ kmh}^{-1}}.$$

3.6. *A car moving along a straight highway with speed of 126 kmh^{-1} is brought to a stop within a distance of 200 m. What is the retardation of the car (assumed uniform), and how long does it take for the car to stop ?*

Ans. Here $u = 126 \text{ kmh}^{-1} = 126 \times \frac{5}{18} = 35 \text{ ms}^{-1}$,

$$v = 0, \quad s = 200 \text{ m}$$

As $v^2 - u^2 = 2as$

∴ $0^2 - 35^2 = 2a \times 200$

or $$a = -\frac{35 \times 35}{2 \times 200} = -\frac{49}{16} = -3.06 \text{ ms}^{-2}$$

∴ Retardation = **3.06 ms^{-2}.**

Required time,

$$t = \frac{v-u}{a} = \frac{0-35}{-49/16} = \frac{35 \times 16}{49} = \frac{80}{7} = \mathbf{11.43 \text{ s.}}$$

3.7. *Two trains A and B of length 400 m each are moving on two parallel tracks with a uniform speed of 72 kmh^{-1} in the same direction, with A ahead of B. The driver of B decides to overtake A and accelerates by 1 ms^{-2}. If after 50 s, the guard of B just brushes past the driver of A, what was the original distance between them ?*

Ans. Let x be the distance between the driver of train A and the guard of train B. Initially, both trains are moving in the same direction with the same speed of 72 km h^{-1}. So relative velocity of B w.r.t. $A = v_B - v_A = 0$. Hence the train B needs to cover a distance with

$$a = 1 \text{ ms}^{-2}, \quad t = 50 \text{ s}, \quad u = 0$$

As $$s = ut + \frac{1}{2}at^2$$

∴ $$x = 0 \times 50 + \frac{1}{2} \times 1 \times (50)^2 = \mathbf{1250 \text{ m.}}$$

3.8. *On a two-lane road, car A is travelling with a speed of 36 kmh^{-1}. Two cars B and C approach car A in opposite directions with a speed of 54 kmh^{-1} each. At a certain instant, when the distance AB is equal to AC, both being 1 km, B decides to overtake A before C does. What minimum acceleration of car B is required to avoid an accident ?*

Ans. At the instant when B decides to overtake A, the speeds of three cars are

$$v_A = 36 \text{ kmh}^{-1} = 36 \times \frac{5}{18} = +10 \text{ ms}^{-1}$$

$$v_B = +54 \text{ kmh}^{-1} = +54 \times \frac{5}{18} = +15 \text{ ms}^{-1}$$

$$v_C = -54 \text{ kmh}^{-1} = -15 \text{ ms}^{-1}$$

Relative velocity of C w.r.t. A,

$$v_{CA} = v_C - v_A = -15 - 10 = -25 \text{ ms}^{-1}$$

∴ Time that C requires to just cross A

$$= \frac{1 \text{ km}}{v_{CA}} = \frac{1000 \text{ m}}{25 \text{ ms}^{-1}} = 40 \text{ s}$$

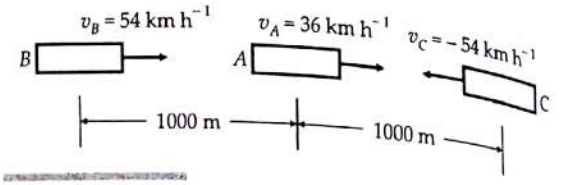

Fig. 3.85

In order to avoid the accident, B must overtake A in a time less than 40 s. So, for car B we have

Relative velocity of car B w.r.t. A,

$$v_{BA} = v_B - v_A = 15 - 10 = 5 \text{ ms}^{-1}$$

∴ $$s = 1 \text{ km} = 1000 \text{ m}, \quad u = 5 \text{ ms}^{-1}, t = 40 \text{ s}$$

As $$s = ut + \frac{1}{2}at^2$$

∴ $$1000 = 5 \times 40 + \frac{1}{2}a \times (40)^2$$

or $$1000 = 200 + 800a$$

or $$a = \mathbf{1 \text{ ms}^{-2}}$$

Thus 1 ms^{-2} is the minimum acceleration that car B requires to avoid an accident.

3.9. *Two towns A and B are connected by a regular bus service with a bus leaving in either direction every T min. A man cycling with a speed of 20 kmh^{-1} in the direction A to B notices that a bus goes past him every 18 min in the direction of his motion, and every 6 min in the opposite direction. What is the period T of the bus service and with what speed (assumed constant) do the buses ply on the road ?*

Ans. Let speed of each bus = v kmh^{-1}

For buses going from town A to B :

Relative speed of a bus in the direction of motion of the man $= (v - 20) \text{ kmh}^{-1}$

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buses plying in this direction go past the cyclist after every 18 min.

Distance covered $= (v - 20)\frac{18}{60}$ km

Since a bus leaves the town after every T min, so the above distance covered

$= v \times \frac{T}{60}$ km

$\therefore \quad (v - 20)\frac{18}{60} = v \times \frac{T}{60}$ ...(i)

For buses going from town B to A :

Relative speed of bus in the direction opposite to the motion of the man

$= (v + 20)$ kmh^{-1}

Buses going in this direction go past the cyclist after every 6 min, therefore

$(v + 20)\frac{6}{60} = v \times \frac{T}{60}$ ...(ii)

Dividing (i) by (ii), we get

$$\frac{(v - 20)\,18}{(v + 20)\,6} = 1$$

$$3v - 60 = v + 20$$

or $\quad v = \mathbf{40\ kmh^{-1}}$.

or

From equation (ii),

$$(40 + 20)\frac{6}{60} = \frac{40 \times T}{60}$$

$$T = \frac{60 \times 6}{40} = \mathbf{9\ min.}$$

or

3.10. *A player throws a ball upwards with an initial speed of 29.4 ms^{-1}.*

(i) *What is the direction of acceleration during the upward motion of the ball ?*

(ii) *What are the velocity and acceleration of the ball at the highest point of its motion ?*

(iii) *Choose the $x = 0$ and $t = 0$ to be the location and time of the ball at its highest point, vertically downward direction to be the positive direction of X-axis, and give the signs of position, velocity and acceleration of the ball during its upward, and downward motion.*

(iv) *To what height does the ball rise and after how long does the ball return to the player's hands ?*

(Take $g = 9.8$ ms^{-2}, and neglect air resistance).

Solution. (i) The ball moves under the effect of gravity. The direction of acceleration due to gravity is always *vertically downwards.*

(ii) At the highest point, velocity of the ball = **0**.

Acceleration = Acceleration due to gravity 'g'

$= \mathbf{9.8\ ms^{-2}}$ (vertically downwards)

(iii) When the highest point is chosen as the location for $x = 0$ and $t = 0$ and vertically downward direction to be the positive direction of X-axis :

During upward motion. Position is positive, velocity is negative and acceleration is positive.

During downward motion. Position is positive, velocity is positive and acceleration is positive.

(iv) *For upward motion.*

$u = -29.4$ ms^{-1}, $g = +9.8$ ms^{-2}, $v = 0$

If s is the height to which the ball rises, then

$$v^2 - u^2 = 2as$$

or $\quad 0^2 - (-29.4)^2 = 2 \times 9.8 \times s$

or $\quad s = -\frac{(29.4)^2}{2 \times 9.8} = \mathbf{-44.1\ m}$

Negative sign shows that the distance is covered in upward direction.

If the ball reaches the highest point in time t, then

$$v = u + at$$

or $\quad 0 = -29.4 + 9.8\,t$

or $\quad t = \frac{29.4}{9.8} = 3$ s

As time of ascent = time of descent

$\therefore$ Total time taken = $3 + 3 = \mathbf{6}$ s.

3.11. *Read each statement below carefully and state with reasons and examples, if it is true or false. A particle in one-dimensional motion :*

(a) *with zero speed at an instant may have non-zero acceleration at the instant,*

(b) *with zero speed may have non-zero velocity,*

(c) *with constant speed must have zero acceleration,*

(d) *with positive value of acceleration must be speeding up.*

Ans.

(a) **True.** When a body begins to fall freely under gravity, its speed is zero but it has non-zero acceleration of 9.8 ms^{-2}.

(b) **False.** Speed is the magnitude of velocity and the magnitude of non-zero velocity cannot be zero.

(c) **True.** When a particle moves with a constant speed in the same direction, neither the magnitude nor the direction of velocity changes and so acceleration is zero. In case a particle rebounds instantly with the same speed, its acceleration will be infinite which is physically not possible.

(d) **False.** If the initial velocity of a body is negative, then even in case of positive acceleration, the body speeds down. A body speeds up when the acceleration acts in the direction of motion.

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3.12. *A ball is dropped from a height of 90 m on a floor. At each collision with the floor, the ball loses one-tenth of its speed. Plot the speed-time graph of its motion between t = 0 to 12 s.*

Ans. (*i*) Time taken by the ball to fall through a height of 90 m is obtained as follows :

$$x = v(0)t + \frac{1}{2}gt^2$$

$$90 = 0 + \frac{1}{2} \times 9.8 t^2$$

or $$t = \sqrt{\frac{2 \times 90}{9.8}} = \frac{30}{7} \text{ s} = 4.3 \text{ s}$$

Now $$v(t) = v(0) + gt$$

$$\therefore \quad v(4.3) = 0 + 9.8 \times \frac{30}{7} = 42 \text{ ms}^{-1}$$

From time $t = 0$ to $t = 4.3$ s,

$$v(t) = gt = 9.8t \quad \text{or} \quad v(t) \propto t$$

In this duration speed increases linearly with time t from 0 to 42 ms^{-1} during the downward motion of the ball and this speed-time variation has been shown by straight line OA in Fig. 3.86.

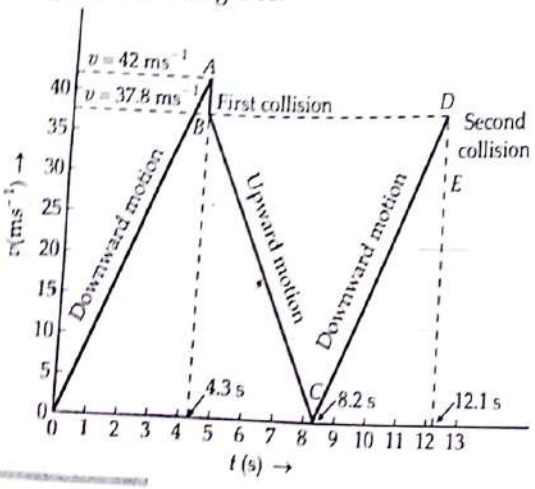

Fig. 3.86

(*ii*) At first collision with the floor, speed lost by ball

$$= \frac{1}{10} \times 42 = 4.2 \text{ ms}^{-1}.$$

Thus, the ball rebounds with a speed of $42 - 4.2 = 37.8 \text{ ms}^{-1}$. For the further upward motion, the speed at any instant t is given by

$$v(t) = v(0) - gt = 37.8 - 9.8 \times t$$

Now the speed decreases linearly with time and becomes zero after time

$$t = \frac{37.8}{9.8} = 3.9 \text{ s}$$

Thus, the ball reaches the highest point again after time $t = 4.3 + 3.9 = 8.2$ s from the start. Straight line BC represents the speed-time graph for this upward motion.

(*iii*) At highest point, speed of ball is zero. It again starts falling. At any instant t, its speed is given by

$$v(t) = 0 + 9.8t$$

Again the speed of the ball increases linearly with time t from 0 to 37.8 ms^{-1} (initial speed of the previous upward motion) in the next time-interval of 3.9 s. Total time taken from the start = 4.3 + 3.9 + 3.9 = 12.1 s. This part of motion has been shown by straight line CD.

Here, we have assumed a negligible time of collision between the ball and the floor.

3.13. *Explain clearly, with examples, the distinction between :*

(a) magnitude of displacement (sometimes called distance) over an interval of time, and the total length of path covered by a particle over the same interval ;

(b) magnitude of average velocity over an interval of time, and the average speed over the same interval. [Average speed of a particle over an interval of time is defined as the total path length divided by the time interval].

(c) Show in both (a) and (b) that the second quantity is either greater than or equal to the first. When is the equality sign true ? [For simplicity, consider one-dimensional motion only].

Solution.

(*a*) Suppose a body moves from point A and to point B along a straight path and then returns back to the point A along the same path.

As the body returns back to its initial position A, so magnitude of displacement = 0.

Distance covered

= Total length of the path covered

$= AB + BA = AB + AB = 2AB.$

(*b*) In the above example, suppose the body takes time t to complete the whole journey. Then

Magnitude of average velocity

$$= \frac{\text{Magnitude of displacement}}{\text{Time taken}} = \frac{0}{t} = 0.$$

$$\text{Average speed} = \frac{2AB}{t}.$$

(*c*) In example (*a*), distance covered > magnitude of displacement.

In example (*b*), average speed > magnitude of average velocity.

The sign of equality will hold when the body moves along a straight line path in a fixed direction.

3.14. *A man walks on a straight road from his home to a market 2.5 km away with a speed of 5 km h^{-1}. Finding the market closed, he instantly turns and walks back home with a speed of 7.5 km h^{-1}. What is the*

(a) magnitude of average velocity, and

(b) average speed of the man over the interval of time

(i) 0 to 30 min, (ii) 0 to 50 min, (iii) 0 to 40 min ?

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Ans. ***Case (i) : 0 to 30 min***

Speed = 5 kmh^{-1}

Distance covered in 30 min

$= 5 \text{ kmh}^{-1} \times \frac{30}{60} \text{ h} = 2.5 \text{ km}$

Displacement covered = 2.5 km

(*a*) Average velocity

$= \frac{\text{Displacement}}{\text{Time}} = \frac{2.5 \text{ km}}{30/60 \text{ h}} = \mathbf{5 \text{ kmh}^{-1}}.$

(*b*) Average speed

$= \frac{\text{Distance}}{\text{Time}} = \frac{2.5 \text{ km}}{30/60 \text{ h}} = \mathbf{5 \text{ kmh}^{-1}}.$

Case (ii) : 0 to 50 min

Displacement covered in first 30 min in going to market

$= 5 \text{ kmh}^{-1} \times \frac{30}{60} \text{ h} = 2.5 \text{ km}$

Displacement covered in next 20 min in coming back to home

$= 7.5 \text{ kmh}^{-1} \times \frac{20}{60} \text{ h} = 2.5 \text{ km}$

Net displacement = 2.5 – 2.5 = 0

Total distance covered = 2.5 + 2.5 = 5 km

(*a*) Average velocity

$= \frac{\text{Net displacement}}{\text{Time taken}} = \frac{0}{50/60 \text{ h}} = \mathbf{0}.$

(*b*) Average speed

$= \frac{\text{Total distance}}{\text{Time taken}} = \frac{5 \text{ km}}{50/60 \text{ h}} = \mathbf{6 \text{ kmh}^{-1}}.$

Case (iii) : 0 to 40 min

Displacement covered in first 30 min in going to market = 2.5 km

Displacement covered in next 10 min in coming back to home

$= 7.5 \text{ kmh}^{-1} \times \frac{10}{60} \text{ h} = 1.25 \text{ km}$

Net displacement = 2.5 – 1.25 = 1.25 km

Total distance travelled = 2.5 + 1.25 = 3.75 km

(*a*) Average velocity

$= \frac{\text{Net displacement}}{\text{Time taken}}$

$= \frac{1.25 \text{ km}}{40/60 \text{ h}} = \mathbf{1.875 \text{ kmh}^{-1}}.$

(*b*) Average speed

$= \frac{\text{Total distance}}{\text{Time taken}} = \frac{3.75 \text{ km}}{40/60 \text{ h}} = \mathbf{5.625 \text{ kmh}^{-1}}.$

3.15. *The instantaneous speed is always equal to the magnitude of instantaneous velocity. Why ?*

Ans. Instantaneous speed,

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

Instantaneous velocity,

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t}$$

For an arbitrary small interval of time (Δt), the magnitude of displacement $|\Delta \vec{x}|$ is equal to the length of the path Δx. So instantaneous speed is always equal to the magnitude of instantaneous velocity.

3.16. *Look at the graphs (a) to (d) [Fig. 3.87] carefully and state, with reasons, which of these cannot possibly represent one-dimensional motion of a particle.*

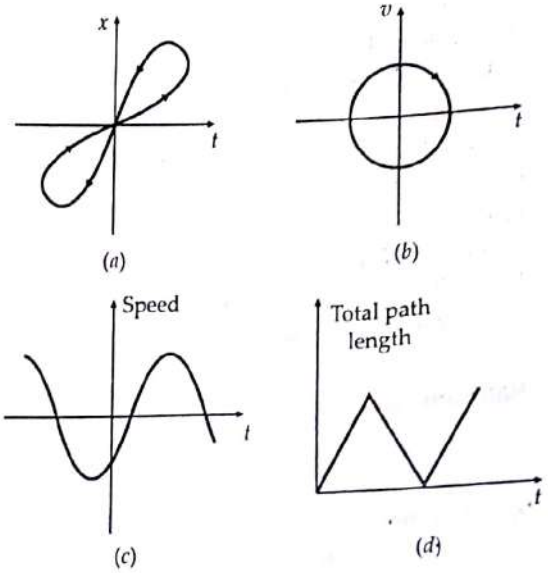

Fig. 3.87

Ans. All the four graphs are impossible.

(*a*) If we draw a line parallel to the position-axis, it intersects the x-t graph at two points. This means that the particle occupies two different positions at the same time which is not possible.

(*b*) If we draw a line parallel to the velocity-axis, it meets the circle in two points. This means the particle has two velocities (positive and negative) in opposite directions at the same time. This is not possible.

(*c*) The graph indicates that speed is negative in some time intervals. But speed cannot be negative.

(*d*) The graph indicates that the total path length decreases after a certain time. But total path length can never decrease with time.

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3.17. *Fig. 3.88 shows the x-t plot of one-dimensional motion of a particle. Is it correct to say from the graph that the particle moves in a straight line for $t < 0$ and on a parabolic path for $t > 0$? If not, suggest a suitable physical context for this graph.*

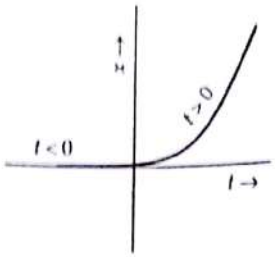

Fig. 3.88

Ans. No, it is wrong to say that the particle moves in a straight line for $t < 0$ and on a parabolic path for $t > 0$, because a position-time (x - t) graph does not represent the trajectory of a moving particle.

This graph can represent the motion of a freely falling particle dropped from a tower when we take its initial position as $x = 0$, at $t = 0$.

3.18. *A police van moving on a highway with a speed of 30 kmh^{-1} fires a bullet at a thief's car speeding away in the same direction with a speed of 192 kmh^{-1}. If the muzzle speed of the bullet is 150 ms^{-1}, with what speed does the bullet hit the thief's car ?*

Solution. Speed of police van,

$$v_p = 30\ \text{kmh}^{-1} = \frac{25}{3}\ \text{ms}^{-1}$$

Speed of bullet,

$$v_b = 150\ \text{ms}^{-1}$$

Speed of the police van is shared by the bullet.

$\therefore$ Relative speed of bullet w.r.t. ground

$$= v_b + v_p$$

$$= 150 + \frac{25}{3}$$

$$= \frac{475}{3}\ \text{ms}^{-1}$$

Speed of thief's car,

$$v_t = 192\ \text{kmh}^{-1} = \frac{160}{3}\ \text{ms}^{-1}$$

Relative speed of bullet w.r.t. thief's car

$$= (v_b + v_p) - v_t$$

$$= = \frac{475}{3} - \frac{160}{3}$$

$$= 105\ \text{ms}^{-1}.$$

Hence the speed of the bullet with which it hits the thief's car = **105 ms^{-1}**.

3.19. *Suggest a suitable physical situation for each of the following graphs [Fig. 3.89] :*

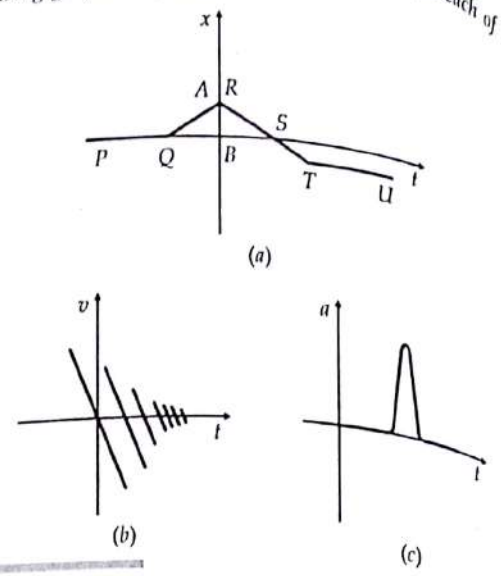

Fig. 3.89

Ans.

(*a*) A ball lies at rest on a smooth floor, as indicated by straight line PQ of the x-t graph. It is kicked towards a wall. Slope of line QR gives speed of kicking. The ball rebounds from the wall with a reduced speed (given by the slope of line RS). At S, the sign of position coordinate changes sign, it indicates that the ball moves to the opposite wall which stops it. The line TU indicates the final rest position of the ball.

(*b*) The velocity-time graph represents the motion of a ball thrown up with some initial velocity, it hits the ground and gets rebounded with a reduced speed. It goes on hitting the ground and after each hit its speed decreases unit it becomes zero and the ball comes to rest finally.

(*c*) The acceleration-time graph represents the motion of a uniformly moving cricket ball turned back by hitting it with a bat for a very short time interval.

3.20. *Figure 3.90 gives the x-t plot of a particle executing one-dimensional simple harmonic motion. Give the signs of position, velocity and acceleration variables of the particle at $t = 0.3\ s,\ 1.2s,\ -1.2s$.*

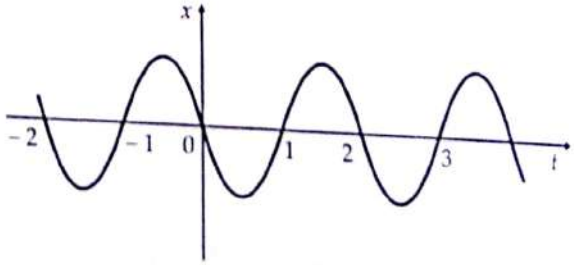

Fig. 3.90

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Ans. The acceleration of a particle executing S.H.M. is given by $a = -\omega^2 x$

where ω (angular frequency) is a constant.

At time $t = 0.3$ s

As is obvious from the graph, $x < 0$

As slope of x-t graph is negative, so $v < 0$

As $a = -\omega^2 x$, so $a > 0$.

At time $t = 1.2$ s

As is obvious from the graph, $x > 0$

As slope of $x - t$ graph is positive, so $v > 0$

As $a = -\omega^2 x$, so $a < 0$.

At time $t = -1.2$ s

As is obvious from the graph, $x < 0$

As slope of $x - t$ graph is positive, so $v > 0$

As $a = -\omega^2 x$, so $a > 0$.

3.21. *Figure 3.91 gives the x-t plot of a particle in one-dimensional motion. Three different equal intervals of time are shown. In which interval is the average speed greatest, and in which is it the least ? Give the sign of average velocity for each interval.*

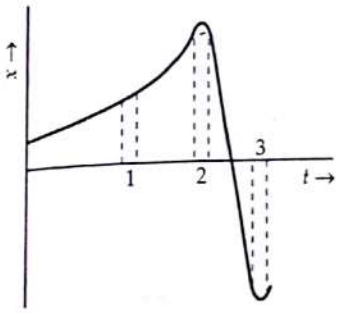

Fig. 3.91

Solution. Slope of x-t graph in a small time interval = Average speed in that interval

As the slope of x-t is greatest in interval 3 and least in interval 1, so **the average speed is greatest in interval 3 and least in interval 1.**

As the slope of x-t is positive in intervals 1 and 2 and negative in interval 3, **so average velocity is positive in intervals 1 and 2 and negative in interval 3.**

3.22. *Figure 3.92 gives a speed-time graph of a particle in motion along a constant direction. Three equal intervals of time are shown. (a) In which interval is the average acceleration greatest in magnitude ? (b) In which interval is the average speed greatest ? (c) Choosing the positive direction as the constant direction of motion, give the signs of v and a in the three intervals. (d) What are the accelerations at the points A, B, C and D ?*

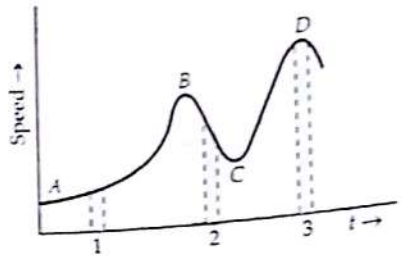

Fig. 3.92

Ans.

(*a*) As the change in speed is greatest in interval 2, so **magnitude of average acceleration is greatest in interval 2.**

(*b*) Obviously from the graph, **average speed is greatest in interval 3.**

(*c*) v is positive in all three intervals. a is positive in the intervals 1 and 3 as speed is increasing in these intervals. But a is negative in interval 2 as speed is decreasing in this interval.

(*d*) At the points A, B, C and D, the v-t graph is parallel to the time-axis or has a zero slope, so $a = 0$ at all these points.

3.23. *A three-wheeler starts from rest, accelerates uniformly with* $1\ ms^{-2}$ *on a straight road for 10 s, and then moves with uniform velocity. Plot the distance covered by the vehicle during the nth second* ($n = 1, 2, 3\ldots$) *versus n. What do you expect this plot to be during accelerated motion : a straight line or a parabola ?*

Ans. Distance travelled in nth second,

$$s_{n\,th} = u + \frac{a}{2}(2n - 1)$$

As $u = 0$, $a = 1\ ms^{-2}$

$$\therefore \quad s_{n\,th} = 0 + \frac{1}{2}(2n - 1)$$

$$= \frac{1}{2}(2n - 1)\ m$$

Thus the distances travelled by the three-wheeler at the end of each second are given by

n(s)	1	2	3	4	5	6	7	8	9	10
$s_{n\,th}$ (m)	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5

Now, velocity of the three-wheeler at the end of 10th s is given by

$$v = u + at = 0 + 1 \times 10 = 10\ ms^{-1}.$$

Upto $n = 10$ s, the motion is accelerated and the graph between s_{nth} and n is a straight line AB inclined to

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time-axis as shown in Fig. 3.93. After 10th second, the three-wheeler moves with uniform velocity of 10 ms^{-1}, so graph is straight line BC parallel to time-axis.

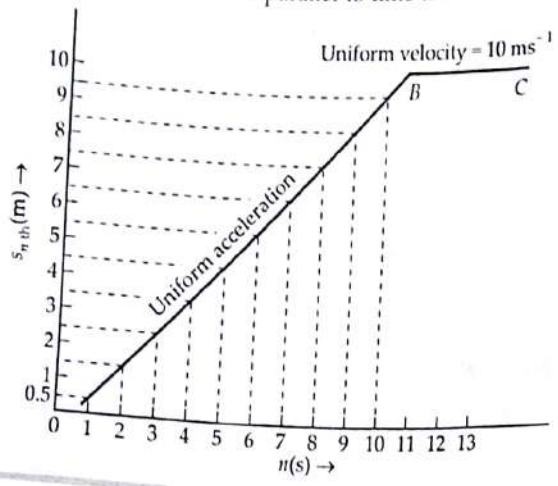

Fig. 3.93

3.24. *A boy standing on a stationary lift (open from above) throws a ball upwards with the maximum initial speed he can, equal to 49 ms^{-1}. (i) How much time does the ball take to return to his hands ? (ii) If the lift starts moving up with a uniform speed of 5 ms^{-1}, and the boy again throws the ball up with the maximum speed he can, how long does the ball take to return to his hands ?*

Ans. (*i*) *When the lift is stationary* : For upward motion of the ball, we have

$$u = 49\ ms^{-1},\quad g = -9.8\ ms^{-2},\quad v = 0,\quad t = ?$$

As $\quad v = u + at$

$\therefore \quad 0 = 49 - 9.8t \quad$ or $\quad t = \dfrac{49}{9.8} = 5\ s.$

As time of ascent = time of descent

$\therefore$ Total time taken = 5 + 5 = **10 s.**

(*ii*) *When the lift moves up with uniform speed* : The uniform speed of the lift does not change the relative velocity of the ball w.r.t. the boy *i.e.* it still remains 49 ms^{-1}.

Hence total time after which the ball returns = **10 s.**

3.25. *On a long horizontally moving belt, a child runs to and fro with a speed 9 kmh^{-1} (with respect to the belt) between his father and mother located 50 m apart on the moving belt. The belt moves with a speed of 4 kmh^{-1}. For an observer on a stationary platform outside, what is the*

(*i*) *speed of the child running in the direction of motion of the belt,*

(*ii*) *speed of the child running opposite to the direction of motion of the belt, and*

(*iii*) *time taken by the child in* (*i*) *and* (*ii*) ?

Which of the answers alter if motion is viewed by one of the parents ?

Ans. (*i*) Speed of the child running in the direction of motion of the belt

$$= (9 + 4)\ kmh^{-1} = \mathbf{13\ kmh^{-1}}.$$

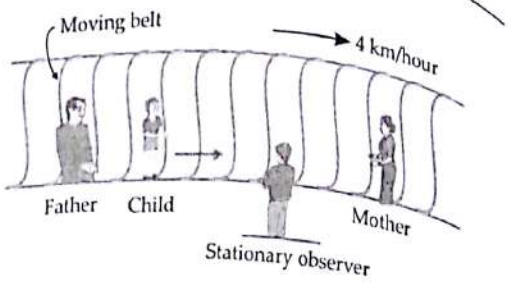

Fig. 3.94

(*ii*) Speed of the child running opposite to the direction of motion of the belt

$$= (9 - 4)\ kmh^{-1} = \mathbf{5\ kmh^{-1}}.$$

(*iii*) Speed of the child w.r.t. either parent

$$= 9\ kmh^{-1} = 2.5\ ms^{-1}$$

Distance to be covered = 50 m

$$\text{Time taken} = \frac{50}{2.5} = \mathbf{20\ s.}$$

If the motion is viewed by one of the parents, answers to (*i*) and (*ii*) are altered but answer to (*iii*) remains unchanged.

3.26. *Two stones are thrown up simultaneously from the edge of a cliff 200 m high with initial speeds of 15 ms^{-1} and 30 ms^{-1}. Verify that the following graph correctly represents the time variation of the relative position of the second stone with respect to the first. Neglect air resistance and assume that the stones do not rebound after hitting the ground. Take $g = 10\ ms^{-2}$. Give the equations for the linear and curved parts of the plot.*

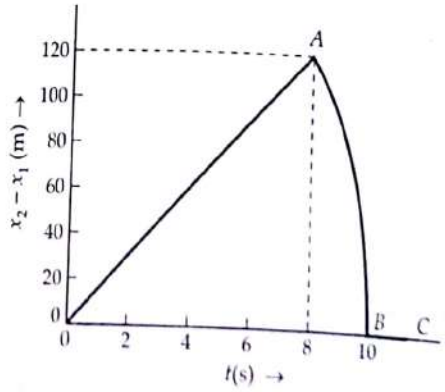

Fig. 3.95

Ans. $\quad x(t) = x(0) + v(0)t + \dfrac{1}{2}gt^2$

If we take origin for position measurement on the ground, then the positions of the two stones at any instant t will be

$$x_1 = 200 + 15t - \frac{1}{2} \times 10t^2 \qquad ...(1)$$

$$x_2 = 200 + 30t - \frac{1}{2} \times 10t^2 \qquad ...(2)$$

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When the first stone hits the ground,

$$x_1 = 0$$

$$200 + 15t - 5t^2 = 0$$

or $$5t^2 - 15t - 200 = 0$$

or $$t^2 - 3t + 40 = 0$$

or $$\therefore \quad t = \frac{3 \pm \sqrt{9 + 160}}{2} = \frac{3 \pm 13}{2} = 8\text{ s or } -5\text{ s}$$

As time cannot be negative, so $t = 8$ s

i.e. the first stone hits the ground after 8 s.

From (1) and (2), the relative position of second stone *w.r.t.* first is given by

$$x_2 - x_1 = 15t$$

As there is a linear relationship between $x_2 - x_1$ and t, so the graph is straight line OA upto $t = 8$ s. After $t = 8$ s, only the second stone is in motion. So the graph is parabolic (AB) in accordance with quadratic equation,

$$x_2 = 200 + 30t - 5t^2$$

The second stone will hit the ground, when $x_2 = 0$

$$200 + 30t - 5t^2 = 0$$

or On solving, $$t = 10\text{ s}$$

After $t = 10$ s, the separation between the balls is zero, which explains the part BC of the graph.

3.27. *The speed-time graph of a particle moving along a fixed direction is shown in Fig. 3.96. Obtain the distance travelled by the particle between (i) $t = 0$ to 10 s (ii) $t = 2$ to 6 s. What is the average speed of the particle in intervals in (i) and (ii) ?*

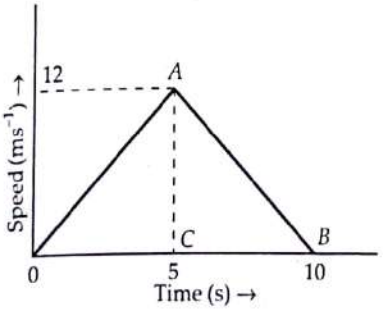

Fig. 3.96

Ans. (*i*) Distance travelled by the particle between $t = 0$ to 10 s is given by

$$s = \text{Area of } \Delta OAB = \frac{1}{2} OB \times AC$$

$$= \frac{1}{2} \times 10 \times 12 = 60\text{ m}$$

Average speed

$$= \frac{\text{Total distance covered}}{\text{Total time taken}} = \frac{60}{10} = \mathbf{6\ ms^{-1}}.$$

(*ii*) Acceleration of the particle during journey OA is given by

$$v = u + at \quad \text{or} \quad 12 = 0 + a \times 5$$

or $$a = +2.4\text{ ms}^{-2}$$

Similarly, acceleration of the particle during journey AB is given by

$$v = u + at \quad \text{or} \quad 0 = 12 + a \times 5$$

or $$a = -2.4\text{ ms}^{-2}$$

Velocity of the particle after 2 s from start will be

$$v = u + at = 0 + 2.4 \times 2 = 4.8\text{ ms}^{-1}$$

$\therefore$ Distance covered by the particle between $t = 2$ to 5 s (in 3 s) is given by

$$s_1 = ut + \frac{1}{2} at^2$$

$$= 4.8 \times 3 + \frac{1}{2} \times 2.4 \times 3^2 = 25.2\text{ m}$$

Distance covered by the particle in $t = 5$ to 6 s (in 1 s) is given by

$$s_2 = ut + \frac{1}{2} at^2$$

$$= 12 \times 1 + \frac{1}{2} \times (-2.4) \times 1^2 = 10.8\text{ m}$$

Total distance travelled in $t = 2$ to 6 s,

$$s = s_1 + s_2 = 25.2 + 10.8 = 36\text{ m}$$

Average speed in the interval $t = 2$ to 6 s

$$= \frac{\text{Total distance covered}}{\text{Total time taken}} = \frac{36}{4} = \mathbf{9\ ms^{-1}}.$$

3.28. *The velocity-time graph of particle in one-dimensional motion is shown in Fig. 3.97.*

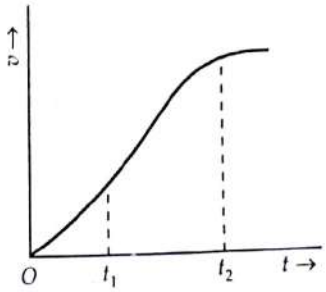

Fig. 3.97

Which of the following formulae are correct for describing the motion of the particle over the time interval t_1 to t_2 :

(*a*) $x(t_2) = x(t_1) + v(t_1) \times (t_2 - t_1) + \frac{1}{2} a (t_2 - t_1)^2$

(*b*) $v(t_2) = v(t_1) + a(t_2 - t_1)$

(*c*) $v_{av} = \{x(t_2) - x(t_1)\} / (t_2 - t_1)$

(*d*) $a_{av} = \{v(t_2) - v(t_1)\} / (t_2 - t_1)$

(*e*) $x(t_2) = x(t_1) + v_{av}(t_2 - t_1) + (\frac{1}{2}) a_{av} (t_2 - t_1)^2$.

(*f*) $x(t_2) - x(t_1)$ = *area under the v-t curve bounded by the t-axis and the dotted line shown.*

Ans. (*a*) It is not correct because in the time interval between t_1 and t_2, a is not constant.

(*b*) This relation is also not correct for the same reason as in (*a*).

(*c*) This relation is correct.

(*d*) This relation is also correct.

(*e*) This relation is not correct because average acceleration cannot be used in this relation.

(*f*) This relation is correct.

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Text Based Exercises

Type A : Very Short Answer Questions — 1 Mark Each

1. Are rest and motion absolute or relative terms ?
2. Can an object be at rest as well as in motion at the same time ?
3. Is it true that a body is at rest in a frame within which it has been fixed ?
4. Under what condition can an object in motion be considered a point object ?
5. Give an example of a physical phenomenon in which earth cannot be regarded as a point mass.
6. Under what condition will the distance and displacement of moving object have the same magnitude ? [Chandigarh 08]
7. A bullet fired vertically upwards falls at the same place after some time. What is the displacement of the bullet ?
8. A particle is moving along a circular track of radius r. What is the distance traversed by particle in half revolution ? What is its displacement ?
9. Will the displacement of an object change on shifting the position of origin of the coordinate system ? [Himachal 06C]
10. What does the speedometer of a car measure-average speed or instantaneous speed ?
11. What is the numerical ratio of velocity to speed of an object ?
12. A ball hits a wall with a velocity of $30\ ms^{-1}$ and rebounces with the same velocity. What is the change in its velocity ?
13. Why does time occur twice in the unit of acceleration ?
14. Give an example which shows that a positive acceleration can be associated with a slowing down object.
15. Give an example which shows that a negative accele ration can be associated with a speeding up object.
16. Is the acceleration of a car greater than when the accelerator is pushed to the floor or when brake pedal is pushed hard ?
17. The v-t graphs of two objects make angles of $30°$ and $60°$ with the time-axis. Find the ratio of their accelerations.
18. Is it possible that your cycle has a northward velocity but southward acceleration ? If yes, how ?
19. If the instantaneous velocity of a particle is zero, will its instantaneous acceleration be necessarily zero ?
20. A woman standing on the edge of a cliff throws a ball straight up with a speed of $8\ kmh^{-1}$ and then throws another ball straight down with a speed of $8\ kmh^{-1}$ from the same position. What is the ratio of the speeds with which the balls hit the ground ?
21. A body travels, with uniform acceleration a_1 for time t_1 and with uniform acceleration a_2 for time t_2. What is the average acceleration ?
22. What is the nature of position-time graph for a uniform motion ? [Chandigarh 03]
23. What does the slope of position-time graph indicate ? [Himachal 07]
24. What is the nature of velocity-time graph for uniform motion ?
25. If the displacement-time graph for a particle is parallel to displacement axis, what should be the velocity of the particle ?
26. If the displacement-time graph for a particle is parallel to time-axis, how much is the velocity of the particle ?
27. How can the distance travelled be calculated from the velocity-time graph in a uniform one-dimensional motion ?
28. Suppose the acceleration of a body varies with time. Then what does the area under its acceleration-time graph for any time interval represent ?
29. What is the area under the velocity-time curve in the case of a body projected vertically upwards from the ground after reaching the ground ?
30. Can a particle with zero acceleration speed up ?
31. Is the formula : $s = vt - \frac{1}{2}at^2$ correct, when the body is moving with uniform acceleration ?
32. A body projected up reaches a point P of its path at the end of 4 seconds and the highest point at the end of 12 seconds. After how many seconds from the start will it reach P again ?
33. Can a body subjected to a uniform acceleration always move in a straight line ?

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36. Suggest a suitable physical situation for the graph shown in Fig. 3.89(*b*) on page 3.50.

37. A uniformly moving cricket ball is turned back by hitting it with a bat for a very short time- interval. Suggest acceleration-time graph for the situation.

38. The position coordinate of a moving particle is given by $x = 6 + 18t + 9t^2$ (x in metres and t in seconds). What is its velocity at $t = 2$ sec. ? **[Central Schools 14]**

39. A player throws a ball upwards with an initial speed of 29.4 ms^{-1}. What are the velocity and acceleration of the ball at the highest point of its motion ? **[Delhi 05 ; Central Schools 13, 14]**

40. Under what condition will the distance and displacement of a moving object have the same magnitude ? **[Chandigarh 08]**

41. State the condition when the magnitude of velocity and speed of an object are equal. **[Delhi 08]**

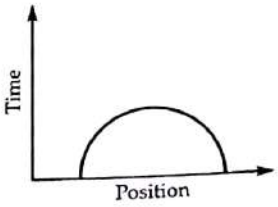
42. What does the slope of velocity-time graph represent ? **[Delhi 10 ; Central Schools 13]**

43. What does the area under velocity-time graph represent ? **[Central Schools 13]**

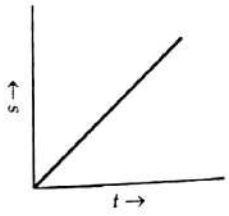
44. What does the area under acceleration-time graph represent ?

45. The displacement-time graphs for the two particles A and B are straight lines inclined at angles of 30° and 45° with the time-axis. What is the ratio of the velocities $v_A : v_B$? **[Delhi 1]2]**

46. Is the time variation of position, shown in the adjacent figure, observed in nature ? **[Central Schools 12]**



47. Displacement-time graph of any object is shown in the adjacent figure. Draw velocity-time graph for this motion. **[Delhi 14 ; Central Schools 14]**



Answers

1. Yes, both rest and motion are relative terms.
2. Yes. A body may be at rest relative to one object and at the same time it may be in motion relative to another object.
3. Yes.
4. An object can be considered a point object if its size is much smaller than the distance travelled by it.
5. Solar or Lunar eclipse.
6. When the object moves along a straight line in the same fixed direction.
7. Zero.
8. Distance travelled = πr.
   Displacement covered = $2r$.
9. No, the displacement of the object will remain unaltered even on shifting the position of the origin.
10. The speedometer measures the instantaneous speed of the car.
11. Less than or equal to one.
12. Change in velocity = $v - u = -30 - 30 = -60$ **ms^{-1}**.
13. Acceleration
$$= \frac{\text{Change in velocity}}{\text{Time taken}} = \frac{\text{Displacement / Time taken}}{\text{Time taken}}$$
$$= \frac{\text{Displacement}}{\text{Time taken}^2}$$
Hence time occurs twice in the unit of acceleration.
14. An object with positive acceleration is slowing down if its initial velocity is negative.
15. An object in simple harmonic motion speeds up while moving from an extreme position to the mean position but its acceleration is negative.
16. Acceleration is greater in the second case, because car suddenly comes to halt, the rate of change of velocity is large.
17. $\frac{a_1}{a_2} = \frac{\text{Slope of } v\text{-}t \text{ graph of first object}}{\text{Slope of } v\text{-}t \text{ graph of second object}}$
$$= \frac{\tan 30°}{\tan 60°} = \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3} = 1:3.$$
18. Suppose brakes are applied to a cycle moving northward. At that instant, it has a northward velocity and a southward acceleration.
19. No. When a stone is thrown vertically upwards, its velocity is zero at the highest point but it has a non-zero acceleration of 9.8 ms^{-2} at the same instant.
20. Both the balls will hit the ground with the same speed, so ratio of their speeds = 1 : 1 This is because the first fall will cross the point of projection with the same speed of 8 kmh^{-1} while moving downward.
21. Average acceleration = $\frac{a_1 t_1 + a_2 t_2}{t_1 + t_2}$.

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22. For uniform motion, the position-time graph is a straight line inclined to time-axis.
23. The slope of position-time graph gives velocity of the object.
24. For uniform motion, velocity-time graph is a straight line parallel to time-axis.
25. Infinity.
26. Zero.
27. The distance travelled by an object in any time interval can be determined by finding area between the velocity-time graph and time-axis for the given time interval.
28. Area under acceleration time graph for any time interval
= Change of velocity of the body in that interval.
29. Zero.
30. Not possible.
31. Yes.

$$s = \left(\frac{u+v}{2}\right) \times t = \left(\frac{v - at + v}{2}\right) \times t = vt - \frac{1}{2}at^2.$$

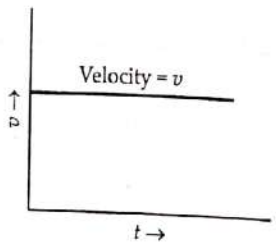
32. $12 + (12 - 4) = 20$ seconds.
33. No. The path of a projectile is a parabola even when it has a uniform acceleration.
34. Refer to the answer of Exercise 3.19(*b*) on page 3.50.
35. The given velocity-time graph shown in Fig. 3.89(*b*) represents the motion of a ball thrown up with some initial velocity and rebounding from the floor with reduced speed after each hit.
36. Given $x = 6 + 18t + 9t^2$

$$v = \frac{dx}{dt} = 18 + 18t$$

At $t = 2$ s,

$$v = 18 + 18 \times 2 = 54 \text{ ms}^{-1}.$$

37. At the highest point, the velocity of the ball is zero and its acceleration is equal to acceleration due to gravity acting in the downward direction.
38. When the body moves along a straight line path.
39. When the body moves along a straight line path.
40. Acceleration.
41. Displacement.
42. Change in velocity in the given time interval.
43. $\frac{v_A}{v_B} = \frac{\tan 30°}{\tan 45°} = \frac{1/\sqrt{3}}{1} = 1 : \sqrt{3}$
44. No, a body cannot occupy two different positions at the same instant of time.
45. The v-t graph for the given uniform motion is shown below :



Type B : Short Answer Questions

2 or 3 Marks Each

1. Define the following terms and write the SI units :
   (*a*) Displacement, and
   (*b*) Instantaneous velocity. **[Delhi 03C]**
2. Distinguish between average velocity and instantaneous velocity. If the velocity does not change from instant to instant, will the average velocities be different for the different intervals ?
3. Differentiate between average speed and instantaneous speed of an object. **[Delhi 04]**
4. What are the characteristics of displacement ? **[Himachal 09]**
5. What are the characteristics of uniform motion ? **[Himachal 09]**
6. Derive the equations of motion given below :
   (*i*) $v = u + at$
   (*ii*) $s = ut + \frac{1}{2}at^2$
   where symbols have their usual meanings. **[Himachal 09, 09C ; Central Schools 07]**
7. Deduce the following relation : $v^2 - u^2 = 2as$, where symbols have their usual meaning. **[Central Schools 04]**
8. Using integration technique, prove that
   $v^2 - u^2 = 2as$. **[Himachal 04, 09]**
9. Prove that
   $x = x_0 + v_0t + \frac{1}{2}at^2$. **[Delhi 1998 ; Himachal 03, 04]**
10. Use integration technique to prove that the distance travelled in nth second,
    $s_{nth} = u + \frac{a}{2}(2n - 1)$ **[Himachal 03, 04, 09, 09C]**

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11. Show that the slope of displacement-time graph is equal to the velocity of uniform motion.
[Himachal 04, 07C]

12. Draw velocity-time graph of uniform motion and prove that the area under the velocity-time graph of a particle gives the displacement of the particle in a given time.
[Himachal 04, 09]

13. Show that area under the velocity-time graph of an object moving with constant acceleration in a straight line in certain time interval is equal to the distance covered by the object in that interval.
[Himachal 06, 06C, 09C]

14. Derive the three kinematic equations for uniformly accelerated motion graphically.
[Delhi 06]

15. Derive the relation graphically :

$$s = ut + \frac{1}{2}at^2$$

where symbols have their usual meanings.
[Himachal 07 ; Chandigarh 07]

16. Using velocity-time graph, prove that

$$v^2 - u^2 = 2as$$

where symbols have their usual meanings.
[Himachal 07]

17. Explain with example the distinction between the magnitude of average velocity over an interval of time and the average speed over the same period. Show that the average speed is either greater than or equal to the average velocity. When is the equality sign true ?
[Delhi 03C]

18. Draw the following graphs for an object projected upward with a velocity v_0, which comes back to the same point after some time :
(*i*) Acceleration versus time graph.
(*ii*) Speed versus time graph.
(*iii*) Velocity versus time graph.
[Central Schools 03 ; Delhi 12, 13]

19. Acceleration-time graph of a moving object is shown in figure. Draw the velocity-time graph and displacement-time graph corresponding to this type of motion.

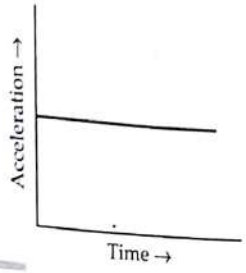

Fig. 3.98

20. Draw the following graphs (expected nature only) between distance and time of an object in case of
(*i*) For a body at rest
(*ii*) For a body moving with uniform velocity
(*iii*) For a body moving with constant acceleration.
[Central Schools 09]

21. Draw the following graphs (expected nature only) representing motion of an object under free fall. Neglect air resistance.
(*i*) Variation of position with respect to time.
(*ii*) Variation of velocity with respect to time.
(*iii*) Variation of acceleration with respect to time.
[Central Schools 08]

Answers

1. Refer to points 13 and 23 of Glimpses.
2. For average velocity and instantaneous velocity, refer answer to Q. 11 on page 3.5. No. The average velocity will be same for different time intervals if the instantaneous velocity does not change from instant to instant.
3. Refer to points 17 and 18 of Glimpses.
4. Refer answer to Q.7 on page 3.3.
5. Refer answer to Q. 15 on page 3.5.
6. Refer answer to Q. 21(*i*) and (*ii*) on page 3.11.
7. Refer answer to Q. 21 (*iii*) on page 3.11.
8. Refer answer to Q. 21(*iii*) on page 3.11.
9. Refer answer to Q. 20(*ii*) on page 3.10.
10. Refer answer to Q. 21(*iv*) on page 3.12.
11. Refer answer to Q. 24 on page 3.22.
12. Refer answer to Q. 26 on page 3.23.
13. Refer answer to Q. 28 on page 3.23.
14. Refer answer to Q. 29 on page 3.24.
15. Refer answer to Q. 29 on page 3.24.
16. Refer answer to Q. 29 on page 3.24.
17. Refer answer to Exercise 3.13 on page 3.48.

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18. (*a*) See Fig. 3.52 on page 3.27.
    (*b*) See Fig. 3.50 on page 3.27.
    (*c*) See Fig. 3.48 on page 3.27.

19. The object is moving with a constant acceleration.
    (*i*) For velocity-time graph, see Fig. 3.43 on page 3.26.
    (*ii*) For displacement-time graph, see Fig. 3.38 on page 3.25.

20. (*i*) See Fig. 3.29 on page 3.25
    (*ii*) See Fig. 3.30 on page 3.25.
    (*iii*) See Fig. 3.31 on page 3.25.

21. The object falls with uniform acceleration equal to '*g*'.

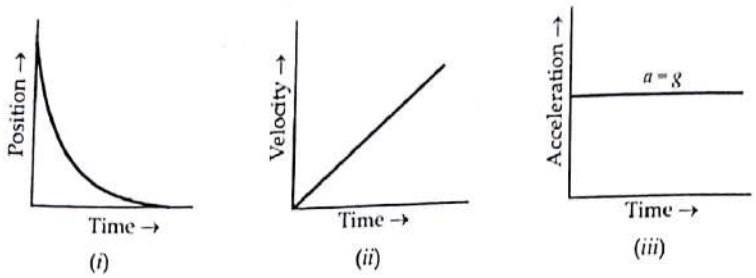

Fig. 3.99

Type C : Long Answer Questions

5 Marks Each

1. (*a*) With the help of a simple case of an object moving with a constant velocity, show that the area under velocity-time curve represents the displacement over a given time interval.
   (*b*) Establish the relation $x = v_0 t + \frac{1}{2} at^2$ graphically.
   (*c*) A car moving with a speed of 126 km h^{-1} is brought to a stop within a distance of 200 m. Calculate the retardation of the car and the time required to stop it. **[Delhi 03]**

2. Derive an equation for the distance covered by a uniformly accelerated body in *n*th second of its motion. A body travels half its total path in the last second of its fall from rest, calculate the time of its fall. **[Central Schools 05]**

3. Derive the following equations of motion for uniformly accelerated motion from velocity-time graph : **[Delhi 14 ; Central Schools 14]**
   (*a*) $v = u + at$ (*b*) $s = ut + \frac{1}{2}at^2$ (*c*) $v^2 - u^2 = 2as$

4. Define relative velocity of one object w.r.t. another object. Draw position-time graphs for two objects moving along a straight line ; when their relative velocity is (*i*) zero and (*ii*) positive. **[Himachal 06 ; Central Schools 12]**

Answers

1. (*a*) Refer answer to Q. 26 on page 3.23.
   (*b*) Refer answer to Q. 29 (*ii*) on page 3.24.
   (*c*) Refer answer to Exercise 3.6 on page 3.46.

2. For derivation, refer answer to Q. 21 (*iv*) on page 3.12.

   Here $u = 0$,

   $\therefore \quad h = 0 + \frac{1}{2} gt^2$ ...(*i*)

   and $\frac{h}{2} = 0 + \frac{1}{2} g(2t - 1)$ ...(*ii*)

   Dividing (*i*) by (*ii*), we get

   $$2 = \frac{t^2}{2t - 1}$$

   or $t^2 - 4t + 2 = 0$

   or $t = \frac{4 \pm \sqrt{16 - 8}}{2} = \frac{4 \pm 2\sqrt{2}}{2}$

   or $t = 2 \pm \sqrt{2}$

   or $t = 3.41$ s or 0.59 s (neglected)

3. Refer answer to Q. 29 on page 3.24.

4. Refer answer to Q.35 on page 3.33. See Fig. 3.72 and Fig. 3.73 on page 3.34.

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Competition Section

Motion in a Straight Line

GLIMPSES

1. **Mechanics.** It is the branch of physics that *deals with the conditions of rest or motion of the material objects around us.*
2. **Statics.** It is the branch of mechanics that *deals with the study of objects at rest or in equilibrium.*
3. **Kinematics.** It is the branch of mechanics that *deals with the study of motion of objects without considering the cause of motion.*
4. **Dynamics.** It is the branch of mechanics that *deals with the study of motion of objects taking into consideration the cause of their motion.*
5. **Rest.** An object is at rest if it does not change its position w.r.t. its surroundings with the passage of time.
6. **Motion.** An object is in motion if it changes its position w.r.t. its surroundings with the passage of time.
7. **Rest and motion are relative terms.** No body can exist in a state of absolute rest or of absolute motion.
8. **Point object.** If the position of an object changes by distances much greater than its own size in a reasonable duration of time, then the object may be regarded as a point object.
9. **One dimensional motion.** The motion of an object is said to be one dimensional motion if only one out of the three coordinates specifying the position of the object changes with time. In such a motion, an object moves along a straight line path.
10. **Two dimensional motion.** The motion of an object is said to be two dimensional motion if two out of the three coordinates specifying the position of the object change with time. In such a motion, the object moves in a plane.
11. **Three dimensional motion.** The motion of an object is said to be three dimensional motion if all the three coordinates specifying the position of the object change with time. In such a motion, the object moves in space.
12. **Distance or path length.** It is the length of the actual path traversed by a body between its initial and final positions. It is a scalar quantity. Its SI unit is metre. It is always positive or zero.
13. **Displacement.** It is defined as the change in the position of an object in a fixed direction. It is given by the vector drawn from the initial position to the final position of the object. It is a vector quantity. It can be positive, negative or zero. Its SI unit is metre.

    The magnitude of displacement is less than or equal to the actual distance travelled by the object in the given time interval.
14. **Speed.** It is rate of change of position of a body in any direction.

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$$

    Speed is a scalar quantity. Its SI unit is ms^{-1} and its dimensional formula is $[M^{\circ}L^{1}T^{-1}]$.
15. **Uniform speed.** An object is said to be moving with uniform speed if it covers equal distances in equal intervals of time, however small these time intervals may be.
16. **Variable speed.** An object is said to be moving with variable speed if it covers unequal distances in equal intervals of time.
17. **Average speed.** It is equal to the total distance travelled by the object divided by the total time taken to cover that distance.

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

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18. **Instantaneous speed.** The speed of an object at any particular instant of time or at a particular point of its path is called the instantaneous speed of the object.

$$\text{Instantaneous speed, } v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

19. **Velocity.** It is the rate of change of position of an object in a particular direction. It is equal to the displacement covered by a body per unit time.

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time}}$$

Velocity is a vector quantity. Its SI unit is ms^{-1} and dimensional formula is $[M^0L^1T^{-1}]$.

20. **Uniform velocity.** A body is said to be moving with uniform velocity if it covers equal displacements in equal intervals of time, however small these time intervals may be.

21. **Variable velocity.** A body is said to be moving with variable velocity if either its speed changes or direction of motion changes or both change with time.

22. **Average velocity.** It is equal to the net displacement covered divided by the total time taken.

$$v_{av} = \frac{\text{Net displacement}}{\text{Total time taken}} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

23. **Instantaneous velocity.** The velocity of an object at a particular instant of time or at a particular point of its path is called its instantaneous velocity.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t} = \frac{d\vec{x}}{dt}$$

24. **Uniform motion.** An object is said to be in uniform motion if it covers equal distances in equal intervals of time, however small these intervals may be, in the same fixed direction.

25. **Displacement in uniform motion.** If at time $t = 0$, the displacement is x_0, then

$$x = x_0 + vt \quad \text{or} \quad x - x_0 = vt \quad \text{or} \quad s = vt.$$

26. **Non-uniform motion.** A body is said to be in non-uniform motion if its velocity changes with time.

27. **Acceleration.** The rate of change of velocity of an object is called its acceleration.

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time taken}}$$

Acceleration is a vector quantity. Its SI unit is ms^{-2} and its dimensional formula is $[M^0L^1T^{-2}]$.

28. **Uniform acceleration.** If the velocity of an object changes by equal amounts in equal intervals of time, however small these intervals may be, then the object is said to move with uniform or constant acceleration.

29. **Variable acceleration.** If the velocity of an object changes by unequal amounts in equal intervals of time, then the object is said to be in variable acceleration.

30. **Average acceleration.** The average acceleration of an object between two points on the path of its motion is defined as ratio of the change in velocity to the total time interval in which that change has taken place.

$$a_{av} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

31. **Instantaneous acceleration.** The acceleration of an object at a given instant of time or at a given point of its motion is called its instantaneous acceleration.

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{x}}{dt^2}$$

32. **Positive acceleration.** If the velocity of an object increases with time, its acceleration is positive.

33. **Negative acceleration.** If the velocity of an object decreases with time, its acceleration is negative. Negative acceleration is also called *retardation* or *deceleration*.

34. **Equations of motion for constant acceleration**

(a) *In conventional form.* Let u be the initial velocity of a particle, a the uniform acceleration, v its velocity after time t and s is the distance travelled in time t, then the following equations hold good :

(i) $v = u + at$ (ii) $s = \frac{u + v}{2} \times t$

(iii) $s = ut + \frac{1}{2}at^2$ (iv) $v^2 - u^2 = 2as$

(v) $s = vt - \frac{1}{2}at^2$ (vi) $s_{n\,th} = u + \frac{a}{2}(2n - 1)$

where $s_{n\,th}$ = the distance travelled in n^{th} second.

(b) *In cartesian form.* Suppose a particle moves with uniform acceleration a along X-axis. Let x_0 and x be its position co-ordinates and v_0 and v be its velocities at times $t = 0$ and t respectively. Then the following equations hold good :

(i) $v = v_0 + at$

(ii) $x = x_0 + v_0t + \frac{1}{2}at^2$ or $s = v_0t + \frac{1}{2}at^2$

(iii) $v^2 = v_0^2 + 2a(x - x_0) = v_0^2 + 2as$

(iv) $s_{n\,th} = v_0 + \frac{a}{2}(2n - 1)$

35. **Free fall.** In the absence of air resistance, all bodies fall with the same acceleration near the surface of the earth. This motion of a body falling towards the earth from a small height is called free fall. The acceleration with which a body falls is called acceleration due to gravity and is denoted by g. Near the surface of the earth, $g = 9.8\ ms^{-2}$.

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36. **Motion under gravity.** For a freely falling body, the following equations of motion hold good :

(*i*) $v = u + gt$ (*ii*) $s = ut + \frac{1}{2}gt^2$

(*iii*) $v^2 - u^2 = 2gs$

When a body falls freely under the action of gravity, its velocity increases and the value of g is taken *positive*.

When a body is thrown vertically upward, its velocity decreases and the value of g is taken *negative*.

For a body thrown vertically upward with initial velocity u, we have

(*i*) Maximum height reached, $h = \frac{u^2}{2g}$

(*ii*) Time of ascent = Time of descent = $\frac{u}{g}$

(*iii*) Total time of flight to come back to the point of projection = $\frac{2u}{g}$

(*iv*) Velocity of fall at the point of projection = u

(*v*) Velocity attained by a body dropped through height h, $v = \sqrt{2gh}$.

37. **Position-time graph.** It is the graph between the time t and position x of a particle relative to a fixed origin. Its slope at any point gives the instantaneous velocity at that point.

(*i*) For a stationary object, the position-time graph is a straight line parallel to the time-axis.

(*ii*) For a body in uniform motion, the position-time graph is a straight line inclined to the time-axis.

(*iii*) For uniformly accelerated motion, the position-time graph is a parabola.

38. **Velocity-time graph.** It is a graph of time versus velocity. Its slope at any point gives the acceleration at the corresponding instant. Distance covered in time t equals area under the velocity-time graph bounded by the time-axis.

(*i*) For uniform motion, the velocity-time graph is a straight line parallel to the time-axis.

(*ii*) For uniform acceleration, the velocity-time graph is a straight line inclined to the time-axis.

(*iii*) For variable acceleration, the velocity-time graph is a curve.

39. **Relative velocity.** The relative velocity of an object B with respect to object A when both are in motion is the rate of change of position of object B with respect to object A.

Relative velocity of object B w.r.t. object A,

$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$$

Relative velocity of object A w.r.t. object B,

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

When both the objects A and B move in the same direction,

$$v_{AB} = v_A - v_B$$

When the object B moves in the opposite direction of A,

$$v_{AB} = v_A + v_B$$

When v_A and v_B are inclined to each other at angle θ,

$$v_{AB} = \sqrt{v_A^2 + v_B^2 + 2v_Av_B\cos(180° - \theta)}$$

$$= \sqrt{v_A^2 + v_B^2 - 2v_Av_B\cos\theta}$$

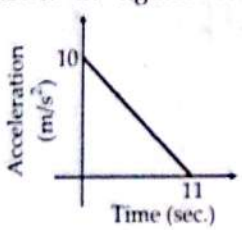
If v_{AB} makes angle β with v_A, then

$$\tan\beta = \frac{v_B\sin\theta}{v_A - v_B\cos\theta}.$$

IIT Entrance Exam

MULTIPLE CHOICE QUESTIONS WITH ONE CORRECT ANSWER

1. A body starts from rest at time $t = 0$, the acceleration-time graph is shown in figure. The maximum velocity attained by the body will be



(*a*) 110 m/s (*b*) 55 m/s

(*c*) 650 m/s (*d*) 550 m/s

[IIT 04]

2. A small block slides, without friction, down an inclined plane starting from rest. Let s_n be the distance travelled from $t = (n-1)$ to $t = (n)$.

Then $\frac{s_n}{s_{n+1}}$ is

(*a*) $\frac{2n-1}{2n}$ (*b*) $\frac{2n+1}{2n-1}$

(*c*) $\frac{2n-1}{2n+1}$ (*d*) $\frac{2n}{2n+1}$ [IIT 04]

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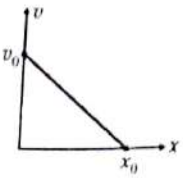
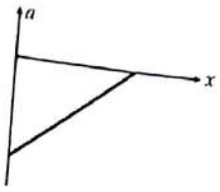
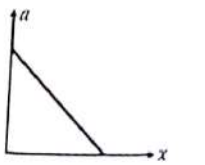
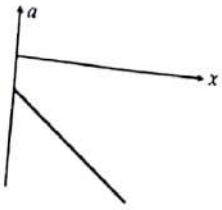
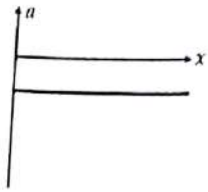
3. The velocity-displacement graph of a particle moving along a straight line is shown. The most suitable acceleration-displacement graph will be [IIT 05]



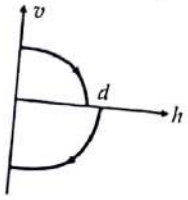
(a) 

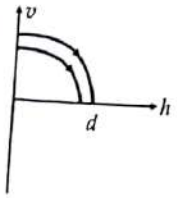
(b) 

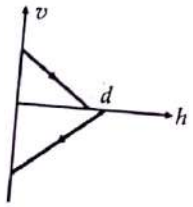
(c) 

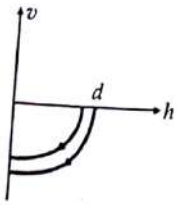
(d) 

4. A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height $d/2$. Neglecting subsequent motion and air resistance, its velocity v varies with the height h above the ground as [IIT 2000]

(a) 

(b) 

(c) 

(d) 

MULTIPLE CHOICE QUESTIONS WITH ONE OR MORE THAN ONE CORRECT ANSWER

5. A particle is moving eastwards with a velocity of 5 m/s. In 10 s the velocity changes to 5 m/s northwards. The average acceleration in this time is

(a) zero

(b) $\frac{1}{\sqrt{2}}$ m/s^2 towards north-west

(c) $\frac{1}{\sqrt{2}}$ m/s^2 towards north-east

(d) $\frac{1}{2}$ m/s^2 towards north-west

6. A particle of mass m moves on the x-axis as follows : it starts from rest at $t = 0$ from the point $x = 0$, and comes to rest at $t = 1$ at the point $x = 1$. No other information is available about its motion at intermediate times $(0 < t < 1)$. If α denotes the instantaneous acceleration of the particle, then [IIT 92]

(a) α cannot remain positive for all t in the interval $0 \le t \le 1$.

(b) $|\alpha|$ cannot exceed 2 at any point in its path.

(c) $|\alpha|$ must be ≥ 4 at some point or points in its path.

(d) α must change sign during the motion, but no other assertion can be made with the information given. [IIT 93]

REASONING TYPE

7. **Statement 1.** For an observer looking out through the window of a fast moving train, the nearby objects appear to move in the opposite direction to the train, while the distant objects appear to be stationary.

Statement 2. If the observer and the object are moving at velocities $\vec{V}_1$ and $\vec{V}_2$ respectively with reference to a laboratory frame, the velocity of the object with respect to the observer is $\vec{V}_2 - \vec{V}_1$.

(a) Statement 1 is true ; Statement 2 is true ; Statement 2 is a correct explanation for Statement 1.

(b) Statement 1 is true ; Statement 2 is true ; Statement 2 is not a correct explanation for Statement 1.

(c) Statement 1 is true ; Statement 2 is false.

(d) Statement 1 is false ; Statement 2 is true. [IIT 08]

Answers and Explanations

1. (b) Change in velocity

= Area under acceleration-time graph

$= \frac{1}{2} \times 10 \times 11 = 55$ m/s

But initial velocity = 0

∴ Maximum velocity attained

= **55 m/s**.

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2. (*c*) Here $s_n = u + \frac{a}{2}(2n-1)$

As $u = 0$, $s_n = \frac{a}{2}(2n-1)$

$$s_{n+1} = \frac{a}{2}[2(n+1)-1] = \frac{a}{2}(2n+1)$$

$$\therefore \quad \frac{s_n}{s_{n+1}} = \frac{2n-1}{2n+1}.$$

3. (*a*) From the given velocity-displacement graph,

$$\text{Slope} = -\frac{v_0}{x_0}, \text{ intercept on } y\text{-axis} = v_0$$

Thus the equation for this graph is

$$v = -\frac{v_0}{x_0}.x + v_0$$

$$\therefore \quad \frac{dv}{dt} = -\frac{v_0}{x_0}.\frac{dx}{dt}$$

$$a = -\frac{v_0}{x_0}.v$$

or

$$= -\frac{v_0}{x_0}\left(-\frac{v_0}{x_0}x + v_0\right)$$

$$a = -\frac{v_0^2}{x_0^2}.x - \frac{v_0^2}{x_0}$$

or

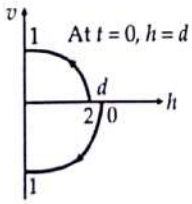
Clearly, the a-x graph must have a positive slope (v_0^2/x_0^2) and negative intercept $(-v_0^2/x_0)$ on y-axis.

Hence the correct option is (*a*).

4. (*a*) For the uniformly accelerated/decelerated motion,

$$v^2 = u^2 \pm 2gh$$

Thus the v-h graph is a parabola.



Initially, velocity is downwards (−*ve*). After collision, it reverses the direction with a smaller magnitude. The velocity is upwards (+ ve). Graph (*a*) satisfies all these conditions.

Moreover, at time $t = 0$, $h = d$.

$0 \to 1$, v increases downwards

At $1 \to v$, reverses its direction

$1 \to 2$, v decreases upwards

Afterwards, the next collision occurs.

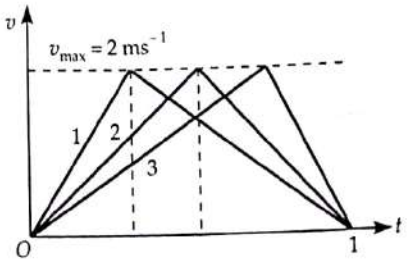
5. (*b*) Refer to the solution of Example 56 on page 4.36.

6. (*a*), (*c*) and (*d*)

The body is at rest at $t = 0$ and $t = 1$. Initially α is positive, the body acquires some velocity. Then α is negative, the body comes to rest.

Hence α cannot remain positive for all t in the interval $0 < t < 1$.

Options (*a*) and (*d*) are correct.



s = Area under v-t graph.

or $$s = \frac{1}{2} \times t \times v_{max}$$

$$\therefore \quad v_{max} = \frac{2s}{t}$$

$$= \frac{2 \times 1\text{m}}{1\text{s}}$$

$$= 2\ \text{ms}^{-1}$$

In case 1 :

first $\alpha > 4\ \text{ms}^{-2}$, then $\alpha < -4\ \text{ms}^{-2}$

In case 2 :

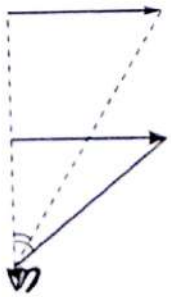
first $\alpha = 4\ \text{ms}^{-2}$, then $\alpha < -4\ \text{ms}^{-2}$

In case 3 :

first $\alpha < 4\ \text{ms}^{-2}$, then $\alpha < -4\ \text{ms}^{-2}$

Hence $|\alpha| \geq 4$ at some point or points in the path. Option (*b*) is wrong and option (*c*) is correct.

7. (*b*) Both statements are true but statement 2 is not correct explanation for statement 1. Distance appeared to move depends upon angle subtended on the eye as illustrated in the figure.



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AIEEE

1. A bullet fired into a fixed target loses half of its velocity after penetrating 3 cm. How much further will it penetrate before coming to rest assuming that it faces constant resistance in motion ?

(a) 1.5 cm (b) 1.0 cm
(c) 3.0 cm (d) 2.0 cm [AIEEE 05]

2. A car moving with the speed of 50 kmh^{-1} can be stopped by brakes after atleast 6 m. If the same car is moving at a speed of 100 kmh^{-1}, the minimum stopping distance is

(a) 12 m (b) 18 m
(c) 24 m (d) 6 m [AIEEE 03]

3. An automobile travelling with a speed of 60 kmh^{-1} can brake to stop within a distance of 20 m. If the car is going twice as fast *i.e.*, 120 kmh^{-1}, the stopping distance will be

(a) 20 m (b) 40 m
(c) 60 m (d) 80 m [AIEEE 04]

4. Speeds of two identical cars are u and $4u$ at a specific instant. The ratio of the respective distances at which the two cars are stopped from that instant is

(a) 1 : 1 (b) 1 : 4
(c) 1 : 8 (d) 1 : 16 [AIEEE 02]

5. If a body loses half of its velocity on penetrating 3 cm in a wooden block, then how much will it penetrate more before coming to rest ?

(a) 1 cm (b) 2 cm
(c) 3 cm (d) 4 cm [AIEEE 02]

6. A car, starting from rest, accelerates at the rate f through a distance s, then continues at constant speed for time t and then decelerates at the rate $f/2$ to come to rest. If the total distance traversed is $5s$, then

(a) $s = ft$ (b) $s = \frac{1}{6} ft^2$
(c) $s = \frac{1}{2} ft^2$ (d) $s = \frac{1}{4} ft^2$ [AIEEE 05]

7. The relation between time t and distance x is $t = ax^2 + bx$, where a and b are constants. The acceleration is

(a) $-2abv^2$ (b) $-2bv^3$
(c) $-2av^3$ (d) $-2av^2$ [AIEEE 05]

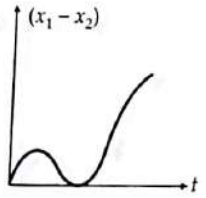
8. A particle located at $x = 0$ at time $t = 0$, starts moving along the positive X-direction with a velocity v that varies with time as

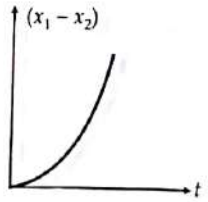
(a) $t^{1/2}$ (b) t^3
(c) t^2 (d) t [AIEEE 06]

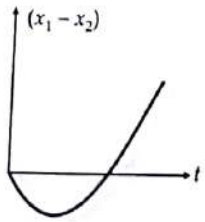
9. The velocity of a particle is $v = v_0 + gt + ft^2$. If its position is $x = 0$ at $t = 0$, then its displacement after time $(t = 1)$ is

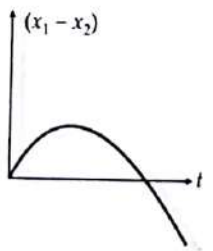
(a) $v_0 + \frac{g}{2} + f$ (b) $v_0 + 2g + 3f$
(c) $v_0 + \frac{g}{2} + \frac{f}{3}$ (d) $v_0 + g + f$ [AIEEE 07]

10. A body is at rest at $x = 0$. At $t = 0$, it starts moving in the positive x-direction with a constant acceleration. At the same instant another body passes through $x = 0$ moving in the positive x-direction with a constant speed. The position of the first body is given by $x_1(t)$ after time t and that of second body by $x_2(t)$ after the same time interval. Which of the following graphs correctly describes $(x_1 - x_2)$ as a function of time t ?

(a) 

(b) 

(c) 

(d) 

[AIEEE 08]

11. From a building two balls A and B are thrown such that A is thrown upwards and B downwards (both vertically). If v_A and v_B are their respective velocities on reaching the ground, then

(a) $v_B > v_A$ (b) $v_A = v_B$
(c) $v_A > v_B$
(d) their velocities depend on their masses. [AIEEE 02]

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12. A ball is released from the top of a tower of height h metres. It takes T seconds to reach the ground. What is the position of the ball in $T/3$ seconds ?

(*a*) $h/9$ metres from the ground
(*b*) $7h/9$ metres from the ground
(*c*) $8h/9$ metres from the ground
(*d*) $17h/18$ metres from the ground [AIEEE 04]

13. A parachutist after bailing out falls 50 m without friction. When parachute opens, it decelerates at $2\ ms^{-2}$. He reaches the ground with a speed of $3\ ms^{-2}$. At what height, did he bail out ?

(*a*) 293 m (*b*) 111 m
(*c*) 91 m (*d*) 182 m [AIEEE 05]

14. An object, moving with a speed of 6.25 m/s, is decelerated at a rate given by

$$\frac{dv}{dt} = -2.5\sqrt{v}$$

where v is the instantaneous speed. The time taken by the object, to come to rest, would be

(*a*) 1 s (*b*) 2 s
(*c*) 4 s (*d*) 8 s [AIEEE 2011]

Answers and Explanations

1. (*b*) Refer to the solution of Problem 7 on page 3.43.

2. (*c*) Refer to the solution of Problem 9 on page 3.43.

3. (*d*) In first case,

Velocity, $v = \frac{60\times1000}{3600} = \frac{50}{3}\ ms^{-1}$

$\therefore$ Retardation, $a = \frac{v^2}{2s} = \frac{\frac{50}{3}\times\frac{50}{3}}{2\times20} = \frac{250}{36}\ ms^{-2}$

In second case,

Velocity, $v = \frac{120\times1000}{3600} = \frac{100}{3}\ ms^{-1}$

$\therefore$ $s = \frac{v^2}{2a} = \frac{\frac{100}{3}\times\frac{100}{3}}{2}\times\frac{36}{200} = 80$ **m.**

4. (*d*) **For first car :** $u = u,\ a = -a,\ v = 0$

As $v^2 - u^2 = 2as$

$\therefore$ $v^2 - u^2 = 2(-a)s_1$ or $s_1 = \frac{u^2}{2a}$

For second car : $u = 4u,\ a = -a,\ v = 0$

$\therefore$ $s_2 = \frac{(4u)^2}{2a} = \frac{8u^2}{a}$

Hence, $\frac{s_1}{s_2} = \frac{u^2}{2a}\cdot\frac{a}{8u^2} = \frac{1}{16} = 1:16.$

5. (*a*) **In first case :** $v = \frac{u}{2},\ s = 3$ cm.

As $v^2 - u^2 = 2as$

$\therefore$ $\left(\frac{u}{2}\right)^2 - u^2 = 2a\times3$ or $a = -\frac{u^2}{8}$

In second case : $v = 0,\ u = \frac{u}{2},\ a = -\frac{u^2}{8}$

$\therefore$ $0^2 - \left(\frac{u}{2}\right)^2 = 2\left(-\frac{u^2}{8}\right)s$ or $s = 1$ **cm.**

6. (*c*) Refer to the solution of Problem 8 on page 3.43.

7. (*c*) Refer to the solution of Problem 10 on page 3.44.

8. (*c*) Given : $v = \alpha\sqrt{x}$

or $\frac{dx}{dt} = \alpha x^{1/2}$

or $x^{-1/2}dx = \alpha\,dt$

Integrating both sides,

$$\int x^{-1/2}dx = \alpha\int dt$$

$$\frac{x^{1/2}}{1/2} = \alpha t \quad \text{or} \quad x = \frac{\alpha^2t^2}{4}$$

Hence, $x \propto t^2$.

9. (*c*) $v = \frac{dx}{dt} = v_0 + gt + ft^2$

$\therefore$ $\int_0^x dx = \int_0^1 (v_0 + gt + ft^2)dt$

$$x = \left[v_0t + \frac{gt^2}{2} + \frac{ft^3}{3}\right]_0^1 = v_0 + \frac{g}{2} + \frac{f}{3}.$$

10. (*c*) For particle 1, $v = 0,\ v_1 = at.$

For particle 2, $v_2 =$ constant.

Initially, both are at $x = 0$.

Relative velocity of 1 w.r.t. $2 = v_1 - v_2$.

At first, v_1 is less than v_2.

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The distance travelled by particle 1 increases with time as

$$x_1 = \frac{1}{2}at^2$$

The distance travelled by second particle is proportional to time t.

$$x_2 = v_1 t \quad \therefore \quad x_1 - x_2 = \frac{1}{2}at^2 - v_1 t$$

Hence graph of $(x_1 - x_2)$ versus t is a parabola even after crossing x-axis.

11. *(b)* Suppose the ball A is thrown upward with velocity u and ball B is thrown downward with the same velocity u. After reaching the highest point, the ball A comes back to its position of projection attaining velocity u in the downward direction. As the two balls fall down from the same position of projection with the same velocity, both attain the same velocity on reaching the ground.

12. *(c)* Refer to the solution of problem 11 on page 3.44.

13. *(a)* Velocity attained after falling freely through first 50 m,

$$v = \sqrt{2gh} = \sqrt{2 \times 98 \times 50} = \sqrt{980} \text{ m/s}$$

After the parashute opens,

$$u = \sqrt{980} \text{ m/s}, \quad v = 3 \text{ m/s}, \quad a = -2 \text{ m/s}^2$$

As $v^2 - u^2 = 2gh$

$\therefore \quad 9 - 980 = 2 \times 2 \times h \quad$ or $\quad h = \frac{971}{4} \simeq 243 \text{ m}$

Total height $= 50 + 243 = 293$ m.

14. *(b)* Given $\frac{dv}{dt} = -2.5\sqrt{v}$

or $\frac{1}{\sqrt{v}}dv = -2.5\,dt$

Integrating, $\int_{6.25}^{0} v^{-1/2}\,dv = -2.5\int_0^t dt$

$$[2\sqrt{v}]_{6.25}^{0} = -2.5[t]_0^t$$

$$0 - 2\sqrt{6.25} = -2.5[t - 0]$$

$$-2 \times 2.5 = -2.5t \quad \text{or} \quad t = 2 \text{ s}$$

DCE and G.G.S. Indraprastha University Engineering Entrance Exam

(Similar Questions)

1. What is the relation among displacement, time and acceleration in case of a body having uniform acceleration f ?

(a) $s = ut + \frac{1}{2}ft^2$ (b) $s = (u + f)t$

(c) $s = v^2 - 2fs$ (d) none of these [DCE 99]

2. The motion of a particle is described by the equation $u = at$. The distance travelled by particle in first 4 s is

(a) $4a$ (b) $12a$

(c) $6a$ (d) $8a$ [DCE 2K]

3. The displacement x of a particle varies with time t as $x = ae^{-\alpha t} + be^{\beta t}$

where a, b, α and β are positive constants. The velocity of the particle will

(a) go on decreasing with time

(b) be independent of α and β

(c) drop to zero when $\alpha = \beta$

(d) go on increasing with time [DCE 01]

4. A train started from rest from a station and accelerated at 2 ms^{-2} for 10 s. Then, it ran at constant speed for 30 s and thereafter it decelerated at 4 ms^{-2} until it stopped at the next station. The distance between two station is

(a) 650 m (b) 700 m

(c) 750 m (d) 800 m [DCE 03]

5. A ball falls from 20 m height on floor and rebounds to 5 m. Time of contact is 0.02 sec. Find acceleration during impact.

(a) 1200 m / s^2 (b) 1000 m / s^2

(c) 2000 m / s^2 (d) 1500 m / s^2 [DCE 06]

6. A ball is dropped from top of a tower of 100 m height. Simultaneously another ball was thrown upward from bottom of the tower with a speed of 50 m / s. They will cross each other after ($g = 10$ m / s^2):

(a) 1 sec (b) 2 sec

(c) 3 sec (d) 4 sec [IIPUEE 04]

7. A ball thrown upward from the top of a tower with speed v reaches the ground in t_1 second. If this ball is thrown downward from the top of the same tower with speed v, it reaches the ground in t_2 seconds. In what time will the ball reach the ground if it is allowed to fall freely under gravity from the top of the tower?

(a) $\frac{t_1 + t_2}{2}$ (b) $\frac{t_1 - t_2}{2}$

(c) $\sqrt{t_1 t_2}$ (d) $t_1 + t_2$ [DCE 07]

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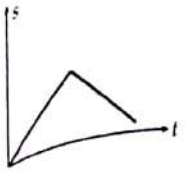
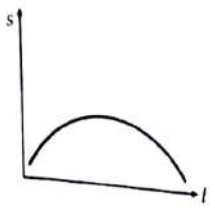
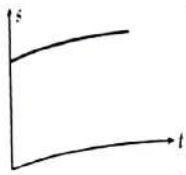
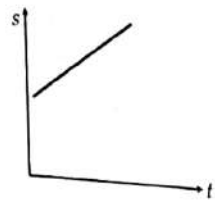
8. Which one of the following graphs represents uniform motion ? [DCE 99]

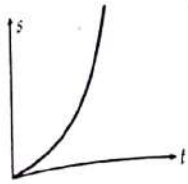
(a) 

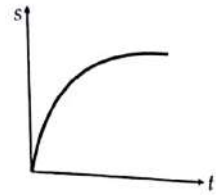
(b) 

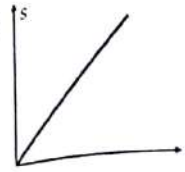
(c) 

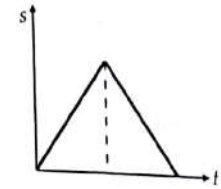
(d) 

9. Which graph pertains to uniform acceleration ?

(a) 

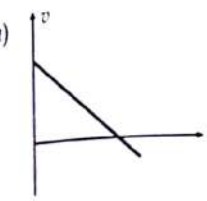
(b) 

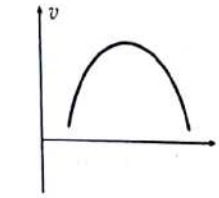
(c) 

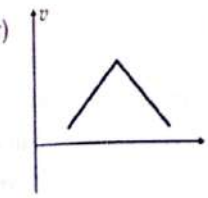
(d) 

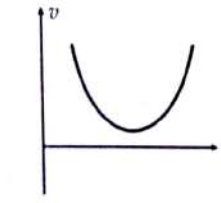
[DCE 2K, 03]

10. A particle is thrown above, then correct v-t graph will be [DCE 07]

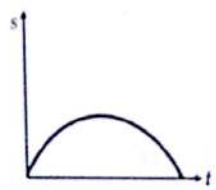
(a) 

(b) 

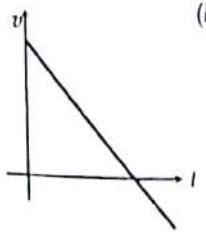
(c) 

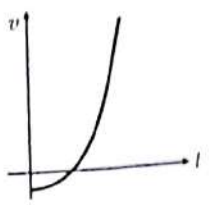
(d) 

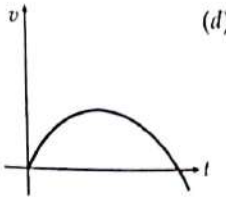
11. The graph of displacement *vs.* time is

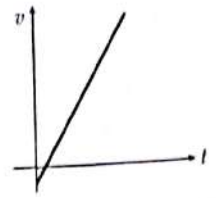


The corresponding velocity-time graph will be

(a) 

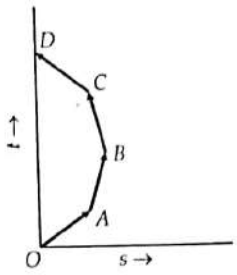
(b) 

(c) 

(d) 

[DCE 01]

12. Which of the following options is correct for the object having a straight line motion represented by the following graph ?



(a) The object moves with constantly increasing velocity from O to A and then it moves with constant velocity.

(b) Velocity of the object increases uniformly

(c) Average velocity is zero

(d) The graph shown is impossible. [DCE 04]

13. The driver of a train travelling at 115 kmh^{-1} sees on the same track, 100 m in front of him, a slow train travelling in the same direction at 25 kmh^{-1}. The least retardation that must be applied to faster train to avoid collision is

(a) 25 ms^{-2} (b) 50 ms^{-2}

(c) 75 ms^{-2} (d) 3.125 ms^{-2} [DCE 07]

14. The acceleration 'a' of a particle starting from rest varies with time according to relation $a = \alpha t + \beta$. The velocity of the particle after a time 't' will be

(a) $\frac{\alpha t^2}{2} + \beta$ (b) $\frac{\alpha t^2}{2} + \beta t$

(c) $\alpha t^2 + \frac{1}{2}\beta t$ (d) $\frac{\alpha t^2 + \beta}{2}$ [DCE 09]

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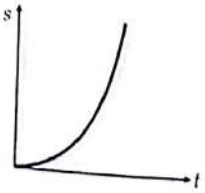
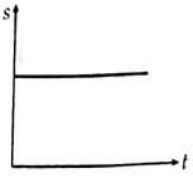
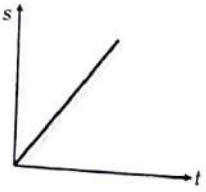
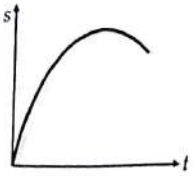
15. A body is travelling in a straight line with a uniformly increasing speed. Which one of the plots represents the changes in distance (s) travelled with time (t) ? [DCE 09]

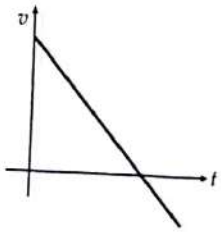
(a) 

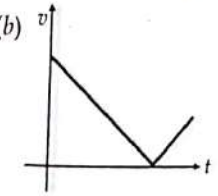
(b) 

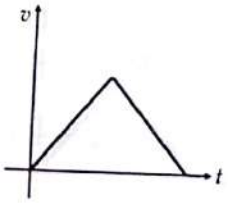
(c) 

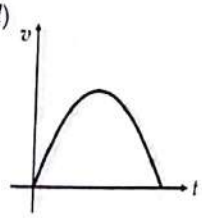
(d) 

16. A ball is thrown vertically upward. Ignoring the air resistance, which one of the following plots represents the velocity-time plot for the period ball remains in air ? [DCE 09]

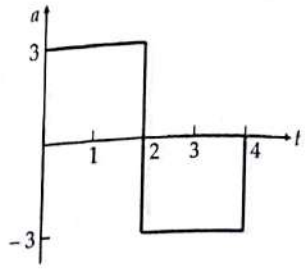
(a) 

(b) 

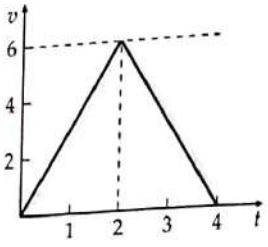
(c) 

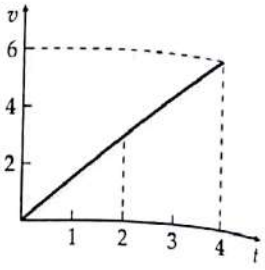
(d) 

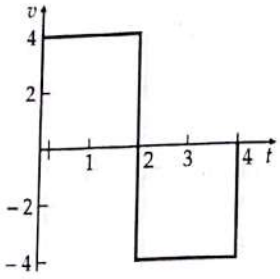
17. A particle starts from rest at $t = 0$ and undergoes an acceleration 'a' in ms^{-2} with time 't' in seconds which is as shown below :

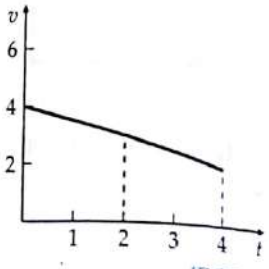


Which one of one following plots represents velocity v in ms^{-1} verses time t in seconds ?

(a) 

(b) 

(c) 

(d) 

[DCE 09]

Answers and Explanations

1. (a) $s = ut + \frac{1}{2}ft^2$

2. (d) $u = \frac{dx}{dt} = at$

$\therefore \quad \int_0^x dx = \int_0^4 at\, dt$

$$x = a\left[\frac{t^2}{2}\right]_0^4 = \frac{a}{2}[16 - 0] = 8a.$$

3. (d) Refer to the solution of Problem 10 on page 3.74 .

4. (c) Refer to the solution of Problem 3 on page 3.70.

5. (d) $a = \frac{v_2 - (-v_1)}{t} = \frac{v_2 + v_1}{t} = \frac{\sqrt{2gh_2} + \sqrt{2gh_1}}{t}$

$$= \frac{\sqrt{2 \times 10 \times 20} + \sqrt{2 \times 10 \times 5}}{0.02}\ \text{m/s}^2$$

$$= \frac{20 + 10}{0.02} = 1500\ \text{m/s}^2.$$

6. (b) Suppose the two balls cross each other after time t. Then the distances covered by the two balls will be

$$s_1 = \frac{1}{2}gt^2$$

and $$s_2 = ut - \frac{1}{2}gt^2$$

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$$s_1 + s_2 = \frac{1}{2}gt^2 + ut - \frac{1}{2}gt^2 = 100 \text{ m}$$

but $ut = 100$

$$\therefore \quad t = \frac{100}{u} = \frac{100}{50} = 2 \text{ s.}$$

7. *(c)* For upward motion,

$$h = -vt_1 + \frac{1}{2}gt_1^2$$

for downward motion,

$$h = vt_2 + \frac{1}{2}gt_2^2$$

$$\therefore \quad \frac{h}{t_1} + \frac{h}{t_2} = \frac{1}{2}g(t_1 + t_2) \qquad \text{or} \qquad h = \frac{1}{2}gt_1t_2$$

For freely falling ball,

$$h = \frac{1}{2}gt^2$$

$$\therefore \quad \frac{1}{2}gt^2 = \frac{1}{2}gt_1t_2 \qquad \text{or} \qquad t = \sqrt{t_1t_2}$$

8. *(d)* Graph (*d*) represents uniform motion because here $s \propto t$.

9. *(a)* For uniform acceleration, displacement-time graph is a parabola symmetric about displacement-axis.

10. *(a)* In upward motion, velocity (+ve) decreases and becomes zero at highest point. In downward motion, velocity (– ve) increases.

11. *(a)* The given displacement-time graph is a parabola. It represents the motion of a body thrown up against gravity and then returning to the ground. As explained in the above problem, its v-t graph will be (*a*).

12. *(c)* In the given graph, net displacement is zero.

$$\therefore \text{ Average velocity} = \frac{\text{Net displacement}}{\text{Time taken}} = \frac{0}{t} = 0.$$

13. *(d)* Relative velocity $= 115 - 25 = 90 \text{ kmh}^{-1}$

$$= 90 \times \frac{5}{18} \text{ ms}^{-1} = 25 \text{ ms}^{-1}.$$

As $v^2 - v^2 = 2as$

$$0^2 - 25^2 = 2a \times 100$$

or $$a = -\frac{625}{200} = -3.125 \text{ ms}^{-2}.$$

14. *(b)* $$a = \frac{dv}{dt} = \alpha t + \beta$$

$$\therefore \quad \int_0^v dv = \int_0^t (\alpha t + \beta)\, dt$$

or $$v = \frac{\alpha t^2}{2} + \beta t.$$

15. *(a)* $$s = ut + \frac{1}{2}at^2$$

for $u = 0$, $$s = \frac{1}{2}at^2.$$

Hence, s-t graph is a parabola as given in option (*a*).

16. *(a)* During upward motion the magnitude of (+ve) velocity decreases while during downward motion the magnitude of (–ve) velocity increases.

17. *(a)* Area under a-t curve = change in velocity

For constant acceleration, $v \propto t$

As a is first +ve and then –ve, so the correct v-t graph is the one given in option (*a*).

AIIMS Entrance Exam

1. The displacement of a body is given to be proportional to the cube of time elapsed. The magnitude of the acceleration of the body is

(*a*) increasing with time

(*b*) decreasing with time

(*c*) constant but not zero

(*d*) zero [AIIMS 96]

2. The velocity of a bullet is reduced from 200 ms^{-1} to 100 ms^{-1} while travelling through a wooden plank of thickness 10 cm. The retardation, assuming it to be uniform, will be

(*a*) $10 \times 10^4 \text{ ms}^{-2}$ (*b*) $12 \times 10^4 \text{ ms}^{-2}$

(*c*) $13.5 \times 10^4 \text{ ms}^{-2}$ (*d*) $15 \times 10^4 \text{ ms}^{-2}$ [AIIMS 01]

3. A particle starts from rest and has an acceleration of 2 ms^{-2} for 10 s. After that, the particle travels for 30 s with constant speed and then undergoes a retardation of 4 ms^{-2} and comes back to rest. The total distance covered by the particle is

(*a*) 650 m (*b*) 700 m

(*c*) 750 m (*d*) 800 m [AIIMS 02]

4. The body A starts from rest with an acceleration a_1. After 2 s, another body B starts from rest with an

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acceleration a_2. If they travel equal distances in the 5th second after the start of A, then the ratio $a_1 : a_2$ is equal to

(a) 5 : 9 (b) 5 : 7
(c) 9 : 5 (d) 9 : 7 [AIIMS 01]

5. Velocity of a body on reaching the point, from which it was projected upwards, is

(a) $v = 0$ (b) $v = 2u$
(c) $v = 0.5u$ (d) $v = u$ [AIIMS 99]

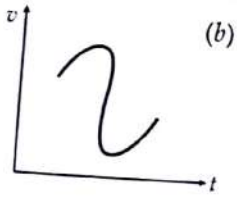
6. Three different objects of masses m_1, m_2 and m_3 are allowed to fall from the same point O along three different frictionless paths. The speeds of the three objects, on reaching the ground, will be in the ratio of

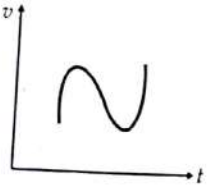
(a) $m_1 : m_2 : m_3$ (b) $m_1 : 2m_2 : 3m_3$
(c) $1/m_1 : 1/m_2 : 1/m_3$ (d) 1 : 1 : 1 [AIIMS 02]

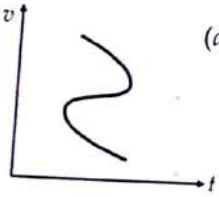
7. When a ball is thrown up vertically with velocity v_0, it reaches a maximum height of h. If one wishes to triple the maximum height, then the ball should be thrown with velocity

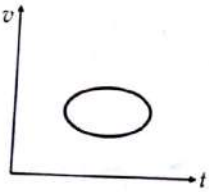
(a) $\sqrt{3}v_0$ (b) $3v_0$
(c) $9v_0$ (d) $3v_0/2$ [AIIMS 05]

8. Which of the following velocity-time graph shows a realistic situation for a body in motion ? [AIIMS 04, 07]

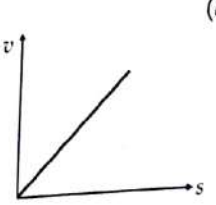
(a) 

(b) 

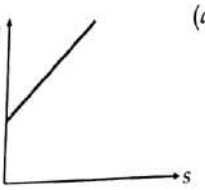
(c) 

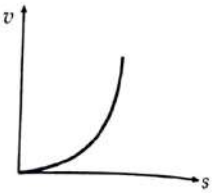
(d) 

9. A body starting from rest moves along a straight line with constant acceleration. The variation of speed v with distance s is [AIIMS 03]

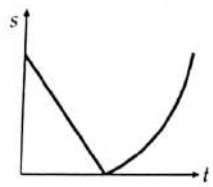
(a) 

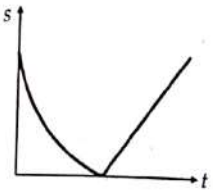
(b)

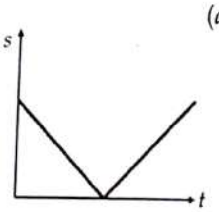
(c) 

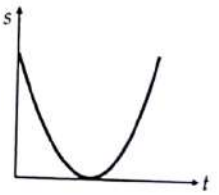
(d) 

10. A ball is thrown vertically upwards. Which of the following plots represents the speed-time graph of the ball during its flight, if the air resistance is not ignored ? [AIIMS 03]

(a) 

(b) 

(c) 

(d) 

Answers and Explanations

1. (a) Refer to the solution of Problem 23 on page 3.38.

2. (d) As $v^2 - u^2 = 2as$

$\therefore \quad 100^2 - 200^2 = 2a \times 0.10$

or $\quad a = -15 \times 10^4 \text{ ms}^{-2}$.

3. (c) *For motion with acceleration :* $u = 0$

$a = 2 \text{ ms}^{-2}$, $t = 10$ s.

$\therefore \quad s_1 = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 2 \times (10)^2 = 100$ m

$v = u + at = 0 + 10 \times 2 = 20 \text{ ms}^{-1}$

For motion with constant speed :

$t = 30$ s

$s_2 = vt = 20 \times 30 = 600$ m.

For motion with retardation :

$u = 20 \text{ ms}^{-1}$, $a = -4 \text{ ms}^{-2}$, $v = 0$

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As $v^2 - u^2 = 2as$

$0 - 20^2 = 2(-4) \times s_3$

$\therefore \quad s_3 = 50$ m

Total distance covered

$s = s_1 + s_2 + s_3$

$= 100 + 600 + 50 = \mathbf{750}$ m.

4. (a) Here $s_{5th}(A) = s_{3rd}(B)$

$$0 + \frac{a_1}{2}(2 \times 5 - 1) = 0 + \frac{a_2}{2}(2 \times 3 - 1)$$

or $$\frac{a_1 \times 9}{2} = \frac{a_2 \times 5}{2}$$

or $$\frac{a_1}{a_2} = \frac{5}{9} = \mathbf{5 : 9}.$$

or

5. (d) When a body is projected upwards with a velocity u, it returns to the point of projection with the same velocity u.

6. (d) When allowed to fall from the same height, three objects attain the same speed on reaching the ground, independent of their masses or the frictionless paths.

7. (a) $0^2 - v_0^2 = 2(-g)h$

or $\quad v_0^2 = 2gh \quad$ *i.e.*, $\quad v_0^2 \propto h$

$\therefore \quad v_0'^2 \propto 3h \quad \therefore \quad \dfrac{v_0'^2}{v_0^2} = \dfrac{3h}{h} = 3$

or $\quad v_0' = \sqrt{3}\, v_0$.

8. (b) In options (a), (c) and (d), the body possesses more than one velocity at any given time which is not physically possible.

9. (d) $v^2 - 0^2 = 2as \quad$ *i.e.*, $\quad v^2 \propto s$

Hence the speed versus distance or v-s graph is a parabola symmetric about v-axis.

10. (a) As the ball goes up, its speed decreases with time and becomes zero at the highest point. As the ball falls down, its speed increases. The ball attains the same speed with which it was thrown up.

CBSE PMT Prelims and Final Exams

1. A car covers the first half of the distance between two places at 40 km/h and another half at 60 km/h. The average speed of the car is

(a) 40 km/h (b) 48 km/h

(c) 50 km/h (d) 60 km/h [CBSE PMT 90]

2. A car moves a distance of 200 m. It covers the first half of the distance at speed 40 km/h and the second half of distance at speed v. The average speed is 48 km/h. The value of v is

(a) 56 km/h (b) 60 km/h

(c) 50 km/h (d) 48 km/h [CBSE PMT 91]

3. A bus travelling the first one-third distance at a speed of 10 km/h, the next one-third at 20 km/h and at last one-third at 60 km/h. The average speed of bus is

(a) 9 km/h (b) 16 km/h

(c) 18 km/h (d) 48 km/h [CBSE PMT 91]

4. A car moves from X to Y with a uniform speed v_u and returns to Y with a uniform speed v_d. The average speed of this sound trip is

(a) $\sqrt{v_u v_d}$ (b) $\dfrac{v_d v_u}{v_d + v_u}$

(c) $\dfrac{v_u + v_d}{2}$ (d) $\dfrac{2v_d v_u}{v_d + v_u}$ [CBSE PMT 07]

5. A car moves along a straight line, whose equation of motion is given by

$$s = 12t + 3t^2 - 2t^3$$

where s is in metres and t in seconds. The velocity of the car at the start will be

(a) 7 ms^{-1} (b) 9 ms^{-1}

(c) 12 ms^{-1} (d) 16 ms^{-1} [CBSE PMT 98]

6. A particle moves along a straight line OX. At a time t (in seconds) the distance x (in metres) of the particle from O is given by $x = 40 + 12t - t^3$. How long would the particle travel before coming to rest ?

(a) 16 m (b) 24 m

(c) 40 m (d) 56 m [CBSE PMT 06]

7. The position x of a particle varies with time t as $x = at^2 - bt^3$. The acceleration will be zero at time t equal to

(a) $\dfrac{a}{3b}$ (b) zero

(c) $\dfrac{2a}{3b}$ (d) $\dfrac{a}{b}$ [CBSE PMT 97]

8. Motion of a particle is given by equation $s = (3t^3 + 7t^2 + 14t + 8)$ m. The value of acceleration of the particle at $t = 1$ sec is

(a) 10 m / s^2 (b) 32 m / s^2

(c) 23 m / s^2 (d) 16 m / s^2 [CBSE PMT 2K]

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9. A particle moves along a straight line such that its displacement at any time t is given by $s=(t^3-6t^2+3t+4)$ metres. The velocity when the acceleration is zero is

(a) 3 m/s (b) 42 m/s
(c) −9 m/s (d) −15 m/s [CBSE PMT 94]

10. The displacement x of a particle varies with time t as $x=ae^{-\alpha t}+be^{\beta t}$, where a, b, α and β are positive constants. The velocity of the particle will

(a) be independent of β
(b) drop to zero when $\alpha=\beta$
(c) go on decreasing with time
(d) go on increasing with time [CBSE PMT 05]

11. The position x of a particle with respect to time t along x-axis is given by

$$x=9t^2-t^3,$$

where x is in metres and t in seconds.

What will be the position of this particle when it achieves maximum speed along the $+x$ direction ?

(a) 54 m (b) 81 m
(c) 24 m (d) 32 m [CBSE PMT 07]

12. The acceleration experienced by a moving motor boat, after its engine is cut off, is given by

$$\frac{dv}{dt}=-kv^3$$

where k is constant. If v_0 is the magnitude of velocity at cut-off, the magnitude of the velocity at a time t after the cut off is

(a) $v_0/2$ (b) v_0
(c) $\dfrac{v_0}{\sqrt{2v_0^2kt+1}}$ (d) v_0e^{-kt}
[CBSE PMT 94]

13. A particle moving along x-axis has acceleration f at time t, given by

$$f=f_0\left(1-\frac{t}{T}\right),$$

where f_0 and T are constants.

The particle at $t=0$ has zero velocity. In the time interval between $t=0$ and the instant when $f=0$, the particle's velocity v_x is

(a) $\frac{1}{2}f_0T^2$ (b) f_0T^2
(c) $\frac{1}{2}f_0T$ (d f_0T [CBSE PMT 07]

14. The acceleration of a particle is increasing linearly with time t as bt. The particle starts from origin with an initial velocity v_0. The distance travelled by the particle in time t will be

(a) $v_0t+\frac{1}{3}bt^2$ (b) $v_0t+\frac{1}{2}bt^2$
(c) $v_0t+\frac{1}{6}bt^3$ (d) $v_0t+\frac{1}{3}bt^3$ [CBSE PMT 95]

15. If a car at rest accelerates uniformly to a speed of 144 km/h in 20 sec, it covers a distance of

(a) 1440 cm (b) 2980 cm
(c) 20 m (d) 400 m [CBSE PMT 97]

16. The velocity of train increases uniformly from 20 km/h to 60 km/h in 4 hours. The distance travelled by the train during this period is

(a) 160 km (b) 180 km
(c) 100 km (d) 120 km [CBSE PMT 94]

17. A car moving with a speed of 40 km/h can be stopped by applying brakes after atleast 2 m. If the same car is moving with a speed of 80 km/h, what is the minimum stopping distance ?

(a) 4 m (b) 6 m
(c) 8 m (d) 2 m [CBSE PMT 98]

18. A car is moving along a straight road with a uniform acceleration. It passes through two points P and Q separated by a distance with velocity 30 km/h and 40 km/h respectively. The velocity of the car midway between P and Q is

(a) 33.3 km/h (b) $20\sqrt{2}$ km/h
(c) $25\sqrt{2}$ km/h (d) 35 km/h
[CBSE PMT 90]

19. A car accelerates from rest at a constant rate α for some time after which it decelerates at a constant rate β and comes to rest. If total time elapsed is t, then maximum velocity acquired by car will be

(a) $\dfrac{(\alpha^2-\beta^2)t}{\alpha\beta}$ (b) $\dfrac{(\alpha^2+\beta^2)t}{\alpha\beta}$
(c) $\dfrac{(\alpha+\beta)t}{\alpha\beta}$ (d) $\dfrac{\alpha\beta t}{\alpha+\beta}$ [CBSE PMT 94]

20. If a ball is thrown vertically upwards with speed u, the distance covered during the last t seconds of its ascent is

(a) ut (b) $\frac{1}{2}gt^2$
(c) $ut-\frac{1}{2}gt^2$ (d) $(u+gt)t$
[CBSE PMT 03]

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21. What will be the ratio of the distances moved by a freely falling body from rest in 4th and 5th seconds of journey ?

(a) 4 : 5 (b) 7 : 9
(c) 16 : 25 (d) 1 : 1 [CBSE PMT 89, 93]

22. Two bodies A (of mass 1 kg) and B (of mass 3 kg) are dropped from heights of 16 m and 25 m, respectively. The ratio of the time taken by them to reach the ground is

(a) 4/5 (b) 5/4
(c) 12/5 (d) 5/12 [CBSE PMT 06]

23. A ball is thrown vertically upward. It has a speed of 10 m/sec when it has reached one half of its maximum height. How high does the ball rise ? Take $g = 10 \text{m/s}^2$.

(a) 10 m (b) 5 m
(c) 15 m (d) 20 m [CBSE PMT 05]

24. A body dropped from top of a tower falls through 40 m during the last two seconds of its fall. The height of tower ($g = 10 \text{ms}^{-2}$) is

(a) 60 m (b) 45 m
(c) 80 m (d) 50 m [CBSE PMT 92]

25. A body dropped from a height h with initial velocity zero, strikes the ground with a velocity 3 m/s. Another body of same mass dropped from the same height h with an initial velocity of 4 m/s. The final velocity of second mass, with which it strikes the ground is

(a) 5 m/s (b) 12 m/s
(c) 3 m/s (d) 4 m/s [CBSE PMT 06]

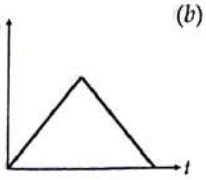
26. The water drop falls at regular intervals from a tap 5 m above the ground. The third drop is leaving the tap at instant the first drop touches the ground. How far above the ground is the second drop at that instant ?

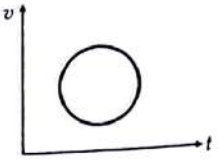
(a) 3.75 m (b) 4.00 m
(c) 1.25 m (d) 2.50 m [CBSE PMT 95]

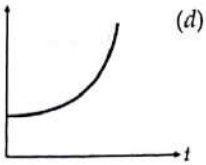
27. A rubber ball is dropped from a height of 5 m on a plane. On bouncing it rises to 1.8 m. The ball loses its velocity on bouncing by a factor of

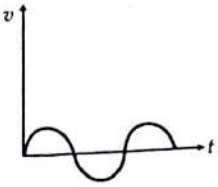
(a) $\frac{3}{5}$ (b) $\frac{2}{5}$
(c) $\frac{16}{25}$ (d) $\frac{9}{25}$ [CBSE PMT 98]

28. Which of the following curves does not represent motion in one dimension ? [CBSE PMT 92]

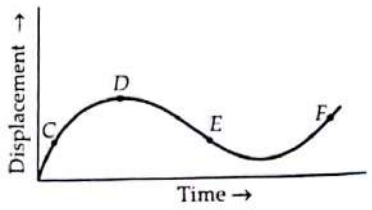
(a) 

(b) 

(c) 

(d) 

29. The displacement-time graph of a moving particle is shown ahead. The instantaneous velocity of the particle is negative at the point



(a) E (b) F
(c) C (d) D [CBSE PMT 94]

30. A train of 150 metre length is going towards north direction at a speed of 10 m/s. A parrot flies at the speed of 5 m/s towards south direction parallel to the railways track. The time taken by the parrot to cross the train is

(a) 12 sec (b) 8 sec
(c) 15 sec (d) 10 sec. [CBSE PMT 88]

31. A particle starts its motion from rest under the action of a constant force. If the distance covered in first 10 seconds is S_1 and that covered in the first 20 seconds is S_2, then

(a) $S_2 = S_1$ (b) $S_2 = 2S_1$
(c) $S_2 = 3S_1$ (d) $S_2 = 4S_1$ [CBSE PMT 09]

32. A bus is moving with a speed of 10 ms^{-1} on a straight road. A scooterist wishes to overtake the bus in 100 s. If the bus is at a distance of 1 km from the scooterist, with what speed should the scooterist chase the bus ?

(a) 10 ms^{-1} (b) 20 ms^{-1}
(c) 40 ms^{-1} (d) 25 ms^{-1} [CBSE PMT 09]

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33. A particle covers half of its total distance with speed v_1 and the rest half distance with speed v_2. Its average speed during the complete journey is

(a) $\frac{v_1^2 v_2^2}{v_1^2 + v_2^2}$ (b) $\frac{v_1 + v_2}{2}$

(c) $\frac{v_1 v_2}{v_1 + v_2}$ (d) $\frac{2v_1 v_2}{v_1 + v_2}$ [CBSE Final 2011]

34. A particle moves a distance x in time t according to equation $x = (t+5)^{-1}$. The acceleration of particle is proportional to

(a) (velocity)$^{3/2}$ (b) (distance)^2

(c) (distance)$^{-2}$ (d) (velocity)$^{2/3}$

[CBSE Pre 2010]

35. A boy standing at the top of a tower of 20 m height drops a stone. Assuming $g = 10\,ms^{-2}$, the velocity with which it hits the ground is

(a) 10.0 m/s (b) 20.0 m/s

(c) 40.0 m/s (d) 5.0 m/s [CBSE Pre 2011]

36. A ball is dropped from a high rise platform at $t = 0$ starting from rest. After 6 seconds another ball is thrown downwards from the same platform with a speed v. The two balls meet at $t = 18$ s. What is the value of v ?

(take $g = 10\,m/s^2$)

(a) 75 m/s (b) 55 m/s

(c) 40 m/s (d) 60 m/s [CBSE Pre 2010]

Answers and Explanations

1. (b) $v_{av} = \frac{2v_1v_2}{v_1+v_2} = \frac{2\times 40\times 60}{40+60}$

$= \mathbf{48\ kmh^{-1}}$.

2. (b) $v_{av} = \frac{2v_1v_2}{v_1+v_2}$

$\therefore \quad 48 = \frac{2\times 40\times v}{40+v}$

or $\quad v = \mathbf{60\ km/h}$.

3. (c) Total distance travelled $= s$

Total time taken,

$$t = \frac{s/3}{10} + \frac{s/3}{20} + \frac{s/3}{60}$$

$$= \frac{10s}{180} = \frac{s}{18}$$

$$v_{av} = \frac{s}{t} = \frac{s}{s/18}$$

$= \mathbf{18\ km/h}$.

4. (d) $v_{av} = \frac{s+s}{\frac{s}{v_u} + \frac{s}{v_d}} = \frac{\mathbf{2v_u v_d}}{v_d + v_u}$.

5. (c) $s = 12t + 3t^2 - 2t^3$

$$v = \frac{ds}{dt} = 12 + 6t - 6t^2$$

At $t = 0$, $v = \mathbf{12\,ms^{-1}}$.

6. (d) Given : $x = 40 + 12t - t^3$

$$v = \frac{dx}{dt} = 0 + 12 - 3t^2$$

Velocity becomes zero, when

$$12 - 3t^2 = 0$$

or $\quad t = 2$ s

Distance travelled by particle before coming to rest,

$$x = 40 + 12\times 2 - 2^3 = \mathbf{56\ m}.$$

7. (c) $x = at^2 - bt^3$

$$v = \frac{dx}{dt} = 2at - 3bt^2$$

Acceleration $= \frac{dv}{dt} = 2a - 6bt = 0$

or $\quad t = \frac{\boldsymbol{a}}{\mathbf{3b}}$.

8. (b) $s = 3t^3 + 7t^2 + 14t + 8$

$$v = \frac{ds}{dt} = 9t^2 + 14t + 14$$

$$a = 18t + 14$$

At $t = 1$ s,

$$a = 18\times 1 + 14 = \mathbf{32\ ms^{-2}}.$$

9. (c) $s = t^3 - 6t^2 + 3t + 4$

$$v = \frac{ds}{dt} = 3t^2 - 12t + 3$$

$$a = 6t - 12 = 0$$

or $\quad t = 2$ s

At $t = 2$ s,

$$v = 3\times 2^2 - 12\times 2 + 3 = \mathbf{-9\ m/s}.$$

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10. (*d*) $x = ae^{-\alpha t} + be^{\beta t}$

$$v = \frac{dx}{dt} = -a\alpha e^{-\alpha t} + b\beta e^{\beta t}$$

Clearly, v is not independent of α and β.

Moreover, it does not become zero for $\alpha = \beta$. Infact, the velocity increases with time because

$$\frac{dv}{dt} = a\alpha^2 e^{-\alpha t} + b\beta^2 e^{\beta t} > 0$$

11. (*a*) $x = 9t^2 - 3t^3$

$$v = \frac{dx}{dt} = 18t - 9t^2$$

For maximum speed, $\frac{dv}{dt} = 0$

or $18t - 9t^2 = 0 \quad \Rightarrow \quad t = 3$ s.

$\therefore$ At maximum speed, position of the particle is

$$x = 9 \times 3^2 - 3^3 = 81 - 27 = \mathbf{54\ m}.$$

12. (*c*) Given : $\frac{dv}{dt} = -kv^3$

or $v^{-3}\,dv = -k\,dt$

On integration,

$$\int v^{-3}\,dv = -k\int dt$$

or $$\frac{v^{-2}}{-2} = -kt + C$$

or $$\frac{1}{2v^2} = kt - C$$

where C is constant of integration.

At $t = 0, \quad v = v_0$

$\therefore$ $$\frac{1}{2v_0^2} = 0 - C$$

Hence $$\frac{1}{2v^2} = kt + \frac{1}{2v_0^2} = \frac{2v_0^2 kt + 1}{2v_0^2}$$

or $$v = \frac{v_0}{\sqrt{2v_0^2 kt + 1}}.$$

13. (*c*) At time $t = 0$, velocity $v = 0$.

Acceleration, $f = f_0\left(1 - \frac{t}{T}\right)$

When $f = 0$, $f_0\left(1 - \frac{t}{T}\right) = 0$

As f_0 is constant

$$1 - \frac{t}{T} = 0 \quad \text{or} \quad t = T$$

Now acceleration,

$$f = \frac{dv}{dt}$$

$\therefore$ $$\int_0^{v_x} dv = \int_0^T f\,dt = \int_0^T f_0\left(1 - \frac{t}{T}\right)dt$$

or $$[v]_0^{v_x} = \left[f_0 t - \frac{f_0 t^2}{2T}\right]_0^T = f_0 T - \frac{f_0 T^2}{2T}$$

$$v_x = \frac{1}{2} f_0 T.$$

14. (*c*) $a = \frac{dv}{dt} = bt$

$\therefore$ $$v = \int bt\,dt = \frac{bt^2}{2} + C$$

At $t = 0, \quad v = v_0$

$\therefore$ $v_0 = C$

Hence $$v = \frac{dx}{dt} = \frac{bt^2}{2} + v_0$$

or $$x = \int\left(\frac{bt^2}{2} + v_0\right)dt$$

$$= \frac{bt^3}{6} + v_0 t + C'$$

At $t = 0, \quad x = 0$

$\Rightarrow$ $C' = 0$

$\therefore$ $$x = v_0 t + \frac{1}{6} bt^3.$$

15. (*d*) $u = 0, \ v = 144$ kmh^{-1} $= 40$ ms^{-1}, $t = 20$ s

$$a = \frac{v - u}{t} = \frac{40 - 0}{20} = 2\ \text{ms}^{-1}.$$

$$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 2 \times (20)^2 = \mathbf{400\ m}.$$

16. (*a*) $$a = \frac{v - u}{t} = \frac{60 - 20}{4} = 10\ \text{km/h}^2$$

$$s = ut + \frac{1}{2}at^2 = 20 \times 4 + \frac{1}{2} \times 10 \times 4^2$$

$$= \mathbf{160\ km}.$$

17. (*c*) $v^2 - u^2 = 2as$

$\therefore$ $0^2 - u_1^2 = 2as_1$ and $0^2 - u_2^2 = 2as_2$

or $$\frac{s_2}{s_1} = \frac{u_2^2}{u_1^2} = \frac{(80)^2}{(40)^2} = 4$$

$$s_2 = 4s_1 = 4 \times 2\ \text{m} = \mathbf{8\ m}.$$

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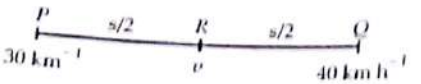
18. (*c*) The situation is shown below :



As $v^2 - u^2 = 2as$

$\therefore \quad (40)^2 - (30)^2 = 2as$

$$a = \frac{1600 - 900}{2s} = \frac{350}{s}$$

Again, $v^2 - (30)^2 = 2a \times \frac{s}{2}$

or $v^2 - (30)^2 = 2 \times \frac{350}{s} \times \frac{s}{2}$

or $v = 25\sqrt{2}$ **km/h.**

19. (*d*) Refer to the solution of Example 21(*i*) on Page 3.14.

20. (*b*) Suppose the ball takes time T to reach the maximum height H.

As $v = u + at$

$\therefore \quad 0 = u - gT$

or $T = \frac{u}{g}$

Also, $s = ut + \frac{1}{2}at^2$

$\therefore \quad H = uT - \frac{1}{2}gT^2$

Height attained by the ball in time $(T - t)$,

$$H' = u(T-t) - \frac{1}{2}g(T-t)^2$$

Height covered by the ball in last t seconds is given by

$h = H - H'$

$$= uT - \frac{1}{2}gT^2 - \left[u(T-t) - \frac{1}{2}g(T-t)^2\right]$$

or $h = ut + \frac{1}{2}gt^2 - gtT = ut + \frac{1}{2}gt^2 - gt \cdot \frac{u}{g}$

or $h = \mathbf{\frac{1}{2}gt^2}$.

21. (*b*) $s_{nth} = u + \frac{a}{2}(2n-1)$

Here $u = 0, \ a = g$

$\therefore \quad s_{4th} = \frac{g}{2}(2 \times 4 - 1) = \frac{7g}{2}$

$$s_{5th} = \frac{g}{2}(2 \times 5 - 1) = \frac{9g}{2}$$

$$\frac{s_{4th}}{s_{5th}} = \frac{7}{9} = 7:9.$$

22. (*a*) $s = ut + \frac{1}{2}gt^2 = 0 + \frac{1}{2}gt^2$

$$\therefore \quad \frac{t_1}{t_2} = \sqrt{\frac{s_1}{s_2}} = \sqrt{\frac{16}{25}} = \frac{4}{5}.$$

23. (*a*) Speed becomes zero at maximum height.

$v^2 - u^2 = 2as$

$0 - (10)^2 = -2 \times 10 \times \frac{h}{2}$

$\therefore \quad h = \mathbf{10\ m}.$

24. (*b*) Height covered in t seconds,

$$h = \frac{1}{2}gt^2$$

Height covered in $(t-2)$ seconds,

$$h' = \frac{1}{2}g(t-2)^2$$

Height covered in last two seconds,

$$h - h' = \frac{1}{2}g[t^2 - (t-2)^2]$$

$$40 = \frac{1}{2} \times 10(4t - 4)$$

or $t = 3$ s.

$\therefore \quad h = \frac{1}{2} \times 10 \times (3)^2 = \mathbf{45\ m}.$

25. (*a*) We use, $v^2 - u^2 = 2as$

For first body : $3^2 - 0^2 = 2gh$

For second body : $v^2 - 4^2 = 2gh$

$\therefore \quad v^2 - 4^2 = 3^2$

or $v = \sqrt{25} = \mathbf{5\ m/s}.$

26. (*a*) Time t taken by a drop to reach the ground is given by

$$s = ut + \frac{1}{2}at^2$$

$$5 = 0 + \frac{1}{2} \times 10 \times t^2 \quad \text{or} \quad t = 1 \text{ s}$$

When the third drop leaves the tap, the first drop reaches the ground. So a drop leaves the tap at interval of $\frac{1}{2}$s. Distance covered by second drop during this time,

$$s = 0 + \frac{1}{2} \times 10 \times \left(\frac{1}{2}\right)^2 = 1.25 \text{ m}$$

Height of the second drop from the ground

$= 5 - 1.25 = \mathbf{3.75\ m}.$

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27. (d) When the ball falls down,

$v^2 - 0^2 = 2 \times 10 \times 5$ or $v = 10$ m/s

When the ball rises up,

$0^2 - u^2 = -2 \times 10 \times 1.8$ or $u = 6$ m/s

The factor by which ball loses velocity

$$= \frac{10-6}{10} = \frac{2}{5}.$$

28. (b) In one-dimensional motion, a body can have only one value of velocity at any time and not two values of velocity.

29. (a) Instantaneous velocity,

$$v = \frac{ds}{dt}$$

= Slope of the displacement-time curve at any instant.

Slope and hence instantaneous velocity is negative at point E.

30. (d) $v_T = +10\,\text{ms}^{-1}$, $v_p = -5\,\text{ms}^{-1}$

Relative velocity of parrot w.r.t. train,

$$= v_p - v_T = -5 - 10 = -15\ \text{ms}^{-1}.$$

Time taken by parrot to cross the train

$$= \frac{150\,\text{m}}{15\,\text{ms}^{-1}} = 10\ \text{s}.$$

31. (d) $S_1 = \frac{1}{2}at_1^2$ and $S_2 = \frac{1}{2}at_2^2$

$$\therefore \quad \frac{S_2}{S_1} = \left(\frac{t_2}{t_1}\right)^2 = \left(\frac{20}{10}\right)^2 = 4$$

or $S_2 = 4S_1$.

32. (b) Relative velocity of scooter w.r.t. bus,

$$v_{SB} = \frac{1\,\text{km}}{100\,\text{s}} = \frac{1000\,\text{m}}{100\,\text{s}} = 10\ \text{ms}^{-1}$$

But $v_{SB} = v_S - v_B$

$\therefore \quad v_S = v_{SB} + v_B = 10 + 10 = 20\ \text{ms}^{-1}$.

33. (d) $v_{av} = \dfrac{\text{Total distance}}{\text{Total time}}$

$$= \frac{d}{\frac{d/2}{v_1} + \frac{d/2}{v_2}} = \frac{2v_1v_2}{v_1 + v_2}$$

34. (a) $x = (t+5)^{-1}$

$$v = \frac{dx}{dt} = \frac{-1}{(t+5)^2}$$

$$a = \frac{dv}{dt} = \frac{2}{(t+5)^3}$$

As $\frac{1}{t+5} \propto v^{1/2}$

$$\therefore \quad a \propto \frac{1}{(t+5)^3} \propto v^{3/2}$$

35. (b) $v = \sqrt{2gh} = \sqrt{2 \times 10 \times 20} = \sqrt{400}$

$= 20\,\text{ms}^{-1}$

36. (a) Distance moved in 18 s by first ball

$$= \frac{1}{2} \times 10 \times 18^2$$

$$= 90 \times 18 = 1620\ \text{m}$$

Distance moved in 12 s by second ball

$$= ut + \frac{1}{2}gt^2$$

$$\therefore \quad 1620 = 12v + 5 \times 144$$

$$v = 132 - 60 = 75\,\text{ms}^{-1}$$

Delhi PMT and VMMC Entrance Exam

(Similar Questions)

1. A particle is constrained to move on a straight line path. It returns to the starting point after 10 s. The total distance covered by the particle during this time is 30 m. Which of the following statements about the motion of the particle is false ?

(a) displacement of the particle is zero

(b) displacement of the particle is 30 m

(c) average speed of the particle is 3 m/s

(d) both (a) and (c) [DPMT 92]

2. A car travels first half of the distance between two places with a speed of 30 km/hr and remaining half with a speed of 50 km/hr. The average speed of the car is

(a) 37.5 km/hr (b) 42 km/hr

(c) 40 km/h (d) 49 km/h [DPMT 07]

3. If the first one-third of a journey is travelled at 20 km/h, next one-third at 40 km/h and the last one-third at 60 km/h, the average speed of whole journey will be

(a) 32.7 km/h (b) 35 km/h

(c) 40 km/h (d) 45 km/h [VMMC 07]

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4. A car travelling on a straight track moves with uniform velocity of v_1 for some time and with uniform velocity of v_2 for the next equal time. Average velocity of the car is

(a) $\sqrt{v_1 v_2}$ (b) $\left(\frac{1}{v_1}+\frac{1}{v_2}\right)^{-1}$

(c) $\frac{v_1+v_2}{2}$ (d) $2\left(\frac{1}{v_1}+\frac{1}{v_2}\right)^{-1}$ [DPMT 93, 96]

5. Given $a=2t+5$. Calculate the velocity of the body after five sec if it starts from rest.

(a) 50 m/s (b) 25 m/s

(c) 100 m/s (d) 75 m/s [DPMT 07]

6. The displacement of a particle moving in straight line is given by $x=2t^2+t+5$, where x is expressed in metres and t in seconds. The acceleration at $t=2$ sec is

(a) $4\,m/s^2$ (b) $10\,m/s^2$

(c) $8\,m/s^2$ (d) $15\,m/s^2$ [DPMT 96]

7. The velocity of a particle at an instant is 10 m/s. After 3 s its velocity will become 16 m/s. The velocity at 2 s, before the given instant will be

(a) 6 m/s (b) 4 m/s

(c) 2 m/s (d) 1 m/s [VMMC 06]

8. A body initially at rest is moving with uniform acceleration a. Its velocity after n seconds is v. The displacement of the body in last 2 s is

(a) $\frac{2v(n-1)}{n}$ (b) $\frac{v(n-1)}{n}$

(c) $\frac{v(n+1)}{n}$ (d) $\frac{2v(n+1)}{n}$ [VMMC 06]

9. Velocity of a body on reaching the point from which it was projected upwards, is

(a) $v=0$ (b) $v=0.5u$

(c) $v=2u$ (d) $v=u$ [DPMT 99]

10. If a ball is thrown vertically upwards with 40 m/s, its velocity after two sec will be

(a) 10 m/s (b) 30 m/s

(c) 20 m/s (d) 40 m/s [DPMT 95, 99]

11. A stone released with zero velocity from top of the tower reaches the ground in 4 sec. The height of the tower is about

(a) 20 m (b) 80 m

(c) 40 m (d) 160 m [DPMT 99, 04]

12. A stone is thrown vertically up from the ground. It reaches a maximum height of 50 m in 10 sec. After what time it will reach the ground from maximum height position ?

(a) 1.2 sec (b) 10 sec

(c) 5 sec (d) 25 sec [DPMT 94, 98]

13. A stone falls freely such that the distance covered by it in the last second of its motion is equal to the distance covered by it in the first 5 seconds. It remained in air for

(a) 12 sec (b) 13 sec

(c) 25 sec (d) 26 sec [VMMC 03]

14. A body sliding on a smooth inclined plane requires 4 seconds to reach the bottom, starting from rest at the top. How much time does it take to cover one-fourth the distance starting from rest at the top ?

(a) 1 sec (b) 4 sec

(c) 2 sec (d) 16 sec [DPMT 93]

15. A body falling from rest describes distances s_1, s_2 and s_3 in the first, second and third seconds of its fall, then the ratio $s_1 : s_2 : s_3$ is

(a) 1 : 1 : 1 (b) 1 : 3 : 5

(c) 1 : 2 : 3 (d) 1 : 4 : 9 [DPMT 90]

16. When a ball is thrown vertically upwards, at the maximum height

(a) the velocity is zero and therefore there is no acceleration acting on the particle

(b) the acceleration is present and therefore velocity is not zero

(c) the acceleration depends on the velocity as $a=\frac{dv}{dt}$

(d) the acceleration is independent of the velocity [DPMT 06]

17. A police jeep is chasing with velocity 45 km/h. A thief in another jeep is moving with 155 km/h. Police fires a bullet with a muzzle velocity 180 m/s. The bullet strikes the jeep of the thief with a velocity

(a) $27\,ms^{-1}$ (b) $150\,ms^{-1}$

(c) $250\,ms^{-1}$ (d) $450\,ms^{-1}$ [DPMT 06]

18. Velocity-time curve for a body projected vertically upwards is

(a) ellipse (b) hyperbola

(c) parabola (d) straight line [DPMT 95]

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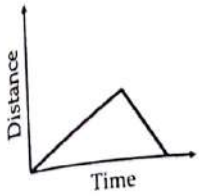
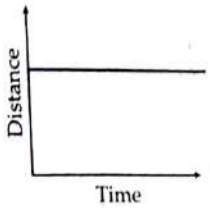
19. Which of the following distance-time graph shows accelerated motion ?

(a)

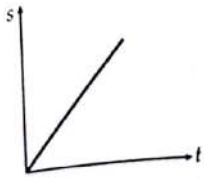
(b)

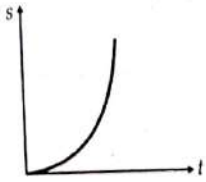
(c)


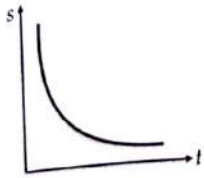
(d)


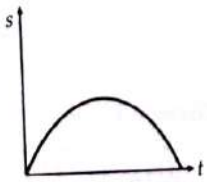
[DPMT 2K]

20. Which of the following graph represents uniformly accelerated motion ? [DPMT 01]

(a)


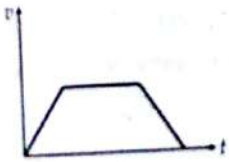
(b)


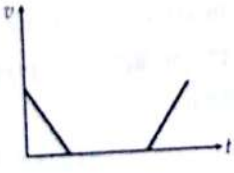
(c)


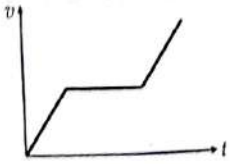
(d)


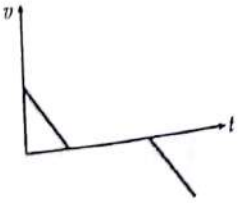
21. Acceleration-time graph of a body is shown.

The corresponding velocity-time graph of the same body is :

(a)


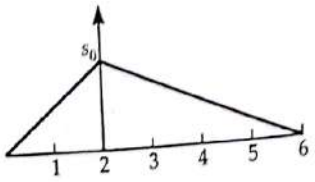
(b)


(c)


(d)


[DPMT 04]

22. What will be ratio of speed in first two seconds to the speed in next 4 seconds ?



(a) $\sqrt{2} : 1$ (b) 3 : 1
(c) 2 : 1 (d) 1 : 2 [VMMC 02]

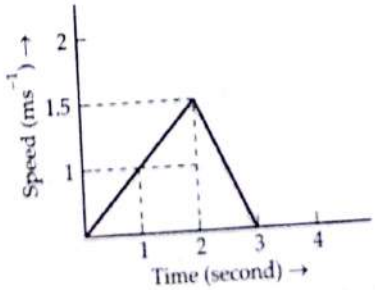
23. You drive a car at a speed of 70 km/h in a straight road for 8.4 km, and then the car runs out of petrol. You walk for 30 min to reach a petrol pump at a distance of 2 km. The average velocity from the beginning of your drive till you reach the petrol pump is

(a) 16.8 km/h (b) 35 km/h
(c) 64 km/h (d) 18.6 km/h. [DPMT 09]

24. Tom and Dick are running forward with the same speed. They are following a rubber ball to each other at a constant speed v as seen by the thrower. According to Sam who is standing on the ground the speed of ball is

(a) same as v (b) greater than v
(c) less than v (d) none of these. [DPMT 09]

25. The speed-time graph of a particle moving along a solid curve is shown below. The distance traversed by the particle from $t = 0$ to $t = 3$ is



(a) $\frac{9}{2}$ m (b) $\frac{9}{4}$ m
(c) $\frac{9}{3}$ m (d) $\frac{9}{5}$ m [DPMT 2011]

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Answers and Explanations

1. (*d*) As the particle returns to the starting point, its displacement is zero.

$$\text{Average speed} = \frac{30\text{ m}}{10\text{ s}} = 3\text{ m/s}.$$

2. (*a*) $$v_{av} = \frac{2v_1v_2}{v_1 + v_2}$$

$$= \frac{2 \times 30 \times 50}{30 + 50} = 37.5\text{ km/h}.$$

3. (*a*) $$v_{av} = \frac{3v_1v_2v_3}{v_1v_2 + v_2v_3 + v_1v_3}$$

$$= \frac{3 \times 20 \times 40 \times 60}{20 \times 40 + 40 \times 60 + 20 \times 60}$$

$$= 32.7\text{ km/h}.$$

4. (*d*) Let total distance $= 2x$

$$\text{Total time taken} = \frac{x}{v_1} + \frac{x}{v_2} = x\left(\frac{1}{v_1} + \frac{1}{v_2}\right)$$

$$\text{Average speed} = \frac{2x}{x\left(\frac{1}{v_1} + \frac{1}{v_2}\right)} = 2\left(\frac{1}{v_1} + \frac{1}{v_2}\right)^{-1}$$

5. (*a*) $$a = \frac{dv}{dt} = 2t + 5$$

$$\int_0^v dv = \int_0^5 (2t + 5)\,dt$$

$$v = [t^2]_0^5 + 5[t]_0^5$$

$$= (25 - 0) + 5(5 - 0)$$

$$= 50\text{ m/s}.$$

6. (*a*) Given : $x = 2t^2 + t + 5$

$$v = \frac{dx}{dt} = 4t + 1$$

$$a = \frac{dv}{dt} = 4\text{ m/s}^2.$$

7. (*a*) Here $u = 10$ m/s, $t = 3$ s, $v = 16$ m/s.

$$\therefore \quad a = \frac{v - u}{t} = \frac{16 - 10}{3} = 2\text{ m/s}^2$$

Velocity at 2 s before the given instant,

$$10 = u + 2 \times 2 \qquad [v = u + at]$$

$$u = 10 - 4 = 6\text{ m/s}.$$

8. (*a*) $$a = \frac{v - u}{t} = \frac{v - 0}{n} = \frac{v}{n}$$

Displacement in last 2 s,

$$s_n - s_{n-2} = \frac{1}{2}a^2 - \frac{1}{2}a(n-2)^2$$

$$= 2a(n-1) = 2 \times \frac{v}{n}(n-1)$$

$$= \frac{2v(n-1)}{n}.$$

9. (*d*) $v = u$.

10. (*c*) $v = u + at = 40 - 10 \times 2 = 20$ m/s.

11. (*b*) $h = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times (4)^2 = 80$ m.

12. (*b*) Time of ascent = Time of decent, in the absence of air friction.

13. (*b*) $s_{nth} = u + \frac{g}{2}(2n - 1)$

For a freely falling body, $u = 0$

$$\therefore \quad s_{nth} = \frac{g}{2}(2n - 1)$$

Distance travelled in first 5 s,

$$s = ut + \frac{1}{2}gt^2 = 0 + \frac{1}{2}g(5)^2 = \frac{25}{2}g$$

Given : $s_{nth} = s$

$$\text{or} \quad \frac{g}{2}(2n - 1) = \frac{25}{2}g$$

$$\text{or} \quad n = 13\text{ s}.$$

14. (*c*) Here : $u = 0$, $s = \frac{1}{2}at^2$ or $s \propto t^2$

$$\therefore \quad \frac{t_2}{t_1} = \sqrt{\frac{s_2}{s_1}} = \sqrt{\frac{s/4}{s}} = \frac{1}{2}$$

$$\text{or} \quad t_2 = \frac{1}{2} \times 4 = 2\text{ s}.$$

15. (*d*) $s \propto t^2$

$$\therefore \quad s_1 : s_2 : s_3 = 1^2 : 2^2 : 3^2 = 1 : 4 : 9.$$

16. (*d*) The acceleration is independent of velocity.

17. (*b*) Relative speed of bullet with respect to thief's jeep,

$$= (v_b + v_p) - v_t$$

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$= 180\,ms^{-1} + 45\,kmh^{-1} - 155\,kmh^{-1}$

$= 180\,ms^{-1} + 45 \times \frac{5}{18}\,ms^{-1} - 155 \times \frac{5}{18}\,ms^{-1}$

$= 180\,ms^{-1} - 30.5\,ms^{-1}$

$= \mathbf{150\ ms^{-1}}$.

18. (*d*) As acceleration (g) remains constant throughout, the velocity-time graph is a straight line.

19. (*a*) For accelerated motion, distance-time graph is a parabola about distance-axis.

20. (*b*) For uniformly accelerated motion, s-t graph is a parabola symmetric about distance-axis.

21. (*c*) When acceleration a is constant, velocity v increases uniformly with time t and when $a = 0$, velocity v remains constant.

22. (*c*) $\frac{v_1}{v_2} = \frac{\frac{s_0}{2-0}}{\frac{s_0}{6-2}}$

$= \frac{4}{2} = \mathbf{2:1}$.

23. (*a*) Average speed

$= \frac{\text{Total distance}}{\text{Total time}}$

$= \frac{s_1 + s_2}{t_1 + t_2} = \frac{8.4\text{ km} + 2\text{ km}}{\frac{8.4}{70}\text{ h} + \frac{30}{60}\text{ h}}$

$= \frac{10.4}{0.12 + 0.5} = \frac{10.4}{0.62}$

$\simeq \mathbf{16.8\ km/h}$.

24. (*b*) As they are moving in the same direction, the relative velocity of ball w.r.t. Tom or Dick will be

$v = v_B - v_{Tom}$

or $v = v_B - v_{Dick}$

$\therefore \quad v_B = v + v_{Tom} = v + v_{Dick}$

For Sam, the speed of the ball will be greater than v.

25. (*b*)

Distance traversed = Area under speed-time graph

$= \frac{1}{2} \times 1.5 \times 3 = \frac{9}{4}\text{ m}$

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CHAPTER

4

MOTION IN A PLANE

4.1 NEED FOR VECTORS

1. *Briefly describe the necessity for introducing the concept of vectors.*

Need for vectors. In one-dimensional motion, only two directions are possible. So the directional aspect of the quantities like position, displacement, velocity and acceleration can be taken care of by using + and – signs. But in case of motion in two-dimensions (plane) or three dimensions (space), an object can have a large number of directions. In order to deal with such situations effectively, we need to introduce the concept of new physical quantities, called vectors, in which we take care of both magnitude and direction.

4.2 SCALARS AND VECTORS

2. *What are scalar and vector quantities ? Give examples.*

Scalar quantities. *The physical quantities which have only magnitude and no direction are called scalar quantities or scalars.* A scalar quantity can be specified by a single number, along with the proper unit.

Examples : Mass, volume, density, time, temperature, electric current, etc.

Vector quantities. *The physical quantities which have both magnitude and direction and obey the laws of vector addition are called vector quantities or vectors.* A vector quantity is specified by a number with a unit and its direction.

Examples : Displacement, velocity, force, momentum, etc.

3. *Give some points of difference between scalars and vectors.*

Scalars	Vectors
1. Scalars have only magnitude.	Vectors have both magnitude and direction.
2. They change if their magnitude changes.	They change if either their magnitude, direction or both change.
3. They can be added according to ordinary laws of algebra.	They can be added only by using special laws of vector addition.

4.3 REPRESENTATION OF A VECTOR

4. *With the help of a suitable example, explain how is a vector quantity represented.*

Representation of a vector. *A vector quantity is represented by a straight line with an arrowhead over it.* The length of the line gives the magnitude and the arrowhead gives the direction. Suppose a body has a velocity of 40 kmh^{-1} due east. If 1 cm is chosen to

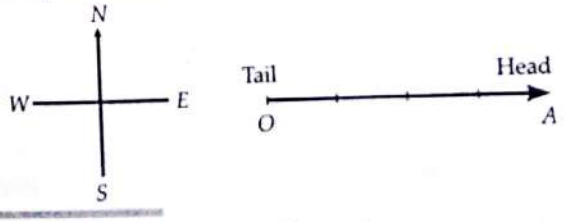

Fig. 4.1 Representation of a vector.

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represent a velocity of 10 kmh^{-1}, a line OA, 4 cm in length and drawn towards east with arrowhead at A will completely represent the velocity of the body. The point A is called head or terminal point and point O is called tail or initial point of the vector $\vec{OA}$ (Fig. 4.1).

In a simpler notation, a vector is represented by single letter of alphabet either in bold face or with an arrow over it. For example, a force vector can be represented as $\vec{F}$ or **F**.

4.4 POSITION AND DISPLACEMENT VECTORS

5. *Distinguish between position vector and displacement vector.*

Position vector. *A vector which gives position of an object with reference to the origin of a co-ordinate system is called position vector.* Consider the motion of an object in X-Y plane with origin at O. Suppose an object is at point P at any instant t, as shown in Fig. 4.2. Then $\vec{OP}$ is the position vector of the object at point P. The position vector provides two informations :

(*i*) It tells the straight line distance of the object from the origin O.

(*ii*) It tells the direction of the object with respect to the origin.

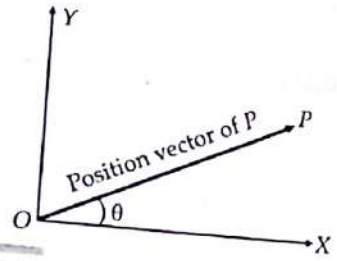

Fig. 4.2 Position vector.

Displacement vector. *It is that vector which tells how much and in which direction an object has changed its position in a given time interval.* Consider an object moving in the XY-plane. Suppose it is at point P at any instant t and at point Q at any later instant t', as shown in Fig. 4.3. Then vector $\vec{PQ}$ is the displacement vector of the object in time t to t'.

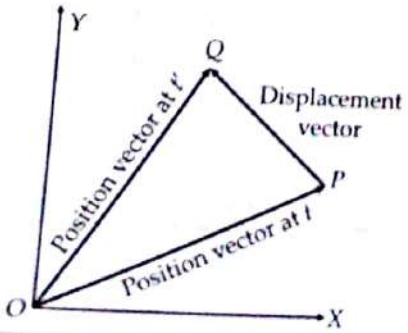

Fig. 4.3. Displacement vector.

4.5 POLAR AND AXIAL VECTORS

6. *What is meant by polar and axial vectors ? Give examples.*

Broadly speaking, vectors are of *two* types :

Polar vectors. *The vectors which have a starting point or a point of application are called polar vectors.*

Examples : Displacement, velocity, force, etc., are polar vectors.

Axial vectors. *The vectors which represent rotational effect and act along the axis of rotation in accordance with right hand screw rule are called axial vectors.*

Examples : Angular velocity, torque, angular momentum, etc. are axial vectors.

As shown in Fig. 4.4, axial vector will have its direction along its axis of rotation depending on its anticlockwise or clockwise rotational effect.

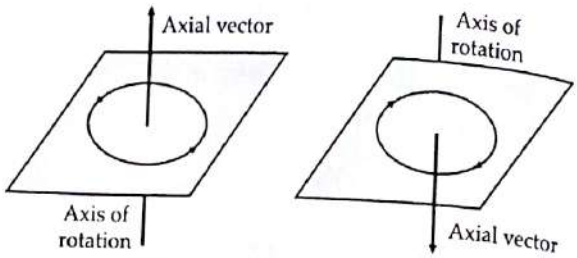

Fig. 4.4

For Your Knowledge

- The Latin word *vector* means *carrier*.
- The physical quantities which have no specified direction and have different values in different directions are called *tensors*. For example, moment of inertia.

4.6 SOME DEFINITIONS IN VECTOR ALGEBRA

7. *Define and illustrate the following terms :*

(*i*) *Equal vectors* (*ii*) *Negative of a vector*
(*iii*) *Modulus of a vector* (*iv*) *Unit vector*
(*v*) *Fixed vector* (*vi*) *Free vector*
(*vii*) *Collinear vectors* (*viii*) *Coplanar vectors*
(*ix*) *Co-initial vectors* (*x*) *Co-terminus vectors.*

(*i*) **Equal vectors.** *Two vectors are said to be equal if they have the same magnitude and same direction.* In Fig. 4.5, $\vec{A}$ and $\vec{B}$ are two equal vectors.

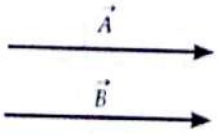

Fig. 4.5 Equal Vectors.

Note If a vector is displaced parallel to itself, it remains equal to itself.

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(ii) **Negative of a vector.** *The negative of a vector is defined as another vector having the same magnitude but having an opposite direction.* In Fig. 4.6, vector $\vec{A}$ is the negative of vector $\vec{B}$ or vice versa.

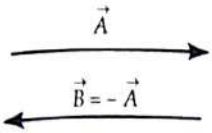

Fig. 4.6 Negative of a vector.

(iii) **Modulus of a vector.** *The modulus of a vector means the length* or *the magnitude of that vector.* It is a scalar quantity.

Modulus of vector $\vec{A} = |\vec{A}| = A$

(iv) **Unit vector.** *A unit vector is a vector of unit magnitude drawn in the direction of a given vector.* A unit vector in the direction of a given vector is found by dividing the given vector by its modulus. Thus a unit vector in the direction of vector $\vec{A}$ is given by

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{\vec{A}}{A}$$

A unit vector in the direction of a given vector $\vec{A}$ is written as $\hat{A}$ and is pronounced as 'A carat' or 'A hat' or 'A cap'.

Any vector can be expressed as the magnitude times the unit vector along its own direction.

$$\vec{A} = |\vec{A}|\,\hat{A}$$

Note The magnitude of a unit vector is unity. It just gives the direction of a vector. A unit vector has no units or dimensions.

(v) **Fixed vector.** *The vector whose initial point is fixed is called a fixed vector or a localised vector.* For example, the position vector of a particle is a fixed vector because its initial point lies at the origin.

(vi) **Free vector.** *A vector whose initial point is not fixed is called a free vector or a non-localised vector.* For example, the velocity vector of a particle moving along a straight line is a free vector.

(vii) **Collinear vectors.** *The vectors which either act along the same line or along parallel lines are called collinear vectors.* Two collinear vectors having the same direction ($\theta = 0°$) are called *like* or *parallel vectors.* Two collinear vectors having the opposite directions ($\theta = 180°$) are called *unlike* or *antiparallel vectors.*

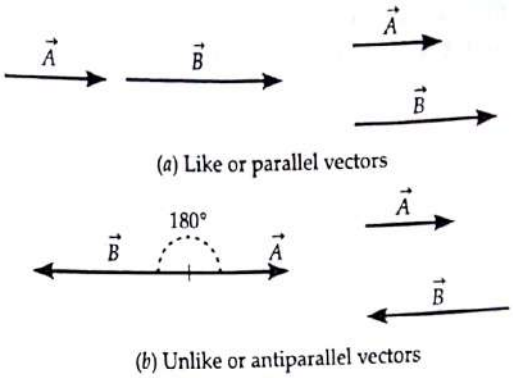

Fig. 4.7 Collinear vectors

(viii) **Coplanar vectors.** *The vectors which act in the same plane are called coplanar vectors.*

(ix) **Co-initial vectors.** *The vectors which have the same initial point are called co-initial vectors.* In Fig. 4.8, $\vec{A}$, $\vec{B}$ and $\vec{C}$ are co-initial vectors.

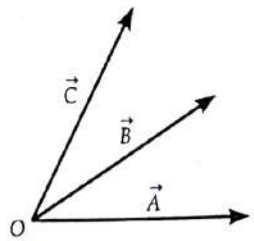

Fig. 4.8 Co-initial vectors.

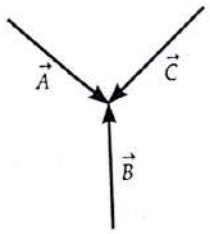

Fig. 4.9 Co-terminus vectors.

(ix) **Co-terminus vectors.** *The vectors which have the common terminal point are called co-terminus vectors.* In Fig. 4.9, $\vec{A}$, $\vec{B}$ and $\vec{C}$ are co-terminus vectors.

4.7 ZERO VECTOR AND ITS PROPERTIES

8. *What is a zero vector ? Explain the need of a zero vector. Give the important properties and physical examples of zero vectors.*

Zero vector. *A zero or null vector is a vector that has zero magnitude and an arbitrary direction.* It is represented by $\vec{0}$ (arrow over the number 0).

Need of a zero vector. The need of a zero vector arises due to the following situations :

(i) If $\vec{A} = \vec{B}$, then what is $\vec{A} - \vec{B}$?

(ii) If $\mu = -\lambda$, then what is $(\lambda + \mu)\vec{A}$?

In all these cases the resultant has to be a vanishingly vector and not a scalar. Hence there is need for introducing the concept of zero vector.

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Properties of zero vectors :

(*i*) When a vector is added to zero vector, we get the same vector

$$\vec{A} + \vec{0} = \vec{A}$$

(*ii*) When a real number is multiplied by a zero vector, we get a zero vector

$$\lambda\,\vec{0} = \vec{0}$$

(*iii*) When a vector is multiplied by zero, we get zero vector

$$0\,\vec{A} = \vec{0}$$

(*iv*) If λ and μ are two different non-zero real numbers, then the relation

$$\lambda\,\vec{A} = \mu\,\vec{B}$$

can hold only if both $\vec{A}$ and $\vec{B}$ are zero vectors.

Physical examples of zero vector :

(*i*) The position vector of a particle lying at the origin is a zero vector.

(*ii*) The velocity vector of a stationary object is a zero vector.

(*iii*) The acceleration vector of an object moving with uniform velocity is a zero vector.

4.8 MULTIPLICATION OF A VECTOR BY A REAL NUMBER

9. *What do you mean by multiplication of a vector by a real number ? Illustrate it by some examples.*

Multiplication of a vector by a real number. When a vector $\vec{A}$ is multiplied by a real number λ, we get another vector $\lambda\,\vec{A}$. The magnitude of $\lambda\,\vec{A}$ is λ times the magnitude of $\vec{A}$. If λ is positive, then the direction of $\lambda\,\vec{A}$ is same as that of $\vec{A}$. If λ is negative, then the direction of $\lambda\,\vec{A}$ is opposite to that of $\vec{A}$. Fig. 4.10 shows multiplication of vector $\vec{A}$ by different real numbers.

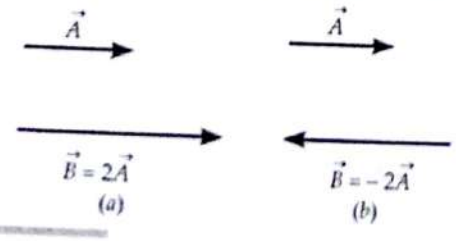

Fig. 4.10 Multiplication of a vector by a real number when (*a*) $\lambda = 2$ and (*b*) $\lambda = -2$

Examples :

(*i*) If a vector $\vec{A}$ is multiplied by real number $\lambda = 2$, then we get another vector $\vec{B}$ such that $\vec{B} = 2\,\vec{A}$. As shown in Fig. 4.10(*a*), the magnitude of $\vec{B}$ is twice the magnitude of $\vec{A}$ while the direction of $\vec{B}$ is the same as that of $\vec{A}$.

(*ii*) If a vector $\vec{A}$ is multiplied by real number $\lambda = -2$, then the new vector is such that $\vec{B} = -2\,\vec{A}$. As shown in Fig. 4.10(*b*), the magnitude of $\vec{B}$ is again twice the magnitude of $\vec{A}$ but direction is opposite to that of $\vec{A}$.

10. *How are the units of a vector affected when it is multiplied by a scalar having units or dimensions ? Give example.*

Multiplication of a vector by scalar. If λ is a pure number having no units or dimensions, then the units of $\lambda\,\vec{A}$ are the same as that of $\vec{A}$. However, when a vector $\vec{A}$ is multiplied by a scalar λ which has certain units, the units of resultant $\lambda\,\vec{A}$ are obtained by multiplying the units of $\vec{A}$ by the units of λ. For example, when velocity vector is multiplied by mass (a scalar), we get momentum. The units of momentum are obtained by multiplying the units of velocity by units of mass.

4.9 ADDITION OR COMPOSITION OF VECTORS

11. *What do you mean by resultant of two or more vectors ? Mention the various laws of vector addition.*

Composition of vectors. *The resultant of two or more vectors is that single vector which produces the same effect as the individual vectors together would produce. The process of adding two or more vectors is called composition of vectors.*

As the vectors have both magnitude and direction, so they cannot be added by using ordinary rules of algebra. Vectors can be added *geometrically*. The following three *laws of vector addition* can be used to add two or more vectors having any inclination to each other.

(*i*) *Triangle law of vector addition* for adding two vectors.

(*ii*) *Parallelogram law of vector addition* for adding two vectors.

(*iii*) *Polygon law of vector addition* for adding more than two vectors.

12. *State and illustrate triangle law of vector addition.*

Triangle law of vector addition. *If two vectors can be represented both in magnitude and direction by the two sides of a triangle taken in the same order, then their resultant is represented completely, both in magnitude and direction, by the third side of the triangle taken in the opposite order.*

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Illustration. Suppose we wish to add two vectors $\vec{A}$ and $\vec{B}$ as shown in Fig. 4.11(*a*). Draw a vector $\vec{OP}$ equal and parallel to vector $\vec{A}$, as shown in Fig. 4.11(*b*). From head P of $\vec{OP}$, draw a vector $\vec{PQ}$ equal and parallel to vector $\vec{B}$. Then the resultant vector is given by $\vec{OQ}$ which joins the tail of $\vec{A}$ and head of $\vec{B}$.

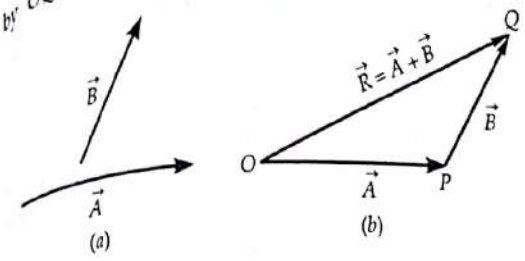

Fig. 4.11 Triangle law of vector addition.

According to triangle law of vector addition,

$$\vec{OQ} = \vec{OP} + \vec{PQ}$$

$$\vec{R} = \vec{A} + \vec{B}$$

or

13. *State and illustrate parallelogram law of vector addition.*

Parallelogram law of vector addition. *If two vectors can be represented both in magnitude and direction by the two adjacent sides of a parallelogram drawn from a common point, then their resultant is completely represented, both in magnitude and direction, by the diagonal of the parallelogram passing through that point.*

Illustration. Suppose we wish to add two vectors $\vec{A}$ and $\vec{B}$, as shown in Fig. 4.12(*a*). From a common point O, draw a vector $\vec{OP}$ equal and parallel to $\vec{A}$ and vector $\vec{OQ}$ equal and parallel to $\vec{B}$. Complete the parallelogram $OPSQ$. According to the parallelogram law of vector addition, the diagonal $\vec{OS}$ gives the resultant vector $\vec{R}$.

Thus $\vec{OS} = \vec{OP} + \vec{OQ}$ or $\vec{R} = \vec{A} + \vec{B}$

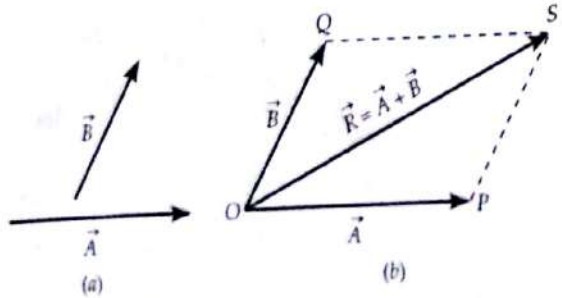

Fig. 4.12 Parallelogram law of vector addition.

14. *State and prove the polygon law of vector addition.*

Polygon law of vector addition. *If a number of vectors are represented both in magnitude and direction by the sides of an open polygon taken in the same order, then their resultant is represented both in magnitude and direction by the closing side of the polygon taken in opposite order.*

Illustration. Suppose we wish to add four vectors $\vec{A}$, $\vec{B}$, $\vec{C}$ and $\vec{D}$, as shown in Fig. 4.13(*a*). Draw vector $\vec{OP} = \vec{A}$. Move vectors $\vec{B}$, $\vec{C}$ and $\vec{D}$ parallel to themselves so that the tail of $\vec{B}$ touches the head of $\vec{A}$, the tail of $\vec{C}$ touches the head of $\vec{B}$ and the tail of $\vec{D}$ touches the head of $\vec{C}$, as shown in Fig. 4.13(*b*). According to the polygon law, the closing side OT of the polygon taken in the reverse order represents the resultant $\vec{R}$. Thus

$$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$$

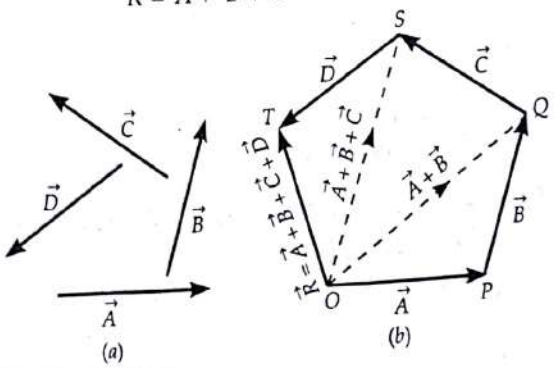

Fig. 4.13 Polygon law of vector addition.

Proof. We apply triangle law of vector addition to different triangles of the polygon shown in Fig. 4.13(*b*),

In ΔOPQ, $\vec{OQ} = \vec{OP} + \vec{PQ} = \vec{A} + \vec{B}$

In ΔOQS, $\vec{OS} = \vec{OQ} + \vec{QS} = \vec{A} + \vec{B} + \vec{C}$

In ΔOST, $\vec{OT} = \vec{OS} + \vec{ST} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$

or $\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$.

This proves the polygon law of vector addition.

15. *Prove that the vector addition is commutative.*

Vector addition is commutative. In Fig. 4.14, the sides OP and OQ of a parallelogram $OPSQ$ represent vectors $\vec{A}$ and $\vec{B}$ respectively. According to parallelogram law of vector addition, diagonal OS gives the resultant. Thus

$$\vec{OP} + \vec{OQ} = \vec{OS}$$

or $$\vec{A} + \vec{B} = \vec{OS} \quad \ldots(1)$$

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As $OP = QS$ and $OP \parallel QS$, therefore

$$\vec{QS} = \vec{OP} = \vec{A}$$

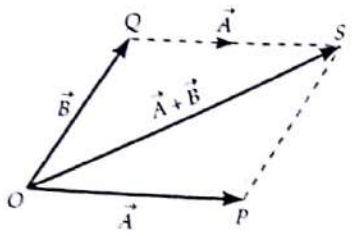

Fig. 4.14 Vector addition is commutative.

Using triangle law of vector addition in ΔOQS, we get

$$\vec{OQ} + \vec{QS} = \vec{OS}$$

or
$$\vec{B} + \vec{A} = \vec{OS} \quad ...(2)$$

From equations (1) and (2), we find that

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

This proves that the vector addition is commutative.

16. *Prove that vector addition is associative.*

Vector addition is associative. Suppose three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ are represented by the sides $\vec{OP}$, $\vec{PQ}$ and $\vec{QS}$ of a polygon $OPQS$. According to polygon law, $\vec{OS}$ represents the resultant $\vec{R}$ both in magnitude and direction. Join O to Q and P to S.

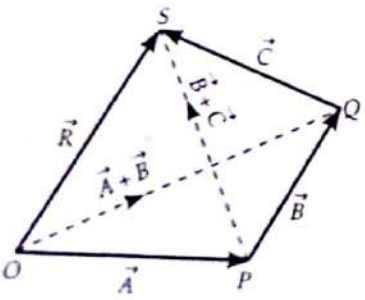

Fig. 4.15. Vector addition is associative.

Using triangle law of vector addition,

In ΔOPQ $\quad \vec{OQ} = \vec{OP} + \vec{PQ} = \vec{A} + \vec{B}$

In ΔOQS, $\quad \vec{OS} = \vec{OQ} + \vec{QS} = (\vec{A} + \vec{B}) + \vec{C}$ ...(1)

In ΔPQS, $\quad \vec{PS} = \vec{PQ} + \vec{QS} = \vec{B} + \vec{C}$

In ΔOPS, $\quad \vec{OS} = \vec{OP} + \vec{PS} = \vec{A} + (\vec{B} + \vec{C})$ ...(2)

From equations (1) and (2), we find that

$$(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$$

This proves that vector addition is associative.

17(a) *Is the flying of a bird an example of composition of vectors ? Explain.*

Flight of a bird. When a bird flies, it pushes the air with forces F_1 and F_2 in the downward direction with its wings W_1 and W_2. The lines of action of these two forces meet at point O. In accordance with Newton's third law of motion, the air exerts equal and opposite reactions R_1 and R_2. According to the parallelogram law, the resultant R of the reactions R_1 and R_2 acts on the bird in the upward direction and helps the bird to fly upward.

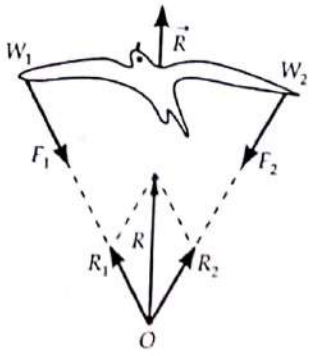

Fig. 4.16 Flight of a bird.

17(b) *Is the working of a sling based on the parallelogram law of vector addition ? Explain.*

Working of a sling. A sling consists of a Y-shaped wooden or metallic frame, to which a rubber band is attached, as shown in Fig. 4.17. When a stone held at the point O on the rubber band is pulled, the tensions T_1 and T_2 are produced along OA and OB in the two segments of the rubber band. According to the parallelogram law of forces, the resultant T of the tensions T_1 and T_2 acts on the stone along OC. As the stone is released, it moves under the action of the resultant tension T in forward direction with a high speed.

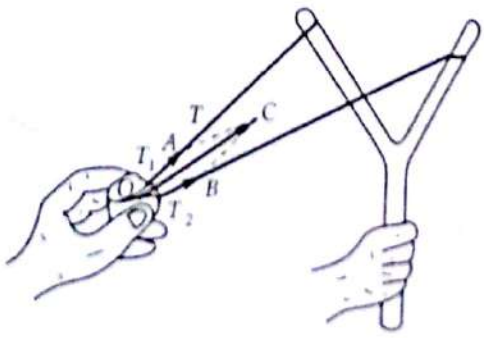

Fig. 4.17

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4.10 ANALYTICAL METHOD OF VECTOR ADDITION

18. Two vectors $\vec{A}$ and $\vec{B}$ are inclined to each other at an angle θ. Using triangle law of vector addition, find the magnitude and direction of their resultant.

Analytical treatment of the triangle law of vector addition. Let the two vectors $\vec{A}$ and $\vec{B}$ be represented both in magnitude and direction by the sides $\vec{OP}$ and $\vec{PQ}$ of ΔOPQ taken in the same order. Then according to the triangle law of vector addition, the resultant $\vec{R}$ is given by the closing side OQ taken in the reverse order, as shown in Fig. 4.18.

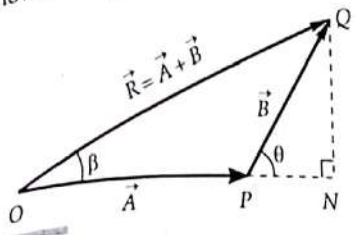

Fig. 4.18 Analytical treatment of Δ law of vector addition.

Magnitude of the resultant $\vec{R}$. From Q, draw QN perpendicular to OP produced.

Then $\angle QPN = \theta$, $OP = A$, $PQ = B$, $OQ = R$.

From right angled ΔQNP, we have

$$\frac{QN}{PQ} = \sin\theta \quad \text{or} \quad QN = PQ\sin\theta = B\sin\theta$$

and $$\frac{PN}{PQ} = \cos\theta \quad \text{or} \quad PN = PQ\cos\theta = B\cos\theta$$

Using Pythagoras theorem in right angled ΔONQ, we get

$$OQ^2 = ON^2 + QN^2$$
$$= (OP + PN)^2 + QN^2$$

or $$R^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$$
$$= A^2 + B^2\cos^2\theta + 2AB\cos\theta + B^2\sin^2\theta$$
$$= A^2 + B^2(\cos^2\theta + \sin^2\theta) + 2AB\cos\theta$$
$$= A^2 + B^2 + 2AB\cos\theta$$

or $$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

Direction of the resultant $\vec{R}$. Let the resultant $\vec{R}$ make an angle β with the direction of $\vec{A}$. Then from right angled ΔONQ, we get

$$\tan\beta = \frac{QN}{ON} = \frac{QN}{OP + PN}$$

or $$\tan\beta = \frac{B\sin\theta}{A + B\cos\theta}$$

19. Two vectors $\vec{A}$ and $\vec{B}$ are inclined to each other at an angle θ. Using parallelogram law of vector addition, find the magnitude and direction of their resultant. Discuss the special cases, when (i) $\theta = 0°$, (ii) $\theta = 180°$ and (iii) $\theta = 90°$.

Analytical treatment of the parallelogram law of vector addition. Let the two vectors $\vec{A}$ and $\vec{B}$ inclined to each other at an angle θ be represented both in magnitude and direction by the adjacent sides $\vec{OP}$ and $\vec{OQ}$ of the parallelogram $OPSQ$. Then according to the parallelogram law of vector addition, the resultant of $\vec{A}$ and $\vec{B}$ is represented both in magnitude and direction by the diagonal $\vec{OS}$ of the parallelogram.

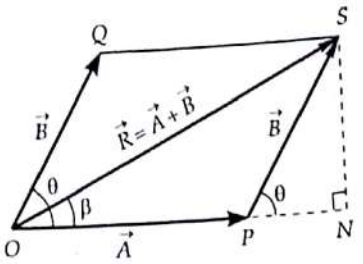

Fig. 4.19 Analytical treatment of parallelogram law.

Magnitude of resultant $\vec{R}$. Draw SN perpendicular to OP produced.

Then $\angle SPN = \angle QOP = \theta$, $OP = A$, $PS = OQ = B$, $OS = R$

From right angled ΔSNP, we have

$$\frac{SN}{PS} = \sin\theta \quad \text{or} \quad SN = PS\sin\theta = B\sin\theta$$

$$\frac{PN}{PS} = \cos\theta \quad \text{or} \quad PN = PS\cos\theta = B\cos\theta$$

Using Pythagoras theorem in right-angled ΔONS, we get

$$OS^2 = ON^2 + SN^2 = (OP + PN)^2 + SN^2$$

or $$R^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$$
$$= A^2 + B^2(\cos^2\theta + \sin^2\theta) + 2AB\cos\theta$$
$$= A^2 + B^2 + 2AB\cos\theta$$

or $$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

Direction of resultant $\vec{R}$. Let the resultant $\vec{R}$ make angle β with the direction of $\vec{A}$. Then from right angled ΔONS, we get

$$\tan\beta = \frac{SN}{ON} = \frac{SN}{OP + PN} \quad \text{or} \quad \tan\beta = \frac{B\sin\theta}{A + B\cos\theta}.$$

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Special cases. (*i*) If the two vectors $\vec{A}$ and $\vec{B}$ are acting along the same direction, $\theta = 0°$. Therefore the magnitude of the resultant is given by

$$R = \sqrt{A^2 + B^2 + 2\,AB.\cos 0°}$$

$$= \sqrt{A^2 + B^2 + 2\,AB} \qquad [\because \cos 0° = 1]$$

$$= \sqrt{(A+B)^2}$$

or $$R = A + B$$

$\therefore$ Magnitude of the resultant vector is equal to the sum of magnitudes of two vectors acting along the same direction and the resultant vector also acts along that direction.

(*ii*) If the vectors $\vec{A}$ and $\vec{B}$ are acting along mutually opposite directions, $\theta = 180°$ and therefore,

$$R = \sqrt{A^2 + B^2 + 2\,AB\cos 180°}$$

$$= \sqrt{A^2 + B^2 - 2\,AB} \qquad [\because \cos 180° = -1]$$

or $$R = (A \sim B)$$

$\therefore$ Resultant is equal to the positive difference between magnitudes of two vectors and acts along the direction of bigger vector.

(*iii*) When the two vectors are acting at right angle to each other, $\theta = 90°$ and therefore,

$$R = \sqrt{A^2 + B^2 + 2\,AB\cos 90°}$$

$$= \sqrt{A^2 + B^2} \qquad [\because \cos 90° = 0]$$

Also $$\tan\beta = \frac{B\sin 90°}{A + B\cos 90°} = \frac{B}{A}$$

$\therefore$ $$\beta = \tan^{-1}\left(\frac{B}{A}\right)$$

Examples based on Composition of Vectors

Formulae Used

1. By triangle law or parallelogram law of vector addition, the magnitude of resultant $\vec{R}$ of two vectors $\vec{P}$ and $\vec{Q}$ inclined to each other at angle θ, is given by

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

2. If resultant $\vec{R}$ makes an angle β with $\vec{P}$, then

$$\tan\beta = \frac{Q\sin\theta}{P + Q\cos\theta}.$$

Example 1. ABCD is a parallelogram and $\vec{AC}$ and $\vec{BD}$ are its diagonals. Prove that :

(i) $\vec{AC} + \vec{BD} = 2\,\vec{BC}$ and (ii) $\vec{AC} - \vec{BD} = 2\,\vec{AB}$.

Solution. Using triangle law of vector addition in Fig. 4.20,

$$\vec{AC} = \vec{AB} + \vec{BC}$$

$$\vec{BD} = \vec{BC} + \vec{CD}$$

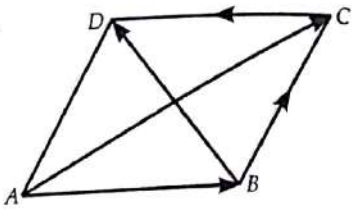

Fig. 4.20

(*i*) $\vec{AC} + \vec{BD} = \vec{AB} + \vec{BC} + \vec{BC} + \vec{CD}$

But $\vec{AB} = -\vec{CD}$

$\therefore$ $\vec{AC} + \vec{BD} = -\vec{CD} + 2\,\vec{BC} + \vec{CD} = 2\,\vec{BC}$.

(*ii*) $\vec{AC} - \vec{BD} = \vec{AB} + \vec{BC} - \vec{BC} - \vec{CD}$

$= \vec{AB} - \vec{CD} = \vec{AB} - (-\vec{AB}) = 2\,\vec{AB}$.

Example 2. A body is simultaneously given two velocities, one 30 ms^{-1} due east and other 40 ms^{-1} due north. Find the resultant velocity.

Solution. Let the body start moving from O, as shown in Fig. 4.21.

$\vec{v}_A = \vec{OA} = 30\ \text{ms}^{-1}$, due east

$\vec{v}_B = \vec{OB} = 40\ \text{ms}^{-1}$, due north

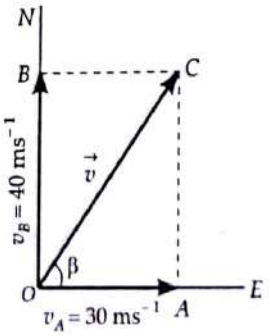

Fig. 4.21

According to parallelogram law, $\vec{OC}$ is the resultant velocity.

Its magnitude is

$$v = \sqrt{v_A^2 + v_B^2} = \sqrt{30^2 + 40^2} = \mathbf{50\ ms^{-1}}.$$

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suppose the resultant velocity $\vec{v}$ makes angle β with the east direction. Then

$$\tan\beta = \frac{CA}{OA} = \frac{40}{30} = 1.3333$$

$$\beta = \tan^{-1}(1.3333) = 53°8'.$$

EXAMPLE 3. *A particle has a displacement of 12 m towards east and 5 m towards the north and then 6 m vertically upward. Find the magnitude of the sum of these displacements.*

Solution. As shown in Fig. 4.22, suppose initially the particle is at origin O. Then its displacement vectors are

$\vec{OA} = 12$ m, due east

$\vec{AB} = 5$ m, due north

$\vec{CD} = 6$ m, vertically upwards.

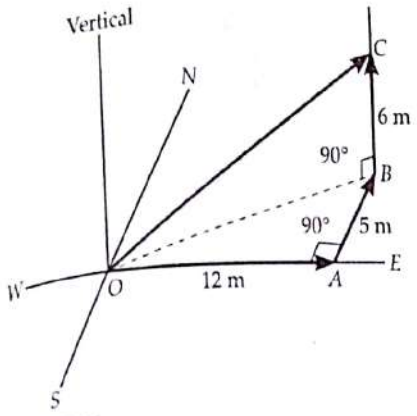

Fig. 4.22

According to polygon law, $\vec{OC}$ is the resultant displacement. From right ΔOAB,

$$OB = \sqrt{OA^2 + AB^2} = \sqrt{12^2 + 5^2}$$

$$= \sqrt{144 + 25} = 13 \text{ m}$$

From right ΔOBC,

$$OC = \sqrt{OB^2 + BC^2} = \sqrt{13^2 + 6^2}$$

$$= \sqrt{169 + 36} = \sqrt{205} = \mathbf{14.32\ m}.$$

EXAMPLE 4. *Two forces of 5 N and 7 N act on a particle with an angle of 60° between them. Find the resultant force.*

Solution. Here $P = 5$ N, $Q = 7$ N, $\theta = 60°$

The magnitude of the resultant force is

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$= \sqrt{5^2 + 7^2 + 2 \times 5 \times 7 \times \cos 60°}$$

$$= \sqrt{109} = \mathbf{10.44\ N}.$$

If $\vec{R}$ makes angle β with the force $\vec{P}$, then

$$\tan\beta = \frac{Q\sin\theta}{P + Q\cos\theta} = \frac{7\sin 60°}{5 + 7\cos 60°} = 0.7132$$

or $\beta = \tan^{-1} 0.7132 = \mathbf{35°29'}$.

EXAMPLE 5. *Two vectors, both equal in magnitude, have their resultant equal in magnitude of the either. Find the angle between the two vectors.* **[Central Schools 04]**

Or

Two equal forces have their resultant equal to either. What is the inclination between them ? **[Delhi 02, 14]**

Solution. Here $P = Q = R$

As $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$\therefore$ $P = \sqrt{P^2 + P^2 + 2P.P\cos\theta}$

or $P^2 = 2P^2(1 + \cos\theta)$

or $1 + \cos\theta = \frac{1}{2}$

or $\cos\theta = -\frac{1}{2} = \cos 120°$ or $\theta = \mathbf{120°}$.

EXAMPLE 6. *Two forces whose magnitudes are in the ratio of 3:5 give a resultant of 35 N. If the angle of inclination be 60°, calculate the magnitude of each force.* **[Himachal 04]**

Solution. Let $P = 3x$ newton, $Q = 5x$ newton,

$R = 35$ N, $\theta = 60°$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

or $$35 = \sqrt{(3x)^2 + (5x)^2 + 2 \times 3x \times 5x \cos 60°}$$

or $35 = 7x$ or $x = \frac{35}{7} = 5$

$\therefore$ $P = 3 \times 5 = \mathbf{15\ N}$ and $Q = 5 \times 5 = \mathbf{25\ N}$.

EXAMPLE 7. *Two forces equal to P and 2 P newton act on a particle. If the first be doubled and the second be increased by 20 newton, the direction of the resultant is unaltered. Find the value of P.*

Solution. Let the resultant make angle β with the force P.

$\therefore$ In first cast, $\tan\beta = \dfrac{2P\sin\theta}{P + 2P\cos\theta}$.

In second case, $\tan\beta = \dfrac{(2P + 20)\sin\theta}{2P + (2P + 20)\cos\theta}$

Hence $$\frac{(2P + 20)\sin\theta}{2P + (2P + 20)\cos\theta} = \frac{2P\sin\theta}{P + 2P\cos\theta}$$

or $$\frac{2P\sin\theta}{P + 2P\cos\theta} = \frac{20\sin\theta}{P + 20\cos\theta} \quad \left[\because \frac{a}{b} = \frac{c}{d} = \frac{a-c}{b-d}\right]$$

From the above equation,

$2P = 20$

or $P = \mathbf{10\ N}$.

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EXAMPLE 8. The greatest and the least resultant of two forces acting at a point are 29 N and 5 N respectively. If each force is increased by 3 N, find the resultant of two new forces acting at right angle to each other.

Solution. Let P and Q be the two forces. Then

Greatest resultant $= P + Q = 29$ N ...(*i*)

Least resultant $= P - Q = 5$ N ...(*ii*)

On solving (*i*) and (*ii*), we get

$$P = 17 \text{ N}, \quad Q = 12 \text{ N}$$

When each force is increased by 3 N, new forces are

$$p = P + 3 = 17 + 3 = 20 \text{ N}$$

$$q = Q + 3 = 12 + 3 = 15 \text{ N}$$

As the new forces act at right angle to each other, their resultant is

$$R = \sqrt{p^2 + q^2} = \sqrt{20^2 + 15^2} = \sqrt{625} = 25 \text{ N}$$

If the resultant R makes angle β with the force p, then

$$\tan\beta = \frac{q}{p} = \frac{15}{20} = 0.75$$

or $$\beta = \tan^{-1}(0.75) = 36°52'.$$

EXAMPLE 9. The sum of the magnitudes of two forces acting at a point is 18 N and the magnitude of their resultant is 12 N. If the resultant makes an angle of 90° with the force of smaller magnitude, what are the magnitudes of the two forces ? **[AIEEE 02]**

Solution. Let the two individual forces be $\vec{P}$ and $\vec{Q}$ and θ be the angle between them. Let $P < Q$. If the resultant $\vec{R}$ makes angle β with the force $\vec{P}$, then

$$\tan\beta = \frac{Q\sin\theta}{P + Q\cos\theta}$$

But $\beta = 90°$

$$\therefore \quad \frac{Q\sin\theta}{P + Q\cos\theta} = \tan 90° = \infty \quad \text{or} \quad P + Q\cos\theta = 0$$

Also $P + Q = 18$ N

As $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} = 12$

$$\therefore \quad P^2 + Q^2 + 2PQ\cos\theta = 144$$

or $$P^2 + (18 - P)^2 + 2P(-P) = 144$$

$$[\because \quad Q = 18 - P, \; Q\cos\theta = -P]$$

or $$P^2 + (324 + P^2 - 36P) - 2P^2 = 144$$

or $$6P = 180$$

or $$P = 5 \text{ N} \quad \text{and} \quad Q = 18 - 5 = 13 \text{ N}.$$

EXAMPLE 10. A motorboat is racing towards north at 25 km h^{-1} and the water current in that region is 10 km h^{-1} in the direction of 60° east of south. Find the resultant velocity of the boat. **[NCERT]**

Solution. Let the motorboat start moving from O, as shown in Fig. 4.23.

$\vec{v_b}$ = velocity of motorboat

= 25 km h^{-1}, due north

$\vec{v_c}$ = velocity of water current

= 10 km h^{-1}, 60° east of south

By parallelogram law, the resultant velocity $\vec{v}$ is equal to the diagonal $\vec{OC}$.

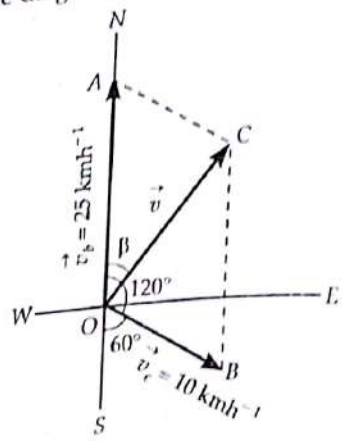

Fig. 4.23

Its magnitude is

$$v = \sqrt{v_b^2 + v_c^2 + 2v_b v_c \cos 120°}$$

$$= \sqrt{25^2 + 10^2 + 2 \times 25 \times 10\,(-1/2)}$$

$$= \mathbf{21.8 \text{ km h}^{-1}}.$$

Suppose the resultant velocity $\vec{v}$ makes angle β with the north direction. Then

$$\tan\beta = \frac{v_c \sin 120°}{v_b + v_c \cos 120°} = \frac{10 \times (\sqrt{3}/2)}{25 + 10 \times (-1/2)}$$

$$= \frac{\sqrt{3}}{4} = 0.433$$

$$\therefore \quad \beta = \tan^{-1}(0.433) = \mathbf{23.4°}$$

EXAMPLE 11. On a certain day, rain was falling vertically with a speed of 35 ms^{-1}. A wind started blowing after some time with a speed of 12 ms^{-1} in east to west direction. In which direction should a boy waiting at a bus stop hold his umbrella ? **[NCERT ; Delhi 02 ; Central Schools 14]**

Solution. In Fig. 4.24,

Velocity of rain,

$$\vec{v_R} = \vec{OA} = 35 \text{ ms}^{-1}, \quad \text{(vertically downward)}$$

Velocity of wind,

$$\vec{v_W} = \vec{OB} = 12 \text{ ms}^{-1}, \text{ east to west.}$$

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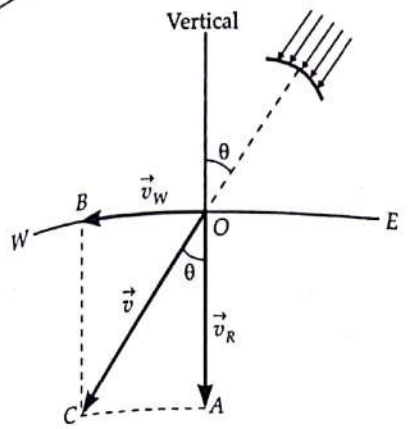

Fig. 4.24

The magnitude of the resultant velocity is

$$v = \sqrt{v_R^2 + v_W^2} = \sqrt{35^2 + 12^2} = 37 \text{ ms}^{-1}$$

Let the resultant velocity $\vec{v}$ $(= \vec{OC})$ make an angle θ with the vertical. Then

$$\tan\theta = \frac{AC}{OA} = \frac{v_W}{v_R} = \frac{12}{35} = 0.343$$

$$\therefore \quad \theta = \tan^{-1}(0.343) = 19°.$$

Hence the boy should hold umbrella bending it towards east making an angle of about 19° with the vertical.

Example 12. A river 800 m wide flows at the rate of 5 kmh^{-1}. A swimmer who can swim at 10 kmh^{-1} in still water, wishes to cross the river straight. (i) Along what direction must he strike ? (ii) What should be his resultant velocity ? (iii) How much time he would take ?

Solution. (*i*) The situation is shown in Fig. 4.25.

$\vec{OA} = \vec{v_1}$ = Velocity of river = 5 kmh^{-1}

$\vec{OB} = \vec{v_2}$ = Velocity of swimmer in still water.

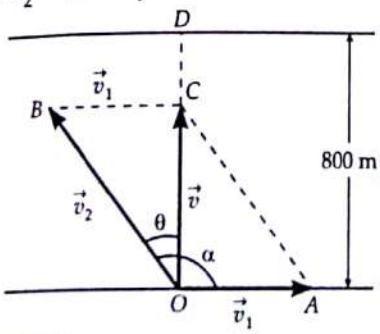

Fig. 4.25

The swimmer will cross the river straight if the resultant velocity $\vec{v}$ is perpendicular to the bank of the river. This will be possible if the swimmer moves making an angle θ with the upstream of the river.

In right ΔOCB,

$$\sin\theta = \frac{BC}{OB} = \frac{v_1}{v_2} = \frac{5}{10} = 0.5$$

$$\theta = \sin^{-1}(0.5) = 30°.$$

(*ii*) Resultant velocity of the swimmer,

$$v = \sqrt{v_2^2 - v_1^2} = \sqrt{10^2 - 5^2} = \sqrt{75} = 8.66 \text{ kmh}^{-1}$$

$$= \frac{8.66 \times 5}{18} = 2.4 \text{ ms}^{-1}.$$

(*iii*) Time taken to cross the river,

$$t = \frac{\text{width of river}}{v} = \frac{800 \text{ m}}{2.4 \text{ ms}^{-1}} = 333.3 \text{ s}.$$

Example 13. A boatman can row with a speed for 10 kmh^{-1} in still water. If the river flows steadily at 5 kmh^{-1}, in which direction should the boatman row in order to reach a point on the other bank directly opposite to the point from where he started ? The width of the river is 2 km.

Solution. As shown in Fig. 4.26, the boatman starts from *S*. He should reach *Q*. Since the river flows along *PQ* with a velocity 5 kmh^{-1}, he should travel along *SP*.

$$\vec{PQ} = 5 \text{ kmh}^{-1}$$

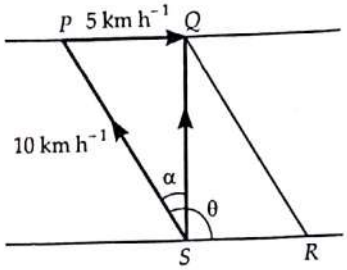

Fig. 4.26

Speed of boatman is shown by vector $\vec{SP}$.

$$\vec{SP} = 10 \text{ kmh}^{-1}$$

$\vec{SQ}$ is the resultant of $\vec{SP}$ and $\vec{PQ}$

$$\angle QSP = \alpha, \quad \angle PQS = 90°$$

Since *Q* and *S* are directly opposite.

$$\sin\alpha = \frac{PQ}{SP} = \frac{5}{10} = \frac{1}{2} \quad \text{i.e.,} \quad \alpha = 30°$$

$$\therefore \quad \theta = 90° + \alpha = 120°$$

Thus the boatman must row the boat in a direction at an angle of 120° with the direction of river flow. The direction does not depend on width of the river.

Example 14. A car travelling at a speed of 20 ms^{-1} due north along the highway makes a right turn on to a side road that heads due east. It takes 50 s for the car to complete the

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turn. At the end of 50 seconds, the car has a speed of 15 ms^{-1} along the side road. Determine the magnitude of average acceleration over the 50 second interval.

Solution. In Fig. 4.27,

Initial velocity $\vec{v_1} = \vec{OA} = 20$ ms^{-1}, due north

Final velocity $\vec{v_2} = \vec{OB} = 15$ ms^{-1}, due east

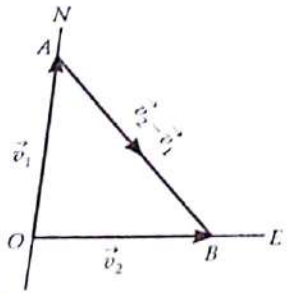

Fig. 4.27

As $\vec{OA} + \vec{AB} = \vec{OB}$

$\therefore$ $\vec{AB} = \vec{OB} - \vec{OA} = \vec{v_2} - \vec{v_1}$

= change in velocity

$\therefore$ $|\vec{v_2} - \vec{v_1}| = AB = \sqrt{OA^2 + OB^2}$

$= \sqrt{20^2 + 15^2} = \sqrt{625} = 25$ ms^{-1}

Average acceleration $= \dfrac{|\vec{v_2} - \vec{v_1}|}{t} = \dfrac{25}{50} = 0.5$ ms^{-2}.

✕ PROBLEMS FOR PRACTICE

1. *ABCDE is a pentagon. Prove that*

$$\vec{AB} + \vec{BC} + \vec{CD} + \vec{DE} + \vec{EA} = \vec{0}.$$

2. *In Fig. 4.28, ABCDEF is a regular hexagon. Prove that* $\vec{AB} + \vec{AC} + \vec{AD} + \vec{AE} + \vec{AF} = 6\vec{AO}$.

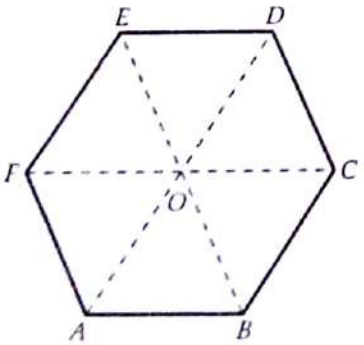

Fig. 4.28

3. *A boy travels 10 m due north and then 7 m due east. Find the displacement of the boy.*

(Ans. 12.21 m, 35° east of north)

4. *Find the resultant of two forces, one 6 N due east and other 8 N due north.*

(Ans. 10 N, 53°8′ with 6 N force)

5. Calculate the angle between a 2 N force and a 3 N force so that their resultant is 4 N. **(Ans.** 75°31′)

6. The resultant vector of $\vec{P}$ and $\vec{Q}$ is $\vec{R}$. On reversing the direction of $\vec{Q}$, the resultant vector becomes $\vec{S}$. Show that : $R^2 + S^2 = 2(P^2 + Q^2)$.

7. Two equal forces have the square of their resultant equal to three times their product. Find the angle between them. **(Ans.** 60°)

8. When the angle between two vectors of equal magnitude is $2\pi/3$, prove that the magnitude of the resultant is equal to either.

9. At what angle do the two forces $(P + Q)$ and $(P - Q)$ act so that the resultant is $\sqrt{3P^2 + Q^2}$. **(Ans.** 60°) **[Himachal 04]**

10. The resultant of two equal forces acting at right angles to each other is 1414 dyne. Find the magnitude of either force. **[Himachal 04]**

(Ans. 1000 dyne)

11. A particle is acted upon by four forces simultaneously :
   (*i*) 30 N due east
   (*ii*) 20 N due north
   (*iii*) 50 N due west and
   (*iv*) 40 N due south. Find the resultant force on the particle. **(Ans.** $20\sqrt{2}$ N, 45° south of west)

12. Two boys raising a load pull at an angle to each other. If they exert forces of 30 N and 60 N respectively and their effective pull is at right angles to the direction of the pull of the first boy, what is the angle between their arms ? What is the effective pull ? **(Ans.** 120°, $30\sqrt{3}$ N)

13. A ship is steaming due east at 12 ms^{-1}. A woman runs across the deck at 5 ms^{-1} in a direction at right angles to the direction of motion of the ship and then towards north. Calculate the velocity of the woman relative to sea.

(Ans. 13 ms^{-1}, 22°37′ north of east)

14. Find the angle between two vectors $\vec{P}$ and $\vec{Q}$ if resultant of the vectors is given by $R^2 = P^2 + Q^2$.

[Central Schools 07] **(Ans.** 90°)

✕ HINTS

1. Refer to Fig. 4.29. Applying triangle law of vector addition,

$$\text{L.H.S.} = (\vec{AB} + \vec{BC}) + \vec{CD} + \vec{DE} + \vec{EA}$$

$$= \vec{AC} + \vec{CD} + \vec{DE} + \vec{EA}$$

$$= (\vec{AC} + \vec{CD}) + \vec{DE} + \vec{EA}$$

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$= \vec{AD} + \vec{DE} + \vec{EA} = (\vec{AD} + \vec{DE}) + \vec{EA}$

$= \vec{AE} + \vec{EA} = -\vec{EA} + \vec{EA} = \vec{0}$ = R.H.S.

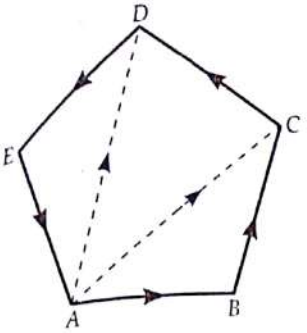

Fig. 4.29

2. We use triangle law of vector addition.

$\vec{AB} + \vec{AC} + \vec{AD} + \vec{AE} + \vec{AF}$

$= \vec{AB} + (\vec{AD} + \vec{DC}) + \vec{AD} + (\vec{AD} + \vec{DE}) + \vec{AF}$

$= 3\vec{AD} + (\vec{AB} + \vec{DE}) + (\vec{DC} + \vec{AF})$

$= 3 \times 2\vec{AO} + \vec{O} + \vec{O} = 6\vec{AO}$

$[\because \vec{AD} = 2\vec{AO}, \vec{DE} = -\vec{AB}, \vec{AF} = -\vec{DC}]$

3. Let the boy start moving from point O as shown in Fig. 4.30.

$\vec{OA}$ = 10 m, due north ; $\vec{AB}$ = 7 m, due east

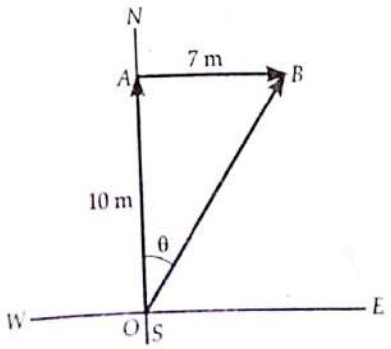

Fig. 4.30

By triangle law of vector addition, the resultant displacement is

$\vec{OA} + \vec{AB} = \vec{OB}$

$|\vec{OB}| = OB = \sqrt{OA^2 + AB^2}$

$= \sqrt{10^2 + 7^2} = 12.21$ m

$\tan\theta = \dfrac{AB}{OA} = \dfrac{7}{10} = 0.7$

$\theta = \tan^{-1}(0.7) = 35°$

∴ Displacement = **12.21 m, along N 35° E.**

4. As shown in Fig. 4.31,

$P = 6$ N, $Q = 8$ N, $R = ?$

$R = \sqrt{P^2 + Q^2} = \sqrt{6^2 + 8^2} = \mathbf{10\ N.}$

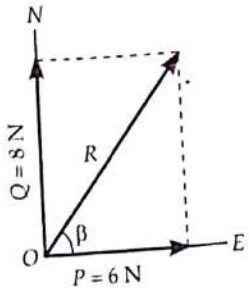

Fig. 4.31

If the resultant R makes angle β with the force of 6 N, then

$\tan\beta = \dfrac{Q}{P} = \dfrac{8}{6} = 1.3333$

∴ $\beta = \tan^{-1}(1.3333) = \mathbf{53°8'}$.

5. Here $P = 2$ N, $Q = 3$ N, $R = 4$ N, $\theta = ?$

As $R^2 = P^2 + Q^2 + 2PQ\cos\theta$

∴ $4^2 = 2^2 + 3^2 + 2 \times 2 \times 3\cos\theta$

or $16 = 4 + 9 + 12\cos\theta$

or $\cos\theta = \dfrac{3}{12} = \dfrac{1}{4} = 0.25$

$\theta = \cos^{-1}(0.25) = \mathbf{75°31'}$.

6. Let θ be the angle between $\vec{P}$ and $\vec{Q}$.

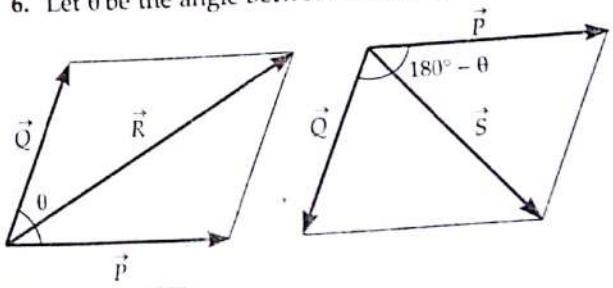

Fig. 4.32

As the resultant of $\vec{P}$ and $\vec{Q}$ is $\vec{R}$, therefore

$R^2 = P^2 + Q^2 + 2PQ\cos\theta$ ...(i)

When the direction of $\vec{Q}$ is reversed, the resultant becomes $\vec{S}$, therefore

$S^2 = P^2 + Q^2 + 2PQ\cos(180° - \theta)$

$= P^2 + Q^2 - 2PQ\cos\theta$ ...(ii)

Adding (i) and (ii), we get :

$R^2 + S^2 = 2P^2 + 2Q^2 = 2(P^2 + Q^2)$.

8. Here $P = Q$,

$\theta = \dfrac{2\pi}{3} = 120°$

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$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

or $$R = \sqrt{P^2 + P^2 + 2PP\cos 120^\circ}$$

$$= \sqrt{2P^2 + 2P^2 \times (-1/2)} = P$$

9. Here $F_1 = P + Q$, $F_2 = P - Q$, $R = \sqrt{3P^2 + Q^2}$

As $R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta$

$\therefore\ 3P^2 + Q^2 = (P+Q)^2 + (P-Q)^2 + 2(P+Q)(P-Q)\cos\theta$

or $P^2 - Q^2 = 2(P^2 - Q^2)\cos\theta$

or $\cos\theta = \frac{1}{2}$

$\therefore\ \theta = 60^\circ$

10. Here $P = Q$, $\theta = 90^\circ$, $R = 1414$ dyne

As $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$\therefore\ 1411 = \sqrt{P^2 + P^2 + 2P^2\cos 90^\circ} = \sqrt{2}\,P$

or $P = \frac{1414}{\sqrt{2}} = \frac{1414}{1.414} = 1000$ **dyne.**

11. Net force due west, $P = 50 - 30 = 20$ N

Net force due south, $Q = 40 - 20 = 20$ N

$\therefore\ R = \sqrt{P^2 + Q^2} = \sqrt{20^2 + 20^2} = 20\sqrt{2}$ N

$\tan\beta = \frac{Q}{P} = \frac{20}{20} = 1$ or $\beta = 45^\circ$.

12. Here $P = 30$ N, $Q = 60^\circ$, $\beta = 90^\circ$, $R = ?$, $\theta = ?$

As $\tan\beta = \frac{Q\sin\theta}{P + Q\cos\theta}$

$\therefore\ \tan 90^\circ = \frac{60\sin\theta}{30 + 60\cos\theta} = \infty$

or $30 + 60\cos\theta = 0$ or $\cos\theta = -\frac{1}{2}$

$\therefore\ \theta = 120^\circ$

$R = \sqrt{30^2 + 60^2 + 2 \times 30 \times 60 \times (-1/2)} = 30\sqrt{3}$ **N.**

13. In Fig. 4.33, $\vec{v}_S = 12\ \text{ms}^{-1}$, due east

$\vec{v}_W = 5\ \text{ms}^{-1}$, due north

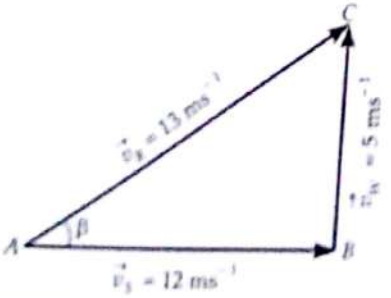

Fig. 4.33

Resultant velocity of woman is

$\vec{v}_R = \vec{v}_S + \vec{v}_W$

$\therefore\ v_R = \sqrt{12^2 + 5^2} = 13\ \text{ms}^{-1}$.

Also $\tan\beta = \frac{5}{12} = 0.4167$

$\therefore\ \beta = 22^\circ 37'$, **north of east.**

14. If θ is the angle between $\vec{P}$ and $\vec{Q}$, then

$$R^2 = P^2 + Q^2 + 2PQ\cos\theta$$

Given $R^2 = P^2 + Q^2$

$\therefore\ P^2 + Q^2 + 2PQ\cos\theta = P^2 + Q^2$

or $2PQ\cos\theta = 0$

or $\cos\theta = 0$

or $\theta = 90^\circ$

4.11 RESOLUTION OF A VECTOR

20. *What is meant by resolution of a vector ? Prove that a vector can be resolved along two given directions in one and only one way.*

Resolution of a vector. *It is the process of splitting a vector into two or more vectors in such a way that their combined effect is same as that of the given vector. The vectors into which the given vector is splitted are called* **component vectors**.

A component of a vector in any direction gives a measure of the effect of the given vector in that direction. The resolution of a vector is just opposite to the process of vector addition.

Resolution of a vector along two given directions. Suppose we wish to resolve a vector $\vec{R}$ in the direction of two coplanar and non-parallel vectors $\vec{A}$ and $\vec{B}$, as shown in Fig. 4.34.

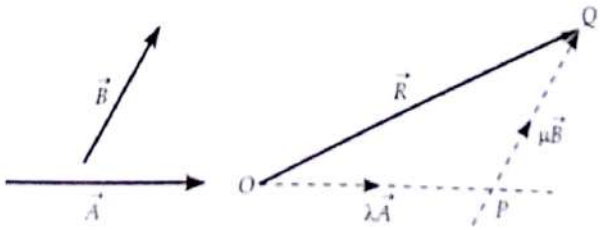

Fig. 4.34 Resolving a vector $\vec{R}$ in the directions of $\vec{A}$ and $\vec{B}$.

Suppose $\vec{OQ}$ represents vector $\vec{R}$. Through O and Q draw lines parallel to vectors $\vec{A}$ and $\vec{B}$ respectively to meet at point P. From triangle law of vector addition,

$$\vec{OQ} = \vec{OP} + \vec{PQ}$$

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$\vec{OP} \parallel \vec{A}$ therefore, $\vec{OP} = \lambda \vec{A}$

As $\vec{PQ} \parallel \vec{B}$ therefore, $\vec{PQ} = \mu \vec{B}$

As Here λ and μ are scalars. Hence

$$\vec{R} = \lambda \vec{A} + \mu \vec{B} \quad ...(1)$$

Thus the vector $\vec{R}$ has been resolved in the directions of $\vec{A}$ and $\vec{B}$. Here $\lambda \vec{A}$ is the component of $\vec{R}$ in the direction $\vec{A}$ and $\mu \vec{B}$ is the component in the direction of $\vec{B}$.

Uniqueness of resolution. Let us assume that $\vec{R}$ can be resolved in the directions of $\vec{A}$ and $\vec{B}$ in another way.

Then $$\vec{R} = \lambda' \vec{A} + \mu' \vec{B} \quad ...(2)$$

From equations (1) and (2), we have

$$\lambda \vec{A} + \mu \vec{B} = \lambda' \vec{A} + \mu' \vec{B}$$

$$(\lambda - \lambda') \vec{A} = (\mu' - \mu) \vec{B}$$

or

But $\vec{A}$ and $\vec{B}$ are non-zero vectors acting along different directions. The above equation is possible only if

$$\lambda - \lambda' = 0 \quad \text{and} \quad \mu' - \mu = 0$$

$$\lambda' = \lambda \quad \text{and} \quad \mu' = \mu.$$

or

Hence there is one and only one way in which a vector $\vec{R}$ can be resolved in the directions of vectors $\vec{A}$ and $\vec{B}$.

4.12 ORTHOGONAL TRIAD OF UNIT VECTORS : BASE VECTORS

21. *What do you mean by orthogonal triad of unit vectors or base vectors ? Show them in a diagram.*

Orthogonal triad of unit vectors or base vectors. In a right-handed Cartesian coordinate system, three unit vectors $\hat{i}, \hat{j}, \hat{k}$ are used to represent the positive directions of X-axis, Y-axis and Z-axis respectively. These three mutually perpendicular unit vectors $\hat{i}, \hat{j}, \hat{k}$ are collectively known as *orthogonal triad of unit vectors or base vectors.* Thus : $|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$

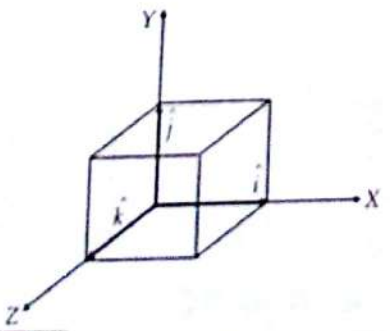

Fig. 4.35 Orthogonal triad of unit vectors.

4.13 RECTANGULAR COMPONENTS OF A VECTOR

22. *What do you mean by rectangular components of a vector ? Explain how a vector can be resolved into two rectangular components in a plane.*

Rectangular components. *When a vector is resolved along two mutually perpendicular directions, the components so obtained are called rectangular components of the given vector.*

Rectangular components of a vector in a plane. Suppose we wish to resolve vector $\vec{A}$ along X- and Y-axes. Taking the initial point of $\vec{A}$ as the origin O, draw two axes OX and OY perpendicular to each other. From the head P of $\vec{A}$, draw $PM \perp OX$ and $PN \perp OY$, as shown in Fig. 4.36 (*a*).

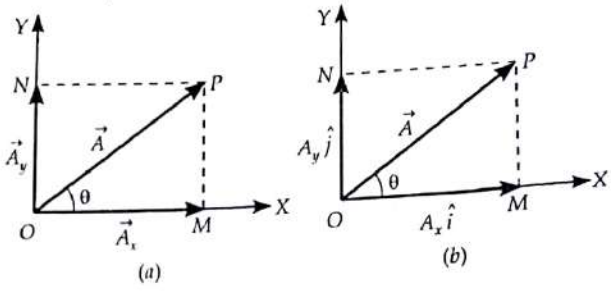

Fig. 4.36. Rectangular components of $\vec{A}$.

According to parallelogram law of vector addition,

$$\vec{OP} = \vec{OM} + \vec{ON} \quad \text{or} \quad \vec{A} = \vec{A}_x + \vec{A}_y$$

Thus $\vec{A}_x$ is the *horizontal* or X-*component* of $\vec{A}$ and $\vec{A}_y$ is the *vertical* or Y-*component* of $\vec{A}$.

Now, let $\hat{i}$ and $\hat{j}$ be the unit vectors along X- and Y-axes respectively, and A_x and A_y be the scalar magnitudes of $\vec{A}_x$ and $\vec{A}_y$ respectively. Then, we can write

$$\vec{A}_x = A_x \hat{i} \quad \text{and} \quad \vec{A}_y = A_y \hat{j}$$

$$\therefore \quad \vec{A} = A_x \hat{i} + A_y \hat{j}$$

This is the equation for vector $\vec{A}$ in terms of its rectangular components.

If vector $\vec{A}$ makes an angle θ with X-axis, then

$$A_x = A \cos\theta \quad \text{and} \quad A_y = A \sin\theta$$

Conversely, if A_x and A_y are given, we can find A and θ as follows :

$$A_x^2 + A_y^2 = A^2 (\cos^2\theta + \sin^2\theta) = A^2$$

or $$A = \sqrt{A_x^2 + A_y^2} \quad \text{and} \quad \tan\theta = \frac{A_y}{A_x} \quad \text{or} \quad \theta = \tan^{-1}\frac{A_y}{A_x}.$$

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23. *How can the position vector $\vec{r}$ of any point P (x, y) be expressed in terms of rectangular components ?*

Resolution of a position vector into rectangular components. As shown in Fig. 4.37, draw $PM \perp X$-axis and $PN \perp Y$-axis. Then $OM = x$ and $ON = y$. According to parallelogram law of vector addition,

$$\vec{OP} = \vec{OM} + \vec{ON} \quad \text{or} \quad \vec{r} = x\hat{i} + y\hat{j}$$

This equation expresses position vector $\vec{r}$ in terms of its rectangular components along X- and Y-axes.

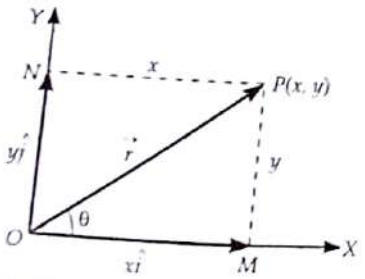

Fig. 4.37 Rectangular resolution of a position vector.

Clearly,

$$|\vec{r}| = \sqrt{OM^2 + ON^2} = \sqrt{x^2 + y^2}.$$

If $\vec{r}$ makes angle θ with X-axis, then

$$x = r\cos\theta \quad \text{and} \quad y = r\sin\theta.$$

24. *Explain how a vector can be resolved into its rectangular components in three dimensions.*

Rectangular components of a vector in three dimensions. Suppose vector $\vec{A}$ is represented by $\vec{OP}$, as shown in Fig. 4.38. Taking O as origin, construct a rectangular parallelopiped with its three edges along the three rectangular axes *i.e.*, X-, Y- and Z-axes.

Clearly, $\vec{A}$ represents the diagonal of the parallelopiped whose intercepts along X-, Y- and Z-axes are $\vec{A}_x$, $\vec{A}_y$ and $\vec{A}_z$ respectively.

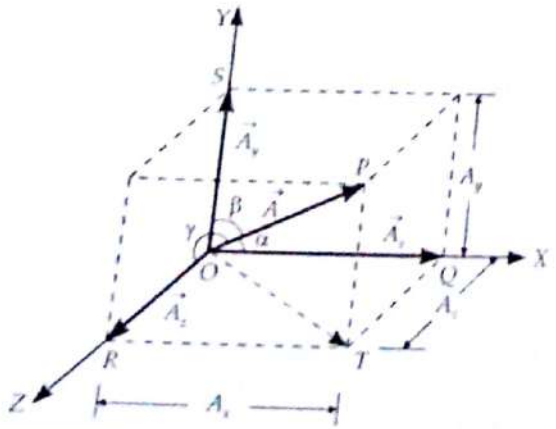

Fig. 4.38

Thus $\vec{A}_x$, $\vec{A}_y$ and $\vec{A}_z$ are the three rectangular components of $\vec{A}$.

Applying triangle law of vectors,

$$\vec{OP} = \vec{OT} + \vec{TP}$$

Applying parallelogram law of vectors,

$$\vec{OT} = \vec{OR} + \vec{OQ}$$

$$\therefore \quad \vec{OP} = \vec{OR} + \vec{OQ} + \vec{TP}$$

But $\vec{TP} = \vec{OS}$

Hence $\vec{OP} = \vec{OR} + \vec{OQ} + \vec{OS}$

or $$\vec{A} = \vec{A}_x + \vec{A}_y + \vec{A}_z = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

If α, β and γ are the angles between $\vec{A}$ and X-, Y- and Z-axes respectively, then

$$A_x = A\cos\alpha, \quad A_y = A\cos\beta, \quad A_z = A\cos\gamma$$

Magnitude of $\vec{A}$. We note that

$$OP^2 = OT^2 + TP^2 = OQ^2 + QT^2 + TP^2$$

or $$A^2 = A_x^2 + A_y^2 + A_z^2$$

or $$A = \sqrt{A_x^2 + A_y^2 + A_z^2}.$$

Note A position vector $\vec{r}$ in three dimensions can be expressed as

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

where x, y and z are the components of $\vec{r}$ along X-, Y-, and Z-axes respectively.

25. *Can the walk of a man be an example of resolution of vectors ? If yes, how ?*

Walking of a man is an example of resolution of forces. While walking, a person presses the ground with his feet slightly slanted in the backward direction, as shown in Fig. 4.39. The ground exerts upon him an equal and opposite reaction R. Its horizontal component $H = R\cos\theta$ enables the person to move forward while the vertical component $V = R\sin\theta$ balances his weight.

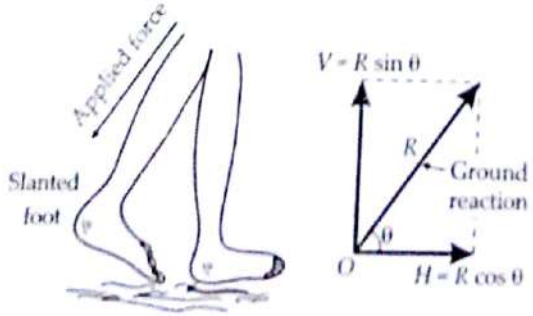

Fig. 4.39 Example of resolution of a vector.

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Examples based on Expressing the Vectors in terms of Base Vectors and Rectangular Components of Vectors

FORMULAE USED

1. If A_x, A_y, A_z are the rectangular components of $\vec{A}$ and $\hat{i}$, $\hat{j}$, $\hat{k}$ are the unit vectors along X-, Y- and Z-axes respectively, then $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

2. $|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

3. $\hat{A} = \dfrac{\vec{A}}{|\vec{A}|} = \dfrac{A_x\hat{i} + A_y\hat{j} + A_z\hat{k}}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$

4. If vector $\vec{A}$ makes angle θ with the horizontal, then horizontal component of $\vec{A} = A_x = A\cos\theta$

vertical component of $\vec{A} = A_y = A\sin\theta$

and $A = \sqrt{A_x^2 + A_y^2}$

UNITS USED

Units of A_x, A_y and A_z are same as that of A and angle θ is in radians.

EXAMPLE 15. Find the vector $\vec{AB}$ and its magnitude if it has initial point $A(1, 2, -1)$ and final point $B(3, 2, 2)$.

Solution. Here $\vec{OA} = \hat{i} + 2\hat{j} - \hat{k}$

$\vec{OB} = 3\hat{i} + 2\hat{j} + 2\hat{k}$

$\therefore \vec{AB} = \vec{OB} - \vec{OA}$

$= (3-1)\hat{i} + (2-2)\hat{j} + [2-(-1)]\hat{k} = 2\hat{i} + 3\hat{k}$

$AB = \sqrt{2^2 + 0^2 + 3^2} = \sqrt{13}$

EXAMPLE 16. Find unit vector parallel to the resultant of the vectors $\vec{A} = \hat{i} + 4\hat{j} - 2\hat{k}$ and $\vec{B} = 3\hat{i} - 5\hat{j} + \hat{k}$.

Solution. The resultant of $\vec{A}$ and $\vec{B}$ is

$\vec{R} = \vec{A} + \vec{B} = (\hat{i} + 4\hat{j} - 2\hat{k}) + (3\hat{i} - 5\hat{j} + \hat{k})$

$= (1+3)\hat{i} + (4-5)\hat{j} + (-2+1)\hat{k} = 4\hat{i} - \hat{j} - \hat{k}$

$|\vec{R}| = \sqrt{4^2 + (-1)^2 + (-1)^2} = \sqrt{16 + 1 + 1} = 3\sqrt{2}$

The unit vector parallel to $\vec{R}$ is

$$\hat{R} = \frac{\vec{R}}{|\vec{R}|} = \frac{1}{3\sqrt{2}}(4\hat{i} - \hat{j} - \hat{k})$$

EXAMPLE 17. A Vector $\vec{X}$, when added to the resultant of the vectors $\vec{A} = 3\hat{i} - 5\hat{j} + 7\hat{k}$ and $\vec{B} = 2\hat{i} + 4\hat{j} - 3\hat{k}$ gives a unit vector along Y-axis. Find the vector $\vec{X}$.

Solution. Resultant of $\vec{A}$ and $\vec{B}$ is

$\vec{R} = \vec{A} + \vec{B} = (3\hat{i} - 5\hat{j} + 7\hat{k}) + (2\hat{i} + 4\hat{j} - 3\hat{k})$

$= 5\hat{i} - \hat{j} + 4\hat{k}$

Unit vector along Y-axis $= \hat{j}$

$\therefore$ Required vector,

$\vec{X} = \hat{j} - \vec{R} = \hat{j} - (5\hat{i} - \hat{j} + 4\hat{k})$

$= -5\hat{i} + 2\hat{j} - 4\hat{k}$.

EXAMPLE 18. Two forces $\vec{F_1} = 3\hat{i} + 4\hat{j}$ and $\vec{F_2} = 3\hat{j} + 4\hat{k}$ are acting simultaneously at a point. What is the magnitude of the resultant force ?

Solution. The resultant force is

$\vec{R} = \vec{F_1} + \vec{F_2} = (3\hat{i} + 4\hat{j}) + (3\hat{j} + 4\hat{k})$

$= 3\hat{i} + 7\hat{j} + 4\hat{k}$

Magnitude of the resultant force is

$|\vec{R}| = \sqrt{3^2 + 7^2 + 4^2} = \sqrt{9 + 49 + 16}$

$= \sqrt{74}$ **units of force.**

EXAMPLE 19. If $\vec{A} = 3\hat{i} + 4\hat{j}$ and $\vec{B} = 7\hat{i} + 24\hat{j}$, find a vector having the same magnitude as $\vec{B}$ and parallel to $\vec{A}$. **[EAMCET, 89]**

Solution. $|\vec{A}| = \sqrt{3^2 + 4^2} = 5$

Unit vector in the direction of $\vec{A}$ is

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{1}{5}(3\hat{i} + 4\hat{j})$$

Also, $|\vec{B}| = \sqrt{7^2 + 24^2} = 25$

The vector having the same magnitude as $\vec{B}$ and parallel to $\vec{A}$

$= |\vec{B}|\hat{A} = 25 \times \frac{1}{5}(3\hat{i} + 4\hat{j}) = 15\hat{i} + 20\hat{j}$.

EXAMPLE 20. A bird moves with velocity 20 ms^{-1} in a direction making an angle of 60° with the eastern line and 60° with vertical upward. Represent the velocity vector in rectangular form.

Solution. Let eastern line be taken as X-axis, northern as Y-axis and vertical upward as Z-axis. Let the velocity vector $\vec{v}$ make angles α, β and γ with X-, Y- and Z-axis respectively. Then $\alpha = 60°$, $\gamma = 60°$.

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As $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\therefore \quad \cos^2 60° + \cos^2\beta + \cos^2 60° = 1$

or $\left(\frac{1}{2}\right)^2 + \cos^2\beta + \left(\frac{1}{2}\right)^2 = 1$

or $\cos^2\beta = 1 - \frac{1}{2} = \frac{1}{2}$

or $\cos\beta = \frac{1}{\sqrt{2}}$

$\therefore \quad \vec{v} = v\cos\alpha\,\hat{i} + v\cos\beta\,\hat{j} + v\cos\gamma\,\hat{k}$

$= 20 \times \frac{1}{2}\hat{i} + 20 \times \frac{1}{\sqrt{2}}\hat{j} + 20 \times \frac{1}{2}\hat{k}$

$= 10\,\hat{i} + 10\sqrt{2}\,\hat{j} + 10\,\hat{k}$.

Example 21. One of the rectangular components of a velocity of 80 kmh^{-1} is 40 kmh^{-1}. Find the other component.

Solution. Let $v = 80$ kmh^{-1}, $v_x = 40$ kmh^{-1}, then $v_y = ?$

As $v = \sqrt{v_x^2 + v_y^2}$

$\therefore \quad v_y = \sqrt{v^2 - v_x^2} = \sqrt{80^2 - 40^2} = \sqrt{6400 - 1600}$

$= \sqrt{4800} = \mathbf{69.28\ kmh^{-1}}$.

Example 22. A force is inclined at 50° to the horizontal. If its rectangular component in the horizontal direction be 50 N, find the magnitude of the force and its vertical component.

Solution. Here $F_x = 50$ N, $\theta = 50°$

But $F_x = F\cos\theta$

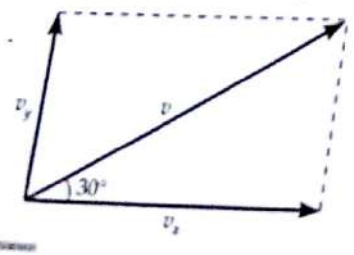
$\therefore \quad F = \frac{F_x}{\cos\theta} = \frac{50}{\cos 50°} = \frac{50}{0.6428} = \mathbf{77.78\ N}$.

Also, $F_y = F\sin\theta = 77.78 \times \sin 50°$

$= 77.78 \times 0.7660 = \mathbf{59.58\ N}$.

Example 23. An aeroplane takes off at angle of 30° to the horizontal. If the component of its velocity along the horizontal is 250 kmh^{-1}, what is the actual velocity ? Find also the vertical component of the velocity.

Solution. Let v_x and v_y be the horizontal and vertical components of actual velocity v (Fig. 4.40).



4.40

Then $v_x = v\cos 30° = 250$ kmh^{-1}

$\therefore \quad v = \frac{250}{\cos 30°} = \frac{250 \times 2}{\sqrt{3}} = \mathbf{288.67\ kmh^{-1}}$.

$v_y = v\sin 30° = 288.67 \times 0.5 = \mathbf{144.33\ kmh^{-1}}$.

Example 24. A man rows a boat with a speed of 18 kmh^{-1} in the north-west direction. The shoreline makes an angle of 15° south of west. Obtain the component of the velocity of the boat along the shoreline and perpendicular to the shoreline.

Solution. As shown in Fig. 4.41, the boat makes an angle of 45° with the west direction while the shorline makes an angle of 15° with it. Thus the boat makes angle of 45° + 15° = 60° with the shoreline.

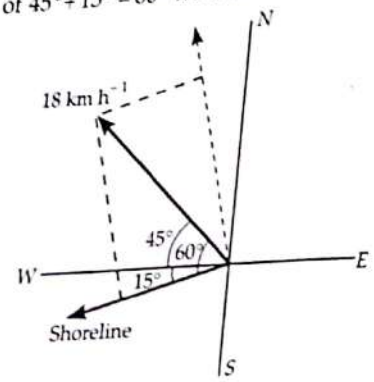


Fig. 4.41

∴ Component of the velocity of the boat along the shoreline

$= 18\cos 60° = 18 \times \frac{1}{2} = \mathbf{9\ kmh^{-1}}$.

Component of the velocity of the boat perpendicular to the shoreline

$= 18\sin 60° = \frac{18 \times \sqrt{3}}{2} = \mathbf{15.6\ kmh^{-1}}$.

Example 25. Two billiard balls are rolling on a flat table. One has the velocity components $v_x = 1\ ms^{-1}$, $v_y = \sqrt{3}\ ms^{-1}$ and the other has components $v'_x = 2\ ms^{-1}$ and $v'_y = 2\ ms^{-1}$. If both the balls start moving from the same point, what is the angle between their paths ?

Solution. *For first ball :*

$OC = v_x = 1\ ms^{-1}$, $OE = v_y = \sqrt{3}\ ms^{-1}$.

Let θ be the angle which the resultant OA of v_x and v_y makes with the X-axis. Then

$\tan\theta = \frac{v_y}{v_x} = \frac{\sqrt{3}}{1} = \sqrt{3}$

$\therefore \quad \theta = 60°$

For second ball :

$OD = v'_x = 2\ ms^{-1}$, $OF = v'_y = 2\ ms^{-1}$

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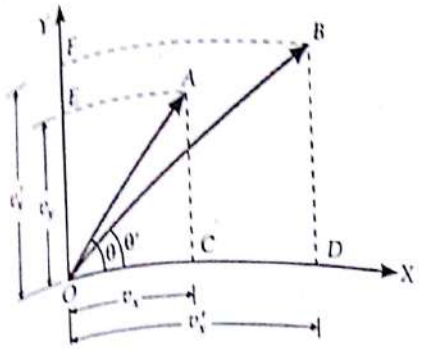

Fig. 4.42

Let θ' be the angle which the resultant OB of v'_x and v'_y makes with the X-axis. Then

$$\tan\theta' = \frac{v'_y}{v'_x} = \frac{2}{2} = 1$$

$$\theta' = 45°$$

∴ Angles between the paths of two balls

$$= \theta - \theta' = 60° - 45° = \mathbf{15°}.$$

PROBLEMS FOR PRACTICE

1. If $\vec{A} = 3\hat{i} + 2\hat{j}$ and $\vec{B} = \hat{i} - 2\hat{j} + 3\hat{k}$, find the magnitudes of $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$. **[Ans. 5, $\sqrt{29}$]**

2. Find the unit vector parallel to the resultant of the vectors $\vec{A} = 2\hat{i} - 6\hat{j} - 3\hat{k}$ and $\vec{B} = 4\hat{i} + 3\hat{j} - \hat{k}$.

[Ans. $1/\sqrt{61}\,(6\hat{i} - 3\hat{j} - 4\hat{k})$]

3. Determine the vector which when added to the resultant of $\vec{A} = 2\hat{i} - 4\hat{j} - 6\hat{k}$ and $\vec{B} = 4\hat{i} + 3\hat{j} + 3\hat{k}$ gives the unit vector along z-axis. **[Ans. $-6\hat{i} + \hat{j} + 4\hat{k}$]**

4. Find the value of λ in the unit vector $0.4\hat{i} + 0.8\hat{j} + \lambda\hat{k}$. **[Ans. $\sqrt{0.2}$]**

5. Given three coplanar vectors $\vec{a} = 4\hat{i} - \hat{j}$, $\vec{b} = -3\hat{i} + 2\hat{j}$ and $\vec{c} = -3\hat{j}$. Find the magnitude of the sum of the three vectors. **[Ans. $\sqrt{5}$]**

6. A force is inclined at 30° to the horizontal. If its rectangular component in the horizontal direction is 50 N, find the magnitude of the force and its vertical component. **[Ans. 57.74 N, 28.87 N]**

7. A velocity of 10 ms^{-1} has its Y-component $5\sqrt{2}$ ms^{-1}. Calculate its X-component. **[Ans. $5\sqrt{2}$ ms^{-1}]**

8. An aeroplane takes off at an angle of 30° to the horizontal. If the component of its velocity along the horizontal is 200 km h^{-1}, what is its actual velocity ? Also find the vertical component of its velocity.

[Ans. 230.94 km h^{-1}, 115.47 km h^{-1}]

9. A child pulls a rope attached to a stone with a force of 60 N. The rope makes an angle of 40° to the ground. (*i*) Calculate the effective value of the pull tending to move the stone along the ground. (*ii*) Calculate the force tending to lift the stone.

[Ans. (*i*) 45.96 N (*ii*) 38.57 N]

HINTS

4. As the given vector is a unit vector, so

$$|0.4\hat{i} + 0.8\hat{j} + \lambda\hat{k}| = 1$$

or $$\sqrt{(0.4)^2 + (0.8)^2 + \lambda^2} = 1$$

or $$\lambda^2 = 1 - (0.4)^2 - (0.8)^2$$

$$= 1 - 0.16 - 0.64 = 0.2$$

or $$\lambda = \sqrt{0.2}.$$

9. From Fig. 4.43,

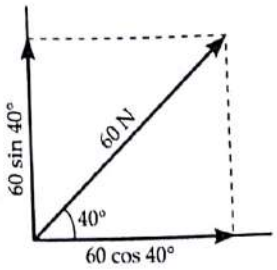

Fig. 4.43

(*i*) Pull along the ground

$$= 60\cos 40° = \mathbf{45.96\ N}.$$

(*ii*) Force tending to lift the stone vertically up

$$= 60\sin 40° = \mathbf{38.57\ N}.$$

4.14 SCALAR PRODUCT OF TWO VECTORS

26. *What are the different ways in which a vector can be multiplied by another vector ?*

These are *two* ways in which a vector can be multiplied by another vector :

(*i*) One way produces a scalar and is known as **scalar product.**

(*ii*) Another way produces a new vector and is known as **vector product.**

27. *Define scalar product of two vectors. Give its geometrical interpretation.*

Scalar or dot product. *The scalar or dot product of two vectors $\vec{A}$ and $\vec{B}$ is defined as the product of the magnitudes of $\vec{A}$ and $\vec{B}$ and cosine of the angle θ between them.* Thus

$$\vec{A} \cdot \vec{B} = |\vec{A}|\,|\vec{B}|\cos\theta = AB\cos\theta$$

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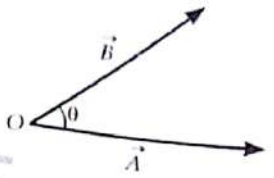

Fig. 4.44 Scalar product of $\vec{A}$ and $\vec{B}$ is a scalar : $\vec{A} \cdot \vec{B} = AB\cos\theta$.

As A, B and $\cos\theta$ are all scalars, so the dot product of $\vec{A}$ and $\vec{B}$ is a scalar quantity. Both $\vec{A}$ and $\vec{B}$ have directions, but their dot product has no direction.

Geometrical interpretation of scalar product. As shown in Fig. 4.45(*a*), suppose two vectors $\vec{A}$ and $\vec{B}$ are represented by $\vec{OP}$ and $\vec{OQ}$ and $\angle POQ = \theta$.

$$\vec{A} \cdot \vec{B} = AB\cos\theta = A(B\cos\theta) = A(OR)$$

$= A \times$ Magnitude of component of $\vec{B}$ in the direction of $\vec{A}$

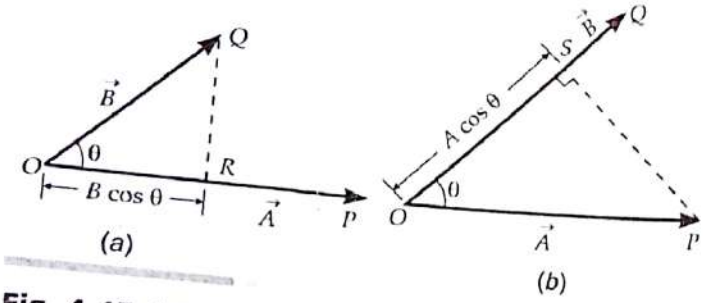

Fig. 4.45 (*a*) $B\cos\theta$ is the projection of $\vec{B}$ onto $\vec{A}$.
(*b*) $A\cos\theta$ is the projection of $\vec{A}$ onto $\vec{B}$.

From Fig. 4.45(*b*), we have

$$\vec{A} \cdot \vec{B} = AB\cos\theta = B(A\cos\theta)$$
$$= B(OS)$$
$= B \times$ Magnitude of component of $\vec{A}$ in the direction of $\vec{B}$.

Thus *the scalar product of two vectors is equal to the product of magnitude of one vector and the magnitude of component of other vector in the direction of first vector.*

28. *Give some examples of physical quantities that may be expressed as the scalar product of two vectors.*

Physical examples of scalar product of two vectors :

(*i*) **Work done (W).** *It is defined on the scalar product of the force ($\vec{F}$), acting on the body and the displacement ($\vec{s}$) produced.* Thus

$$W = \vec{F} \cdot \vec{s}$$

(*ii*) **Instantaneous power (P).** It is defined as the scalar product of force ($\vec{F}$) and the instantaneous velocity ($\vec{v}$) of the body. Thus

$$P = \vec{F} \cdot \vec{v}$$

(*iii*) **Magnetic flux (ϕ).** The magnetic flux linked with a surface is defined as the scalar product of magnetic induction ($\vec{B}$) and the area vector ($\vec{A}$). Thus

$$\phi = \vec{B} \cdot \vec{A}$$

Note As the scalar product of two vectors is a scalar quantity, so *work, power and magnetic flux are all scalar quantities.*

4.15 PROPERTIES OF SCALAR PRODUCT OF TWO VECTORS

29. *Mention important properties of the scalar product of vectors.*

Properties of scalar product :

(*i*) The scalar product is commutative *i.e.*,

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

(*ii*) The scalar product is distributive over addition *i.e.*,

$$\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$$

(*iii*) If $\vec{A}$ and $\vec{B}$ are two vectors perpendicular to each other, then their scalar product is zero.

$$\vec{A} \cdot \vec{B} = A\,B\cos 90° = 0.$$

(*iv*) If $\vec{A}$ and $\vec{B}$ are two parallel vectors having same direction, then their scalar product has the maximum positive magnitude.

$$\vec{A} \cdot \vec{B} = A\,B\cos 0° = AB$$

(*v*) If $\vec{A}$ and $\vec{B}$ are two parallel vectors having opposite directions, then their scalar product has the maximum negative magnitude.

$$\vec{A} \cdot \vec{B} = A\,B\cos 180° = -A\,B$$

(*vi*) The scalar product of a vector with itself is equal to the square of its magnitude.

$$\vec{A} \cdot \vec{A} = A \cdot A\cos 0° = A \cdot A = A^2 = |\vec{A}|^2$$

(*vii*) Scalar product of two similar base vectors is unity and that of two different base vectors is zero.

$$\hat{i} \cdot \hat{i} = (1)(1)\cos 0° = 1$$

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$$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$$

$$\hat{i}\cdot\hat{j} = (1)(1)\cos 90^\circ = 0$$

$$\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$$

(vii) *Scalar product of two vectors $\vec{A}$ and $\vec{B}$ is equal to the sum of the products of their corresponding rectangular components.*

$$\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z$$

(ix) *The cosine of the angle θ between $\vec{A}$ and $\vec{B}$ is given by*

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \frac{A_x B_x + A_y B_y + A_z B_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}\sqrt{B_x^2 + B_y^2 + B_z^2}}$$

30. *Show that the scalar product of two vectors $\vec{A}$ and $\vec{B}$ is commutative.*

Scalar product of two vectors is commutative. We know that

$$\vec{A}\cdot\vec{B} = AB\cos\theta \qquad ...(i)$$

where θ is the angle between $\vec{A}$ and $\vec{B}$ measured anticlockwise, as shown in Fig. 4.46.

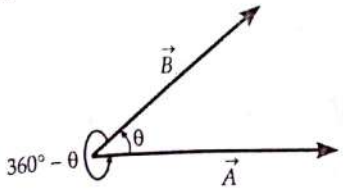

Fig. 4.46

Also, $\vec{B}\cdot\vec{A} = BA\cos(360^\circ - \theta)$

$$= BA\cos\theta = AB\cos\theta \qquad ...(ii)$$

where $(360^\circ - \theta)$ is the angle between $\vec{B}$ and $\vec{A}$ measured anticlockwise.

From equations (*i*) and (*ii*), we get

$$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$$

Hence *the scalar product of two vectors is commutative.*

31. *Prove that the scalar product of vectors is distributive over addition.*

Scalar product of vectors is distributive over addition. In Fig. 4.47, *OP*, *PQ* and *OQ* are the

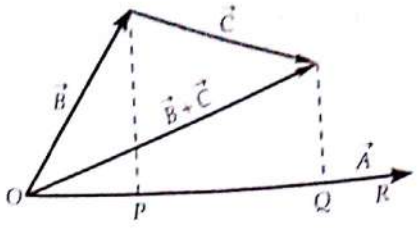

Fig. 4.47

projections or components of $\vec{B}$, $\vec{C}$ and $(\vec{B}+\vec{C})$ in the direction of vector $\vec{A}$. By definition of scalar product,

$\vec{A}\cdot(\vec{B}+\vec{C})$

= Magnitude of $\vec{A}$ × Magnitude of component of $(\vec{B}+\vec{C})$ in the direction of $\vec{A}$

$= (OR)(OQ) = (OR)(OP + PQ)$

$= (OR)(OP) + (OR)(PQ)$

= Magnitude of $\vec{A}$ × Magnitude of component of $\vec{B}$ in the direction of $\vec{A}$

+ Magnitude of $\vec{A}$ × Magnitude of component of $\vec{C}$ in the direction of $\vec{A}$

$= \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$ [By definition of scalar product]

Hence *the scalar product is distributive over addition.*

32. *Show that the scalar product of two vectors is equal to the sum of the products of their corresponding rectangular components.*

Scalar product in terms of rectangular components. We can express $\vec{A}$ and $\vec{B}$ in terms of their rectangular components as

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$$\vec{A}\cdot\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$= A_x\hat{i}\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) + A_y\hat{j}\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) + A_z\hat{k}\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$= A_x B_x\,\hat{i}\cdot\hat{i} + A_x B_y\,\hat{i}\cdot\hat{j} + A_x B_z\,\hat{i}\cdot\hat{k} + A_y B_x\,\hat{j}\cdot\hat{i} + A_y B_y\,\hat{j}\cdot\hat{j} + A_y B_z\,\hat{j}\cdot\hat{k} + A_z B_x\,\hat{k}\cdot\hat{i} + A_z B_y\,\hat{k}\cdot\hat{j} + A_z B_z\,\hat{k}\cdot\hat{k}$$

$$= A_x B_x(1) + A_x B_y(0) + A_x B_z(0) + A_y B_x(0) + A_y B_y(1) + A_y B_z(0) + A_z B_x(0) + A_z B_y(0) + A_z B_z(1)$$

or $\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z$

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Examples based on Scalar or Dot Product of two Vectors

Formulae Used

1. $\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta = AB\cos\theta$
2. If $\vec{A}\perp\vec{B}$, $\theta = 90°$ and $\vec{A}\cdot\vec{B} = 0$
3. Angle θ between $\vec{A}$ and $\vec{B}$ is given by
$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}$$
4. In terms of rectangular components,
$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$
5. Work done, $W = \vec{F}\cdot\vec{S}$
6. Power, $P = \vec{F}\cdot\vec{v}$

Example 26. *Find the angle between the vectors* $\vec{A} = \hat{i} + 2\hat{j} - \hat{k}$ *and* $\vec{B} = -\hat{i} + \hat{j} - 2\hat{k}$. [Delhi 1995]

Solution.
$$|\vec{A}| = \sqrt{1^2 + 2^2 + (-1)^2} = \sqrt{6}$$
$$|\vec{B}| = \sqrt{(-1)^2 + 1^2 + (-2)^2} = \sqrt{6}$$
$$\vec{A}\cdot\vec{B} = 1\times(-1) + 2\times 1 + (-1)\times(-2)$$
$$= -1 + 2 + 2 = 3$$
$$\therefore \quad \cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \frac{3}{\sqrt{6}\times\sqrt{6}} = \frac{3}{6} = \frac{1}{2}$$
Hence $\theta = 60°$.

Example 27. *Prove that the vectors* $\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}$ *and* $\vec{B} = 2\hat{i} - \hat{j}$ *are perpendicular to each other.*

Solution. $\vec{A}\cdot\vec{B} = (\hat{i} + 2\hat{j} + 3\hat{k})\cdot(2\hat{i} - \hat{j} + 0\hat{k})$
$$= 1\times 2 + 2\times(-1) + 3\times 0 = 0$$
Hence $\vec{A}\perp\vec{B}$.

Example 28. *Find the value of* λ *so that the vectors* $\vec{A} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ *and* $\vec{B} = 4\hat{i} - 2\hat{j} - 2\hat{k}$ *are perpendicular to each other.*

Solution. As $\vec{A}\perp\vec{B}$, so $\vec{A}\cdot\vec{B} = 0$
or $(2\hat{i} + \lambda\hat{j} + \hat{k})\cdot(4\hat{i} - 2\hat{j} - 2\hat{k}) = 0$
or $2\times 4 + \lambda\times(-2) + 1\times(-2) = 0$
or $\lambda = 3$.

Example 29. *If the magnitudes of two vectors are 3 and 4 and their scalar product is 6, then find the angle between the two vectors.*

Solution. Here $|\vec{A}| = 3$, $|\vec{B}| = 4$, $\vec{A}\cdot\vec{B} = 6$
$$\therefore \quad \cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \frac{6}{3\times 4} = \frac{1}{2}$$
Hence $\theta = 60°$.

Example 30. *A body constrained to move along the z-axis of a co-ordinate system is subjected to a constant force* $\vec{F}$ *given by* $\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$ *newton where* $\hat{i}$, $\hat{j}$ *and* $\hat{k}$ *represent unit vectors along x-, y- and z-axis of the system respectively. Calculate the work done by the force in displacing the body through a distance of 4 m along the z-axis.* [NCERT]

Solution. As the body moves 4 m along the z-axis, so the displacement vector is
$$\vec{S} = 4\hat{k} = 0\hat{i} + 0\hat{j} + 4\hat{k} \text{ metre}$$
Also $\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$ newton
$$\therefore \quad W = \vec{F}\cdot\vec{S}$$
$$= (-\hat{i} + 2\hat{j} + 3\hat{k})\cdot(0\hat{i} + 0\hat{j} + 4\hat{k})$$
$$= -1\times 0 + 2\times 0 + 3\times 4 = \mathbf{12\ joule.}$$

Example 31. *A force of* $7\hat{i} + 6\hat{k}$ *newton makes a body move on a rough plane with a velocity of* $3\hat{j} + 4\hat{k}$ ms^{-1}. *Calculate the power in watt.*

Solution. Power
$$P = \vec{F}\cdot\vec{v} = (7\hat{i} + 0\hat{j} + 6\hat{k})\cdot(0\hat{i} + 3\hat{j} + 4\hat{k})$$
$$= 7\times 0 + 0\times 3 + 6\times 4 = \mathbf{24\ W.}$$

Example 32. *Three vectors* $\vec{A}$, $\vec{B}$ *and* $\vec{C}$ *are such that* $\vec{A} = \vec{B} + \vec{C}$ *and their magnitudes are 5, 4 and 3 respectively. Find the angle between* $\vec{A}$ *and* $\vec{C}$.

Solution. Given $\vec{A} = \vec{B} + \vec{C}$ or $\vec{B} = \vec{A} - \vec{C}$
Now $\vec{B}\cdot\vec{B} = (\vec{A} - \vec{C})\cdot(\vec{A} - \vec{C})$
$$= \vec{A}\cdot\vec{A} - \vec{A}\cdot\vec{C} - \vec{C}\cdot\vec{A} + \vec{C}\cdot\vec{C}$$
$$= \vec{A}\cdot\vec{A} + \vec{C}\cdot\vec{C} - 2\,\vec{A}\cdot\vec{C}$$
$[\because \vec{C}\cdot\vec{A} = \vec{A}\cdot\vec{C}]$
or $B^2 = A^2 + C^2 - 2AC\cos\theta$
where θ is the angle between $\vec{A}$ and $\vec{C}$.

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Thus $\cos\theta = \dfrac{A^2 + C^2 - B^2}{2AC} = \dfrac{5^2 + 3^2 - 4^2}{2 \times 5 \times 3} = \dfrac{18}{30} = 0.6$

$\theta = \cos^{-1}(0.6) = 53°$.

EXAMPLE 33. If $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$, find the angle between $\vec{A}$ and $\vec{B}$. [CBSE PMT 2K]

Solution. Given :

$$|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$$

$$|\vec{A} + \vec{B}|^2 = |\vec{A} - \vec{B}|^2$$

or $(\vec{A} + \vec{B}).(\vec{A} + \vec{B}) = (\vec{A} - \vec{B}).(\vec{A} - \vec{B})$ $[\because |\vec{A}|^2 = \vec{A}.\vec{A}]$

or $\vec{A}.\vec{A} + \vec{A}.\vec{B} + \vec{B}.\vec{A} + \vec{B}.\vec{B}$

$= \vec{A}.\vec{A} - \vec{A}.\vec{B} - \vec{B}.\vec{A} + \vec{B}.\vec{B}$

or $A^2 + 2\vec{A}.\vec{B} + B^2 = A^2 - 2\vec{A}.\vec{B} + B^2$ $[\because \vec{B}.\vec{A} = \vec{A}.\vec{B}]$

or $4\vec{A}.\vec{B} = 0$ or $4AB\cos\theta = 0$

As $\vec{A}$ and $\vec{B}$ are non-zero vectors, so

$\cos\theta = 0$ or $\theta = 90°$

EXAMPLE 34. If vectors $\vec{P}$, $\vec{Q}$ and $\vec{R}$ have magnitudes 5, 12 and 13 units and $\vec{P} + \vec{Q} = \vec{R}$, find the angle between $\vec{Q}$ and $\vec{R}$.

Solution. As $\vec{P} + \vec{Q} = \vec{R}$.

$\therefore \quad \vec{R} - \vec{Q} = \vec{P}$

and $(\vec{R} - \vec{Q}).(\vec{R} - \vec{Q}) = \vec{P}.\vec{P}$

or $\vec{R}.\vec{R} - \vec{R}.\vec{Q} - \vec{Q}.\vec{R} + \vec{Q}.\vec{Q} = \vec{P}.\vec{P}$

or $R^2 - 2\vec{R}.\vec{Q} + Q^2 = P^2$

or $\cos\theta = \dfrac{R^2 + Q^2 - P^2}{2RQ}$

$= \dfrac{13^2 + 12^2 - 5^2}{2 \times 13 \times 12} = \dfrac{288}{2 \times 13 \times 12} = \dfrac{12}{13}$

$\therefore \quad \theta = \cos^{-1} 12/13.$

EXAMPLE 35. Determine the angles which the vector $\vec{A} = 5\hat{i} + 0\hat{j} + 5\hat{k}$, makes with X-, Y- and Z-axes.

Solution. Here $A = |\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

$= \sqrt{5^2 + 0^2 + 5^2} = 5\sqrt{2}$

If vector $\vec{A}$ makes angles α, β and γ with X-, Y- and Z-axis respectively, then

$\cos\alpha = \dfrac{A_x}{A} = \dfrac{5}{5\sqrt{2}} = \dfrac{1}{\sqrt{2}} \quad \therefore \quad \alpha = 45°.$

$\cos\beta = \dfrac{A_y}{A} = \dfrac{0}{5\sqrt{2}} = 0 \quad \therefore \quad \beta = 90°$

$\cos\gamma = \dfrac{A_z}{A} = \dfrac{5}{5\sqrt{2}} = \dfrac{1}{\sqrt{2}} \quad \therefore \quad \gamma = 45°.$

EXAMPLE 36. If unit vectors $\hat{a}$ and $\hat{b}$ are inclined at angle θ, then prove that

$$|\hat{a} - \hat{b}| = 2\sin\frac{\theta}{2}.$$ [NCERT]

Solution. For any vector $\vec{a}$,

$|\vec{a}|^2 = \vec{a}.\vec{a}$

$\therefore \quad |\hat{a} - \hat{b}|^2 = (\hat{a} - \hat{b}).(\hat{a} - \hat{b})$

$= \hat{a}.\hat{a} - \hat{a}.\hat{b} - \hat{b}.\hat{a} + \hat{b}.\hat{b}$

$= 1 - 2\hat{a}.\hat{b} + 1$ $[\because \hat{a}.\hat{a} = 1 \times 1 \times \cos 0° = 1]$

$= 2 - 2 \times 1 \times 1 \times \cos\theta = 2(1 - \cos\theta)$

$= 2.2\sin^2\dfrac{\theta}{2} = 4\sin^2\dfrac{\theta}{2}$ $[\because 1 - \cos 2\theta = 2\sin^2\theta]$

Hence $|\hat{a} - \hat{b}| = 2\sin\dfrac{\theta}{2}.$

EXAMPLE 37. If $\vec{A} + \vec{B} = \vec{C}$ and $A^2 + B^2 = C^2$, then prove that $\vec{A}$ and $\vec{B}$ are perpendicular to each other.

Solution. Given $\vec{A} + \vec{B} = \vec{C}$

$\therefore \quad (\vec{A} + \vec{B}).(\vec{A} + \vec{B}) = \vec{C}.\vec{C}$

or $A^2 + 2\vec{A}.\vec{B} + B^2 = C^2$

But $A^2 + B^2 = C^2$

$\therefore \quad 2\vec{A}.\vec{B} = 0$

or $2AB\cos\theta = 0$

or $\cos\theta = 0$ $[\because A \neq 0, B \neq 0]$

$\therefore \quad \theta = 90°.$

EXAMPLE 38. A point P lies in the x-y plane. Its position can be specified by its x, y coordinates or by a radially directed vector $\vec{r} = (x\hat{i} + y\hat{j})$ making an angle θ with the x-axis. Find a vector $\hat{i}_r$ of unit magnitude in the direction of vector $\vec{r}$ and a vector $\hat{i}_\theta$ of unit magnitude normal to the vector $\hat{i}_r$ and lying in the x-y plane. [NCERT]

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Solution. The unit vector in the direction of vector $\vec{r}$ is given by

$$\hat{i}_r = \frac{\vec{r}}{r} = \frac{x\,\hat{i} + y\,\hat{j}}{r} = \hat{i}\,\frac{x}{r} + \hat{j}\,\frac{y}{r}$$

or $$\hat{i}_r = \hat{i}\cos\theta + \hat{j}\sin\theta$$

$[\because\ x = r\cos\theta,\ y = r\sin\theta]$

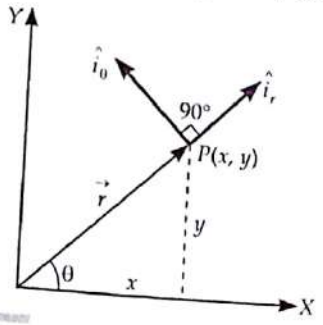

Fig. 4.48

Let the unit vector normal to the unit vector $\hat{i}_r$ be given by

$$\hat{i}_\theta = \hat{i}\,\alpha + \hat{j}\,\beta$$

where the coefficients α and β are to be determined. As $\hat{i}_r \perp \hat{i}_\theta$, so

$$\hat{i}_r \cdot \hat{i}_\theta = 0$$

or $$(\hat{i}\cos\theta + \hat{j}\sin\theta).(\hat{i}\,\alpha + \hat{j}\,\beta) = 0$$

or $$\alpha\cos\theta + \beta\sin\theta = 0$$

or $$\frac{\alpha}{\beta} = -\frac{\sin\theta}{\cos\theta}$$

and $$\alpha = -\beta\frac{\sin\theta}{\cos\theta}$$

Since $\hat{i}_\theta$ is a vector of unit magnitude, so

$$\alpha^2 + \beta^2 = 1$$

or $$\left(\frac{\alpha}{\beta}\right)^2 + 1 = \frac{1}{\beta^2}$$

or $$\left(-\frac{\sin\theta}{\cos\theta}\right)^2 + 1 = \frac{1}{\beta^2} \quad \text{or} \quad \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta} = \frac{1}{\beta^2}$$

or $$\beta^2 = \cos^2\theta$$

$\Rightarrow$ $$\beta = +\cos\theta \quad \text{and} \quad \alpha = -\sin\theta$$

The other solution, $\beta = -\cos\theta$ is rejected because we define a system of $\hat{i}_r$, $\hat{i}_\theta$ vectors in the increasing directions of $\vec{r}$ and θ.

Hence $\hat{i}_\theta = -\hat{i}\sin\theta + \hat{j}\cos\theta$.

✕ PROBLEMS FOR PRACTICE

1. Find the angle between the vectors $\vec{A} = 2\hat{i} - 4\hat{j} + 6\hat{k}$ and $\vec{B} = 3\hat{i} + \hat{j} + 2\hat{k}$. **(Ans. 60°)**
2. Find the angles between the vectors $\vec{A} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{B} = -\hat{i} - \hat{j} + 2\hat{k}$. [Delhi 95] **(Ans. 90°)**
3. Find the value of m so that the vector $3\hat{i} - 2\hat{j} + \hat{k}$ is perpendicular to the vector $2\hat{i} + 6\hat{j} + m\hat{k}$. **(Ans. 6)**
4. For what value of a are the vectors $\vec{A} = a\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{B} = 2a\hat{i} + a\hat{j} - 4\hat{k}$ perpendicular to each other? **(Ans. 2, – 1)**
5. Find the angles between the following pairs of vectors :
   (*i*) $\vec{A} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{B} = -2\hat{i} - 2\hat{j} - 2\hat{k}$. **(Ans. 180°)**
   (*ii*) $\vec{A} = -2\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{B} = 3\hat{i} + 6\hat{j} + 2\hat{k}$ **(Ans. 79°)**
   (*iii*) $\vec{A} = 4\hat{i} + 6\hat{j} - 3\hat{k}$ and $\vec{B} = -2\hat{i} - 5\hat{j} + 7\hat{k}$. **(Ans. 148.9°)**
6. Calculate the values of (*i*) $\hat{j}.(2\hat{i} - 3\hat{j} + \hat{k})$ and (*ii*) $(2\hat{i} - \hat{j}).(3\hat{i} + \hat{k})$. [**Ans.** (*i*) – 3 (*ii*) 6]
7. A force $\vec{F} = 4\hat{i} + \hat{j} + 3\hat{k}$ newton acts on a particle and displaces it through displacement $\vec{S} = 11\hat{i} + 11\hat{j} + 15\hat{k}$ metre. Calculate the work done by the force. **(Ans. 100 J)**
8. Under a force of $10\hat{i} - 3\hat{j} + 6\hat{k}$ newton, a body of mass 5 kg is displaced from the position $6\hat{i} + 5\hat{j} - 3\hat{k}$ to the position $10\hat{i} - 2\hat{j} + 7\hat{k}$. Calculate the work done. **(Ans. 121 J)**
9. The sum and difference of two vectors $\vec{A}$ and $\vec{B}$ are $\vec{A} + \vec{B} = 2\hat{i} + 6\hat{j} + \hat{k}$ and $\vec{A} - \vec{B} = 4\hat{i} + 2\hat{j} - 11\hat{k}$. Find the magnitude of each vector and their scalar product $\vec{A}.\vec{B}$. **(Ans. $\sqrt{50}$, $\sqrt{41}$, – 25)**
10. A force $\vec{F} = 5\hat{i} + 4\hat{j}$ newton displaces a body through $\vec{S} = 3\hat{i} + 4\hat{k}$ metre in 3 s. Find the power. **(Ans. 5 W)**

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11. If the resultant of the vectors $3\hat{i}+4\hat{j}+5\hat{k}$ and $5\hat{i}+3\hat{j}+4\hat{k}$ makes an angle θ with x-axis, then find $\cos\theta$. **(Ans. 0.5744)**

12. Show that the vectors $\vec{A}=3\hat{i}-2\hat{j}+\hat{k}$, $\vec{B}=\hat{i}-3\hat{j}+5\hat{k}$ and $\vec{C}=2\hat{i}+\hat{j}-4\hat{k}$ form a right angle triangle.

13. If vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ have magnitudes 8, 15 and 17 units and $\vec{A}+\vec{B}=\vec{C}$, find the angle between $\vec{A}$ and $\vec{B}$. **(Ans. 90°)**

14. If $\vec{A}=\vec{B}-\vec{C}$, then determine the angle between $\vec{A}$ and $\vec{B}$. $\left(\textbf{Ans. } \theta=\cos^{-1}\dfrac{A^2+B^2-C^2}{2AB}\right)$

15. For two vectors $\vec{A}$ and $\vec{B}$ if $\vec{A}+\vec{B}=\vec{C}$ and $A+B=C$, then prove that $\vec{A}$ and $\vec{B}$ are parallel to each other.

16. Prove that :
$(\vec{A}+2\vec{B}).(2\vec{A}-3\vec{B})=2A^2+AB\cos\theta-6B^2$.

4.16 VECTOR PRODUCT OF TWO VECTORS

33. *Define vector or cross product of two vectors. How is its direction determined ?*

Vector or cross product. *The vector or cross product of two vectors is defined as the vector whose magnitude is equal to the product of the magnitudes of two vectors and sine of the angle between them and whose direction is perpendicular to the plane of the two vectors and is given by right hand rule.* Mathematically, if θ is the angle between vectors $\vec{A}$ and $\vec{B}$, then

$$\vec{A}\times\vec{B}=AB\sin\theta\,\hat{n}$$

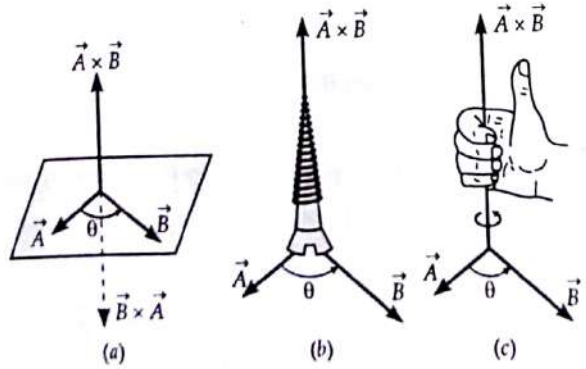

Fig. 4.49 Right hand rules for direction of vector product.

where $\hat{n}$ is a unit vector perpendicular to the plane of $\vec{A}$ and $\vec{B}$ and its direction is given by right hand **rule.** Thus, the direction of $\vec{A}\times\vec{B}$ is same as **that of unit** vector $\hat{n}$.

Rules for determining the direction of $\vec{A}\times\vec{B}$:

(*i*) **Right handed screw rule.** As shown in Fig. 4.49(*b*), *if a right handed screw is placed with its axis perpendicular to the plane of vectors $\vec{A}$ and $\vec{B}$ and is rotated from $\vec{A}$ to $\vec{B}$ through the smaller angle, then the direction in which the screw advances gives the direction of $\vec{A}\times\vec{B}$.*

(*ii*) **Right hand thumb rule.** As shown in Fig. 4.49(*c*), *curl the fingers of the right hand in such a way that they point in the direction of rotation from vector $\vec{A}$ to $\vec{B}$ through the smaller angle, then the stretched thumb points in the direction of $\vec{A}\times\vec{B}$.*

34. *Give geometrical interpretation of vector product of two vectors.*

Geometrical interpretation of vector product. Suppose two vectors $\vec{A}$ and $\vec{B}$ are represented by the sides OP and OQ of a parallelogram $OPRQ$, as shown in Fig. 4.50.

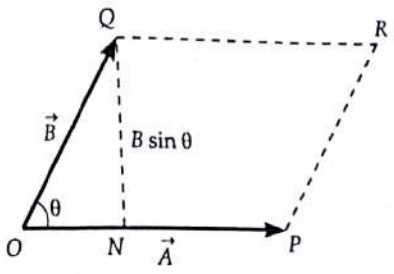

Fig. 4.50 Geometrical significance of vector product.

Let $\angle POQ=\theta$. Draw $QN\perp OP$. The magnitude of vector product $\vec{A}\times\vec{B}$ is

$$|\vec{A}\times\vec{B}|=AB\sin\theta=(OP)(OQ)\sin\theta$$
$$=(OP)(QN) \qquad [\because QN=OQ\sin\theta]$$
$$=\text{Area of parallelogram } OPRQ$$

Thus *the magnitude of the vector product of two vectors is equal to area of the parallelogram formed by the two vectors as its adjacent sides.* Moreover,

$$|\vec{A}\times\vec{B}|=2\times\frac{1}{2}(OP)(QN)=2\times\text{Area of }\Delta POQ$$

Thus *the magnitude of the vector product of two vectors is equal to twice the area of the triangle formed by the two vectors as its adjacent sides.*

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35. *Give some examples of physical quantities which can be expressed as the vector product of two vectors.*

Physical examples of vector product :

(*i*) **Torque** $\vec{\tau}$. The torque acting on a particle is equal to the vector product of its position vector ($\vec{r}$) and force vector ($\vec{F}$). Thus

$$\vec{\tau} = \vec{r} \times \vec{F}$$

(*ii*) **Angular momentum** $\vec{L}$. The angular momentum of a particle is equal to the cross product of its position vector ($\vec{r}$) and linear momentum ($\vec{p}$). Thus

$$\vec{L} = \vec{r} \times \vec{p}$$

(*iii*) **Instantaneous velocity** $\vec{v}$. The instantaneous velocity of a particle is equal to the cross product of its angular velocity ($\vec{\omega}$) and the position vector ($\vec{r}$). Thus

$$\vec{v} = \vec{\omega} \times \vec{r}$$

4.17 PROPERTIES OF VECTOR PRODUCT

36. *Mention some important properties of vector product.*

Properties of vector product :

(*i*) **Vector product is anti-commutative** *i.e.*,

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

(*ii*) **Vector product is distributive over addition** *i.e.*,

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

(*iii*) **Vector product of two parallel or antiparallel vectors is a null vector.** Thus

$$\vec{A} \times \vec{B} = AB \sin(0° \text{ or } 180°)\, \hat{n} = \vec{0}$$

(*iv*) **Vector product of a vector with itself is a null vector.**

$$\vec{A} \times \vec{A} = AA \sin 0°\, \hat{n} = \vec{0}$$

(*v*) **The magnitude of the vector product of two mutually perpendicular vectors is equal to the product of their magnitudes.**

$$|\vec{A} \times \vec{B}| = AB \sin 90° = AB$$

(*vi*) **Vector product of orthogonal unit vectors.** The magnitude of each of the vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$ is 1 or and the angle between any of two of them is 90°.

$$\therefore \quad \hat{i} \times \hat{j} = (1)(1) \sin 90° \hat{n} = \hat{n}$$

As $\hat{n}$ is a unit vector perpendicular to the plane of $\hat{i}$ and $\hat{j}$, so it is just the third vector $\hat{k}$

$$\therefore \quad \hat{i} \times \hat{j} = \hat{k}$$

$$\hat{i} \times \hat{j} = \hat{k}, \quad \hat{j} \times \hat{k} = \hat{i}, \quad \hat{k} \times \hat{i} = \hat{j}$$

$$\hat{j} \times \hat{i} = -\hat{k}, \quad \hat{k} \times \hat{j} = -\hat{i}, \quad \hat{i} \times \hat{k} = -\hat{j}$$

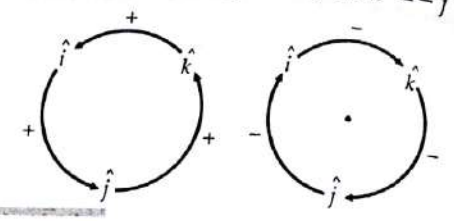

Fig. 4.51 Vector product of base vectors is cyclic (*a*) Anticlock-wise positive (*b*) Clockwise negative.

Aid to memory. Write $\hat{i}$, $\hat{j}$ and $\hat{k}$ cyclically round a circle, as shown in Fig. 4.51. Multiplying two unit vectors anticlockwise, we get positive value of third unit vector (*e.g.*, $\hat{i} \times \hat{j} = +\hat{k}$) ; and multiplying two unit vectors clockwise, we get negative value of third unit vector (*e.g.*, $\hat{j} \times \hat{i} = -\hat{k}$).

Also, $\hat{i} \times \hat{i} = (1)(1) \sin 0° \hat{n} = \vec{0}$

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = \vec{0}$$

(*vii*) **The vector product of two vectors can be expressed in terms of their rectangular components as a determinant.**

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

(*viii*) **Sine of the angle between two vectors.** If θ is the angle between two vectors $\vec{A}$ and $\vec{B}$, then

$$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin\theta$$

or

$$\sin\theta = \frac{|\vec{A} \times \vec{B}|}{|\vec{A}||\vec{B}|}$$

(*ix*) **Unit vector perpendicular to the plane of two vectors.** If $\hat{n}$ is a unit vector perpendicular to the plane of vectors $\vec{A}$ and $\vec{B}$, then

$$\vec{A} \times \vec{B} = AB \sin\theta\, \hat{n} = |\vec{A} \times \vec{B}|\, \hat{n}$$

$$\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$$

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37. *Show that the cross product of two vectors is not commutative but anticommutative.*

Cross product is anticommutative. As shown in Figs. 4.52 (*a*) and (*b*), consider two vectors $\vec{A}$ and $\vec{B}$ such that the small angle between them is θ.

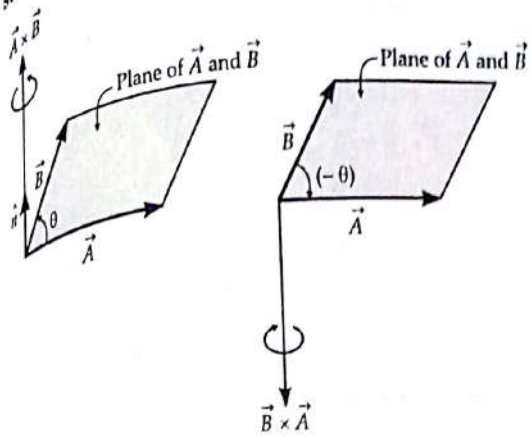

Fig. 4.52 Direction of $\vec{B} \times \vec{A}$ is opposite to that $\vec{A} \times \vec{B}$.

Now, $\vec{A} \times \vec{B} = AB \sin\theta\, \hat{n}$

and $\vec{B} \times \vec{A} = AB \sin(-\theta)\, \hat{n} = -AB \sin\theta\, \hat{n}$

Clearly, both $\vec{A} \times \vec{B}$ and $\vec{B} \times \vec{A}$ have equal magnitudes ($AB \sin\theta$) but they have opposite directions. Thus

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A} \quad \text{and} \quad \vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

Hence *cross product of two vectors is not commutative, instead it is anticommutative.*

38. *Show how the vector product of two vectors can be expressed in terms of their rectangular components as a determinant.*

Vector product in terms of rectangular components.

We can express $\vec{A}$ and $\vec{B}$ in terms of their rectangular components as

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

and $\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$

$$\vec{A} \times \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \times (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

$$= A_x \hat{i} \times (B_x \hat{i} + B_y \hat{j} + B_z \hat{k}) + A_y \hat{j} \times (B_x \hat{i} + B_y \hat{j} + B_z \hat{k}) + A_z \hat{k} \times (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

$$= A_x B_x (\hat{i} \times \hat{i}) + A_x B_y (\hat{i} \times \hat{j}) + A_x B_z (\hat{i} \times \hat{k}) + A_y B_x (\hat{j} \times \hat{i}) + A_y B_y (\hat{j} \times \hat{j}) + A_y B_z (\hat{j} \times \hat{k}) + A_z B_x (\hat{k} \times \hat{i}) + A_z B_y (\hat{k} \times \hat{j}) + A_z B_z (\hat{k} \times \hat{k})$$

$$= A_x B_x (\vec{0}) + A_x B_y (\hat{k}) + A_x B_z (-\hat{j}) + A_y B_x (-\hat{k}) + A_y B_y (\vec{0}) + A_y B_z (\hat{i}) + A_z B_x (\hat{j}) + A_z B_y (-\hat{i}) + A_z B_z (\vec{0})$$

or $$= \hat{i}(A_y B_z - A_z B_y) - \hat{j}(A_x B_z - A_z B_x) + \hat{k}(A_x B_y - A_y B_x)$$

or $$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

This equation expresses $\vec{A} \times \vec{B}$ in a determinant form.

Examples based on Vector or Cross Product of two Vectors

Formulae Used

1. $\vec{A} \times \vec{B} = AB \sin\theta\, \hat{n}$
2. Unit vector $\hat{n}$ perpendicular to the plane of vectors $\vec{A}$ and $\vec{B}$ is given by $\hat{n} = \dfrac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$
3. Angle θ between vectors $\vec{A}$ and $\vec{B}$ is given by $\sin\theta = \dfrac{|\vec{A} \times \vec{B}|}{|\vec{A}||\vec{B}|}$
4. In terms of rectangular components, we have

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

or $$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\hat{i} + (A_z B_x - A_x B_z)\hat{j} + (A_x B_y - A_y B_x)\hat{k}$$

5. For parallel vectors, $\vec{A} \times \vec{B} = \vec{0}$
6. Moment of a force or torque, $\vec{\tau} = \vec{r} \times \vec{F}$.

Example 39. *Prove that the vectors* $\vec{A} = 2\hat{i} - 3\hat{j} - \hat{k}$ *and* $\vec{B} = -6\hat{i} + 9\hat{j} + 3\hat{k}$ *are parallel.*

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Solution. $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & -1 \\ -6 & 9 & 3 \end{vmatrix}$

$= \hat{i}\begin{vmatrix} -3 & -1 \\ 9 & 3 \end{vmatrix} - \hat{j}\begin{vmatrix} 2 & -1 \\ -6 & 3 \end{vmatrix} + \hat{k}\begin{vmatrix} 2 & -3 \\ -6 & 9 \end{vmatrix}$

$= \hat{i}(-9+9) - \hat{j}(6-6) + \hat{k}(18-18) = \vec{0}$

Hence $\vec{A} \parallel \vec{B}$.

EXAMPLE 40. Calculate the area of the parallelogram whose two adjacent sides are formed by the vectors $\vec{A} = 3\hat{i} + 4\hat{j}$ *and* $\vec{B} = -3\hat{i} + 7\hat{j}$.

Solution. $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ -3 & 7 & 0 \end{vmatrix}$

$= \hat{i}\begin{vmatrix} 4 & 0 \\ 7 & 0 \end{vmatrix} - \hat{j}\begin{vmatrix} 3 & 0 \\ -3 & 0 \end{vmatrix} + \hat{k}\begin{vmatrix} 3 & 4 \\ -3 & 7 \end{vmatrix}$

$= \hat{i}(0-0) - \hat{j}(0-0) + \hat{k}(21+12) = 33\hat{k}$

Area of parallelogram

$= |\vec{A} \times \vec{B}| = \sqrt{0^2 + 0^2 + 33^2} = 33$ **sq. units.**

EXAMPLE 41. If $\vec{A}$ *and* $\vec{B}$ *denote the sides of a parallelogram and its area is AB/2, find the angle between* $\vec{A}$ *and* $\vec{B}$.

Solution. Area of parallelogram

$= |\vec{A} \times \vec{B}| = AB \sin\theta = AB/2$

$\therefore \quad \sin\theta = \frac{1}{2}$ or $\theta = 30°$

EXAMPLE 42. Determine a unit vector perpendicular to both $\vec{A} = 2\hat{i} + \hat{j} + \hat{k}$ *and* $\vec{B} = \hat{i} - \hat{j} + 2\hat{k}$. **[Chandigarh 04]**

Solution. The perpendicular unit vector $\hat{n}$ is given by

$\vec{A} \times \vec{B} = AB \sin\theta\, \hat{n} = |\vec{A} \times \vec{B}|\hat{n}$

$\therefore \quad \hat{n} = \dfrac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$

Now $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ 1 & -1 & 2 \end{vmatrix}$

$= \hat{i}\begin{vmatrix} 1 & 1 \\ -1 & 2 \end{vmatrix} - \hat{j}\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} + \hat{k}\begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix}$

$= \hat{i}(2+1) - \hat{j}(4-1) + \hat{k}(-2-1)$

$= 3\hat{i} - 3\hat{j} - 3\hat{k}$

$\therefore |\vec{A} \times \vec{B}| = \sqrt{3^2 + (-3)^2 + (-3)^2} = \sqrt{27} = 3\sqrt{3}$

Hence $\hat{n} = \dfrac{3\hat{i} - 3\hat{j} - 3\hat{k}}{3\sqrt{3}} = \dfrac{1}{\sqrt{3}}(\hat{i} - \hat{j} - \hat{k})$.

EXAMPLE 43. Find a vector whose length is 7 and which is perpendicular to each of the vectors :

$\vec{A} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ *and* $\vec{B} = \hat{i} + \hat{j} - \hat{k}$

Solution. Here

$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 6 \\ 1 & 1 & -1 \end{vmatrix}$

$= \hat{i}(3-6) - \hat{j}(-2-6) + \hat{k}(2+3)$

$= -3\hat{i} + 8\hat{j} + 5\hat{k}$

$|\vec{A} \times \vec{B}| = \sqrt{(-3)^2 + 8^2 + 5^2} = \sqrt{98} = 7\sqrt{2}$.

Unit vector perpendicular to $\vec{A}$ and $\vec{B}$ is

$\hat{n} = \dfrac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|} = \dfrac{-3\hat{i} + 8\hat{j} + 5\hat{k}}{7\sqrt{2}}$

Required vector

$= 7\hat{n} = 7\left(\dfrac{-3\hat{i} + 8\hat{j} + 5\hat{k}}{7\sqrt{2}}\right)$

$= \dfrac{1}{\sqrt{2}}(-3\hat{i} + 8\hat{j} + 5\hat{k})$.

EXAMPLE 44. Determine the sine of the angle between the vectors $3\hat{i} + \hat{j} + 2\hat{k}$ *and* $2\hat{i} - 2\hat{j} + 4\hat{k}$.

Solution. Let

$\vec{A} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{B} = 2\hat{i} - 2\hat{j} + 4\hat{k}$

Then $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 2 & -2 & 4 \end{vmatrix}$

$= \hat{i}\begin{vmatrix} 1 & 2 \\ -2 & 4 \end{vmatrix} - \hat{j}\begin{vmatrix} 3 & 2 \\ 2 & 4 \end{vmatrix} + \hat{k}\begin{vmatrix} 3 & 1 \\ 2 & -2 \end{vmatrix}$

$= \hat{i}(4+4) - \hat{j}(12-4) + \hat{k}(-6-2)$

$= 8\hat{i} - 8\hat{j} - 8\hat{k}$

$|\vec{A} \times \vec{B}| = \sqrt{8^2 + (-8)^2 + (-8)^2} = 8\sqrt{3}$

$|\vec{A}| = \sqrt{3^2 + 1^2 + 2^2} = \sqrt{14}$

$|\vec{B}| = \sqrt{2^2 + (-2)^2 + 4^2} = \sqrt{24}$

$\therefore \quad \sin\theta = \dfrac{|\vec{A} \times \vec{B}|}{|\vec{A}||\vec{B}|} = \dfrac{8\sqrt{3}}{\sqrt{14} \times \sqrt{24}} = \dfrac{2}{\sqrt{7}}$.

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EXAMPLE 45. Show that
$(\vec{A}-\vec{B})\times(\vec{A}+\vec{B})=2(\vec{A}\times\vec{B})$.

Solution.
$$L.H.S.=(\vec{A}-\vec{B})\times(\vec{A}+\vec{B})$$
$$=\vec{A}\times\vec{A}+\vec{A}\times\vec{B}-\vec{B}\times\vec{A}-\vec{B}\times\vec{B}$$
$$=\vec{0}+\vec{A}\times\vec{B}+\vec{A}\times\vec{B}+\vec{0}$$
$$[\because \vec{A}\times\vec{B}=-\vec{B}\times\vec{A}]$$
$$=2(\vec{A}\times\vec{B})=R.H.S.$$

EXAMPLE 46. For any three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$, prove that
$\vec{A}\times(\vec{B}+\vec{C})+\vec{B}\times(\vec{C}+\vec{A})+\vec{C}\times(\vec{A}+\vec{B})=\vec{0}$.

Solution. L.H.S.
$$=\vec{A}\times(\vec{B}+\vec{C})+\vec{B}\times(\vec{C}+\vec{A})+\vec{C}\times(\vec{A}+\vec{B})$$
$$=\vec{A}\times\vec{B}+\vec{A}\times\vec{C}+\vec{B}\times\vec{C}+\vec{B}\times\vec{A}+\vec{C}\times\vec{A}+\vec{C}\times\vec{B}$$
$$=\vec{A}\times\vec{B}+\vec{A}\times\vec{C}+\vec{B}\times\vec{C}-\vec{A}\times\vec{B}-\vec{A}\times\vec{C}-\vec{B}\times\vec{C}$$
$$[\because \vec{B}\times\vec{A}=-\vec{A}\times\vec{B}]$$
$$=\vec{0}=R.H.S.$$

EXAMPLE 47. For any two vectors $\vec{A}$ and $\vec{B}$, prove that
$$(\vec{A}\times\vec{B})^2=A^2B^2-(\vec{A}.\vec{B})^2$$
[Himachal 07, 09C]

Solution. L.H.S. $=(\vec{A}\times\vec{B})^2$
$$=|\vec{A}\times\vec{B}|^2=(AB\sin\theta)^2$$
$$=A^2B^2(1-\cos^2\theta)$$
$$=A^2B^2-(AB\cos\theta)^2$$
$$=A^2B^2-(\vec{A}.\vec{B})^2=R.H.S.$$

EXAMPLE 48. Find $\vec{A}.\vec{B}$ if $|\vec{A}|=2$, $|\vec{B}|=5$ and $|\vec{A}\times\vec{B}|=8$.

Solution. As
$$|\vec{A}\times\vec{B}|^2=|\vec{A}|^2|\vec{B}|^2-(\vec{A}.\vec{B})^2$$
$$\therefore\quad 8^2=2^2\times5^2-(\vec{A}.\vec{B})^2$$
or $$(\vec{A}.\vec{B})^2=100-64=36$$
$$\therefore\quad \vec{A}.\vec{B}=\pm 6.$$

EXAMPLE 49. Find the area of the triangle formed by the tips of the vectors $\vec{a}=\hat{i}-\hat{j}-3\hat{k}$, $\vec{b}=4\hat{i}-3\hat{j}+\hat{k}$ and $\vec{c}=3\hat{i}-\hat{j}+2\hat{k}$.

Solution. Let ABC the triangle formed by the tips of the given vectors.

Then
$$\vec{BA}=\vec{a}-\vec{b}=(\hat{i}-\hat{j}-3\hat{k})-(4\hat{i}-3\hat{j}+\hat{k})$$
$$=-3\hat{i}+2\hat{j}-4\hat{k}$$
$$\vec{BC}=\vec{c}-\vec{b}=(3\hat{i}-\hat{j}+2\hat{k})-(4\hat{i}-3\hat{j}+\hat{k})$$
$$=-\hat{i}+2\hat{j}+\hat{k}$$

Now
$$\vec{BA}\times\vec{BC}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\-3&2&-4\\-1&2&1\end{vmatrix}$$
$$=\hat{i}(2+8)-\hat{j}(-3-4)+\hat{k}(-6+2)$$
$$=10\hat{i}+7\hat{j}-4\hat{k}$$
$$|\vec{BA}\times\vec{BC}|=\sqrt{(10)^2+7^2+(-4)^2}=\sqrt{165}=12.8$$
Ar. of $\Delta ABC=\frac{1}{2}|\vec{BA}\times\vec{BC}|$
$$=\frac{1}{2}\times12.8=\textbf{6.4 sq. units.}$$

EXAMPLE 50. Find the moment about the point $(1,-1,-1)$ of the force $3\hat{i}+4\hat{j}-5\hat{k}$ acting at the point $(1,0,-2)$.

Solution. Here $\vec{F}=3\hat{i}+4\hat{j}-5\hat{k}$

Let P be the point about which moment is to be obtained and A be the point at which force is applied. If O is the origin, then
$$\vec{OP}=\hat{i}-\hat{j}-\hat{k},\quad \vec{OA}=\hat{i}+0\hat{j}-2\hat{k}$$
$$\therefore\quad \vec{PA}=\vec{OA}-\vec{OP}$$
$$=(\hat{i}-0\hat{j}-2\hat{k})-(\hat{i}-\hat{j}-\hat{k})$$
$$=\hat{j}-\hat{k}$$

Moment of force $\vec{F}$ about the point P is
$$\vec{\tau}=\vec{PA}\times\vec{F}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\0&1&-1\\3&4&-5\end{vmatrix}$$
$$=i(-5+4)-\hat{j}(0+3)+k(0-3)$$
$$=-\hat{i}-3\hat{j}-3\hat{k}.$$

EXAMPLE 51. The diagonals of a parallelogram are given by the vectors $3\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}-3\hat{j}+4\hat{k}$. Find the area of the parallelogram.

Solution. If $\vec{A}$ and $\vec{B}$ are the adjacent sides of the parallelogram, then its diagonals will be
$$\vec{A}+\vec{B}=3\hat{i}+\hat{j}+2\hat{k}$$
and $$\vec{A}-\vec{B}=\hat{i}-3\hat{j}+4\hat{k}$$

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Now $\vec{A} \times \vec{B} = \frac{1}{2}(\vec{A} - \vec{B}) \times (\vec{A} + \vec{B})$

$$= \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 4 \\ 3 & 1 & 2 \end{vmatrix}$$

$$= \frac{1}{2}[\hat{i}(-6-4) - \hat{j}(2-12) + \hat{k}(1+9)]$$

$$= \frac{1}{2}(-10\hat{i} + 10\hat{j} + 10\hat{k})$$

Area of parallelogram

$$= |\vec{A} \times \vec{B}| = \frac{1}{2}\sqrt{(-10)^2 + 10^2 + 10^2}$$

$$= \frac{1}{2} \times 17.32 = \mathbf{8.66\ sq.\ units.}$$

EXAMPLE 52. In any ΔABC, prove that

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

Solution. As shown in Fig. 4.53, the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are cyclic, therefore

$$\vec{a} + \vec{b} + \vec{c} = 0 \quad \text{or} \quad \vec{a} + \vec{b} = -\vec{c}$$

or $(\vec{a} + \vec{b}) \times \vec{c} = -\vec{c} \times \vec{c}$ or $\vec{a} \times \vec{c} + \vec{b} \times \vec{c} = \vec{0}$

or $-\vec{c} \times \vec{a} + \vec{b} \times \vec{c} = \vec{0}$

or $\vec{b} \times \vec{c} = \vec{c} \times \vec{a}$ ...(i)

Similarly, $\vec{a} \times \vec{b} = \vec{b} \times \vec{c}$ ...(ii)

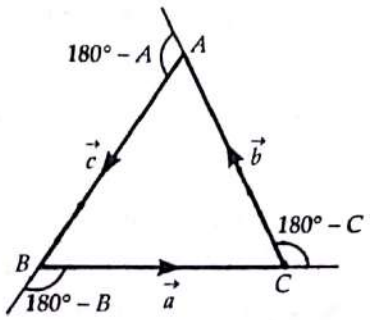

Fig. 4.53

From (i) and (ii), we get

$$\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$$

or $|\vec{a} \times \vec{b}| = |\vec{b} \times \vec{c}| = |\vec{c} \times \vec{a}|$

or $ab \sin(180° - C) = bc \sin(180° - A)$

$= ca \sin(180° - B)$

or $ab \sin C = bc \sin A = ca \sin B$

Dividing throughout by abc, we get

$$\frac{\sin C}{c} = \frac{\sin A}{a} = \frac{\sin B}{b} \quad \text{or} \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

EXAMPLE 53. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three vectors such that

$$\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}, \quad \vec{a} \times \vec{b} = \vec{a} \times \vec{c}, \quad \vec{a} \neq \vec{0}$$

then prove that $\vec{b} = \vec{c}$.

Solution. Given : $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$

or $\vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{c} = 0$

or $\vec{a} \cdot (\vec{b} - \vec{c}) = 0$

But $\vec{a} \neq \vec{0}$

$\therefore$ Either $\vec{b} - \vec{c} = \vec{0}$ or $\vec{a} \perp (\vec{b} - \vec{c})$

$\Rightarrow$ Either $\vec{b} = \vec{c}$, $\vec{a} \perp (\vec{b} - \vec{c})$ ...(i)

or Also, $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$

or $\vec{a} \times \vec{b} - \vec{a} \times \vec{c} = \vec{0}$ or $\vec{a} \times (\vec{b} - \vec{c}) = \vec{0}$.

But $\vec{a} \neq \vec{0}$

$\therefore$ Either $\vec{b} - \vec{c} = \vec{0}$ or $\vec{a} \parallel (\vec{b} - \vec{c})$

$\Rightarrow$ Either $\vec{b} = \vec{c}$, $\vec{a} \parallel (\vec{b} - \vec{c})$ ...(ii)

or

But $\vec{a}$ cannot be simultaneously perpendicular and parallel to $(\vec{b} - \vec{c})$, so equations (i) and (ii) will hold simultaneously if $\vec{b} = \vec{c}$.

EXAMPLE 54. If $\vec{a} = \hat{i} - 2\hat{j} - 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = \hat{i} + 3\hat{j} - 2\hat{k}$, then find $\vec{a} \times (\vec{b} \times \vec{c})$.

Solution. $\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 3 & -2 \end{vmatrix}$

$$= \hat{i}(-2+3) - \hat{j}(-4+1) + \hat{k}(6-1)$$

$$= \hat{i} + 3\hat{j} + 5\hat{k}$$

Now

$$\vec{a} \times (\vec{b} \times \vec{c}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & -3 \\ 1 & 3 & 5 \end{vmatrix}$$

$$= \hat{i}(-10+9) - \hat{j}(5+3) + \hat{k}(3+2)$$

$$= -\hat{i} - 8\hat{j} + 5\hat{k}.$$

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PROBLEMS FOR PRACTICE

1. If $\vec{A} = \hat{i} + 3\hat{j} + 2\hat{k}$ and $\vec{B} = 3\hat{i} + \hat{j} + 2\hat{k}$, then find the vector product $\vec{A} \times \vec{B}$. (Ans. $4\hat{i} + 4\hat{j} - 8\hat{k}$)

2. Prove that the vectors $\vec{A} = 4\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{B} = 12\hat{i} + 9\hat{j} + 3\hat{k}$ are parallel to each other.

3. If $\vec{A} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{B} = 3\hat{i} + 2\hat{j} + 4\hat{k}$, then find the value of $(\vec{A} + \vec{B}) \times (\vec{A} - \vec{B})$.

(Ans. $-20\hat{i} + 10\hat{j} + 10\hat{k}$)

4. Find the value of a for which the vectors $3\hat{i} + 3\hat{j} + 9\hat{k}$ and $\hat{i} + a\hat{j} + 3\hat{k}$ are parallel.

(Ans. $a = 1$)

5. Find a unit vector perpendicular to the vectors $\vec{A} = 4\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{B} = -2\hat{i} + \hat{j} - 2\hat{k}$.

[Ans. $\frac{1}{3}(-\hat{i} + 2\hat{j} + 2\hat{k})$]

6. Find the sine of the angle between the vectors $\vec{A} = 3\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{B} = \hat{i} - \hat{j} + \hat{k}$. (Ans. 1/5)

7. Find a vector of magnitude 18 which is perpendicular to both the vectors $4\hat{i} - \hat{j} + 3\hat{k}$ and $-2\hat{i} + \hat{j} - 2\hat{k}$.

(Ans. $-6\hat{i} + 12\hat{j} + 12\hat{k}$)

8. Determine the area of the parallelogram whose adjacent sides are formed by the vectors $\vec{A} = \hat{i} - 3\hat{j} + \hat{k}$ and $\vec{B} = \hat{i} + \hat{j} + \hat{k}$.

(Ans. $4\sqrt{2}$ square units)

9. Find the area of the triangle formed by points O, A and B such that $\vec{OA} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{OB} = -3\hat{i} - 2\hat{j} + \hat{k}$. (Ans. $3\sqrt{5}$ square units)

10. Find with the help of vectors, the area of the triangle with vertices $A(3, -1, 2)$, $B(1, -1, -3)$ and $C(4, -3, 1)$. (Ans. $\sqrt{165}/2$ sq. units)

11. If $\vec{A}$ and $\vec{B}$ are two such vectors that $|\vec{A}| = 2$, $|\vec{B}| = 7$ and $\vec{A} \times \vec{B} = 3\hat{i} + 2\hat{j} + 6\hat{k}$, find the angle between $\vec{A}$ and $\vec{B}$. (Ans. $\pi/6$)

12. Find the moment about the point $\hat{i} + 2\hat{j} - \hat{k}$ of a force represented by $3\hat{i} + \hat{k}$ acting through the point $2\hat{i} - \hat{j} + 3\hat{k}$.

(Ans. $-3\hat{i} + 11\hat{j} + 9\hat{k}$)

13. Prove that $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = 2(\vec{b} \times \vec{a})$.

14. Prove that $|\vec{a} \times \vec{b}| = \sqrt{a^2b^2 - (\vec{a} \cdot \vec{b})^2}$.

15. Find $|\vec{A} \times \vec{B}|$ if $|\vec{A}| = 10$, $|\vec{B}| = 2$ and $\vec{A} \cdot \vec{B} = 12$.

(Ans. 16)

16. If $\vec{a} = \hat{i} - 2\hat{j} - 3\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} - \hat{k}$ and $\vec{c} = \hat{i} + 3\hat{j} - 2\hat{k}$, find $(\vec{a} \times \vec{b}) \times \vec{c}$.

(Ans. $\hat{i} + \hat{j} + 2\hat{k}$)

17. Calculate the area of the triangle determined by the two vectors : $\vec{A} = 3\hat{i} + 4\hat{j}$ and $\vec{B} = -3\hat{i} + 7\hat{j}$.

[Central Schools 08, 09]

(Ans. 16.5 square units)

18. Find the area of a parallelogram formed by vectors $\vec{A} = 3\hat{i} + 2\hat{j}$, $\vec{B} = -3\hat{i} + 7\hat{j}$

[Central Schools 07]

(Ans. $3\sqrt{3}$ square units)

19. The diagonals of a parallelogram are represented by $\vec{R}_1 = 3\hat{i} + 2\hat{j} - 7\hat{k}$ and $\vec{R}_2 = 5\hat{i} + 6\hat{j} - 3\hat{k}$. Find the area of the parallelogram. [Chandigarh 07]

(Ans. $2\sqrt{509}$ square units)

4.18 MOTION IN A PLANE

39. *Show the position vector for a particle in two dimensional motion. Write an expression for this position vector.*

Expression for position vector. Fig. 4.54 shows the position vector $\vec{OP}$ of a particle located at P with respect to the origin O. If (x, y) are the coordinates of point P, then

$$\vec{OP} = \vec{OA} + \vec{OB}$$

or $$\vec{r} = x\hat{i} + y\hat{j}$$

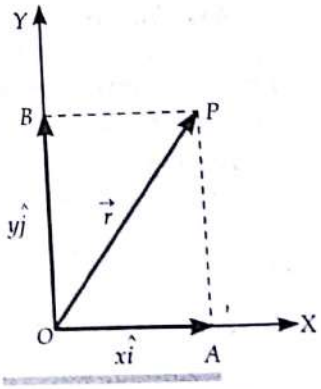

Fig. 4.54 Position vector.

This equation expresses position vector $\vec{r}$ in terms of its rectangular components x and y.

40. *Show graphically the displacement vector for a motion in two dimensions. Write an expression for displacement vector in terms of its rectangular components.*

Expression for displacement vector. Suppose a particle moves in the X-Y plane along the curved path

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shown in Fig. 4.55. The particle is at point $P(x, y)$ at time t and at $P'(x', y')$ at time t'.

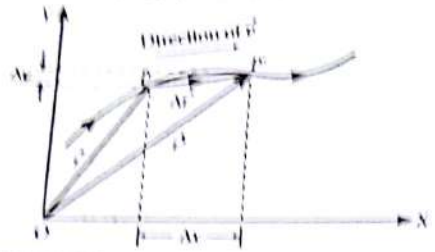

Fig. 4.55 Displacement $\Delta \vec{r}$ and average velocity $\vec{v}$.

Using triangle law of vectors addition,

$$\vec{OP'} = \vec{OP} + \vec{PP'}$$

So the displacement vector is

$$\vec{PP'} = \vec{OP'} - \vec{OP}$$

or $$\Delta \vec{r} = \vec{r'} - \vec{r}$$

The direction of $\Delta \vec{r}$ is from P to P'.

In terms of rectangular components,

$$\Delta \vec{r} = (x' \hat{i} + y' \hat{j}) - (x \hat{i} + y \hat{j})$$

$$= (x' - x) \hat{i} + (y' - y) \hat{j}$$

or $$\Delta \vec{r} = \Delta x \hat{i} + \Delta y \hat{j}$$

41. *Write an expression for average velocity in terms of its rectangular components. Hence write an expression for instantaneous velocity. Find the direction of instantaneous velocity.*

Average velocity. Refer to Fig. 4.55. Suppose the particle moves from point P to P' in time t to t'. *The average velocity of an object is the ratio of the displacement and the corresponding time interval.* So it is given by

$$\vec{v} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\Delta x \hat{i} + \Delta y \hat{j}}{\Delta t} = \frac{\Delta x}{\Delta t} \hat{i} + \frac{\Delta y}{\Delta t} \hat{j}$$

[Here $\Delta t = t' - t$]

r $$\vec{v} = \bar{v}_x \hat{i} + \bar{v}_y \hat{j},$$

This equation expresses average velocity $\bar{v}$ in terms its rectangular components $\bar{v}_x$ and $\bar{v}_y$.

As $\bar{v} = \frac{\Delta \vec{r}}{\Delta t}$, so the direction of the average city is same as that of displacement vector $\Delta \vec{r}$, as n in Fig. 4.55.

Instantaneous velocity. *The instantaneous velocity of a particle is equal to the limiting value of its average velocity when the time interval approaches zero.* It is given by

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{dr}}{dt}$$

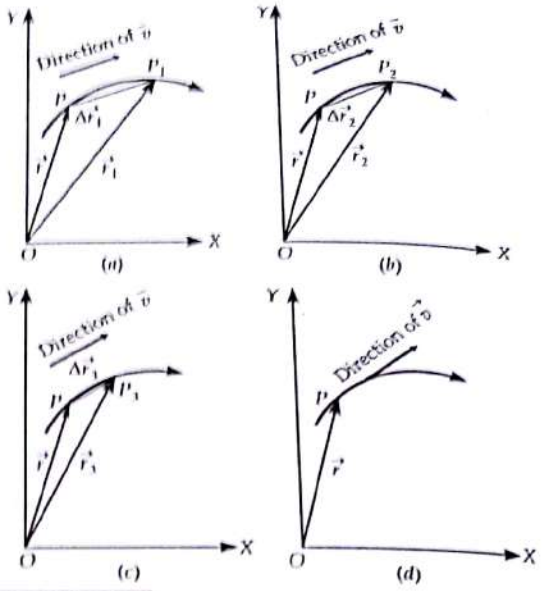

Fig. 4.56 As $\Delta t \to 0$, the average velocity approaches the velocity $\vec{v}$. The direction of $\vec{v}$ is parallel to the line tangent to the path.

Direction of instantaneous velocity. In Figs. 4.56(*a*) to (*d*), the thick line curve represents the path of an object. The object is at point P at time t. P_1, P_2 and P_3 are the positions of the object after time intervals Δt_1, Δt_2 and Δt_3; where $\Delta t_1 > \Delta t_2 > \Delta t_3$. The directions of average velocity $\bar{v}$ are shown parallel to the corresponding displacement vector $\Delta \vec{r}_1$, $\Delta \vec{r}_2$ and $\Delta \vec{r}_3$, for the decreasing values of Δt. As $\Delta t \to 0$, $\Delta \vec{r} \to 0$, and the direction of velocity is along the tangent to the path, as shown in Fig. 4.56(*d*). *Hence the direction of (instantaneous) velocity at any point on the path of an object is tangent to the path at that point and is in the direction of motion.*

42. *Obtain an expression for instantaneous velocity in terms of its rectangular components.*

Velocity in terms of rectangular components. The instantaneous velocity is given by

$$\vec{v} = \frac{\vec{dr}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta x \hat{i} + \Delta y \hat{j}}{\Delta t}$$

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$$= \left(\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}\right)\hat{i} + \left(\lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t}\right)\hat{j} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{z}$$

$$\vec{v} = v_x\hat{i} + v_y\hat{j}.$$

If the coordinates x and y are known as functions of time t, then we can determine v_x and v_y. The magnitude of $\vec{v}$ will be

$$v = \sqrt{v_x^2 + v_y^2}$$

Figure 4.57 shows the rectangular components v_x and v_y of velocity $\vec{v}$. If $\vec{v}$ makes angle θ with X-axis, then

$$\tan\theta = \frac{v_y}{v_x} \quad \text{or} \quad \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right).$$

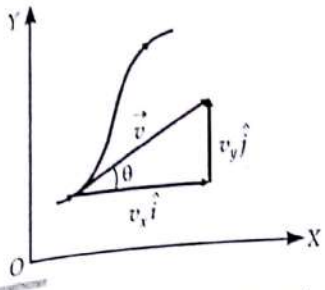

Fig. 4.57 Components v_x and v_y of velocity $\vec{v}$.

43. *Write an expression for average acceleration in terms of its rectangular components. Hence write an expression for instantaneous acceleration. Find the direction of instantaneous acceleration.*

Average acceleration. *The average acceleration of an object is the ratio of the change in velocity and the corresponding time interval.* If the velocity of an object changes from $\vec{v}$ to $\vec{v'}$ in time Δt, then the average acceleration is given by

$$\bar{a} = \frac{\vec{v'} - \vec{v}}{\Delta t} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\Delta(v_x\hat{i} + v_y\hat{j})}{\Delta t} = \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j}$$

or $\bar{a} = \bar{a}_x\hat{i} + \bar{a}_y\hat{j}$

This equation expresses average acceleration $\bar{a}$ in terms of its rectangular components $\bar{a}_x$ and $\bar{a}_y$. As $\bar{a} = \frac{\Delta\vec{v}}{\Delta t}$, so the direction of average acceleration is same as that of the change in velocity $\Delta\vec{v}$.

Instantaneous acceleration. *The instantaneous acceleration of an object is equal to the limiting value of its average acceleration when the time interval approaches zero.* It is given by

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}.$$

Direction of instantaneous acceleration. In Figs. 4.58(*a*) to (*d*), the thick line curve represents the path of object's motion. The object is at point P at time t. P_1, P_2 and P_3 are the positions after time intervals Δt_1, Δt_2 and Δt_3; where $\Delta t_1 > \Delta t_2 > \Delta t_3$. The velocity vectors at points P, P_1, P_2 and P_3 are shown in the figures. In each case, the change in velocity $\Delta\vec{v}$ is obtained by using triangle law of vector addition. Again, in each case the direction of average acceleration $\bar{a}$ is shown parallel to $\Delta\vec{v}$.

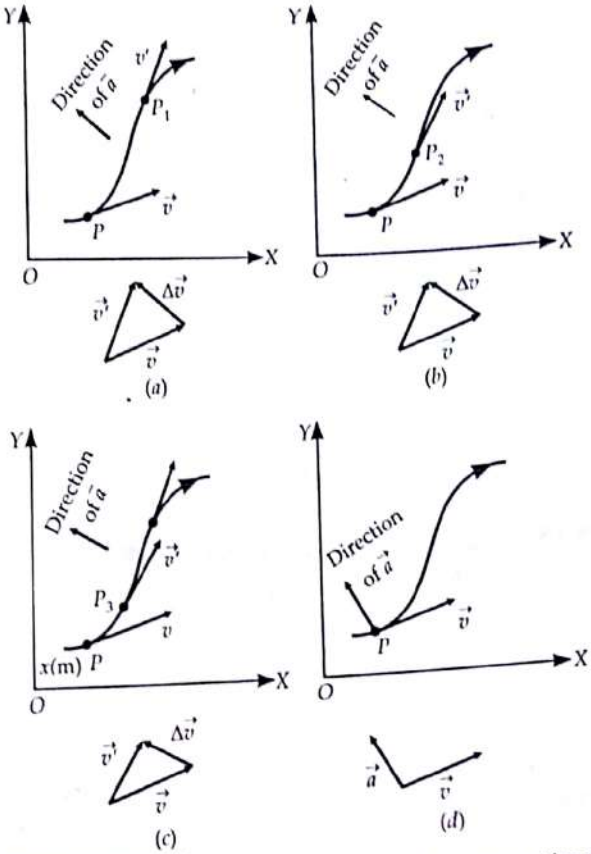

Fig. 4.58 The average acceleration for three time intervals (*a*) Δt_1, (*b*) Δt_2 and (*c*) Δt_3, $(\Delta t_1 > \Delta t_2 > \Delta t_3)$. (*d*) In the limit $\Delta t \to 0$, the average acceleration becomes the acceleration.

As Δt decreases from (*a*) to (*d*), *the direction of* $\Delta\vec{v}$ *and hence that of* $\bar{a}$ *changes. In Fig. 4.58(d), as* $\Delta t \to 0$, the average acceleration becomes the instantaneous acceleration and its direction is as shown.

For Your Knowledge

- For motion in one dimension, the velocity and acceleration are always along the same line either in same direction (for accelerated motion) or in opposite direction (for decelerated motion).
- For motion in two or three dimensions, the angle between velocity and acceleration vectors may have any value between 0° and 180°.

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44. *Obtain an expression for instantaneous acceleration in terms of its rectangular components.*

Acceleration in terms of rectangular components. The instantaneous acceleration is given by

$$\vec{a} = \frac{d\vec{v}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta (v_x \hat{i} + v_y \hat{j})}{\Delta t}$$

$$= \left(\lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} \right) \hat{i} + \left(\lim_{\Delta t \to 0} \frac{\Delta v_y}{\Delta t} \right) \hat{j}$$

$$= \frac{dv_x}{dt} \hat{i} + \frac{dv_y}{dt} \hat{j}$$

or $$\vec{a} = a_x \hat{i} + a_y \hat{j}$$

This equation expresses acceleration $\vec{a}$ in terms of its rectangular components a_x and a_y. We can express a_x and a_y in terms of coordinates x and y as follows :

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

and $$a_y = \frac{dv_y}{dt} = \frac{d}{dt}\left(\frac{dy}{dt}\right) = \frac{d^2y}{dt^2}.$$

4.19 MOTION IN A PLANE WITH UNIFORM VELCOITY

45. *Define uniform velocity. Show that two dimensional uniform velocity motion is equivalent to two one dimensional uniform velocity motions along two perpendicular directions.*

Uniform velocity. *A body is said to be moving with uniform velocity if it suffers equal displacements in equal intervals of time, however small.*

Position vector for uniform velocity. Consider an object moving with uniform velocity $\vec{v}$ in XY-plane.

Let $\vec{r}(0)$ and $\vec{r}(t)$ be its position vectors at times $t = 0$ and $t = t$ respectively. Then

$$\vec{v} = \frac{\vec{r}(t) - \vec{r}(0)}{t - 0}$$

or $$\vec{r}(t) = \vec{r}(0) + \vec{v}\,t \qquad ...(1)$$

In terms of rectangular components, we can write

$$\vec{v} = v_x \hat{i} + v_y \hat{j}, \text{ where } v = \sqrt{v_x^2 + v_y^2}$$

$$\vec{r}(0) = x(0)\hat{i} + y(0)\hat{j}$$

$$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j}$$

Substituting these values in equation (1), we get

$$x(t)\hat{i} + y(t)\hat{j} = x(0)\hat{i} + y(0)\hat{j} + (v_x \hat{i} + v_y \hat{j})\,t$$

or $$x(t)\hat{i} + y(t)\hat{j} = [x(0) + v_x t]\hat{i} + [y(0) + v_y t]\hat{j} \qquad ...(2)$$

Equating the coefficients of $\hat{i}$ and $\hat{j}$ on both sides, we get

$$x(t) = x(0) + v_x t$$

$$y(t) = y(0) + v_y t$$

The above two equations represent uniform motions along X-axis and Y-axis respectively. Thus equation (2) shows that *a uniform motion in two-dimensions can be expressed as the sum of two uniform motions along two mutually perpendicular directions.*

4.20 MOTION IN A PLANE WITH CONSTANT ACCELERATION

46(a) *Define uniform acceleration. Show that in two-dimensional motion with uniform acceleration, each rectangular component of velocity is similar to that of uniformly accelerated motion along one dimension.*

Uniform acceleration. *A body is said to be moving with uniform acceleration if its velocity vector suffers the same change in the same interval of time, however small.*

Velocity vector for uniform acceleration. If $\vec{v}_0$ and $\vec{v}$ be the velocity vectors at times $t = 0$ and $t = t$ respectively, then the acceleration is given by

$$\vec{a} = \frac{\vec{v} - \vec{v}_0}{t - 0} = \frac{\vec{v} - \vec{v}_0}{t}$$

or $$\vec{v} = \vec{v}_0 + \vec{a}\,t$$

Writing the above equation in terms of rectangular components, we get

$$v_x \hat{i} + v_y \hat{j} = v_{0x} \hat{i} + v_{0y} \hat{j} + (a_x \hat{i} + a_y \hat{j})\,t$$

or $$v_x \hat{i} + v_y \hat{j} = (v_{0x} + a_x t)\hat{i} + (v_{0y} + a_y t)\hat{j}$$

Comparing the coefficients of $\hat{i}$ and $\hat{j}$ on both sides of the above equation, we get

$$v_x = v_{0x} + a_x t \quad \text{and} \quad v_y = v_{0y} + a_y t$$

The above two equations show that *each rectangular component of velocity of an object moving with uniform acceleration in a plane depends upon time as if it were the velocity vector of one- dimensional uniformly accelerated motion.*

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46(b) *Write an expression for the instantaneous position vector of an object having two dimensional motion under uniform acceleration. Hence show that a two dimensional uniformly accelerated motion is a combination of two rectangular uniformly accelerated motions.*

Position vector for uniform acceleration. Consider a particle moving with uniform acceleration $\vec{a}$. Let $\vec{r}_0$ and $\vec{r}$ be its position vectors at times 0 and t and let the velocities at these instants be $\vec{v}_0$ and $\vec{v}$. Now

Displacement = Average velocity × time interval

$$\text{or} \quad \vec{r} - \vec{r}_0 = \frac{\vec{v}_0 + \vec{v}}{2} \times t = \frac{\vec{v}_0 + (\vec{v}_0 + \vec{a}\,t)}{2} \times t$$

$$= \vec{v}_0\, t + \frac{1}{2}\vec{a}\; t^2$$

$$\text{or} \quad \vec{r} = \vec{r}_0 + \vec{v}_0\, t + \frac{1}{2}\vec{a}\, t^2$$

This equation gives position of a uniformly accelerated particle at time t. Writing the above equation in terms of rectangular components, we get

$$x\hat{i} + y\hat{j} = x_0\hat{i} + y_0\hat{j} + (v_{0x}\hat{i} + v_{0y}\hat{j})\,t + \frac{1}{2}(a_x\hat{i} + a_y\hat{j})\,t^2$$

Equating the coefficients of $\hat{i}$ and $\hat{j}$ on both sides, we get

$$x = x_0 + v_{0x}\, t + \frac{1}{2} a_x\, t^2$$

$$\text{and} \quad y = y_0 + v_{0y}\, t + \frac{1}{2} a_y\, t^2$$

The above two equations show that the motions in x and y directions can be treated independently of each other. Thus, *the motion in a plane with uniform acceleration can be treated as the superposition of two separate simultaneous one-dimensional motions along two perpendicular directions.*

Note In uniform acceleration, the position vector at time t,

$$\vec{r} = \vec{r}_o + \vec{v}_o t + \frac{1}{2}\vec{a}\, t^2$$

Similarly, position vector at time t' is

$$\vec{r}' = \vec{r}_o + \vec{v}_o t' + \frac{1}{2}\vec{a}\, t'^2$$

$$\therefore \quad \vec{r}' - \vec{r} = \vec{v}_o (t' - t) + \frac{1}{2}\vec{a}\,(t'^2 - t^2)$$

$$\text{or} \quad \vec{r}' = \vec{r} + \vec{v}_o (t' - t) + \frac{1}{2}\vec{a}\,(t'^2 - t^2)$$

Prove that $v^2 - v_0^2 = 2\vec{a}\,.(\vec{r} - \vec{r}_0)$.

From first equation of motion,

$$\vec{v} = \vec{v}_0 + \vec{a}\, t \quad \text{or} \quad \vec{v} - \vec{v}_0 = \vec{a}\, t \qquad ...(1)$$

Now,

displacement = Average velocity × time interval

$$\text{or} \quad \vec{r} - \vec{r}_0 = \frac{\vec{v} + \vec{v}_0}{2} \times t$$

$$\text{or} \quad \vec{v} + \vec{v}_0 = \frac{2(\vec{r} - \vec{r}_0)}{t} \qquad ...(2)$$

Taking the dot products of the corresponding sides of the equations (1) and (2), we get

$$(\vec{v} - \vec{v}_0).(\vec{v} + \vec{v}_0) = \vec{a}\, t\,.2\,\frac{(\vec{r} - \vec{r}_0)}{t}$$

$$\text{or} \quad v^2 - v_0^2 = 2\vec{a}\,.(\vec{r} - \vec{r}_0).$$

Examples based on Motion in a Plane

CONCEPTS USED

1. Distance is the length of actual path traversed by a moving body between its initial and final positions.
2. Displacement is the shortest distance between the initial and final positions of a body.
3. Average speed = $\dfrac{\text{Distance travelled}}{\text{Time taken}}$
4. Average velocity = $\dfrac{\text{Displacement covered}}{\text{Time taken}}$
5. Instantaneous velocity, $\vec{v} = \dfrac{d\vec{r}}{dt}$
6. Instantaneous acceleration, $\vec{a} = \dfrac{d\vec{v}}{dt}$

UNITS USED

Displacement and distance are in metre, average speed and average velocity are in ms^{-1}.

Example 55. A cyclist moves along a circular path of radius 70 m. If he completes one round in 11 s, calculate (i) total length of path, (ii) magnitude of the displacement, (iii) average speed, and (iv) magnitude of average velocity.

Solution. Radius of the circular path, $r = 70$ m

Time taken to complete one round, $t = 11$ s

(*i*) Total length of path,

$$s = 2\pi r = 2 \times \frac{22}{7} \times 70 = \mathbf{440\ m}..$$

(*ii*) As the cyclist returns to the initial position after one round, magnitude of the displacement = **0**.

(*iii*) Average speed = $\dfrac{s}{t} = \dfrac{440\,\text{m}}{11\,\text{s}} = \mathbf{40\ ms^{-1}}$.

(*iv*) Average velocity = $\dfrac{\text{Displacement}}{\text{Time}} = \dfrac{0}{11} = \mathbf{0}$.

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Example 56. *A particle is moving eastwards with a velocity of 5 ms^{-1}. In 10 seconds, the velocity changes to 5 ms^{-1} northwards. Find the average acceleration of the particle in this time interval.* **[AIEEE 05]**

Solution. In Fig. 4.59,

$\vec{OA}$ = Initial velocity $\vec{v}_1 = 5\ ms^{-1}$, due east

$\vec{OB}$ = Final velocity $\vec{v}_2 = 5\ ms^{-1}$, due north

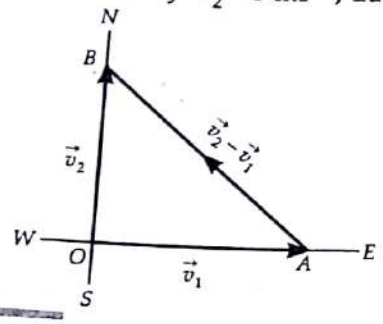

Fig. 4.59

By Δ law of vector addition,

$\vec{OA} + \vec{AB} = \vec{OB}$

$\therefore \quad \vec{AB} = \vec{OB} - \vec{OA} = \vec{v}_2 - \vec{v}_1$

= Change in velocity

$|\vec{v}_2 - \vec{v}_1| = AB = \sqrt{OA^2 + OB^2}$

$= \sqrt{5^2 + 5^2} = 5\sqrt{2}$ m

Average acceleration

$$= \frac{|\vec{v}_2 - \vec{v}_1|}{t} = \frac{5\sqrt{2}}{10}$$

$$= \frac{1}{\sqrt{2}}\ ms^{-2}, \text{ along N-W direction.}$$

Example 57. *The position of a particle is given by*

$$\vec{r} = 3.0\,t\,\hat{i} + 2.0\,t^2\,\hat{j} + 5.0\,\hat{k}$$

where t is in seconds and the coefficients have the proper units for r to be in metres. (a) Find v (t) and a (t) of the particle. (b) Find the magnitude and direction of v (t) at t = 3.0 s. **[NCERT]**

Solution. Here $\vec{r} = 3.0t\hat{i} + 2.0t^2\hat{j} + 5.0\hat{k}$

$$\therefore \quad \vec{v}(t) = \frac{d\vec{r}}{dt} = \frac{d}{dt}(3.0\,t\,\hat{i} + 2.0\,t^2\,\hat{j} + 5.0\,\hat{k})$$

$$= 3.0\,\hat{i} + 4.0\,t\,\hat{j}$$

$$\vec{a}(t) = \frac{d\vec{v}}{dt} = +4.0\,\hat{j}$$

At $t = 3.0$ s, $\quad \vec{v} = 3.0\,\hat{i} + 12.0\,\hat{j}$

It's magnitude is $v = \sqrt{3^2 + 12^2} = 12.4\ ms^{-1}$ and

ction is $\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{12}{3}\right) \simeq 76°$.

EXAMPLE 58. *If the position vector of a particle is given by: $\vec{r} = (4\cos 2t)\,\hat{i} + (4\sin 2t)\,\hat{j} + (6t)\,\hat{k}$ m, calculate its acceleration at $t = \pi/4$.* **[S.C.R.A. 91]**

Solution. Position,

$$\vec{r} = (4\cos 2t)\,\hat{i} + (4\sin 2t)\,\hat{j} + 6t\,\hat{k}.$$

Velocity, $\vec{v} = \frac{d\vec{r}}{dt} = [4(-\sin 2t).(2)]\,\hat{i}$

$$+ [4(\cos 2t).(2)]\,\hat{j} + 6\,\hat{k}$$

$$= (-8\sin 2t)\,\hat{i} + (8\cos 2t)\,\hat{j} + 6\,\hat{k}$$

Acceleration,

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$= [-8(\cos 2t)(2)]\,\hat{i} + [8(-\sin 2t)(2)]\,\hat{j}$$

$$= (-16\cos 2t)\,\hat{i} + (-16\sin 2t)\,\hat{j}$$

When $t = \pi/4$

$$\vec{a} = (-16\cos \pi/2)\,\hat{i} + (-16\sin \pi/2)\,\hat{j}$$

$$= (-16 \times 0)\,\hat{i} + (-16 \times 1)\,\hat{j} = -16\,\hat{j}\ ms^{-2}.$$

PROBLEMS FOR PRACTICE

1. A body is moving with a uniform velocity of 10 ms^{-1} on a circular path of diameter 2.0 m. Calculate (*i*) the difference between the magnitude of the displacement of the body and the distance covered in half a round and (*ii*) the magnitude of the change in velocity of the body in half a round.

   (Ans. 1.14 m, 20 ms^{-1}**)**

2. A particle starts from origin at $t = 0$ with a velocity $5.0\,\hat{i}$ m/s and moves in *x*-*y* plane under action of a force which produces a constant acceleration of $(3.0\,\hat{i} + 2.0\,\hat{j})$ m/s^2. (*a*) What is the *y*-coordinate of the particle at the instant its *x*-coordinate is 84 m ? (*b*) What is the speed of the particle at this time ?

   [NCERT, Central Schools 03]

   [Ans. (*a*) 36.0 m (*b*) 25.9 ms^{-1}**]**

HINTS

1. (*i*) In Fig. 4.60, let *A* and *B* be the initial and final positions of the body after half around.

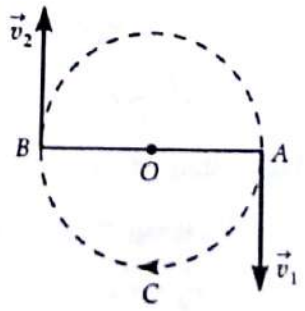

Fig. 4.60

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Distance covered = Half circumference
$= \pi r = 3.14 \times 1.0 = 3.14$ m

Magnitude of displacement = $AB = 2.0$ m
Required difference = $3.14 - 2.0 = \mathbf{1.14\ m}$.

(ii) Let $\vec{v}_1$ and $\vec{v}_2$ be the velocity vectors at A and B.

Then $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1 = 10 - (-10) = \mathbf{20\ ms^{-1}}$.

2. The position of the particle is given by

$$\vec{r}\,(t) = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$

$$= 5.0\,\hat{i}\,t + (1/2)(3.0\,\hat{i} + 2.0\,\hat{j})\,t^2$$

or $x(t)\,\hat{i} + y(t)\,\hat{j} = (5.0t + 1.5t^2)\,\hat{i} + 1.0t^2\,\hat{j}$

$\therefore \quad x(t) = 5.0t + 1.5t^2; \qquad y(t) = 1.0t^2$

Given $x(t) = 84$ m, $t = ?$

$5.0t + 1.5t^2 = 84 \quad \Rightarrow \quad t = 6$ s

At $t = 6$ s, $\quad y = 1.0(6)^2 = \mathbf{36.0\ m}$.

Velocity, $\quad \vec{v} = \dfrac{d\vec{r}}{dt} = (5.0 + 3.0t)\,\hat{i} + 2.0t\,\hat{j}$

At $t = 6$ s, $\quad \vec{v} = 23.0\,\hat{i} + 12.0\,\hat{j}$

Speed $= |\vec{v}| = \sqrt{23^2 + 12^2} \simeq \mathbf{25.9\ ms^{-1}}$.

4.21 RELATIVE VELOCITY IN TWO DIMENSIONS

47. *Define the term relative velocity. How can it be obtained vectorially ?*

Relative velocity. *The relative velocity of an object A with respect to object B, when both are in motion, is the rate of change of position of object A with respect to object B.* Suppose two objects A and B are moving with velocities $\vec{v}_A$ and $\vec{v}_B$, with respect to ground or the earth. Then

Relative velocity of object A w.r.t. object B,

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

Relative velocity of object B w.r.t. object A,

$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$$

Clearly,

$$\vec{v}_{AB} = -\vec{v}_{BA} \quad \text{and} \quad |\vec{v}_{AB}| = |\vec{v}_{BA}|$$

Now, relative velocity of object $A = \vec{v}_A + (-\vec{v}_B)$

= Velocity vector of A + Negative velocity vector of $\vec{B}$

Hence the relative velocity of object A with respect to object B is equal to the vector addition of velocity vector of A and the negative velocity vector of B.

48. *A man moving in rain holds his umbrella inclined to the vertical even though the rain drops are falling vertically downwards. Why ?*

Rain and man. The man experiences the velocity of rain relative to himself. To protect himself from the rain, the man should hold umbrella in the direction of relative velocity of rain w.r.t. the man.

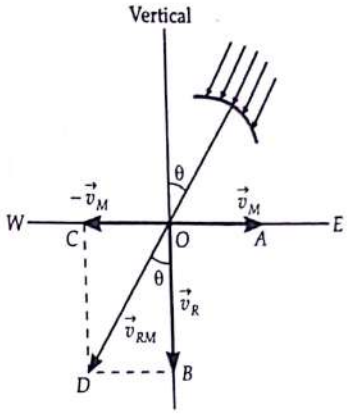

Fig. 4.61 Rain and man.

As shown in Fig. 4.61, consider a man moving due east with velocity $\vec{v}_M$. Suppose the rain falls vertically with velocity $\vec{v}_R$. The relative velocity of rain w.r.t. the man is

$$\vec{v}_{RM} = \vec{v}_R - \vec{v}_M = \vec{v}_R + (-\vec{v}_M) = \vec{OB} + \vec{OC} = \vec{OD}$$

If $\vec{OD}$ makes angle θ with the vertical, then

$$\tan\theta = \frac{DB}{OB} = \frac{v_M}{v_R}$$

So the man can protect himself from rain by holding his umbrella at an angle θ with the vertical in the direction of his motion.

Examples based on
Relative Velocity of Two Inclined Motions

FORMULAE USED

1. The relative velocity of A w.r.t. B, $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$
2. The relative velocity of B w.r.t. A, $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$
3. For two objects moving with velocities v_A and v_B at an angle θ, the relative velocity of an object A w.r.t. B is given by
$$v_{AB} = \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos\theta}$$
4. If velocity v_{AB} makes angle β with v_A, then
$$\tan\beta = \frac{v_B \sin\theta}{v_A - v_B\cos\theta}$$

UNIT USED

Velocities v_A, v_B and v_{AB} are in ms^{-1}.

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EXAMPLE 59. A boat is moving with a velocity $(3\hat{i} + 4\hat{j})$ with respect to ground. The water in the river is moving with a velocity $-3\hat{i} - 4\hat{j}$ with respect to ground. What is the relative velocity of boat with respect to river ? **[EAMCET 91]**

Solution. Velocity of boat, $\vec{v_B} = 3\hat{i} + 4\hat{j}$

Velocity of water, $\vec{v_W} = -3\hat{i} - 4\hat{j}$

$\therefore$ Relative velocity of boat with respect to water is

$$\vec{v_{BW}} = \vec{v_B} - \vec{v_W}$$

$$= (3\hat{i} + 4\hat{j}) - (-3\hat{i} - 4\hat{j}) = 6\hat{i} + 8\hat{j}.$$

EXAMPLE 60. A particle P is moving along a straight line with a velocity of 3 ms^{-1} and another particle Q has a velocity of 4 ms^{-1} at an angle of 30° to the path of P. Find the speed of Q relative to P.

Solution. The situation is shown in Fig. 4.62. Here

$\vec{OA} = \vec{v_P} = 3\ ms^{-1}$

$\vec{OB} = \vec{v_Q} = 4\ ms^{-1}$, inclined to $\vec{v_P}$ at 30°

$\vec{OC} = -\vec{v_P}$ = opposite velocity of P.

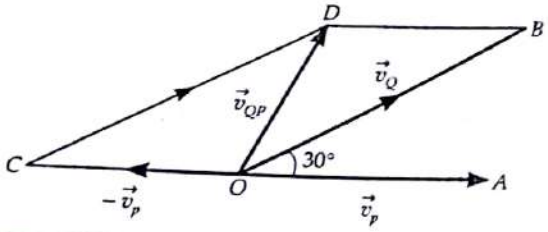

Fig. 4.62

Relative velocity of Q w.r.t. P is

$$\vec{v_{QP}} = \vec{v_Q} - \vec{v_P} = \vec{v_Q} + (-\vec{v_P}) = \vec{OB} + \vec{OC} = \vec{OD}$$

In parallelogram OBDC,

$OC = 3\ ms^{-1}$, $OB = 4\ ms^{-1}$, $\angle BOC = 150°$

$\therefore$ $\vec{v_{QP}} = OD$

$$= \sqrt{OC^2 + OB^2 + 2OC.OB\cos 150°}$$

$$= \sqrt{3^2 + 4^2 + 2 \times 3 \times 4 \times (-\sqrt{3}/2)}$$

$$= \sqrt{9 + 16 - 12 \times 1.732} = \sqrt{25 - 20.784}$$

$$= \sqrt{4.216} = \mathbf{2.1\ ms^{-1}}.$$

EXAMPLE 61. To a driver going east in a car with a velocity of 40 km h^{-1}, a bus appears to move towards north with a velocity of $40\sqrt{3}$ km h^{-1}. What is the actual velocity and direction of motion of the bus ?

Solution. In Fig. 4.63, the true velocity of the car is

$\vec{v_C} = \vec{OA} = 40\ km\ h^{-1}$, due east

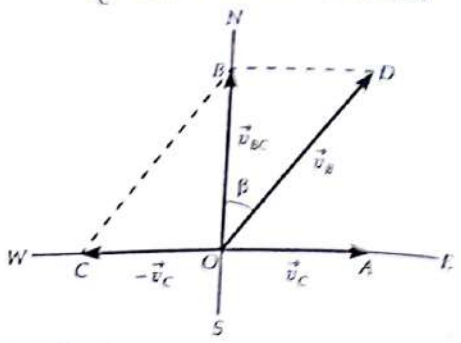

Fig. 4.63

Relative velocity of bus w.r.t. car is

$$\vec{v_{BC}} = \vec{OB} = 40\sqrt{3}\ km\ h^{-1}, \text{ due north}$$

Let the true velocity of bus be along $\vec{OD}$ and $\angle BOD = \beta$. Then

$$v_B = OD = \sqrt{OA^2 + OB^2}$$

$$= \sqrt{40^2 + (40\sqrt{3})^2} = \mathbf{80\ km\ h^{-1}}$$

$$\tan\beta = \frac{BD}{OB} = \frac{40}{40\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$\therefore$ $\beta = \mathbf{30°}$, **east of north.**

EXAMPLE 62. A man rows directly across a flowing river in time t_1 and rows an equal distance down the stream in time t_2. If u be the speed of man in still water and v that of stream, then show that :

$$t_1 : t_2 = \sqrt{u + v} : \sqrt{u - v}$$

Solution. The situation is shown in Fig. 4.64. While rowing directly across the river, the resultant velocity is perpendicular to the velocity v of the stream.

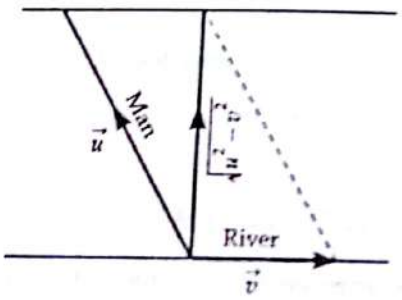

Fig. 4.64

$\therefore$ Resultant velocity across the river $= \sqrt{u^2 - v^2}$

Resultant velocity down the stream $= u + v$

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Let s be the distance covered in each case. Then

$$t_1 = \frac{s}{\sqrt{u^2 - v^2}} \quad \text{and} \quad t_2 = \frac{s}{u + v}$$

$$\frac{t_1}{t_2} = \frac{u+v}{\sqrt{u^2 - v^2}} = \frac{\sqrt{u+v} \times \sqrt{u+v}}{\sqrt{u+v} \times \sqrt{u-v}} = \frac{\sqrt{u+v}}{\sqrt{u-v}}$$

$$\therefore \quad t_1 : t_2 = \sqrt{u+v} : \sqrt{u-v}.$$

PROBLEMS FOR PRACTICE

1. A train is moving with a velocity of 30 km h^{-1} due east and a car is moving with a velocity of 40 km h^{-1} due north. What is the velocity of car as appears to a passenger in the train ?

   (**Ans.** 50 km h^{-1}, 36°52′ west of north)

2. Rain is falling vertically with a speed of 35 ms^{-1}. A woman rides a bicycle with a speed of 12 ms^{-1} in east to west direction. What is the direction in which she should hold her umbrella ?

   (**Ans.** At an angle of 19° with the vertical towards the west)

3. To a person moving eastwards with a velocity of 4.8 km h^{-1}, rain appears to fall vertically downwards with a speed of 6.4 km h^{-1}. Find the actual speed and direction of the rain.

   (**Ans.** 8 kmh^{-1}, 53°7′33″ with the horizontal)

4. A ship is streaming towards east with a speed of 12 ms^{-1}. A woman runs across the deck at a speed of 5 ms^{-1} in the direction at right angles to the direction of motion of the ship *i.e.*, towards north. What is the velocity of the woman relative to the sea ? (**Ans.** 13 ms^{-1}, 22°37′ north of east)

5. A plane is travelling eastward at a speed of 500 km h^{-1}. But a 90 km h^{-1} wind is blowing southward. What is the direction and speed of the plane relative to the ground ?

   (**Ans.** 10.2° south of east, 508 km h^{-1})

6. A reckless drunk is playing with a gun in an airplane that is going directly east at 500 km h^{-1}. The drunk shoots the gun straight up at the ceiling of the plane. The bullet leaves the gun at a speed of 1000 km h^{-1}. Relative to an observer on earth, what angle does the bullet make with the vertical ?

   (**Ans.** 26.6°)

HINTS

1. In Fig. 4.65,

$$\vec{OA} = \vec{v}_T = 30 \text{ km h}^{-1}$$

$$\vec{OC} = \vec{v}_C = 40 \text{ km h}^{-1}$$

$$\vec{v}_{CT} = \vec{v}_C + (-\vec{v}_T) = \vec{OC} + \vec{OB} = \vec{OD}$$

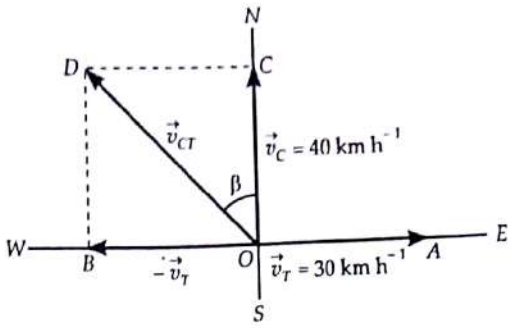

Fig. 4.65

3. The situation is shown in Fig. 4.66. Here

$\vec{OA}$ = Velocity of person
= 4.8 kmh^{-1}, due east

$\vec{OB}$ = Relative velocity of rain w.r.t. the person
= 6.4 kmh^{-1}, vertically downwards

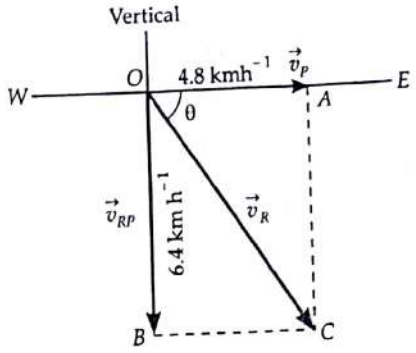

Fig. 4.66

If $\vec{v}_R$ is the actual velocity of rain, then

$$\vec{v}_{RP} = \vec{v}_R - \vec{v}_P$$

or

$$\vec{v}_R = \vec{v}_{RP} + \vec{v}_P$$

Complete the parallelogram $OACB$. Then $\vec{OC}$ represents the rain velocity $\vec{v}_R$.

$$\therefore \quad v_R = OC = \sqrt{OA^2 + AC^2}$$

$$= \sqrt{v_P^2 + v_{RP}^2} = \sqrt{4.8^2 + 6.4^2} = 8 \text{ kmh}^{-1}.$$

If $\vec{v}_R$ makes angle θ with OE, then

$$\tan \theta = \frac{AC}{OA} = \frac{v_{RP}}{v_P} = \frac{6.4}{4.8} = \frac{4}{3} = 1.333$$

$$\therefore \quad \theta = 53° \, 7'33''.$$

4. As shown in Fig. 4.67, the woman has two velocities simultaneously : (*i*) a velocity equal to the velocity of ship which is 12 ms^{-1} due east and (*ii*) a velocity of 5 ms^{-1} due north.

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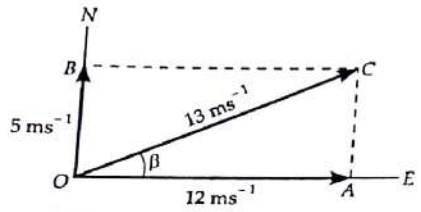

Fig. 4.67

Velocity of woman relative to the sea
= Resultant velocity of woman

$$= OC = \sqrt{OA^2 + OB^2} = \sqrt{12^2 + 5^2} = 13 \text{ ms}^{-1}.$$

If the resultant velocity makes angle β with the direction of the motion of the ship, then

$$\tan \beta = \frac{AC}{OA} = \frac{5}{12} = 0.4167$$

$$\therefore \quad \beta = 22°37', \text{ north of east}$$

4.22 PROJECTILE MOTION

49. *What is a projectile ? Give some examples of projectile motion.*

Projectile. *A projectile is the name given to any body which once thrown into space with some initial velocity, moves thereafter under the influence of gravity alone without being propelled by any engine or fuel. The path followed by a projectile is called its trajectory.*

Examples of projectile motion :

(*i*) A javelin thrown by an athlete.

(*ii*) An object dropped from an aeroplane.

(*iii*) A bullet fired from a rifle.

(*iv*) A jet of water coming out from the side hole of a vessel.

(*v*) A stone thrown horizontally from the top of a building.

50. *State the principle of physical independence of motions used in projectile motion. Give an illustration.*

Principle of physical independence of motions. In the absence of air resistance, the motion of a projectile is considered as the combination of the following two independent motions :

(*i*) Motion along horizontal direction with *uniform velocity.*

(*ii*) Motion along vertical direction under gravity *i.e., with uniform acceleration equal to g.*

The two motions of a projectile along horizontal and vertical directions are independent of each other. This is called the **principle of physical independence of motions.** For example, if a ball is dropped downward from the roof a building and simultaneously another ball is thrown in a horizontal direction, then both the balls will reach the ground at the same time but at different places. Clearly, the vertical motion is not being affected by the horizontal motion.

For Your Knowledge

▲ It was *Galileo* who first stated the principle of physical independence of the horizontal and vertical motions of a projectile in his book "*Dialogue on the great world systems*" in 1632.

51. *State the assumptions made in the study of projectile motion.*

Assumptions used in projectile motion. While studying the motion of a projectile, we use following assumptions :

(*i*) There is no air resistance on the projectile.

(*ii*) The effect due to curvature of the earth is negligible.

(*iii*) The effect due to rotation of the earth is negligible.

(*iv*) For all points of the trajectory, the acceleration due to gravity is constant both in magnitude and direction.

4.23 PROJECTILE GIVEN HORIZONTAL PROJECTION

52. *A projectile is fired horizontally with a velocity u. Show that its trajectory is a parabola. Also obtain expressions for its (i) time of flight (ii) horizontal range and (iii) velocity at any instant.*

Projectile fired parallel to horizontal. As shown in Fig. 4.68, suppose a body is projected horizontally with velocity *u* from a point *O* at a certain height *h* above the ground level. The body is under the influence of two simultaneous independent motions :

(*i*) Uniform horizontal velocity *u*.

(*ii*) Vertically downward accelerated motion with constant acceleration *g*.

Under the combined effect of the above two motions, the body moves along the path *OPA*.

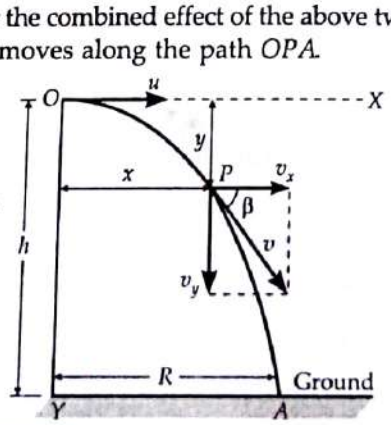

Fig. 4.68 Horizontal projection of a projectile.

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Trajectory of the projectile. After the time t, suppose the body reaches the point $P(x, y)$.

The horizontal distance covered by the body in time t is

$$x = ut \quad \therefore \quad t = \frac{x}{u}$$

The vertical distance travelled by the body in time t is given by

$$s = ut + \frac{1}{2}at^2$$

or
$$y = 0 \times t + \frac{1}{2}gt^2 = \frac{1}{2}gt^2 \qquad \text{[For vertical motion, } u = 0\text{]}$$

or
$$y = \frac{1}{2}g\left(\frac{x}{u}\right)^2 = \left(\frac{g}{2u^2}\right)x^2 \qquad \left[\because\ t = \frac{x}{u}\right]$$

or
$$y = k\,x^2 \qquad [\text{Here } k = \frac{g}{2u^2} = \text{a constant}]$$

As y is a quadratic function of x, so *the trajectory of the projectile is a parabola.*

Time of flight. *It is the total time for which the projectile remains in its flight* (from O to A). Let T be its time of flight.

For the vertical downward motion of the body, we use

$$s = ut + \frac{1}{2}at^2 \quad \text{or} \quad h = 0 \times T + \frac{1}{2}gT^2$$

or
$$T = \sqrt{\frac{2h}{g}}.$$

Horizontal range. *It is the horizontal distance covered by the projectile during its time of flight.* It is equal to $OA = R$. Thus

$$R = \text{Horizontal velocity} \times \text{time of flight} = u \times T$$

or
$$R = u\sqrt{\frac{2h}{g}}.$$

Velocity of the projectile at any instant. At the instant t (when the body is at point P), let the velocity of the projectile be v. The velocity v has two rectangular components :

Horizontal component of velocity, $v_x = u$.

Vertical component of velocity, $v_y = 0 + gt = gt$

$\therefore$ The resultant velocity at point P is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + g^2t^2}$$

If the velocity v makes an angle β with the horizontal, then

$$\tan\beta = \frac{v_y}{v_x} = \frac{gt}{u}$$

or
$$\beta = \tan^{-1}\left(\frac{gt}{u}\right).$$

Examples based on Projectile Fired Horizontally

Formulae Used

1. Position of the projectile after time t :
$$x = ut\ ,\ y = \frac{1}{2}gt^2$$
2. Equation of trajectory : $y = \frac{g}{2u^2}\cdot x^2$
3. Velocity after time t : $v = \sqrt{u^2 + g^2t^2}$
$$\beta = \tan^{-1}\frac{gt}{u}$$
4. Time of flight : $T = \sqrt{\frac{2h}{g}}$
5. Horizontal range : $R = u \times T = u\sqrt{\frac{2h}{g}}$

Units Used

Distances x, y, h and R are in metres, velocities, u and v in ms^{-1}, acceleration due to gravity g in ms^{-2}, and times t and T in second.

Example 63. A hiker stands on the edge of a cliff 490 m above the ground and throws a stone horizontally with an initial speed of $15 ms^{-1}$. Neglecting air resistance, find the time taken by the stone to reach the ground, and the speed with which it hits the ground. (Take $g = 9.8\ ms^{-2}$). **[NCERT]**

Solution. Suppose the stone is thrown from the edge of a cliff with speed $u = 15\ ms^{-1}$ along the horizontal OX. It hits the ground at point P after time t.

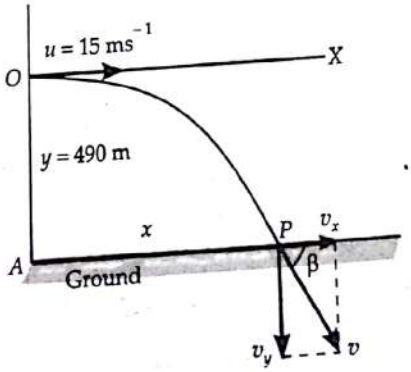

Fig. 4.69

Initial velocity in the downward direction $= 0$

Vertical distance, $OA = y = 490$ m

As $y = \frac{1}{2}gt^2 \ \therefore\ 490 = \frac{1}{2} \times 9.8\,t^2$ or $t = \sqrt{100} = \mathbf{10\ s}$.

The horizontal and vertical components of speed v of the stone at point P are

$$v_x = u = 15\ ms^{-1}$$

$$v_y = u_y + gt = 0 + 9.8 \times 10 = 98\ ms^{-1}$$

$$\therefore\quad v = \sqrt{v_x^2 + v_y^2} = \sqrt{15^2 + 98^2} = \mathbf{99.1\ ms^{-1}}.$$

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EXAMPLE 64. A projectile is fired horizontally with a velocity of 98 ms^{-1} from the top of a hill 490 m high. Find (i) the time taken to reach the ground (ii) the distance of the target from the hill and (iii) the velocity with which the projectile hits the ground.

Solution. (*i*) As shown in Fig. 4.69, the projectile is fired from the top O of a hill with velocity $u = 98\ ms^{-1}$ along the horizontal OX. It reaches the target P in time t.

Initial velocity in the downward direction $= 0$

Vertical distance,

$$OA = y = 490\ m$$

As $\quad y = \frac{1}{2} g t^2$

$\therefore \quad 490 = \frac{1}{2} \times 9.8\, t^2$

or $\quad t = \sqrt{100} = \mathbf{10\ s.}$

(*ii*) Distance of the target from the hill,

$AP = x =$ Horizontal velocity $\times$ time

$= 98 \times 10 = \mathbf{980\ m.}$

(*iii*) The horizontal and vertical components of velocity v of the projectile at point P are

$$v_x = u = 98\ ms^{-1}$$

$$v_y = u_x + gt = 0 + 9.8 \times 10 = 98\ ms^{-1}$$

$\therefore \quad v = \sqrt{v_x^2 + v_y^2}$

$$= \sqrt{98^2 + 98^2} = 98\sqrt{2} = \mathbf{138.59\ ms^{-1}.}$$

If the resultant velocity v makes angle β with the horizontal, then

$$\tan\beta = \frac{v_y}{v_x} = \frac{98}{98} = 1 \quad \therefore \quad \beta = \mathbf{45°.}$$

EXAMPLE 65. A body is thrown horizontally from the top of a tower and strikes the ground after three seconds at an angle of 45° with the horizontal. Find the height of the tower and the speed with which the body was projected. Take $g = 9.8\ ms^{-2}$.

Solution. As shown in Fig. 4.69, suppose the body is thrown horizontally from the top O of a tower of height with velocity u. The body hits the ground after 3 s. Considering vertically downward motion of the body,

$$y = u_y t + \frac{1}{2} g t^2 = 0 \times 3 + \frac{1}{2} \times 9.8 \times (3)^2 = \mathbf{44.1\ m}$$

$[\because$ Initial vertical velocity, $u_y = 0]$

Final vertical velocity,

$$v_y = u_y + gt = 0 + 9.8 \times 3 = 29.4\ ms^{-1}$$

Final horizontal velocity, $v_x = u$

As the resultant velocity v makes an angle of 45° the horizontal, so

$$\tan 45° = \frac{v_y}{v_x} \quad \text{or} \quad 1 = \frac{29.4}{u} \quad \text{or} \quad u = \mathbf{29.4\ ms^{-1}.}$$

EXAMPLE 66. A bomb is dropped from an aeroplane when it is directly above a target at a height of 1000 m. The aeroplane is moving horizontally with a speed of 500 kmh^{-1}. By how much distance will the bomb miss the target ?

Solution. As the aeroplane is moving horizontally, the initial downward velocity of the bomb, $u_y = 0$.

Also $\quad y = 1000\ m,\ g = 9.8\ ms^{-2},\ t = ?$

Now $\quad y = u_y t + \frac{1}{2} g t^2$

$\therefore \quad 1000 = 0 + \frac{1}{2} \times 9.8\, t^2$

or $\quad t = \sqrt{\frac{1000}{4.9}} = \frac{100}{7}\ s$

Horizontal velocity of the aeroplane

$$= 500\ kmh^{-1}$$

$$= 500 \times \frac{5}{18}\ ms^{-1} = \frac{1250}{9}\ ms^{-1}$$

Distance by which the bomb misses the target

= Horizontal distance covered by the bomb before it hits the ground

= Horizontal velocity $\times$ time

$$= \frac{1250}{9} \times \frac{100}{7} = \mathbf{1984.13\ m.}$$

EXAMPLE 67. A body is projected horizontally from the top of a cliff with a velocity of 9.8 ms^{-1}. What time elapses before horizontal and vertical velocities become equal ? Take $g = 9.8\ ms^{-2}$.

Solution. Horizontal velocity at any instant, $v_x = u = 9.8\ ms^{-1}$

Vertical velocity at any instant, $v_y = 0 + gt = 9.8\, t$

According to the question, $v_x = v_y$

$\therefore \quad 9.8 = 9.8\, t \quad \text{or} \quad t = \mathbf{1\ s.}$

EXAMPLE 68. A marksman wishes to hit a target just in the same level as the line of sight. How high from the target he should aim, if the distance of the target is 1600 m and the muzzle velocity of the gun is 800 ms^{-1} ? Take $g = 9.8\ ms^{-2}$.

Solution. Let u be the speed of the bullet. In time t, it covers a horizontal distance,

$$x = 1600\ m$$

But $\quad x = ut$

$\therefore \quad 1600 = 800 \times t \quad \text{or} \quad t = 2\ s$

Distance through which the bullet is pulled down by the force of gravity in 2 s is

$$y = \frac{1}{2} g t^2 = \frac{1}{2} \times 9.8 \times (2)^2 = 19.6\ m$$

$\therefore$ Height of the gun from the target = **19.6 m.**

EXAMPLE 69. Two tall building
distance of 180 m from each oth
a ball be thrown horizontally fr
ground in one building, so that i
the ground in the second build

Solution. In Fig. 4.70, A
which are 180 m apart. W_1
in A and B respectively.

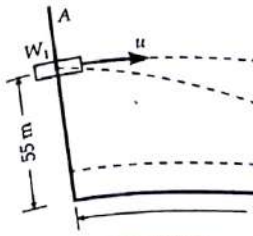

Fig. 4.70

Vertical downwar
= H
= 5
Initial vertical
$u_y =$
As $\quad y =$
$\therefore \quad 44.1 =$
or $\quad t^2$
Required ho
= Hor

EXAMPLE 70.
speed u from t
horizontal. H
particle strike

Solution

Fig. 4

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EXAMPLE 69. *Two tall buildings face each other and are at a distance of 180 m from each other. With what velocity must a ball be thrown horizontally from a window 55 m above the ground in one building, so that it enters a window 10.9 m above the ground in the second building ?* **[Central Schools 14]**

Solution. In Fig. 4.70, A and B are two tall buildings which are 180 m apart. W_1 and W_2 are the two windows in A and B respectively.

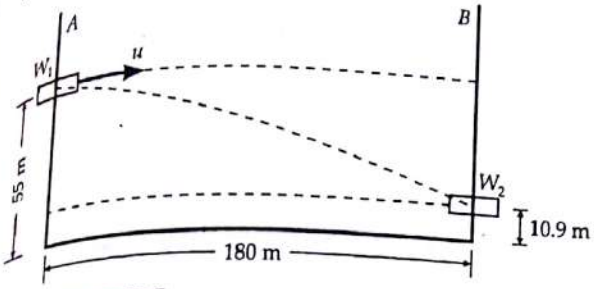

Fig. 4.70

Vertical downward distance to be covered by the ball
$= \text{Height of } W_1 - \text{Height of } W_2$
$= 55 - 10.9 = 44.1 \text{ m}$

Initial vertical velocity of ball,

$$u_y = 0$$

As $\quad y = u_y\, t + \frac{1}{2} g t^2$

$\therefore \quad 44.1 = 0 + \frac{1}{2} \times 9.8\, t^2$

or $\quad t^2 = \dfrac{44.1 \times 2}{9.8} = 9 \quad \text{or} \quad t = 3 \text{ s}$

Required horizontal velocity

$$= \frac{\text{Horizontal distance}}{\text{Time}} = \frac{180 \text{ m}}{3 \text{ s}} = 60 \text{ ms}^{-1}.$$

EXAMPLE 70. *A particle is projected horizontally with a speed u from the top of plane inclined at an angle θ with the horizontal. How far from the point of projection will the particle strike the plane ?*

Solution. The horizontal distance covered in time t,

$$x = ut \quad \text{or} \quad t = \frac{x}{u} \qquad \ldots(1)$$

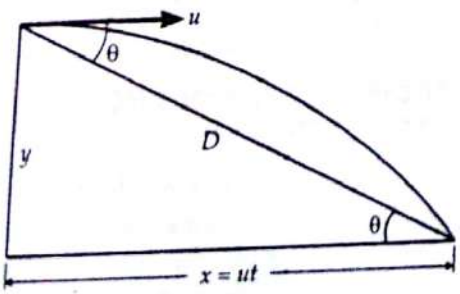

Fig. 4.71

The vertical distance covered in time t,

$$y = 0 + \frac{1}{2} g t^2 = \frac{1}{2} g \times \frac{x^2}{u^2} \qquad \text{[using (1)]}$$

Also $\quad \dfrac{y}{x} = \tan\theta \quad \text{or} \quad y = x \tan\theta$

$\therefore \quad \dfrac{g x^2}{2 u^2} = x \tan\theta$

or $\quad x\left(\dfrac{g x}{2 u^2} - \tan\theta\right) = 0$

As $x = 0$ is not possible, so $x = \dfrac{2u^2 \tan\theta}{g}$

The distance of the point of strike from the point of projection is

$$D = \sqrt{x^2 + y^2} = \sqrt{x^2 + (x \tan\theta)^2}$$
$$= x\sqrt{1 + \tan^2\theta} = x \sec\theta$$

or $\quad D = \dfrac{2u^2}{g} \tan\theta \sec\theta.$

EXAMPLE 71. *A helicopter on a flood relief mission flying horizontally with a speed u at an altitude h, has to drop a food packet for a victim standing on the ground. At what distance from the victim should the food packet be dropped ?*

Solution. In Fig. 4.72, H represents position of the helicopter and V that of the victim. For vertical motion of the packet

$$h = 0 + \frac{1}{2} g t^2 \quad \text{or} \quad t = \sqrt{\frac{2h}{g}}$$

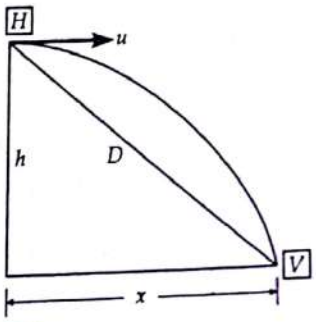

Fig. 4.72

Horizontal distance covered by the food packet in time t,

$$x = ut = u\sqrt{\frac{2h}{g}}$$

The distance of the point of projection from the food packet is

$$D = \sqrt{h^2 + x^2} = \sqrt{h^2 + \frac{2u^2 h}{g}}.$$

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✕ PROBLEMS FOR PRACTICE

1. A plane is flying horizontally at a height of 1000 m with a velocity of 100 ms^{-1} when a bomb is released from it. Find *(i)* the time taken by it to reach the ground *(ii)* the velocity with which the bomb hits the target and *(iii)* the distance of the target. **[Ans.** *(i)* 14.28 s *(ii)* 172.1 ms^{-1}, $\beta = 54°28'$ *(iii)* 1428.5 m]

2. From the top of a building 19.6 m high, a ball is projected horizontally. After how long does it strike the ground ? If the line joining the point of projection to the point where it hits the ground makes an angle of 45° with the horizontal, what is the initial velocity of the ball ? **(Ans.** 2 s, 9.8 ms^{-1})

3. A body is thrown horizontally from the top of a tower and strikes the ground after two seconds at angle of 45° with the horizontal. Find the height of the tower and the speed with which the body was thrown. Take $g = 9.8\ ms^{-2}$. **(Ans.** 19.6 m, 19.6 ms^{-1})

4. Two tall buildings are situated 200 m apart. With what speed must a ball be thrown horizontally from the window 540 m above the ground in one building, so that it will enter a window 50 m above the ground in the other ? **(Ans.** 20 ms^{-1})

5. A stone is dropped from the window of a bus moving at 60 kmh^{-1}. If the window is 1.96 m high, find the distance along the track, which the stone moves before striking the ground. **(Ans.** 10.54 m)

6. An aeroplane is flying in a horizontal direction with a velocity of 600 kmh^{-1} and at a height of 1960 m. When it is vertically above a point *A* on the ground, a body is dropped from it. The body strikes the ground at a point *B*. Calculate the distance *AB*. **(Ans.** 3333.3 m)

7. A mailbag is to be dropped into a post office from an aeroplane flying horizontally with a velocity of 270 kmh^{-1} at a height of 176.4 m above the ground. How far must the aeroplane be from the post office at the time of dropping the bag so that it directly falls into the post office ? **(Ans.** 450 m)

8. In between two hills of heights 100 m and 92 m respectively, there is a valley of breadth 16 m. If a vehicle jumps from the first hill to the second, what must be its minimum horizontal velocity so that it may not fall into the valley ? Take $g = 9\ ms^{-2}$. **(Ans.** 12 ms^{-1})

9. A ball is projected horizontally from a tower with a velocity of 4 ms^{-1}. Find the velocity of the ball after 0.7 s. Take $g = 10\ ms^{-2}$. **(Ans.** 8.06 ms^{-1}, 60°15')

✕ HINTS

1. *(i) For vertical motion :*

$$y = \frac{1}{2} g t^2 \qquad \therefore 1000 = \frac{1}{2} \times 9.8\, t^2$$

$$\text{or} \quad t^2 = \frac{10000}{49} \qquad \therefore \quad t = \frac{100}{7}\text{ s} = \mathbf{14.28\ s}$$

(ii) Velocity with which the bomb hits the target,

$$v = \sqrt{u^2 + g^2 t^2} = \sqrt{(100)^2 + \left(9.8 \times \frac{100}{7}\right)^2}$$

$$= \sqrt{100^2 + 140^2} = \mathbf{172.05\ ms^{-1}}.$$

$$\tan\beta = \frac{gt}{u} = \frac{9.8 \times 100}{100 \times 7} = 1.4$$

$$\therefore \qquad \beta = \mathbf{54°28'}.$$

(iii) Distance of the target,

$$x = ut = 1000 \times \frac{100}{7} = \mathbf{1428.51\ m}.$$

7. For vertical motion :

$$y = \frac{1}{2} g t^2 \qquad \therefore \quad 176.4 = \frac{1}{2} \times 9.8\, t^2$$

$$\text{or} \qquad t^2 = \frac{176.4}{4.9} = 36 \quad \therefore \qquad t = 6\text{ s}$$

$$\text{Also} \qquad u = 270\ kmh^{-1} = \frac{270 \times 5}{18} = 75\ ms^{-1}$$

$$\therefore \qquad x = ut = 75 \times 6 = \mathbf{450\ m}.$$

8. For vertical motion : $u = 0$,

y = Difference of heights = 8 m, $g = 9\ ms^{-2}$

$$\text{As} \quad y = \frac{1}{2} g t^2 \quad \therefore t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 8}{9}} = \frac{4}{3}\text{ s}$$

For horizontal motion :

x = Distance between the two hills = 16 m

$$\text{As} \qquad x = ut \qquad \therefore \quad u = \frac{x}{t} = \frac{16}{4/3} = \mathbf{12\ ms^{-1}}.$$

9. Here $v_x = u_x = 4\ ms^{-1}$

$$v_y = u_y + a_y t = 0 + 10 \times 0.7 = 7\ ms^{-1}$$

Resultant velocity,

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{4^2 + 7^2} = \sqrt{65} = \mathbf{8.06\ ms^{-1}}.$$

If velocity v makes angle β with the horizontal, then

$$\tan\beta = \frac{v_y}{v_x} = \frac{7}{4} = 1.75 \quad \text{or} \quad \beta = \mathbf{60°15'}.$$

4.24 ▼ PROJECTILE GIVEN ANGULAR PROJECTION

53. *A projectile is fired with a velocity u making an angle θ with the horizontal. Show that its trajectory is a parabola. Derive expressions for (i) time of maximum height (ii) time of flight (iii) maximum height (iv) horizontal range.*

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Projectile fired at an angle θ with the horizontal. As shown in Fig. 4.73, suppose a body is projected with initial velocity u, making an angle θ with the horizontal. The velocity u has two rectangular components :

(*i*) The horizontal component $u \cos\theta$, which remains constant throughout the motion.

(*ii*) The vertical component $u \sin\theta$, which changes with time under the effect of gravity. This component first decreases, becomes zero at the highest point A, after which it again increases, till the projectile hits the ground.

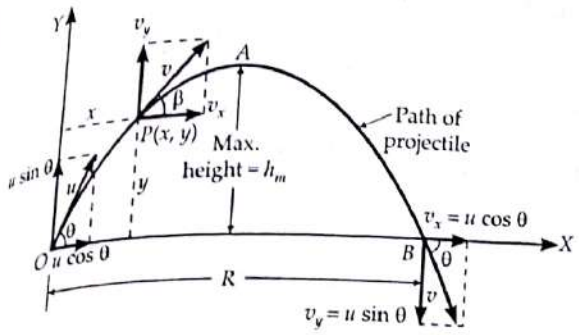

Fig. 4.73 Trajectory of a projectile fired at an angle θ with the horizontal.

Under the combined effect of the above two components, the body follows the parabolic path OAB as shown in the figure.

Equation of trajectory of a projectile. Suppose the body reaches the point $P(x, y)$ after time t.

∴ The horizontal distance covered by the body in time t,

$$x = \text{Horizontal velocity} \times \text{time} = u \cos\theta \, . \, t$$

or $$t = \frac{x}{u \cos\theta}$$

For vertical motion : $u = u \sin\theta$, $a = -g$, so the vertical distance covered in time t is given by

$$s = ut + \frac{1}{2} at^2$$

or $$y = u \sin\theta \, . \, \frac{x}{u \cos\theta} - \frac{1}{2} g \, . \, \frac{x^2}{u^2 \cos^2\theta}$$

or $$y = x \tan\theta - \frac{g}{2u^2 \cos^2\theta} x^2$$

or $$y = px - qx^2$$

where p and q are constants.

Thus y is a quadratic function of x. Hence the *trajectory of a projectile is a parabola.*

Time of maximum height. Let t_m be the time taken by the projectile to reach the maximum height h_m .

At the highest point, vertical component of velocity $= 0$

As $v = u + at$ ∴ $0 = u \sin\theta - g t_m$

or $$t_m = \frac{u \sin\theta}{g}$$

Time of flight. *It is the time taken by the projectile from the instant it is projected till it reaches a point in the horizontal plane of its projection.* The body reaches the point B after the time of flight T_f .

∴ Net vertical displacement covered during the time of flight $= 0$

As $$s = ut + \frac{1}{2} at^2$$

∴ $$0 = u \sin\theta \, . \, T_f - \frac{1}{2} g T_f^2$$

or $$T_f = \frac{2u \sin\theta}{g}$$

Obviously, $T_f = 2t_m$. This is expected because the time of ascent is equal to the time of descent for the symmetrical parabolic path.

Maximum height of a projectile. *It is the maximum vertical distance attained by the projectile above the horizontal plane of projection.* It is denoted by h_m .

At the highest point A, vertical component of velocity $= 0$

As $$v^2 - u^2 = 2as$$

∴ $$0^2 - (u \sin\theta)^2 = 2(-g) h_m$$

or $$h_m = \frac{u^2 \sin^2\theta}{2g}$$

Horizontal range (R). *It is the horizontal distance travelled by the projectile during its time of flight.* So

Horizontal range = Horizontal velocity × time of flight

or $$R = u \cos\theta \times \frac{2u \sin\theta}{g} = \frac{u^2}{g} \, . \, 2 \sin\theta \cos\theta$$

or $$R = \frac{u^2 \sin 2\theta}{g} \quad \therefore \quad [\because \; 2 \sin\theta \cos\theta = \sin 2\theta]$$

54. *A body is projected at an angle θ with the horizontal. Derive an expression for its horizontal range. Determine the condition for maximum horizontal range. Show that there are two angles of projections for the same horizontal range. Also calculate the velocity of the projectile at any instant t and at the end point of the flight.*

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Horizontal range. Refer answer to the above question.

Condition for the maximum horizontal range. The horizontal range is given by

$$R = \frac{u^2 \sin 2\theta}{g}$$

Clearly, R will be maximum when

$$\sin 2\theta = 1 = \sin 90°$$

or $$2\theta = 90° \quad \text{or} \quad \theta = 45°$$

Thus *the horizontal range of a projectile is maximum when it is projected an angle of 45° with the horizontal.*

The maximum horizontal range is given by

$$R_m = \frac{u^2 \sin 90°}{g} = \frac{u^2 \times 1}{g}$$

or $$R_m = u^2/g$$

Two angles of projection for the same horizontal range. The horizontal range of a projectile projected at an angle θ with the horizontal with velocity u is given by

$$R = \frac{u^2 \sin 2\theta}{g}$$

Replacing θ by $(90° - \theta)$, we get

$$R' = \frac{u^2 \sin 2\,(90° - \theta)}{g}$$

$$= \frac{u^2 \sin (180° - 2\theta)}{g} = \frac{u^2 \sin 2\theta}{g}$$

i.e., $$R' = R$$

Hence *for a given velocity of projection, a projectile has the same horizontal range for the angles of projection θ and $(90° - \theta)$.* As shown in Fig. 4.74, the horizontal range is maximum for 45°. Clearly, R is same for $\theta = 15°$ and 75° but less than R_m. Again R is same for $\theta = 30°$ and 60°.

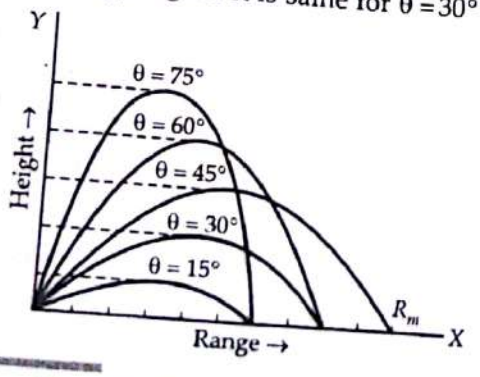

Fig. 4.74 R is same for θ and $(90° - \theta)$. R is maximum for $\theta = 45°$.

When the angle of projection is $(90° - \theta)$ with the horizontal, the angle of projection with the vertical is θ. This indicates that the *horizontal range is same whether θ is the angle of projection with the horizontal or with the vertical, as shown in Fig. 4.75.*

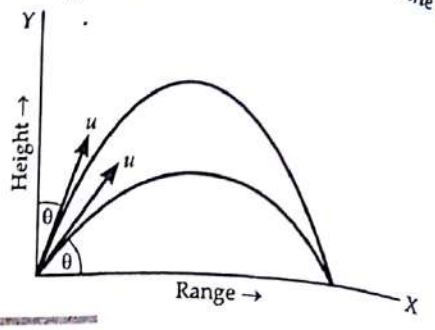

Fig. 4.75 R is same for angle of projection θ with horizontal or with vertical.

Velocity of projectile at any instant. As shown in Fig. 4.73, suppose the projectile has velocity v at the instant t when it is at point $P(x, y)$. The velocity v has two rectangular components :

Horizontal component of velocity,

$$v_x = u \cos \theta$$

Vertical component of velocity,

$$v_y = u \sin \theta - gt \qquad [\text{Using } v = u + at]$$

The resultant velocity at point P is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(u \cos \theta)^2 + (u \sin \theta - gt)^2}$$

or $$v = \sqrt{u^2 + g^2t^2 - 2ugt \sin \theta}$$

If the velocity v makes an angle β with the horizontal, then

$$\tan \beta = \frac{v_y}{v_x} = \frac{u \sin \theta - gt}{u \cos \theta}$$

Velocity of projectile at the end point. At the end of flight,

$$t = \text{total time of flight} = \frac{2u \sin \theta}{g}$$

So the resultant velocity is

$$v' = \sqrt{u^2 + g^2 \cdot \frac{4u^2 \sin^2 \theta}{g^2} - 2ug \cdot \frac{2u \sin \theta}{g} \cdot \sin \theta}$$

$$= \sqrt{u^2} = u$$

Also, $$\tan \beta = \frac{u \sin \theta - g \cdot \dfrac{2u \sin \theta}{g}}{u \cos \theta}$$

$$= -\frac{u \sin \theta}{u \cos \theta} = -\tan \theta = \tan (-\theta)$$

or $$\beta = -\theta.$$

The negative sign shows that the projectile is moving downwards. Thus *in projectile motion, a body returns to the ground at the same angle and with the same speed at which it was projected.*

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For Your Knowledge

- A body is said to be projectile if it is projected into space with some initial velocity and then it continues to move in a vertical plane such that its horizontal acceleration is zero and vertical downward acceleration is equal to g.
- In projectile motion, the horizontal motion and the vertical motion are independent of each other *i.e.*, neither motion affects the other.
- The horizontal range is maximum for $\theta = 45°$ and $R_m = u^2/g$.
- The horizontal range is same when the angle of projection is θ and $(90° - \theta)$.
- Again, the horizontal range is same for the angles of projection of $(45° + \theta)$ and $(45° - \theta)$.
- At the highest point of the parabolic path, the velocity and acceleration of a projectile are perpendicular each other.
- The velocity at the end of flight of an oblique projectile is the same in magnitude as at the beginning but the angle that it makes with the horizontal is negative of the angle of projection.
- In projectile motion, the kinetic energy is maximum at the point of projection or point of reaching the ground and is minimum at the highest point.
- There are two values of time for which the projectile is at the same height. The sum of these two times is equal to the time of flight.
- The maximum horizontal range is four times the maximum height attained by the projectile, when fired at $\theta = 45°$. Thus $h_m = R_m/4 = u^2/4g$.
- If a body is projected from a place above the surface of the earth, then for the maximum range the angle of projection should be slightly less than 45°. For javelin throw and discus throw, the athlete throws the projectile at an angle slightly less than 45° to the horizontal for achieving maximum range.
- The trajectory of a projectile is parabolic only when the acceleration of the projectile is constant and the direction of acceleration is different from the direction of projectile's initial velocity. The acceleration of a projectile thrown from the earth is equal to acceleration due to gravity (g) which remains constant if
  - (*i*) the projectile does not go to a very large height.
  - (*ii*) the range of the projectile is not very large.
  - (*iii*) the initial velocity of the projectile is not large.

  Thus the trajectory of a bullet fired from a gun will be parabolic, but not so the trajectory of a missile.
- The shape of the trajectory of the motion of an object is not determined by position alone but also depends on its initial position and initial velocity. Under the same acceleration due to gravity, the trajectory of an object can be a straight line or a parabola depending on the initial conditions.

Examples based on Projectile Fired at an Angle with the Horizontal

Formulae Used

1. For a projectile fired with velocity u at an angle θ with the horizontal : $u_x = u\cos\theta$, $u_y = u\sin\theta$, $a_x = 0$, $a_y = -g$
2. Position after time t :
$$x = (u\cos\theta)t, \quad y = (u\sin\theta)t - \frac{1}{2}gt^2$$
3. Equation of trajectory :
$$y = x\tan\theta - \frac{g}{2u^2\cos^2\theta}\cdot x^2$$
4. Maximum height : $H = \dfrac{u^2\sin^2\theta}{2g}$
5. Time of flight, $T = \dfrac{2u\sin\theta}{g}$
6. Horizontal range, $R = \dfrac{u^2\sin 2\theta}{g}$
7. Maximum horizontal range is attained at $\theta = 45°$ and its value is $R_{max} = \dfrac{u^2}{g}$
8. Velocity after time t, $v_x = u\cos\theta$, $v_y = u\sin\theta - gt$
$$\therefore \quad v = \sqrt{v_x^2 + v_y^2} \quad \text{and} \quad \tan\beta = \frac{v_y}{v_x}$$

Units Used

Distances x, y, R and R_{max} are in metres, velocities u, v_x, v_y and v are in ms^{-1}, accelerations a_x, a_y and g are in ms^{-2} and times t and T in second.

EXAMPLE 72. A cricket ball is thrown at a speed of 28 ms^{-1} in a direction 30° above the horizontal. Calculate (a) the maximum height, (b) the time taken by the ball to return to the same level, and (c) the horizontal distance from the thrower to the point where the ball returns to the same level.

[NCERT ; Central Schools 08, 12]

Solution. Here $u = 28\ ms^{-1}$, $\theta = 30°$

(*a*) Maximum height,
$$H = \frac{u^2\sin^2\theta}{2g} = \frac{28^2\sin^2 30°}{2\times 9.8} = \mathbf{10.0\ m}.$$

(*b*) The time taken by the ball to return to the same level,
$$T = \frac{2u\sin\theta}{g} = \frac{2\times 28\times\sin 30°}{9.8} = \mathbf{2.9\ s}.$$

(*c*) The distance from the thrower to the point where the ball returns to the same level,
$$R = \frac{u^2\sin 2\theta}{g} = \frac{28\times 28\times\sin 60°}{9.8} = \mathbf{69.3\ m}.$$

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EXAMPLE 73. A body is projected with a velocity of 30 ms^{-1} at an angle of 30° with the vertical. Find the maximum height, time of flight and the horizontal range.

Solution. Here $u = 30\ ms^{-1}$,

Angle of projection, $\theta = 90° - 30° = 60°$

Maximum height,

$$H = \frac{u^2 \sin^2 \theta}{2g} = \frac{30^2 \sin^2 60°}{2 \times 9.8} = \mathbf{34.44\ m}$$

Time of flight,

$$T = \frac{2\ u \sin \theta}{g} = \frac{2 \times 30 \sin 60°}{9.8} = \mathbf{5.3\ s}$$

Horizontal range,

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{30^2 \sin 120°}{9.8}$$

$$= \frac{30^2 \sin 60°}{9.8} = \mathbf{79.53\ m.}$$

EXAMPLE 74. A projectile has a range of 50 m and reaches a maximum height of 10 m. Calculate the angle at which the projectile is fired.

Solution. Here $R = 50$ m, $H = 10$ m, $\theta = ?$

Horizontal range,

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{2u^2 \sin \theta \cos \theta}{g} \quad ...(1)$$

Maximum height,

$$H = \frac{u^2 \sin^2 \theta}{2g} \quad ...(2)$$

Dividing (2) by (1), we get

$$\frac{H}{R} = \frac{u^2 \sin^2 \theta}{2g} \times \frac{g}{2u^2 \sin \theta \cos \theta} = \frac{1}{4} \tan \theta$$

or $$\tan \theta = \frac{4H}{R} = \frac{4 \times 10}{50} = 0.8$$

or $$\theta = \tan^{-1}(0.8) = \mathbf{38.66°}.$$

EXAMPLE 75. A boy stands at 39.2 m from a building and throws a ball which just passes through a window 19.6 m above the ground. Calculate the velocity of projection of the ball.

Solution. Here $H = 19.6$ m

$$R = 39.2 + 39.2 = 78.4\ \text{m}$$

As proved in the above example,

$$\frac{H}{R} = \frac{1}{4} \tan \theta$$

$$\tan \theta = \frac{4H}{R} = \frac{4 \times 19.6}{78.4} = 1$$

$$\theta = 45°$$

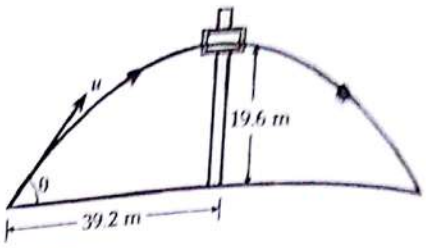

Fig. 4.76

As $$\frac{u^2 \sin 2\theta}{g} = R$$

$$\therefore \quad \frac{u^2 \sin 90°}{9.8} = 78.4$$

or $$u = \sqrt{78.4 \times 9.8} = 19.6\sqrt{2} = \mathbf{27.72\ ms^{-1}}.$$

EXAMPLE 76. Find the angle of projection for which the horizontal range and the maximum height are equal.

Solution. Given

horizontal range = maximum height

or $$\frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin^2 \theta}{2g}$$

or $$2 \sin \theta \cos \theta = \frac{\sin^2 \theta}{2}$$

or $$\frac{\sin \theta}{\cos \theta} = 4 \quad \text{or} \quad \tan \theta = 4$$

$$\therefore \quad \theta = \mathbf{75°58'}.$$

EXAMPLE 77. Prove that the maximum horizontal range is four times the maximum height attained by the projectile, when fired at an inclination so as to have maximum horizontal range. **[Chandigarh 08]**

Solution. For $\theta = 45°$, the horizontal range is maximum and is given by

$$R_{max} = \frac{u^2}{g}$$

Maximum height attained,

$$H_{max} = \frac{u^2 \sin^2 45°}{2g} = \frac{u^2}{4g} = \frac{R_{max}}{4}$$

or $$R_{max} = 4H_{max}.$$

EXAMPLE 78. A ball is kicked at an angle of 30° with the vertical. If the horizontal component of its velocity is 19.6 ms^{-1}, find the maximum height and horizontal range.

Solution. Here $\theta = 90° - 30° = 60°$

Horizontal velocity $= u \cos 60° = 19.6\ ms^{-1}$

$$\therefore \quad u = \frac{19.6}{\cos 60°} = \frac{19.6}{0.5} = 39.2\ ms^{-1}$$

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∴ Maximum height,

$$H = \frac{u^2 \sin^2 60°}{2g} = \frac{(39.2)^2}{2 \times 9.8} \times \left(\frac{\sqrt{3}}{2}\right)^2 = \mathbf{58.8\ m.}$$

Horizontal range,

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{(39.2)^2 \times \sin 120°}{9.8}$$

$$= \frac{(39.2)^2}{9.8} \times \left(\frac{\sqrt{3}}{2}\right) = \mathbf{135.8\ m}$$

EXAMPLE 79. Show that a given gun will shoot three times as high when elevated at an angle of 60° as when fired at angle of 30° but will carry the same distance on a horizontal plane.

Solution. The vertical height attained by a projectile is given by

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

When $\theta = 60°$,

$$H_1 = \frac{u^2 \sin^2 60°}{2g} = \frac{u^2}{2g}\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3u^2}{8g}$$

When $\theta = 30°$,

$$H_2 = \frac{u^2 \sin^2 30°}{2g} = \frac{u^2}{2g}\left(\frac{1}{2}\right)^2 = \frac{u^2}{8g}$$

$$\therefore \quad H_1 : H_2 = \frac{3u^2}{8g} \times \frac{8g}{u^2} = \mathbf{3:1}$$

Thus the gun will shoot three times as high when elevated at an angle of 60° as when fired at an angle of 30°.

Horizontal range of a projectile,

$$R = \frac{u^2 \sin 2\theta}{g}$$

When $\theta = 60°$, $\quad R_1 = \dfrac{u^2 \sin 120°}{g} = \dfrac{\sqrt{3}\,u^2}{2g}$

When $\theta = 30°$ $\quad R_2 = \dfrac{u^2 \sin 60°}{g} = \dfrac{\sqrt{3}\,u^2}{2g}$

Thus $R_1 = R_2$, *i.e.* the horizontal distance covered will be same in both cases.

EXAMPLE 80. A ball is thrown at an angle θ and another ball is thrown at an angle (90° − θ) with the horizontal direction from the same point with velocity 39.2 ms^{-1}. The second ball reaches 50 m higher than the first ball. Find their individual heights. Take $g = 9.8$ ms^{-2}.

Solution. *For the first ball* : Angle of projection = θ, Velocity of projection, $u = 39.2$ ms^{-1}

As maximum height,

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

$$\therefore \quad H = \frac{(39.2)^2 \sin^2 \theta}{2 \times 9.8} \qquad \ldots(1)$$

For the second ball : Angle of projection = 90° − θ, velocity of projection, $u = 39.2$ ms^{-1}, maximum height reached = $(H + 50)$ m

$$\therefore \quad H + 50 = \frac{(39.2)^2 \sin^2 (90° - \theta)}{2 \times 9.8}$$

or $$H + 50 = \frac{(39.2)^2 \cos^2 \theta}{2 \times 9.8} \qquad \ldots(2)$$

Adding (1) and (2), we get

$$2H + 50 = \frac{(39.2)^2}{2 \times 9.8}(\sin^2 \theta + \cos^2 \theta)$$

$$= \frac{(39.2)^2}{2 \times 9.8} = 78.4$$

or $\quad 2H = 78.4 - 50 = 28.4 \quad$ or $\quad H = 14.2$ m

∴ Height of first ball = H = **14.2 m**

Height of second ball

$= H + 50 = 14.2 + 50 =$ **64.2 m.**

EXAMPLE 81. If R is the horizontal range for θ inclination and h is the maximum height reached by the projectile, show that the maximum range is given by $\dfrac{R^2}{8h} + 2h$.

Solution. Horizontal range, $R = \dfrac{u^2 \sin 2\theta}{g}$

Maximum height, $h = \dfrac{u^2 \sin^2 \theta}{2g}$

$$\therefore \quad \frac{R^2}{8h} + 2h = \frac{u^4 (\sin 2\theta)^2}{g^2} \times \frac{2g}{8u^2 \sin^2 \theta} + \frac{2u^2 \sin^2 \theta}{2g}$$

$$= \frac{u^2 (2 \sin \theta \cos \theta)^2}{4g \sin^2 \theta} + \frac{u^2 \sin^2 \theta}{g}$$

$$= \frac{u^2 \cos^2 \theta}{g} + \frac{u^2 \sin^2 \theta}{g}$$

$$= \frac{u^2}{g}(\cos^2 \theta + \sin^2 \theta) = \frac{u^2}{g}.$$

or $$\frac{R^2}{8h} + 2h = R_{max}.$$

EXAMPLE 82. Show that there are two angles of projection for which the horizontal range is the same. Also show that the sum of the maximum heights for these two angles is independent of the angle of projection.

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Solution. When a projectile is fired with velocity u at an angle θ with the horizontal, its horizontal range is

$$R = \frac{u^2 \sin 2\theta}{g}$$

Replacing θ by $(90° - \theta)$, we get

$$R' = \frac{u^2 \sin 2(90° - \theta)}{g} = \frac{u^2 \sin (180° - 2\theta)}{g} = \frac{u^2 \sin 2\theta}{g}$$

i.e., $R' = R$

Hence there are two angles of projection θ and $(90° - \theta)$ for which the horizontal range R is same.

Now $$H = \frac{u^2 \sin^2 \theta}{2g} \quad ...(1)$$

and $$H' = \frac{u^2 \sin^2 (90° - \theta)}{2g} = \frac{u^2 \cos^2 \theta}{2g} \quad ...(2)$$

Adding equations (1) and (2), we get

$$H + H' = \frac{u^2}{2g} (\sin^2 \theta + \cos^2 \theta)$$

or $$H + H' = \frac{u^2}{2g}$$

Clearly, the sum of the heights for the two angles of projection is independent of the angle of projection.

EXAMPLE 83. Show that there are two values of time for which a projectile is at the same height. Also show that the sum of these two times is equal to the time of flight.

Solution. For vertically upward motion of a projectile,

$$y = u \sin \theta \, . \, t - \tfrac{1}{2} g t^2$$

or $$\tfrac{1}{2} g t^2 - u \sin \theta \, . \, t + y = 0$$

This is a quadratic equation in t. Its roots are

$$t_1 = \frac{u \sin \theta - \sqrt{u^2 \sin^2 \theta - 2gy}}{g} \quad \text{(Lower value)}$$

and $$t_2 = \frac{u \sin \theta + \sqrt{u^2 \sin^2 \theta - 2gy}}{g} \quad \text{(Higher value)}$$

These are the two values of time for which the vertical height y is same, first while going up and second while going down.

Now $$t_1 + t_2 = \frac{u \sin \theta - \sqrt{u^2 \sin^2 \theta - 2gy}}{g} + \frac{u \sin \theta + \sqrt{u^2 \sin^2 \theta - 2gy}}{g}$$

$$t_1 + t_2 = \frac{2u \sin \theta}{g} = T, \text{ the time of flight.}$$

EXAMPLE 84. Two projectiles are thrown with different velocities and at different angles so as to cover the same maximum height. Show that the sum of the times taken by each to reach the highest point is equal to the total time taken by either of the projectiles.

Solution. If the two projectiles are thrown with velocities u_1 and u_2 at angle θ_1 and θ_2 with horizontal, then their maximum heights will be

$$H_1 = \frac{u_1^2 \sin^2 \theta_1}{2g} \quad \text{and} \quad H_2 = \frac{u_2^2 \sin^2 \theta_2}{2g}$$

But $H_1 = H_2$

$$\therefore \quad \frac{u_1^2 \sin^2 \theta_1}{2g} = \frac{u_2^2 \sin^2 \theta_2}{2g}$$

or $$u_1 \sin \theta_1 = u_2 \sin \theta_2 \quad ...(1)$$

Times of flight for the two projectiles are

$$T_1 = \frac{2u_1 \sin \theta_1}{g} \quad \text{and} \quad T_2 = \frac{2u_2 \sin \theta_2}{g}$$

Making use of equation (1), we get

$$T_1 = T_2 = \frac{2u_1 \sin \theta_1}{g} = \frac{2u_2 \sin \theta_2}{g}$$

Times taken to reach the highest point in the two cases will be

$$t_1 = \frac{u_1 \sin \theta_1}{g} \quad \text{and} \quad t_2 = \frac{u_2 \sin \theta_2}{g}$$

$$\therefore \quad t_1 + t_2 = \frac{u_1 \sin \theta_1}{g} + \frac{u_2 \sin \theta_2}{g}$$

$$= \frac{2u_1 \sin \theta_1}{g} \text{ or } \frac{2u_2 \sin \theta_2}{g} \quad \text{[using (1)]}$$

or $t_1 + t_2$ = Time of flight of either projectile.

EXAMPLE 85. A hunter aims his gun and fires a bullet directly at a monkey on a tree. At the instant the bullet leaves the barrel of the gun, the monkey drops. Will the bullet hit the monkey ? Substantiate your answer with proper reasoning.

Solution. As shown in Fig. 4.77, the gun at O is directed towards the monkey at position M. Suppose the bullet leaves the barrel of the gun with velocity u at an angle θ with the horizontal. Let the bullet cross the vertical line MB at A after time t. Horizontal distance travelled,

$$OB = x = u \cos \theta \, . \, t$$

or $$t = \frac{x}{u \cos \theta} \quad ...(1)$$

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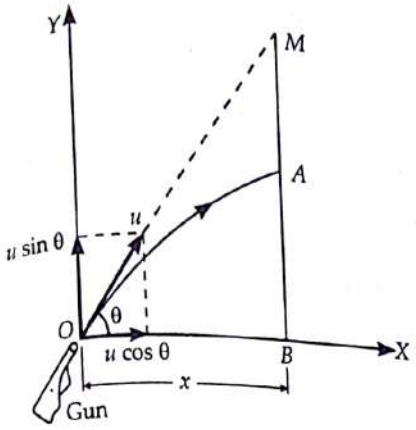

Fig. 4.77

For motion of the bullet from O to B, the vertical range is

$$AB = u \sin\theta \cdot t - \frac{1}{2} gt^2 = u \sin\theta \cdot \frac{x}{u \cos\theta} - \frac{1}{2} gt^2$$

$$= x \tan\theta - \frac{1}{2} gt^2 \qquad \text{[using (1)]}$$

Also $MB = x \tan\theta$

$\therefore$ $MA = MB - AB$

$$= x \tan\theta - \left[x \tan\theta - \frac{1}{2} gt^2\right] = \frac{1}{2} gt^2$$

Thus in a time t the bullet passes through A a vertical distance $\frac{1}{2} gt^2$ below M.

The vertical distance through which the monkey falls in time t

$$= 0 + \frac{1}{2} gt^2 = \frac{1}{2} gt^2$$

Thus the bullet and the monkey will always reach the point A at the same time. Hence the bullet will always hit the monkey whatever be the velocity of the bullet.

EXAMPLE 86. A machine gun is mounted on the top of a tower 100 m high. At what angle should the gun be inclined to cover a maximum range of firing on the ground below ? The muzzle speed of the bullet is 150 ms^{-1}, take $g = 10$ ms^{-2}.

Solution. Let u be the muzzle speed of the bullet fired from the gun (on the top of the tower) at an angle θ with the horizontal, as shown in Fig. 4.78.

Clearly, the total range of firing on the ground is

$$x = \frac{u^2 \sin 2\theta}{g} + 100 \cot\theta$$

$$\therefore \quad \frac{dx}{d\theta} = \frac{u^2 \times 2 \cos 2\theta}{g} + 100 \times (-\operatorname{cosec}^2\theta)$$

$$= \frac{2u^2}{g}(1 - 2\sin^2\theta) - \frac{100}{\sin^2\theta}$$

$$= 4500 - 9000 \sin^2\theta - \frac{100}{\sin^2\theta}$$

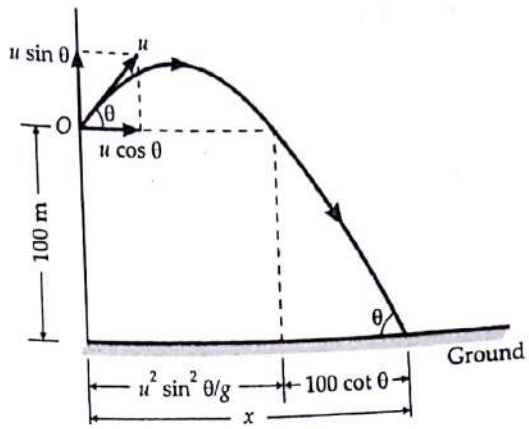

Fig. 4.78

For x to be maximum,

$$\frac{dx}{d\theta} = 0$$

or $$4500 - 9000 \sin^2\theta - \frac{100}{\sin^2\theta} = 0$$

or $$90 \sin^4\theta - 45 \sin^2\theta + 1 = 0$$

or $$\sin^2\theta = \frac{45 \pm \sqrt{(-45)^2 - 4 \times 90 \times 1}}{2 \times 90}$$

$$= \frac{45 \pm 40.80}{180}$$

Taking only positive sign,

$$\sin^2\theta = 0.4767$$

or $$\sin\theta = 0.6904$$

or $$\theta = \mathbf{43.7°}.$$

EXAMPLE 87. At what angle should a body be projected with a velocity 24 ms^{-1} just to pass over the obstacle 16 m high at a horizontal distance of 32 m ? Take $g = 10$ ms^{-2}.

Solution. As shown in Fig. 4.79, if point of projection is taken as the origin of the coordinate system, the projected body must pass through a point having coordinates (32 m, 16 m). If u be the initial velocity of the projectile and θ the angle of projection, then

Horizontal component of initial velocity, $u_x = u \cos\theta$

Vertical component of initial velocity, $u_y = u \sin\theta$

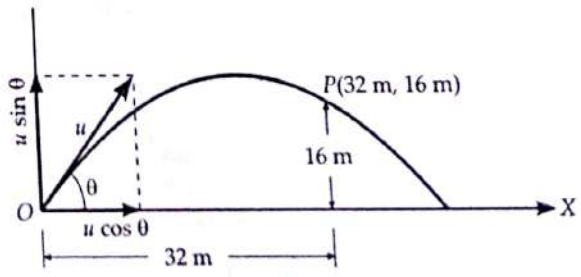

Fig. 4.79

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If the body passes through point P after time t, then horizontal distance covered,

$$x = (u \cos \theta)\, t$$

or $$32 = (24 \cos \theta)\, t \qquad ...(1)$$

Similarly, vertical distance covered,

$$y = (u \sin \theta)\, t - \frac{1}{2} g t^2$$

or $$16 = (24 \sin \theta)\, t - \frac{1}{2} \times 10 \times t^2 \qquad ...(2)$$

From equation (1), $t = \dfrac{32}{24 \cos \theta}$

Putting this value of t in equation (2), we get

$$16 = (24 \sin \theta) \frac{32}{24 \cos \theta} - \frac{1}{2} \times 10 \times \left(\frac{32}{24 \cos \theta} \right)^2$$

or $$16 = 32 \tan \theta - 5 \times \frac{16}{9 \cos^2 \theta}$$

or $$1 = 2 \tan \theta - \frac{5}{9} \sec^2 \theta$$

or $$9 = 18 \tan \theta - 5 (1 + \tan^2 \theta)$$

or $$5 \tan^2 \theta - 18 \tan \theta + 14 = 0$$

$$\therefore \quad \tan \theta = \frac{18 \pm \sqrt{(18)^2 - 4 \times 5 \times 14}}{10}$$

$$= 2.462 \text{ or } 1.137$$

Hence $\theta = 67°54'$ or $48°40'$.

EXAMPLE 88. A target is fixed on the top of a pole 13 metre high. A person standing at a distance of 50 metre from the pole is capable of projecting a stone with a velocity $10 \sqrt{g}$ ms^{-1}. If he wants to strike the target in shortest possible time, at what angle should he project the stone ?

Solution. The trajectory of a projectile is given by

$$y = x \tan \theta - \frac{g x^2}{2 u^2 \cos^2 \theta}$$

As the target lies on the path of projectile, so

$$13 = 50 \tan \theta - \frac{g \times (50)^2}{2 (10 \sqrt{g})^2} \sec^2 \theta$$

$$[\because \; u = 10 \sqrt{g} \text{ ms}^{-1}]$$

or $$13 = 50 \tan \theta - \frac{25}{2} \sec^2 \theta$$

$$= 50 \tan \theta - \frac{25}{2} (1 + \tan^2 \theta)$$

or $$25 \tan^2 \theta - 100 \tan \theta + 51 = 0$$

$$\therefore \quad \tan \theta = \frac{100 \pm \sqrt{4900}}{50} = \frac{17}{5} \text{ or } \frac{3}{5}$$

The horizontal distance covered in time t is given by

$$x = (u \cos \theta)\, t$$

$$50 = (10 \sqrt{g})(\cos \theta)\, t$$

or $$t = \frac{50}{10 \sqrt{g} \cos \theta}$$

or

For t to be minimum, $\cos \theta$ should be maximum, $\cos \theta$ is greatest if $\tan \theta$ is least, *i.e.*

$$\tan \theta = \frac{3}{5} \quad \text{or} \quad \theta = \tan^{-1} \left(\frac{3}{5} \right) = 30°58'.$$

EXAMPLE 89. A particle is projected over a triangle from one end of a horizontal base and grazing the vertex falls on the other end of the base. If α and β be the base angles and θ the angle of projection, prove that $\tan \theta = \tan \alpha + \tan \beta$.

Solution. The situation is shown in Fig. 4.80.

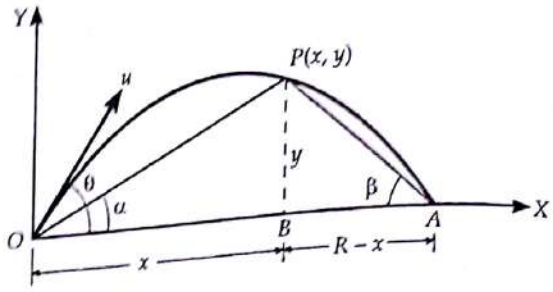

Fig. 4.80

If R is the range of the particle, then from the figure we have

$$\tan \alpha + \tan \beta = \frac{y}{x} + \frac{y}{R - x} = \frac{y (R - x) + xy}{x (R - x)}$$

or $$\tan \alpha + \tan \beta = \frac{y}{x} \times \frac{R}{(R - x)} \qquad ...(1)$$

Also, the trajectory of the particle is

$$y = x \tan \theta - \frac{1}{2} g \frac{x^2}{u^2 \cos^2 \theta}$$

$$= x \tan \theta \left[1 - \frac{g x}{2 u^2 \cos^2 \theta \tan \theta} \right]$$

$$= x \tan \theta \left[1 - \frac{g}{u^2 \sin 2\theta} x \right]$$

$$= x \tan \theta \left[1 - \frac{x}{R} \right]$$

or $$\tan \theta = \frac{y}{x} \times \frac{R}{(R - x)} \qquad ...(2)$$

From equations (1) and (2), we get

$$\tan \theta = \tan \alpha + \tan \beta.$$

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EXAMPLE 90. A shell is fired from a gun from the bottom of a hill along its slope. The slope of the hill is $\alpha = 30°$, and the angle of the barrel to the horizontal $\beta = 60°$. The initial velocity u of the shell is 21 ms^{-1}. Find the distance from the gun to the point at which the shell falls.

Solution. The situation is shown in Fig. 4.81. The horizontal and vertical distances covered by the shell in time t are given by

$$x = u \cos \beta \, . \, t$$

and

$$y = u \sin \beta \, . \, t - \frac{1}{2} g t^2 \quad ...(1)$$

...(2)

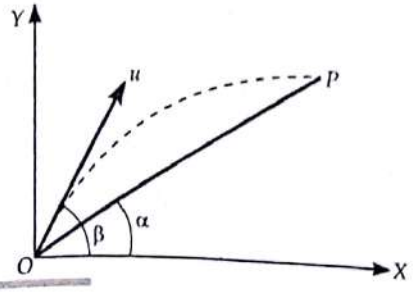

Fig. 4.81

If the shell falls on the inclined plane at distance $OP = s$, then

$$x = s \cos \alpha \quad ...(3)$$

$$y = s \sin \alpha \quad ...(4)$$

From (1) and (3),

$$s \cos \alpha = u \cos \beta \, . \, t \quad \text{or} \quad t = \frac{s \cos \alpha}{u \cos \beta}$$

From (2) and (4),

$$s \sin \alpha = u \sin \beta \, . \, t - \frac{1}{2} g t^2$$

$$= u \sin \beta \, . \, \frac{s \cos \alpha}{u \cos \beta} - \frac{1}{2} g \, . \, \frac{s^2 \cos^2 \alpha}{u^2 \cos^2 \beta}$$

$$\therefore \quad s = \frac{2u^2}{g} \left[\frac{\sin \beta \cos \alpha - \cos \beta \sin \alpha}{\cos^2 \alpha} \right] \cos \beta$$

$$= \frac{2u^2 \sin (\beta - \alpha) \cos \beta}{g \cos^2 \alpha}$$

$$= \frac{2 \times (21)^2 \sin (60° - 30°) \cos 60°}{9.8 \times \cos^2 30°} = 30 \text{ m.}$$

✕ PROBLEMS FOR PRACTICE

1. A football player kicks a ball at an angle of 37° to the horizontal with an initial speed of 15 ms^{-1}. Assuming that the ball travels in a vertical plane, calculate (*i*) the time at which the ball reaches the highest point (*ii*) the maximum height reached (*iii*) the horizontal range of the projectile and (*iv*) the time for which the ball is in air.

[**Ans.** (*i*) 0.92 s (*ii*) 4.16 m (*iii*) 21.2 m (*iv*) 1.84 s]

2. A body is projected with a velocity of 20 ms^{-1} in a direction making an angle of 60° with the horizontal. Calculate its (*i*) position after 0.5 s and (*ii*) velocity after 0.5 s. [**Ans.** (*i*) $x = 5$ m, $y = 7.43$ m (*ii*) 15.95 ms^{-1}, $\beta = 51.16°$]

3. The maximum vertical height of a projectile is 10 m. If the magnitude of the initial velocity is 28 ms^{-1}, what is the direction of the initial velocity ? Take $g = 9.8$ ms^{-2}. (**Ans.** 30°)

4. A bullet fired from a gun with a velocity of 140 ms^{-1} strikes the ground at the same level as the gun at a distance of 1 km. Find the angle of inclination with the horizontal at which the bullet is fired. Take $g = 9.8$ ms^{-2}. (**Ans.** 15°)

5. A bullet is fired at an angle of 15° with the horizontal and hits the ground 6 km away. Is it possible to hit a target 10 km away by adjusting the angle of projection assuming the initial speed to be the same ? (**Ans.** Yes)

6. A cricketer can throw a ball to maximum horizontal distance of 160 m. Calculate the maximum vertical height to which he can throw the ball. Given $g = 10$ ms^{-2}. (**Ans.** 80 m)

7. A football is kicked 20 ms^{-1} at a projection angle of 45°. A receiver on the goal line 25 metres away in the direction of the kick runs the same instant to meet the ball. What must be his speed, if he is to catch the ball before it hits the ground ? (**Ans.** 5.483 ms^{-1})

8. A bullet fired at an angle of 60° with the vertical hits the ground at a distance of 2 km. Calculate the distance at which the bullet will hit the ground when fired at an angle of 45°, assuming the speed to be the same. (**Ans.** 2.31 km)

9. A person observes a bird on a tree 39.6 m high and at a distance of 59.2 m. With what velocity the person should throw an arrow at an angle of 45° so that it may hit the bird ? (**Ans.** 41.86 ms^{-1})

10. A ball is thrown from the top of a tower with an initial velocity of 10 ms^{-1} at an angle of 30° with the horizontal. If it hits the ground at a distance of 17.3 m from the base of the tower, calculate the height of the tower. Given $g = 10$ ms^{-2}. (**Ans.** 10 m)

11. Prove that the time of flight T and the horizontal range R of a projectile are connected by the equation : $gT^2 = 2R \tan \theta$, where θ is the angle of projection.

12. Show that the range of a projectile for two angles of projection α and β is same, where $\alpha + \beta = 90°$.

13. A body is projected with velocity of 40 ms^{-1}. After 2 s it crosses a vertical pole of height 20.4 m. Calculate the angle of projection and horizontal range. (**Ans.** 30°, 141.39 m)

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14. From the top of a tower 156.8 m high, a projectile is thrown up with velocity of 39.2 ms^{-1} making an angle of 30° with the horizontal direction. Find the distance from the foot of the tower where it strikes the ground and the time taken by it to do so.

(Ans. 271.6 m, 8 s)

15. As shown in Fig. 4.82, a body is projected with velocity u_1 from the point A. At the same time

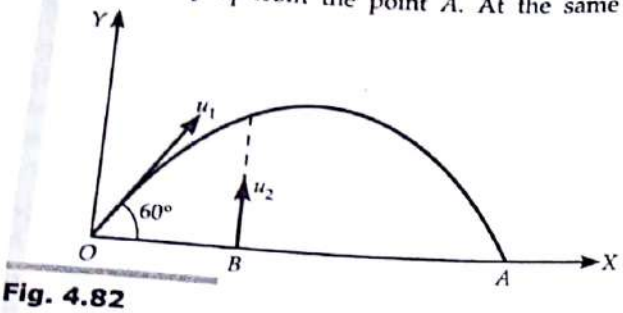

Fig. 4.82

another body is projected vertically upwards with the velocity u_2 from the point B. What should be the value of u_1 / u_2 for both the bodies to collide ?

(Ans. $2/\sqrt{3}$)

16. A body is projected such that its kinetic energy at the top is 3/4th of its initial kinetic energy. What is the initial angle of projection of the projectile with the horizontal ?

(Ans. 30°)

17. A projectile is projected in the upward direction making an angle of 60° with the horizontal direction with a velocity of 147 ms^{-1}. After what time will its inclination with the horizontal be 45° ?

(Ans. 5.49 s)

✕ HINTS

1. *(i)* Time of ascent $= \dfrac{u\sin\theta}{g} = \dfrac{15\times\sin 37°}{9.8} = \mathbf{0.92\ s}$

(ii) $H = \dfrac{u^2\sin^2\theta}{2g} = \dfrac{(15)^2\sin^2 37°}{9.8} = \mathbf{4.16\ m}$

(iii) $R = \dfrac{u^2\sin 2\theta}{g} = \dfrac{(15)^2\sin 74°}{9.8} = \mathbf{21.2\ m}$

(iv) $T = \dfrac{2u\sin\theta}{g} = \dfrac{2\times 15\sin 37°}{9.8} = \mathbf{1.84\ s.}$

2. Here $u = 20\ ms^{-1}$, $\theta = 60°$, $t = 0.5\ s$

(i) $x = (u\cos\theta)t = (20\cos 60°)\times 0.5 = \mathbf{5\ m}$

$y = (u\sin\theta)t - \frac{1}{2}gt^2$

$= (20\times\sin 60°)\times 0.5 - \frac{1}{2}\times 9.8\times(0.5)^2$

$= \mathbf{7.43\ m}$

(ii) $v_x = u\cos\theta = 20\cos 60° = 10\ ms^{-1}$

$v_y = u\sin\theta - gt = 20\sin 60° - 9.8\times 0.5$

$= 12.42\ ms^{-1}$

$\therefore\quad v = \sqrt{v_x^2 + v_y^2} = \sqrt{(10)^2 + (12.42)^2} = \mathbf{15.95\ ms^{-1}}.$

$\tan\beta = \dfrac{v_y}{v_x} = \dfrac{12.42}{10} = 1.242$

$\therefore\quad \beta = \tan^{-1} 1.242 = \mathbf{51.16°}.$

5. As $R = \dfrac{u^2\sin 2\theta}{g}$ $\therefore\quad 6 = \dfrac{u^2\sin 30°}{g}$

or $\dfrac{u^2}{g} = \dfrac{6}{\sin 30°} = 12$

Also $R_{max} = \dfrac{u^2}{g} = 12$ km

As $R_{max} > 10$ km, the bullet can hit the target at a distance of 10 km by adjusting the angle of projection.

7. Horizontal range,

$$R = \frac{u^2\sin 2\theta}{g} = \frac{(20)^2\times\sin 90°}{9.8} = 40.82\ \text{m}$$

Distance required to be covered by the receiver to meet the football $= 40.82 - 25 = 15.82$ m. This distance must be covered during the time of flight of the ball (2.886 s).

∴ Velocity with which player should run to catch the ball

$$= \frac{15.82}{2.886} = \mathbf{5.483\ ms^{-1}}.$$

8. *In the first case :* $\theta = 90° - 60° = 30°$, $R = 2$ km

As $R = \dfrac{u^2\sin 2\theta}{g}$ $\therefore\quad 2 = \dfrac{u^2\sin 60°}{g}$ ...(1)

In the second case : $\theta = 45°$

$\therefore\quad R' = \dfrac{u^2\sin 90°}{g}$ ...(2)

From (1) and (2),

$$\frac{R'}{2} = \frac{\sin 90°}{\sin 60°} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$\therefore\quad R' = \dfrac{4\times 1.732}{3} = \mathbf{2.31\ km.}$

9. For horizontal motion of the arrow :

$$u\cos 45°\,.\,t = 59.2 \text{ or } u = \frac{59.2\sqrt{2}}{t}$$

For vertical motion of the arrow :

$$39.6 = u\sin 45°\,.\,t - \frac{1}{2}\times 9.8\times t^2$$

$\therefore\quad 39.6 = \dfrac{59.2\sqrt{2}}{t}\,.\,\dfrac{1}{\sqrt{2}}\,t - \dfrac{1}{2}\times 9.8\,t^2$

or $t^2 = \dfrac{59.2 - 39.6}{4.9} = \dfrac{19.6}{4.9} = 4$ or $t = 2$ s

Hence $u = \dfrac{59.2\sqrt{2}}{2} = 29.6\sqrt{2} = \mathbf{41.86\ ms^{-1}}.$

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10. Here $\theta = 30°$, $u = 10\ ms^{-1}$, $R = 17.3\ m$, $g = 10\ ms^{-2}$

For horizontal motion,

$$R = u\cos\theta \cdot t$$

or $$t = \frac{R}{u\cos\theta} = \frac{17.3}{10\cos 30°} = \frac{17.3 \times 2}{10 \times \sqrt{3}}$$

$$= \frac{17.3 \times 2}{10 \times 1.73} = 2\ s$$

For vertical motion,

$$y = u\sin\theta \cdot t - \frac{1}{2} g t^2$$

$$= 10\sin 30° \times 2 - \frac{1}{2} \times 10 \times 2^2$$

$$= 10 - 20 = -10\ m$$

Height of tower = **10 m.**

11. As $$R = \frac{u^2 \sin 2\theta}{g} \quad \text{and} \quad T = \frac{2u\sin\theta}{g}$$

$$\therefore \quad gT^2 = \frac{g \times 4u^2 \sin^2\theta}{g^2}$$

$$= \frac{2u^2 \times 2\sin\theta\cos\theta}{g} \times \frac{\sin\theta}{\cos\theta}$$

$$= \frac{2u^2 \sin 2\theta}{g} \times \tan\theta = 2R\tan\theta.$$

13. Here $u = 40\ ms^{-1}$, $t = 2\ s$, $y = 20.4\ m$

As $$y = u\sin\theta \times t - \frac{1}{2} g t^2$$

$$\therefore \quad 20.4 = 40 \times \sin\theta \times 2 - \frac{1}{2} 9.8 \times 2^2$$

or $$20.4 + 19.6 = 80\sin\theta \quad \text{or} \quad 40 = 80\sin\theta.$$

$$\therefore \quad \sin\theta = \frac{1}{2} \text{ and } \theta = 30°$$

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{(40)^2 \sin 60°}{9.8} = \mathbf{141.39\ m.}$$

14. As $$y = u\sin\theta \times t - \frac{1}{2} g t^2$$

$$\therefore \quad -156.8 = 39.2 \times \frac{1}{2} \times t - 4.9 t^2$$

or $$t^2 - 4t - 32 = 0$$

$$\therefore \quad t = 8, -4$$

But $t = -4$ is meaningless, so $t = \mathbf{8\ s}$

Hence $$R = u\cos\theta \times t = 39.2\cos 30° \times 8$$

$$= \mathbf{271.6\ m.}$$

15. For two bodies to collide in air, $y_1 = y_2$

$$u_1 \sin 60° \times t - \frac{1}{2} g t^2 = u_2 t - \frac{1}{2} g t^2$$

$$u_1 \sin 60° \times t = u_2 t$$

or $$\frac{u_1}{u_2} = \frac{1}{\sin 60°} = \frac{2}{\sqrt{3}}.$$

16. Initial K.E. $= \frac{1}{2} mu^2$

K.E. at the top $= \frac{1}{2} mu^2 \cos^2\theta$

$$\therefore \quad \frac{\frac{1}{2} mu^2 \cos^2\theta}{\frac{1}{2} mu^2} = \frac{3}{4} \quad \text{or} \quad \cos^2\theta = \frac{3}{4}$$

or $$\cos\theta = \frac{\sqrt{3}}{2} \quad \text{or} \quad \theta = 30°.$$

17. Suppose the body is projected with velocity u at an angle of 60° with the horizontal and after time t, its velocity becomes v making an angle of 45° with the horizontal. Equating the horizontal and vertical components of u and v,

$$u\cos 60° = v\cos 45° \text{ or } v = \frac{u\cos 60°}{\cos 45°}$$

Also, $$v\sin 45° = u\sin 60° - gt$$

or $$\frac{u\cos 60°}{\cos 45°} \times \sin 45° = u\sin 60° - gt$$

or $$t = \frac{u}{g}[\sin 60° - \cos 60°] = \frac{147}{9.8}\left[\frac{\sqrt{3}}{2} - \frac{1}{2}\right]$$

$$= 5.49\ s.$$

4.25 UNIFORM CIRCULAR MOTION

55. *Define uniform circular motion. Give some examples.*

Uniform circular motion. *If a particle moves along a circular path with a constant speed (i.e., it covers equal distances along the circumference of the circle in equal intervals of time), then its motion is said to be a uniform circular motion.*

Examples :

(*i*) Motion of the tip of the second hand of a clock.

(*ii*) Motion of a point on the rim of a wheel rotating uniformly.

56. *Justify that a uniform circular motion is an accelerated motion.*

Uniform circular motion is an accelerated motion. In uniform circular motion, the speed of the body remains the same but the direction of motion changes at every point. Fig. 4.83 shows the different velocity vectors at different positions of the particle. At each position, the velocity vector $\vec{v}$ is perpendicular to the radius vector $\vec{r}$. Thus the velocity of the body changes continuously due to the continuous change in the direction of motion of the body. As the rate of change of velocity is acceleration, so a uniform circular motion is an accelerated motion.

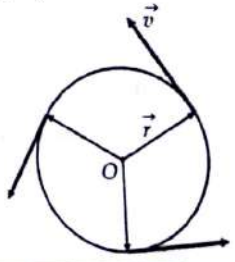

Fig. 4.83 Direction of velocity changes at every point.

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57. *Define the terms : (i) angular displacement (ii) angular velocity (iii) time period and (iv) frequency, in connection with a circular motion.*

(i) **Angular displacement.** *The angular displacement of a particle moving along a circular path is defined as the angle swept out by its radius vector in the given time interval.*

As shown in Fig. 4.84, suppose a particles starts from the position P_0. Its angular position is θ_1 at time t_1 and θ_2 at time t_2. Let the particle cover a distance Δs in the time interval $t_2 - t_1 (= \Delta t)$. It revolves through angle $\theta_2 - \theta_1 (= \Delta\theta)$ in this interval. The angle of revolution $\Delta\theta$ is the angular displacement of the particle. If r is the radius of the circle, then

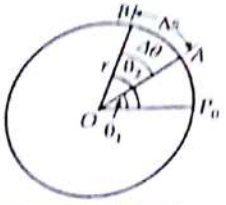

Fig. 4.84 Angular displacement.

$$\Delta\theta = \frac{\Delta s}{r} \qquad \left[\because \text{Angle} = \frac{\text{Arc}}{\text{Radius}}\right]$$

The unit of angular displacement is radian. It is a dimensionless quantity.

(ii) **Angular velocity.** *The time rate of change of angular displacement of a particle is called its angular velocity.* It is denoted by ω. It is measured in **radian per second** (rad s^{-1}) and its dimensional formula is $[M^0L^0T^{-1}]$.

As shown in Fig. 4.84, if $\Delta\theta$ is the angular displacement of a particle is time Δt, then its average angular velocity is

$$\overline{\omega} = \frac{\Delta\theta}{\Delta t}$$

When the time interval $\Delta t \to 0$, the limiting value of the average velocity is called the instantaneous angular velocity, which is given by

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

(iii) **Time period.** *The time taken by a particle to complete one revolution along its circular path is called its period of revolution.* It is denoted by T and is measured **second**.

(iv) **Frequency.** *The frequency of an object in circular motion is defined as the number of revolutions completed per unit time.* It is denoted by ν (nu).

If ν is the frequency of revolution of a particle, then time taken to complete ν revolutions = 1 second, time to complete 1 revolution $= \frac{1}{\nu}$ second

But time taken to complete 1 revolution is the time period T, so

$$T = \frac{1}{\nu} \quad \text{or} \quad \nu = \frac{1}{T}$$

58. *Deduce relations between angular velocity, frequency and time period.*

Relations between ω, ν and T. By definition of time period, a particle completes one revolution in time T *i.e.*, it traverses an angle of 2π radian in time T.

$\therefore$ When time $t = T$

angular displacement $\theta = 2\pi$ radian

$$\text{Angular velocity} = \frac{\text{Angular displacement}}{\text{Time}}$$

or $$\omega = \frac{\theta}{t} = \frac{2\pi}{T} = 2\pi\nu \qquad \left[\because \frac{1}{T} = \nu\right]$$

59. *Derive the relation between linear velocity and angular velocity.*

Relation between linear velocity and angular velocity. Consider a particle moving along a circular path of radius r. As shown in Fig. 4.85, suppose the particle moves from A to B in time Δt covering distance Δs along the arc AB. Hence the angular displacement of the particle is

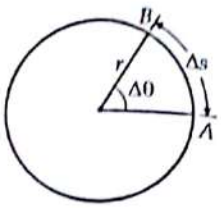

Fig. 4.85 Velocity in uniform circular motion

$$\Delta\theta = \frac{\Delta s}{r}$$

Dividing both sides by Δt, we get

$$\frac{\Delta\theta}{\Delta t} = \frac{1}{r}\frac{\Delta s}{\Delta t}$$

Taking the limit $\Delta t \to 0$ on both sides,

$$\lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{1}{r} \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$$

But $$\lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} = \omega,$$

is the instantaneous angular velocity

and $$\lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt} = v,$$

is the instantaneous linear velocity.

$\therefore$ $$\omega = \frac{1}{r} \cdot v$$

or $$v = \omega r$$

Linear velocity = Angular velocity × radius

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In vector notation, we have the relation

$$\vec{v} = \vec{\omega} \times \vec{r}$$

For a given angular velocity (ω), *the linear velocity v of a particle is directly proportional to its distance from the centre.*

60. *Define angular acceleration. Deduce its relation with linear acceleration.*

Angular acceleration. *The time rate of change of angular velocity of a particle is called its angular acceleration.* If $\Delta\omega$ is the change in angular velocity in time Δt, then the *average angular acceleration* is

$$\bar{\alpha} = \frac{\Delta\omega}{\Delta t}$$

The *instantaneous acceleration* is equal to the limiting value of the average acceleration $\Delta\omega/\Delta t$ when Δt approaches zero. It is given by

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

The angular acceleration is measured in *radian per second*^{2} ($rad\ s^{-2}$) and has the dimensions $[M^{\circ}L^{\circ}T^{-2}]$.

The relation between linear velocity v and angular velocity ω is $v = r\omega$

Differentiating both sides w.r.t. time t, we get

$$\frac{dv}{dt} = \frac{d}{dt}(r\omega)$$

or $$\frac{dv}{dt} = r\left(\frac{d\omega}{dt}\right) \qquad [\because r \text{ is constant}]$$

or $$a = \alpha r$$

Linear acceleration = Angular acceleration × radius

In vector notation, we have the relation

$$\vec{a} = \vec{\alpha} \times \vec{r}\ .$$

61. *Define centripetal acceleration. Derive an expression for the centripetal acceleration of a particle moving with uniform speed v along a circular path of radius r. Discuss the direction of this acceleration.*

Centripetal acceleration. When a body is in uniform circular motion, its speed remains constant but its velocity changes continuously due to the change in its direction. Hence the motion is accelerated. *A body undergoing uniform circular motion is acted upon by an acceleration which is directed along the radius towards the centre of the circular path. This acceleration is called centripetal (centre seeking) acceleration.*

Expression for centripetal acceleration. Consider a particle moving on a circular path of radius r and centre O, with a uniform speed v. As shown in Fig. 4.86(*a*), suppose at time t the particle is at P and at time $t + \Delta t$, the particle is at Q. Let $\vec{v}_1$ and $\vec{v}_2$ be the velocity vectors at P and Q, directed along the tangents at P and Q respectively.

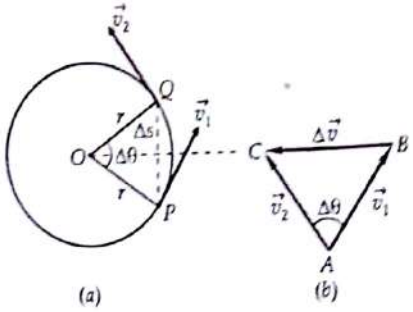

Fig. 4.86 Determination of centripetal acceleration.

To determine the change in velocity, take an external point A. Draw $\vec{AB}$ equal to and parallel to $\vec{v}_1$ and $\vec{AC}$ equal to and parallel to $\vec{v}_2$. Draw the vector $\vec{BC}$ to close the triangle, as shown in Fig. 4.86(*b*).

Applying triangle law of vector addition in $\Delta\ BAC$,

$$\vec{AB} + \vec{BC} = \vec{AC}$$

$$\therefore \qquad \vec{BC} = \vec{AC} - \vec{AB} = \vec{v}_2 - \vec{v}_1$$

Thus the change in velocity in time Δt is given by

$$\vec{BC} = \Delta\vec{v}$$

If Δt is small, the chord PQ becomes equal to arc PQ. Then OPQ can be considered as a triangle. $\angle POQ = \angle BAC = \Delta\theta$. This is because the angle between the radii PO and QO is same as the angle between the tangents at P and Q.

Also $OP = OQ = r$, radius of the circle.

$$|\vec{v}_1| = |\vec{v}_2| = v \qquad i.e.,\ AB = AC = v$$

And $\angle POQ = \Delta\theta$, $\angle BAC = \Delta\theta$

Thus the two triangles POQ and BAC are similar. Hence

$$\frac{PQ}{OP} = \frac{BC}{AB}$$

or $$\frac{\Delta s}{r} = \frac{\Delta v}{v}$$

or $$\Delta v = \frac{v}{r}\Delta s$$

Dividing both sides by Δt, we get

$$\frac{\Delta v}{\Delta t} = \frac{v}{r}\frac{\Delta s}{\Delta t}$$

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Taking the limit $\Delta t \to 0$ on both sides, we get

$$\lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{v}{r} \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$$

But $$\lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = a,$$

is the instantaneous acceleration

and $$\lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt} = v,$$

is the instantaneous velocity

$$\therefore \quad a = \frac{v}{r} \cdot v$$

or $$a = \frac{v^2}{r} = \omega^2 r \qquad [\because \ v = \omega r]$$

This gives the magnitude of the acceleration of a particle in uniform circular motion.

Direction of acceleration. As Δt tends to zero, the angle $\Delta\theta$ also approaches zero. In this limit, as $AB = AC$, so $\angle ABC = \angle ACB = 90°$. Thus the change in velocity $\Delta \vec{v}$ and hence the acceleration $\vec{a}$ is perpendicular to the velocity vector $\vec{v}_1$. But $\vec{v}_1$ is directed along tangent at point P, so acceleration $\vec{a}$ acts along the radius towards the centre of the circle. Such an acceleration is called *centripetal acceleration*. Its magnitude remains constant $(= v^2/r)$ but its direction continuously changes and remains perpendicular to the velocity vector at all positions.

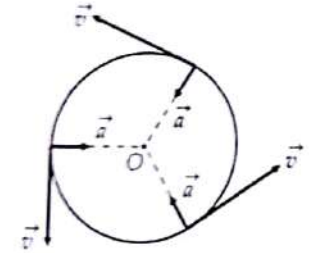

Fig. 4.87 Centripetal acceleration.

61. *Write an expression for the resultant acceleration of a particle having non-uniform circular motion.*

Circular motion with variable speed. Consider a particle moving along a circular path of radius r with a variable speed v.

As the speed of the particle changes, so acceleration has a tangential component,

$$a_T = \frac{dv}{dt} = r\alpha$$

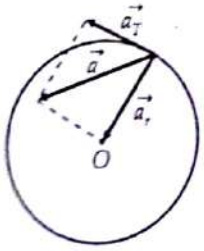

Fig. 4.88 Resultant acceleration in circular motion.

As the direction of motion changes continuously, so the acceleration has a radial component,

$$a_r \quad \text{or} \quad a_c = \frac{v^2}{r}$$

The resultant acceleration of the particle will be

$$a = \sqrt{a_T^2 + a_r^2}\,.$$

For Your Knowledge

- It was the Dutch scientist *Christiaan Huygens* who first gave a thorough analysis of centripetal acceleration in 1673.
- In uniform circular motion, the direction of velocity vector which acts along the tangent to the path, changes continuously but its magnitude always remains constant $(v = r\omega)$. So circular motion is an accelerated motion.
- As v and r are constant, so the magnitude of the centripetal acceleration is a constant $(= v^2/r)$. But the direction of a_c changes continuously, pointing always towards the centre. So centripetal acceleration is not a constant vector.
- The resultant acceleration of an object in circular motion is towards the centre only if the speed is constant.
- For a body moving with a constant angular velocity, the angular acceleration is zero.
- In projectile motion, both the magnitude and direction of acceleration (g) remain constant, while in uniform circular motion the magnitude remains constant but the direction continuously changes. Hence the equations of motion $v = u + at$, etc., are not applicable to circular motion. These equations hold only when both the magnitude and direction of acceleration are constant.

Examples based on Uniform Circular Motion

Formulae Used

1. Angular displacement, $\theta = \frac{s}{r}$
2. Angular velocity, $\omega = \frac{\theta}{t}$
3. Also, $\omega = \frac{2\pi}{T} = 2\pi\nu$
4. Linear velocity, $v = r\,\omega$
5. Centripetal acceleration, $a = \frac{v^2}{r} = r\,\omega^2$
6. Linear acceleration, $a = r\,\alpha$

Units Used

Here θ is in radian, ω in rad s^{-1}, v in ms^{-1}, a in ms^{-2} and angular acceleration α in rad s^{-2}.

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EXAMPLE 91. Which is greater : the angular velocity of hour hand of a watch or angular velocity of earth around its own axis ? Give their ratio. **[Delhi 12]**

Solution. The hour hand completes one rotation in 12 hours. Its angular speed is

$$\therefore\ \omega_1 = \frac{\theta}{t} = \frac{2\pi \text{ rad}}{12 \text{ h}} = \frac{2\pi \text{ rad}}{12 \times 60 \times 60 \text{ s}} = \frac{\pi}{21600} \textbf{ rad s}^{-1}.$$

The earth completes one rotation about its axis in 24 hours. Its angular speed is

$$\therefore\ \omega_2 = \frac{\theta}{t} = \frac{2\pi \text{ rad}}{24 \text{ h}} = \frac{2\pi \text{ rad}}{24 \times 60 \times 60} = \frac{\pi}{43200} \textbf{ rad s}^{-1}.$$

Clearly, $\omega_1 > \omega_2$ and $\omega_1 : \omega_2 = 2 : 1$

EXAMPLE 92. Calculate the angular speed of flywheel making 420 revolutions per minute.

Solution. Here $\nu = 420$ revolutions/min

$$= \frac{420}{60} \text{ revolutions/s}$$

$$\omega = 2\pi\nu = 2 \times \frac{22}{7} \times \frac{420}{60} = \textbf{44 rad s}^{-1}.$$

EXAMPLE 93. A body of mass 10 kg revolves in a circle of diameter 0.40 m making 1000 revolutions per minute. Calculate its linear velocity and centripetal acceleration.

Solution. Here $m = 10$ kg, $r = 0.20$ m, $\nu = \frac{1000}{60}$ s^{-1}

Angular speed,

$$\omega = 2\pi\nu = 2\pi \times \frac{1000}{60} = \frac{100\pi}{3} \text{ rad s}^{-1}$$

Linear velocity,

$$v = r\omega = 0.20 \times \frac{100\pi}{3} = \frac{20\pi}{3} \textbf{ ms}^{-1}.$$

Centripetal acceleration,

$$a = r\omega^2 = 0.20 \times \left(\frac{100\pi}{3}\right)^2 = \frac{2000\pi^2}{9} \textbf{ ms}^{-2}.$$

Example 94. Find the magnitude of the centripetal acceleration of a particle on the tip of a fan blade, 0.30 metre in diameter, rotating at 1200 rev/minute. **[BIT Ranchi 97]**

Solution. Here $\nu = \frac{1200}{60} = 20$ rps

$$\omega = 2\pi\nu = 2\pi \times 20 = 40\pi \text{ rad s}^{-1}$$

$$a_c = r\omega^2 = 0.15 \times (40\pi)^2 = \textbf{2368.8 ms}^{-2}.$$

EXAMPLE 95. An insect trapped in a circular groove of radius 12 cm moves along the groove steadily and completes 7 revolutions in 100 s. (i) What is the angular speed and the linear speed of the motion ? (ii) Is the acceleration vector a constant vector ? What is the magnitude ? (iii) What is its linear displacement ? **[NCERT, Central Schools 03]**

Solution. Here $r = 12$ cm, $\nu = \frac{7}{100}$ s^{-1}

(*i*) Angular speed,

$$\omega = 2\pi\nu = 2 \times \frac{22}{7} \times \frac{7}{100} = \textbf{0.44 rads}^{-1}.$$

Linear speed,

$$v = r\omega = 12 \times 0.44 = \textbf{5.28 cms}^{-1}.$$

(*ii*) Acceleration is always directed towards the centre of the circular groove. As the insect moves, the direction of the acceleration vector changes. So acceleration vector is not a constant vector.

Magnitude of acceleration,

$$a = r\omega^2 = 12 \times (0.44)^2 = \textbf{2.32 cms}^{-2}.$$

(*iii*) After completing 7 revolutions, the insect comes back to its initial position. So its linear displacement is zero.

EXAMPLE 96. The radius of the earth's orbit around the sun is 1.5×10^{11} m. Calculate the angular and linear velocity of the earth. Through how much angle does the earth revolve in 2 days ?

Solution. Here $r = 1.5 \times 10^{11}$ m,

Period of revolution of the earth,

$$T = 365 \text{ days} = 365 \times 24 \times 60 \times 60 \text{ s}$$

$\therefore$ Angular velocity,

$$\omega = \frac{2\pi}{T} = \frac{2 \times 3.14}{365 \times 24 \times 60 \times 60}$$

$$= \textbf{1.99} \times \textbf{10}^{-7} \textbf{ rad s}^{-1}$$

Linear velocity,

$$v = r\omega = 1.5 \times 10^{11} \times 1.99 \times 10^{-7} = \textbf{2.99} \times \textbf{10}^{4} \textbf{ ms}^{-1}$$

In 365 days, the earth revolves through an angle of 2π radians.

$\therefore$ Angle through which the earth revolves in 2 days

$$= \frac{2\pi}{365} \times 2 = \frac{2 \times 3.14 \times 2}{365} = \textbf{0.0344 rad.}$$

EXAMPLE 97. A particle moves in a circle of radius 4.0 cm clockwise at constant speed of 2 cms^{-1}. If $\hat{x}$ and $\hat{y}$ are unit acceleration vectors along X-axis and Y-axis respectively (in cms^{-2}), find the acceleration of the particle at the instant half way between P and Q. Refer to Fig. 4.89(a).

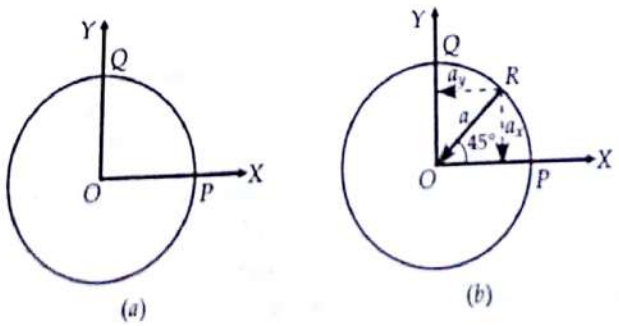

Fig. 4.89

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Solution. As shown in Fig. 4.89(*b*), let R be the midpoint of arc PQ. Then $\angle POR = 45°$.

Magnitude of acceleration at R,

$$a = \frac{v^2}{r} = \frac{(2)^2}{4} = 1 \text{ cms}^{-2}$$

The acceleration a acts along RO.

Magnitude of component of a along X-axis,

$$a_x = a \cos 45° = 1 \times \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \text{ cms}^{-2}.$$

$$\therefore \quad \hat{a}_x = -\frac{1}{\sqrt{2}} \hat{x}$$

Magnitude of component of a along Y-axis,

$$a_y = 1 \times \sin 45° = \frac{1}{\sqrt{2}} \text{ cms}^{-2}.$$

$$\therefore \quad \hat{a}_y = -\frac{1}{\sqrt{2}} \hat{y}$$

Hence $\hat{a} = \hat{a}_x + \hat{a}_y = -\frac{1}{\sqrt{2}} (\hat{x} + \hat{y})$.

PROBLEMS FOR PRACTICE

1. What is the angular velocity of a second hand and minute hand of a clock ? **[Himachal 06C]**

(**Ans.** 0.1047 rad s^{-1}, 0.0017 rad s^{-1})

2. A body of mass 0.4 kg is whirled in a horizontal circle of radius 2 m with a constant speed of 10 ms^{-1}. Calculate its (*i*) angular speed (*ii*) frequency of revolution (*iii*) time period and (*iv*) centripetal acceleration. [**Ans.** (*i*) 5 rad s^{-1} (*ii*) 0.8 Hz (*iii*) 1.25 s (*iv*) 50 ms^{-2}]

3. A circular wheel of 0.50 m radius is moving with a speed of 10 ms^{-1}. Find the angular speed.

(**Ans.** 20 rads^{-1})

4. Assuming that the moon completes one revolution in a circular orbit around the earth in 27.3 days, calculate the acceleration of the moon towards the earth. The radius of the circular orbit can be taken as 3.85×10^5 km. (**Ans.** 2.73×10^{-3} ms^{-2})

5. The angular velocity of a particle moving along a circle of radius 50 cm is increased in 5 minutes from 100 revolutions per minute to 400 revolutions per minute. Find (*i*) angular acceleration and (*ii*) linear acceleration. $\left[\text{**Ans.** } (i) \frac{\pi}{30} \text{ rad s}^{-2} \ (ii) \frac{5\pi}{3} \text{ cms}^{-2}\right]$

6. Calculate the linear acceleration of a particle moving in a circle of radius 0.4 m at the instant when its angular velocity is 2 rad s^{-1} and its angular acceleration is 5 rad s^{-2}.

(**Ans.** 2.6 ms^{-2}, 38°40′ with a_T)

7. A threaded rod with 12 turns per cm and diameter 1.18 cm is mounted horizontally. A bar with a threaded hole to match the rod is screwed onto the rod. The bar spins at the rate of 216 rpm. How long will it take for the bar to move 1.50 cm along the rod ? (**Ans.** 5 s)

HINTS

1. The second hand of a clock completes one rotation in 60 s.

$t = 60$ s and $\theta = 2\pi$ rad

Hence $\omega = \frac{\theta}{t} = \frac{2\pi}{60} = \frac{2 \times 3.14}{60} = 0.1047$ rad s^{-1}.

(*ii*) The minute hand of a clock completes one rotation in 60 minutes.

$\therefore \quad t = 60 \text{ min} = 3600 \text{ s}$ and $\theta = 2\pi$ rad

Hence $\omega = \frac{\theta}{t} = \frac{2\pi}{3600} = \frac{2 \times 3.14}{3600} = 0.0017$ rad s^{-1}.

2. Here $m = 0.4$ kg, $r = 2$ m, $v = 10$ ms^{-1}

(*i*) Angular speed, $\omega = \frac{v}{r} = \frac{10}{2} = 5$ rad s^{-1}.

(*ii*) Frequency, $\nu = \frac{\omega}{2\pi} = \frac{5 \times 7}{2 \times 22} = 0.795$ Hz

$= 0.8$ Hz.

(*iii*) Time period, $T = \frac{1}{\nu} = \frac{1}{0.8} = 1.25$ s.

(*iv*) Centripetal acceleration,

$$a = r\omega^2 = 2 \times (5)^2 = 50 \text{ ms}^{-2}.$$

3. Here $r = 0.50$ m, $v = 10$ ms^{-1}

As $v = r\omega$

$$\therefore \quad \omega = \frac{v}{r} = \frac{10 \text{ ms}^{-1}}{0.50 \text{ m}} = 20.0 \text{ rad s}^{-1}.$$

4. $$a = r\omega^2 = r\left(\frac{2\pi}{T}\right)^2$$

$$= 3.85 \times 10^8 \times \left[\frac{2 \times 3.14}{27.3 \times 24 \times 60 \times 60}\right]^2$$

$$= 2.73 \times 10^{-3} \text{ ms}^{-2}.$$

5. Here $\nu_1 = \frac{100}{60}$ rps, $\nu_2 = \frac{400}{60}$ rps, $r = 50$ cm,

$t = 5 \text{ min} = 300 \text{ s}$

(*i*) Angular acceleration,

$$\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{2\pi\nu_2 - 2\pi\nu_1}{t} = 2\pi\left(\frac{\nu_2 - \nu_1}{t}\right)$$

$$= 2\pi\left(\frac{400 - 100}{60 \times 300}\right) = \frac{\pi}{30} \text{ rad s}^{-1}.$$

(*ii*) Linear acceleration,

$$a = r\alpha = 50 \times \frac{\pi}{30} = \frac{5\pi}{3} \text{ cms}^{-2}.$$

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6. Here $r = 0.4$ m, $\omega = 2$ rad s^{-1}, $\alpha = 5$ rad s^{-2}

Tangential acceleration,

$$a_T = r\alpha = 0.4 \times 5 = 2 \text{ ms}^{-2}$$

Centrepetal acceleration,

$$a_C = r\omega^2 = 0.4 \times (2)^2 = 1.6 \text{ ms}^{-2}$$

$\therefore$ Total linear acceleration,

$$a = \sqrt{a_T^2 + a_C^2} = \sqrt{2^2 + (1.6)^2} = 2.6 \text{ ms}^{-2}.$$

If $\vec{a}$ makes angle θ with a_T, then

$$\tan\theta = \frac{a_C}{a_T} = \frac{1.6}{2} = 0.8$$

$\therefore$ $\theta = 38°40'$.

7. Pitch of threaded screw $= \frac{1}{12}$ cm

Number of rotations required to move a distance of 1.5 cm,

$$n = \frac{\text{Distance}}{\text{Pitch}} = \frac{1.5}{1/12} = 18$$

$\therefore$ $\theta = 2\pi n = 2\pi \times 18 = 36\pi$ rad

Angular speed of the bar,

$$\omega = 2\pi\nu = 2\pi \times \frac{216}{60} = 7.2\pi \text{ rad s}^{-1}$$

$\therefore$ Required time,

$$t = \frac{\theta}{\omega} = \frac{36\pi}{7.2\pi} = 5 \text{ s}.$$

Very Short Answer Coceptual Problems

Problem 1. Why cannot be vectors added algebraically ?

Solution. Apart from magnitude, the vectors also have directions, so they cannot be added algebraically.

Problem 2. State the essential condition for the addition of vectors.

Solution. The essential condition for the addition of vectors is that they must represent the physical quantities of same nature.

Problem 3. Is time a vector quantity ? Give reason.

Solution. No, time flows from past to present and present to future. Thus the direction of time flow is unique and does not need to be specified. Hence time is not a vector, though it has a direction.

Problem 4. Is pressure a vector ? Give reason.

Solution. No. Pressure is always taken to be normal to the plane of the area on which it is acting. As this direction is unique, it does need any specification. So pressure is not a vector.

Problem 5. Can two vectors of different magnitudes be combined to give zero resultant ?

Solution. No, two vectors of different magnitudes cannot be combined to give zero resultant.

Problem 6. When is the magnitude of the resultant of two vectors equal to either of them ?

Solution. When two vectors of equal magnitude are inclined to each other at an angle of 120°, the magnitude of their resultant is equal to that of the either vector.

$$a^2 = a^2 + a^2 + 2a^2\cos^2\theta$$

$$\Rightarrow \quad \cos\theta = -\frac{1}{2} \quad \Rightarrow \quad \theta = 120°$$

Problem 7. Are the magnitude and direction of $(\vec{A} - \vec{B})$ same as that of $(\vec{B} - \vec{A})$? *[Delhi 10]*

Solution. The vectors $(\vec{A} - \vec{B})$ and $(\vec{B} - \vec{A})$ have equal magnitude but opposite directions.

Problem 8. Can $\vec{A} + \vec{B} = \vec{A} - \vec{B}$?

Solution. Yes. The equality holds when $\vec{B}$ is a null vector.

Problem 9. When is the magnitude of $(\vec{A} + \vec{B})$ equal to the magnitude of $(\vec{A} - \vec{B})$?

Solution. When the vectors $\vec{A}$ and $\vec{B}$ are perpendicular to each other, $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$.

Problem 10. Under what condition will the directions of sum and difference of two vectors be same ?

Solution. When the two vectors are unequal in magnitude and are in the same direction.

Problem 11. If $\vec{A} + \vec{B} = \vec{B} + \vec{A}$, what can you say about the angle between $\vec{A}$ and $\vec{B}$?

Solution. $\vec{A}$ and $\vec{B}$ can have any angle between them because commutative law holds good for any two coplanar vectors.

Problem 12. Can we add a velocity vector to a displacement vector ?

Solution. No, only vectors representing physical quantities of same nature can be added together.

Problem 13. What is the minimum number of coplanar vectors of different magnitudes which can give zero resultant ?

Solution. Three. If three vectors can be represented by the three sides of a triangle taken in the same order, then their resultant is a zero vector.

Problem 14. If $\vec{a} + \vec{b} = \vec{c}$ and $|\vec{a}| + |\vec{b}| = |\vec{c}|$, what can we say about the direction of these vectors ?

Solution. The three vectors have the same direction.

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Problem 15. What is the resultant of vector $\vec{A}$ multiplied by real number m ?

Solution. The resultant vector $m\vec{A}$ has magnitude m times that of $\vec{A}$. It has same direction as that of $\vec{A}$ if m is positive. It has direction opposite to that of $\vec{A}$ if m is negative.

Problem 16. Can a vector be multiplied by both dimensional and non-dimensional scalars ?

Solution. Yes. When a vector is multiplied by a dimensional scalar, the resultant has different dimensions. When the vector is multiplied by a non-dimensional scalar, its dimensions remain unchanged.

Problem 17. What is the maximum number of components into which a vector can be resolved ?

Solution. Infinite.

Problem 18. Can the magnitude of the rectangular component of a vector be greater than the magnitude of that vector ?

Solution. No. For example, the rectangular components of vector $\vec{A}$ are $A_x = A\cos\theta$ and $A_y = A\sin\theta$. As both $\sin\theta$ and $\cos\theta$ can take values between -1 and $+1$, so the magnitudes of both A_x and A_y cannot be greater than A.

Problem 19. Can a vector be zero when one of the components is not zero while all the other components are zero ?

Solution. No. Any vector $\vec{A}$ in three dimensions can be written as

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

where $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$.

Clearly, if any of the components A_x, A_y or A_z is not zero, the vector $\vec{A}$ will not be a zero vector.

Problem 20. Can the increment Δa in the magnitude of vector $\vec{a}$ be greater than the modulus of the increment of the vector, that is, $|\Delta\vec{a}|$? Can they be equal ?

Solution. No, the increment Δa cannot be greater than the increment $|\Delta\vec{a}|$, as shown in Fig. 4.90. The two will be equal if $\vec{a}$ and $\Delta\vec{a}$ have the same direction.

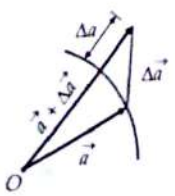

Fig. 4.90

Problem 21. The direction of vector $\vec{a}$ is reversed. Find $\Delta\vec{a}$, $|\Delta\vec{a}|$ and Δa.

Solution. $\Delta\vec{a} = -\vec{a} - \vec{a} = -2\vec{a}$

$$|\Delta\vec{a}| = 2a$$

Problem 22. If $\vec{a}$ and $\Delta\vec{a}$ are directed opposite to each other, what is the relation between Δa and $|\Delta\vec{a}|$?

Solution. $\Delta a = -|\Delta\vec{a}|$.

Problem 23. A vector $\vec{a}$ is turned through a small angle $d\theta$ without a change in its length. What are $|\Delta\vec{a}|$ and Δa ?

Solution. $|\Delta\vec{a}| = a\,d\theta$ and $\Delta a = 0$.

Problem 24. Give two conditions necessary for a given quantity to be a vector.

Solution. (*i*) The quantity must have both magnitude and direction.

(*ii*) It must obey the laws of vector addition.

Problem 25. Is finite rotation a vector ?

Solution. No. This is because the addition of two finite rotations about different axes does not obey commutative law.

Problem 26. Can the scalar product of two vectors be negative ?

Solution. Yes, the scalar product is negative when the angle between two vectors lies between 90° and 270°.

Problem 27. What is the dot product of two perpendicular vectors $\vec{A}$ and $\vec{B}$? *[Himachal 01, 02, 03, 04]*

Solution. Zero, as $\vec{A}\cdot\vec{B} = AB\cos 90° = 0$.

Problem 28. What is the dot product of two similar unit vectors ?

Solution. Unity. For example, $\hat{i}\cdot\hat{i} = (1)(1)\cos 0° = 1$.

Problem 29. What is the dot product of two dissimilar unit vectors ?

Or

Calculate the value of $\hat{i}\cdot\hat{j}$. *[Himachal 02]*

Solution. Zero.

For example, $\hat{i}\cdot\hat{j} = (1)(1)\cos 90° = 0$.

Problem 30. If $\vec{A}\cdot\vec{B} = \vec{A}\cdot\vec{C}$, is it correct to conclude that $\vec{B} = \vec{C}$?

Solution. No. We can write $\vec{A}\cdot(\vec{B} - \vec{C}) = 0$ which implies that $\vec{A}$ may be perpendicular to $(\vec{B} - \vec{C})$.

Problem 31. If $\vec{A}$, $\vec{B}$ and $\vec{A}\cdot\vec{B} = 0$ and $\vec{B}\cdot\vec{C} = 0$, then

Solution. $\vec{A}\cdot\vec{B} = 0$ =

$\vec{B}\cdot\vec{C} = 0$ =

$\therefore$ $\vec{A} \| \vec{C}$ and $\vec{A}\cdot\vec{C}$ =

Problem 32. What is th $\hat{i} + \hat{j}$?

Solution. $|\hat{i} + \hat{j}| = \sqrt{1^2}$

If the vector $\hat{i} + \hat{j}$ mak

$$\tan\beta = \frac{\text{coeff.}}{\text{coeff.}}$$

$\therefore$ $\beta = 45°$.

Problem 33. What $\vec{A} = 2\hat{i} + 2\hat{j}$ with x-axis

Solution. The angle θ

$$\cos\theta = \frac{\vec{A}\cdot\hat{i}}{|\vec{A}||\hat{i}|} = \frac{(2\hat{i}}{|2\hat{i}}$$

or $\cos\theta = \frac{1}{\sqrt{2}}$

$\therefore$ $\theta = 45°$.

Problem 34. What vectors $\vec{A}$ and $\vec{B}$ for t

Solution. $R = \sqrt{A}$

R will be maximu

$R_{max} = A$

Problem 35. Wha vectors $\vec{A}$ and $\vec{B}$ for

Solution. $R = \sqrt{}$

R will be minim

Problem 36. W the resultant of tw them is increased

Solution. $R =$

As the angle cos θ decreases, s decreases.

Problem 37. string in such a

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Problem 31. If $\vec{A}$, $\vec{B}$ and $\vec{C}$ are non-zero vectors and $\vec{A}\cdot\vec{B}=0$ and $\vec{B}\cdot\vec{C}=0$, then find out the value of $\vec{A}\cdot\vec{C}$.

Solution. $\vec{A}\cdot\vec{B}=0 \Rightarrow \vec{A}\perp\vec{B}$

$\vec{B}\cdot\vec{C}=0 \Rightarrow \vec{B}\perp\vec{C}$

$\therefore \vec{A}\parallel\vec{C}$ and $\vec{A}\cdot\vec{C}=AC\cos 0^\circ=AC$.

Problem 32. What is the magnitude and direction of $\hat{i}+\hat{j}$?

Solution. $|\hat{i}+\hat{j}|=\sqrt{1^2+1^2}=\sqrt{2}$

If the vector $\hat{i}+\hat{j}$ makes angle β with x-axis, then

$$\tan\beta=\frac{\text{coeff. of }\hat{i}}{\text{coeff. of }\hat{j}}=\frac{1}{1}=1$$

$\therefore \beta=45^\circ$.

Problem 33. What is the angle made by vector $\vec{A}=2\hat{i}+2\hat{j}$ with x-axis?

Solution. The angle θ between $\vec{A}$ and x-axis is given by

$$\cos\theta=\frac{\vec{A}\cdot\hat{i}}{|\vec{A}||\hat{i}|}=\frac{(2\hat{i}+2\hat{j})\cdot\hat{i}}{|2\hat{i}+2\hat{j}||\hat{i}|}=\frac{2\times1+2\times0}{\sqrt{2^2+2^2}\sqrt{1^2}}=\frac{2}{2\sqrt{2}}$$

or $$\cos\theta=\frac{1}{\sqrt{2}}$$

$\therefore \theta=45^\circ$.

Problem 34. What should be the angle θ between two vectors $\vec{A}$ and $\vec{B}$ for their resultant $\vec{R}$ to be maximum?

Solution. $R=\sqrt{A^2+B^2+2AB\cos\theta}$

R will be maximum when $\cos\theta=+1$ or $\theta=0^\circ$.

$$R_{max}=A+B.$$

Problem 35. What should be the angle θ between two vectors $\vec{A}$ and $\vec{B}$ for their resultant $\vec{R}$ to be minimum?

Solution. $R=\sqrt{A^2+B^2+2AB\cos\theta}$

R will be minimum when $\cos\theta=-1$ or $\theta=180^\circ$.

$$R_{min}=A-B.$$

Problem 36. What is the effect on the magnitude of the resultant of two vectors when the angle θ between them is increased from 0° to 180°?

Solution. $R=\sqrt{A^2+B^2+2AB\cos\theta}$

As the angle θ increases from 0° to 180°, the value of $\cos\theta$ decreases, so the magnitude R of the resultant also decreases.

Problem 37. Two persons are pulling the ends of a string in such a way that the string is stretched horizontally. When a weight of 10 kg is suspended in the middle of the string, the string does not remain horizontal. Can the persons make it horizontal again by putting it with a greater force?

Solution. No, the vertical weight cannot be balanced by the horizontal force, however large the two forces may be.

Problem 38. What is the vector sum of n coplanar forces, each of magnitude F, if each force makes an angle of $2\pi/n$ with the preceding force?

Solution. Total angle between n coplanar forces $=(2\pi/n)\times n=2\pi$. This shows that the n forces can be represented by the n sides of a closed polygon taken in the same order. So the resultant force is **zero**.

Problem 39. Two vectors $\vec{A}$ and $\vec{B}$ are in the same plane and angle between them is θ. What is the magnitude and direction of $\vec{A}\times\vec{B}$?

Solution. $|\vec{A}\times\vec{B}|=AB\sin\theta$

The direction of $\vec{A}\times\vec{B}$ is perpendicular to the plane of $\vec{A}$ and $\vec{B}$ and points in the same direction in which a right handed screw would advance when rotated from $\vec{A}$ to $\vec{B}$.

Problem 40. If $\vec{A}$ and $\vec{B}$ are two length vectors, then what is the geometrical significance of $|\vec{A}\times\vec{B}|$?

Solution. $|\vec{A}\times\vec{B}|=AB\sin\theta$, this gives area of the parallelogram with adjacent sides $\vec{A}$ and $\vec{B}$.

Problem 41. What is the unit vector perpendicular to the plane of vectors $\vec{A}$ and $\vec{B}$?

Solution. $\hat{n}=\dfrac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}=\dfrac{\vec{A}\times\vec{B}}{AB\sin\theta}$.

Problem 42. What is the value of $\vec{A}\times\vec{A}$?

[Himachal 02]

Solution. Zero, because $\vec{A}\times\vec{A}=AA\sin 0^\circ=0$.

Problem 43. What is the condition for two vectors to be collinear?

Solution. For two given vectors to be collinear, their cross-product must be zero.

Problem 44. If $\vec{A}\times\vec{B}=\vec{C}\times\vec{B}$, show that $\vec{C}$ need not be equal to $\vec{A}$.

Solution. As $\vec{A}\times\vec{B}=\vec{C}\times\vec{B}$

$\therefore \vec{A}\times\vec{B}-\vec{C}\times\vec{B}=0$

or $(\vec{A}-\vec{C})\times\vec{B}=0$

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This implies that if $\vec{C} \neq \vec{A}$, then either $\vec{B} = \vec{0}$

or $(\vec{A} - \vec{C}) \perp \vec{B}$.

Problem 45. Find the value of $\hat{i} \times \hat{j}$.

Solution. $\hat{i} \times \hat{j} = (1)(1)\sin 90^\circ \hat{k} = \hat{k}$.

Problem 46. What is $\hat{i} \cdot (\hat{j} \times \hat{k})$?

Solution. $\hat{i} \cdot (\hat{j} \times \hat{k}) = \hat{i} \cdot \hat{i} = (1)(1)\cos 0^\circ = 1$

Problem 47. Under what condition will the equality $|\vec{A} \times \vec{B}| = \vec{A} \cdot \vec{B}$ hold good ?

Solution. As $|\vec{A} \times \vec{B}| = \vec{A} \cdot \vec{B}$

$$\therefore \quad AB\sin\theta = AB\cos\theta \quad \text{or} \quad \tan\theta = 1$$

$$\Rightarrow \quad \theta = 45^\circ.$$

Problem 48. What is the angle between $(\vec{A} + \vec{B})$ and $(\vec{A} \times \vec{B})$?

Solution. The resultant vector $(\vec{A} + \vec{B})$ lies in the plane of $\vec{A}$ and $\vec{B}$, while the cross product $(\vec{A} \times \vec{B})$ is perpendicular to this plane. So the angle between $(\vec{A} + \vec{B})$ and $(\vec{A} \times \vec{B})$ is 90°.

Problem 49. Can there be motion in two dimensions with an acceleration only in one dimension ?

Solution. Yes, in a projectile motion the acceleration acts vertically downwards while the projectile follows a parabolic path.

Problem 50. Can the direction of velocity of a body change when the acceleration is constant ?

Solution. Yes, in projectile motion the direction of velocity changes from point to point, but its acceleration is always constant and acts vertically downwards.

Problem 51. Is the rocket in flight an example of projectile ?

Solution. No, because it is propelled by combustion of fuel and does not move under the effect of gravity alone.

Problem 52. A stone is thrown vertically upwards and then it returns to the thrower. Is it a projectile ?

Solution. It is not a projectile, because a projectile should have two component velocities in two mutually perpendicular directions, but in this case the body has velocity only in one direction while going up or coming downwards.

Problem 53. Why does a projectile fired along the horizontal not follow a straight line path ?

Solution. Because the projectile fired horizontally is constantly acted upon by acceleration due to gravity acting vertically downwards.

Problem 54. A body projected horizontally moves with the same horizontal velocity although it is under the action of force of gravity. Why ?

Solution. The force of gravity acts in the vertically downward direction and has no effect on the horizontal component of velocity, and this makes the body to move with constant horizontal velocity.

Problem 55. What is the angle between the direction of velocity and acceleration at the highest point of a projectile path ?

Solution. 90°. At the highest point the vertical component of velocity becomes zero, the projectile has only a horizontal velocity while the acceleration due to gravity acts vertically downwards.

Problem 56. A bullet is dropped from a certain height and at the same time, another bullet is fired horizontally from the same height. Which one will hit the ground earlier and why ?

Solution. Since the heights of both bullets from the ground is the same, so the time taken by both of them to reach the ground will be the same.

Problem 57. A stone dropped from the window of a stationary bus takes 5 seconds to reach the ground. In what time the stone will reach the ground when the bus is moving with (a) constant velocity of 80 kmh^{-1} (b) constant acceleration of 2 kmh^{-2} ?

Solution. (a) 5 seconds (b) 5 seconds. In both cases, initial vertical velocity is zero, downward acceleration is equal to 'g' and also vertical distance covered is same.

Problem 58. A bomb thrown as projectile explodes in mid-air. What is the path traced by the centre of mass of the fragments assuming the friction to be negligible ?

Solution. The path traced by the centre of mass of the fragments is a parabola.

Problem 59. At what point in its trajectory does a projectile have its (i) minimum speed (ii) maximum speed ?

Solution. For a projectile given angular projection, the horizontal component of velocity remains constant throughout while the vertical component first decreases, becomes zero at the highest point and then increases again. Hence

(i) Projectile has *minimum speed at the highest point* of its trajectory.

(ii) Projectile has *maximum speed at the point of projection* and at the point where it returns to the horizontal plane of projection.

Problem 60. What will be the effect on horizontal range of a projectile when its initial velocity is doubled, keeping the angle of projection same ? *[Delhi 02]*

Solution. As $R = \dfrac{u^2 \sin 2\theta}{g}$ *i.e.*, $R \propto u^2$

When u is doubled, the horizontal range becomes four times the original horizontal range.

Problem 61. A projectile is fired the horizontal with the speed v. If projected with the same speed, the the horizontal it must be projecte same range ?

Solution. For same R and v, the of projection is 90°. As one angle is $90^\circ - 15^\circ = 75^\circ$.

Problem 62. Is the maximum projectile is largest when its horizon

Solution. No. Horizontal rang $\theta = 45^\circ$ and maximum height att largest when $\theta = 90^\circ$.

Problem 63. What will be th height of a projectile when its changed from 30° to 60°, keep velocity of projection ?

Solution. Maximum height,

$$\therefore \quad \frac{H_2}{H_1} = \frac{\sin^2 60^\circ}{\sin^2 30^\circ} = \frac{(\sqrt{3}/2)^2}{(1/2)^2}$$

Thus the maximum height original maximum height.

Problem 64. A body is proje angle θ to the horizontal to hav is the velocity at the highest po

Solution. For maximum hor

Velocity at highest point = velocity = $u\cos 45^\circ = u/\sqrt{2}$.

Problem 65. What is the projectile motion whose ra maximum height H ?

Solution. Here $R = nH$

or $$\frac{u^2 \sin 2\theta}{g} = n.$$

or $$\frac{u^2 \times 2\sin\theta\cos\theta}{g} = n.$$

$$\Rightarrow \quad \theta = t$$

Problem 66. The greatest throw a stone is h. What w upto which he can throw th

Solution. Maximum hei

Clearly, H will be maxi

$$\therefore \quad H_{max} = \frac{u^2}{2g} = h$$

Maximum horizontal ra

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Problem 61. A projectile is fired at an angle of 15° to the horizontal with the speed v. If another projectile is projected with the same speed, then at what angle with the horizontal it must be projected so as to have the same range ?

Solution. For same R and v, the sum of the two angles of projection is 90°. As one angle is of 15°, other should be $90° - 15° = 75°$.

Problem 62. Is the maximum height attained by projectile is largest when its horizontal range is maximum ?

Solution. No. Horizontal range is maximum when $\theta = 45°$ and maximum height attained by projectile is largest when $\theta = 90°$.

Problem 63. What will be the effect on maximum height of a projectile when its angle of projection is changed from 30° to 60°, keeping the same initial velocity of projection ?

Solution. Maximum height, $H = \dfrac{u^2 \sin^2 \theta}{2g}$

$$\therefore \quad \frac{H_2}{H_1} = \frac{\sin^2 60°}{\sin^2 30°} = \frac{(\sqrt{3}/2)^2}{(1/2)^2} = 3$$

Thus the maximum height becomes three times the original maximum height.

Problem 64. A body is projected with speed u at an angle θ to the horizontal to have maximum range. What is the velocity at the highest point ?

Solution. For maximum horizontal range, $\theta = 45°$

Velocity at highest point = Horizontal component of velocity $= u \cos 45° = u/\sqrt{2}$.

Problem 65. What is the angle of projection for a projectile motion whose range R is n times the maximum height H ?

Solution. Here $R = nH$

$$\text{or} \quad \frac{u^2 \sin 2\theta}{g} = n \cdot \frac{u^2 \sin^2 \theta}{2g}$$

$$\text{or} \quad \frac{u^2 \times 2 \sin\theta \cos\theta}{g} = n \cdot \frac{u^2 \sin^2 \theta}{2g} \quad \text{or} \quad \tan\theta = \frac{4}{n}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{4}{n}\right).$$

Problem 66. The greatest height to which a man can throw a stone is h. What will be the greatest distance upto which he can throw the stone ?

Solution. Maximum height attained, $H = \dfrac{u^2 \sin^2 \theta}{2g}$

Clearly, H will be maximum when $\sin\theta = 1$

$$\therefore \quad H_{max} = \frac{u^2}{2g} = h$$

Maximum horizontal range, $R_{max} = \dfrac{u^2}{g} = 2h$.

Problem 67. A particle is projected at an angle θ from the horizontal with kinetic energy K. What is the kinetic energy of the particle at the highest point ?

Solution. If the particle is projected with velocity v, then its initial kinetic energy is

$$K = \frac{1}{2} mv^2$$

Velocity of the projectile at the highest point

$$= v \cos\theta$$

$\therefore$ K.E. of the particle at the highest point

$$= \frac{1}{2} m (v\cos\theta)^2 = (\frac{1}{2} mv^2) \cos^2\theta = K \cos^2\theta$$

Problem 68. A man can jump on the moon six times as high as on the earth. Why ?

Solution. Maximum height attained on earth,

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

Acceleration due to gravity on moon, $g' = g/6$

$\therefore$ Maximum height attained on moon,

$$H' = \frac{u^2 \sin^2 \theta}{2g'} = \frac{6u^2 \sin^2 \theta}{2g} = 6H$$

So one can jump on moon six times as high as on earth.

Problem 69. A projectile of mass m is fired with velocity v at an angle θ with the horizontal. What is the change in momentum as it rises to the highest point of the trajectory ?

Solution. Horizontal component of velocity remains constant. The vertical component is $v \sin\theta$ at the point of projection and zero at the highest point.

Magnitude of change in momentum

$= m(v \sin\theta - 0) = mv \sin\theta$ (in the vertical direction)

Problem 70. A projectile of mass m is thrown with velocity v from the ground at an angle of 45° with the horizontal. What is the magnitude of change in momentum between leaving and arriving back at the ground ?

Solution. The vertical component of velocity at the point of projection is $v \sin\theta$ and at the point of return is $-v \sin\theta$.

$\therefore$ Magnitude of change in momentum

$$= m(v\sin\theta + v\sin\theta) = 2mv\sin\theta$$

$$= 2mv \sin 45° = \sqrt{2}\, mv$$

(in the vertical direction).

Problem 71. A block slides down a smooth inclined plane when released from the top while another falls freely from the same point. Which one of them will strike the ground earlier ?

Solution. Acceleration of the freely falling block is g while that of the block sliding down the smooth inclined plane is $g \sin\theta$. As $g > g \sin\theta$, so the block falling freely will reach the ground earlier.

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Problem 72. Why does a tennis ball bounce higher on hills than in plains ?

Solution. Maximum height attained by a projectile

$$\propto \frac{1}{g}$$

As the value of g is less on hills than on plains, so a tennis ball bounces higher on hills than on plains.

Problem 73. Two projectiles A and B are projected with velocities $\sqrt{2}v$ and v respectively. They have the same range. If A is thrown at angle of 15° with the horizontal, then what is the angle of projection of B ?

Solution. Given $R_A = R_B$

$$\text{or} \quad \frac{(\sqrt{2}v)^2 \sin(2 \times 15°)}{g} = \frac{v^2 \sin 2\theta}{g}$$

$$\text{or} \quad \sin 2\theta = 1 = \sin 90°$$

$$\text{or} \quad \theta = 45°.$$

Problem 74. A stone is thrown horizontally with a speed $\sqrt{2gh}$ from the top of a wall of height h. It strikes the level ground through the foot of the wall at a distance x from the wall. What is the value of x ?

Solution. As $h = \frac{1}{2} g t^2$

$$\therefore \quad t = \sqrt{\frac{2h}{g}}$$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\sqrt{2gh} = \frac{x}{\sqrt{2h/g}}$$

$$\text{or} \quad x = \sqrt{2gh} \times \sqrt{\frac{2h}{g}} = 2h.$$

Problem 75. The velocity of a particle is constant in magnitude but not in direction. What is the nature of trajectory ?

Solution. The particle moves on a curved path. The instantaneous direction will be tangential to the path at every point.

Problem 76. What is the angle between velocity vector and acceleration vector in uniform circular motion ?

Solution. 90°.

Problem 77. For uniform circular motion, does the direction of centripetal acceleration depend upon the sense of rotation ?

Solution. The direction of centripetal acceleration does not depend on the clockwise or anticlockwise sense of rotation of the body. It always acts along the radius towards the centre of the circle.

Problem 78. If both the speed and radius of the circular path of a body are doubled, how will the centripetal acceleration change ?

Solution. Original centripetal acceleration, $a = \frac{v^2}{r}$

New centripetal acceleration, $a' = \frac{(2v)^2}{2r} = \frac{2v^2}{r} = 2a.$

Problem 79. A stone tied to the end of a string is whirled in a circle. If the string breaks, the stone flies away tangentially. Why ?

Solution. The instantaneous velocity of the stone going around the circular path is always along tangent to the circle. When the string breaks, the centripetal force ceases. Due to inertia, the stone continues its motion along tangent to the circular path.

Problem 80. When a knife is sharpened with the help of a rotating grinding stone, the spark always travels tangentially to it. Why ?

Solution. When a knife is sharpened, the red hot particles of the grinding stone get separated as sparks. As these particles are in circular motion, hence their velocities are directed tangentially to the grinding stone.

Short Answer Conceptual Problems

Problem 1. Which of the following quantities are independent of the choice of orientation of the coordinate axes :

$$\vec{a} + \vec{b}, \quad 3a_x + 2b_y, \quad |\vec{a} + \vec{b} - \vec{c}|,$$

angle between $\vec{b}$ and $\vec{c}$, $\lambda \vec{a}$, where λ is a scalar ?

Solution. A vector, its magnitude and the angle between two vectors do not depend on the choice of the orientation of the coordinate axes, so $\vec{a} + \vec{b}$, $|\vec{a} + \vec{b} - \vec{c}|$, angle between $\vec{b}$ and $\vec{c}$ and $\lambda \vec{a}$ are independent of the orientation of the coordinate axes.

But the quantity $3a_x + 2b_y$ depends upon the magnitude of the components along x- and y-axes, so it will change with the change in coordinate axes.

Problem 2. Do $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ lie in the same plane. Give reason.

Solution. Yes, because $\vec{a} + \vec{b}$ is represented by the diagonal of the parallelogram drawn with $\vec{a}$ and $\vec{b}$ as adjacent sides. The diagonal passes through the common tail of $\vec{a}$ and $\vec{b}$. However, $\vec{a} - \vec{b}$ is represented by the other diagonal of the same parallelogram not passing through the common tail of $\vec{a}$ and $\vec{b}$. Thus both $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ lie in plane of the same parallelogram.

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Problem 3. Can we apply the commutative and associative laws to vector subtraction also ?

Solution. (*i*) No, we cannot apply commutative law to vector subtraction because $\vec{a} - \vec{b} \neq \vec{b} - \vec{a}$

(*ii*) Yes, association law can be applied to vector subtraction because $(\vec{a} + \vec{b}) - \vec{c} = \vec{a} + (\vec{b} - \vec{c})$.

Problem 4. Can three vectors not in one plane give a zero resultant ? Can four vectors do so ?

Solution. No, three vectors not in one plane cannot give a zero resultant because the resultant of two vectors will not lie in the plane of the third vector and hence cannot cancel its effect.

The resultant of four vectors not in one plane may be a zero vector.

Problem 5. What is the difference between the following two data :

(*i*) 8 (5 km/hr, east) (*ii*) (8 hr) (5 km/hr, east) ?

Solution. (*i*) It is the product of a pure number and a vector (velocity), hence the unit of product is the same as that of vector *i.e.*, the product is a velocity of 40 km/hr, towards east.

(*ii*) It is the product of a scalar (time) and a vector (velocity). Hence the unit of the product will be hr × (km/hr). Thus the product is a displacement of magnitude 40 km, towards east.

Problem 6. Is $|\vec{a} + \vec{b}|$ greater than or less than $|\vec{a}| + |\vec{b}|$? Give reason.

Solution. $|\vec{a} + \vec{b}|^2 - (|\vec{a}| + |\vec{b}|)^2$

$= |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta - |\vec{a}|^2 - |\vec{b}|^2 - 2|\vec{a}||\vec{b}|$

$= -2|\vec{a}||\vec{b}|(1 - \cos\theta)$

$= -2|\vec{a}||\vec{b}|\sin^2\frac{\theta}{2}$ = a negative quantity

Hence $|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$.

Problem 7. Is $|\vec{a} - \vec{b}|$ greater than or less than $|\vec{a}| + |\vec{b}|$?

Solution. $|\vec{a} - \vec{b}|^2 - (|\vec{a}| + |\vec{b}|)^2$

$= |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos\theta - |\vec{a}|^2 - |\vec{b}|^2 - 2|\vec{a}||\vec{b}|$

$= -4|\vec{a}||\vec{b}|\cos^2\frac{\theta}{2}$ = a negative quantity

Hence $|\vec{a} - \vec{b}| \leq |\vec{a}| + |\vec{b}|$.

Problem 8. Under what condition does the equality : $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$ hold good ?

Solution. $(\vec{A} + \vec{B}) = (\vec{A} - \vec{B})$

$\therefore \quad (\vec{A} + \vec{B})^2 = (\vec{A} - \vec{B})^2$

or $\quad A^2 + B^2 + 2\vec{A}.\vec{B} = A^2 + B^2 - 2\vec{A}.\vec{B}$

or $\quad 4\vec{A}.\vec{B} = 0 \quad$ or $\quad \vec{A}.\vec{B} = 0$

This implies that $\vec{A}$ and $\vec{B}$ are perpendicular to each other.

Problem 9. The sum and difference of two vectors are perpendicular to each other. Prove that the vectors are equal in magnitude.

Solution. As the vectors $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$ are perpendicular to each other, therefore

$$(\vec{A} + \vec{B}).(\vec{A} - \vec{B}) = 0$$

$$\vec{A}.\vec{A} - \vec{A}.\vec{B} + \vec{B}.\vec{A} - \vec{B}.\vec{B} = 0$$

or $\quad A^2 - B^2 = 0 \Rightarrow A = B \qquad [\because \vec{A}.\vec{B} = \vec{B}.\vec{A}]$

Problem 10. Find the conditions for two vectors to be (*i*) parallel and (*ii*) perpendicular to each other.

[Punjab 91]

Solution. (*i*) If $\vec{A} \parallel \vec{B}$, then $\theta = 0°$

and $\quad \vec{A} \times \vec{B} = |\vec{A}||\vec{B}|\sin 0°\,\hat{n} = \vec{0}$

Hence *if two vectors are parallel, then their cross product must be zero.*

(*ii*) If $\vec{A} \perp \vec{B}$, then $\theta = 90°$ and

$$\vec{A}.\vec{B} = |\vec{A}||\vec{B}|\cos 90° = 0$$

Hence *if two vectors are perpendicular, then their dot product must be zero.*

Problem 11. The resultant of two vectors $\vec{P}$ and $\vec{Q}$ is perpendicular to $\vec{P}$ and its magnitude is half that of $\vec{Q}$. What is the angle between $\vec{P}$ and $\vec{Q}$? *[Delhi 13]*

Solution. Clearly, $P^2 + \left(\frac{Q}{2}\right)^2 = Q^2$

or $\quad P^2 = \frac{3}{4}Q^2 \quad$ or $\quad P = \frac{\sqrt{3}}{2}Q$

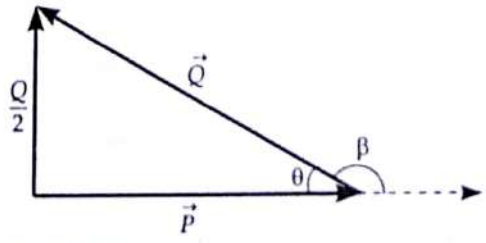

Fig. 4.91

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$$\therefore \quad \tan\theta = \frac{Q/2}{P} = \frac{Q/2}{\sqrt{3}\,Q/2} = \frac{1}{\sqrt{3}}$$

or $\theta = 30°$

Angle between $\vec{P}$ and $\vec{Q}$,

$$\beta = 180° - \theta = 180° - 30° = 150°.$$

Problem 12. The magnitude of vector $\vec{A}$ is 2.5 metre and is directed towards east. What will be the magnitudes and directions of the following :

(a) $-\vec{A}$ (b) $\vec{A}/2$ (c) $2.5\vec{A}$ (d) $4\vec{A}$?

Solution. (a) 2.5 m due west. (b) 1.25 due east.
(c) 6.25 m due west. (d) 10 m due east.

Problem 13. A person sitting in a moving train throws a ball vertically upwards. How will the ball appear to move to an observer (i) sitting inside the train (ii) standing outside the train ? Give reason.

Solution. (i) To the observer inside the train, the ball will appear to move *straight vertically upwards and then downwards* because with respect to an observer sitting inside the train, the ball has only one velocity acting vertically downward.

(ii) To the observer outside the train, the ball will appear to move along the *parabolic path* because with respect to the observer outside the train, the ball has both horizontal and vertical components of velocity.

Problem 14. A bob hung from the celling of a room by a string is performing simple harmonic oscillations. What will be the trajectory of the bob, if the string is cut, when bob is (i) at one of its extreme positions, (ii) at its mean position ?

Solution. (i) When the string is cut in one of the extreme positions of the bob, the bob will fall *vertically downwards*. The reason is that when the bob reaches its extreme positions, it comes to rest for a moment. If the string is cut, it will be simply the case of a body falling freely under gravity.

(ii) When the string is cut in mean position of the bob, then the bob will move on parabolic path. The reason is that when the bob is at its mean position, it has a velocity parallel to the horizontal. If the string is cut, it will be simply the case of a projectile fired parallel to the horizontal.

Problem 15. Prove that the horizontal range is same when angle of projection is (i) greater than 45° by certain value and (ii) less than 45° by the same value.

[Central Schools 08]

Solution. (i) When the angle of projection, $\theta = 45° + \alpha$, the horizontal range is

$$R = \frac{u^2 \sin 2(45° + \alpha)}{g} = \frac{u^2 \sin(90° + 2\alpha)}{g}$$

$$= \frac{u^2 \cos 2\alpha}{g}$$

(ii) When the angle of projection, $\theta = 45° - \alpha$, the horizontal range is

$$R' = \frac{u^2 \sin 2(45° - \alpha)}{g} = \frac{u^2 \sin(90° - 2\alpha)}{g}$$

$$= \frac{u^2 \cos 2\alpha}{g}$$

Clearly, $R' = R$.

Problem 16. Two bombs of 5 kg and 10 kg are thrown from a cannon with the same velocity in the same direction. (i) Which bomb will reach the ground first ? (ii) If the bombs are thrown in the same direction with different velocity, what would be the effect ?

Solution. (i) Both the bombs will reach the ground simultaneously because the time of flight does not depend upon mass of the projectile.

(ii) On throwing with different velocities, the bomb thrown with lesser velocity will reach the ground earlier.

$$T = \frac{2u\sin\theta}{g} \quad i.e., \quad T \propto u.$$

Problem 17. Two bodies are thrown with the same initial velocity at angles α and $(90° - \alpha)$ with the horizontal. What will be the ratio of (i) maximum heights attained by them and (ii) their horizontal ranges ?

Solution. (i) When the angle of projection α, the maximum height is

$$H_1 = \frac{u^2 \sin^2\alpha}{2g}$$

When the angle of projection is $(90° - \alpha)$, the maximum height is

$$H_2 = \frac{u^2 \sin^2(90° - \alpha)}{2g} = \frac{u^2 \cos^2\alpha}{2g}$$

$$\therefore \quad \frac{H_1}{H_2} = \frac{\sin^2\alpha}{\cos^2\alpha} = \tan^2\alpha = \tan^2\alpha : 1.$$

(ii) When the angle of projection is α, the horizontal range is

$$R_1 = \frac{u^2 \sin 2\alpha}{g}$$

When the angle of projection is $(90° - \alpha)$, the horizontal range is

$$R_2 = \frac{u^2 \sin 2(90° - \alpha)}{g} = \frac{u^2 \sin(180° - 2\alpha)}{g}$$

$$= \frac{u^2 \sin 2\alpha}{g}$$

$$\therefore \quad R_1 : R_2 = 1 : 1.$$

Problem 18. At which points on the projectile trajectory is the (i) potential energy maximum (ii) kinetic energy minimum and (iii) total energy maximum ?

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Solution. (*i*) P.E. of a projectile is maximum at its highest point because of its maximum height. It is given by

$$(P.E.)_H = mg\,H = mg\cdot\frac{u^2\sin^2\theta}{2g} = \frac{1}{2}mu^2\sin^2\theta$$

(*ii*) K.E. of a projectile is minimum (not zero) at its highest point because of its minimum velocity.

$$(K.E.)_H = \frac{1}{2}mu_H^2 = \frac{1}{2}m(u\cos\theta)^2$$
$$= \frac{1}{2}mu^2\cos^2\theta$$

(*iii*) Total energy at highest point

$$= (P.E.)_H + (K.E.)_H$$
$$= \frac{1}{2}mu^2(\sin^2\theta + \cos^2\theta) = \frac{1}{2}mu^2$$

= Total energy at the point of projection

Hence total energy of a projectile is conserved at all points of its motion. The K.E. is maximum at the point of projection.

Problem 19. A body *A* of mass *m* is thrown with velocity *v* at an angle 30° to the horizontal and another body *B* of the same mass is thrown with velocity *v* at an angle of 60° to the horizontal, find the ratio of the horizontal range and maximum height of *A* and *B*.

Solution. *For body A* : $\theta = 30°$

Horizontal range,

$$R_A = \frac{u^2\sin^2(2\times30°)}{g} = \frac{u^2}{g}\times\frac{\sqrt{3}}{2}.$$

Maximum height,

$$H_A = \frac{u^2\sin^2 30°}{2g} = \frac{u^2}{2g}\times\frac{1}{4}$$

For body B : $\theta = 60°$

$$\therefore\quad R_B = \frac{u^2\sin(2\times60°)}{g} = \frac{u^2}{g}\times\frac{\sqrt{3}}{2}$$

and
$$H_B = \frac{u^2\sin^2 60°}{2g} = \frac{u^2}{2g}\times\frac{3}{4}$$

Hence $R_A : R_B = 1:1$ and $H_A : H_B = 1:3$.

Problem 20. A ball of mass *m* is thrown vertically up. Another ball of mass 2*m* is thrown at an angle θ with the vertical. Both of them remain in air for the same period of time. What is the ratio of the heights attained by the two balls ?

Solution. Time of flight of the ball thrown vertically upwards,

$$T_1 = \frac{2u_1}{g}$$

Time of flight of the ball thrown at an angle θ with the vertical,

$$T_2 = \frac{2u_2\cos\theta}{g}$$

As $T_1 = T_2$

$$\therefore\quad \frac{2u_1}{g} = \frac{2u_2\cos\theta}{g}$$

or
$$u_1 = u_2\cos\theta$$

Now $H_1 = \dfrac{u_1^2}{2g}$ and $H_2 = \dfrac{u_2^2\cos^2\theta}{2g}$

$$\therefore\quad \frac{H_1}{H_2} = \frac{u_1^2}{u_2^2\cos^2\theta} = \frac{u_2^2\cos^2\theta}{u_2^2\cos^2\theta} = 1:1.$$

Problem 21. A skilled gunman always keeps his gun slightly tilted above the line of sight while shooting. Why ?

Solution. When a bullet is fired from a gun with its barrel directed towards the target, it starts falling downwards on account of acceleration due to gravity. Due to which the bullet hits below the target. Just to avoid it, the barrel of the gun is lined up little above the target, so that the bullet, after travelling in parabolic path hits the distant target, as shown in Fig. 4.92.

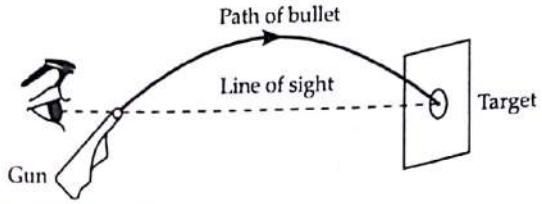

Fig. 4.92

Problem 22. Is it important in the long jump that how much height you take for jumping ? What factors determine the span of a jump ?

Solution. Yes, for the longest jump the player should throw himself at an angle of 45° with respect to horizontal. The vertical height for this angle is

$$H = u^2\sin^2 45°/2g = u^2/4g,$$

where *u* is the velocity of projection. If the vertical height is different from $u^2/4g$, then the angle will be different from 45° and the horizontal distance covered will also be less.

Clearly, the span of jump depends upon

(*i*) the initial velocity of the jump,

(*ii*) the angle of projection.

Problem 23. A body is thrown horizontally with a velocity *v* from a tower of height *H*. After how much time and at what distance from the base of the tower will the body strike the ground ?

Solution. Clearly, $H = \dfrac{1}{2}gt^2$

$$\therefore\quad t = \sqrt{\frac{2H}{g}}.$$

Distance from the base of the tower,

$$x = vt = v\sqrt{2H/g}.$$

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Problem 24. A ball is thrown horizontally and at the same time another ball is dropped down from the top of a tower (i) Will both the balls reach the ground at the same time ? (ii) Will both strike the ground with the same velocity ?

Solution. (i) Yes, because the initial vertical velocity of both the balls is zero, and both will cover the same vertical distance under the same vertical acceleration g.

(ii) No, because on striking the ground although their vertical velocities will be same but horizontal velocities will be different. Hence their resultant velocities will be different.

Problem 25. A horizontal stream of water leaves an opening in the side of a tank. If the opening is h metre above the ground and the stream hits the ground D metre away, then what is the speed of water as it leaves the tank in terms of g, h and D ?

Solution. The stream of water leaving the tank behaves like a horizontal projectile and follows parabolic path.

$$h = \frac{1}{2} g t^2$$

$$\therefore \quad t = \sqrt{\frac{2h}{g}}$$

$$v = \frac{D}{t} = D\sqrt{\frac{g}{2h}}.$$

Problem 26. A railway carriage moves over a straight track with acceleration a. A passenger in the carriage drops a stone. What is the acceleration of the stone w.r.t. the carriage and the earth ?

Solution. When the stone is dropped, it falls freely under the acceleration due to gravity g. With respect to the earth, the acceleration of the stone is g.

Inside the carriage, the stone possesses two accelerations :

(i) Horizontal acceleration 'a' due to the motion of the carriage.

(ii) Vertical downward acceleration 'g' due to gravity.

The acceleration of the stone w.r.t. the carriage

$$= \sqrt{a^2 + g^2}.$$

Problem 27. A glass marble slides from rest from the topmost point of a vertical circle of radius r along a smooth chord. Does the time of descent depend upon the chord chosen ?

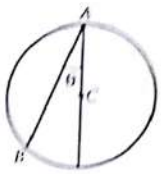

Fig. 4.93

Solution. As shown in Fig. 4.92, let C be the centre of the circle. Suppose chord AB makes angle θ with the vertical.

If the marble takes time t to slide along chord AB, then

$$AB = 0 \times t + \frac{1}{2} g \cos\theta \times t^2$$

[$\because$ Acceleration along $AB = g\cos\theta$]

or $$t^2 = \frac{2\,AB}{g\cos\theta} = \frac{2 \times 2r\cos\theta}{g\cos\theta} = \frac{4r}{g}$$

or $$t = 2\sqrt{\frac{r}{g}}$$

As t is independent of θ, so the time of descent does not depend on the chord chosen.

HOTS

Problems on Higher Order Thinking Skills

Problem 1. *Four persons K, L, M and N are initially at rest at the four corners of a square of side d. Each person now moves with a uniform speed v in such a way that K always moves directly towards L, L directly towards M, M directly towards N and N directly towards K. Show that the four persons meet at a time d/v.*

Solution. All the four persons will meet at the centre O of the square. Each person covers a displacement,

$$s = \frac{1}{2} \cdot \sqrt{d^2 + d^2} = \frac{d}{\sqrt{2}}$$

Component of velocity v towards O,

$$v' = v\cos 45° = v/\sqrt{2}$$

$\therefore$ Required time,

$$t = \frac{s}{v'} = \frac{d/\sqrt{2}}{v/\sqrt{2}} = \frac{d}{v}.$$

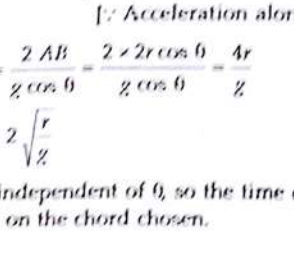

Fig. 4.94

Problem 2. *The x- and y*
6 m respectively. The x-
($\vec{A} + \vec{B}$) are 10 m and 9 m
vector $\vec{B}$ (i) its x- and y-
(iii) the angle it makes with

Solution. Here $A_x = 4$

$A_y + B_y = 9$ m

(i) $B_x = 10 - 4 = 6$ m,

(ii) $B = \sqrt{B_x^2 + B_y^2} = \sqrt{}$

(iii) $\theta = \tan^{-1}\dfrac{B_y}{B_x} = t$

Problem 3. *A bird is*
(4 m, – 1 m, 5 m). The bird
having coordinates (–1
respectively. At time $t = 0$,
positions, with a constan
perpendicular to the straig
collinear at time t. Calcul

Solution. The situa
bird flies in a directio
Suppose it reaches the
from point P) where it

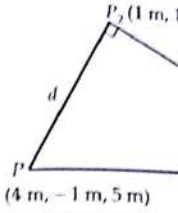

Fig. 4.95

Let $\vec{PP_1} = \vec{A}$, $\vec{P_1}$

As $|\vec{A} \times \vec{B}| = |\vec{A}$

$\therefore \quad \sin\theta = \dfrac{|\vec{A}}{|\vec{A}}$

Now $d = |\vec{A}$

But $\vec{A} = (-1$

$= -5$

and $\vec{B} = (1$

$= 2\,\hat{}$

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Problem 2. *The x- and y- components of $\vec{A}$ are 4 m and 6 m respectively. The x- and y- components of vector $(\vec{A}+\vec{B})$ are 10 m and 9 m respectively. Calculate for the vector $\vec{B}$ (i) its x- and y- components (ii) its length and (iii) the angle it makes with the x-axis.* **[REC 89]**

Solution. Here $A_x = 4$ m, $A_y = 6$ m, $A_x + B_x = 10$ m, $A_y + B_y = 9$ m

(i) $B_x = 10 - 4 = 6$ m, $\quad B_y = 9 - 6 = 3$ m.

(ii) $B = \sqrt{B_x^2 + B_y^2} = \sqrt{36+9} = \sqrt{45}$ m.

(iii) $\theta = \tan^{-1}\dfrac{B_y}{B_x} = \tan^{-1}\dfrac{3}{6} = 26.6°$.

Problem 3. *A bird is at a point P whose coordinates are (4 m, −1 m, 5 m). The bird observes two points P_1 and P_2 having coordinates (−1 m, 2 m, 0 m) and (1 m, 1 m, 4 m) respectively. At time t = 0, it starts flying in a plane of three positions, with a constant speed of 5 ms^{-1} in a direction perpendicular to the straight line P_1P_2 till it sees P_1 and P_2 collinear at time t. Calculate t.*

Solution. The situation is shown in Fig. 4.95. The bird flies in a direction perpendicular to line P_1P_2. Suppose it reaches the point Q in time t (after starting from point P) where it sees P_1 and P_2 as collinear.

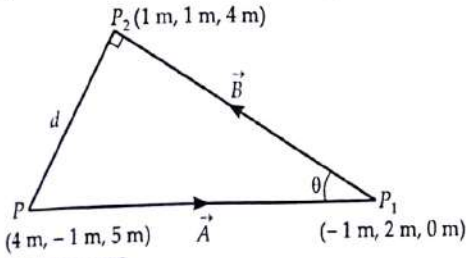

Fig. 4.95

Let $\vec{PP_1} = \vec{A}$, $\vec{P_1P_2} = \vec{B}$, $\angle PP_1P_2 = \theta$ and $PQ = d$.

As $|\vec{A}\times\vec{B}| = |\vec{A}||\vec{B}|\sin\theta$

$$\therefore \quad \sin\theta = \frac{|\vec{A}\times\vec{B}|}{|\vec{A}||\vec{B}|}$$

Now $$d = |\vec{A}|\sin\theta = |\vec{A}|\frac{|\vec{A}\times\vec{B}|}{|\vec{A}||\vec{B}|} = \frac{|\vec{A}\times\vec{B}|}{\vec{B}}$$

But $$\vec{A} = (-1-4)\hat{i} + (2+1)\hat{j} + (0-5)\hat{k}$$
$$= -5\hat{i} + 3\hat{j} - 5\hat{k}$$

and $$\vec{B} = (1+1)\hat{i} + (1-2)\hat{j} + (4-0)\hat{k}$$
$$= 2\hat{i} - \hat{j} + 4\hat{k}$$

$$\therefore \quad \vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -5 & 3 & -5 \\ 2 & -1 & 4 \end{vmatrix}$$

$$= (12-5)\hat{i} + (-10+20)\hat{j} + \hat{k}(5-6)$$

$$= 7\hat{i} + 10\hat{j} - \hat{k}$$

$$\therefore \quad |\vec{A}\times\vec{B}| = \sqrt{7^2 + 10^2 + 1^2} = 12.25 \text{ m}^2$$

and $$|\vec{B}| = \sqrt{2^2 + 1^2 + 4^2} = 4.583 \text{ m}$$

$$\therefore \quad d = \frac{12.25}{4.583} = 2.67 \text{ m}$$

Time taken by bird to reach the point Q will be

$$t = \frac{d}{v} = \frac{2.67}{5} = 0.5346 \text{ s}.$$

Problem 4. *A person travelling eastward with a speed of 3 $km\,h^{-1}$ finds that wind seems to blow from north. On doubling his speed, the wind appears to flow from north-east. Find the magnitude of the actual velocity of the wind.*

Solution. The situation is shown in Fig. 4.96.

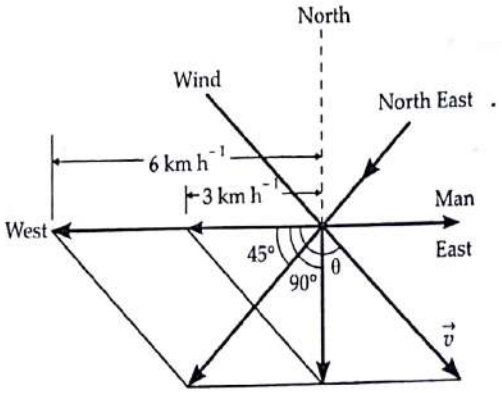

Fig. 4.96

Let the wind velocity be v.

In the first case, relative velocity of wind makes an angle of 90° with opposite velocity of man.

From parallelogram law,

$$\tan 90° = \frac{v\sin\theta}{3 + v\cos\theta} = \infty$$

or $$3 + v\cos\theta = 0 \qquad ...(1)$$

In the second case, relative velocity of wind makes an angle of 45° with double opposite velocity of man

$$\therefore \quad \tan 45° = \frac{v\sin\theta}{6 + v\cos\theta} = 1$$

or $$6 + v\cos\theta = v\sin\theta \qquad ...(2)$$

From (1), $\quad v\cos\theta = -3$

From (2), $\quad v\sin\theta = 6 + v\cos\theta = 6 - 3 = 3$

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$\therefore \quad v^2 = (v\cos\theta)^2 + (v\sin\theta)^2$

$= (-3)^2 + (3)^2 = 9 + 9 = 18$

or $\quad v = 3\sqrt{2}$ km h^{-1}.

Problem 5. *Consider a collection of a large number of particles each with speed v. The direction of velocity is randomly distributed in the collection. Show that the magnitude of the relative velocity between a pair of particles averaged over all the pairs in the collection is greater than v.* [NCERT]

Solution. As shown in Fig. 4.97, let $\vec{v}_1$ and $\vec{v}_2$ be velocities of any two particles and θ be the angle between them. As each particle has speed v, so

$$|\vec{v}_1| = |\vec{v}_2| = v$$

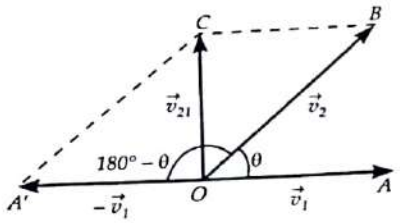

Fig. 4.97

The relative velocity $\vec{v}_{21}$ of particle 2 w.r.t. 1 is given by

$$v_{21} = \sqrt{|-\vec{v}_1|^2 + |\vec{v}_2|^2 + 2|-\vec{v}_1||\vec{v}_2|\cos(180°-\theta)}$$

$$= \sqrt{v^2 + v^2 - 2v\,v\cos\theta} = \sqrt{2v^2(1-\cos\theta)}$$

$$= \sqrt{2v^2 \times 2\sin^2\frac{\theta}{2}} = 2v\sin\frac{\theta}{2}$$

$$\left[1 - \cos 2\theta = 2\sin^2\theta \quad \therefore \quad 1 - \cos\theta = 2\sin^2\frac{\theta}{2}\right]$$

As the velocities of the particles are randomly distributed, so θ can vary from 0 to 2π. If $\bar{v}_{21}$ is the magnitude of the average velocity when averaged over all pairs, then

$$\bar{v}_{21} = \frac{\int_0^{2\pi} 2v\sin\frac{\theta}{2}\,dv}{\int_0^{2\pi} d\theta} = \frac{2v\left[\frac{-\cos\theta/2}{1/2}\right]_0^{2\pi}}{[\theta]_0^{2\pi}}$$

$$= \frac{-4v\left[\cos\frac{\theta}{2}\right]_0^{2\pi}}{2\pi - 0} = -\frac{2v}{\pi}[\cos\pi - \cos 0]$$

$$= -\frac{2v}{\pi}[-1-1] = \frac{4v}{\pi} = 1.273\,v$$

Clearly, $\bar{v}_{21} > v$

Problem 6. *A ball rolls off the top of a stairway with a constant horizontal velocity u. If the steps are h metre high and w metre wide, show that the ball will just hit the edge of nth step if* $n = \frac{2hu^2}{gw^2}$.

Solution. Refer to Fig. 4.98. For nth step,

net vertical displacement = nh

net horizontal displacement = nw

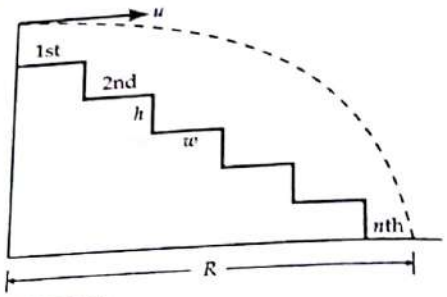

Fig. 4.98

Let t be the time taken by the ball to reach the nth step. Then

$$R = ut$$

or $\quad nw = ut$

or $\quad t = \frac{nw}{u}$

Also, $\quad y = u_y\,t + \frac{1}{2}gt^2$

or $\quad nh = 0 + \frac{1}{2}gt^2 = \frac{1}{2}g\left(\frac{nw}{u}\right)^2$

or $\quad n = \frac{2hu^2}{gw^2}$.

Problem 7. *Show that the motion of one projectile as seen from another projectile will always be a straight line motion.*

Solution. As shown in Fig. 4.99, suppose two projectiles are thrown from the origin O of the XY-plane with velocities u_1 and u_2, making angles θ_1 and θ_2

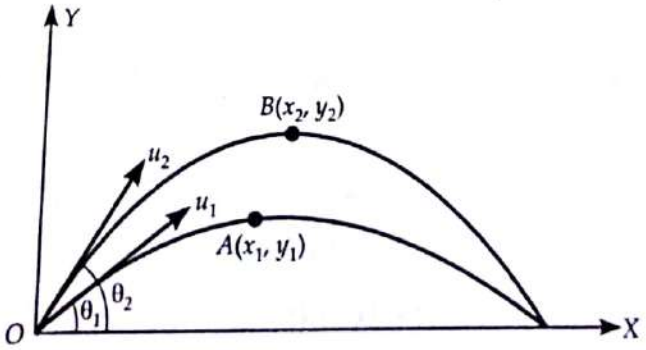

Fig. 4.99

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respectively with X-axis. After time t, let the two projectiles occupy positions $A(x_1, x_1)$ and $B(x_2, y_2)$. Then

$$x_1 = u_1 \cos\theta_1 \,.\, t$$

and $$y_1 = u_1 \sin\theta_1 \,.\, t - \tfrac{1}{2} g t^2$$

Also, $$x_2 = u_2 \cos\theta_2 \,.\, t$$

and $$y_2 = u_2 \sin\theta_2 \,.\, t - \tfrac{1}{2} g t^2$$

$$\therefore \quad x_2 - x_1 = (u_2 \cos\theta_2 - u_1 \cos\theta_1)\, t$$

$$y_2 - y_1 = (u_2 \sin\theta_2 - u_1 \sin\theta_1)\, t$$

or $$\frac{y_2 - y_1}{x_2 - x_1} = \frac{u_2 \sin\theta_2 - u_1 \sin\theta_1}{u_2 \cos\theta_2 - u_1 \cos\theta_1} = m \text{ (a constant)}$$

If (x, y) be the coordinates of point B relative to the point A, then

$$x_2 - x_1 = x \quad \text{and} \quad y_2 - y_1 = y$$

$$\therefore \quad \frac{y}{x} = m \quad \text{or} \quad y = mx$$

This is the equation of a straight line. Hence the motion of a projectile as seen from another projectile is a straight line motion.

Problem 8. *A boy playing on the roof a 10 m high building throws a ball with a speed of 10 ms^{-1} at an angle of 30° with the horizontal. How far from the throwing point will the ball be at the height of 10 m from the ground ? Given $g = 10\ ms^{-2}$.* **[AIEEE 03]**

Solution. When the ball thrown from the height of 10 m reaches again at the height of 10 m, it covers a distance equal to its horizontal range R given by

$$R = \frac{u^2 \sin 2\theta}{g}$$

But $u = 10\ \text{ms}^{-1}$, $\theta = 30°$, $g = 10\ \text{ms}^{-2}$

$$\therefore \quad R = \frac{(10)^2 \sin 60°}{10} = 10 \times 0.866 = \mathbf{8.66\ m.}$$

Problem 9. *A ball is thrown from a point with a speed of v_0 at an angle of projection θ. From the same point and at the same instant, a person starts running with a constant speed of $v_0/2$ to catch the ball. Will the person be able to catch the ball ? If yes, what should be the angle of projection ?* **[AIEEE 04]**

Solution. Yes, the person will be able to catch up the ball if the horizontal component of the velocity of the ball is equal to the constant speed of the person, *i.e.*,

$$v_0 \cos\theta = \frac{v_0}{2} \quad \text{or} \quad \cos\theta = \frac{1}{2}$$

or $$\theta = 60°.$$

Problem 10. *A projectile can have the same range R for two angles of projection. If t_1 and t_2 be the time of flight in the two cases, then prove that*

$$t_1 t_2 = \frac{2R}{g}.$$

[AIEEE 05]

Solution. For equal range, the particle should either be projected at an angle θ or $(90° - \theta)$.

$$\therefore \quad t_1 = \frac{2u \sin\theta}{g}$$

and $$t_2 = \frac{2u \sin(90° - \theta)}{g} = \frac{2u \cos\theta}{g}$$

Hence $$t_1 t_2 = \frac{2u \sin\theta}{g} \cdot \frac{2u \cos\theta}{g}$$

$$= \frac{2}{g} \frac{u^2 \sin 2\theta}{g} = \frac{2R}{g}.$$

Guidlines to NCERT Exercises

4.1. *State for each of the following physical quantities, if it is a scalar or a vector : volume, mass, speed, acceleration, density, number of moles, velocity, angular frequency, displacement, angular velocity.*

Ans. Scalars : volume, mass, speed, density, number of moles and angular frequency.

Vectors : Acceleration, velocity, displacement and angular velocity.

4.2. *Pick out the two scalar quantities in the following list : force, angular momentum, work, current, linear momentum, electric field, average velocity, magnetic moment, reaction as per Newton's third law, relative velocity.*

Solution. Work and current.

4.3. *Pick out the only vector quantity in the following list : Temperature, pressure, impulse, time, power, total path length, energy, gravitational potential, coefficient of friction, charge.*

Ans. Impulse.

4.4. *State with reasons, whether the following algebraic operations with scalar and vector physical quantities are meaningful :*

(a) *Adding any two scalars.*

(b) *Adding a scalar to a vector of the same dimensions.*

(c) *Multiplying any vector by any scalar.*

(d) *Multiplying any two scalars.*

(e) *Adding any two vectors.*

(f) *Adding a component of a vector to the same vector.*

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Ans. (*a*) **No.** Only two such scalars can be added which represent the same physical quantity.

(*b*) **No.** A scalar cannot be added to a vector even of same dimensions because a vector has a direction while a scalar has no direction *e.g.*, speed cannot be added to velocity.

(*c*) **Yes.** We can multiply any vector by a scalar. For example, when mass (scalar) is multiplied with acceleration (vector), we get force (vector) *i.e.*, $\vec{F} = m\vec{a}$.

(*d*) **Yes.** We can multiply any two scalars. When we multiply power (scalar) with time (scalar), we get work done (scalar) *i.e.*, $W = Pt$.

(*e*) **No.** Only two vectors of same nature can be added by using the law of vector addition.

(*f*) **No.** A component of a vector can be added to the same vector only by using the law of vector addition. So the addition of a component of a vector to the same vector is not a meaningful algebraic operation.

4.5. *Read each statement below carefully and state with reasons, if it is true or false :*

(*a*) *The magnitude of a vector is always a scalar.*

(*b*) *Each component of a vector is always a scalar.*

(*c*) *The total path length is always equal to the magnitude of the displacement vector of a particle.*

(*d*) *The average speed of a particle (defined as total path length divided by the time taken to cover the path) is either greater or equal to the magnitude of average velocity of the particle over the same interval of time.*

(*e*) *Three vectors not lying in a plane can never add up to give a null vector.*

Ans. (*a*) **True.** The magnitude of a vector is a pure number and has no direction.

(*b*) **False.** Each component of a vector is also a vector.

(*c*) **False.** The displacement depends only on the end points while the path length depends on the actual path. The two quantities are equal only if the direction of motion of the object does not change. In all other cases, path length is greater than the magnitude of displacement.

(*d*) **True.** This is because the total path length is either greater than or equal to the magnitude of displacement over the same interval of time.

(*e*) **True.** This is because the resultant of two vectors will not lie in the plane of third vector and hence cannot cancel its effect to give null vector.

4.6. *Establish the following vector inequalities geometrically or otherwise :*

(*a*) $|\vec{a} + \vec{b}| \le |\vec{a}| + |\vec{b}|$ (*b*) $|\vec{a} + \vec{b}| \ge |\vec{a}| - |\vec{b}|$

(*c*) $|\vec{a} - \vec{b}| \le |\vec{a}| + |\vec{b}|$ (*d*) $|\vec{a} - \vec{b}| \ge |\vec{a}| - |\vec{b}|$

When does the equality sign above apply ?

Ans. (*a*) If θ be the angle between $\vec{a}$ and $\vec{b}$, then

$$|\vec{a} + \vec{b}| = \sqrt{|\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta}$$

Now $|\vec{a} + \vec{b}|$ will be maximum when

$\cos\theta = 1$ or $\theta = 0$

$$\therefore \quad |\vec{a} + \vec{b}|_{max} = \sqrt{|\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos 0^\circ}$$

$$= \sqrt{(|\vec{a}| + |\vec{b}|)^2} = |\vec{a}| + |\vec{b}|$$

Hence $|\vec{a} + \vec{b}| \le |\vec{a}| + |\vec{b}|$

The equality sign is applicable when $\theta = 0°$ *i.e.*, when $\vec{a}$ and $\vec{b}$ are in the same direction.

(*b*) Again

$$|\vec{a} + \vec{b}| = \sqrt{|\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta}$$

The value of $|\vec{a} + \vec{b}|$ will be minimum when

$\cos\theta = -1$

or $\theta = 180°$

$$\therefore \quad |\vec{a} + \vec{b}|_{min} = \sqrt{|\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos 180^\circ}$$

$$= \sqrt{(|\vec{a}| - |\vec{b}|)^2} = |\vec{a}| - |\vec{b}|$$

Hence $|\vec{a} + \vec{b}| \ge |\vec{a}| - |\vec{b}|$

The equality sign is applicable when $\theta = 180°$ *i.e.*, when $\vec{a}$ and $\vec{b}$ are in opposite directions.

(*c*) If θ is the angle between $\vec{a}$ and $\vec{b}$, then the angle between $\vec{a}$ and $-\vec{b}$ will be $(180° - \theta)$, as shown in Fig. 4.100.

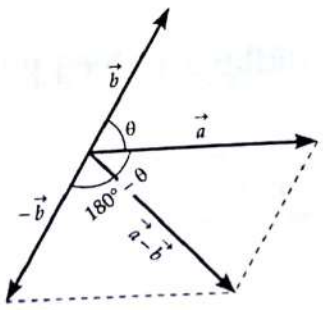

Fig. 4.100

$$\therefore \quad |\vec{a} - \vec{b}| = |\vec{a} + (-\vec{b})|$$

$$= \sqrt{|\vec{a}|^2 + |-\vec{b}|^2 + 2|\vec{a}||-\vec{b}|\cos(180° - \theta)}$$

$$= \sqrt{|\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos\theta}$$

$[\because \ |-\vec{b}| = |\vec{b}|, \cos(180° - \theta) = -\cos\theta]$

$|\vec{a} - \vec{b}|$ will be maximu

$\therefore \quad |\vec{a} - \vec{b}|_{max} = \sqrt{|\vec{a}|^2 + |}$

$= \sqrt{(|\vec{a}| + |}$

Hence $|\vec{a} - \vec{b}| \le |\vec{a}| +$

The equality sign is ap

(*d*) $|\vec{a} - \vec{b}|$ will be mi

$\cos\theta = 1$ or θ

$\therefore \quad |\vec{a} - \vec{b}|_{min} = \sqrt{|\vec{a}|^2}$

$= \sqrt{(|\vec{a}|}$

Hence $|\vec{a} - \vec{b}| \ge |\vec{a}|$

The equality sign is a

4.7. *Given* $\vec{a} + \vec{b} + \vec{c}$ *statements are correct :*

(*a*) $\vec{a}$, $\vec{b}$, $\vec{c}$, *and*

(*b*) *The magnitude* $(\vec{b} + \vec{d})$,

(*c*) The magnitu the sum of th

(*d*) $\vec{b} + \vec{c}$ must $\vec{d}$ are not col they are coll

Ans. (*a*) $\vec{a}$, $\vec{b}$, $\vec{c}$ a

The resultant of four vector in many way vectors may be equal but has the opposit $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ must

(*b*) Because $\vec{a} + \vec{b}$ *i.e.*, the magnitude of $(\vec{b} + \vec{d})$ but their dire statement is **correct.**

(*c*) Because, $\vec{a}$ +

Hence, magnitude o vector $(\vec{b} + \vec{c} + \vec{d})$. $\vec{b}$, $\vec{c}$ and $\vec{d}$ may be

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$|\vec{a} - \vec{b}|$ will be maximum when $\cos\theta = -1$ or $\theta = 180°$

$$\therefore |\vec{a} - \vec{b}|_{max} = \sqrt{|\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos 180°}$$

$$= \sqrt{(|\vec{a}| + |\vec{b}|)^2} = |\vec{a}| + |\vec{b}|$$

Hence $|\vec{a} - \vec{b}| \le |\vec{a}| + |\vec{b}|$

The equality sign is applicable when $\theta = 180°$.

(*d*) $|\vec{a} - \vec{b}|$ will be minimum when

$\cos\theta = 1$ or $\theta = 0°$

$$\therefore |\vec{a} - \vec{b}|_{min} = \sqrt{|\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos 0°}$$

$$= \sqrt{(|\vec{a}| - |\vec{b}|)^2} = |\vec{a}| - |\vec{b}|$$

Hence $|\vec{a} - \vec{b}| \ge |\vec{a}| - |\vec{b}|$

The equality sign is applicable when $\theta = 0°$.

4.7. *Given* $\vec{a} + \vec{b} + \vec{c} + \vec{d} = 0$, *which of the following statements are correct :*

(*a*) $\vec{a}$, $\vec{b}$, $\vec{c}$, *and* $\vec{d}$ *must each be a null vector,*

(*b*) *The magnitude of* $(\vec{a} + \vec{c})$ *equals the magnitude of* $(\vec{b} + \vec{d})$,

(*c*) The magnitude of $\vec{a}$ can never be greater than the sum of the magnitudes of $\vec{b}$, $\vec{c}$, and $\vec{d}$,

(*d*) $\vec{b} + \vec{c}$ must lie in the plane of $\vec{a}$ and $\vec{d}$ if $\vec{a}$ and $\vec{d}$ are not collinear, and in the line of $\vec{a}$ and $\vec{d}$, if they are collinear ?

Ans. (*a*) $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ need not each be a null vector. The resultant of four non-zero vectors can be a null vector in many ways *e.g.*, the resultant of any three vectors may be equal to the magnitude of fourth vector but has the opposite direction. Hence the statement $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ must each be a null vector, is **not correct.**

(*b*) Because $\vec{a} + \vec{b} + \vec{c} + \vec{d} = 0$, hence $\vec{a} + \vec{c} = -(\vec{b} + \vec{d})$ *i.e.*, the magnitude of $(\vec{a} + \vec{c})$ is equal to the magnitude of $(\vec{b} + \vec{d})$ but their directions are opposite. Hence the given statement is **correct.**

(*c*) Because, $\vec{a} + \vec{b} + \vec{c} + \vec{d} = 0$ or $\vec{a} = -(\vec{b} + \vec{c} + \vec{d})$. Hence, magnitude of vector $\vec{a}$ is equal to magnitude of vector $(\vec{b} + \vec{c} + \vec{d})$. The sum of the magnitudes of vectors $\vec{b}$, $\vec{c}$ and $\vec{d}$ may be greater than or equal to that of vector $\vec{a}$ (or vector $\vec{b} + \vec{c} + \vec{d}$). Hence the statement that the magnitude of $\vec{a}$ can never be greater than the sum of the magnitudes of $\vec{b}$, $\vec{c}$ and $\vec{d}$ is **correct.**

(*d*) Because $\vec{a} + \vec{b} + \vec{c} + \vec{d} = 0$, hence $(\vec{b} + \vec{c}) + \vec{a} + \vec{d} = 0$. The resultant sum of three vectors $\vec{b} + \vec{c}$, $\vec{a}$ and $\vec{d}$ can be zero only if $\vec{b} + \vec{c}$ is in plane of $\vec{a}$ and $\vec{d}$. In case $\vec{a}$ and $\vec{d}$ are collinear, $\vec{b} + \vec{c}$ must be in line of $\vec{a}$ and $\vec{d}$. Hence the given statement is **correct.**

4.8. *Three girls skating on a circular ice ground of radius 200 m start from a point P on the edge of the ground and reach a point Q diametrically opposite to P following different paths as shown in Fig. 4.101. What is the magnitude of the displacement vector for each ? For which girl is this equal to the actual length of path skated ?*

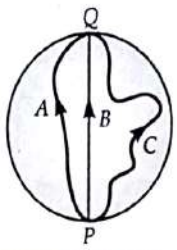

Fig. 4.101

Solution. Displacement of each girl = $\vec{PQ}$

Magnitude of displacement vector for each girl

$$= |\vec{PQ}| = 2 \times \text{radius} = 2 \times 200 = \mathbf{400\ m}.$$

For girl *B*, the magnitude of displacement vector = actual length of path.

4.9. *A cyclist travels from centre O of a circular park of radius 1 km and reaches point P. After cycling along 1/4 th of the circumference along PQ, he returns to the centre of the park along QO. If the total time taken is 10 minutes, calculate (i) net displacement (ii) average velocity and (ii) average speed of the cyclist.*

Ans. (*i*) Net displacement is zero as both initial and final positions are same.

(*ii*) Average velocity = $\dfrac{\text{displacement}}{\text{time taken}}$

As displacement is zero, average velocity of the cyclist is also **zero.**

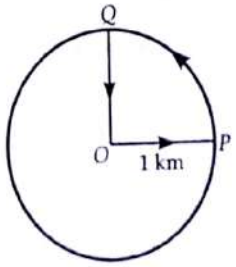

Fig. 4.102

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(iii) Total distance covered

$$= OP + \text{Arc } PQ + QQ = r + \frac{2\pi r}{4} + r$$

$$= 1 + \frac{2 \times 22 \times 1}{7 \times 4} + 1 = \frac{25}{7} \text{ km}$$

$$\text{Time taken} = 10 \text{ min} = \frac{1}{6} \text{ h}$$

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Time taken}}$$

$$= \frac{25/7 \text{ km}}{1/6 \text{ h}} = 21.43 \text{ kmh}^{-1}.$$

4.10. *On an open ground, a motorist follows a track that turns to his left by an angle of 60° after every 500 m. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of the displacement with the total path length covered by the motorist in each case.*

Ans. As shown in Fig. 4.103, suppose motorist starts from point *A*.

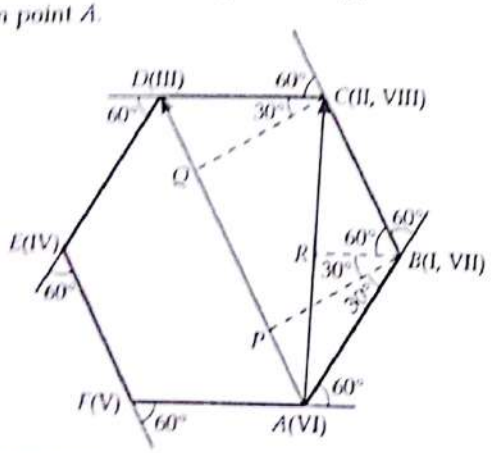

Fig. 4.103

Clearly, he will follow the hexagonal path *ABCDEFA*. The orders of the turns taken by him are indicated at the vertices of the hexagon.

(i) At the third turn, the motorist will be at *D*. The magnitude of displacement $\vec{AD}$ will be

$$= AP + PQ + QD$$

$$= AB \sin 30° + BC + CD \sin 30°$$

$$= 500 \times \frac{1}{2} + 500 + 500 \times \frac{1}{2} = 1000 \text{ m} = \mathbf{1 \text{ km}}.$$

The direction of $\vec{AD}$ is 60° left of the initial direction $\vec{AB}$.

Total path length

$$= AB + BC + CD = 500 \times 3 = 1500 \text{ m} = \mathbf{1.5 \text{ km}}.$$

(ii) At the sixth turn, the motorist comes back to the starting point *A*, so magnitude of displacement is *zero*.

Total path length

$$= AB + BC + CD + DE + EF + FA$$

$$= 500 \times 6 = 3000 \text{ m} = \mathbf{3 \text{ km}}.$$

(iii) At the eighth turn, the motorist will be at C. The magnitude of his displacement $\vec{AC}$ is

$$|\vec{AC}| = AR + RC = AB \sin 60° + BC \sin 60°$$

$$= 500 \times \frac{\sqrt{3}}{2} + 500 \times \frac{\sqrt{3}}{2} = 500\sqrt{3} = \mathbf{866 \text{ m}}.$$

The direction of $\vec{AC}$ is 30° left of the initial direction $\vec{AB}$.

Total path length $= 500 \times 8 = 4000 \text{ m} = \mathbf{4 \text{ km}}$.

4.11. *A passenger arriving in a new town wishes to go from the station to a hotel located 10 km away on a straight road from the station. A dishonest cabman takes him along a circuitous path 23 km long and reaches the hotel in 28 minutes. What is (i) the average speed of the taxi and (ii) the magnitude of average velocity ? Are the two equal ?*

Solution. Magnitude of displacement = 10 km

Total path length = 23 km

$$\text{Time taken} = 28 \text{ min} = \frac{28}{60} \text{ h} = \frac{7}{15} \text{ h}$$

(i) $$\text{Average speed} = \frac{\text{Total path length}}{\text{Time taken}}$$

$$= \frac{23 \text{ km}}{\frac{7}{15} \text{ h}} = \mathbf{49.3 \text{ kmh}^{-1}}.$$

(ii) Magnitude of average velocity

$$= \frac{\text{Displacement}}{\text{Time taken}} = \frac{10 \text{ km}}{\frac{7}{15} \text{ h}} = \mathbf{21.43 \text{ kmh}^{-1}}.$$

Clearly, the average speed and magnitude of the average velocity are not equal. They will be equal only for straight path.

4.12. *Rain is falling vertically with a speed of 30 ms^{-1}. A woman rides a bicycle with a speed of 10 ms^{-1} in the north to south direction. What is the relative velocity of rain with respect to the woman ? What is the direction in which she should hold her umbrella to protect herself from the rain ?*

Solution. The situation is shown in Fig. 4.104.

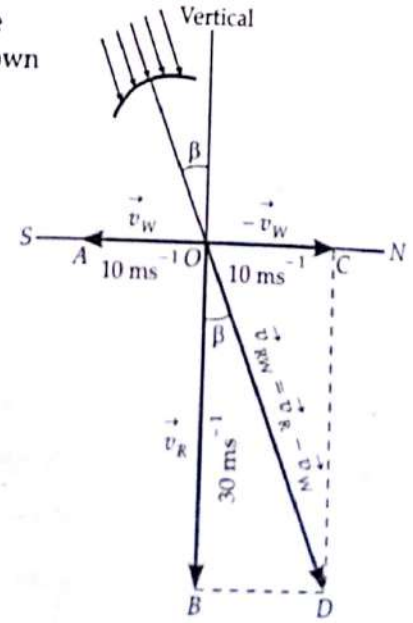

Fig. 4.104

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Here

$\vec{OA} = \vec{v}_W$ = velocity of woman cyclist

$= 10\ ms^{-1}$, due south

$\vec{OB} = \vec{v}_R$ = velocity of rain

$= 30\ ms^{-1}$, vertically downward

$\vec{OC} = -\vec{v}_W$

= Opposite velocity of the woman cyclist.

$\vec{OD} = \vec{v}_R + (-\vec{v}_W) = \vec{v}_R - \vec{v}_W = \vec{v}_{RW}$

= Velocity of rain relative to woman cyclist

$v_{RW} = OD = \sqrt{OC^2 + OB^2} = \sqrt{10^2 + 30^2}$

$= 10\sqrt{10} = \mathbf{31.6\ ms^{-1}}$

If OD makes angle β with the vertical, then

$$\tan\beta = \frac{BD}{OB} = \frac{OC}{OB} = \frac{10}{30} = 0.3333 \quad \text{or} \quad \beta = 18°26'.$$

The woman should hold her umbrella at 18°26' with the vertical in the direction of her motion *i.e.*, towards south.

4.13. *A man can swim with a speed of 4 kmh^{-1} in still water. How long does he take to cross the river 1 km wide, if the river flows steadily at 3 kmh^{-1} and he makes his strokes normal to the river current ? How far from the river does he go, when he reaches the other bank ?*

Solution. In Fig. 4.105, $\vec{v}_M$ and $\vec{v}_R$ represent the velocities of man and river. Clearly $\vec{v}$ is the resultant of these velocities. If the man begins to swim along AB, he will be deflected to the path AC by the flowing river.

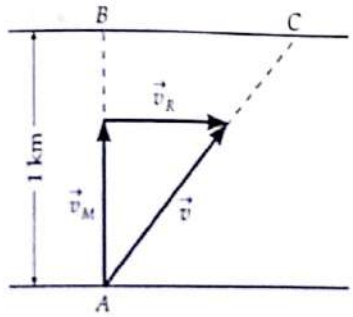

Fig. 4.105

Time taken to cover distance AC with velocity $\vec{v}$ will be same as the time taken to cover distance AB with velocity $\vec{v}_M$.

∴ Time taken by the man to cross the river is

$$t = \frac{AB}{v_M} = \frac{1\ km}{4\ km\ h^{-1}} = \frac{1}{4}\ h = \mathbf{15\ min.}$$

Distance through which the man goes down the river is

$$BC = v_R \times t = 3\ kmh^{-1} \times \frac{1}{4}\ h = \mathbf{0.75\ km.}$$

4.14. *In a harbour, wind is blowing at the speed of 72 kmh^{-1} and the flag on the mast of a boat anchored in the harbour flutters along the N-E direction. If the boat starts moving at a speed of 51 kmh^{-1} to the north, what is the direction of the flag on the mast of the boat ?*

Solution. When the boat is stationary, the flag flutters along N-E direction. This shows that velocity of the wind is along N-E direction. When the boat moves, the flag flutters along the direction of relative velocity of wind w.r.t the boat. Thus $\vec{v}_W = \vec{OA}$ = wind velocity = 72 kmh^{-1}, due N-E direction

Boat velocity = $\vec{v}_B = \vec{OB} = 51\ kmh^{-1}$, due north

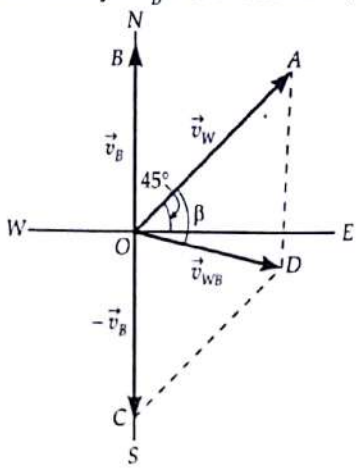

Fig. 4.106

Relative velocity of wind w.r.t. boat is given by

$$\vec{v}_{WB} = \vec{v}_W - \vec{v}_B = \vec{v}_W + (-\vec{v}_B)$$

$$= \vec{OA} + \vec{OC} = \vec{OD}$$

Clearly, the flag will flutter in the direction of $\vec{OD}$ on the mast of the moving boat.

Angle between $\vec{v}_W$ and $-\vec{v}_B$, $\theta = 45° + 90° = 135°$

If $\vec{v}_{WB}$ makes angle β with $\vec{v}_W$, then

$$\tan\beta = \frac{v_B \sin\theta}{v_W + v_B\cos\theta} = \frac{51\sin 135°}{72 + 51\cos 135°}$$

$$= \frac{51\sin 45°}{72 + 51(-\cos 45°)} = \frac{51 \times \frac{1}{\sqrt{2}}}{72 - 51 \times \frac{1}{\sqrt{2}}}$$

$$= \frac{51}{72\sqrt{2} - 51} = 1.0037$$

or $\beta = 45.01°$

Angle w.r.t east direction = 45.01° − 45° = 0.01°

Hence the flag will flutter almost in the east direction.

4.15. *The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 ms^{-1} can go without hitting the ceiling of the hall ?*

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Solution. Here $H = 25$ m, $u = 40\ \text{ms}^{-1}$

If the ball is thrown at an angle θ with the horizontal, then maximum height of flight,

$$H = \frac{u^2 \sin^2\theta}{2g}$$

$$\therefore \quad 25 = \frac{(40)^2 \sin^2\theta}{2 \times 9.8}$$

or $$\sin^2\theta = \frac{25 \times 2 \times 9.8}{(40)^2} = 0.306$$

or $$\sin\theta = \sqrt{0.306} = 0.554$$

and $$\cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - 0.306}$$

$$= \sqrt{0.694} = 0.833$$

The maximum horizontal distance is given by

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{2u^2 \sin\theta\cos\theta}{g}$$

$$= \frac{2 \times (40)^2 \times 0.554 \times 0.833}{9.8} = \mathbf{150.7\ m.}$$

4.16. *A cricketer can throw a ball to a maximum horizontal distance of 100 m. How high above the ground can the cricketer throw the same ball ?* **[Delhi 98]**

Solution. Let u be the velocity of projection. Then

$$R_{max} = \frac{u^2}{g} = 100\ \text{m}$$

or $$u^2 = 100g \quad \text{or} \quad u = \sqrt{100g}$$

For upward throw of the ball, we have

$$u = \sqrt{100g}, \ v = 0, \ a = -g, \ s = ?$$

As $$v^2 - u^2 = 2as$$

$$\therefore \quad 0 - 100g = 2(-g)s$$

or $$s = \frac{-100g}{-2g} = \mathbf{50\ m.}$$

Thus the cricketer can throw the same ball to a height of 50 m.

4.17. *A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 seconds, what is the magnitude and direction of acceleration of the stone ?*

Solution. Here $r = 80$ cm, $\nu = \frac{14}{25}$ rps

$$\therefore \quad \omega = 2\pi\nu = 2 \times \frac{22}{7} \times \frac{14}{25} = \frac{88}{25}\ \text{rad s}^{-1}$$

The acceleration of the stone is

$$a = r\omega^2 = 80 \times \left(\frac{88}{25}\right)^2 = \mathbf{991.2\ cm\ s^{-2}.}$$

This acceleration is directed along the radius of the ular path towards the centre of the circle.

4.18. *An aircraft executes a horizontal loop of radius 1 km with a steady speed of 900 km h^{-1}. Compare its centripetal acceleration with the acceleration due to gravity.*

Solution. Here $r = 1$ km $= 1000$ m

$$v = 900\ \text{kmh}^{-1} = \frac{900 \times 5}{18} = 250\ \text{ms}^{-1}$$

Centripetal acceleration,

$$a = \frac{v^2}{r} = \frac{(250)^2}{1000} = 62.5\ \text{ms}^{-2}$$

$$\therefore \quad \frac{\text{Centripetal acceleration}}{\text{Acceleration due to gravity}} = \frac{62.5}{9.8} = 6.38$$

4.19. *Read each statement below carefully and state, with reasons, if it is true or false :*

(a) The net acceleration of a particle in circular motion is always along the radius of the circle towards the centre.

(b) The velocity vector of a particle at a point is always along the tangent to the path of the particle at that point.

(c) The acceleration vector of a particle in uniform circular motion averaged over one cycle is a null vector.

Solution. *(a)* **False.** The net acceleration of a particle in circular motion is towards the centre only if its speed is constant.

(b) **True.** A particle released at any point of its path will always move along the tangent to the path at that point.

(c) **True.** For any two diametrically opposite points on the circumference, the acceleration vectors are equal and opposite. Hence the acceleration vector averaged over one complete cycle is a null vector.

4.20. *The position of a particle is given by*

$$\vec{r} = 3.0t\,\hat{i} - 2.0t^2\,\hat{j} + 4.0\,\hat{k}\ \text{m}$$

where t is in seconds and the coefficients have the proper units for $\vec{r}$ to be in metres.

(a) Find the $\vec{v}$ and $\vec{a}$ of the particle. (b) What is the magnitude and direction of velocity of the particle at $t = 2$ s ? **[Delhi 10]**

Solution. *(a)* Given :

$$\vec{r}(t) = 3.0t\,\hat{i} - 2.0t^2\,\hat{j} + 4.0\,\hat{k}\ \text{m}$$

$$\vec{v}(t) = \frac{d\vec{r}}{dt} = \frac{d}{dt}(3.0t\,\hat{i} - 2.0t^2\,\hat{j} + 4.0\,\hat{k}\,)$$

$$= \mathbf{3.0\,\hat{i} - 4.0t\,\hat{j}}$$

$$\vec{a}(t) = \frac{d\vec{v}}{dt} = \frac{d}{dt}(3.0\,\hat{i} - 4.0t\,\hat{j}\,) = \mathbf{-4.0\,\hat{j}}\ .$$

(b) At $t = 2$ s, $\quad \vec{v} = 3.0\,\hat{i} - 8.0\,\hat{j}$

The magnitude of the velocity is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{3^2 + (-8)^2} = \sqrt{73} = \mathbf{8.54\ ms^{-1}.}$$

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The direction of velocity is given by

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(-\frac{8}{3}\right)$$

$$= -\tan^{-1}(-2.6667)$$

$$= -70^\circ \text{ with } x\text{-axis.}$$

4.21. *A particle starts from the origin at $t = 0$ s with a velocity of $10.0\,\hat{j}$ m/s and moves in the x-y plane with a constant acceleration of $(8.0\,\hat{i} + 2.0\,\hat{j})$ ms^{-2}. (a) At what time is the x-coordinate of the particle 16 m ? What is the y-coordinate of the particle at the time ? (b) What is the speed of the particle at the time ?*

Solution. (*a*) Here initial velocity,

$$\vec{v}_0 = 10.0\,\hat{j} \text{ ms}^{-1}$$

Acceleration,

$$\vec{a} = (8.0\,\hat{i} + 2.0\,\hat{j}) \text{ ms}^{-2}$$

The position of the particle at any instant t will be

$$\vec{r}(t) = \vec{v}_0 t + \frac{1}{2}\vec{a}\, t^2$$

$$= 10.0\,\hat{j}\, t + \frac{1}{2}(8.0\,\hat{i} + 2.0\,\hat{j})\, t^2$$

or $x(t)\,\hat{i} + y(t)\,\hat{j} = 4.0t^2\,\hat{i} + (10.0t + 1.0t^2)\,\hat{j}$

$\therefore \quad x(t) = 4.0t^2$

$y(t) = 10.0t + 1.0t^2$

Given $x(t) = 16$ m, $t = ?$

$4.0t^2 = 16$

$\Rightarrow \quad t = \mathbf{2\ s.}$

At $t = 2$ s, $y = 10.0 \times 2 + 1.0 \times 2^2 = 24$ m.

(*b*) Velocity,

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$= \frac{d}{dt}[4.0t^2\,\hat{i} + (10.0t + 1.0t^2)\,\hat{j}]$$

$$= 8.0t\,\hat{i} + (10.0 + 2.0t)\,\hat{j}$$

At $t = 2$ s, $\vec{v} = 16.0\,\hat{i} + 14.0\,\hat{j}$

Speed, $v = \sqrt{v_x^2 + v_y^2} = \sqrt{16^2 + 14^2}$

$$= \sqrt{256 + 196} = \sqrt{452} = \mathbf{21.26\ ms^{-1}}$$

4.22. (*a*) *If $\hat{i}$ and $\hat{j}$ are unit vectors along X- and Y-axis respectively, then what is the magnitude and direction of $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$?*

(*b*) *Find the components of $\vec{a} = 2\,\hat{i} + 3\,\hat{j}$ along the directions of vectors $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$.* **[Delhi 14]**

Solution. (*a*) In Fig. 4.107, $\vec{OA} = \hat{i}$, $\vec{AB} = \hat{j}$, $\vec{AC} = -\hat{j}$.

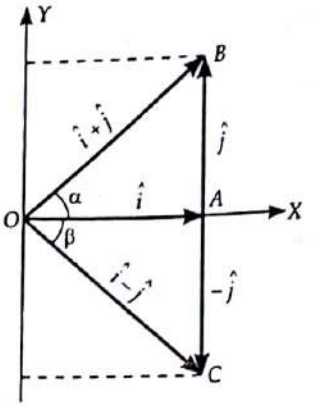

Fig. 4.107

Applying Δ law of vector addition in ΔOAB,

$$\vec{OB} = \vec{OA} + \vec{AB} = \hat{i} + \hat{j}$$

$$\therefore \quad |\hat{i} + \hat{j}| = OB = \sqrt{OA^2 + AB^2} = \sqrt{1^2 + 1^2} = \sqrt{2}.$$

Direction of $\hat{i} + \hat{j}$ is given by

$$\tan\alpha = \frac{BA}{OA} = \frac{1}{1} = 1$$

$$\therefore \quad \alpha = \mathbf{45^\circ}.$$

Again, $\vec{OC} = \vec{OA} + \vec{AC} = \hat{i} + (-\hat{j}) = \hat{i} - \hat{j}$

$$\therefore \quad |\hat{i} - \hat{j}| = OC = \sqrt{OA^2 + AC^2} = \sqrt{1^2 + 1^2} = \sqrt{2}.$$

Direction of $\hat{i} - \hat{j}$ is given by

$$\tan\beta = \frac{AC}{OA} = \frac{-1}{1} = -1$$

$$\therefore \quad \beta = \mathbf{-45^\circ}.$$

(*b*) Given $\vec{a} = 2\,\hat{i} + 3\,\hat{j}$ and let $\vec{b} = \hat{i} + \hat{j}$ and $\hat{c} = \hat{i} - \hat{j}$. Then component of $\vec{a}$ in the direction $\vec{b}$

$$= (a\cos\theta)\,\hat{b}$$

$$= \frac{ab\cos\theta}{b}\frac{\vec{b}}{b} = \frac{\vec{a}\,.\,\vec{b}}{|\vec{b}|^2}\vec{b}$$

$$= \frac{(2\,\hat{i} + 3\,\hat{j})\,.\,(\hat{i} + \hat{j})}{[\sqrt{1^2 + 1^2}]^2}(\hat{i} + \hat{j})$$

$$= \frac{2 \times 1 + 3 \times 1}{2}(\hat{i} + \hat{j}) = \frac{5}{2}(\hat{i} + \hat{j}).$$

Component of $\vec{a}$ in the direction of $\vec{c}$

$$= \frac{\vec{a}\,.\,\vec{c}}{|\vec{c}|^2}\vec{c} = \frac{(2\,\hat{i} + 3\,\hat{j})\,.\,(\hat{i} - \hat{j})}{[\sqrt{1^2 + 1^2}]^2}(\hat{i} - \hat{j})$$

$$= \frac{2 \times 1 + 3 \times (-1)}{2}(\hat{i} - \hat{j}) = -\frac{1}{2}(\hat{i} - \hat{j}).$$

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4.23. *For any arbitrary motion in space, which of the following relations are true :*

(a) $\vec{v}_{av} = \dfrac{\vec{v}(t_1) + \vec{v}(t_2)}{2}$ (b) $\vec{v}_{av} = \dfrac{\vec{r}(t_2) - \vec{r}(t_1)}{t_2 - t_1}$

(c) $\vec{v}(t) = \vec{v}(0) + \vec{a}t$ (d) $\vec{r}(t) = \vec{r}(0) + \vec{v}(0)\,t + \frac{1}{2}\vec{a}t^2$

(e) $\vec{a}_{av} = \dfrac{\vec{v}(t_2) - \vec{v}(t_1)}{t_2 - t_1}$.

Ans. As the motion is arbitrary, the acceleration may not be uniform. So the relations (c) and (d) cannot be true.

For an arbitrary motion, the average velocity cannot be defined as in equation (a), so relation (a) is not true.

Only relations (b) and (e) are true.

4.24. *Read each statement below carefully and state, with reasons and examples, if it is true or false :*

A scalar quantity is one that (a) is conserved in a process, (b) can never take negative values, (c) must be dimensionless, (d) does not vary from one point to another in space, (e) has the same value for observers with different orientations of axes.

Solution. (*a*) **False.** Kinetic energy (scalar) is not conserved in an inelastic collision. Moreover, vector quantities like linear momentum, angular momentum, etc., are also conserved.

(*b*) **False.** Scalar quantities such as electric potential, temperature, etc., can take negative values.

(*c*) **False.** Scalar quantities like mass, density, energy, etc., are not dimensionless.

(*d*) **False.** Density (scalar) varies from point to point in the atmosphere.

(*e*) **True.** The mass (scalar) of a body as measured by different observers with different orientations of axes has the same value.

4.25. *An aircraft is flying at a height of 3400 m above the ground. If the angle subtended at a ground observation point by the aircraft positions 10 s apart is 30°, what is the speed of the aircraft ?* **[Central Schools 12]**

Solution. Let *A* and *B* represent the two aircraft positions separated 10 s apart (Fig. 4.108).

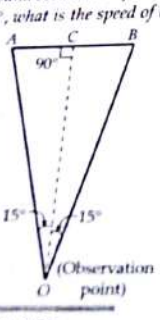

Fig. 4.108

Then $\tan 15° = \dfrac{x}{3400}$

or $x = 3400 \tan 15°$

$= 3400 \times 0.2679$

$= 910.86$ m.

Speed of aircraft

$= \dfrac{910.86 \text{ m}}{5 \text{ s}} = \mathbf{182.2\ ms^{-1}}$.

4.26. *A vector has magnitude and direction. Does it have a location in space ? Can it vary with time ? Will two equal vectors $\vec{a}$ and $\vec{b}$ at different locations in space necessarily have identical physical effects ? Give examples in support of your answer.*

Solution. In addition to magnitude and direction, each vector also has a definite location in space. For example, a velocity vector has definite location at every point of uniform circular motion.

A vector can vary with time. For example, increase in velocity produces acceleration.

Two equal vectors $\vec{a}$ and $\vec{b}$ having different locations may not produce identical physical effects. For example, two equal forces (vectors) acting at two different points may not produce equal turning effects.

4.27. *A vector has both magnitude and direction. Does it mean that anything that has magnitude and direction is necessarily a vector ? The rotation of a body can be specified by the direction of the axis of rotation, and the angle of rotation about the axis. Does that make any rotation a vector ?*

Solution. No, anything that has both magnitude and direction is not necessarily a vector. It must obey the laws of vector addition.

Rotation is not generally considered a vector even though it has magnitude and direction because the addition of two finite rotations does not obey commutative law. However, infinitesimally small rotations obey commutative law and hence an infinitesimally small rotation is considered a vector.

4.28. *Can you associate vectors with (a) the length of a wire bent into a loop, (b) a plane area, (c) a sphere ? Explain.*

Solution. Out of these, only a plane area can be associated with a vector. The direction of this *area vector* is taken normal to the plane.

4.29. *A bullet fired at an angle of 30° with the horizontal hits the ground 3 km away. By adjusting the angle of projection, can one hope to hit a target 5 km away ? Assume the muzzle speed to be fixed and neglect air resistance.* **[Central Schools 14]**

Solution. In the first case, $R = 3$ km $= 3000$ m, $\theta = 30°$

Horizontal range, $R = \dfrac{u^2 \sin 2\theta}{g}$

$\therefore \quad 3000 = \dfrac{u^2 \sin 60°}{g}$

or $\dfrac{u^2}{g} = \dfrac{3000}{\sin 60°} = \dfrac{3000 \times 2}{\sqrt{3}} = 2000\sqrt{3}$

Maximum horizontal range,

$$R_{max} = \frac{u^2}{g} = 2000\sqrt{3} \text{ m} = 3464 \text{ m} = \mathbf{3.46\ km}.$$

But distance of the target (5 km) is greater than the maximum horizontal range of 3.46 km, so the target cannot be hit by adjusting the angle of projection.

4.30. *A fighter plane flying horizontally at an altitude of 1.5 km with a speed 720 kmh^{-1} passes directly overhead an antiaircraft gun. At what angle from the vertical should the gun be fired for the shell muzzle speed 600 ms^{-1} to hit the plane ? At what maximum altitude should the pilot fly the plane to avoid being hit ? Take $g = 10$ ms^{-2}.*

Solution. Speed of plane $= 720$ kmh^{-1} $= 200$ ms^{-1}.

The shell moves along curve *OL*. The plane moves along *PL*. Let them hit after a time *t*.

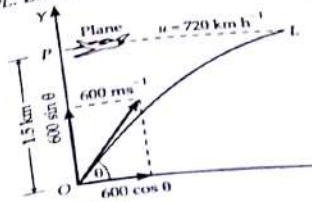

Fig. 4.109

For hitting, horizontal distance travelled by t

= Horizontal distance travelled by th

Horizontal velocity of plane × t

or = Horizontal velocity of shell × t

$200 \times t = 600 \cos\theta \times t$

$\cos\theta = \dfrac{200}{600} = \dfrac{1}{3}$ or $\theta = 70°30'$

The shell should be fired at an angle of 7 horizontal or 19°30' with the vertical.

The maximum height of flight of the sh

$$h = \frac{u^2 \sin^2\theta}{2g} = \frac{u^2(1 - \cos^2\theta)}{2g}$$

$$= \frac{(600)^2 \times (1 - \frac{1}{9})}{2 \times 10} = 16000 \text{ m} =$$

Thus the pilot should fly the plan altitude of 16 km to avoid being hit by

4.31. *A cyclist is riding with a speed approaches a circular turn on the road of r brakes and reduces his speed at the co What is the magnitude and direction of the cyclist on the circular turn ?*

Solution. Here $r = 80$ m

$v = 27$ kmh^{-1} $= \dfrac{27}{1}$

Centripetal acceleration, $a_c =$

Suppose the cyclist applies br circular turn, then, tangential a will act opposite to velocity.

Given $a_T = 0.5$ ms^{-2}

As the accelerations a_C and a_T are perpendicular to each other, so the net acceleration of the cyclist is

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Solution. Speed of plane = 720 kmh^{-1} = 200 ms^{-1}.

The shell moves along curve OA. The plane moves along PA. Let them hit after a time t.

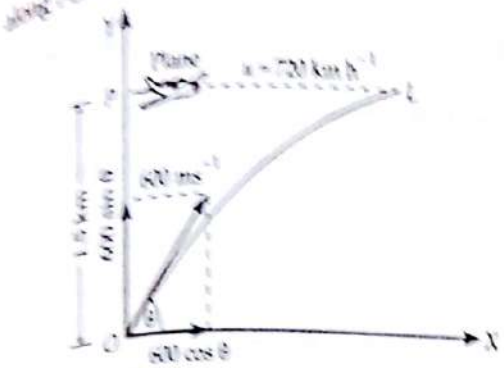

Fig. 4.109

For hitting, horizontal distance travelled by the plane = Horizontal distance travelled by the shell.

Horizontal velocity of plane × t = Horizontal velocity of shell × t

or $200 \times t = 600 \cos\theta \times t$

$$\cos\theta = \frac{200}{600} = \frac{1}{3} \quad \text{or} \quad \theta = 70°30'$$

The shell should be fired at an angle of 70°30' with the horizontal or 19°30' with the vertical.

The maximum height of flight of the shell is

$$h = \frac{u^2 \sin^2\theta}{2g} = \frac{u^2(1-\cos^2\theta)}{2g}$$

$$= \frac{(600)^2 \times (1-\frac{1}{9})}{2 \times 10} = 16000 \text{ m} = \mathbf{16 \text{ km}}.$$

Thus the pilot should fly the plane at a minimum altitude of 16 km to avoid being hit by the shell.

4.31. *A cyclist is riding with a speed of 27 km h^{-1}. As he approaches a circular turn on the road of radius 80 m, he applies brakes and reduces his speed at the constant rate 0.5 ms^{-2}. What is the magnitude and direction of the net acceleration of the cyclist on the circular turn ?*

Solution. Here r = 80 m

$$v = 27 \text{ kmh}^{-1} = \frac{27 \times 5}{18} \text{ ms}^{-1} = 7.5 \text{ ms}^{-1},$$

Centripetal acceleration, $a_c = \frac{v^2}{r} = \frac{(7.5)^2}{80} = 0.7 \text{ ms}^{-2}$

Suppose the cyclist applies brakes at the point A of the circular turn, then, tangential acceleration a_T (negative) will act opposite to velocity.

Given $a_T = 0.5 \text{ ms}^{-2}$

As the accelerations a_C and a_T are perpendicular to each other, so the net acceleration of the cyclist is

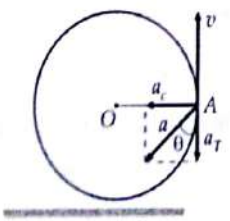

Fig. 4.110

$$a = \sqrt{a_c^2 + a_t^2} = \sqrt{(0.7)^2 + (0.5)^2}$$

$$= \sqrt{0.49 + 0.25} = \sqrt{0.74} = 0.86 \text{ ms}^{-2},$$

If θ is the angle between the total acceleration and the velocity of the cyclist, then

$$\tan\theta = \frac{a_c}{a_t} = \frac{0.7}{0.5} = 1.4 \quad \text{or} \quad \theta = 54°28'.$$

4.32. *(a) Show that for a projectile the angle between the velocity and the X-axis as a function of time is given by*

$$\theta(t) = \tan^{-1}\frac{(v_{oy} - gt)}{v_{ox}}$$

(b) Show that the projection angle θ_0 for a projectile launched from the origin is given by :

$$\theta_0 = \tan^{-1}\left(\frac{4h_m}{R}\right)$$

where the symbols have their usual meaning.

Ans. *(a)* As shown in Fig. 4.111, suppose the projectile is thrown with a velocity v_0 at an angle θ_0 with the X-axis.

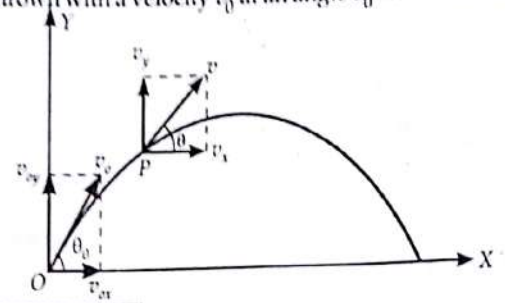

Fig. 4.111

Let v_{ox} and v_{oy} be the components of velocity v_0 along X- and Y-directions respectively.

At any time t, suppose the particle is at point P. Its velocity v has components v_x and v_y along X- and Y-directions. Then

$$v_x = v_{0x} \qquad \text{[Horizontal component}$$

and $$v_y = v_{0y} - gt \qquad \text{remains unchanged]}$$

If the velocity v makes angle θ with X-axis, then

$$\tan\theta = \frac{v_y}{v_x} = \frac{v_{0y} - gt}{v_{0x}} \quad \therefore \quad \theta = \tan^{-1}\left(\frac{v_{0y} - gt}{v_{0x}}\right)$$

(b) Maximum height attained by the projectile ,

$$h_m = \frac{v_0^2 \sin^2\theta_0}{2g}$$

Horizontal range of the projectile,

$$R = \frac{v_0^2 \sin 2\theta_0}{g} = \frac{v_0^2 \times 2\sin\theta_0 \cos\theta_0}{g}$$

$$\therefore \quad \frac{h_m}{R} = \frac{v_0^2 \sin^2\theta_0}{2g} \times \frac{g}{v_0^2 \times 2\sin\theta_0\cos\theta_0} = \frac{\tan\theta_0}{4}$$

or $$\tan\theta_0 = \frac{4h_m}{R}$$

Hence, $$\theta_0 = \tan^{-1}\frac{4h_m}{R}.$$

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Text Based Exercises

Type A : Very Short Answer Questions — 1 Mark Each

1. Does a scalar quantity depend on the frame of reference chosen ?
2. A given quantity has both magnitude and direction. Is it necessarily a vector ?
3. When is the sum of two vectors (*i*) maximum and (*ii*) minimum ?
4. When can the addition of two vectors be zero ?
5. Is the magnitude of $(\vec{A} + \vec{B})$ same as that of $(\vec{B} + \vec{A})$?
6. Under what condition the sum and difference of two vectors will be equal in magnitude ?
7. What is a resultant vector ?
8. The magnitude of the resultant of two vectors of magnitudes 5 and 3 is 2. What is the angle between the two vectors ?
9. Can three vectors of different magnitudes be combined to give a zero resultant ?
10. What is the minimum number of forces acting on an object in a plane that can produce a zero resultant force ? **[Delhi 04]**
11. What is the minimum number of forces of equal magnitude whose vector sum is zero ?
12. Does it make a sense to call a physical quantity a vector, when its magnitude is zero ?
13. Give an example of a zero vector.
14. If $|\vec{P}| > |\vec{Q}|$, what is the angle between the maximum resultant and minimum resultant of the two vectors $\vec{P}$ and $\vec{Q}$?
15. Is the division of vectors defined ?
16. Can the sum of two vectors be a scalar ?
17. Can any of the components of a given vector have greater magnitude than that of the vector itself ?
18. Fifteen vectors, each of magnitude 5 units, are represented by the sides of a closed polygon, all taken in same order. What will be their resultant ?
19. Under what condition is the scalar product of two non-zero vectors zero ?
20. Give one example of dot product of vectors. Also give its unit. **[Delhi 98]**
21. What is the magnitude of the vector $2\hat{i} - 3\hat{j} + \sqrt{3}\,\hat{k}$?
22. Show that $\vec{A} \cdot \vec{A} = A^2$.
23. Can the magnitude of the resultant vector of two given vectors be less than the magnitude of either vector ?
24. The magnitudes of vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ are 12, 5 and 13 units respectively and $\vec{A} + \vec{B} = \vec{C}$. What is the angle between $\vec{A}$ and $\vec{B}$? **[Delhi 96]**
25. If $\vec{A}$, $\vec{B}$ and $\vec{C}$ are mutually perpendicular vectors, then what is the value of $\vec{A} \cdot (\vec{B} + \vec{C})$?
26. What is the angle between $(\hat{i} + \hat{j})$ and $(\hat{i} - \hat{j})$?
27. A unit vector is represented by $a\hat{i} + b\hat{j} + c\hat{k}$. If the values of a and b are 0.6 and 0.8 respectively, find the value of c. **[Delhi 05]**
28. Given four forces :

$$\vec{F}_1 = 3\hat{i} - \hat{j} + 9\hat{k}, \qquad \vec{F}_2 = 2\hat{i} - 2\hat{j} + 16\hat{k},$$

$$\vec{F}_3 = 9\hat{i} + \hat{j} + 18\hat{k} \quad \text{and} \quad \vec{F}_4 = \hat{i} + 2\hat{j} - 18\hat{k}.$$

If all these forces act on a particle at rest at the origin of a coordinate system, then identify the plane in which the particle would begin to move ?

29. Two vectors of magnitudes 3 and 4 give a resultant of magnitude 2. What must be the dot product of these two vectors ?
30. Two vectors of magnitudes 5 and 3 give a resultant of magnitude 2. What must be the angle between the two vectors ?
31. When vectors are added or substrated, their resultant is a vector. Is it also true in case of multiplication of two vectors ?
32. Give an example of a physical quantity which can be represented as a vector product of two vectors.
33. What is the angle between the vectors $\vec{A} \times \vec{B}$ and $\vec{B} \times \vec{A}$?
34. What is the angle between two vectors if the ratio of their dot product and the magnitude of cross product is $\sqrt{3}$?
35. If $\vec{A} \times \vec{B} = \vec{0}$, what can be said about the non-zero vectors $\vec{A}$ and $\vec{B}$? **[Delhi 97]**
36. What will be the angle between $\vec{A}$ and $\vec{B}$, if the area of the parallelogram drawn with $\vec{A}$ and $\vec{B}$ as its adjacent sides is $\frac{1}{2} AB$?

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37. Two vectors of magnitude A and $\sqrt{3}A$ are perpendicular to each other. What is the angle which their resultant makes with $\vec{A}$?

38. If $\vec{A} + \vec{B} + \vec{C} + \vec{D} = \vec{0}$, can the magnitude of $\vec{A} + \vec{B} + \vec{C}$ be equal to the magnitude of $\vec{D}$?

39. What are horizontal and vertical components of acceleration of a body thrown horizontally with uniform speed ?

40. Velocity of a projectile is $10\ ms^{-1}$. At what angle to the horizontal should it be projected so that it covers maximum horizontal distance ?

[Delhi 99]

41. Can a body possess velocity at the same time in horizontal and vertical directions ?

42. Name the quantity which remains unchanged during the flight of an oblique projectile.

43. A football is thrown in a parabolic path. Is there any point at which the acceleration is perpendicular to the velocity ?

44. A projectile fired from the ground follows a parabolic path. The speed of the projectile is minimum at the top of its path. State whether this statement is true or false.

45. Is it true that the instantaneous velocity of a projectile is tangential to its parabolic path ?

46. A body is projected at an angle of 45° with a velocity of $9.8\ ms^{-1}$. What will be its horizontal range ?

47. At what angle to the horizontal should a body be projected so that the maximum height reached is equal to the horizontal range ?

48. Why does the direction of motion of a projectile become horizontal at the highest point of its trajectory ?

49. A projectile is fired with kinetic energy 1 kJ. If its range is maximum, what will be its kinetic energy at the highest point of its trajectory ?

50. A body is projected so that it has maximum range R. What is the maximum height reached during the flight ?

51. What is the maximum vertical height to which a cricketer can throw a ball if he can throw it to a maximum horizontal distance of 160 m ?

52. Two bodies are projected at angles θ and $(90° - \theta)$ to the horizontal with the same speed. Find the ratio of their time of flight.

53. A particle moves in a plane with uniform acceleration in a direction different from its initial velocity. What will be the trajectory followed by the particle ?

54. Name two quantities which have maximum values when the maximum height attained by the projectile is the largest.

55. The range of a projectile when launched at an angle of 15° to the horizontal is 1.5 km. What is the range of the projectile when launched at an angle of 45° to the horizontal ?

56. What are the angles of projection of a projectile projected with velocity $30\ ms^{-1}$, so that the horizontal range is 45 m ?
Take $g = 10\ ms^{-2}$.

57. A bomb is dropped from an aeroplane flying horizontally with a speed of $720\ kmh^{-1}$, at an altitude of 980 m. After what time, the bomb will hit the ground ?

58. A ball is thrown at an angle of 45° to the horizontal with kinetic energy K. What is the kinetic energy at the highest point of trajectory ?

59. A ball is thrown at an angle θ with the horizontal. Its time of flight is 2 s. Its horizontal range is 100 m. What is the horizontal component of velocity of projection ?
Take $g = 10\ ms^{-2}$.

60. The maximum range of a projectile is $2/\sqrt{3}$ times its actual range. What is the angle of projection for actual range ? **[Central Schools 08]**

61. What is the angular velocity of the hour hand of a clock ?

62. What furnishes the centripetal acceleration for the earth to go round the sun ?

63. Express the unit vector $\hat{A}$ in a mathematical form.

[Himachal 03]

64. Find a unit vector parallel to the vector $3\hat{i} + 7\hat{j} + 4\hat{k}$. **[Chandigarh 02]**

65. Evaluate $\hat{k} \cdot \hat{k}$. **[Himachal 01]**

66. Can two non-zero vectors give zero resultant when they multiply with each other ? If yes, give condition for the same. **[Delhi 06]**

67. A vector $\vec{A}$ is expressed as $\vec{A} = A_x\hat{i} + A_y\hat{j}$ where A_x and A_y are its components along x-axis and y-axis respectively. If vector $\vec{A}$ makes an angle θ with x-axis, then θ is given by which expression ?

[Delhi 08]

68. What is the value of $\vec{A} \times \vec{A}$? **[Central Schools 12]**

69. What can be the maximum and minimum values of $(\vec{A} + \vec{B})$ and $(\vec{A} - \vec{B})$? **[Delhi 14]**

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Answers

1. No. A scalar quantity does not depend on the frame of reference.
2. No. The given quantity will be a vector only if it obeys the laws of vector addition.
3. (*i*) When two vectors have same direction, their sum is maximum.
   (*ii*) When two vectors have opposite directions, their sum is minimum.
4. The addition of two vectors can be zero only when these vectors have equal magnitude and opposite directions.
5. Yes. $|\vec{A} + \vec{B}| = |\vec{B} + \vec{A}|$.
6. When the two vectors are perpendicular to each other.
7. The resultant of two or more vectors is that single vector which produces the same effect as the individual vectors together produce.
8. 180°, *i.e.*, the two vectors have opposite directions.
9. Yes, when three vectors are represented by the three sides of a triangle taken in order, they will have resultant null-vector.
10. Three forces, provided they can be represented by the three sides of a triangle taken in the same order.
11. Two forces, provided the two forces have opposite directions.
12. Yes. A zero vector has a definite physical significance.
13. The displacement of a ball thrown up and received back by the thrower is a zero vector.
14. 0°.
15. No.
16. No, the sum of two vectors is always a vector.
17. Yes, provided the components are not rectangular.
18. Zero vector. The vector sum of all the vectors represented by the sides of a closed polygon taken in the same order is zero.
19. When they are mutually perpendicular to each other.
20. Work done is equal to the dot product of force and displacement vectors.

$W = \vec{F} \cdot \vec{S}$. The SI unit of work is joule.

1. $|2\hat{i} - 3\hat{j} + \sqrt{3}\hat{k}| = \sqrt{2^2 + (-3)^2 + (\sqrt{3})^2}$

$= \sqrt{4 + 9 + 3} = 4$

$\vec{A} \cdot \vec{A} = |\vec{A}||\vec{A}| \cos 0° = |\vec{A}|^2 = A^2$.

Yes. This is possible when the angle θ between the two given vectors lies between 90° and 270°, because in that case cos θ is negative.

24. Clearly $12^2 + 5^2 = 13^2$ *i.e.*, $A^2 + B^2 = C^2$. Hence the angle between $\vec{A}$ and $\vec{B}$ is 90°.

25. $\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$

$= AB \cos 90° + AC \cos 90° = 0$.

26. 90°, because $(\hat{i} + \hat{j}) \cdot (\hat{i} - \hat{j}) = 1.1 - 1.1 = 0$.

27. $\sqrt{0.6^2 + 0.8^2 + c^2} = 1 \quad \therefore \quad c = 0$.

28. Resultant force,

$\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4} = 15\hat{i} + 0\hat{j} + 25\hat{k}$.

Clearly, the particle will move in the X-Z plane.

29. As $3^2 + 4^2 = 5^2$, so the two vectors are perpendicular to each other. Their dot product will be **zero**.

30. As 5 – 3 = 2, this indicates that the two vectors have opposite directions and angle between them = 180°.

31. May or may not be. The dot product of two vectors gives a scalar quantity while the cross product gives a vector quantity.

32. Torque $\vec{\tau}$ is the vector product of position vector $\vec{r}$ and force vector $\vec{F}$, *i.e.*, $\vec{\tau} = \vec{r} \times \vec{F}$.

33. 180°, because $\vec{A} \times \vec{B}$ and $\vec{B} \times \vec{A}$ are antiparallel.

34. $\dfrac{\vec{A} \cdot \vec{B}}{|\vec{A} \times \vec{B}|} = \dfrac{AB \cos \theta}{AB \sin \theta} = \cot \theta = \sqrt{3}$.

or $\tan \theta = \dfrac{1}{\sqrt{3}} \Rightarrow \theta = 30°$.

35. The vectors $\vec{A}$ and $\vec{B}$ are parallel vectors.

36. Area of parallelogram

$= |\vec{A} \times \vec{B}| = AB \sin \theta = \frac{1}{2} AB$

$\therefore \quad \sin \theta = \dfrac{1}{2} \Rightarrow \theta = 30°$.

37. Here $\tan \theta = \dfrac{\sqrt{3}A}{A} = \sqrt{3} \Rightarrow \theta = 60°$.

38. Clearly, $\vec{A} + \vec{B} + \vec{C} = \vec{0} - \vec{D} = -\vec{D}$

Hence the magnitude of $(\vec{A} + \vec{B} + \vec{C})$ will be equal to that of $\vec{D}$.

39. For a body thrown horizontally with a uniform speed,
acceleration along horizontal = 0
acceleration along vertical = g.

40. At an angle of 45° to the horizontal.

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41. Yes, a body in projectile motion possesses both horizontal and vertical velocities.

42. Horizontal component of velocity.

43. Yes, at the highest point of the parabolic path.

44. The statement is true. The horizontal component of the velocity will remain constant throughout the motion whereas the vertical component of the velocity will be zero at the top. Hence the resultant velocity of the projectile will be minimum at the top.

45. Yes.

46. Horizontal range will be maximum at 45°. It is given by $R_{max} = \frac{u^2}{g} = \frac{9.8 \times 9.8}{9.8} = \mathbf{9.8\ m}$.

47. Maximum height = Horizontal range

or $\frac{u^2 \sin^2\theta}{2g} = \frac{u^2 \sin 2\theta}{g}$

On solving, $\tan\theta = 4$ or $\theta = \mathbf{76°}$.

48. At the highest point, the vertical component of projectile velocity becomes zero. The projectile has only the horizontal component of velocity.

49. K.E. at the highest point

$= \text{Initial K.E.} \times \cos^2\theta = 1\text{ kJ} \times \cos^2 45° = \mathbf{500\ J}$.

50. $H_{max} = R/4$.

51. Max. horizontal range $= u^2/g = 160$ m

Max. vertical height $= u^2/2g = 80$ m.

52. $T_1 = \frac{2u\sin\theta}{g}$ and $T_2 = \frac{2u\sin(90° - \theta)}{g} = \frac{2u\cos\theta}{g}$

$\therefore \frac{T_1}{T_2} = \frac{\sin\theta}{\cos\theta} = \mathbf{\tan\theta : 1}$.

53. Parabolic path.

54. (*i*) Vertical component of initial velocity.

(*ii*) Time of flight.

55. When $\theta = 15°$,

$R = \frac{u^2 \sin 30°}{g} = \frac{u^2}{g} \times \frac{1}{2} = 1.5$ or $\frac{u^2}{g} = 3$ km

When $\theta = 45°$, $R = \frac{u^2 \sin 90°}{g} = \frac{u^2}{g} = 3$ km.

56. $R = \frac{u^2 \sin 2\theta}{g}$ $\therefore$ $45 = \frac{30^2 \sin 2\theta}{10}$

or $\sin 2\theta = 1/2 = \sin 30°$ or $\sin 150°$

$\therefore$ $\theta = 15°$ or $75°$.

57. $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 980}{9.8}} = 10\sqrt{2} = \mathbf{14.14\ s}$

58. K.E. at the highest point

$= K\cos^2\theta = K\cos^2 45° = K/2$

59. $T = \frac{2u\sin\theta}{g}$ and $R = \frac{2u^2 \sin\theta\cos\theta}{g}$

Horizontal component

$= u\cos\theta = \frac{R}{T} = \frac{100\text{ m}}{2\text{ s}} = 50\ \text{ms}^{-1}$.

60. $R_{max} = \frac{2}{\sqrt{3}} R$ or $\frac{u^2}{g} = \frac{2}{\sqrt{3}} \times \frac{u^2 \sin 2\theta}{g}$

or $\sin 2\theta = \frac{\sqrt{3}}{2} = \sin 60°$ $\therefore$ $\theta = \mathbf{30°}$

61. $\omega = \frac{2\pi \text{ radian}}{12 \text{ hour}} = \frac{\pi}{6}\ \text{rad h}^{-1}$.

62. The gravitational pull of the sun on the earth.

63. $\hat{A} = \frac{\vec{A}}{|\vec{A}|}$.

64. The unit vector parallel to the vector $3\hat{i} + 7\hat{j} + 4\hat{k}$

$= \frac{3\hat{i} + 7\hat{j} + 4\hat{k}}{\sqrt{3^2 + 7^2 + 4^2}} = \frac{3\hat{i} + 7\hat{j} + 4\hat{k}}{\sqrt{74}}$.

65. $\hat{k} \cdot \hat{k} = (1)(1)\cos 0° = 1$

66. Yes. For example, the cross product of two non-zero vectors will be zero when angle $\theta = 0°$ or $180°$.

67. $\tan\theta = \frac{A_y}{A_x}$.

68. $\vec{A} \times \vec{A} = \vec{0}$.

69. $|\vec{A} + \vec{B}|_{max} = |\vec{A}| + |\vec{B}|$ (For $\theta = 0°$)

$|\vec{A} + \vec{B}|_{min} = |\vec{A}| - |\vec{B}|$ (For $\theta = 180°$)

$|\vec{A} - \vec{B}|_{max} = |\vec{A}| + |\vec{B}|$ (For $\theta = 180°$)

$|\vec{A} - \vec{B}|_{min} = |\vec{A}| - |\vec{B}|$ (For $\theta = 0°$)

Type B : Short Answer Questions — 2 or 3 Marks Each

1. Distinguish between position and displacement vectors.

2. Define (*i*) unit vector (*ii*) null vector (*iii*) cross product of two vectors $\vec{A}$ and $\vec{B}$. **[Manipur 98]**

3. State and prove the commutative property of vector addition. **[Himachal 07 ; Central Schools 03]**

4. State and prove the polygon law of vector addition. **[Himachal 01C, 07, 07C, 09C ; Chandigarh 03]**

5. Suppose you have two forces $\vec{F}$ and $\vec{F}$. How would you combine them in order to have resultant force of magnitudes

(*a*) zero, (*b*) $2\vec{F}$, and (*c*) $\vec{F}$? **[Delhi 03]**

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6. Give one example of dot product of vectors. Also give its unit. [Delhi 98]

7. Given that [NCERT]

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$. Find $\vec{A} \times \vec{B}$.

8. Find the scalar and vector product of two vectors, $\vec{a} = (3\hat{i} - 4\hat{j} + 5\hat{k})$ and $\vec{b} = (-2\hat{i} + \hat{j} + 3\hat{k})$. [NCERT]

9. Find the work done in moving a particle along a vector $\vec{S} = (4\hat{i} - \hat{j} + 7\hat{k})$ if the applied force is $\vec{F} = (\hat{i} + 2\hat{j} - \hat{k})$ newton. $\vec{S}$ is in metre. [Delhi 99]

10. The angle between vectors $\vec{A}$ and $\vec{B}$ is 60°. What is the ratio of $\vec{A}.\vec{B}$ and $|\vec{A} \times \vec{B}|$? [C.Schools 05]

11. State parallelogram law of vector addition. Show that resultant of two vectors $\vec{A}$ and $\vec{B}$ inclined at an angle θ is $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$. [Delhi 03 ; Himachal 03, 04, 07, 09C ; Chandigarh 08]

12. A man moving in rain holds his umbrella inclined to the vertical even though the rain drops are falling vertically downwards. Why ? [Delhi 1999]

13. Obtain an expression for the area of a triangle in terms of the cross product of two vectors representing the two sides of the triangle. [NCERT]

14. Show that two dimensional uniform velocity motion is equivalent to two one dimensional uniform velocity motion along two coordinate axes. [Himachal 07C, 08C]

15. Define uniform acceleration. Show that in two-dimensional motion with uniform acceleration, each rectangular component of velocity is similar to that of uniformly accelerated motion along one dimension.

16. Write an expression for the instantaneous position vector of an object having two dimensional motion under uniform acceleration. Hence, show that a two dimensional uniformly accelerated motion is a combination of two rectangular uniformly accelerated motions.

17. Show that for a particle moving in two dimensions,

$$\vec{r'} = \vec{r} + \vec{v}(t' - t) + \frac{1}{2}\vec{a}(t' - t)^2$$

where $\vec{r}$ and $\vec{r'}$ are the position vectors of the particle at time t and t' respectively. [Himachal 06C]

18. A projectile is fired with a velocity 'u' making an angle θ with the horizontal. Show that its trajectory is a parabola. [Central Schools 12, 14 ; Delhi 06, 08]

19. A projectile is projected with velocity u making angle θ with horizontal direction, find :
(*a*) time of flight (*b*) horizontal range [Central Schools 04]

20. Show that there are two angles of projection for which the horizontal range is same for a projectile. [Delhi 95 ; Himachal 06 ; Central Schools 14]

21. Show that there are two angles of projection for a projectile to have the same horizontal range. What will be the maximum heights attained in the two cases ? Compare the two heights for θ = 30° and 60°. [Himachal 07C]

22. Find the angle of projection at which the horizontal range and maximum height of a projectile are equal.

23. What is a projectile ? Find its horizontal range and show it is maximum for an angle of 45°. [Himachal 01C]

24. A projectile is fired at an angle θ with the horizontal with velocity u. Deduce the expression for the maximum height reached by it. [Delhi 12]

25. A projectile is fired with velocity 'u', making an angle θ with the horizontal from the surface of earth. Prove that the projectile will hit the surface of earth with same velocity and at the same angle. [Chandigarh 07]

26. Prove the following statement "For Elevations which exceed or fall short of 45° by equal amounts, the ranges are equal". [Central Schools 08]

27. Prove that the maximum horizontal range is four tims the maximum height attained by the projectile when fired at an inclination so as to have maximum horizontal range. [Himachal 06, 07 ; Chandigarh 08]

28. Justify the statement that a uniform circular motion is an accelerated motion. [Delhi 99]

29. Derive the relation between linear displacement and angular displacement. [Himachal 07]

30. Establish a relation between linear velocity and angular velocity in a uniform circular motion and explain the direction of linear velocity. [Himachal 06, 07, 09 ; Chandigarh 03]

31. Define angular acceleration. Establish its relation with linear acceleration. [Himachal 06]

32. Derive an expression for the centripetal acceleration of a body moving in a circular path of radius 'r' with uniform speed 'v'. [Delhi 14]

33. Define Angular velocity and Angular acceleration. Derive an expression for centripetal acceleration of a uniform circular motion of an object. [Central Schools 13, 14]

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34. Derive an expression for centripetal acceleration of an object in uniform circular motion in a plane. What will be the direction of the velocity and acceleration at any instant ? **[Himachal 04, 06C ; Chandigarh 08 ; Central Schools 08]**

35. For a uniform circular motion show that :
(*i*) $v = r\omega$
(*ii*) $a = r\omega^2$ **[Central Schools 12]**

Answers

1. Refer answer to Q. 5 on page 4.2.
2. Refer answer to Q. 7 on page 4.2 and Q. 33 on page 4.25.
3. Refer answer to Q. 15 on page 4.5.
4. Refer answer to Q. 14 on page 4.5.
5. (*i*) When the two forces act in opposite directions, the magnitude of their resultant is zero.
(*ii*) When the two forces act in the same direction, the magnitude of their resultant = Magnitude of $2\vec{F}$.
(*iii*) When angle between $\vec{F}$ and $\vec{F}$ is 120°, the magnitude of their resultant = Magnitude of $\vec{F}$.
For details, refer to the solution of Example 5 on page 4.9.
6. Refer answer to Very Short Answer Q. 20 on page 4.84.
7. Refer answer to Q. 38 on page 4.27.
8. $\vec{a}.\vec{b} = -25$ and $\vec{a} \times \vec{b} = 7\hat{i} - \hat{j} - 5\hat{k}$.
9. $W = \vec{F} . \vec{S}$
$= (\hat{i} + 2\hat{j} - \hat{k}).(4\hat{i} - \hat{j} + 7\hat{k})$
$= 4 - 2 - 7 = -5$ J.
10. $\frac{\vec{A}.\vec{B}}{|\vec{A} \times \vec{B}|} = \frac{AB\cos\theta}{AB\sin\theta} = \cot\theta = \cot 60° = \frac{1}{\sqrt{3}}$.
11. Refer answer to Q. 19 on page 4.7.
12. Refer answer to Q. 48 on page 4.37.
13. Refer answer to Q. 34 on page 4.25.
14. Refer answer to Q. 45 on page 4.34.
15. Refer answer to Q. 46(*a*) on page 4.34.
16. Refer answer to Q. 46(*b*) on page 4.35.
17. Refer answer to Q. 46(*b*) on page 4.35.
18. Refer answer to Q. 53 on page 4.44.
19. Refer answer to Q. 53 on page 4.44.
20. Refer answer to Q. 54 on page 4.45.
21. Refer answer to Q.54 on page 4.45.
Max. height attained in first case,
$$H_1 = \frac{u^2 \sin^2 0}{2g}$$
Max. height attained in second case,
$$H_2 = \frac{u^2 \sin^2 (90° - 0)}{2g} = \frac{u^2 \cos^2 0}{2g}$$
Again,
$$\frac{H(30°)}{H(60°)} = \frac{u^2 \sin^2 30°}{2g} \cdot \frac{2g}{u^2 \sin^2 60°} = \frac{(1/2)^2}{(\sqrt{3}/2)^2} = \frac{1}{3} = \mathbf{1:3}.$$
22. Refer to the solution of Example 76 on page 4.48.
23. Refer answer to Q. 54 on page 4.45.
24. Refer answer to Q. 53 on page 4.44
25. Refer answer to Q. 54 on page 4.45.
26. Refer to the solution of Problem 15 on page 4.68.
27. Refer to the solution of Example 77 on page 4.48.
28. Refer answer to Q. 56 on page 4.55.
29. Refer answer to Q.57(*i*) on page 4.56.
30. Refer answer to Q. 59 on page 4.56.
31. Refer answer to Q.60 on page 4.57.
32. Refer answer to Q. 61 on page 4.57.
33. Refer answer to Q. 57 and Q. 58 on page 4.56 and Q. 61 on page 4.57.
34. Refer answer to Q. 61 on page 4.57.
35. (*i*) Refer answer to Q. 59 on page 4.56.
(*ii*) Refer answer to Q. 61 on page 4.57.

Type C : Long Answer Questions

5 Marks Each

1. What is meant by resolution of a vector ? Prove that a vector can be resolved along two given directions in one and only one way.
2. State the parallelogram law of vector addition and find the magnitude and direction of the resultant of two vectors $\vec{P}$ and $\vec{Q}$ inclined at an angle θ with each other. What happens, when $\theta = 0°$ and $\theta = 90°$? **[Himachal 02, 04, 06, 09; Delhi 13]**
3. (*a*) Analytically, find the resultant $\vec{R}$ of two vectors $\vec{A}$ and $\vec{B}$ inclined at an angle θ.

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(*b*) Find the angle between two vectors $\vec{P}$ and $\vec{Q}$ if resultant of the vectors is given by $R^2 = P^2 + Q^2$. **[Central Schools 07]**

4. State triangle law of vector addition. Give analytical treatment to find the magnitude and direction of a resultant vector by using this law. **[Himachal 03, 08]**

5. (*a*) Pick out only the vector quantities from the following : Temperature, pressure, impulse, time, power, charge.

(*b*) Show by drawing a neat diagram that the flight of a bird is an example of composition of vectors.

(*c*) A man is travelling at 10.8 km h^{-1} in a topless car on a rainy day. He holds his umbrella at an angle 37° to the vertical to protect himself from the rain which is falling vertically downwards. What is the velocity of the rain ?

[Given $\cos 37° = \frac{4}{5}$] **[Central Schools 08]**

6. Establish the following vector inequalities :

(*i*) $|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$ (*ii*) $|\vec{a} - \vec{b}| \leq |\vec{a}| + |\vec{b}|$

When does the equality sign apply ?

7. A projectile is projected horizontally with a velocity u Show that its trajectory is parabolic. And obtain expressions for : (*i*) Time of flight (*ii*) Horizontal range (*iii*) Velocity at any instant t. **[Delhi 13]**

8. What is a projectile ? Derive the expression for the trajectory, time of flight, maximum height and horizontal range for a projectile thrown upwards, making an angle θ with the horizontal direction. **[Himachal 07C ; Central Schools 07, 13, 14]**

9. A body is projected at an angle θ with the horizontal.

(*a*) Obtain the condition for the maximum horizontal range.

(*b*) Prove that the horizontal range of a projectile is same when fired at an angle θ and $(90° - \theta)$ with the horizontal.

(*c*) Obtain an expression for velocity of projectile at any instant t. **[Delhi 14]**

10. Define projectile. Show that the path of projectile is parabola. Find the angle of projection at which the horizontal range and maximum height of the projectile are equal. **[Central Schools 05]**

11. (*a*) Show that for two complementary angles of projection of a projectile thrown with the same velocity, the horizontal ranges are equal.

(*b*) For what angle of projection of a projectile, is the range maximum ?

(*c*) For what angle of projection of a projectile, are the horizontal range and maximum height attained by the projectile equal ? **[Central Schools 14]**

12. (*a*) What is projectile motion ?

(*b*) The maximum range of projectile is $2/\sqrt{3}$ times actual range. What is the angle of projection for the actual range ?

(*c*) Two balls are thrown with the same initial velcoity at angles α and $(90° - \alpha)$ with the horizontal. What will be the ratio of the maximum heights attained by them ? **[Central Schools 08]**

13. Define centripetal acceleration. Derive an expression for the centripetal acceleration of a body moving with uniform speed v along a circular path of radius r. Explain how it acts along the radius towards the centre of the circular path. **[Himachal 04, 08, 09C]**

Answers

1. Refer answer to Q. 20 on page 4.14.
2. Refer answer to Q. 19 on page 4.7.
3. (*a*) Refer answer to Q. 19 on page 4.7.
   (*b*) Refer to the hint of Problem 14 on page 4.14.
4. Refer answer to Q. 18 on page 4.7.
5. (*a*) Impulse.
   (*b*) Refer answer to Q.17(*a*) on page 4.6.
   (*c*) $v_R = 10.8 \text{ km h}^{-1} = 3 \text{ ms}^{-1}$

Given : $\cos 37° = \frac{4}{5} \quad \therefore \quad \tan 37° = \frac{3}{4}$

But $\tan 37° = \frac{v_R}{v_M}$ or $\frac{3}{4} = \frac{v_R}{3 \text{ms}^{-1}}$

or $v_R = \frac{9}{4} = \mathbf{2.25 \text{ ms}^{-1}}$.

6. Refer to solution of Example 10 on page 4.10.
7. Refer answer to Q. 52 on page 4.40.
8. Refer answer to Q. 53 on page 4.44.
9. Refer answer to Q. 53 and Q. 54 on page 4.44.
10. Refer answer to Q. 53 on page 4.44 and see solution of Example 76 on page 4.48.
11. (*a*) Refer answer to Q. 54 on page 4.44.
    (*b*) $\theta = 45°$
    (*c*) $\theta = 75°58'$.
12. (*a*) See point 46 of Glimpses.
    (*b*) Refer to the answer of Q.60 on page 4.85.
    (*c*) Refer to the solution of Problem 17(*i*) on page 4.78.
13. Refer answer to Q. 61 on page 4.57.

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Competition Section

Motion in a Plane

GLIMPSES

1. **Scalars.** The physical quantities which have only magnitude and no direction are called scalars *e.g.*, mass, length, time, speed, work, power, etc.
2. **Vectors.** The physical quantities which have both magnitude and direction are called vectors *e.g.*, displacement, velocity, acceleration, force, momentum, etc.
3. **Representation of a vector.** A vector is represented by a straight line with an arrowhead over it. The length of the line gives the magnitude and the arrowhead gives the direction of the vector.
4. **Position vector.** A vector which gives position of an object with reference to the origin of a coordinate system is called position vector.
5. **Displacement vector.** It is that vector which tells how much and in which direction an object has changed its position in a given time interval.
6. **Polar vectors.** These are the vectors which have a starting point or a point of application *e.g.*, displacement, force, velocity, etc.
7. **Axial vectors.** The vectors which represent rotational effect and act along the axis of rotation in accordance with right hand screw rule are called axial vectors *e.g.*, torque, angular momentum, etc.
8. **Equal vectors.** Two vectors are said to be equal if they have the same magnitude and direction.
9. **Negative vector.** The negative of a vector is defined as another vector having the same magnitude but having an opposite direction.
10. **Zero vector.** A vector having zero magnitude and an arbitrary direction is called a zero or null vector.
11. **Collinear vectors.** The vectors which either act along the same line or along parallel lines are called collinear vectors.
12. **Coplanar vectors.** The vectors which act in the same plane are called coplanar vectors.
13. **Modulus of a vector.** The magnitude or length of a vector is called its modulus.

    Modulus of vector $\vec{A} = |\vec{A}| = A$
14. **Fixed vector.** The vector whose initial vector is fixed is called a *fixed vector* or *localised vector*.
15. **Unit vector.** A unit vector is a vector of unit magnitude drawn in the direction of a given vector.

    A unit vector in the direction of $\vec{A}$ is given by

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

16. **Free vector.** A vector whose initial point is not fixed is called a *free vector* or *non-localised vector*.
17. **Co-initial vectors.** The vectors which have the same initial point are called co-initial vectors.
18. **Co-terminus vectors.** The vectors which have the common terminal point are called co-terminus vectors.
19. **Properties of zero vector.** A zero vector has the following properties :

$$\vec{A} + \vec{O} = \vec{A}; \quad \lambda\vec{O} = \vec{O}; \quad 0\vec{A} = \vec{O}$$

20. **Multiplication of vector by a real number.** When a vector $\vec{A}$ is multiplied by a real number λ, we get another vector $\lambda\vec{A}$. The magnitude of $\lambda\vec{A}$ is λ times the magnitude of $\vec{A}$. If λ is positive, then the direction of $\lambda\vec{A}$ is same as that of $\vec{A}$. If λ is negative, then the direction of $\lambda\vec{A}$ is opposite to that of $\vec{A}$.
21. **Multiplication of a vector by a scalar.** When a vector $\vec{A}$ is multiplied by a scalar λ, which has

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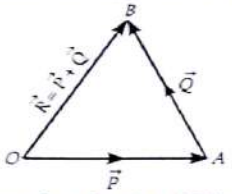
certain units, the units of $\lambda \vec{A}$ are obtained by multiplying the units of $\vec{A}$ by the units of λ.

22. **Composition of vectors.** The resultant of two or more vectors is that single vector which produces the same effect as the individual vectors together would produce. The process of adding two or more vectors is called the composition of vectors.

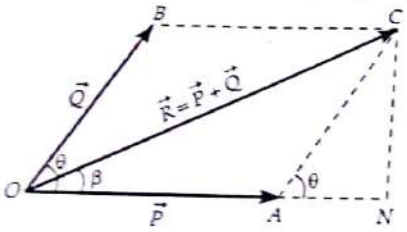
23. **Triangle law of vector addition.** If two vectors can be represented both in magnitude and direction by the two sides of a triangle taken in the same order, then their resultant is represented completely both in magnitude and direction by the third side of the triangle taken in the opposite order. In the figure below,

$$\vec{OA} + \vec{AB} = \vec{OB} \quad \text{or} \quad \vec{P} + \vec{Q} = \vec{R}$$



24. **Parallelogram law of vector addition.** If two vectors acting simultaneously at a point can be represented both in magnitude and direction by the two adjacent sides of a parallelogram, then their resultant is represented completely both in magnitude and direction by the diagonal of the parallelogram passing through that point. In the figure,

$$\vec{OA} + \vec{OB} = \vec{OC} \quad \text{or} \quad \vec{P} + \vec{Q} = \vec{R}$$



The magnitude of the resultant $\vec{R}$ is given by

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

where θ is the angle between $\vec{P}$ and $\vec{Q}$. If $\vec{R}$ makes angle β with $\vec{P}$, then

$$\tan\beta = \frac{Q\sin\theta}{P + Q\cos\theta}$$

25. **Polygon law of vector addition.** If a number of vectors are represented both in magnitude and direction by the sides of an open polygon taken in the same order, then their resultant is represented both in magnitude and direction by the closing side of the polygon taken in opposite order.

26. **Properties of vector addition :**

(i) Vectors representing physical quantities of same nature can only be added.

(ii) Vector addition is *commutative*.

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

(iii) Vector addition is *associative*.

$$(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$$

27. **Subtraction of vectors.** The subtraction of a vector $\vec{B}$ from vector $\vec{A}$ is defined as the addition of vector $-\vec{B}$ to $\vec{A}$. Thus $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$.

28. **Resolution of a vector.** The process of splitting a vector into two or more vectors is known as resolution of the vector. The vectors into which the given vector is splitted are called component vectors. A vector $\vec{A}$ can be resolved into components along two given vectors $\vec{a}$ and $\vec{b}$ lying in the same plane in one and only one way :

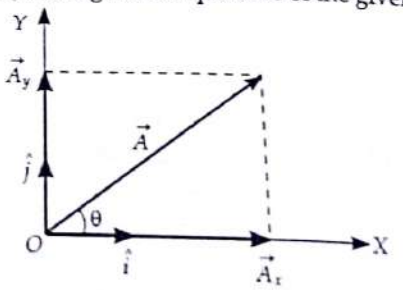
$$\vec{A} = \lambda\vec{a} + \mu\vec{b}$$

where λ and μ are real numbers.

29. **Orthogonal triad of unit vectors : Base vectors.** The unit vectors $\hat{i}$, $\hat{j}$, $\hat{k}$ are vectors of unit magnitude and point in the direction of the x-, y-, and z-axes, respectively in a right-handed coordinate system. These are collectively known as the orthogonal triad of unit vectors or base vectors.

$$|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$$

30. **Rectangular components of a vector.** When a vector is resolved along two mutually perpendicular directions, components so obtained are called rectangular components of the given vector.



As shown in figure, if $\vec{A}$ makes angle θ with X-axis and $\vec{A}_x$ and $\vec{A}_y$ are the rectangular components of $\vec{A}$ along X- and Y-axis respectively, then

$$\vec{A} = \vec{A}_x + \vec{A}_y = A_x\hat{i} + A_y\hat{j}$$

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Also, $A_x = A\cos\theta$, $A_y = A\sin\theta$

$$A = \sqrt{A_x^2 + A_y^2} \text{ and } \tan\theta = \frac{A_y}{A_x}.$$

Any vector in three dimensions can be expressed in terms of its rectangular components as

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

Its magnitude, $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$.

31. **Scalar or dot product.** The scalar or dot product of two vectors $\vec{A}$ and $\vec{B}$ is defined as the product of the magnitudes of $\vec{A}$ and $\vec{B}$ and cosine of the angle θ between them. Thus

$$\vec{A}.\vec{B} = |\vec{A}||\vec{B}|\cos\theta = AB\cos\theta$$

It can be positive, negative or zero depending upon the value of θ.

32. **Geometrical interpretation of scalar product.** The scalar product of two vectors can be interpreted as the product of magnitude of one vector and component of the other vector along the first vector.

33. **Properties of dot product of two vectors :**

(*i*) For parallel vectors,

$$\theta = 0°, \quad \cos\theta = 1, \quad \vec{A}.\vec{B} = AB$$

(*ii*) For antiparallel vectors,

$$\theta = 180°, \quad \cos\theta = -1, \quad \vec{A}.\vec{B} = -AB$$

(*iii*) For perpendicular vectors,

$$\theta = 90°, \quad \cos\theta = 0, \quad \vec{A}.\vec{B} = 0$$

(*iv*) $\vec{A}.\vec{B} = \vec{B}.\vec{A}$ (*Commutative law*)

(*v*) $\vec{A}.(\vec{B} + \vec{C}) = \vec{A}.\vec{B} + \vec{A}.\vec{C}$ (*Distributive law*)

(*vi*) $\vec{A}.\vec{A} = A^2$

(*vii*) $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$

(*viii*) $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$.

34. **Dot product in Cartesian co-ordinates.** For $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$,

$$\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

Angle θ between $\vec{A}$ and $\vec{B}$ is given by

$$\cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|} = \frac{A_xB_x + A_yB_y + A_zB_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}\sqrt{B_x^2 + B_y^2 + B_z^2}}$$

35. **Examples of dot product.**

(*i*) Work done, $W = \vec{F}.\vec{S}$

(*ii*) Instantaneous power, $P = \vec{F}.\vec{v}$

36. **Vector or cross product.** For two vectors $\vec{A}$ and $\vec{B}$ inclined at an angle θ, the vector or cross product is defined as

$$\vec{A} \times \vec{B} = AB\sin\theta\,\hat{n}$$

where $\hat{n}$ is a unit vector perpendicular to the plane of $\vec{A}$ and $\vec{B}$ and its direction is that in which a right handed screw advances when rotated from $\vec{A}$ to $\vec{B}$.

37. **Geometrical interpretation of vector product.** The magnitude of the vector product of two vectors is equal to (*i*) the area of the parallelogram formed by the two vectors as its adjacent sides and (*ii*) twice the area of the triangle formed by the two vectors as its adjacent sides.

38. **Properties of cross product of two vectors :**

(*i*) For parallel or antiparallel vectors,

$$\theta = 0° \text{ or } 180°$$

$$\vec{A} \times \vec{B} = \vec{0}$$

(*ii*) $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$ (*Anti-commutative law*)

(*iii*) $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$ (*Distributive law*)

(*iv*) $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = \vec{0}$

(*v*) $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$, $\hat{k} \times \hat{i} = \hat{j}$

(*vi*) Unit vector perpendicular to the plane of $\vec{A}$ and $\vec{B}$ is given by

$$\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$$

(*vii*) Angle θ between $\vec{A}$ and $\vec{B}$ is given by

$$\sin\theta = \frac{|\vec{A} \times \vec{B}|}{|\vec{A}||\vec{B}|}$$

39. **Cross product in Cartesian co-ordinates.**

$$\vec{A} \times \vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) \times (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= (A_yB_z - A_zB_y)\hat{i} - (A_zB_x - A_xB_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k}.$$

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40. **Examples of cross product.** (*i*) Moment of force or torque,

$$\vec{\tau} = \vec{r} \times \vec{F}$$

(*ii*) Angular momentum,

$$\vec{L} = \vec{r} \times \vec{p}.$$

41. **Position and displacement vectors.** The *position vector* of an object in x-y plane is given by

$$\vec{r} = x\,\hat{i} + y\hat{j}$$

and the *displacement* from position $\vec{r}$ to position $\vec{r}'$ is given by

$$\Delta\vec{r} = \vec{r}' - \vec{r} = (x' - x)\,\hat{i} + (y' - y)\,\hat{j} = \Delta x\,\hat{i} + \Delta y\,\hat{j}$$

42. **Velocity vector.** If an object undergoes a displacement $\Delta\vec{r}$ in time Δt, its *average velocity* is given by

$$\bar{v} = \frac{\Delta\vec{r}}{\Delta t}$$

The *velocity* of an object at time t is the limiting value of the average velocity as Δt tends to zero. Thus

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

In component form, we have

$$\vec{v} = v_x\,\hat{i} + v_y\,\hat{j} + v_z\,\hat{k}$$

where $v_x = \frac{dx}{dt}$, $v_y = \frac{dy}{dt}$, $v_z = \frac{dz}{dt}$ and

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}.$$

When position of a particle is plotted on a coordinate system, $\vec{v}$ is always tangent to the curve representing the path of the particle.

43. **Acceleration vector.** If the velocity of an object changes from $\vec{v}$ to $\vec{v}'$ in time Δt, then its average acceleration is given by

$$\bar{a} = \frac{\vec{v}' - \vec{v}}{\Delta t} = \frac{\Delta\vec{v}}{\Delta t}$$

The *acceleration* $\vec{a}$ at any time t is the limiting value of $\bar{a}$ as Δt tends to zero. So

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

In component form, we have

$$\vec{a} = a_x\,\hat{i} + a_y\,\hat{j} + a_z\,\hat{k}$$

here, $a_x = \frac{dv_x}{dt}$, $a_y = \frac{dv_y}{dt}$, $a_z = \frac{dv_z}{dt}$

d $$|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

44. **Equations of motion in vector form.** For motion with constant acceleration,

(*i*) $\vec{v} = \vec{v}_0 + \vec{a}\,t$

(*ii*) $\vec{r} = \vec{r}_0 + \vec{v}_0\,t + \frac{1}{2}\vec{a}\,t^2$

(*iii*) $v^2 - v_0^2 = 2\vec{a}\,.\,(\vec{r} - \vec{r}_0)$

The motion in a plane with uniform acceleration can be treated as the superposition of two separate simultaneous one-dimensional motions along two perpendicular directions.

45. **Relative velocity.** The relative velocity of an object A with respect to object B, when both are in motion, is the rate of change of position of object A with respect to object B.

(*i*) If the two objects are moving with velocities $\vec{v}_A$ and $\vec{v}_B$ with respect to the ground, then

Relative velocity of A w.r.t. B,

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

Relative velocity of B w.r.t. A,

$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$$

(*ii*) If the two objects A and B are moving with velocities $\vec{v}_A$ and $\vec{v}_B$ inclined at an angle θ, then magnitude of the relative velocity of A w.r.t. B is given by

$$v_{AB} = \sqrt{|\vec{v}_A|^2 + |-\vec{v}_B|^2 + 2|\vec{v}_A||-\vec{v}_B|\cos(180° - \theta)}$$

$$= \sqrt{v_A^2 + v_B^2 - 2\,v_A\,v_B\cos\theta}$$

If the velocity $\vec{v}_{AB}$ makes angle β with the velocity $\vec{v}_A$, then

$$\tan\beta = \frac{|-\vec{v}_B|\sin(180° - \theta)}{|\vec{v}_A| + |-\vec{v}_B|\cos(180° - \theta)}$$

$$= \frac{v_B \sin\theta}{v_A - v_B\cos\theta}$$

46. **Projectile motion.** Any body projected into space, such that it moves under the effect of gravity alone is called a projectile. The path followed by a projectile is called its trajectory which is always a parabola.

A projectile executes two independent motions simultaneously :

(*i*) uniform horizontal motion and

(*ii*) uniform accelerated downward motion.

47. **Projectile fired horizontally.** Suppose a body is projected horizontally with velocity u from a height h above the ground. Let it reach the point (x, y) after time t.

Then

(i) Posi

(ii) Equ

(iii) Ve

(iv) T

(v)

48. Proj
Sup
ang
(x,
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Then

(*i*) Position of the projectile after time t :

$$x = ut, \quad y = \frac{1}{2} gt^2$$

(*ii*) Equation of trajectory : $y = \frac{g}{2u^2} \cdot x^2$

(*iii*) Velocity after time t :

$$v = \sqrt{u^2 + g^2 t^2}\ ; \quad \beta = \tan^{-1} \frac{gt}{u}$$

(*iv*) Time of flight, $T = \sqrt{\frac{2h}{g}}$

(*v*) Horizontal range, $R = u \times T = u\sqrt{\frac{2h}{g}}$.

48. **Projectile fired at an angle with the horizontal.** Suppose a projectile is fired with velocity u at an angle θ with the horizontal. Let it reach the point (x, y) after time t. Then

(*i*) Components of initial velocity :

$$u_x = u\cos\theta, \quad u_y = u\sin\theta$$

(*ii*) Components of acceleration at any instant :

$$a_x = 0, \ a_y = -g$$

(*iii*) Position after time t :

$$x = (u\cos\theta)\,t\,, \quad y = (u\sin\theta)\,t - \frac{1}{2} gt^2$$

(*iv*) Equation of trajectory :

$$y = x\tan\theta - \frac{g}{2u^2\cos^2\theta} \cdot x^2$$

(*v*) Maximum height, $H = \frac{u^2 \sin^2\theta}{2g}$

(*vi*) Time of flight, $T = \frac{2u\sin\theta}{g}$

(*vii*) Horizontal range, $R = \frac{u^2 \sin 2\theta}{g}$

(*viii*) Maximum horizontal range is attained at $\theta = 45°$ and its value is

$$R_{max} = \frac{u^2}{g}$$

(*ix*) Velocity after time t :

$$v_x = u\cos\theta, \quad v_y = u\sin\theta - gt$$

$$\therefore \quad v = \sqrt{v_x^2 + v_y^2} \quad \text{and} \quad \tan\beta = \frac{v_y}{v_x}$$

(*x*) The velocity with which the projectile reaches the horizontal plane through the point of projection is same as the velocity of projection.

49. **Uniform circular motion.** When a body moves along a circular path with uniform speed, its motion is said to uniform circular motion.

50. **Angular displacement.** It is the angle swept out by a radius vector in a given time interval.

$$\theta(\text{rad}) = \frac{\text{Arc}}{\text{Radius}} = \frac{s}{r}$$

51. **Angular velocity.** The angle swept out by the radius vector per second is called angular velocity.

$$\omega = \frac{\theta}{t} \quad \text{or} \quad \omega = \frac{\theta_2 - \theta_1}{t_2 - t_1}$$

52. **Time period and frequency.** Time taken for one complete revolution is called time period (T). The number of revolutions completed per second is called frequency

$$\omega = \frac{2\pi}{T} = 2\pi\nu$$

53. **Relationship between v and ω.** It is given by

$$v = r\omega$$

i.e. Linear velocity = Radius × angular velocity.

54. **Angular acceleration and its relation with linear acceleration.** The rate of change of angular velocity is called angular acceleration. It is given by

$$\alpha = \frac{\omega_2 - \omega_1}{t_2 - t_1}$$

Also, $a = r\alpha$

i.e. Linear acceleration

= Radius × angular acceleration

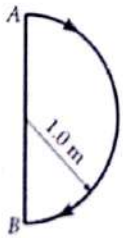
55. **Centripetal acceleration.** A body moving along a circular path is acted upon by an acceleration directed towards the centre along the radius. This acceleration is called centripetal acceleration. It is given by

$$a = \frac{v^2}{r} = r\omega^2.$$

IIT Entrance Exam

MULTIPLE CHOICE QUESTIONS WITH ONE CORRECT ANSWER

1. In 1.0 s, a particle goes from point A to point B, moving in a semicircle of radius 1.0 m as shown in the figure.



The magnitude of the average velocity is

(*a*) 3.14 m/s (*b*) 2.0 m/s

(*c*) 1.0 m/s (*d*) zero [IIT 99]

2. A particle P is sliding down a frictionless hemispherical bowl. It passes the point A at $t = 0$. At this instant of time, the horizontal component of its velocity is v. A bead Q of the same mass as P is ejected

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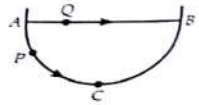
from A at $t=0$ along the horizontal string AB with speed v. Friction between the bead and the string may be neglected. Let t_P and t_Q be the respective times taken by P and Q to reach the point B. Then



(a) $t_P < t_Q$ (b) $t_P = t_Q$

(c) $t_P > t_Q$ (d) $\frac{t_P}{t_Q} = \frac{\text{Lenght of arc } ACB}{\text{Lenght of chord } AB}$

[IIT 93]

3. A river is flowing from west to east at a speed of 5 metre per minute. A man on the south bank of the river, capable of swimming at 10 metre per minute in still water, wants to swim across the river in the shortest time. He should swim in a direction

(a) due north (b) 30° east of north

(c) 30° west of north (d) 60° east of north [IIT 83]

4. A boat which has a speed of 5 km/hr in still water crosses a river of width 1 km along the shortest possible path in 15 minutes. The velocity of the river water in km/hr is

(a) 1 (b) 3

(c) 4 (d) $\sqrt{41}$ [IIT 88]

MULTIPLE CHOICE QUESTIONS WITH ONE OR MORE THAN ONE CORRECT ANSWER

5. The coordinates of a particle moving in a plane are given by $x(t) = a\cos pt$ and $y(t) = b\sin pt$, where a, b $(< a)$ and p are positive constants of appropriate dimensions. Then,

(a) the path of the particle is an ellipse.

(b) the velocity and acceleration of the particle are normal to each other at $t = \pi/2p$

(c) the acceleration of the particle is always directed towards a focus

(d) the distance travelled by the particle in time interval $t = 0$ to $t = \pi/2p$ is a. [IIT 99]

5. (a), (b) and (c)

$x = a\cos pt \Rightarrow$

$y = b\sin pt \Rightarrow$

$\therefore \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Hence the path of the particle
(a) holds good.

Now, $\vec{r} = x\hat{i} + y\hat{j}$

or $\vec{r} = a\cos pt\,\hat{i} + b\sin$

$\therefore \quad \vec{v} = \frac{d\vec{r}}{dt} = -ap\sin$

$\vec{a} = \frac{d\vec{v}}{dt} = -ap^2$

At $t = \frac{\pi}{2p}$, we have

$\vec{v}\cdot\vec{a} = \left(-ap\sin\right.$

Answers and Explanations

1. (b) $|\text{Average velocity}| = \frac{|\text{Displacement}|}{\text{Time}}$

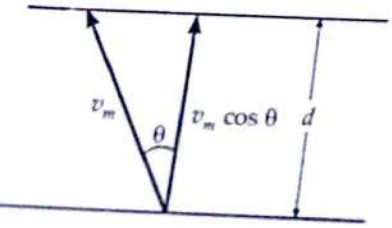
$= \frac{AB}{t} = \frac{2r}{t} = \frac{2\times 1.0\,\text{m}}{1\,\text{s}} = 2\ \text{m/s}.$

2. (a) As the particle P moves from A to C, its zontal velocity increases from v. Again the hori- al velocity decreases to v as P moves from C to B. he horizontal velocity P remains greater than or to v. But the horizontal velocity of bead Q ns constant equal to v. For same horizontal ement AB, P takes takes smaller time than Q i.e.,

$t_P < t_Q$.

Time taken to cross the river

$t = \frac{d}{v_m\cos\theta}$

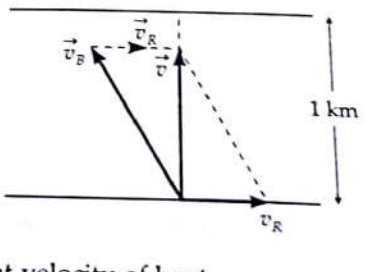


For t to be minimum, $\cos\theta$ should be maximum

i.e., $\cos\theta = 1$ or $\theta = 0°$

Hence the man should swim due north.

4. (b)



Resultant velocity of boat,

$v = \frac{1\ \text{km}}{15\ \text{min}}$

$= \frac{1\ \text{km}}{(1/4)\text{h}} = 4\ \text{km h}^{-1}.$

Velocity of river,

$v_R = \sqrt{v_B^2 - v^2}$

$= \sqrt{5^2 - 4^2} = 3\ \text{km/h}.$

1. Two forces are magnitudes is 18 N ar 12 N and is perpend the magnitudes of th

(a) 12 N, 6 N

(c) 10 N, 8 N

2. A particle is $5\,\text{ms}^{-1}$. In 10 s, th $5\,\text{ms}^{-1}$ northward time is

(a) $\frac{1}{\sqrt{2}}\,\text{ms}^{-2}$ t

(b) $\frac{1}{\sqrt{2}}\,\text{ms}^{-2}$

(c) $\frac{1}{2}\,\text{ms}^{-2}$ t

(d) zero

3. The coo f are given particle at t

(a) $3t\sqrt{}$

(c) $t^2\sqrt{}$

where

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5. (a), (b) and (c)

$$x = a\cos pt \quad \Rightarrow \quad \frac{x}{a} = \cos pt$$

$$y = b\sin pt \quad \Rightarrow \quad \frac{y}{b} = \sin pt$$

$$\therefore \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Hence the path of the particle is an ellipse. Option (a) holds good.

Now, $\vec{r} = x\hat{i} + y\hat{j}$

or $\vec{r} = a\cos pt\,\hat{i} + b\sin pt\,\hat{j}$

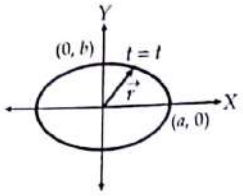
$$\therefore \quad \vec{v} = \frac{d\vec{r}}{dt} = -ap\sin pt\,\hat{i} + bp\cos pt\,\hat{j}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = -ap^2\cos pt\,\hat{i} - bp^2\sin pt\,\hat{j}$$

At $t = \frac{\pi}{2p}$, we have

$$\vec{v}.\vec{a} = \left(-ap\sin\frac{\pi}{2}\hat{i}\right).\left(-bp^2\sin\frac{\pi}{2}\hat{j}\right) = 0 \qquad [\hat{i}.\hat{j} = 0]$$

Thus the velocity and acceleration are perpendicular to each other at $t = \frac{\pi}{2p}$. Option (b) holds good.



Clearly, $\vec{a} = -p(\vec{r})$

Thus the acceleration is always directed towards the origin. Option (c) holds good.

At $t = 0$, the particle is at $(a, 0)$.

At $t = \frac{\pi}{2p}$, the particle is at $(0, b)$.

Thus the distance covered is one-fourth of the elliptical path. It is greater than a and not equal to a. Option (d) does not hold good.

AIEEE

1. Two forces are such that the sum of their magnitudes is 18 N and their resultant has magnitude 12 N and is perpendicular to the smaller force. Then the magnitudes of the forces are

(a) 12 N, 6 N (b) 13 N, 5 N

(c) 10 N, 8 N (d) 16 N, 2N [AIEEE 02]

2. A particle is moving eastward with a velocity of $5\,ms^{-1}$. In 10 s, the velocity of the particle changes to $5\,ms^{-1}$ northward. The average acceleration in this time is

(a) $\frac{1}{\sqrt{2}}\,ms^{-2}$ towards north-west

(b) $\frac{1}{\sqrt{2}}\,ms^{-2}$ towards north-east

(c) $\frac{1}{2}\,ms^{-2}$ towards north-west

(d) zero [AIEEE 05]

3. The coordinates of a moving particle at any time f are given by $x = \alpha t^3$ and $y = \beta t^3$. The speed of the particle at time t is given by

(a) $3t\sqrt{\alpha^2+\beta^2}$ (b) $3t^2\sqrt{\alpha^2+\beta^2}$

(c) $t^2\sqrt{\alpha^2+\beta^2}$ (d) $\sqrt{\alpha^2+\beta^2}$

where the letters have their usual meanings. [AIEEE 03]

4. A boy playing on the roof of a 10 m high building throws a ball with a speed of $10\,ms^{-1}$ at an angle of 30° with the horizontal. How far from the throwing point will the ball be at the height of 10 m from the ground ? Given that

$g = 10\,ms^{-2}$, $\sin 30° = 1/2$, $\cos 30° = \sqrt{3}/2$.

(a) 5.20 m (b) 4.33 m

(c) 2.60 m (d) 8.66 m [AIEEE 03]

5. A projectile can have the same range R for two angles of projection. If t_1 and t_2 be the times of flight in the two cases, then the product of the two times of flight is directly proportional to

(a) $1/R^2$ (b) $1/R$

(c) R (d) R^2 [AIEEE 04]

6. A ball is thrown from a point with a speed v_0 at an angle of projection θ. From the same point and at the same instant, a person starts running with a constant speed $v_0/2$ to catch the ball. Will the person be able to catch the ball ? If yes, what should be the angle of projection ?

(a) Yes, 60° (b) Yes, 30°

(c) No (d) Yes, 45° [AIEEE 04]

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7. A particle is projected at 60° to the horizontal with a kinetic energy K. The kinetic energy at the highest point is

(a) $K/2$ (b) K

(c) zero (d) $K/4$ [AIEEE 07]

8. Which of the following statements is false for a particle moving in a circle with a constant angular speed ?

(a) The velocity vector is tangent to the circle

(b) The acceleration vector is tangent to the circle

(c) The acceleration vector points to the centre of the circle

(d) The velocity and acceleration vectors are perpendicular to each other [AIEEE 04]

9. A particle is acted upon by a force of constant magnitude, which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane. It follows that

(a) its velocity is constant

(b) its acceleration is constant

(c) its kinetic energy is constant

(d) it moves in a circular path [AIEEE 04]

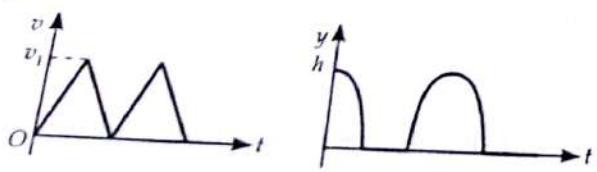
10. A particle has an initial velocity of $3\hat{i}+4\hat{j}$ and an acceleration of $0.4\hat{i}+0.3\hat{j}$. Its speed after 10 s is

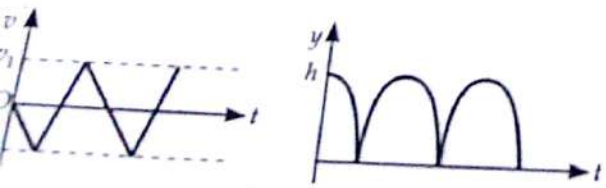
(a) $7\sqrt{2}$ units (b) 7 units

(c) 8.5 units (d) 10 units [AIEEE 09]

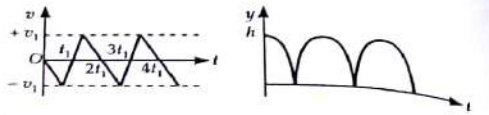
11. Consider a rubber ball freely falling from a height $h = 4.9$ m onto a horizontal elastic plate. Assume that the duration of collision is negligible and the collision with the plate is totally elastic. Then the velocity as a function of time the height as function of time will be

(a)

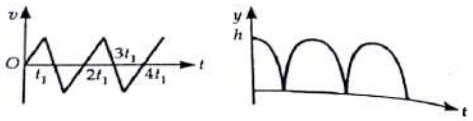




(c)



(d)



[AIEEE 09]

12. A particle is moving with velocity $\vec{v} = K(y\hat{i}+x\hat{j})$, where K is a constant. The general equation for its path is

(a) $y^2 = x^2$ + constant (b) $y = x^2$ + constant

(c) $y^2 = x$ + constant (d) xy = constant

[AIEEE 2010]

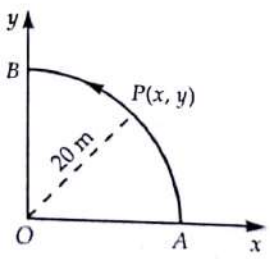
13. For a particle in uniform circular motion, the acceleration $\vec{a}$ at a point $P(R,\theta)$ on the circle of radius R is (Here θ is measured from the x-axis)

(a) $\frac{v^2}{R}\hat{i}+\frac{v^2}{R}\hat{j}$ (b) $-\frac{v^2}{R}\cos\theta\,\hat{i}+\frac{v^2}{R}\sin\theta\,\hat{j}$

(c) $-\frac{v^2}{R}\sin\theta\,\hat{i}+\frac{v^2}{R}\cos\theta\,\hat{j}$

(d) $-\frac{v^2}{R}\cos\theta\,\hat{i}-\frac{v^2}{R}\sin\theta\,\hat{j}$ [AIEEE 2010]

14. A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of P is such that it sweeps out a length



$s = t^3 + 5$, where s in metres and t is in seconds. The radius of the path is 20 m. The acceleration of P when $t = 2$ s is nearly

(a) 14 m / s^2 (b) 13 m / s^2

(c) 12 m / s^2 (d) 7.2 m / s^2 [AIEEE 2010]

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Answers and Explanations

1. (*b*) Refer to the solution of Example 9 on Page 4.10.

2. (*a*) Refer to the solution of Example 56 on page 4.36.

3. (*b*) As $x = \alpha t^3$

$$\therefore \quad v_x = \frac{dx}{dt} = 3\alpha t^2$$

Also, $y = \beta t^3$

$$\therefore \quad v_y = \frac{dy}{dt} = 3\beta t^2$$

Hence $$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(3\alpha t^2)^2 + (3\beta t^2)^2}$$

$$= 3t^2\sqrt{\alpha^2 + \beta^2}$$

4. (*d*) Refer to the solution of Problem 9 on Page 4.73.

5. (*c*) Refer to the solution of Problem 10 on Page 4.73.

6. (*a*) Refer to the solution of Problem 9 on Page 4.73.

7. (*d*) Initial K.E., $K = \frac{1}{2}mv^2$

Velocity at the highest point

= Horizontal component of v

$$= v\cos 60° = \frac{v}{2}$$

$\therefore$ K.E. at the highest point

$$= \frac{1}{2}m\left(\frac{v}{2}\right)^2 = \frac{K}{4}.$$

8. (*b*) The acceleration vector is along the radius of the circle. It is not tangent to the circle.

9. (*c*), (*d*) Such a force makes the particle move along a circular path with a constant speed. So the K.E. also is constant. But velocity and acceleration continuously change due to change in direction.

10. (*a*) $\vec{v} = \vec{u} + \vec{a}t$

$$= (3\hat{i} + 4\hat{j}) + 10(0.4\hat{i} + 0.3\hat{j})$$

$$= 7\hat{i} + 7\hat{j}$$

Speed $= |\vec{v}| = \sqrt{7^2 + 7^2} = \mathbf{7\sqrt{2}}$ **units.**

11. (*c*) We use $v = u + at$

For downward motion,

$$v = 0 - gt = -gt \text{ (negative velocity)}$$

For upward motion,

$$v = +gt \text{ (positive velocity)}$$

Just after collision velocity is upward and then becomes zero after some time and then negative. The same process repeats, so v-t given in option (*c*) is correct.

Again, $$s = ut + \frac{1}{2}gt^2$$

$$h = 4.9 - \frac{1}{2}gt^2$$

So, h-t graph is a downward parabola as given in option (*c*). Moreover, the collision is perfectly elastic. So, the ball reaches the same height again and again with the same velocity.

12. (*a*) $\vec{v} = v_x\hat{i} + v_y\hat{j} = K(y\hat{i} + x\hat{j})$

$$\therefore \quad v_x = \frac{dx}{dt} = Ky$$

$$v_y = \frac{dy}{dt} = Kx$$

Here $$\frac{dx}{dy} = \frac{y}{x} \quad \text{or} \quad y\,dy = x\,dx$$

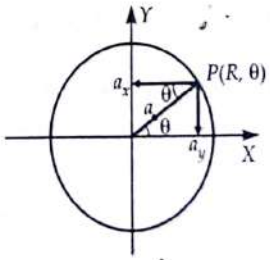
or $$\int y\,dy = \int x\,dx \;\Rightarrow\; y^2 = x^2 + \text{constant}$$

13. (*d*) For a particle in uniform circular motion,

$a = \frac{v^2}{R}$, towards the centre of the circle

$$\therefore \quad \vec{a} = \frac{v^2}{R}(-\cos\theta\,\hat{i} - \sin\theta\,\hat{j})$$

$$= -\frac{v^2}{R}\cos\theta\,\hat{i} - \frac{v^2}{R}\sin\theta\,\hat{j}$$



14. (*a*) $s = t^3 + 5$

$$v = \frac{ds}{dt} = 3t^2$$

$$a_t = \frac{dv}{dt} = 6t$$

At $t = 2$ s ; $v = 12$ m/s

Now $$a_c = \frac{v^2}{R} = \frac{(12)^2}{20} = \frac{144}{20} = 7.2 \text{ m/s}$$

$$a_t = 6 \times 2 = 12 \text{ m/s}$$

$$\therefore \quad a = \sqrt{a_c^2 + a_t^2} = \sqrt{12^2 + (7.2)^2} \approx \mathbf{14\ m/s^2}$$

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DCE and G.G.S. Indraprastha University Engineering Entrance Exam

(Similar Questions)

1. A body moves 6 m north, 8 m east and 10 m vertically upwards, the resultant displacement from its initial position is

(a) $10\sqrt{2}$ m (b) 10 m

(c) $\frac{10}{\sqrt{2}}$ m (d) 20 m [DCE 2K]

2. When $\vec{A}.\vec{B} = -|\vec{A}||\vec{B}|$, then

(a) $\vec{A}$ and $\vec{B}$ are perpendicular to each other

(b) $\vec{A}$ and $\vec{B}$ act in the same direction

(c) $\vec{A}$ and $\vec{B}$ act in the opposite direction

(d) $\vec{A}$ and $\vec{B}$ can act in any direction [IPUEE 05]

3. If the angle between the vectors $\vec{A}$ and $\vec{B}$ is θ, the value of the product $(\vec{B} \times \vec{A}).\vec{A}$ is equal to

(a) $BA^2\cos\theta$ (b) $BA^2\sin\theta$

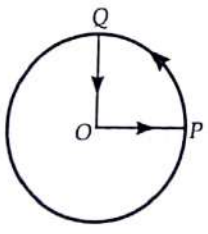
(c) $BA^2\sin\theta\cos\theta$ (d) zero [DCE 06]

4. A particle moves along a parabolic path $y = 9x^2$ in such a way that the x-component of velocity remains constant and has a value $\frac{1}{3}$ m/s. The acceleration of the projectile is

(a) $\frac{1}{3}\hat{j}$ m/s^2 (b) $3\hat{j}$ m/s^2

(c) $\frac{2}{3}\hat{j}$ m/s^2 (d) $2\hat{j}$ m/s^2 [DCE 05]

5. A cyclist starts from the centre O of a circular park of radius 1 km, reaches the edge P of the park, then cycles along the circumference and returns to the centre along QO as shown in the figure. If the round trip takes 10 min, the net displacement and average speed of the cyclist (in metre and kilometre per hour) are



(a) 0, 1 (b) $\frac{\pi+4}{2}$, 0

(c) 21.4, $\frac{\pi+4}{2}$ (d) 0, 21.4 [IPUEE 07]

6. A cyclist moves in such a way that he takes 60° urn after every 100 metres. What is the displacement hen he takes seventh turn ?

(a) 100 m (b) 200 m

(c) $100\sqrt{3}$ m (d) $100/\sqrt{3}$ m [DCE 99]

7. An object is moving in a circle of radius 100 m with a constant speed of 31.4 m/s. What is its average speed for one complete revolution ?

(a) zero (b) 31.4 m/s

(c) 3.14 m/s (d) $\sqrt{2} \times 31.4$ m/s [DCE 04]

8. A car runs at a constant speed on a circular track of radius 100 m, taking 62.8 s for every circular lap. The average velocity and average speed for each circular lap respectively are

(a) 0, 0 (b) 0, 10 m/s

(c) 10 m/s, 10 m/s (d) 10 m/s, 0 [DCE 06]

9. Two boys are standing at the ends A and B of a ground, where $AB = a$. The boy at B starts running simultaneously with velocity v and catches the other boy in a time t, where t is

(a) $a/\sqrt{v^2+v_1^2}$ (b) $\sqrt{a^2/(v^2-v_1^2)}$

(c) $a/(v-v_1)$ (d) $a/(v+v_1)$ [DCE 01]

10. The driver of a car moving towards a rocket launching pad with a speed of 6 m/s observes that the rocket is moving with speed of 10 m/s. The upward speed of the rocket as seen by the stationary observer is nearly

(a) 4 m/s (b) 6 m/s

(c) 8 m/s (d) 11 m/s [IPUEE 04]

11. Consider a collection of large number of particles each with speed v. The direction of velocity is randomly distributed in the collection. The magnitude of the relative velocity between a pair of particles averaged over all the pairs in the collection is

(a) $\frac{4v}{\pi}$ (b) greater than $\frac{4v}{\pi}$

(c) less than $\frac{4v}{\pi}$ (d) zero [DCE 05]

12. Two bullets are fired simultaneously, horizontally and with different speeds from the same place. Which bullet will hit the ground first ?

(a) The faster one (b) Depends on their mass

(c) The slower one

(d) Both will reach simultaneously [IPUEE 05]

13. Three pa same point wit 30°, 45° and 60° of the followir

(a) A, B an

(b) Ranges

(c) Range that o

(d) A, B a

14. A pa relation betv by the parti

(a) $R =$

(c) $2H =$

15. F (maximum

(a) 60°

(c) 45°

16. If 45° and t initial ve

(a) $\sqrt{3}$

(c) 3

17. C gets bur smaller part is

(a)

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13. Three particles A, B and C are projected from the same point with the same initial speeds making angles $30°$, $45°$ and $60°$ respectively with the horizontal. Which of the following statements is correct ?

(a) A, B and C have unequal ranges

(b) Ranges of A and C are equal and less than that of B

(c) Ranges of A and C are equal and greater than that of B

(d) A, B and C have equal ranges. [DCE 05]

14. A particle is projected at an angle 45°. The relation between range and maximum height attained by the particle is

(a) $R = 4H$ (b) $4R = H$

(c) $2H = R$ (d) none of these [DCE 02]

15. For a projectile, $(\text{range})^2$ is 48 times of $(\text{maximum height})^2$ obtained. The angle of projection is

(a) $60°$ (b) $30°$

(c) $45°$ (d) $75°$ [DCE 06]

16. If two balls are projected at an angle of $60°$ and $45°$ and the total heights reached are same, then their initial velocities are in the ratio of

(a) $\sqrt{3} : \sqrt{2}$ (b) $\sqrt{2} : \sqrt{3}$

(c) 3 : 2 (d) 2 : 3 [DCE 98]

17. One projectile moving with velocity v in space gets burst into 2 parts of masses in the ratio of 1 : 2. The smaller part becomes stationary, the velocity of other part is

(a) uv (b) $\frac{3v}{2}$

(c) $\frac{4v}{3}$ (d) $\frac{2v}{3}$ [DCE 99]

18. A body executing uniform circular motion has at any instant its velocity vector and acceleration vector

(a) along the same direction

(b) in opposite direction

(c) normal to each other

(d) not related to each other [DCE 2K, 03]

19. The circular motion of a particle with constant speed is

(a) simple harmonic but not periodic

(b) periodic and simple harmonic

(c) neither periodic nor simple harmonic

(d) periodic but not simple harmonic [DCE 01]

20. A particle moves with constant speed v along a circular path of radius r and completes the circle in time T. The acceleration of the particle is

(a) $\frac{2\pi v}{T}$ (b) $\frac{2\pi r}{T}$

(c) $\frac{2\pi r^2}{T}$ (d) $\frac{2\pi v^2}{T}$ [IITJEE 04]

21. A stone tied to the end of a string 1 m long is whirled in a horizontal circle with a constant speed. If the stone makes 22 revolutions in 44 s, what is the magnitude and direction of acceleration of the stone ?

(a) $\frac{\pi^2}{4}\ ms^{-2}$ and direction along the radius towards the centre

(b) $\pi^2\ ms^{-2}$ and direction along the radius away from the centre

(c) $\pi^2\ ms^{-2}$ and direction along the radius towards the centre

(d) $\pi^2\ ms^{-2}$ and direction along the tangent to the circle [DCE 01]

22. For vectors $\vec{A}$ and $\vec{B}$ making an angle 'θ' which one of the following relations is correct ?

(a) $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$ (b) $\vec{A} \times \vec{B} = AB \sin\theta$

(c) $\vec{A} \times \vec{B} = AB \cos\theta$ (d) $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

[DCE 09]

23. For an object thrown at 45° to the horizontal, the maximum height (H) and horizontal range (R) are related as

(a) $R = 16H$ (b) $R = 8H$

(c) $R = 4H$ (d) $R = 2H$ [DCE 09]

24. A particle is projected with certain velocity at two different angles of projections with respect to horizontal plane so as to have the same range 'R' on a horizontal plane. If 't_1' and 't_2' are the time taken for the two paths, then which one of the following relations is correct ?

(a) $t_1 t_2 = 2R/g$ (b) $t_1 t_2 = R/g$

(c) $t_1 t_2 = R/2g$ (d) $t_1 t_2 = 4R/g$. [DCE 09]

25. A body is tied with a string and is given a circular motion with a velocity v in radius r. The magnitude of the acceleration is

(a) v/r (b) v^2/r

(c) v/r^2 (d) v^2/r^2 [DCE 09]

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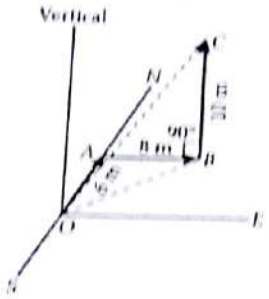
Answers and Explanations

1. *(a)* The situation is shown below



$$OC = \sqrt{OA^2 + AB^2 + BC^2}$$
$$= \sqrt{6^2 + 8^2 + 10^2}$$
$$= \mathbf{10\sqrt{2}\ m}.$$

2. *(d)* $\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|\cos\theta = -|\vec{A}||\vec{B}|$

$\therefore \quad \cos\theta = -1$

or $\quad \theta = \mathbf{180°}$.

3. *(d)* As scalar triple product is cyclic,

$(\vec{B} \times \vec{A}) \cdot \vec{A} = (\vec{A} \times \vec{B}) \cdot \vec{A} = (\vec{A} \times \vec{A}) \cdot \vec{B}$

$= \vec{0} \cdot \vec{B} = 0.$

4. *(d)* Given :

$$v_x = \frac{dx}{dt} = \frac{1}{3}\ \text{ms}^{-2}$$
$$y = 9x^2$$
$$\therefore \quad v_y = \frac{dy}{dt} = 18x\frac{dx}{dt}$$
$$= 18x \times \frac{1}{3} = 6x$$
$$a_y = \frac{dv_y}{dt} = 6\frac{dx}{dt} = 6 \times \frac{1}{3} = 2\ \text{ms}^{-2}$$
$$\therefore \quad \vec{a_y} = 2\hat{j}\ \text{m/s}^2.$$

5. *(d)* As the cyclist returns to initial position O after one round trip, his displacement is zero.

Total distance travelled

$$= OP + OQ + \text{Arc } PQ$$
$$= r + r + \frac{1}{2}\pi r$$
$$= \left(2 + \frac{\pi}{2}\right)r = \frac{4+\pi}{2} \times 1\ \text{km}$$

Time taken $= 10\ \text{min} = \frac{1}{6}\ \text{h}$

$$\text{Average speed} = \frac{4+\pi}{2} \times \frac{6}{1} = (12 + 3\pi)\ \text{kmh}^{-1}$$
$$= \mathbf{21.4\ kmh^{-1}}.$$

6. *(a)* When the cyclist takes seventh turn, he is at 100 m distance from the initial position.

7. *(b)* Average speed = constant speed

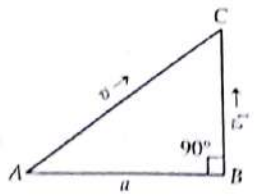
$= \mathbf{31.4\ m/s}.$

8. *(b)* $\text{Average velocity} = \frac{\text{Displacement}}{\text{Time}} = \frac{0}{t} = 0.$

$$\text{Average speed} = \frac{\text{Distance}}{\text{time}} = \frac{2\pi r}{t}$$
$$= \frac{2 \times 3.14 \times 100}{62.8} = \mathbf{10\ ms^{-1}}.$$

9. *(b)* Distance covered by A in time t, $AC = vt$

Distance covered by B in time t, $BC = v_1 t$



By Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$
or $\quad (vt)^2 = a^2 + (v_1 t)^2$
or $\quad t^2(v^2 - v_1^2) = a^2$
or $\quad t = \sqrt{\frac{a^2}{v^2 - v_1^2}}.$

10. *(d)* Observed speed,

$$v = \sqrt{(10)^2 + (6)^2}$$
$$= \sqrt{136} \simeq \mathbf{11\ ms^{-1}}.$$

11. *(a)* Refer to the solution of Problem 5 on Page 4.72.

12. *(d)* Both bullets will reach the ground simultaneously.

$$t = \sqrt{\frac{2h}{g}}$$

Time t does not depend on horizontal speed.

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13. (*b*) $R_A = \frac{u^2 \sin 60°}{g} = \frac{\sqrt{3}}{2}\frac{u^2}{g}$

$$R_B = \frac{u^2 \sin 90°}{g} = \frac{u^2}{g}$$

$$R_C = \frac{u^2 \sin^2 120°}{g} = \frac{\sqrt{3}}{2}\frac{u^2}{g}$$

Ranges of A and C are equal but less than that of B.

14. (*a*) $R = \frac{u^2 \sin(2 \times 45°)}{g} = \frac{u^2}{g}$

$$H = \frac{u^2 \sin^2 45°}{2g} = \frac{u^2}{4g}$$

$\therefore$ $R = 4H$.

15. (*b*) Given : $(\text{Range})^2 = 48\,(\text{Max. Height})^2$

or $$\left(\frac{u^2 \sin 2\theta}{g}\right)^2 = 48\left(\frac{u^2 \sin^2\theta}{2g}\right)^2$$

or $$u^2\frac{\sin 2\theta}{g} = 4\sqrt{3}\frac{u^2 \sin^2\theta}{2g}$$

or $$2\sin\theta\cos\theta = 2\sqrt{3}\sin^2\theta$$

or $$\tan\theta = \frac{1}{\sqrt{3}}$$

$\therefore$ $\theta = 30°$.

16. (*b*) Given : $H_1 = H_2$

or $$\frac{u_1^2 \sin^2 60°}{2g} = \frac{u_2^2 \sin^2 45°}{2g}$$

or $$\frac{u_1}{u_2} = \frac{\sin 45°}{\sin 60°}$$

$$= \frac{1/\sqrt{2}}{\sqrt{3}/2} = \frac{\sqrt{2}}{\sqrt{3}}.$$

17. (*b*) By conservation of momentum,

$$mv = m_1 v_1 + m_2 v_2$$

$$mv = \frac{m}{3} \times 0 + \frac{2m}{3} \times v_2$$

or $$v_2 = \frac{3v}{2}.$$

18. (*c*) Velocity is along the tangent to the circular path and acceleration vector is along the radius.

19. (*d*) Circular motion with constant speed is periodic but not simple harmonic.

20. (*a*) For circular motion,

$$v = \frac{2\pi r}{T}$$

or $$r = \frac{vT}{2\pi}$$

Acceleration in circular motion,

$$a = \frac{v^2}{r} = \frac{v^2}{vT/2\pi}$$

$$= \frac{2\pi v}{T}.$$

21. (*c*) $a = r\omega^2 = r\left(\frac{2\pi n}{t}\right)^2$

$$= r \times \frac{4\pi^2 n^2}{t^2} = \frac{1 \times 4 \times \pi^2 \times (22)^2}{(44)^2}$$

$$= \pi^2 \text{ ms}^{-2}.$$

22. (*d*) The cross product of two vectors is anticommutative *i.e.*,

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}.$$

23. (*c*) As $R = \frac{u^2}{g}$ and $H = \frac{u^2}{4g}$ $\therefore$ $R = 4H$.

24. (*a*) As $t_1 = \frac{2u\sin\theta}{g}$

and $t_2 = \frac{2u\cos\theta}{g}$

$\therefore$ $t_1 t_2 = \frac{4u^2 \sin\theta\cos\theta}{g}$

$$= \frac{2u^2 \sin 2\theta}{g} = \frac{2R}{g}.$$

25. (*b*) Centripetal acceleration,

$$a_c = \frac{v^2}{r}.$$

AIIMS Entrance Exam

1. Two equal vectors have a resultant equal to either. The angle between them is

(*a*) 60° (*b*) 90°

(*c*) 100° (*d*) 120° [AIIMS 01]

2. Angle between two vectors of magnitudes 12 and 18 units, when their resultant is 24 units, is

(*a*) 63°51′ (*b*) 75°52′

(*c*) 82°31′ (*d*) 89°16′ [AIIMS 96]

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3. A body A is dropped vertically from the top of a tower. If another identical body B is thrown horizontally from the same point at the same instant, then
(a) A will reach the ground earlier than B
(b) B will reach the ground earlier than A
(c) both A and B will reach the ground simultaneously
(d) either A or B. [AIIMS 94]

4. Which of the following is constant in a projectile motion ?
(a) Horizontal component of the velocity
(b) Vertical component of the velocity
(c) Velocity of projection
(d) All of these. [AIIMS 96]

5. At the uppermost point of a projectile, its velocity and acceleration are at an angle of
(a) $0°$ (b) $45°$
(c) $90°$ (d) $180°$ [AIIMS 02]

6. Two projectiles are projected with the same velocity. If one is projected at an angle of $30°$ and the other at $60°$ to the horizontal, then the ratio of maximum height reached, is
(a) 3 : 1 (b) 1 : 3
(c) 1 : 2 (d) 2 : 1 [AIIMS 01]

7. A body is projected at such an angle that the horizontal range is three times the greatest height. The angle of projection is
(a) 25°8′ (b) 33°7′
(c) 42°8′ (d) 53°8′ [AIIMS 98]

8. The direction of the angular velocity vector is along
(a) the tangent to the circular path
(b) the inward radius
(c) the outward radius
(d) the axis of rotation [AIIMS 04, 07]

9. A stone tied to the end of string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, what is the magnitude of acceleration of the stone ?
(a) $680\,\text{cm s}^{-2}$ (b) $720\,\text{cm s}^{-2}$
(c) $860\,\text{cm s}^{-2}$ (d) $990\,\text{cm s}^{-2}$ [AIIMS 01]

10. If the equation for the displacement of a particle moving on a circular path is given by $\theta = 2t^3 + 0.5$, where θ is in radians and t in seconds, then, the angular velocity of the particle at $t = 2$ s is
(a) 8 rad s^{-1} (b) 12 rad s^{-1}
(c) 24 rad s^{-1} (d) 36 rad s^{-1} [AIIMS 98]

11. A body is projected horizontally with a velocity of $4\sqrt{2}$ m/sec. The velocity of the body after 0.7 seconds will be nearly (Take $g = 10\ \text{m/s}^2$)
(a) 10 m/sec (b) 9 m/sec
(c) 19 m/sec (d) 11 m/sec [AIIMS 2009]

12. Two projectiles of same mass have their maximum kinetic energies in ratio 4 : 1 and ratio of their maximum heights is also 4 : 1, then what is the ratio of their ranges ?
(a) 2 : 1 (b) 4 : 1
(c) 8 : 1 (d) 16 : 1 [AIIMS 2010]

13. The position of a particle moving in the x-y plane at any time t is given by ; $x = (3t^2 - 6t)$ metres ; $y = (t^2 - 2t)$ metres. Select the correct statement.
(a) acceleration is zero at $t = 0$
(b) velocity is zero at $t = 0$
(c) velocity is zero at $t = 1$s
(d) velocity and acceleration of the particle are never zero. [AIIMS 2010]

ASSERTIONS AND REASONS

Directions. In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as :
(a) If both assertion and reason are true and reason is the correct explanation of assertion
(b) If both assertion and reason are true but reason is not the correct explanation of assertion
(c) If assertion is true but reason is false
(d) If both assertion and reason are false

14. **Assertion.** Two balls of different masses are thrown vertically upward with same speed. They will pass through their point of projection in the downward direction with the same speed.

Reason. The maximum height and downward velocity attained at the point of projection are independent of the mass of the ball. [AIIMS 2010]

15. **Assertion.** In javelin throw, the athlete throws the projectile at an angle slightly more than 45°.

Reason. The maximum range does not depend upon angle of projection. [AIIMS 2010]

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16. **Assertion.** Generally the path of a projectile from the earth is parabolic but it is elliptical for projectiles going to a very great height.

Reason. Up to ordinary height the projectile moves under a uniform gravitational force, but for great heights, projectile moves under a variable force.

[AIIMS 1998]

Answers and Explanations

1. (d) Refer to the solution of Example 5 on page 4.9.

2. (b) As $R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$\therefore \quad 24^2 = 12^2 + 18^2 + 2 \times 12 \times 18\cos\theta$

or $\quad 576 - (144 + 324) = 432\cos\theta$

or $\quad \cos\theta = \dfrac{108}{432} = 0.25.$

$\therefore \quad \theta = \cos^{-1}(0.25) = 75°52'.$

3. (c) Both A and B will reach the ground simultaneously. Both have zero initial vertical velocity, cover the same height under the acceleration due to gravity 'g'.

4. (a) Horizontal component of velocity is not affected by the force of gravity.

5. (c) At the highest point, the projectile has only a horizontal velocity while the acceleration due to gravity acts vertically downwards.

6. (b) Maximum height, $H = \dfrac{u^2\sin^2\theta}{2g}$

$\therefore \quad \dfrac{H_1}{H_2} = \dfrac{\sin^2 30°}{\sin^2 60°}$

$= \dfrac{(1/2)^2}{(\sqrt{3}/2)^2} = \dfrac{1}{3} = \mathbf{1:3}.$

7. (d) Here $\quad R = 3H$

or $\quad \dfrac{u^2\sin 2\theta}{g} = 3\dfrac{u^2\sin^2\theta}{2g}$

or $\quad \dfrac{u^2 \times 2\sin\theta\cos\theta}{g} = 3\dfrac{u^2\sin^2\theta}{2g}$

or $\quad \tan\theta = \dfrac{4}{3} = 1.3333$

$\theta = \tan^{-1}(1.3333) = \mathbf{53°8'}.$

8. (d) $\quad \vec{v} = \vec{\omega} \times \vec{r}$

As linear velocity vector $\vec{v}$ is along the tangent to the circular path and angular velocity vector $\vec{\omega}$ is perpendicular to $\vec{v}$, so $\vec{\omega}$ is along the axis of rotation.

9. (d) Refer answer to NCERT Exercise 4.17.

10. (c) $\quad \theta = 2t^3 + 0.5$

$\omega = \dfrac{d\theta}{dt} = 6t^2$

At $t = 2$ s,

$\omega = 6 \times 4 = \mathbf{24\ rad\ s^{-1}}.$

11. (b) After 0.7 s,

$v_x = 4\sqrt{2}$ m/s

$v_y = 0 + gt = 10 \times 0.7 = 7$ m/s

$\therefore \quad v = \sqrt{v_x^2 + v_y^2} = \sqrt{32 + 49} = \mathbf{9\ m/s}$

12. (b) $\dfrac{K_1^{max}}{K_2^{max}} = \dfrac{\frac{1}{2}mu_1^2}{\frac{1}{2}mu_2^2} = \dfrac{u_1^2}{u_2^2} = \dfrac{4}{1}$

$\dfrac{H_1}{H_2} = \dfrac{u_1^2\sin^2\theta_1}{2g} \cdot \dfrac{2g}{u_2^2\sin^2\theta_2} = \dfrac{4}{1}$

or $\quad \dfrac{u_1^2}{u_2^2} \cdot \dfrac{\sin^2\theta_1}{\sin^2\theta_2} = \dfrac{4}{1}$ or $\dfrac{4}{1} \cdot \dfrac{\sin^2\theta_1}{\sin^2\theta_2} = \dfrac{4}{1}$

$\therefore \quad \sin\theta_1 = \sin\theta_2 \Rightarrow \theta_1 = \theta_2$

$\Rightarrow \quad \dfrac{R_1}{R_2} = \dfrac{u_1^2\sin^2\theta_1}{u_2^2\sin^2\theta_2} = \dfrac{u_1^2}{u_2^2} = \dfrac{4}{1} = \mathbf{4:1} \quad [\theta_1 = \theta_2]$

13. (c) $\quad x = 3t^2 - 6t$

$\therefore \quad v_x = \dfrac{dx}{dt} = 6t - 6$

At $t = 1$, $v_x = 6 \times 1 - 6 = 0$

Also, $\quad y = t^2 - 2t$

$\therefore \quad v_y = \dfrac{dy}{at} = 2t - 2$

At $t = 1$s, $v_y = 2 \times 1 - 2 = 0.$

14. (a) Maximum height attained,

$h = ut - \dfrac{1}{2}gt^2$

Downward velocity attained,

$v = \sqrt{u^2 - 2gh}$

Both h and v are independent of mass m.

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15. *(d)* When a body is projected from a place above the earth's surface, the angle of projection must be slightly less than 45° for the maximum horizontal angle.

16. *(a)* Assertion is true and the reason is its correct explanation. At greater heights, the gravitational force varies in inverse proportion to the square of the distance of the projectile from the centre of the earth. Under such a variable force, the path of the projectile is elliptical

CBSE PMT Prelims and Final Exams

[Similar Questions]

1. If $|\vec{A}+\vec{B}|=|\vec{A}|+|\vec{B}|$, then angle between A and B will be

(a) 90° (b) 120°
(c) 0° (d) 60° [CBSE PMT 01]

2. A bus is moving on a straight road towards north with a uniform speed of 50 km/hour, then it turns left through 90°. If the speed remains unchanged after turning, the increase in the velocity of bus in the turning process is

(a) 70.7 km/hr along south-west direction
(b) zero
(c) 50 km/hr along west
(d) 70.7 km/hr along north-west direction

[CBSE PMT 90]

3. If a unit vector is represented by $0.5\hat{i}-0.8\hat{j}+c\hat{k}$ then the value of c is

(a) $\sqrt{0.01}$ (b) $\sqrt{0.11}$
(c) 1 (d) $\sqrt{0.39}$ [CBSE PMT 99]

4. The angle between the two vectors $\vec{A}=3\hat{i}+4\hat{j}+5\hat{k}$ and $\vec{B}=3\hat{i}+4\hat{j}-5\hat{k}$ will be

(a) 90° (b) 180°
(c) zero (d) 45° [CBSE PMT 94]

5. If a vector $2\hat{i}+3\hat{j}+8\hat{k}$ is perpendicular to the vector $4\hat{j}-4\hat{i}+\alpha\hat{k}$, then the value of α is

(a) 1/2 (b) −1/2
(c) 1 (d) −1 [CBSE PMT 05]

6. The magnitudes of vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ are 3, 4 and 5 units respectively. If $\vec{A}+\vec{B}=\vec{C}$, the angle between $\vec{A}$ and $\vec{B}$ is

(a) $\pi/2$ (b) $\cos^{-1}0.6$
(c) $\tan^{-1}7/5$ (d) $\pi/4$ [CBSE PMT 88]

7. The vectors $\vec{A}$ and $\vec{B}$ are such that $|\vec{A}+\vec{B}|=|\vec{A}-\vec{B}|$. The angle between the two vectors is

(a) 45° (b) 90°
(c) 60° (d) 75°

[CBSE PMT 91, 96, 2K]

8. The vector sum of two forces is perpendicular to their vector differences. In that case, the forces

(a) are equal to each other
(b) are equal to each other in magnitude
(c) are not equal to each other in magnitude
(d) cannot be predicted

[CBSE PMT 03]

9. $\vec{A}$ and $\vec{B}$ are two vectors and θ is the angle between them, if $|\vec{A}\times\vec{B}|=\sqrt{3}(\vec{A}.\vec{B})$, the value of θ is

(a) 45° (b) 30°
(c) 90° (d) 60° [CBSE PMT 07]

10. If $|\vec{A}\times\vec{B}|=\sqrt{3}\,\vec{A}.\vec{B}$, then the value of $|\vec{A}+\vec{B}|$ is

(a) $(A^2+B^2+AB)^{1/2}$ (b) $\left(A^2+B^2+\frac{AB}{\sqrt{3}}\right)^{1/2}$
(c) $A+B$ (d) $(A^2+B^2+\sqrt{3}AB)^{1/2}$

[CBSE PMT 04]

11. If the angle between the vectors $\vec{A}$ and $\vec{B}$ is θ, the value of the product $(\vec{B}\times\vec{A}).\vec{A}$ is equal to

(a) $BA^2\sin\theta$ (b) $BA^2\cos\theta$
(c) $BA^2\sin\theta\cos\theta$ (d) zero

[CBSE PMT 89, 05]

12. The resultant of $\vec{A}\times 0$ will be equal to

(a) zero (b) A
(c) zero vector (d) unit vector

[CBSE PMT 92]

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13. A boat is sent across a river with a velocity of $8\,km\,h^{-1}$. If the resultant velocity of boat is $10\,km\,h^{-1}$, then velocity of river is

(a) $12.8\,km\,h^{-1}$ (b) $6\,km\,h^{-1}$

(c) $8\,km\,h^{-1}$ (d) $10\,km\,h^{-1}$

[CBSE PMT 93, 94]

14. The width of river is 1 km. The velocity of boat is 5 km/h The boat covered the width of river in shortest time 15 min. Then the velocity of river stream is

(a) 3 km/hr (b) 4 km/hr

(c) $\sqrt{29}$ km/hr (d) $\sqrt{41}$ km/hr

[CBSE PMT 98, 2K]

15. A person aiming to reach exactly opposite point on the bank of a stream is swimming with a speed of 0.5 m/s at an angle of 120° with the direction of flow of water. The speed of water in the stream are

(a) 0.25 m/s (b) 0.5 m/s

(c) 1.0 m/s (d) 0.433 [CBSE PMT 99]

16. A car runs at a constant speed of a circular track of radius 100 m, taking 62.8 seconds for every circular lap. The average velocity and average speed for each circular lap respectively is

(a) 10 m/s, 0 (b) 0, 0

(c) 0, 10 m/s (d) 10 m/s, 10 m/s

[CBSE PMT 01]

17. A particle starting from the origin (0, 0) moves in a straight line in the (x,y) plane. Its coordinates at a later time are $(\sqrt{3},3)$. The path of the particle makes with the x-axis an angle of

(a) 45° (b) 60°

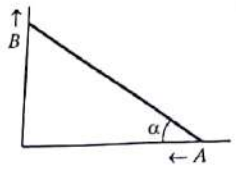
(c) 0° (d) 30° [CBSE PMT 07]

18. Two boys are standing at the ends A and B of a ground where $AB = a$. The boy at B starts running in a direction perpendicular to AB with velocity v_1. The boy at A starts running simultaneously with velocity v and catches the other in a time t, where t is

(a) $\dfrac{a}{\sqrt{v^2+v_1^2}}$ (b) $\dfrac{a}{v+v_1}$

(c) $\dfrac{a}{v-v_1}$ (d) $\sqrt{\dfrac{a^2}{v^2-v_1^2}}$ [CBSE PMT 05]

19. Two particles A and B are connected by a rigid rod AB. The rod slides along perpendicular rails as shown here.



The velocity of A to the left is 10 m/s. What is the velocity of B when angle $\alpha = 60°$?

(a) 10 m/s (b) 9.8 m/s

(c) 5.8 m/s (d) 17.3 m/s [CBSE PMT 98]

20. The position vector of a particle is

$$\vec{r} = (a\cos\omega t)\hat{i} + (a\sin\omega t)\hat{j}$$

The velocity of the particle is

(a) directed towards the origin

(b) directed away from the origin

(c) parallel to the position vector

(d) perpendicular to the position vector

[CBSE PMT 95]

21. Two projectiles of same mass and with same velocity are thrown at an angle 60° and 30° with the horizontal, then which will remain same

(a) time of flight

(b) range of projectile

(c) maximum height acquired

(d) all of them [CBSE PMT 2K]

22. If a body A of mass M is thrown with velocity v at an angle 30° to the horizontal and another body B of the same mass is thrown with the same speed at an angle of 60° to the horizontal, the ratio of horizontal range of A to B will be

(a) 1 : 3 (b) 1 : 1

(c) $1 : \sqrt{3}$ (d) $\sqrt{3} : 1$ [CBSE PMT 90, 92]

23. For angles of projection of a projectile at angle $(45°-\theta)$ and $(45°+\theta)$, the horizontal range described by the projectile are in the ratio of

(a) 2 : 1 (b) 1 : 1

(c) 2 : 3 (d) 1 : 2 [CBSE PMT 06]

24. The maximum range of a gun of horizontal tarrain is 16 km. If $g = 10\ ms^{-2}$, then muzzle velocity of a shell must be

(a) $160\,ms^{-1}$ (b) $200\sqrt{2}\ ms^{-2}$

(c) $400\,ms^{-1}$ (d) $800\,ms^{-1}$

[CBSE PMT 90]

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25. A cricket ball is hit at 45° to the horizontal with a kinetic energy E. The kinetic energy at the highest point is

(a) 0 (b) $E/2$

(c) $E/\sqrt{2}$ (d) E [CBSE PMT 97]

26. A particle A is dropped from a height and another particle B is projected in horizontal direction with speed of 5 m/sec from the same height, then correct statement is

(a) particle A will reach at ground first with respect to particle B

(b) particle B will reach at ground first with respect to particle A

(c) both particles will reach at ground simultaneously

(d) both particles will reach at ground with same speed [CBSE PMT 02]

27. A man throws balls with the same speed vertically upwards one after the other at an interval of 2 seconds. What should be the speed of the throw so that more than two balls are in the sky at any time ? (Given $g = 9.8 \text{ m/s}^2$)

(a) more than 19.6 m/s

(b) atleast 9.8 m/s

(c) any speed less than 19.6 m/s

(d) only with speed 19.6 m/s [CBSE PMT 03]

28. A man is slipping on a frictionless inclined plane and a bag falls down from the same height. Then the velocity of both is related as

(a) $v_B > v_m$

(b) $v_B < v_m$

(c) $v_B = v_m$

(d) v_B and v_m can't be related [CBSE PMT 2K]

29. The angular speed of a flywheel making 120 revolutions/minute is

(a) 4π rad/s (b) $4\pi^2$ rad/s

(c) π rad/s (d) 2π rad/s [CBSE PMT 95]

30. Two racing cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2 respectively. Their speeds are such that each makes a complete circle in the same time t. The ratio of the angular speeds of the first to the second car is

(a) $r_1 : r_2$ (b) $m_1 : m_2$

(c) 1 : 1 (d) $m_1 m_2 : r_1 r_2$ [CBSE PMT 99]

31. Two particles having mass M and m are moving in a circular path having radius R and r. If their time periods are same, then the ratio of angular velocity will be

(a) $\frac{r}{R}$ (b) $\frac{R}{r}$

(c) 1 (d) $\sqrt{\frac{R}{r}}$ [CBSE PMT 01]

32. A body is whirled in a horizontal circle of radius 20 cm. It has an angular velocity of 10 rad/s

What is its linear velocity at any point on circular path ?

(a) 20 m/s (b) $\sqrt{2}$ m/s

(c) 10 m/s (d) 2 m/s [CBSE PMT 96]

33. What is the value of linear velocity, if

$\vec{r} = 3\hat{i} - 4\hat{j} + \hat{k}$ and $\vec{\omega} = 5\hat{i} - 6\hat{j} + 6\hat{k}$?

(a) $4\hat{i} - 13\hat{j} + 6\hat{k}$ (b) $18\hat{i} + 13\hat{j} - 2\hat{k}$

(c) $6\hat{i} + 2\hat{j} - 3\hat{k}$ (d) $6\hat{i} - 2\hat{j} + 8\hat{k}$

[CBSE PMT 99]

34. An electric fan has blades of length 30 cm measured from the axis of rotation. If the fan is rotating at 120 rpm, the acceleration of a point on the tip of the blade is

(a) 1600 ms^{-2} (b) 47.4 ms^{-2}

(c) 23.7 ms^{-2} (d) 50.55 ms^{-2} [CBSE PMT 90]

35. A particle moves along a circle of radius $\left(\frac{20}{\pi}\right)$ m with constant tangential acceleration. If the velocity of the particle is 80 m/s at the end of the second revolution after motion has begun, the tangential acceleration is

(a) 40 m/s^2 (b) $640\pi \text{ m/s}^2$

(c) $160\pi \text{ m/s}^2$ (d) $40\pi \text{ m/s}^2$ [CBSE PMT 03]

36. A stone tied to the end of a string of 1 m length is whirled in a horizontal circle with a constant speed. If the stone makes 22 revolutions in 44 seconds, what is the magnitude and direction of acceleration of the stone ?

(a) $\pi^2 \text{ms}^{-2}$ and direction along the radius towards the centre

(b) $\pi^2 \text{ms}^{-2}$ and direction along the radius away from the centre

(c) $\pi^2 \text{ms}^{-2}$ and direction along the tangent to the circle

(d) $\pi^2/4 \text{ ms}^{-2}$ and direction along the radius towards the centre. [CBSE PMT 05]

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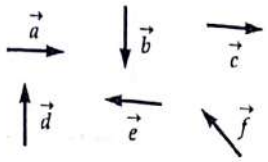
37. Six vectors, $\vec{a}$ through $\vec{f}$ have the magnitudes and directions indicated in the figure. Which of the following statements is true ?



(a) $\vec{b}+\vec{c}=\vec{f}$ (b) $\vec{d}+\vec{c}=\vec{f}$

(c) $\vec{d}+\vec{e}=\vec{f}$ (d) $\vec{b}+\vec{e}=\vec{f}$ [CBSE Pre 2010]

38. A particle has initial velocity $(3\hat{i}+4\hat{j})$ and has acceleration $(0.4\hat{i}+0.3\hat{j})$. Its speed after 10 s is

(a) 7 units (b) $7\sqrt{2}$ units

(c) 8.5 units (d) 10 units [CBSE Pre 2010]

39. A body is moving with velocity 30 m/s towards east. After 10 seconds its velocity becomes 40 m/s towards north. The average acceleration of the body is

(a) 1 m/s^2 (b) 7 m/s^2

(c) $\sqrt{7}$ m/s^2 (d) 5 m/s^2 [CBSE Pre 2011]

40. A missile is fired for maximum range with an initial velocity of 20 m/s. If $g=10\,m/s^2$, the range of the missile is

(a) 40 m (b) 50 m

(c) 60 m (d) 20 m [CBSE Pre 2011]

41. A projectile is fired at an angle of 45° with the horizontal. Elevation angle of the projectile at its highest point as seen from the point of projection, is

(a) $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (b) 45°

(c) 60° (d) $\tan^{-1}\frac{1}{2}$ [CBSE Final 2011]

42. A particle moves in a circle of radius 5 cm with constant speed and time period 0.2π s. The acceleration of the particle is

(a) 15 m/s^2 (b) 25 m/s^2

(c) 36 m/s^2 (d) 5 m/s^2 [CBSE Pre 2011]

Answers and Explanations

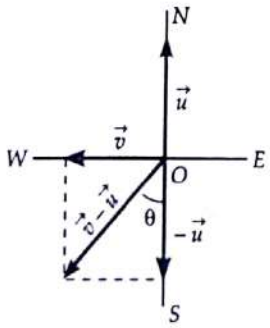
1. (c) When $\theta=0°$,

$$|\vec{A}+\vec{B}|=\sqrt{|\vec{A}|^2+|\vec{B}|^2+2|\vec{A}||\vec{B}|\cos 0°}$$

$$=|\vec{A}|+|\vec{B}|$$

2. (a) Initial velocity, $u=50\ km\,h^{-1}$, due north

Final velocity, $v=50\ kmh^{-1}$, due west



Change in velocity $=\vec{v}-\vec{u}=\vec{v}+(-\vec{u})$, along south-west.

$$|\vec{v}-\vec{u}|=\sqrt{50^2+50^2}=\sqrt{5000}$$

$$=\mathbf{70.71\ km\ h^{-1}}.$$

3. (b) $|0.5\hat{i}-0.8\hat{j}+c\hat{k}|=1$

$$\sqrt{(0.5)^2+(0.8)^2+c^2}=1$$

or $0.25+0.64+c^2=1$

$$c=\sqrt{0.11}.$$

4. (a) $\vec{A}=3\hat{i}+4\hat{j}+5\hat{k}$

$$\vec{B}=3\hat{i}+4\hat{j}-5\hat{k}$$

$$\vec{A}.\vec{B}=3\times3+4\times4+5\times(-5)=0$$

$\Rightarrow$ $\vec{A}\perp\vec{B}$.

5. (b) $\vec{A}\perp\vec{B}\Rightarrow\vec{A}.\vec{B}=0$

$$(2\hat{i}+3\hat{j}+8\hat{k}).(4\hat{j}-4\hat{i}+\alpha\hat{k})=0$$

$$2\times(-4)+3\times4+8\alpha=0$$

as $\alpha=-\frac{1}{2}$.

6. (a) $\vec{A}+\vec{B}=\vec{C}$

$\therefore$ $(\vec{A}+\vec{B}).(\vec{A}+\vec{B})=\vec{C}.\vec{C}$

$$|\vec{A}|^2+|\vec{B}|^2+2|\vec{A}||\vec{B}|\cos\theta=|\vec{C}|^2$$

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$$9+16+2\times3\times4\cos\theta=25$$

$$\cos\theta=0$$

or $$\theta=\frac{\pi}{2}.$$

7. (*b*) Refer to the solution of Example 37 on Page 4.23.

8. (*a*) Given : $(\vec{F}_1+\vec{F}_2)\perp(\vec{F}_1-\vec{F}_2)$

$$\therefore\quad (\vec{F}_1+\vec{F}_2).(\vec{F}_1-\vec{F}_2)=0$$

or $$F_1^2-\vec{F}_1.\vec{F}_2+\vec{F}_2.\vec{F}_1-F_2^2=0$$

or $$F_1^2=F_2^2$$

$$\Rightarrow\quad F_1=F_2.$$

9. (*d*) Given : $|\vec{A}\times\vec{B}|=\sqrt{3}(\vec{A}.\vec{B})$

$$\therefore\quad AB\sin\theta=\sqrt{3}\,AB\cos\theta$$

or $$\tan\theta=\sqrt{3}$$

$$\Rightarrow\quad \theta=60°.$$

10. (*a*) From above problem, $\theta=60°$

$$\therefore\quad |\vec{A}\times\vec{B}|=\sqrt{|\vec{A}|+|\vec{B}|^2+2|\vec{A}||\vec{B}|\cos60°}$$

$$=(A^2+B^2+AB)^{1/2}$$

11. (*d*) As triple scalar product is cyclic.

$$(\vec{B}\times\vec{A})\vec{A}=(\vec{A}\times\vec{B}).\vec{A}$$

$$=(\vec{A}\times\vec{A}).\vec{B}=\vec{0}.\vec{B}=\vec{0}$$

12. (*c*) The resultant of $\vec{A}\times0$ is a vector of zero magnitude. The product of a vector with a scalar gives vector.

13. (*b*) Resultant velocity

$$=\sqrt{v_B^2+v_R^2+2v_Bv_R\cos\theta}$$

$$(10)^2=8^2+V_R^2+2\times8\times V_R\cos90°$$

$$V_R=\sqrt{100-64}$$

$$=\mathbf{6\ km\ h^{-1}}$$

(*a*) $$v_{Resultant}=\frac{1\text{ km}}{15\text{ min}}$$

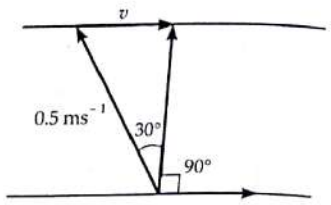
$$=\frac{1\text{km}}{1/4\text{ h}}=4\text{ km h}^{-1}$$

$$v_{River}=\sqrt{v_{Resultant}^2-v_{Boat}^2}$$

$$=\sqrt{5^2-4^2}$$

$$=\mathbf{3\ kmh^{-1}}.$$

15. (*a*) Let v be the speed of river water.



Clearly, $$\frac{v}{0.5\,\text{ms}^{-1}}=\sin30°$$

$$\therefore\quad v=0.5\times0.5=\mathbf{0.25\ ms^{-1}}.$$

16. (*c*) Distance covered in one lap $=2\pi r$

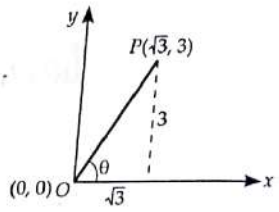
$$\text{Average speed}=\frac{\text{Distance}}{\text{Time}}=\frac{2\pi r}{t}$$

$$=\frac{2\times3.14\times100}{62.8}=10\text{ m}.$$

Net displacement for one lap = 0.

$$\text{Average velocity}=\frac{\text{Displacement}}{\text{Time}}=\frac{0}{t}=0.$$

17. (*b*) The situation is shown in the figure.

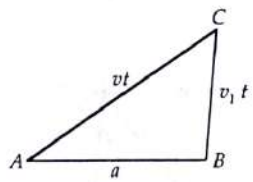


If the path OP makes angle θ with the x-axis, then

$$\tan\theta=\frac{3}{\sqrt{3}}=\sqrt{3}$$

$$\therefore\quad \theta=\mathbf{60°}.$$

18. (*d*) Two boys stand at the points A and B separated by distance a.



From right ΔABC,

$$AB^2+BC^2=AC^2$$

$$a^2+(v_1t)^2=(vt)^2$$

$$a^2=(v^2-v_1^2)t^2$$

$$t=\frac{a}{\sqrt{v^2-v_1^2}}$$

19. (*d*) From the figur

$$\frac{y}{x}=\tan\alpha$$

$$y=x\tan\alpha$$

Differentiating w.r.t. t

$$\frac{dy}{dx}=\frac{dx}{dt}\tan\alpha$$

$$v_y=v_x\tan\alpha$$

$$=10\sqrt{3}=1$$

20. (*d*) Position vector,

$$\vec{r}=(a\cos\omega t)\hat{i}+(a\si$$

Velocity vector,

$$\vec{v}=\frac{d\vec{r}}{dt}=(-a\omega\sin\omega$$

$$\vec{v}.\vec{r}=(-a\omega\sin\omega t)(a\cos$$

$$=0$$

$$\Rightarrow\ \vec{v}\perp\vec{r}.$$

21. (*b*) Here $\theta_1+\theta_2=60$

Hence horizontal range

22. (*b*) Same reason as i

Hence $R_1:R_2=1:1$.

23. (*b*) Here $\theta_1+\theta_2=45$

Hence horizontal range R

24. (*c*) $$R_{max}=\frac{u^2}{g}$$

or $$1600\text{ m}=\frac{u^2}{10\text{ ms}^{-2}}$$

$$u=\sqrt{160000}=$$

25. (*b*) Initial kinetic ene

$$E=\frac{1}{2}mv^2$$

At the highest point, t horizontal component of ve its kinetic energy becomes

$$E'=\frac{1}{2}m(u\cos4$$

$$=\frac{1}{2}mu^2\times\frac{1}{2}=$$

26. (*c*) Initially, both pa velocity. So, both particles through same height.

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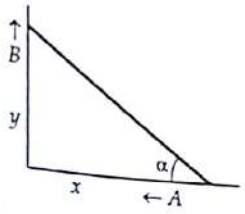
19. (*d*) From the figure,



$$\frac{y}{x} = \tan\alpha$$

$$y = x\tan\alpha$$

Differentiating w.r.t. time t,

$$\frac{dy}{dx} = \frac{dx}{dt}\tan\alpha$$

$$v_y = v_x \tan\alpha = 10\tan 60°$$

$$= 10\sqrt{3} = 10 \times 1.732 = 17.32 \text{ ms}^{-1}.$$

20. (*d*) Position vector,

$$\vec{r} = (a\cos\omega t)\hat{i} + (a\sin\omega t)\hat{j}$$

Velocity vector,

$$\vec{v} = \frac{d\vec{r}}{dt} = (-a\omega\sin\omega t)\hat{i} + (a\omega\cos\omega t)\hat{j}$$

$$\vec{v}.\vec{r} = (-a\omega\sin\omega t)(a\cos\omega t) + (a\omega\cos\omega t)(a\sin\omega t)$$

$$= 0$$

$$\Rightarrow \vec{v} \perp \vec{r}.$$

21. (*b*) Here $\theta_1 + \theta_2 = 60° + 30° = 90°$.

Hence horizontal range R is same.

22. (*b*) Same reason as in the above problem.

Hence $R_1 : R_2 = 1:1$.

23. (*b*) Here $\theta_1 + \theta_2 = 45° - \theta + 45° - \theta = 90°$.

Hence horizontal range R is same and $R_1 : R_2 = 1 : 1$.

24. (*c*) $R_{max} = \dfrac{u^2}{g}$

or $$1600 \text{ m} = \frac{u^2}{10 \text{ ms}^{-2}}$$

$$u = \sqrt{160000} = 400 \text{ ms}^{-1}.$$

25. (*b*) Initial kinetic energy,

$$E = \frac{1}{2}mv^2$$

At the highest point, the particle has only the horizontal component of velocity $u\cos 45°$. Therefore, its kinetic energy becomes

$$E' = \frac{1}{2}m(u\cos 45°)^2$$

$$= \frac{1}{2}mu^2 \times \frac{1}{2} = \frac{E}{2}.$$

26. (*c*) Initially, both particles have zero vertical velocity. So, both particles take same time to fall through same height.

27. (*a*) Time of flight for vertical projection,

$$T = \frac{2u\sin 90°}{g} = \frac{2u}{g}$$

Time interval between two balls thrown = 2 s

For a minimum of three balls to remain in air, time of flight of first ball must be greater than 4 s.

$$\therefore \quad T > 4 \text{ s} \quad \text{or} \quad \frac{2u}{g} > 4 \text{ s}$$

or $$u > 2g \quad \text{or} \quad u > 19.6 \text{ m/s}.$$

28. (*c*) Vertical acceleration of both is equal to g. Also horizontal velocity is constant.

29. (*a*) $$\omega = \frac{\theta}{t} = \frac{2\pi \times 120 \text{ rad}}{60 \text{ s}} = 4\pi \text{ rad s}^{-1}.$$

30. (*c*) $$t = \frac{2\pi}{\omega_1} = \frac{2\pi}{\omega_2} \quad \therefore \quad \frac{\omega_1}{\omega_2} = \frac{1}{1}.$$

31. (*c*) Same reason as in the above problem.

32. (*d*) Here $r = 20 \text{ cm} = 0.20 \text{ m}$,

$$\omega = 10 \text{ rad s}^{-1}.$$

$$\therefore \quad v = r\omega = 0.20 \times 10 = 2 \text{ ms}^{-1}.$$

33. (*b*) $$\vec{v} = \vec{\omega} \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -6 & 6 \\ 3 & -4 & 1 \end{vmatrix}$$

$$= \hat{i}(-6+24) - \hat{j}(5-18) + \hat{k}(-20+18)$$

$$= 18\hat{i} + 13\hat{j} - 2\hat{k}$$

34. (*b*) $$\omega = \frac{2\pi \times 120 \text{ rad}}{60 \text{ s}} = 4\pi \text{ rad s}^{-1}$$

$$a = \omega^2 r = (4\pi)^2 \times 0.30 = 47.4 \text{ ms}^{-2}.$$

35. (*a*) Here $r = \dfrac{20}{\pi}\text{m}$, $v = 80 \text{ ms}^{-1}$, $\omega_0 = 0$

$$\theta = 2 \text{ rev} = 4\pi \text{ rad}$$

As $$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\therefore \quad \left(\frac{v}{r}\right)^2 = 0 + 2\left(\frac{a}{r}\right)\theta$$

or $$a = \frac{v^2}{2r\theta} = \frac{(80)^2}{2 \times 20/\pi \times 4\pi} = 40 \text{ ms}^{-2}.$$

36. (*a*) Period of revolution,

$$T = \frac{44}{22} = 2 \text{ s}$$

$$a = \omega^2 r = \left(\frac{2\pi}{T}\right)^2 r = \left(\frac{2\pi}{2}\right)^2 \times 1 = \pi^2 \text{ ms}^{-2}$$

The centripetal acceleration acts along the radius towards the centre.

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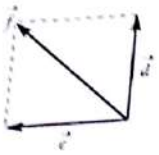
37. (c) Clearly,

$\vec{d} + \vec{c} = \vec{f}$

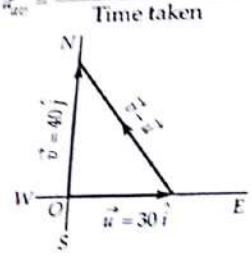


38. (b) $\vec{v} = \vec{u} + \vec{a}t$

$= (3\hat{i} + 4\hat{j}) + (0.4\hat{i} + 0.3\hat{j}) \times 10 = 7\hat{i} + 7\hat{j}$

$\therefore \quad v = \sqrt{v_x^2 + v_y^2} = \sqrt{7^2 + 7^2} = 7\sqrt{2}$ units

39. (d) $a_{av} = \dfrac{\text{Change in velocity}}{\text{Time taken}}$



$= \dfrac{|\vec{v} - \vec{u}|}{t} = \dfrac{|40\hat{j} - 30\hat{i}|}{10}$

$= \dfrac{\sqrt{40^2 + 30^2}}{10} = \dfrac{50}{10} = 5\ \text{m/s}^2$

40. (a) $R_{max} = \dfrac{u^2}{g} = \dfrac{20 \times 20}{10} = 40$ m

41. (d) At $\theta = 45°$,

$y = \dfrac{u^2 \sin^2 45°}{2g} = \dfrac{u^2}{4g}$

$x = \dfrac{1}{2} \cdot \dfrac{u^2 \sin^2 90°}{g} = \dfrac{u^2}{2g}$

$\therefore \quad \tan\beta = \dfrac{y}{x} = \dfrac{1}{2}$

$\Rightarrow \quad \beta = \tan^{-1}\left(\dfrac{1}{2}\right)$

42. (d) Acceleration $= r\omega^2 = r\left(\dfrac{2\pi}{T}\right)^2$

$= 5 \times 10^{-2} \times \left(\dfrac{2\pi}{0.2\pi}\right)^2 = 5 \times 10^{-2} \times 100$

$= 5\ \text{ms}^{-2}$

Delhi PMT and VMMC Entrance Exam

(Similar Questions)

1. If a unit vector is represented by $0.5\hat{i} + 0.8\hat{j} + c\hat{k}$, then the value of c is

(a) 1 (b) $\sqrt{0.8}$

(c) $\sqrt{0.11}$ (d) $\sqrt{0.01}$ [DPMT 94]

2. Two vectors are perpendicular if

(a) $\vec{A} \cdot \vec{B} = 1$ (b) $\vec{A} \times \vec{B} = 0$

(c) $\vec{A} \cdot \vec{B} = 0$ (d) $\vec{A} \cdot \vec{B} = AB$ [DPMT 07]

3. The angle between the two vectors $\vec{A} = 5\hat{i} + 5\hat{j}$ and $\vec{B} = 5\hat{i} - 5\hat{j}$ will be

(a) zero (b) 90°

(c) 45° (d) 0° [DPMT 99]

4. Two equal vectors have a resultant equal to her of the two. The angle between them is

(a) 90° (b) 60°

(c) 120° (d) 0° [VMMC 03]

5. If $|\vec{A} \times \vec{B}| = |\vec{A} \cdot \vec{B}|$, then angle between $\vec{A}$ and $\vec{B}$ will be

(a) 30° (b) 60°

(c) 45° (d) 90° [DPMT 05]

6. If $\vec{P} + \vec{Q} = \vec{R}$ and $|\vec{P}| = |\vec{Q}| = |\vec{R}|$, then angle between $\vec{P}$ and $\vec{Q}$ is

(a) 30° (b) 60°

(c) 90° (d) 120° [DPMT 05]

7. If $|\vec{A} \times \vec{B}| = \sqrt{3}\ \vec{A} \cdot \vec{B}$, then the value of $|\vec{A} + \vec{B}|$ is

(a) $(A^2 + B^2 + AB)^{1/2}$

(b) $\left(A^2 + B^2 + \dfrac{AB}{\sqrt{3}}\right)^{1/2}$

(c) $A + B$

(d) $(A^2 + B^2 + \sqrt{3}AB)^{1/2}$ [VMMC 07]

8. Given the speed of a
the velocity of a river is v,
reach a place downstrea
back to the original place

(a) $\dfrac{2vd}{v^2 - u^2}$

(c) $\dfrac{vd}{v^2 - u^2}$

9. Which of the f
projectile motion ?

(a) horizontal velocit
(b) vertical velocity c
(c) acceleration is no
(d) momentum is co

10. An aeroplane fly
of 360 kmh^{-1} releases a
the ground. If $g = 9.8$ ms

(a) 10 km

(c) 1 km

11. A bomber pla
speed of 500 m/s and
the ground in 10 sec
ground ($g = 10\ \text{m/s}^2$)

(a) $\tan^{-1}\left(\dfrac{1}{5}\right)$

(c) $\tan^{-1}(1)$

12. If range and he
angle of projection w

(a) 60°

(c) 30°

13. The horizonta
its maximum height

The angle of pro

(a) 40°

(c) 30°

14. Two bodi
velocity. If one is
other at 60° to the
heights reached is

(a) 3 : 1

(c) 1 : 3

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8. Given the speed of a boat in still water is u and the velocity of a river is v, the time taken by the boat to reach a place downstream at a distance d and come back to the original place is

(a) $\frac{2vd}{v^2-u^2}$ (b) $\frac{2ud}{u^2-v^2}$

(c) $\frac{vd}{v^2-u^2}$ (d) $\frac{ud}{u^2-v^2}$ [DPMT 06]

9. Which of the following is true regarding projectile motion ?

(a) horizontal velocity of projectile is constant

(b) vertical velocity of projectile is constant

(c) acceleration is not constant

(d) momentum is constant [DPMT 07]

10. An aeroplane flying horizontally with a speed of 360 kmh^{-1} releases a bomb at a height of 490 m from the ground. If $g = 9.8\,ms^{-2}$, it will strike the ground at

(a) 10 km (b) 100 km

(c) 1 km (d) 16 km [VMMC 03]

11. A bomber plane moves horizontally with a speed of 500 m/s and a bomb released from it, strikes the ground in 10 sec. Angle at which it strikes the ground ($g = 10\,m/s^2$) will be

(a) $\tan^{-1}\left(\frac{1}{5}\right)$ (b) $\tan\left(\frac{1}{5}\right)$

(c) $\tan^{-1}(1)$ (d) $\tan^{-1}(5)$

[DPMT 97 ; VMMC 04]

12. If range and height of a projectile are equal, then angle of projection with the horizontal is

(a) 60° (b) $\tan^{-1}(4)$

(c) 30° (d) 45° [VMMC 02]

13. The horizontal range of projectile is $4\sqrt{3}$ times of its maximum height.

The angle of projection will be

(a) 40° (b) 90°

(c) 30° (d) 45° [DPMT 03]

14. Two bodies are projected with the same velocity. If one is projected at an angle of 30° and the other at 60° to the horizontal, then ratio of maximum heights reached is

(a) 3 : 1 (b) 1 : 2

(c) 1 : 3 (d) 2 : 1 [DPMT 96]

15. Two bodies are thrown up at angles of 45° and 60°, respectively, with the horizontal. If both bodies attain same vertical height, then the ratio of velocities with which these are thrown is

(a) $\sqrt{2}/3$ (b) $2/\sqrt{3}$

(c) $\sqrt{3}/2$ (d) $\sqrt{3}/2$ [DPMT 02, 05]

16. A bullet is fired from a gun with a speed of 1000 m/s in order to hit a target 100 m away. At what height above the target should be gun be aimed ? (The resistance of air is negligible and $g = 10\,m/s^2$)

(a) 5 cm (b) 15 cm

(c) 9 cm (d) 23 cm [DPMT 92]

17. A cricketer hits a ball with a velocity 25 m/s at 60° above the horizontal. How far above the ground it passes over a fielder 50 m from the bat (assume the ball is struck very close to the ground) ?

(a) 8.2 m (b) 9.0 m

(c) 11.6 m (d) 12.7 m [VMMC 05]

18. A ball is projected upwards from the top of tower with a velocity 50 ms^{-1} making an angle 30° with the horizontal. The height of tower is 70 m. After how many seconds from the instant of throwing will the ball reach the ground ?

(a) 2 s (b) 5 s

(c) 7 s (d) 9 s [DPMT 04]

19. The angular speed of a fly-wheel making 120 revolutions/minute is

(a) π rad/sec (b) 4π rad/sec

(c) 2π rad/sec (d) $4\pi^2$ rad/sec [DPMT 98]

20. The angular speed of the earth around the sun is

(a) $\frac{2\pi}{365\times24\times60\times60}$ rad s^{-1}

(b) $\frac{365\times24\times60\times60}{2\pi}$ rad s^{-1}

(c) $\frac{2\pi}{24\times60}$ rad/sec (d) $\frac{2\pi}{60}$ rad s^{-1} [DPMT 06]

21. What is the value of linear velocity, if

$\vec{\omega} = 3\hat{i} - 4\hat{j} + \hat{k}$ and $\vec{r} = 5\hat{i} - 6\hat{j} + 6\hat{k}$?

(a) $-6\hat{i} - 2\hat{j} + 3\hat{k}$ (b) $4\hat{i} - 13\hat{j} + 6\hat{k}$

(c) $-18\hat{i} - 13\hat{j} + 2\hat{k}$ (d) $6\hat{i} - 2\hat{j} + 8\hat{k}$ [DPMT 96]

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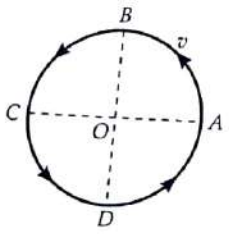
22. Figure along side shows a body of mass m moving with a uniform speed v along a circle of radius r.



The change in velocity in going from A to B is

(a) $v\sqrt{2}$ (b) $v/\sqrt{2}$

(c) v (d) zero [DPMT 04]

23. If a cycle wheel of radius 4 m completes one revolution in two seconds, then acceleration of the cycle is

(a) $\pi \text{m/s}^2$ (b) $2\pi^2 \text{m/s}^2$

(c) $\pi^2 \text{m/s}^2$ (d) $4\pi^2 \text{m/s}^2$

24. A stone tied to the end of string 80 cm long is whirled in horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 sec, what is the magnitude of acceleration of the stone ?

(a) 680 cm/s^2 (b) 860 cm/s^2

(c) 720 cm/s^2 (d) 990 cm/s^2 [DPMT 97]

25. The three initial and final positions of a man on the x-axis are given as

(i) (−8 m, 7 m) (ii) (7 m, −3 m)

(iii) (−7 m, 3 m)

Which pair gives the negative displacement ?

(a) (i) (b) (ii)

(c) (iii) (d) (i) and (iii) [DPMT 09]

26. A bird flies from $(-3m, 4m, -3m)$ to $(7m, -2m, -3m)$ in the xyz coordinates. The bird's displacement in unit vectors is given by

(a) $(4\hat{i} + 2\hat{j} - 6\hat{k})$ (b) $(10\hat{i} - 6\hat{j})$

(c) $(4\hat{i} - 2\hat{j})$ (d) $(10\hat{i} + 6\hat{j} - 6\hat{k})$

[DPMT 09]

27. A boat crosses a river from port A to port B, which are just on the opposite side. The speed of the water is v_W and that of boat is v_B relative to water. Assume $v_B = 2v_W$. What is the time taken by the boat, if it has to cross the river directly on the AB line ?

(a) $\frac{2D}{v_B\sqrt{3}}$ (b) $\frac{\sqrt{3}D}{2v_B}$

(c) $\frac{D}{v_B\sqrt{2}}$ (d) $\frac{D\sqrt{2}}{v_B}$

[DPMT 09]

28. A coastguard ship locates a pirate ship at a distance 560 m. It fires a cannon ball with an initial speed 82 m/s. At what angle from horizontal the ball must be fired so that it hits the pirate ship ?

(a) 54° (b) 125°

(c) 27° (d) 18° [DPMT 09]

29. An object moves at a constant speed along a circular path in a horizontal XY-plane, with the centre at the origin. When the object is at $x = -2$ m, its velocity is $-(4\text{ m/s})\hat{j}$. What is the object's acceleration when it is at $y = 2$ m ?

(a) $-(8\text{ m/s}^2)\hat{j}$ (b) $-(8\text{ m/s}^2)\hat{i}$

(c) $(-4\text{ m/s}^2)\hat{j}$ (d) $(4\text{ m/s}^2)\hat{i}$ [DPMT 09]

30. Which of the following is correct relation between an arbitrary vector $\vec{A}$ and null vector $\vec{O}$?

(a) $\vec{A} + \vec{O} + \vec{A} \times \vec{O} = \vec{A}$

(b) $\vec{A} + \vec{O} + \vec{A} \times \vec{O} \neq \vec{A}$

(c) $\vec{A} + \vec{O} + \vec{A} \times \vec{O} = \vec{O}$

(d) none of these [DPMT 2011]

31. An object is being thrown at a speed of 20 m/s in a direction 45° above the horizontal. The time taken by the object to return to the same level is

(a) $20/g$ (b) $20g$

(c) $20\sqrt{2}/g$ (d) $20\sqrt{2}\,g$ [DPMT 2011]

Answers and Explanations

1. (c) $\sqrt{0.5^2 + 0.8^2 + c^2} = 1$

$c = \sqrt{1 - 0.89} = \sqrt{0.11}$.

2. (c) For two perpendicular vectors,

$\vec{A} \cdot \vec{B} = AB\cos 90° = 0$.

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3. (*b*) $\vec{A}.\vec{B} = (5\hat{i}+5\hat{j}).(5\hat{i}-5\hat{j}) = 0$

$\therefore \quad \theta = 90°.$

4. (*c*) Refer to the solution of Example 5 on Page 4.9.

5. (*c*) Given : $|\vec{A}\times\vec{B}| = |\vec{A}.\vec{B}|$

$\therefore \quad AB\sin\theta = AB\cos\theta$

or $\quad \tan\theta = 1$

$\therefore \quad \theta = 45°$

6. (*d*) As $\vec{R} = \vec{P}+\vec{Q}$

and $\quad |\vec{P}| = |\vec{Q}| = |\vec{R}|,$

$R^2 = P^2+Q^2+2PQ\cos\theta$

or $\quad P^2 = P^2+P^2+2P^2\cos\theta$

or $\quad \cos\theta = -\frac{1}{2}$

$\therefore \quad \theta = 120°.$

7. (*a*) Given : $|\vec{A}\times\vec{B}| = \sqrt{3}\,\vec{A}.\vec{B}$

$\therefore \quad AB\sin\theta = \sqrt{3}\times AB\cos\theta$

or $\quad \tan\theta = \sqrt{3}$

$\therefore \quad \theta = 60°$

Now $\quad |\vec{A}+\vec{B}| = \sqrt{A^2+B^2+2AB\cos 60°}$

$= \sqrt{A^2+B^2+2AB\times\frac{1}{2}}$

$= (A^2+B^2+AB)^{1/2}$

8. (*b*) Velocity downstream $= u+v$

Velocity upstream $= u-v.$

Time taken in one round trip

$$= \frac{d}{u+v}+\frac{d}{u-v} = \frac{2ud}{u^2-v^2}.$$

9. (*a*) Horizontal velocity of a projectile is not affected by gravity.

10. (*c*) Time taken by the bomb to fall through a height of 490 m,

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 490}{9.8}} = 10\text{ s}.$$

Distance at which the bomb strikes the ground

= Horizontal velocity × time

$= 360\text{ kmh}^{-1}\times 10\text{ s}$

$= 360\text{ km h}^{-1}\times\frac{10}{3600}\text{ h} = \mathbf{1\ km}.$

11. (*a*) Height through which the bomb falls,

$$h = \frac{1}{2}gt^2 = \frac{1}{2}\times 10\times(10)^2 = 500\text{ m}$$

Horizontal distance moved by the bomb in 10 s

$= 500\text{ ms}^{-1}\times 10\text{ s} = 5000\text{ m}$

$\therefore \quad \tan\theta = \frac{500}{5000} = \frac{1}{5}$

or $\quad \theta = \tan^{-1}\left(\frac{1}{5}\right).$

12. (*d*) Horizontal range = Max. height

$$\frac{u^2\sin 2\theta}{g} = \frac{u^2\sin^2\theta}{2g}$$

or $\quad 2\sin\theta\cos\theta = \frac{\sin^2\theta}{2}$ or $\tan\theta = 4$

$\therefore \quad \theta = \tan^{-1}(4)$

13. (*c*) Refer to the solution of Problem 15 on page 4.100 .

14. (*c*) $\quad H = \frac{u^2\sin^2\theta}{2g}$

i.e., $\quad H \propto \sin^2\theta$

$\therefore \quad \frac{H_1}{H_2} = \frac{\sin^2 30°}{\sin^2 60°} = \frac{(1/2)^2}{(\sqrt{3}/2)^2} = \frac{1}{3} = \mathbf{1:3}.$

15. (*c*) $\quad H_1 = H_2$

$$\frac{u_1^2\sin^2 45°}{2g} = \frac{u_2^2\sin^2 60°}{2g}$$

or $\quad \frac{u_1}{u_2} = \frac{\sin 60°}{\sin 45°} = \frac{\sqrt{3}/2}{1/\sqrt{2}} = \sqrt{\frac{3}{2}}.$

16. (*a*) Tike taken by the bullet to cover a horizontal distance of 100 m,

$$t = \frac{100\text{ m}}{1000\text{ ms}^{-1}} = \frac{1}{10}\text{ s}$$

Height at which the gun should be aimed,

$$h = \frac{1}{2}gt^2 = \frac{1}{2}\times 10\times\left(\frac{1}{10}\right)^2\text{ m}$$

$= 5\times 10^{-2}\text{ m} = \mathbf{5\ cm}.$

17. (*a*) $\quad u_x = 25\cos 60° = 12.5\text{ ms}^{-1}$

$u_y = 25\sin 60° = 12.5\sqrt{3}\text{ ms}^{-1}$

Time taken by the ball to cover 50 m distance

$$t = \frac{50\text{ m}}{u_x} = \frac{50\text{ m}}{12.5\text{ ms}^{-1}} = 4\text{ s}$$

Vertical distance covered by the ball in 4 s,

$$h = u_y t - \frac{1}{2}gt^2$$

$= 12.5\sqrt{3}\times 4 - \frac{1}{2}\times 9.8\times(4)^2$

$= 50\sqrt{3} - 78.4 = 86.6 - 78.4 = \mathbf{8.2\ m}.$

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18. (*c*) $4y = 50\sin 30° = 25\ \text{ms}^{-1}$

For vertical motion of the ball,

$$h = u_y t - \frac{1}{2}gt^2$$

$$-70 = 25\times t - \frac{1}{2}\times 10\times t^2$$

or $$-14 = 5t - t^2$$

or $$t^2 - 5t - 14 = 0$$

or $$(t+2)(t-7) = 0$$

$\therefore$ $$t = 7\ \text{s}. \qquad (t \neq -2)$$

19. (*b*) $$\omega = \frac{2\pi n}{t} = \frac{2\pi\times 120}{60\,\text{s}}$$

$$= 4\pi\ \text{rad s}^{-1}.$$

20. (*a*) Earth completes one revolution around the sun in one year.

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{365\times 24\times 60\times 60}\ \text{rad s}^{-1}.$$

21. (*c*) $$\vec{v} = \vec{\omega}\times\vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 1 \\ 5 & -6 & 6 \end{vmatrix}$$

$$= -18\hat{i} - 13\hat{j} + 2\hat{k}.$$

22. (*a*) Velocity at A, $\vec{v} = v\hat{j}$

Velocity at B, $\vec{v_2} = -v\hat{i}$

Change in velocity,

$$\Delta\vec{v} = \vec{v_2} - \vec{v_1} = -v\hat{i} - v\hat{j}$$

$$|\Delta\vec{v}| = \sqrt{(-v)^2 + (-v)^2} = v\sqrt{2}.$$

23. (*d*) $$\omega = 2\pi\nu = 2\pi\left(\frac{1}{2}\right) = \pi\ \text{rad s}^{-1}$$

$$a = r\omega^2$$

$$= 4\,\text{m}\times(\pi\,\text{rad s}^{-1})^2 = 4\pi^2\,\text{ms}^{-2}.$$

24. (*d*) $$a = r\omega^2 = r\left(\frac{2\pi n}{t}\right)^2$$

$$= \frac{80\times 4\times(22/7)^2\times(14)^2}{(25)^2}$$

$$= 990\ \text{cm/s}^2.$$

25. (*b*) Displacement is negative only for pair (*ii*) *i.e.*, $(7m, -3m)$.

$$\Delta x = x_f - x_i = -3m - 7m = -10m$$

26. (*b*) Bird's displacement is

$$\Delta\vec{r} = \vec{r_2} - \vec{r_1}$$

$$= (7\hat{i} - 2\hat{j} - 3\hat{k}) - (-3\hat{i} + 4\hat{j} - 3\hat{k})$$

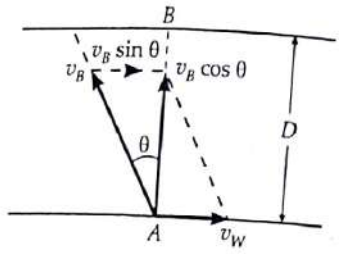
$$= 10\hat{i} - 6\hat{j}.$$

27. (*a*) To move straight along AB,

$$v_B\sin\theta = v_W$$

$$\sin\theta = \frac{v_W}{v_B} = \frac{v_W}{2v_W} = \frac{1}{2}$$

$\therefore$ $$\theta = 30°$$



Time taken to cross the river,

$$t = \frac{D}{v_B\cos\theta} = \frac{D}{v_B\cos 30°} = \frac{2D}{v_B\sqrt{3}}.$$

28. (*c*) Horizontal range,

$$R = \frac{u^2\sin 2\theta}{g}$$

$\therefore$ $$560 = \frac{82\times 82\sin 2\theta}{10}$$

$$\sin 2\theta = \frac{5600}{6724} = 0.82$$

or $$2\theta = 53.8° \Rightarrow \theta = 27°.$$

29. (*a*) Clearly, radius of the circular path is 2 m and velocity has magnitude equal to 4 m/s. Hence

$$a_c = \frac{v^2}{r} = \frac{4\times 4}{2} = 8\ \text{m/s}^2$$

When the object is at $y = 2$ m, the acceleration a_c acts towards the centre *i.e.*, towards the origin along –ve y-axis.

$\therefore$ $$\vec{a_c} = -(8\ \text{m/s}^2)\hat{j}.$$

30. (*a*) $\vec{A} + \vec{O} + \vec{A}\times\vec{O} = \vec{A} + \vec{O} + \vec{O} = \vec{A}$

31. (*c*) $$T = \frac{2u\sin\theta}{g}$$

$$= \frac{2\times 20\times\sin 45°}{g} = \frac{20\sqrt{2}}{g}.$$

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LAWS OF MOTION

5.1 FORCE

1. *Define the term force. State the various effects a force can produce.*

Force. *Force may be defined as an agency (a push or a pull) which changes or tends to change the state of rest or of uniform motion or the direction of motion of a body.*

Effects produced by a force. A force applied on an object can produce *three* types of changes :

1. *Force can change speed of an object*, making it to move slower or faster. For example, a horse by exerting a force on a cart, pulls it from rest and subsequently exerting a larger force, the horse makes the cart move with a larger speed. Similarly, a force exerted by the brakes slows or stops a moving train.

2. *Force can change the direction of motion of an object.* For example, the force exerted on the steering wheel of a car changes the direction of motion.

3. *Force can change the shape of an object.* For example, if we hold a rubber ball between our palms and push the two palms against each other, the ball no longer remains round but gets oblong.

For Your Knowledge

- Force is a polar vector as it has a point of application.
- Forces can be classified as positive and negative. A positive force represents repulsion (*e.g.*, between two like charges) and a negative force represents attraction (*e.g.*, between two unlike charges).

5.2 ARISTOTLE'S FALLACY

2. *What were the views of Greek philosopher Aristotle as regard to maintain the state of uniform motion of objects ?*

Aristotelian law of motion. The Greek philosopher *Aristotle* (384-329 B.C.) asserted that the natural state of a body is the state of rest. Every body in motion tends to slow down and comes to rest. An external force is necessary to maintain its motion. For example, a cart on a road has to be constantly pushed to keep it in motion. A single push will not take it far. From such observations Aristotle concluded the following law :

According to **Aristotelian law of motion**, *an external force is necessary to keep a body moving with uniform velocity.*

However, Aristotle's views were proved wrong by *Galileo Galleio* (1564-1642) about two thousand years later on. It was observed that external forces were necessary to counter the opposing forces of friction to keep bodies in uniform motion. If there were no friction, no external force would be needed to maintain the state of uniform motion of a body.

5.3 THE LAW OF INERTIA

3. *Describe Galileo's experiments concerning motion of objects on inclined planes. Hence state the law of inertia.*

Galileo's experiments on the motion of objects. It was Galileo who first asserted that *objects move with constant speed when no external forces act on them.* He

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arrived at this revolutionary conclusion on the basis of following simple experiments :

(*a*) **Galileo's experiments with single inclined plane.** Galileo first studied the motion of objects on an inclined plane, as shown in Fig. 5.1.

(*i*) Galileo observed that when an object moves down an inclined plane, its speed increases.

(*ii*) When the object is moved up the inclined plane, its speed decreases.

(*iii*) From the above two observations, Galileo argued that when the plane slopes neither upward nor downwards, there should be neither acceleration nor retardation.

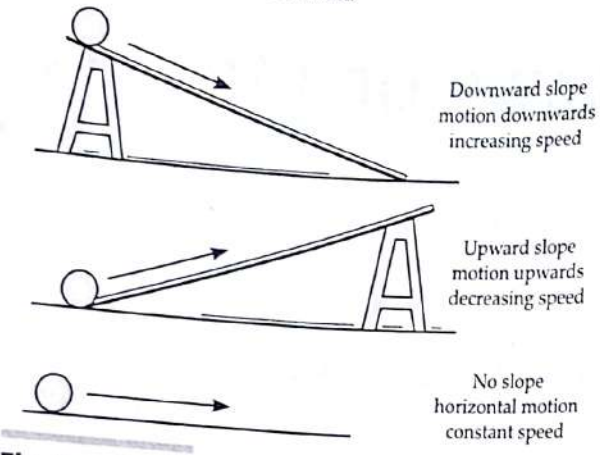

Fig. 5.1 Galileo's observations of motion on a single inclined plane.

Galileo, therefore, concluded that *on a horizontal plane an object should move with a constant velocity in a straight line path.*

(*b*) **Galileo's experiments on two inclined planes combined together.** Galileo noted that in the case of an oscillating pendulum, the bob always reaches the same height on either side of the mean position. From this observation, Galileo thought of an imaginary experiment. In this experiment, two inclined planes are arranged facing each other, as shown in Fig. 5.2(*a*).

(*i*) When an object rolls down one of the inclined planes, it climbs up the other. It almost reaches the same height but not completely because of the presence of friction. If the friction were absent, the object must have reached the same height as the initial height, as shown in Fig. 5.2(*a*).

(*ii*) When the slope of the upward inclined plane is decreased, the object has to travel a longer distance to reach the maximum height, as shown in Fig. 5.2(*b*). The more we decrease the slope of the upward inclined plane, the longer would be the distance that the object is needed to travel to reach the same height.

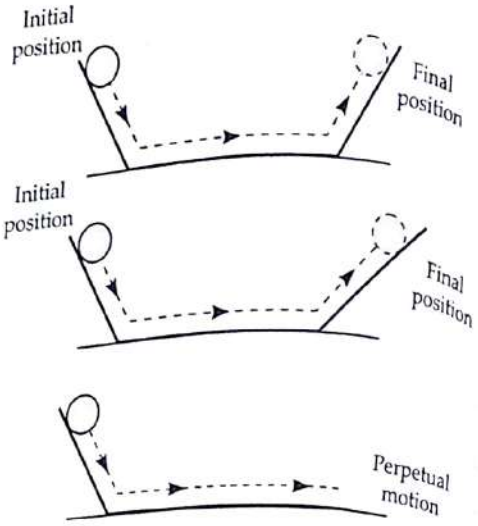

Fig. 5.2 Galileo's observations of motion on a double inclined plane.

(*iii*) From the above two observations, Galileo argued that if the second plane is made horizontal [Fig. 5.2(*c*)], the object will have to travel an infinite distance to reach the same height. This is possible only if the object moves forever with uniform velocity on the horizontal surface.

From the above series of experiments, Galileo formulated the following **law of inertia :**

A body moving with a certain speed along a straight path will continue to move with same speed along the same straight path in the absence of external forces.

5.4 INERTIA AND ITS DIFFERENT TYPES

4. *Define the term inertia. What are different types of inertia ? Give one example of each type.*

Inertia. *The inherent property of a material body by virtue of which it cannot change, by itself, its state of rest or of uniform motion in a straight line is called inertia.*

The term *inertia means resistance to change.* This term was first used by *Galileo.*

Different types of inertia :

(*i*) ***Inertia of rest.*** *The tendency of a body to remain in its position of rest is called inertia of rest.*

Example. *A person standing in a bus falls backward when the bus suddenly starts moving forward.* When the bus moves, the lower part of his body begins to move along with the bus while the upper part of his body continues to remain at rest due to inertia. That is why, a person falls backward when the bus starts.

(*ii*) ***Inertia of motion.*** *The tendency of a body to remain in its state of uniform motion in a straight line is called inertia of motion.*

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Example. *When a moving bus suddenly stops, a person sitting in it falls forward.* As the bus stops, the lower part of his body comes to rest alongwith the bus while the upper part of his body continues to remain in motion due to inertia and falls forward.

(iii) **Inertia of direction.** *The inability of a body to change by itself its direction of motion is called inertia of direction.*

Example. *When a bus takes a sharp turn, a person sitting in the bus experiences a force acting away from the centre of the curved path* due to his tendency to move in the original direction. He has to hold on to a support to prevent himself from swaying away in the turning bus.

5. *How is the inertia of a body measured ?*

Mass as the measure of inertia. Mass of a body is the measure of its inertia. If a body has more mass, it has more inertia *i.e.*, it is more difficult to change its state of rest or of uniform motion. For example, if we kick a football, it flies a long way. If we kick a stone of the same size, it hardly moves. The stone opposes the change in its motion better than the football because of its more mass. Thus stone has more inertia than football.

5.5 LINEAR MOMENTUM

6. *Taking suitable examples, develop the concept of momentum.*

Concept of Momentum. When a small piece of stone is dropped from a small height on a glass pane placed on a table, it does not break the glass pane. But when a heavy stone is dropped from the same height, the glass pane breaks. Here the small and the heavy stones have the same velocity when they fall on the glass pane.

On the other hand, a greater effort is required to stop a bullet fired from the gun than to stop a bullet of the same mass when just thrown by the hand. In the former case, the bullet has large velocity.

The above examples show that the effect of motion of a body depends both on its mass and velocity. The product of mass and velocity of a body is called its momentum.

7. *Define momentum. Is it a scalar or a vector quantity ? Give its units and dimensions.*

Momentum. *Momentum of a body is the quantity of motion possessed by the body. It is equal to the product of mass and velocity of the body.*

$$\text{Momentum} = \text{Mass} \times \text{velocity}$$

$$\vec{p} = m\vec{v}$$

Momentum is a vector quantity because the velocity $\vec{v}$ is a vector and mass m is a scalar. Its direction is same as the direction of the velocity of the body. Its magnitude is given by

$$p = mv$$

SI unit of momentum = kg ms^{-1}

CGS unit of momentum = g cm s^{-1}.

The dimensional formula of momentum is [MLT^{-1}].

8. *Discuss the variation of momentum when two bodies of equal/different masses have different/equal momenta. Show the each situation graphically.*

Case 1. Consider two objects, each of mass m Suppose the two objects are moving with velocities v_1 and v_2 with $v_1 > v_2$. Then

$$p_1 = mv_1 \quad \text{and} \quad p_2 = mv_2$$

$$\therefore \quad \frac{p_1}{p_2} = \frac{mv_1}{mv_2} = \frac{v_1}{v_2}$$

As $v_1 > v_2$, so $p_1 > p_2$

Thus, the linear momenta of bodies having equal masses are proportional to their velocities. This is shown graphically in Fig. 5.3(*a*).

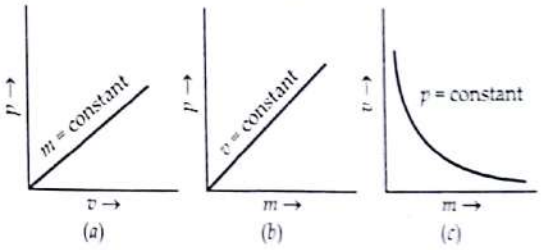

Fig. 5.3

Case 2. Consider a heavier object of mass m_1, and a lighter object of mass m_2. Suppose both the objects are moving with same velocity v. Then

$$p_1 = m_1 v \quad \text{and} \quad p_2 = m_2 v$$

$$\therefore \quad \frac{p_1}{p_2} = \frac{m_1}{m_2}.$$

As $m_1 > m_2$, so $p_1 > p_2$

Thus, *the linear momenta of bodies having equal velocities are proportional to their masses.* This is shown graphically in Fig. 5.3(*b*).

Case 3. Consider two objects having equal linear momenta. Thus

$$p_1 = p_2 \quad \text{or} \quad m_1 v_1 = m_2 v_2$$

or

$$\frac{v_2}{v_1} = \frac{m_1}{m_2}.$$

As $m_1 > m_2$, so $v_1 < v_2$.

Thus, *the velocities of bodies having equal linear momenta are inversely proportional to their masses. i.e.,* when two objects have equal linear momentum, the lighter object will move faster than the heavier one.

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5.6 NEWTON'S LAWS OF MOTION

9. *State the three Newton's laws of motion.*

Newton's laws of motion. Sir *Isaac Newton* (1642 – 1727) made a systematic study of motion and extended the ideas of Galileo. He arrived at three laws of motion which are called Newton's laws of motion. These laws are as follows :

First law. *Every body continues in its state of rest or of uniform motion in a straight line unless it is compelled by some external force to change that state.*

Second law. *The rate of change of linear momentum of a body is directly proportional to the applied force and the change takes place in the direction of the applied force.*

Third law. *To every action, there is always an equal and opposite reaction.*

5.7 DISCUSSION OF THE FIRST LAW OF MOTION

10. *State and explain Newton's first law of motion.*

Newton's first law of motion. *This law states that every body continues in its state of rest or of uniform motion in a straight line unless it is compelled by some external force to change that state.* This law consists of three parts :

(*i*) ***First part*** says that *a body at rest continues in its state of rest.* An external force has to be applied on it to make it move. A chair lying on the floor will continue to remain there unless we displace it.

(*ii*) ***Second part*** says that *a body in motion continues moving in a straight path with a uniform speed.* This part seems to be contrary to our everyday experience. A rolling ball comes to rest on a rough ground. This is because of force of friction. The ball moves through a larger distance on a smooth floor. If the friction were zero, the ball would continue its motion forever. This part also indicates that to *increase or decrease the speed of a body moving in a straight line, a force has to be applied on it in the direction of motion or opposite to the direction of motion.*

(*iii*) ***Third part*** says that *a body moving with a uniform speed in a straight line cannot change itself its direction of motion.* To change its direction of motion, a force has to be applied normal to this direction of motion. Consider the motion of the moon around the earth. The direction of motion of the moon continuously changes. The force needed to change the direction is provided by the gravitational attraction of the earth on the moon.

11. *How does Newton's first law of motion lead to the definition of force ?*

Newton's first law defines force. According to Newton's first law of motion, a body at rest or in uniform motion will remain at rest or in uniform motion unless an external force acts upon it. This shows that force is an agency which changes the state of rest or uniform motion. But sometimes a force applied on a body is not able to change its state of rest or of uniform motion. So it is better to define force as an *external agency which changes or tends to the state of rest or of uniform motion in a straight line.* Hence first law of motion gives a qualitative definition of force.

12. *Why is Newton's first law of motion also called law of inertia ?*

Newton's first law defines inertia. According to Newton's first law of motion, every body continues in its state of rest or uniform motion unless an external force acts upon it. This shows that a body, by itself, cannot change its state of rest or uniform motion. This *inability of a body to change its state of rest or of uniform motion along a straight line is called inertia of the body.* Thus first law defines inertia and hence it is rightly called the law of inertia.

13. *Give some examples from daily life that illustrate the law of inertia.*

Illustrations of Newton's first law of motion :

A. Based on inertia of rest.

(*i*) ***When a horse suddenly starts running, the rider falls backward.*** This is because the lower part of the rider, which is in contact with the horse, comes into motion while his upper part tends to remain at rest due to inertia.

(*ii*) ***Dust is removed from a hanging carpet by beating it with a stick.*** As the carpet is beaten, it suddenly moves forward while the dust particles tend to remain at rest due to inertia of rest and so fall off.

(*iii*) ***When we shake the branch of a tree, its fruits and dry leaves fall down.*** On shaking, the branch comes into motion while the dry fruits and leaves tend to remain at rest (inertia of rest) and so get separated.

(*iv*) ***Coin falls into the tumbler when the card is given a sudden jerk.*** Place a five rupees coin on a playing card covering a glass tumbler. Give a sudden jerk to the card, the card flies off and the coin drops into the tumbler. This is because the coin tends to remain at rest due to inertia.

(*v*) As shown in Fig. 5.4, suspend a body *M* of mass about 2 kg from a rigid support by means of thread *A* and pull it down by another thread *B*. When the force *F* is applied as a

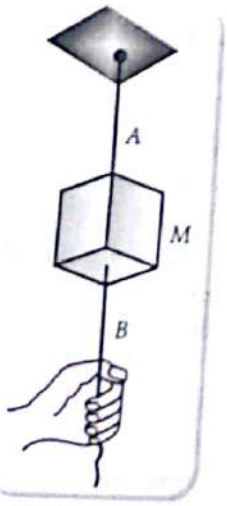

Fig. 5.4

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steady pull, the thread breaks at A but when it is applied as a sudden jerk, the thread breaks at B. In the first case the force gets sufficient time to reach the position A. In the second case, the upper portion of the system is not able to share the force in short time and the block tends to remain at rest.

B. Based on inertia of motion.

(*i*) ***When a horse running fast suddenly stops, the rider is thrown forward*** if he is not firmly seated. This is because the lower part of the rider's body, which is in contact with the horse, comes to rest while the upper part of his body tends to keep moving due to inertia.

(*ii*) ***A person getting out of a moving bus or train falls in the forward direction.*** As the man jumps out of a moving bus, his feet suddenly come to rest on touching the ground while the upper part of his body continues to move forward. That is why he falls with his head forward. In order to save himself from falling down, he should run in the forward direction through some distance.

(*iii*) ***An athlete runs for a certain distance before taking a long jump.*** The inertia of motion gained by him at the time of jumping adds to his muscular effort and helps him in taking a longer jump.

(*iv*) A fireman in a railway engine quickly moves his coal shovel near the furnace and then suddenly stops it. The shovel comes to rest but the coal continues moving due to inertia and falls into the furnace.

(*v*) ***A ball thrown upward in a moving train comes back into the thrower's hands.*** The ball acquires the horizontal velocity of the train and maintains it (inertia of motion) during its upward and downward motion. In this period the ball covers the same horizontal distance as the train, so it comes back into the thrower's hands.

C. Based on inertia of direction.

(*i*) Consider a stone being rotated in a circle at the end of a string. The velocity of the stone at any instant is along the tangent to the circle. When the string is released, the centripetal force whirling the stone vanishes. Due to directional inertia, the stone flies off tangentially.

(*ii*) During the sharpening of a knife, ***the sparks coming from the grind stone fly off tangentially*** to the rim of the rotating stone. This is due to the inertia of direction.

(*iii*) ***When a vehicle moves, the mud sticking to its wheels flies off tangentially.*** This is due to inertia of direction. In order that the flying mud does not spoil the clothes of the passerby, the wheels are provided with mudguards.

(*iv*) ***When a dog chases a hare, the hare runs along a zig-zag path.*** It becomes difficult for the dog to catch the hare. This is because the dog has more mass and hence has more inertia of direction than that of hare.

5.8 DISCUSSION OF NEWTON'S SECOND LAW OF MOTION

14. *Describe the important observation which led Newton to postulate his second law of motion.*

Clue to the second law of motion. Suppose a fixed force is applied on two bodies of different masses for the same duration. The lighter body gains a higher speed than the heavier one. However, the change in momentum in both cases is found to be the same. This shows that the *same force for the same time causes the same change in momentum for bodies of different masses.* This fact was first recognised by Newton who expressed it as his second law of motion.

15. *State and explain Newton's second law of motion.*

Newton's second law of motion. *It states that the rate of change of momentum of a body is directly proportional to the external force applied on the body and the change takes place in the direction of the applied force.* This law can be divided into two parts :

(*i*) ***The rate of change of momentum is directly proportional to the applied force.*** The larger the force acting on a body, greater is the change in its momentum. Since change in momentum is equal to the product of mass and the change in velocity and the mass of the body remains constant, so the rate of change of momentum is directly proportional to the rate of change of velocity *i.e.*, acceleration. Hence force F is proportional to both mass (m) and acceleration (a).

$$F \propto ma$$

(*ii*) ***The change of momentum occurs in the direction of the force.*** If a body is at rest, a force will set it in motion. If a body is moving with a certain velocity, a force will increase or decrease this velocity accordingly as the force acts in its same or opposite direction.

16. *Explain how does Newton's second law of motion give a quantitative definition of force:*

Measurement of force from Newton's second law of motion. If a body of mass m is moving with velocity $\vec{v}$, then its linear momentum is

$$\vec{p} = m\vec{v}$$

Differentiating both sides w.r.t. time t, we get

$$\frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} = m\vec{a}$$

where $\vec{a}$ is the acceleration produced in the body.

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According to Newton's second law,

Applied force $\propto$ Rate of change of momentum

or $$\vec{F} \propto \frac{d\vec{p}}{dt}$$

or $$F \propto m\vec{a}$$

$$\therefore \quad \vec{F} = km\vec{a}$$

The units of m, a and F are so chosen that the proportionality constant $k = 1$. Suppose $m = 1$, $a = 1$ and $F = 1$, then

$$1 = k \times 1 \times 1 \quad \text{or} \quad k = 1$$

$$\therefore \quad \vec{F} = m\vec{a}$$

The above equation can be used to measure force.

In scalar form, Newton's second law can be expressed as

$$F = ma$$

$\therefore$ 1 unit force = 1 unit mass × 1 unit acceleration

Hence a *unit force may be defined as the force which produces unit acceleration in a body of unit mass.*

17. *Is force a scalar or vector quantity ? Write its dimensional formula.*

As force is the product of mass m (scalar) and acceleration $\vec{a}$ (vector), so it is a vector quantity. The direction of force $\vec{F}$ is same as that of acceleration $\vec{a}$.

Dimensional formula of force is $[MLT^{-2}]$.

18. *Name and define the various absolute and gravitational units of force.*

Absolute units of force. *An absolute unit of force is that force which produces a unit acceleration in a body of unit mass.*

(*i*) In SI, the absolute unit of force is **newton (N)**. *One newton is defined as that much force which produces an acceleration of 1 ms^{-2} in a body of mass 1 kg.*

$$1\text{ N} = 1\text{ kg} \times 1\text{ ms}^{-2} = 1\text{ kg ms}^{-2}$$

(*ii*) In CGS system, the absolute unit of force is **dyne (dyn)**. *One dyne is that much force which produces an acceleration of 1 cm s^{-2} in a body of mass 1 gram.*

$$1\text{ dyne} = 1\text{ g} \times 1\text{ cm s}^{-2} = 1\text{ g cm s}^{-2}.$$

Gravitational units of force. *A gravitational unit of force is that force which produces an acceleration equal to 'g' (acceleration due to gravity) in a body of unit mass.* It may also be defined as the weight of a body of unit mass.

(*i*) In SI, the gravitational unit of force is **kilogram weight (kg wt) or kilogram force (kg f)**. *It is defined as that much force which produces an acceleration of 9.80 ms^{-2} in a body of mass 1 kg.*

$$1\text{ kg wt} = 1\text{ kg f} = 9.8\text{ N}$$

(*ii*) In CGS system, the gravitational unit of force is **gram weight (g wt) or gram force (g f)**. *It is defined as that force which produces on acceleration of 980 cm s^{-2} in a body of mass 1 gram,*

$$1\text{ g wt} = 1\text{ g f} = 980\text{ dyne}$$

19. *Find the number of dynes in one newton.*

Relation between newton and dyne.

$$1\text{N} = 1\text{ kg} \times 1\text{ ms}^{-2} = 1000\text{ g} \times 100\text{ cm s}^{-2}$$
$$= 10^5\text{ g cm s}^{-2}$$

or $$1\text{ N} = 10^5\text{ dyne}$$

20. *How do the gravitational units of force differ from the absolute units of force ?*

The absolute units of force remain the same throughout the universe. But the gravitational units of force vary from place to place because they depend on the value of g which takes different values at different places.

For Your Knowledge

- A gravitational unit is 'g' times the corresponding absolute unit.
- A gravitational unit of force is used to express the weight of a body. For example, the weight of a body of mass 10 kg is 10 kg wt or 10 kg f. For this reason, the gravitational units are also called *practical units.*

21. *Express Newton's second law of motion in component form. Give its significance.*

Newton's second law in component form. In terms of their rectangular components, the force, momentum and acceleration vectors can be expressed as

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

$$\vec{p} = p_x\hat{i} + p_y\hat{j} + p_z\hat{k}$$

and $$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

The vector form of Newton's second law is

$$\vec{F} = \frac{d\vec{p}}{dt} = m\vec{a}$$

$$\therefore \quad F_x\hat{i} + F_y\hat{j} + F_z\hat{k} = \frac{d}{dt}(p_x\hat{i} + p_y\hat{j} + p_z\hat{k})$$
$$= m(a_x\hat{i} + a_y\hat{j} + a_z\hat{k})$$

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Equating the components along the three coordinate axes, we get

$$F_x = \frac{dp_x}{dt} = ma_x$$

$$F_y = \frac{dp_y}{dt} = ma_y$$

$$F_z = \frac{dp_z}{dt} = ma_z$$

The above three equations express Newton's second law in component form.

Significance of component form. The component form of Newton's second law indicates that if the applied force makes some angle with the velocity of the body, it changes the component of velocity along the direction of the force. The component of velocity normal to the force remains unchanged. For example, in the motion of a projectile under the vertical gravitational force, the horizontal component of velocity remains unchanged.

22. *Give four important implications of Newton's second law of motion.*

Some important implications of Newton's second law :

(*i*) The second of motion is a vector law expressed as

$$\vec{F} = m\vec{a}$$

(*ii*) In the second law, if $\vec{F} = 0$, then $\vec{a} = 0$. This indicates that a body moves with a uniform velocity in the absence of any external force. Thus second law is consistent with the first law.

(*iii*) The second law is strictly applicable to a point particle, but is also applicable to a body or a system of particles, provided $\vec{F}$ is the total external force on the system and $\vec{a}$ is the acceleration of the system as a whole.

(*iv*) Second law of motion is a local relation. This means $\vec{F}$ at a space point at a certain instant determines $\vec{a}$ at the same point and the same instant.

23. *What does it mean that Newton's second law is a local relation ? Explain it with an example.*

Second law of motion is a local relation. The acceleration $\vec{a}$ at any point (location of the particle) at any instant is determined by the force $\vec{F}$ at the same point at the same instant. That is, acceleration is determined here and now by the force here and now, not by the earlier history of motion of the particle. For example, the moment a stone is released out of an accelerated train, there is no horizontal force or acceleration on the stone, if air resistance is neglected. The stone has only the vertical force of gravity. It carries no memory of its acceleration with the train a moment ago.

5.9 CONSEQUENCES OF NEWTON'S SECOND LAW

24. *How does Newton's second law of motion lead to the concept of inertial mass ?*

Concept of inertial mass. Suppose a force F acting independently on two masses m_1 and m_2 produces accelerations a_1 and a_2 in them. By using Newton's second law, we get

$$F = m_1 a_1 = m_2 a_2 \quad \text{or} \quad \frac{a_1}{a_2} = \frac{m_2}{m_1}$$

If $m_2 < m_1$, then $a_2 > a_1$, *i.e.*, a given force produces a larger acceleration in a lighter mass than that in a heavier mass. Thus *the mass of a body gives a measure of the resistance or the opposition to a given force that tends to change its velocity.* That is why, $m = F / a$ is called the **inertial mass** of the body.

25. *State the three different ways in which accelerated motion of a body can occur under an external force.*

The accelerated motion of a body is always due to an external force. It can occur in the following ways :

(*i*) *Due to the change in the speed.* This happens when the force acts on the body along the direction of motion or opposite to the direction of motion.

(*ii*) *Due to the change in the direction of motion.* This happens when the force acts perpendicular to the direction of motion. Such a force produces circular motion.

(*iii*) *Due to the change in both speed and direction of motion.* This happens when the force is inclined at some angle with the direction of motion. The tangential component changes speed while the transverse component changes direction.

26. *Briefly explain how an unknown force can be measured with the help of a coiled spring.*

Measurement of unknown force. When a spring is stretched by a constant force, the increase in the length of the spring is proportional to the applied force. This fact can be used to measure an unknown force. As shown in Fig. 5.5, consider a spring attached to a block resting on a frictionless surface. When the spring is

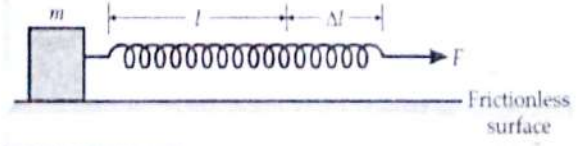

Fig. 5.5 Force due to a spring.

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pulled with a force F, the block acquires acceleration a. If the spring stretches by an amount Δl, then

$$F \propto \Delta l$$

If the mass of the block is 1 kg and the acceleration produced is 1 ms^{-2}, then the applied force would be 1 N. Suppose the extension in the spring is Δl_0 in this case. Then

$$1\text{ N} \propto \Delta l_0 \quad \therefore \quad \frac{F}{1\text{ N}} = \frac{\Delta l}{\Delta l_0} \quad \text{or} \quad F = \frac{\Delta l}{\Delta l_0}\text{N}$$

For Your Knowledge

- Force is not always in the direction of motion. It may be along $\vec{v}$, opposite to $\vec{v}$, normal to $\vec{v}$ or may make some angle with $\vec{v}$.
- In every motion, force $\vec{F}$ is parallel to acceleration $\vec{a}$.
- No force is required to move a body with a uniform velocity. In that case, $a = 0$, so $F = ma = 0$.
- Force can be measured from Newton's second law of motion.

$$F = ma = m\frac{\Delta v}{\Delta t}$$

By knowing mass m and change in velocity Δv in time Δt, the force F can be determined.

- The cause of every accelerated motion is an external force. Internal forces have no role to play.
- If $\vec{v} = 0$ at an instant, *i.e.*, if a body is momentarily at rest, it does not mean that force or acceleration are necessarily zero at that instant. For example, when a ball thrown up reaches its maximum height, $\vec{v} = 0$ but the force continues to be its weight mg and acceleration equal to g.
- The straight line along which a force acts is called the line of action of the force.

Examples based on Linear Momentum and Newton's Second Law of Motion

Formulae Used

1. Linear momentum, $p = mv$
2. According to Newton's second law,

Applied force = Rate of change of linear momentum

or $$F = \frac{dp}{dt} = ma = m\left(\frac{v-u}{t}\right)$$

Units Used

Velocities u and v are in ms^{-1}, time t in second, momentum p in $kgms^{-1}$, acceleration a in ms^{-2} and force F in newton (N).

Conversions Used

$1\text{ N} = 10^5$ dyne, 1 kg wt = 9.8 N, 1 g wt = 980 dyne

Example 1. A car of mass 1000 kg is moving with a velocity of 10 ms^{-1} and is acted upon by a forward force of 1000 N due to engine and retarding force of 500 N due to friction. What will be its velocity after 10 seconds ?

Solution. Here $m = 1000$ kg, $u = 10\text{ ms}^{-1}$, $t = 10$ s, $v = ?$

Net forward force,

F = Forward force − Retarding force

$= 1000 - 500 = 500$ N

Acceleration,

$$a = \frac{F}{m} = \frac{500}{1000} = \frac{1}{2}\text{ ms}^{-2}$$

$$\therefore \quad v = u + at = 10 + \frac{1}{2} \times 10 = \mathbf{15\ ms^{-1}}.$$

Example 2. A bullet of mass 0.04 kg moving with a speed of 90 ms^{-1} enters a heavy wooden block and is stopped after a distance of 60 cm. What is the average resistive force exerted by the block on the bullet ? **[NCERT ; Central Schools 14]**

Solution. Here $m = 0.04$ kg, $u = 90\text{ ms}^{-1}$,

$v = 0$, $s = 60\text{ cm} = 0.60$ m

As $v^2 - u^2 = 2as$

$\therefore \quad 0 - (90)^2 = 2a \times 0.60 \quad \text{or} \quad a = -6750\text{ ms}^{-2}$

i.e. Retardation $= 6750\text{ ms}^{-2}$

$\therefore$ Retarding force = Mass × retardation

$= 0.04 \times 6750 = \mathbf{270\ N}.$

Example 3. A force of 72 dyne is inclined to the horizontal at an angle of 60°. Find the acceleration in a mass of 9 g, which moves in a horizontal direction.

Solution. Here $m = 9$ g, $F = 72$ dyne, $\theta = 60°$

The horizontal component of the force is

$F_x = F\cos\theta = 72 \times \cos 60°$

$= 72 \times 0.5 = 36$ dyne

Acceleration,

$$a = \frac{F_x}{m} = \frac{36}{9} = \mathbf{4\ cm\ s^{-2}}.$$

Example 4. A scooterist moving with a speed of 36 kmh^{-1} sees a child standing in the middle of the road. He applies the brakes and brings the scooter to rest in 5 s just in time to save child. Calculate the average retarding force on the vehicle, if mass of the vehicle and driver is 300 kg.

Solution. Here $u = 36\text{ kmh}^{-1} = 10\text{ ms}^{-1}$, $v = 0$, $t = 5$ s

$$a = \frac{v-u}{t} = \frac{0-10}{5} = -2\text{ ms}^{-2}$$

$\therefore$ Retardation $= 2\text{ ms}^{-2}$

Average retarding force = Mass × retardation

$= 300 \times 2 = \mathbf{600\ N}.$

Example 5. A bus starts from rest acce with 4 ms^{-2}. At $t = 10$ s, a stone is drop of the bus 2 m high. What are the (i) m and (ii) acceleration of the stone $g = 10\text{ ms}^{-2}$.

Solution. (i) Horizontal veloci stone at $t = 10$ s is

$v_x = u + at = 0 + 4 \times$

For vertical motion of the st

$u = 0$, $a = g = 10\text{ ms}^{-2}$,

$\therefore \quad v_y = 0 + 10 \times 0.2 = 2\text{ ms}^{-}$

Magnitude of the resultan

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{40^2 +$

(ii) After the stone is drop horizontal is zero. It has onl 10 ms^{-2}.

Example 6. Forces of $5\sqrt{2}$ body of mass 1000 kg at an the acceleration, distance mass after 10 s.

Solution.

$F = \sqrt{F_1^2 + F_2^2 +$

$= \sqrt{(5\sqrt{2})^2 + ($

$= \sqrt{50 + 72 +$

$a = \frac{F}{m} = \frac{13.49}{1000}$

$v = u + at = 0$

$s = 0 + \frac{1}{2} \times ($

Example 7. A blo frictionless plane. rate of 1 $kg\ s^{-1}$ initial acceleratio

Solution. F block is

$F = \frac{dp}{dt} = \frac{d}{}$

Mass of b

$\therefore$ Accel

Example

that its p

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EXAMPLE 5. A bus starts from rest accelerating uniformly with 4 ms^{-2}. At $t = 10$ s, a stone is dropped out of a window of the bus 2 m high. What are the (i) magnitude of velocity and (ii) acceleration of the stone at 10.2 s ? Take $g = 10\ ms^{-2}$.

Solution. (*i*) Horizontal velocity of the bus or the stone at $t = 10$ s is

$$v_x = u + at = 0 + 4 \times 10 = 40\ ms^{-1}$$

For vertical motion of the stone,

$$u = 0,\ a = g = 10\ ms^{-2},\ t = 10.2 - 10 = 0.2\ s$$

$$\therefore\ v_y = 0 + 10 \times 0.2 = 2\ ms^{-1}$$

Magnitude of the resultant velocity of the stone is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{40^2 + 2^2} = \sqrt{1604} = \mathbf{40.04\ ms^{-1}}.$$

(*ii*) After the stone is dropped, its acceleration along horizontal is zero. It has only a vertical acceleration of 10 ms^{-2}.

EXAMPLE 6. Forces of $5\sqrt{2}$ N and $6\sqrt{2}$ N are acting on a body of mass 1000 kg at an angle to 60° to each other. Find the acceleration, distance covered and the velocity of the mass after 10 s.

Solution.

$$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$= \sqrt{(5\sqrt{2})^2 + (6\sqrt{2})^2 + 2 \times 5\sqrt{2} \times 6\sqrt{2}\cos 60°}$$

$$= \sqrt{50 + 72 + 60} = \sqrt{182} = 13.49\ N$$

$$a = \frac{F}{m} = \frac{13.49}{1000} = \mathbf{0.01349\ ms^{-2}}$$

$$v = u + at = 0 + 0.01349 \times 10 = \mathbf{0.1349\ ms^{-1}}$$

$$s = 0 + \frac{1}{2} \times 0.01349 \times (10)^2 = \mathbf{0.6745\ m}.$$

EXAMPLE 7. A block of metal weighing 2 kg is resting on a frictionless plane. It is struck by a jet releasing water at the rate of 1 $kg\ s^{-1}$ and at a speed of 5 ms^{-1}. Calculate the initial acceleration of the block.

Solution. Force exerted by the jet of water on the block is

$$F = \frac{dp}{dt} = \frac{d}{dt}(mv) = v\frac{dm}{dt} = 5\ ms^{-1} \times 1\ kgs^{-1} = 5\ N$$

Mass of block,

$$m = 2\ kg$$

$\therefore$ Acceleration of the block,

$$a = \frac{F}{m} = \frac{5}{2} = \mathbf{2.5\ ms^{-2}}.$$

EXAMPLE 8. A body of mass m moves along X-axis such that its position co-ordinate at any instant t is

$$x = at^4 - bt^3 + ct,$$

where a, b and c are constants. What is the force acting on the particle at any instant t ?

Solution. Position co-ordinate, $x = at^4 - bt^3 + ct$

$$\text{Velocity} = \frac{dx}{dt} = 4at^3 - 3bt^2 + c$$

$$\text{Acceleration} = \frac{d^2x}{dt^2} = \frac{d}{dt}(4at^3 - 3bt^2 + c)$$

$$= 12\ at^2 - 6\ bt$$

$$\text{Force} = \text{Mass} \times \text{acceleration}$$

$$= m\ (12\ at^2 - 6\ bt).$$

PROBLEMS FOR PRACTICE

1. A force acts for 10 s on a body of mass 10 kg after which the force ceases and the body describes 50 m in the next 5 s. Find the magnitude of the force. **(Ans.** 10 N)

2. A truck starts from rest and rolls down a hill with constant acceleration. It travels a distance of 400 m in 20 s. Calculate the acceleration and the force acting on it if its mass is 7 metric tonnes. **(Ans.** 2 ms^{-2}, 14,000 N)

3. A force of 5 N gives a mass m_1 an acceleration of 8 ms^{-2} and a mass m_2 an acceleration of 24 ms^{-2}. What acceleration would it give if both the masses are tied together ? **(Ans.** 6 ms^{-2})

4. In an X-ray machine, an electron is subjected to a force of 10^{-23} N. In how much time the electron will cover a distance of 0.1 m ? Take mass of the electron = 10^{-30} kg. **(Ans.** 1.4×10^{-4} s)

5. A stone of mass 5 kg falls from top of a cliff 50 m high and buries 1 m in sand. Find the average resistance offered by the sand and the time it takes to penetrate. **(Ans.** 2450 N, 0.064 s)

6. A bullet of mass 100 g moving with 20 m/s strikes a wooden plank and penetrates upto 20 cm. Calculate the resistance offered by the wooden plank. **[Delhi 96] (Ans.** 100 N)

7. A motor car running at the rate of 7 ms^{-1} can be stopped by applying brakes in 10 m. Show that total resistance to the motion, when brakes are on, is one fourth of the weight of the car.

8. A force of 50 N is inclined to the vertical at an angle of 30°. Find the acceleration it produces in a body of mass 2 kg which moves in the horizontal direction. **(Ans.** 12.5 ms^{-2})

9. A ship of mass 3×10^7 kg and initially at rest can be pulled through a distance of 3 m by means of a force of 5×10^4 N. The water resistance is negligible. Find the speed attained by the ship. **[IIT 80]**
(Ans. 0.1 ms^{-1})

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10. A balloon has a mass of 5 g in air. The air escapes from the balloon at a uniform rate with a velocity of 5 cms^{-1}. If the balloon shrinks completely in 2.5 s, find the average force acting on the balloon.

(**Ans.** 10 dyne)

HINTS

1. After the force ceases, the body covers 50 m in 5 s, so final velocity of the body is

$$v = \frac{\text{Distance}}{\text{Time}} = \frac{50\text{ m}}{5\text{ s}} = 10\text{ ms}^{-1}$$

But $v = u + at$

$\therefore$ $10 = 0 + a \times 10$ or $a = 1\text{ ms}^{-2}$

$\therefore$ $F = ma = 10\text{ kg} \times 1\text{ ms}^{-2} = \mathbf{10\text{ N}}$.

2. As $s = ut + \frac{1}{2}at^2$

$\therefore$ $400 = 0 + \frac{1}{2}a(20)^2$

or $a = \mathbf{2\text{ ms}^{-2}}$

and $F = ma = 7000 \times 2 = \mathbf{14{,}000\text{ N}}$.

3. Here $m_1 = \frac{F}{a_1} = \frac{5}{8}\text{ kg}$

and $m_2 = \frac{F}{a_2} = \frac{5}{24}\text{ kg}$

$\therefore m_1 + m_2 = \frac{5}{8} + \frac{5}{24} = \frac{5}{6}\text{ kg}$

Acceleration of the tied masses

$$= \frac{F}{m_1 + m_2} = \frac{5}{5/6} = \mathbf{6\text{ ms}^{-2}}.$$

4. Here $a = \frac{F}{m} = \frac{10^{-23}}{10^{-30}} = 10^7\text{ ms}^{-2}$

As $s = ut + \frac{1}{2}at^2$

$\therefore$ $0.1 = 0 + \frac{1}{2} \times 10^7 \times t^2$

or $t^2 = 2 \times 10^{-8}\text{ s}^2$

or $t = \mathbf{1.4 \times 10^{-4}\text{ s}}$.

5. Velocity attained by the stone as it falls through a height of 50 m is given by

$$v^2 - u^2 = 2as$$

or $v^2 - 0^2 = 2 \times 9.8 \times 50$

or $v = \sqrt{980}\text{ ms}^{-1}$

Now the stone starts burying into sand with a velocity of $\sqrt{980}\text{ ms}^{-1}$ and finally comes to rest after travelling a distance, $s = 1$ m.

$\therefore$ $0^2 - 980 = 2a \times 1$

or $a = -490\text{ ms}^{-2}$

Average resistance offered by sand,

$F = ma = 5 \times 490 = \mathbf{2450\text{ N}}$.

Time taken by stone to penetrate sand,

$$t = \frac{v - u}{a} = \frac{0 - \sqrt{980}}{-490} = \mathbf{0.064\text{ s}}.$$

6. As $v^2 - u^2 = 2as$

$\therefore$ $0 - (20)^2 = 2a \times 0.20$

or $a = -1000\text{ ms}^{-2}$

Retarding force = Mass × retardation

$= 0.100 \times 1000 = \mathbf{100\text{ N}}$.

7. Here $u = 7\text{ ms}^{-1}$, $v = 0$, $s = 10$ m, $a = ?$

As $v^2 - u^2 = 2as$ $\therefore$ $0 - 7^2 = 2a \times 10$

or $a = -2.45\text{ ms}^{-2} = -\frac{9.8\text{ ms}^{-1}}{4} = -\frac{g}{4}$

Total resistance to motion

$$= -ma = \frac{mg}{4} = \frac{1}{4} \times \textbf{Weight of car.}$$

8. Horizontal component of force

$= F\cos(90° - 0) = F\sin 0$

$\therefore$ $a = \frac{F\sin\theta}{m} = \frac{50\sin 30°}{2} = \frac{50 \times 1}{2 \times 2} = \mathbf{12.5\text{ ms}^{-2}}$.

9. Acceleration, $a = \frac{F}{m} = \frac{5 \times 10^4}{3 \times 10^7} = \frac{5}{3} \times 10^{-3}\text{ ms}^{-2}$

Also $u = 0$, $s = 3$ m

As $v^2 - u^2 = 2as$

$\therefore$ $v^2 - 0 = 2 \times \frac{5}{3} \times 10^{-3} \times 3$

or $v = 10^{-1}\text{ ms}^{-1} = \mathbf{0.1\text{ ms}^{-1}}$.

10. Force, $F = \frac{dp}{dt} = \frac{d}{dt}(mv) = v\frac{dm}{dt}$ [$\because$ v is constant]

But $\frac{dm}{dt} = \frac{5\text{ g}}{2.5\text{ s}} = 2\text{ gs}^{-1}$ and $v = 5\text{ cms}^{-1}$

$\therefore$ $F = 5 \times 2 = \mathbf{10\text{ dyne}}$.

5.10 IMPULSE

27. *What are impulsive forces ? Give examples.*

Impulsive force. *A large force acting for a short time to produce a finite change in momentum is called an impulsive force.*

Examples :

(*i*) Force exerted by a bat while hitting a ball.

(*ii*) Blow of a hammer on a nail.

(*iii*) Force experienced by a person when he falls from a certain height on a marble floor.

Impulsive forces usually vary in an abrupt and complicated manner. It is difficult to measure force and time duration separately in such situations. But the product of force and time, which is equal to the change in momentum of the body, is a measurable quantity. This product is given the name *impulse*.

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28. *What do you mean by impulse of a force ? Show that impulse is equal to the product of average force and the time interval for which the force acts. Give the units and dimensions of impulse.*

Impulse of a force. *Impulse is the total effect of a large force which acts for a short time to produce a finite change in momentum. Impulse is defined as the product of the force and the time for which it acts and is equal to the total change in momentum.*

Impulse = Force × time duration
= Total change in momentum

Impulse is a vector quantity denoted by $\vec{J}$. Its direction is same as that of force or the change in momentum. The impulse of a force is positive, negative or zero depending on the momentum of the body increases, decreases or remains unchanged.

Impulse as the product of force and time. Suppose a force $\vec{F}$ acts for a small time dt. The impulse of the force is given by

$$d\vec{J} = \vec{F}\, dt$$

If we consider a finite interval of time from t_1 to t_2, then the impulse will be

$$\vec{J} = \int d\vec{J} = \int_{t_1}^{t_2} \vec{F}\, dt$$

If $\vec{F}_{av}$ is the average force, then

$$\vec{J} = \int_{t_1}^{t_2} \vec{F}_{av}\, dt = \vec{F}_{av} \int_{t_1}^{t_2} dt$$

$$= \vec{F}_{av}\,[t]_{t_1}^{t_2} = \vec{F}_{av}\,(t_2 - t_1)$$

or $$\vec{J} = \vec{F}_{av} \times \Delta t, \quad \text{where} \quad \Delta t = t_2 - t_1$$

Thus, *the impulse of a force is equal to the product of the average force and the time interval for which it acts.*

Dimensions of impulse = $[MLT^{-1}]$
SI unit of impulse = $kg\ ms^{-1}$
CGS unit of impulse = $g\ cm\ s^{-1}$.

29. *Show that the impulse of a force is equal to the change in momentum produced by the force.*

Impulse-momentum theorem. According to Newton's second law of motion,

Applied force = Rate of change of momentum

$$\vec{F} = \frac{d\vec{p}}{dt} \quad \text{or} \quad \vec{F}\, dt = d\vec{p}$$

If in time 0 to t, the momentum of the body changes from $\vec{p}_1$ to $\vec{p}_2$, then integrating the above equation within these limits, we get

$$\int_0^t \vec{F}\, dt = \int_{\vec{p}_1}^{\vec{p}_2} d\vec{p} = [\vec{p}]_{\vec{p}_1}^{\vec{p}_2} = \vec{p}_2 - \vec{p}_1$$

But $\int_0^t \vec{F}\, dt$ = Impulse, $\vec{J}$ $\quad \therefore \quad \vec{J} = \vec{p}_2 - \vec{p}_1$

Thus, *the impulse of a force is equal to the total change in momentum produced by the force.* This relationship between impulse and momentum is known as **impulse-momentum theorem**.

30. *Briefly explain how the impulse of a force can be measured graphically.*

Measurement of impulse by graphical method.

A. ***When a constant force acts on a body.*** Suppose a constant force F acts on a body from time t_1 to t_2. The force-time graph is a straight line AB parallel to the time-axis, as shown in Fig. 5.6.

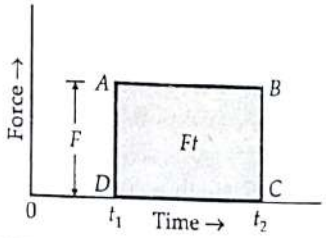

Fig. 5.6 Impulse of a constant force.

Area of rectangle $ABCD$
$= AD \times AB = F(t_2 - t_1) = F \times t$
= Magnitude of impulse of force F in time interval t

B. ***When a variable force acts on the body.*** Suppose a force varying in magnitude acts on a body for time $t_2 - t_1 = t$. The force-time graph is a curve ABC as shown in Fig. 5.7.

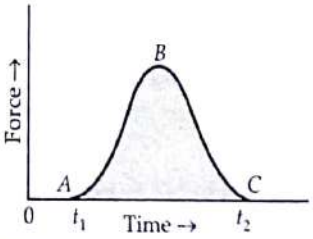

Fig. 5.7 Impulse of a variable force.

Impulse of force F in time interval t,

$J = \int_0^t F dt$ = Area under the force-time curve ABC.

Thus, *the area under the force-time graph gives the magnitude of the impulse of the given force in the given time interval.*

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5.11 APPLICATIONS OF THE CONCEPT OF IMPULSE

31. *Give some practical examples from daily life which make use of the concept of impulse.*

Practical applications of impulse. We know that

Impulse of a force = Force × time
= Change in momentum

If two forces $\vec{F}_1$ and $\vec{F}_2$ act on a body to produce the same impulse (or change in momentum), then their time durations t_1 and t_2 should be such that

$$\vec{F}_1 t_1 = \vec{F}_2 t_2$$

Clearly, if the time duration of an impulse is large, the force exerted will be small. The following examples will make clear this concept.

*(i) **A cricket player lowers his hands while catching a ball.*** When the ball is caught, the impulse received by the hands is equal to the product of the force exerted by the ball and the time taken to complete the catch. By moving the hands backwards, the cricketer increases the time of catch. The force exerted on his hands becomes much smaller and it does not hurt him.

*(ii) **A person falling from a certain height receives more injuries when he falls on a cemented floor than when he falls on a heep of sand.*** In both cases, the impulse or the total change in momentum is same. On the cemented floor, the person is stopped abruptly. So the cemented floor exerts a large force of reaction causing him severe injuries. When the person falls on a heap of sand, the sand yields (gets depressed) under his weight. The person takes longer time to stop. This decreases the force exerted by the floor on the person.

*(iii) **Automobiles (cars, buses, etc) are provided with shockers.*** When a vehicle moves on an uneven road, it receives a jerk. The shocker increases the time of jerk and hence reduces its force. This makes journey comfortable and saves the automobile from damage due to bumps.

*(iv) **Buffers are provided between the boggies of a train.*** Buffers increase the time of jerk during shunting. This decreases the force of impact between the bogies. The bogies are thus prevented from receiving severe jerks.

*(v) **Chinawares are packed in straw paper before packing.*** The straw paper between the chinawares increases the time of experiencing the jerk during transportation. Hence, they strike against each other with a lesser force and are less likely to be damaged.

Examples based on Impulse of a Force

FORMULAE USED

1. Impulse = Force × time = Change in momentum
or $J = F \times t = m(v - u)$
2. $\vec{J} = \int_{t_1}^{t_2} \vec{F}\,.\,dt$ = Area under force-time (F-t) graph

UNITS USED

Velocities u and v are in ms^{-1}, mass m in kg, time t in second, force F in newton and impulse J in Ns or kg ms^{-1}.

EXAMPLE 9. A batsman hits back a ball straight in the direction of the bowler without changing its initial speed of 12 ms^{-1}. If the mass of the ball is 0.15 kg, determine the impulse imparted to the ball. (Assume linear motion of the ball.) [NCERT]

Solution. Here

$m = 0.15$ kg, $u = 12$ ms^{-1}, $v = -12$ ms^{-1}

Impulse $= m(v - u) = 0.15(-12 - 12) = -3.6$ Ns

The negative sign indicates that the direction of the impulse is from the batsman to the bowler.

EXAMPLE 10. A cricket ball of mass 150 g moving with a velocity of 15 ms^{-1} is brought to rest by a player in 0.05 s. Calculate the impulse and the average force exerted by the player.

Solution. Here $m = 150$ g $= 0.15$ kg, $u = 15$ ms^{-1}, $v = 0$, $t = 0.05$ s

Impulse $= m(v - u) = 0.15(0 - 15) = -2.25$ Ns

$$\text{Average force} = \frac{\text{Impulse}}{\text{Time}} = \frac{-2.25}{0.05} = -45 \text{ N}$$

The negative sign indicates the retarding nature of the force.

EXAMPLE 11. A rubber ball of mass 50 g falls from a height of 1 m and rebounds to a height of 0.5 m. Find the impulse and the average force between the ball and the ground if the time for which they are in contact was 0.1 s.

Solution. Refer to Fig. 5.8. Initial velocity of ball at $A = 0$.

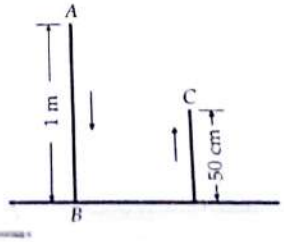

Fig. 5.8

Final velocity of ball at $B = v$

$s = 1$ m

As $v^2 - u^2 = 2as$

$v^2 - 0^2 = 2 \times 9.8 \times 1$ or $v = \sqrt{19.6}$ ms^{-1}

Let u' be the velocity of rebound of the ball

$s' = 50$ cm $= 0.5$ m, $g = -9.8$ ms^{-2}

Final velocity at $C = v' = 0$

As $v'^2 - u'^2 = 2as$

$\therefore\ 0^2 - (u')^2 = 2 \times (-9.8) \times 0.5$

$u' = \sqrt{9.8}$ ms^{-1}

or

Now, Impulse = Change in momentum

$= mv - (-mu') = mv + mu' = m(v + u')$

$= \frac{50}{1000}(\sqrt{19.6} + \sqrt{9.8})$

$= \frac{1}{20}(4.427 + 3.130) = 0.378$ Ns

$$\text{Average force} = \frac{\text{Impulse}}{\text{Time}} = \frac{0.378}{0.1} = 3.78 \text{ N}$$

EXAMPLE 12. While launching a rocket of mass 2 a force of 5×10^5 N is applied for 20 s. Calculate attained by the rocket at the end of 20 s.

Solution. Here $m = 2 \times 10^4$ kg, $F = 5 \times$
$t = 20$ s, $u = 0$, $v = ?$

Impulse of force = $F \times t = m(v - u)$

$\therefore\quad 5 \times 10^5 \times 20 = 2 \times 10^4 (v - 0)$

or $$v = \frac{5 \times 10^5 \times 20}{2 \times 10^4}$$

EXAMPLE 13. A machine gun fires a bull a speed of 1200 ms^{-1}. The person holdin maximum force of 144 N on it. What is that can be fired from the gun per seco

Solution. Let maximum numb be fired per second = n

$\therefore$ Change in momentum of

$= nm(v - u)$

$= n \times \frac{40}{1000}(120$

As impulse = Change in

$\therefore\quad Ft = 48\,n$

or $$n = \frac{Ft}{48} = \frac{144}{4}$$

EXAMPLE 14. A ball movin strikes against a wall at an same angle. Calculate the

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Final velocity of ball at $B = v$

$s = 1$ m

As $v^2 - u^2 = 2as$

$v^2 - 0^2 = 2 \times 9.8 \times 1$ or $v = \sqrt{19.6}$ ms^{-1}

Let u' be the velocity of rebound of the ball.

$s' = 50$ cm $= 0.5$ m, $g = -9.8$ ms^{-2}

Final velocity at $C = v' = 0$

As $v'^2 - u'^2 = 2as$

$\therefore\ 0^2 - (u')^2 = 2 \times (-9.8) \times 0.5$

$u' = \sqrt{9.8}$ ms^{-1}

or

Now, Impulse = Change in momentum

$= mv - (-mu') = mv + mu' = m(v + u')$

$= \frac{50}{1000}(\sqrt{19.6} + \sqrt{9.8})$

$= \frac{1}{20}(4.427 + 3.130) = \mathbf{0.378\ Ns}$

Average force $= \frac{\text{Impulse}}{\text{Time}} = \frac{0.378}{0.1} = \mathbf{3.78\ N.}$

EXAMPLE 12. While launching a rocket of mass 2×10^4 kg, a force of 5×10^5 N is applied for 20 s. Calculate the velocity attained by the rocket at the end of 20 s.

Solution. Here $m = 2 \times 10^4$ kg, $F = 5 \times 10^5$ N,

$t = 20$ s, $u = 0$, $v = ?$

Impulse of force $= F \times t = m(v - u)$

$\therefore\quad 5 \times 10^5 \times 20 = 2 \times 10^4 (v - 0)$

or $\quad v = \frac{5 \times 10^5 \times 20}{2 \times 10^4} = \mathbf{500\ ms^{-1}}.$

EXAMPLE 13. A machine gun fires a bullet of mass 40 g with a speed of 1200 ms^{-1}. The person holding the gun can exert a maximum force of 144 N on it. What is the number of bullets that can be fired from the gun per second ? **[AIEEE 04]**

Solution. Let maximum number of bullets that can be fired per second $= n$

$\therefore$ Change in momentum of n bullets

$= nm(v - u)$

$= n \times \frac{40}{1000}(1200 - 0) = 48\,n$ kg ms^{-1}

As impulse = Change in momentum

$\therefore\quad Ft = 48\,n$

or $\quad n = \frac{Ft}{48} = \frac{144 \times 1}{48} = \mathbf{3\ bullets/s.}$

EXAMPLE 14. A ball moving with a momentum of 5 kg ms^{-1} strikes against a wall at an angle of 45° and is reflected at the same angle. Calculate the change in momentum. **[Chandigarh 09]**

Solution. Initial momentum is along AO. It has two rectangular components :

$p \cos 45°$ along CO and $p \sin 45°$ along DO

Final momentum p is along OB. It has two rectangular components :

$p \cos 45°$ along OC and $p \sin 45°$ along OE

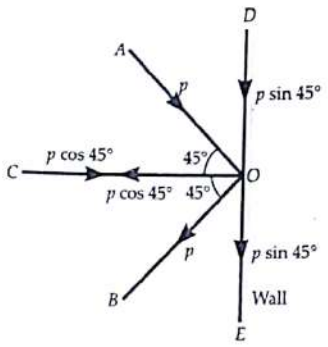

Fig. 5.9

Change in momentum along vertical direction

= Final momentum – Initial momentum

$= p \sin 45° - p \sin 45° = 0$

Change in momentum along horizontal direction

$= -p \cos 45° - p \cos 45° = -2p \cos 45°$

$= -2 \times 5 \times \frac{1}{\sqrt{2}} = -2 \times 5 \times 0.707$

$= \mathbf{-7.07\ kg\ ms^{-1}}$

Negative sign indicates that the direction of change in momentum is away from the wall.

EXAMPLE 15. The initial speed of a body of mass 2.0 kg is 5.0 ms^{-1}. A force acts for 4 s in the direction of motion of the body. The force-time graph is shown in Fig. 5.10. Calculate the impulse of the force and the final speed of the body.

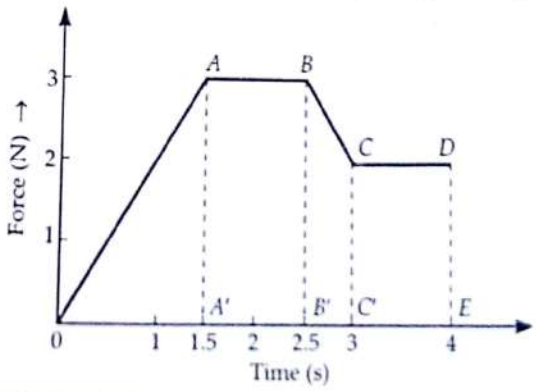

Fig. 5.10

Solution. Impulse of a force

= Area between the force-time graph and the time-axis

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= Area of triangle $OA'A$
+ Area of rectangle $AA'B'B$
+ Area of trapezium $BB'C'C$
+ Area of rectangle $CC'ED$

$= \frac{1}{2} \times 1.5 \times 3 + 1 \times 3 + \frac{1}{2}(3+2)(3-2.5) + 2 \times 1$

$= 2.25 + 3 + 1.25 + 2 = \mathbf{8.50\ Ns.}$

As impulse = Change in momentum $= m\Delta v$

∴ Change in velocity,

$$\Delta v = \frac{\text{Impulse}}{m} = \frac{8.50}{2} = 4.25\ \text{ms}^{-1}$$

Final speed of the body

= Initial speed + $\Delta v = 5.0 + 4.25 = \mathbf{9.25\ ms^{-1}}$.

✕ PROBLEMS FOR PRACTICE

1. A cricket ball of mass 150 g is moving with a velocity of 12 ms^{-1}, and is hit by a bat, so that the ball is turned back with a velocity of 20 ms^{-1}. The force of the blow acts for 0.01 second on the ball. Find the average force exerted by the bat on the ball. [Central Schools 04, 05, 07] (Ans. 480° N)

2. A hammer weighing 1 kg moving with the speed of 10 ms^{-1} strikes the head of a nail driving it 10 cm into a wall. Neglecting the mass of the nail, calculate (*i*) the acceleration during impact (*ii*) the time interval of the impact and (*iii*) the impulse.

[Ans. (*i*) − 500 ms^{-2} (*ii*) 0.02 s (*iii*) − 10 Ns]

3. Two billiard balls of mass 50 g moving in opposite directions with speed of 16 ms^{-1} collide and rebound with the same speed. What is the impulse imparted by each ball to the other ?

(Ans. − 1.6 kg ms^{-1}, + 1.6 kg ms^{-1})

4. A machine gun has a mass of 20 kg. It fires 30 g bullets at the rate of 400 bullets per second with a speed of 400 ms^{-1}. What force must be applied to the gun to keep it in position ? (Ans. 4800 N)

5. A ball of mass 20 g hits a wall at an angle of 45° with a velocity of 15 ms^{-1}. If the ball rebounds at 90° to the direction of incidence, calculate the impulse received by the ball. (Ans. 0.42 kg ms^{-1})

6. A glass ball whose mass is 10 g falls from a height of 40 m and rebounds to a height of 10 m. Find the impulse and the average force between the glass ball and the floor if the time during which they are in contact is 0.1 s. (Ans. 0.42 Ns, 4.2 N)

7. A force acting on a body of mass 2 kg varies with time as shown in Fig. 5.11. Find (*i*) impulse of the force and (*ii*) the final velocity of the body.

[Ans. (*i*) 12 kg ms^{-1} (*ii*) 6 ms^{-1}]

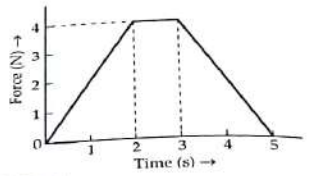

Fig. 5.11

8. Fig. 5.12 shows an estimated force-time graph for a base ball struck by a bat.

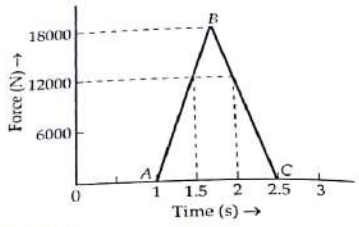

Fig. 5.12

From this curve, determine (*i*) impulse delivered to the ball (*ii*) force exerted on the ball (*iii*) the maximum force on the ball.

[Ans. (*i*) 1.35×10^4 kg ms^{-1} (*ii*) 9000 N (*iii*) 18000 N]

9. A ball moving with a momentum of 15 kg ms^{-1} strikes against the wall at an angle of 30° and is reflected with the same momentum at the same angle. Calculate impulse. [Chandigarh 08]

(Ans. $-15\sqrt{3}$ kg ms^{-1})

10. Calculate the impulse necessary to stop a 1500 kg car travelling at 90 km/h. [Delhi 97]

(Ans. − 37500 Ns)

11. A body of mass 0.25 kg moving with velocity 12 m/s is stopped by applying a force of 0.6 N. Calculate the time taken to stop the body. Also calculate the impulse of this force. [Delhi 99]

(Ans. 5 s, 3 Ns)

✕ HINTS

1. Here $m = 150\ \text{g} = 0.15\ \text{kg}$, $u = 12\ \text{ms}^{-1}$,

$v = -20\ \text{ms}^{-1}$, $t = 0.01\ \text{s}$

Impulse $= m(v-u) = 0.15(-20-12)$

$= -4.8\ \text{kg ms}^{-1}$

$$\text{Average force} = \frac{\text{Impulse}}{\text{Time}} = \frac{4.8}{0.01} = \mathbf{480\ N.}$$

2. Here $u = 10\ ms^{-1}$, $v = 0$, $s = 10$ cm =

(i) As $v^2 - u^2 = 2as$

∴ $0^2 - 10^2 = 2a \times 0.1$ or $a = \frac{-100}{2 \times 0}$

(ii) As $v = u + at$

∴ $t = \frac{v-u}{a} = \frac{0-10}{-500} = \frac{10}{500}$

(iii) Impulse $= F \times t = ma \times t$

$= 1 \times (-500) \times 0.02$

6. For downward motion of the ball

$u = 0$, $h = 40$ m, $g = +$

As $v^2 - u^2 = 2gh$

∴ $v^2 - 0 = 2 \times 9.8 \times 40$

or $v = \sqrt{784} = 28\ ms^{-}$

For upward motion of the b

$u = ?$, $h = 10$ m, $v = 0$

$g = -9.8\ ms^{-2}$

As $v^2 - u^2 = 2gh$

∴ $0 - u^2 = -2 \times 9.8$

or $u = \sqrt{196} = 1$

∴ Change in velocity

$= 28 - (-$

Impulse = Change

= Mass

$= \frac{10}{1000}$

Average force =

8. (i) Impulse = A

$= \frac{1}{}$

$=$

(ii) Force $=$

(iii) Maxi

10. Here m

Impul

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2. Here $u = 10\ \text{ms}^{-1}$, $v = 0$, $s = 10\ \text{cm} = 0.1\ \text{m}$

(*i*) As $v^2 - u^2 = 2as$

$\therefore\ 0^2 - 10^2 = 2a \times 0.1$ or $a = \dfrac{-100}{2 \times 0.1} = -500\ \text{ms}^{-2}$.

(*ii*) As $v = u + at$

$\therefore\ t = \dfrac{v-u}{a} = \dfrac{0-10}{-500} = \dfrac{10}{500} = 0.02\ \text{s}$

(*iii*) Impulse $= F \times t = ma \times t$

$= 1 \times (-500) \times 0.02 = -10\ \text{N}$.

6. *For downward motion of the ball :*

$u = 0,\ h = 40\ \text{m},\ g = +9.8\ \text{ms}^{-1}$

As $v^2 - u^2 = 2gh$

$\therefore\ v^2 - 0 = 2 \times 9.8 \times 40$

or $v = \sqrt{784} = 28\ \text{ms}^{-1}$

For upward motion of the ball :

$u = ?,\ h = 10\ \text{m},\ v = 0$ (at the highest point),

$g = -9.8\ \text{ms}^{-2}$

As $v^2 - u^2 = 2gh$

$\therefore\ 0 - u^2 = -2 \times 9.8 \times 10$

or $u = \sqrt{196} = 14\ \text{ms}^{-1}$

$\therefore$ Change in velocity of the ball

$= 28 - (-14) = 42\ \text{ms}^{-1}$

Impulse = Change in momentum

= Mass × Change in velocity

$= \dfrac{10}{1000}\ \text{kg} \times 42\ \text{ms}^{-1} = 0.42\ \text{Ns}$

Average force $= \dfrac{\text{Impulse}}{\text{Time}} = \dfrac{0.42}{0.1} = 4.2\ \text{N}$.

8. (*i*) Impulse = Area *ABC*

$= \frac{1}{2} \times 18000 \times (2.5 - 1)$

$= 1.35 \times 10^4\ \text{kg ms}^{-1}$.

(*ii*) Force $= \dfrac{\text{Impulse}}{\text{Time}}$

$= \dfrac{1.35 \times 10^4}{(2.5-1)} = 9000\ \text{N}$.

(*iii*) Maximum force = 18000 N.

10. Here $m = 1500\ \text{kg},\ u = 90\ \text{kmh}^{-1} = 25\ \text{ms}^{-1},\ v = 0$

Impulse $= m(v-u) = 1500(0-25)$

$= -37500\ \text{Ns}$

11. Here $m = 0.25\ \text{kg},\ u = 12\ \text{ms}^{-1},\ v = 0,\ F = -0.6\ \text{N}$

$a = \dfrac{F}{m} = -\dfrac{0.6}{0.25} = -24\ \text{ms}^{-2}$

$t = \dfrac{v-u}{a} = \dfrac{0-12}{-24} = 5\ \text{s}$

Impulse $= Ft = -0.6 \times 5 = 3\ \text{Ns}$

5.12 DISCUSSION OF NEWTON'S THIRD LAW OF MOTION

32. *State and explain Newton's third law of motion.*

Newton's third law of motion. It states that

To every action, there is always an equal and opposite reaction.

In simple terms, third law can be stated as follows :

Forces in nature always occur between pairs of bodies. Force on body A by body B is equal and opposite to the force on the body B by A.

As shown in Fig. 5.13, if $\vec{F}_{BA}$ is the force exerted by body *A* on *B* and $\vec{F}_{AB}$ is the force exerted by *B* on *A*, then according to Newton's third law,

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

Force on *A* by *B* = – Force on *B* by *A*

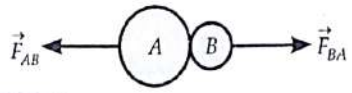

Fig. 5.13 Newton's third law.

For example, while swimming a man pushes water backward and in turn ; he is pushed forward, due to the reaction of water.

The above discussion shows that a *single force can never exist.*

The forces always exist in pairs. The two forces act simultaneously. Any one of them may be called the action and the other reaction. No cause-effect relationship exists between action and reaction.

33. *Give some important implications of the third law of motion.*

Some important points about the third law of motion :

1. Newton's third law of motion is *applicable irrespective of the nature of the forces.* The forces of action and reactions may be mechanical, gravitational, electric or of any other nature.

2. *Action and reaction always act on different bodies.* If they acted on the same body, the resultant force would be zero and there could never be accelerated motion.

3. *The forces of action and reaction cannot cancel each other.* This is because action and reaction, though equal and opposite, always act on different bodies and so cannot balance each other.

4. *No action can occur in the absence of a reaction.* For example, in a tug-of-war, one team can pull the rope only if the other team is pulling the other end of the rope. No force can be exerted if the other end is free. One team exerts the force of action and the other team provides the force of reaction.

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5.13 ILLUSTRATIONS OF NEWTON'S THIRD LAW OF MOTION

34. *Give some examples from daily life which illustrate the use of Newton's law of motion.*

Examples of Newton's third law of motion :

1. ***Book kept on a table.*** Consider a book of weight W resting on a table top. The book exerts a downward force (action) on the table equal to its own weight W. According to Newton's third law, the table also exerts an equal and upward force R (reaction) on the book such that

$$\vec{R} = -\vec{W}$$

As the book is under the action of two equal and opposite forces, it remains in equilibrium.

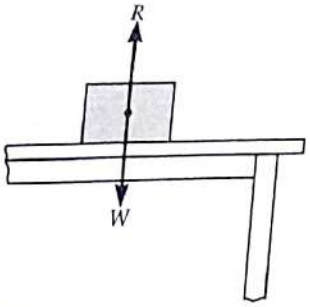

Fig. 5.14 Forces of action and reaction.

2. ***While walking, we press the ground (action) with our feet slightly slanted in the backward direction.*** The ground exerts an equal and opposite force on us. The vertical component of the force of reaction balances our weight and the horizontal component enables us to move forward, as shown in Fig. 5.15.

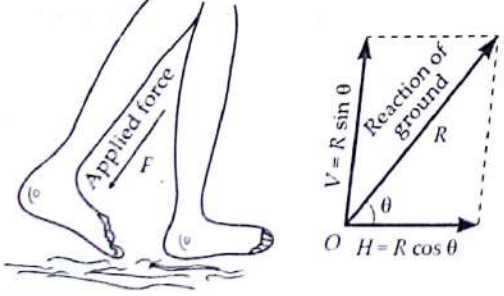

Fig. 5.15

3. ***It is difficult to walk on a slippery ground or*** because we are unable to push such a ground fficiently hard. As a result, the force of reaction is not fficient to help us move forward.

4. ***It is difficult to drive a nail into a wooden block hout supporting it.*** When we hit the nail with a mer, the nail and unsupported block together move forward as a single system. There is no reaction. When the block is rested against a support, the reaction of the support holds the block in position and the nail is driven into the wooden block.

5. ***While swimming, a person pushes water with his hands in the backward direction*** (action) and water, in turn, pushes him forward due to the force of reaction.

6. ***Rotatory lawn sprinkler.*** The action of a rotatory lawn sprinkler is based on third law of motion. As water issues out of the nozzle, it exerts an equal and opposite force in the backward direction, causing the sprinkler to rotate in the opposite direction. Thus water is scattered in all directions.

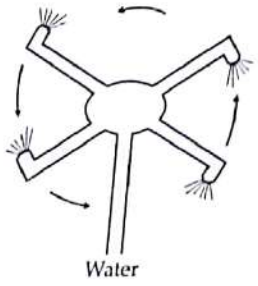

Fig. 5.16 Lawn sprinkler.

5.14 HORSE AND CART PROBLEM

35. *Briefly explain how is a horse able to pull a cart.*

Horse and cart problem. As shown in Fig. 5.17, consider a cart connected to a horse by a string. The horse while pulling the cart produces a tension T in the string in the forward direction (action). The cart, in turn, pulls the horse by an equal force T in the opposite direction.

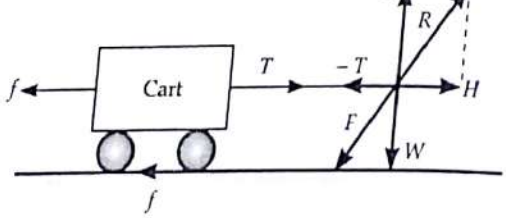

Fig. 5.17

Initially, the horse presses the ground with a force F in an inclined direction. The reaction R of the ground acts on the horse in the opposite direction. The reaction R has two rectangular components :

(*i*) The vertical component V which balances the weight W of the horse.

(*ii*) The horizontal component H which helps the horse to move forward.

Let f be the force of friction.

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The horse moves forward if $H > T$. In that case,

net force acting on the horse $= H - T$

If the acceleration of the horse is a and m is its mass, then

$$H - T = ma \quad ...(1)$$

The cart moves forward if $T > f$. In that case,

net force acting on the cart $= T - f$

The weight of the cart is balanced by the reaction of the ground acting on it.

Obviously, the acceleration acting on the cart is also a. If M is the mass of the cart, then

$$T - f = Ma \quad ...(2)$$

Adding (1) and (2), we get

$$H - f = (M + m)\, a$$

or

$$a = \frac{H - f}{M + m}$$

Obviously, a is positive if $H - f$ is positive, or if $H > f$

Thus the system moves if $H > f$.

5.15 SECOND LAW IS THE REAL LAW OF MOTION

36. *Show that Newton's second law of motion is the real law of motion.*

Or

Show that Newton's first law and third law of motion are contained in the second law.

A. *First law is contained in the second law.* According to Newton's second law of motion, the force acting on a body is given by

$$F = ma$$

In the absence of any external force,

$$F = 0 \quad \text{or} \quad ma = 0$$

As $m \neq 0$, therefore, $a = 0$.

Thus there is no acceleration when no force is applied. That is in the absence of any external force a body at rest will remain at rest and a body in uniform motion will continue to move uniformly along the same straight path. This is nothing but first law of motion. Hence first law of motion is contained in the second law.

B. *Third law is contained in the second law.* Consider an isolated system of two bodies A and B. Suppose the two bodies interact mutually with one another. Let $\vec{F}_{BA}$ be the force (action) exerted by A on B. Let $\vec{F}_{AB}$ be the force (reaction) exerted by B on A. Let $\frac{d\vec{p}_A}{dt}$ be the resulting change of momentum of A. According to Newton's second law,

Let $\frac{d\vec{p}_B}{dt}$ be the resulting change of momentum of B. Let

$$\vec{F}_{BA} = \frac{d\vec{p}_B}{dt} \quad \text{and} \quad \vec{F}_{AB} = \frac{d\vec{p}_A}{dt}$$

$$\therefore \vec{F}_{BA} + \vec{F}_{AB} = \frac{d\vec{p}_B}{dt} + \frac{d\vec{p}_A}{dt} = \frac{d}{dt}(\vec{p}_B + \vec{p}_A)$$

In the absence of external force, the rate of change of momentum must be zero i.e., $\frac{d}{dt}(\vec{p}_B + \vec{p}_A)$ must be zero.

$$\therefore \vec{F}_{BA} + \vec{F}_{AB} = 0$$

or

$$\vec{F}_{BA} = -\vec{F}_{AB} \quad \text{or} \quad \text{Action} = -\text{Reaction}$$

This is nothing but Newton's third law of motion. Hence third law of motion is contained in the second.

As both first and third laws of motion are contained in the second law, we can say that *Newton's second law is the real law of motion.*

5.16 APPARENT WEIGHT OF A MAN IN AN ELEVATOR/LIFT

37. *A man of mass m is standing on the floor of a lift. Find his apparent weight when the lift is (i) moving upwards with uniform acceleration a (ii) moving downwards with uniform acceleration a (iii) at rest or moving with uniform velocity v (iv) falling freely. Take acceleration due to gravity equal to g.*

Apparent weight of a man in a lift. Consider a man of mass m standing on a weighing machine placed in a lift. The actual weight of the man is mg. It acts vertically downwards though the centre of gravity G of the man, it acts on the weighing machine which offers resistance R. The weighing machine reads the reaction R and which is the force experienced by the man. So R is the apparent weight of the man.

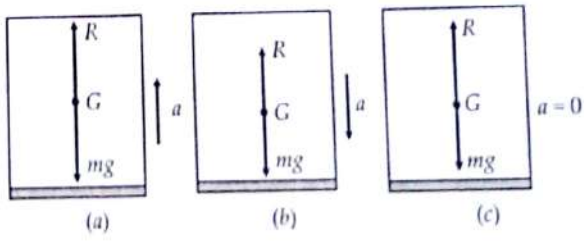

Fig. 5.18 Apparent weight of a man in a lift.

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(i) When the lift moves upwards with acceleration a. As shown in Fig. 5.18(*a*), the net upward force on the man is

$$R - mg = ma$$

$$\therefore \quad \text{Apparent weight,} \qquad R = m(g + a)$$

So when a lift accelerates upwards, the apparent weight of the man inside it increases.

(ii) When the lift moves downwards with acceleration a. As shown in Fig. 5.18(*b*), the net downward force on the man is

$$mg - R = ma$$

$$\therefore \quad \text{Apparent weight,} \qquad R = m(g - a)$$

So when a lift accelerates downwards, the apparent weight of a man inside it decreases.

(iii) When the lift is at rest or moving with uniform velocity v downward/upward. As shown in Fig. 5.18(*c*), the acceleration $a = 0$. Net force on the man is

$$R - mg = m \times 0 = 0$$

$$R = mg$$

or Apparent weight = Actual weight.

(iv) When the lift falls freely. If the supporting cable of the lift breaks, the lift falls freely under gravity. Then $a = g$. The net downward force on the man is

$$R = m(g - g) = 0.$$

Thus the apparent weight of the man becomes zero. This is because both the man and the lift are moving downwards with the same acceleration 'g' and so there are no forces of action and reaction between the man and the lift. Hence a person develops a feeling of **weightlessness** when he falls freely under gravity.

Examples based on Newton's Third Law and Motion in a Lift

Formulae Used

1. Reaction = − Action
2. The apparent weight of a man in a lift :
   (*i*) When the lift moves upwards with acceleration a, $R = m(g + a)$
   (*ii*) When the lift moves downwards with acceleration a, $R = m(g - a)$
   (*iii*) When the lift falls freely, $a = g$, so $R = m(g - a) = m(g - g) = 0$
   (*iv*) When the lift is at rest or moves with uniform velocity, $a = 0$, so $R = m(g - 0) = mg$

Units Used

The absolute SI unit of force is newton and CGS unit is dyne. The gravitational SI unit of force is kg f or kg wt and CGS unit is gf or gwt.

EXAMPLE 16. Two identical billiard balls strike a rigid wall with the same speed but at different angles, and get reflected without any loss of speed, as shown in Fig. 5.19. What is (i) the direction of the force of the wall due to each ball ? and (ii) the ratio of the magnitudes of the impulses imparted on the two balls by the wall ? [NCERT]

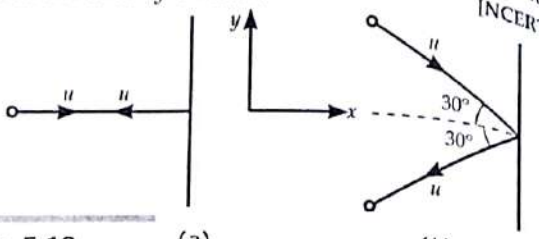

Fig. 5.19 (*a*) (*b*)

Solution. (*i*) Let u be the speed of each ball before and after collision with the wall, and m be the mass of each ball. Choose x-and y-axes as shown.

In Fig. 5.19(*a*),

$$p_x^{\text{initial}} = mu, \qquad p_y^{\text{initial}} = 0$$

$$p_x^{\text{final}} = -mu, \qquad p_y^{\text{final}} = 0$$

As impulse = change in momentum

$\therefore$ x-component of impulse

$$= -mu - mu = -2mu.$$

y-component of impulse $= 0 - 0 = 0$.

Clearly, the direction of the impulse is along the negative x-direction of motion.

As the direction of the force is same as that of impulse, so the force on the ball due to the wall is normal to the wall, along the negative x-direction of motion.

By Newton's third law of motion, the force on the wall due to the ball is normal to the wall along the positive x-direction.

In Fig. 5.19(*b*),

$$p_x^{\text{initial}} = mu\cos 30°, \quad p_y^{\text{initial}} = -mu\sin 30°,$$

$$p_x^{\text{final}} = -mu\cos 30° \quad p_y^{\text{final}} = -mu\sin 30°,$$

$\therefore$ x-component of impulse

$$= -mu\cos 30° - mu\cos 30°$$

$$= -2\,mu\cos 30°$$

y-component of impulse

$$= -mu\sin 30° + mu\sin 30° = 0$$

Again, the direction of the impulse is normal to the wall along the negative x-direction. By Newton's third law, the force on wall due to the ball is normal to the wall along the positive x-direction.

EXAMPLE 17. An elevator weighs 4000 kg. When the upward tension in the supporting cable is 48000 N, what is the upward acceleration ? Starting from rest, how far does it rise in 3 s ?

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Solution. Mass of elevator, $M = 4000$ kg

Weight of elevator

$= Mg = 4000$ kg wt $= 4000 \times 9.8 = 39200$ N

Upward tension, $T = 48000$ N

Net upward force on the elevator,

$F' = T - Mg = 48000 - 39200 = 8800$ N

Upward acceleration, $a = \dfrac{F'}{m} = \dfrac{8800}{4000} = 2.2 \text{ ms}^{-2}$

For upward motion : $u = 0$, $a = 2.2 \text{ ms}^{-2}$, $t = 3$ s

$\therefore \quad s = ut + \frac{1}{2}at^2 = 0 \times 3 + \frac{1}{2} \times 2.2 \times 3^2 = 9.9$ m.

EXAMPLE 18. A lift of mass 2000 kg is supported by thick steel ropes. If maximum upward acceleration of the lift be 1.2 ms^{-2}, and the breaking stress for the ropes be 2.8×10^8 Nm^{-2}, what should be the minimum diameter of rope ?

Solution. Here $m = 2000$ kg, $a = 1.2 \text{ ms}^{-2}$,

breaking stress $= 2.8 \times 10^8 \text{ Nm}^{-2}$.

As the lift moves upwards, so the tension in the rope is

$T = m(g + a) = 2000\,(9.8 + 1.2) = 22{,}000$ N.

Now, breaking stress $= \dfrac{\text{Force}}{\text{Area}} = \dfrac{T}{\pi D^2/4} = \dfrac{4T}{\pi D^2}$

or $\quad 2.8 \times 10^4 = \dfrac{4 \times 22{,}000 \times 7}{22 \times D^2}$

or $\quad D^2 = \dfrac{4 \times 22{,}000 \times 7}{22 \times 2.8 \times 10^8} = 10^{-4}$

or $\quad D = 10^{-2}$ m $= 1$ cm.

✕ PROBLEMS FOR PRACTICE

1. An elevator weighing 5000 kg is moving upward and tension in the supporting cable is 50,000 N. Find the upward acceleration. How far does it rise in a time of 10 seconds starting from rest ?
(**Ans.** 0.2 ms^{-2}, 10 m)

2. A woman weighing 50 kgf stands on a weighing machine placed in a lift. What will be the reading of the machine, when the lift is (*i*) moving upwards with a uniform velocity of 5 ms^{-1} and (*ii*) moving downwards with a uniform acceleration of 1 ms^{-2} ? Take $g = 10 \text{ ms}^{-2}$. **[Punjab 91]**
[**Ans.** (*i*) 500 N (*ii*) 450 N]

3. A 75 kg man stands in a lift. What force does the floor exert on him when the elevator starts moving upwards with an acceleration of 2.0 ms^{-2} ? Take $g = 10 \text{ ms}^{-2}$. (**Ans.** 90 kg f)

4. Find the apparent weight of a man weighing 49 kg on earth when he is standing in a lift which is (*i*) rising with an acceleration of 1.2 ms^{-2} (*ii*) going down with the same acceleration (*iii*) falling freely under the action of gravity and (*iv*) going up or down with uniform velocity. Given $g = 9.8 \text{ ms}^{-2}$.
[**Ans.** (*i*) 55 kg f (*ii*) 43 kg f (*iii*) 0 (*iv*) 49 kg f]

5. A body of mass 15 kg is hung by a spring balance in a lift. What would be the reading of the balance when (*i*) the lift is ascending with an acceleration of 2 ms^{-2} (*ii*) descending with the same acceleration (*iii*) descending with a constant velocity of 2 ms^{-1} ? Take $g = 10 \text{ ms}^{-2}$.
[**Ans.** (*i*) 18 kg f (*ii*) 12 kg f (*iii*) 15 kg f]

6. A mass of 10 kg is suspended from a string, the other end of which is held in hand. Find the tension in the string when the hand is moved up with a uniform acceleration of 2 ms^{-2}. Given $g = 10 \text{ ms}^{-2}$.
(**Ans.** 120 N)

7. The strings of a parachute can bear a maximum tension of 72 kg wt. By what minimum acceleration can a person of 96 kg descend by means of this parachute ? (**Ans.** 2.45 ms^{-2})

8. A 70 kg man in sea is being lifted by a helicopter with the help of a rope which can bear a maximum tension of 100 kg wt. With what maximum acceleration the helicopter should rise so that the rope may not break ? Given $g = 9.8 \text{ ms}^{-2}$. (**Ans.** 4.2 ms^{-2})

9. An elevator and its load weigh a total of 800 kg. Find the tension T in the supporting cable when the elevator, originally moving downwards at 20 ms^{-1} is brought to rest with constant retardation in a distance of 50 m. (**Ans.** 1.014×10^4 N)

✕ HINTS

1. Use $T - mg = ma$

3. $R = m(g + a) = 75\,(10 + 2)$ N $= 900$ N $=$ **90 kg f.**

4. (*i*) $R = m(g + a) = 49\,(1.2 + 9.8) = 49 \times 11$ N

$= \dfrac{49 \times 11}{9.8} =$ **55 kg f.**

(*ii*) $R = m(g - a) = 49\,(9.8 - 1.2)$ N $=$ **43 kg f.**

(*iii*) $R = m(g - g) = 0$.

(*iv*) $R = m(g - 0) = mg =$ **49 kg f.**

5. (*i*) $R = m(g + a) = 15\,(10 + 2) = 180$ N $=$ **18 kg f.**

(*ii*) $R = m(g - a) = 15\,(10 - 2) = 120$ N $=$ **12 kg f.**

(*iii*) Here $a = 0 \quad \therefore \quad R = mg =$ **15 kg f.**

7. For the person to descend, $T = m(g - a)$

$\therefore \quad 72 \times 9.8 = 96\,(9.8 - a)$ or $a =$ **2.45 ms^{-2}.**

8. For the rising helicopter, $T = m(g + a)$

$\therefore \quad 100 \times 9.8 = 70\,(9.8 + a)$ or $a =$ **4.2 ms^{-2}.**

9. Here $m = 800$ kg, $u = 20 \text{ ms}^{-1}$, $v = 0$, $s = 50$ m, $T = ?$

As $v^2 - u^2 = 2as \quad \therefore \quad 0^2 - (20)^2 = 2a \times 50$

or $\quad a = -4 \text{ ms}^{-2}$

For the elevator moving downwards,

$T = m(g - a) = 800\,(9.8 + 4) =$ **1.104×10^4 N.**

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5.17 CONSERVATION OF LINEAR MOMENTUM

38. *State the law of conservation of linear momentum and derive it from Newton's second law of motion.*

Law of conservation of linear momentum. The second and third laws of motion lead to one of the most important and fundamental principles of physics, called the **law of conservation of linear momentum**. It can be stated as follows :

When no external force acts on a system of several interacting particles, the total linear momentum of the system is conserved. The total linear momentum is the vector sum of the linear momenta of all the particles of the system.

Derivation of the law of conservation of linear momentum from Newton's second law of motion. Consider an isolated system (the system on which no external force acts) of n particles. Suppose the n particles have masses m_1 , m_2 , m_3 , ..., m_n and are moving with velocities $\vec{v_1}$, $\vec{v_2}$, $\vec{v_3}$, ... $\vec{v_n}$, respectively. Then total linear momentum of the system is

$$\vec{p} = m_1\vec{v_1} + m_2\vec{v_2} + m_3\vec{v_3} + ... + m_n\vec{v_n}$$

$$= \vec{p_1} + \vec{p_2} + \vec{p_3} + ... + \vec{p_n}$$

If $\vec{F}$ is the external force acting on the system, then according to Newton's second law,

$$\vec{F} = \frac{d\vec{p}}{dt}$$

For an isolated system,

$$\vec{F} = 0 \quad \text{or} \quad \frac{d\vec{p}}{dt} = 0$$

As the derivative of a constant is zero, so

$$\vec{p} = \text{constant}$$

or $\vec{p_1} + \vec{p_2} + \vec{p_3} + ... + \vec{p_n} = \text{constant}$

Thus *in the absence of any external force, the total linear momentum of the system is constant.* This is the law of conservation of linear momentum.

39. *Derive the law of conservation of linear momentum from Newton's third law of motion.*

Derivation of the law of conservation of linear momentum from Newton's third law. As shown in Fig. 5.20, consider two bodies A and B of masses m_1 and m_2, moving in the same direction along a straight line with velocities u_1 and u_2 respectively ($u_1 > u_2$). They collide for time Δt. After collision, let their velocities be v_1 and v_2.

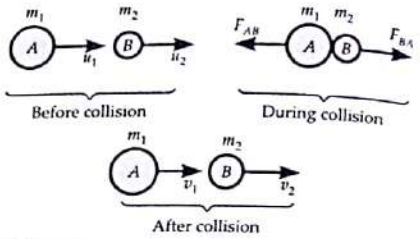

Fig. 5.20

During collision, the body A exerts a force $\vec{F}_{BA}$ on body B. From Newton's third law, the body B also exerts a force $\vec{F}_{AB}$ on body A such that

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

Impulse of $\vec{F}_{AB} = \vec{F}_{AB}\,\Delta t$ = change in momentum of A

$$= m_1\vec{v_1} - m_1\vec{u_1}$$

Impulse of $\vec{F}_{BA} = \vec{F}_{BA}\,.\,\Delta t$ = change in momentum of B

$$= m_2\vec{v_2} - m_2\vec{u_2}$$

But $\vec{F}_{AB}\,.\,\Delta t = -\vec{F}_{BA}\,\Delta t$

$\therefore$ $m_1\vec{v_1} - m_1\vec{u_1} = -(m_2\vec{v_2} - m_2\vec{u_2})$

or $m_1\vec{v_1} + m_2\vec{v_2} = m_1\vec{u_1} + m_2\vec{u_2}$

i.e., Total linear momentum = Total linear momentum
after collision before collision

This proves the law of conservation of linear momentum.

40. *Derive Newton's third law of motion from the law of conservation of momentum.*

Derivation of Newton's third law of motion from the law of conservation of momentum. Consider two bodies of masses m_1 and m_2 moving along a straight line and colliding against each other. The velocities and hence momenta of the two bodies undergo a change. Let Δp_1 and Δp_2 be the changes in the momenta produced in time Δt.

According to the law of conservation of momentum, the net change in the linear momentum in the absence of external force is zero.

$\therefore$ $\Delta p_1 + \Delta p_2 = 0$

or $\Delta p_2 = -\Delta p_1$ or $\frac{\Delta p_2}{\Delta t} = -\frac{\Delta p_1}{\Delta t}$

or Rate of change of momentum of m_2
= – Rate of change of momentum of m_1

or Force on m_2 = – Force on m_1

or Action = – Reaction

This proves the Newton's third law of motion.

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5.18 PRACTICAL APPLICATIONS OF THE LAW OF CONSERVATION OF MOMENTUM

41. *Give some examples from daily life which illustrate the law of conservation of linear momentum.*

Practical applications based on the law of conservation of linear momentum :

(i) Recoil of a gun. Let 'M' be the mass of the gun and 'm' be the mass of the bullet. Before firing, both the gun and the bullet are at rest. After firing, the bullet moves with velocity v and the gun moves with velocity $\vec{V}$. As no external force acts on the system, so according to the principle of conservation of momentum,

Total momentum before firing

= Total momentum after firing

or $0 = m\vec{v} + M\vec{V}$

or $M\vec{V} = -m\vec{v}$

or $\vec{V} = -\frac{m}{M}\vec{v}$

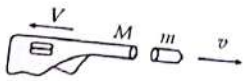

Fig. 5.21

The negative sign shows that $\vec{V}$ and $\vec{v}$ are in opposite directions *i.e.*, the gun gives a kick in the backward direction or the gun recoils with velocity $\vec{V}$. Further, as $M >> m$, so $V << v$ *i.e.*, the recoil velocity of the gun is much smaller than the forward velocity of the bullet.

(ii) ***While firing a bullet, the gun should be held tight to the shoulder.*** The recoiling gun can hurt the shoulder. If the gun is held tightly against the shoulder, then the body and the gun together will constitute one system. Total mass becomes large and the recoil velocity becomes small.

(iii) ***When a man jumps out of a boat to the shore, the boat slightly moves away from the shore.*** Initially, the total momentum of the boat and the man is zero. As the man jumps from the boat to the shore, he gains a momentum in the forward direction. To conserve momentum, the boat also gains an equal momentum in the opposite direction. So the boat slightly moves backwards.

(iv) ***An astronaut in open space, who wants to return to the spaceship, throws some object in a direction opposite to the direction of motion of the spaceship.*** By doing so, he gains a momentum equal and opposite to that of the thrown object and so he moves towards the spaceship.

(v) ***Rocket and jetplanes work on the principle of conservation of momentum.*** Initially, both the rocket and its fuel are at rest. Their total momentum is zero. For rocket propulsion, the fuel is exploded. The burnt gases are allowed to escape through a nozzle with a very high downward velocity. The gases carry a large momentum in the downward direction. To conserve momentum, the rocket also acquires an equal momentum in the upward direction and hence starts moving upwards.

(vi) Explosion of a bomb. Before explosion, suppose the bomb is at rest. Its total momentum is zero. As it explodes, it breaks up into many parts of masses m_1, m_2, m_3, etc., which fly off in different directions with velocities $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$, etc.

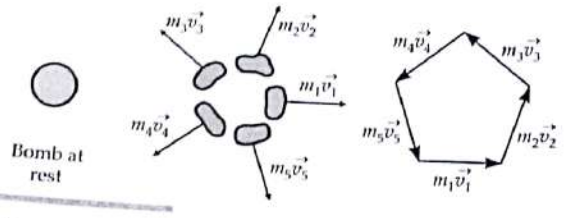

Fig. 5.22 An exploding bomb, $\Sigma m\vec{v} = 0$

The different parts have definite momenta $m_1\vec{v}_1$, $m_2\vec{v}_2$, $m_3\vec{v}_3$, etc.

The momenta of the various parts can be represented by the sides of a closed polygon taken in the same order. This indicates that the total momentum after explosion is zero *i.e.*, momentum is conserved. If bomb explodes into two parts, they will fly off in opposite directions with equal momenta.

Examples based on Conservation of Linear Momentum

FORMULAE USED

1. In the absence of any external force, vector sum of the linear momenta of a system of particles remains constant.

$$m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \ldots\ldots + m_n\vec{v}_n = \text{constant}$$

2. For a two body system, $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

3. Recoil velocity of a gun, $V = -\frac{mv}{M}$

where M is the mass of the gun, m the mass of bullet and v is the velocity of the bullet.

UNITS USED

All masses are in kg, velocities in ms^{-1} and linear momenta in kg ms^{-1}.

EXAMPLE 19. A 30 kg shell is flying at 48 ms^{-1}. When it explodes, its one part of 18 kg stops, while the remaining part flies on. Find the velocity of the later.

Solution. Mass of shell, $M = 30$ kg

Velocity of shell, $u = 48\ ms^{-1}$

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After explosion, mass of first part,

$$m_1 = 18 \text{ kg}$$

Velocity of first part,

$$v_1 = 0$$

Mass of second part,

$$m_2 = 30 - 18 = 12 \text{ kg}$$

If v_2 is the velocity of second part, then from the law of conservation of momentum,

$$Mu = m_1 v_1 + m_2 v_2$$

or $$30 \times 48 = 18 \times 0 + 12 \times v_2$$

or $$v_2 = \frac{30 \times 48}{12} = \mathbf{120 \text{ ms}^{-1}}.$$

EXAMPLE 20. A neutron having a mass 1.67×10^{-27} kg and moving at 10^8 ms^{-1} collides with a deutron at rest and sticks to it. If the mass of duetron is 3.34×10^{-27} kg, find the speed of the combination.

Solution. *For neutron :*

$$m_1 = 1.67 \times 10^{-27} \text{ kg}, \; u_1 = 10^8 \text{ ms}^{-1}$$

For deutron :

$$m_2 = 3.34 \times 10^{-27} \text{ kg}, \; u_2 = 0$$

Let v be the speed of the combination. Then by conservation of momentum,

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2) v$$

$$1.67 \times 10^{-27} \times 10^8 + 3.34 \times 10^{-27} \times 0 = (1.67 + 3.34) \times 10^{-27} \times v$$

or $$v = \frac{1.67 \times 10^{-27} \times 10^8}{5.01 \times 10^{-27}} = \mathbf{0.333 \times 10^8 \text{ ms}^{-1}}.$$

EXAMPLE 21. A car of mass 1000 kg travelling at 32 ms^{-1} dashes into the rear of a truck of mass 8000 kg moving in the same direction with a velocity of 4 ms^{-1}. After the collision, the car bounces with a velocity of 8 ms^{-1}. What is the velocity of truck after the impact ?

Solution. *For the car :*

$$m_1 = 1000 \text{ kg}, \quad u_1 = 32 \text{ ms}^{-1}, \; v_1 = -8 \text{ ms}^{-1}$$

For the truck : $m_2 = 8000$ kg, $u_2 = 4$ ms^{-1}, $v_2 = ?$

By conservation of linear momentum,

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$1000 \times 32 + 8000 \times 4 = 1000 \times (-8) + 8000 \, v_2$$

or $$64000 + 8000 = 8000 \, v_2$$

or $$v_2 = \frac{72000}{8000} = \mathbf{9 \text{ ms}^{-1}}.$$

(in the same direction)

EXAMPLE 22. A hunter has a machine gun that can fire 50 g bullets with a velocity of 150 ms^{-1}. A 60 kg tiger springs at him with a velocity of 10 ms^{-1}. How many bullets must the hunter fire into the tiger in order to stop him in track ?

Solution. Mass of bullet, $m = 50 \text{ g} = 0.05$ kg

Velocity of bullet, $v = 150 \text{ ms}^{-1}$

Mass of tiger, $M = 60$ kg

Velocity of tiger, $V = 10 \text{ ms}^{-1}$

Let n be the number of bullets required to be pumped into the tiger to stop him in his track.

According to the law of conservation of momentum,

Magnitude of the momentum of n bullets

= Magnitude of the momentum of tiger

or $n \times mv = MV$ or $n = \frac{MV}{mv} = \frac{60 \times 10}{0.05 \times 150} = 80.$

EXAMPLE 23. A body of mass 1 kg initially at rest explodes and breaks into three fragments of masses in the ratio 1 : 1 : 3. The two pieces of equal mass fly off perpendicular to each other with a speed of 30 ms^{-1} each. What is the velocity of the heavier fragment ? [IIT 82]

Solution. Here $m_1 + m_2 + m_3 = 1$ kg

As $m_1 : m_2 : m_3 = 1 : 1 : 3$

$\therefore$ $m_1 = m_2 = 0.2$ kg, $m_3 = 0.6$ kg

$v_1 = v_2 = 30 \text{ ms}^{-1}$, $v_3 = ?$

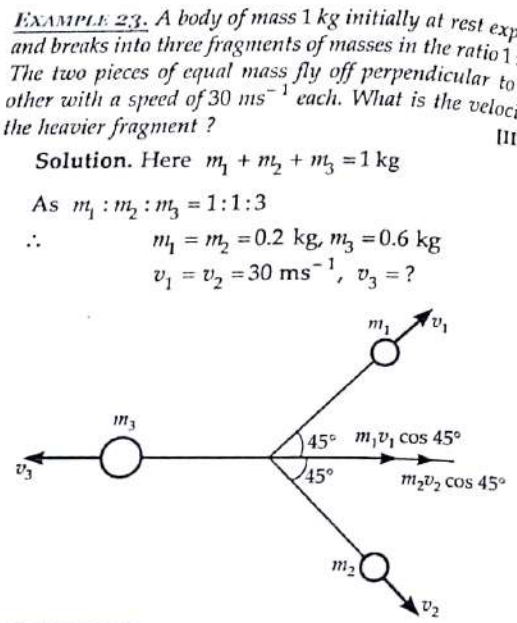

Fig. 5.23

Applying the law of conservation of momentum to the momenta along horizontal direction,

$$m_3 v_3 = m_1 v_1 \cos 45° + m_2 v_2 \cos 45°$$

$$0.6 \, v_3 = 0.2 \times 30 \times 0.707 + 0.2 \times 30 \times 0.707$$

or $$v_3 = \frac{2 \times 0.2 \times 30 \times 0.707}{0.6} = \mathbf{14.14 \text{ ms}^{-1}}.$$

EXAMPLE 24. A disc of mass 10 g is kept floating horizontally by throwing 10 marbles per second against it from below. If the mass of each marble is 5 g, calculate the velocity with which the marbles are striking the disc. Assume that the marbles strike the disc normally and rebound downward with the same speed. [IIT]

Solution. Mass of each marble piece,

$$m = 5 \text{ g} = 5 \times 10^{-3} \text{ kg}$$

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Number of marbles thrown per second = 10

Let velocity of impact of each marble = v

Change in momentum of each marble

$$= mv - (-mv) = 2mv$$

Change in momentum per second $= 2mv \times 10$

$\therefore$ Force exerted by marbles on the disc $= 20\ mv$

But the disc can be kept floating if this force balances the weight of the disc.

$\therefore$ $20\ mv = Mg$

$$20 \times 5 \times 10^{-3} \times v = 10 \times 10^{-3} \times 9.8$$

$$v = \frac{10 \times 9.8}{100}$$

$$= 0.98\ \text{ms}^{-1} = 98\ \text{cms}^{-1}.$$

PROBLEMS FOR PRACTICE

1. A 30 g bullet leaves a rifle with a velocity of $300\ \text{ms}^{-1}$ and the rifle recoils with a velocity of $0.60\ \text{ms}^{-1}$. Find the mass of the rifle. **(Ans.** 15 kg)
2. A 40 kg shell is flying at a speed of $72\ \text{kmh}^{-1}$. It explodes into two pieces. One of the two pieces of mass 15 kg stops. Calculate the speed of the other. [Delhi 06] **(Ans.** $32\ \text{ms}^{-1}$)
3. A gun weighing 10 kg fires a bullet of 30 g with a velocity of $330\ \text{ms}^{-1}$. With what velocity does the gun recoil ? What is the combined momentum of the gun and bullet before firing and after firing ?

   (Ans. $99\ \text{cms}^{-1}$, zero before and after firing)
4. A car of mass 1000 kg moving with a speed of 30 m/s collides with the back of a stationary lorry of mass 9000 kg. Calculate the speed of the vehicles immediately after the collision if they remain jammed together. [Central Schools 08]

   (Ans. $3\ \text{ms}^{-1}$)
5. A bullet of mass 7 g is fired into a block of metal weighing 7 kg. The block is free to move. After the impact, the velocity of the bullet and the block is $70\ \text{ms}^{-1}$. What is the initial velocity of the bullet ?

   (Ans. $700.7\ \text{ms}^{-1}$)
6. A truck of mass 2×10^4 kg travelling at $0.5\ \text{ms}^{-1}$ collides with another truck of half its mass moving in the opposite direction with a velocity of $0.4\ \text{ms}^{-1}$. If the trucks couple automatically on collision, calculate the common velocity with which they move. **(Ans.** $0.2\ \text{ms}^{-1}$)
7. A neutron of mass 1.67×10^{-27} kg moving with a speed of $3 \times 10^6\ \text{ms}^{-1}$ collides with a deutron of mass 3.34×10^{-27} kg which is at rest. After collision, the neutron sticks to the deutron and forms a triton. What is the speed of the triton ? **(Ans.** $10^6\ \text{ms}^{-1}$)
8. A bomb at rest explodes into three fragments of equal masses. Two fragments fly off at right angles to each other with velocities $9\ \text{ms}^{-1}$ and $12\ \text{ms}^{-1}$ respectively. Calculate the speed of the third fragment. **(Ans.** $15\ \text{ms}^{-1}$)
9. A man weighing 60 kg runs along the rails with a velocity of $18\ \text{kmh}^{-1}$ and jumps into a car of mass 1 quintal standing on the rails. Calculate the velocity with which the car will start travelling along the rails. **(Ans.** $1.88\ \text{ms}^{-1}$)
10. A machine gun of mass 10 kg fires 20 g bullets at the rate of 10 bullets per second with a speed of $500\ \text{ms}^{-1}$. What force is required to hold the gun in position ? **(Ans.** 100 N)

HINTS

2. Proceed as in Example 19.
8. Let $\vec{p_1}, \vec{p_2}$ and $\vec{p_3}$ be the momenta and $\vec{v_1}, \vec{v_2}$ and $\vec{v_3}$ be the velocities of the three fragments respectively.

   Then $\vec{p_1} = m\vec{v_1}$, $\vec{p_2} = m\vec{v_2}$, $\vec{p_3} = m\vec{v_3}$,

   $v_1 = 9\ \text{ms}^{-1}$, $v_2 = 12\ \text{ms}^{-1}$, $v_3 = ?$

   As $\vec{p}_1 \perp \vec{p}_2$, so the magnitude of their resultant is

   $$p = \sqrt{p_1^2 + p_2^2} = m\sqrt{v_1^2 + v_2^2}$$

   $$= m\sqrt{9^2 + 12^2} = 15\ m\ \text{kg ms}^{-1}.$$

   By conservation of linear momentum, $\vec{p} + \vec{p}_3 = 0$

   In magnitude, $p_3 = p$ or $mv_3 = p$

   $\therefore$ $v_3 = \frac{p}{m} = \frac{15\ m}{m} = 15\ \text{ms}^{-1}$.
9. Here $m_1 = 60$ kg, $u_1 = 18\ \text{kmh}^{-1} = 5\ \text{ms}^{-1}$,

   $m_2 = 1$ quintal $= 100$ kg, $u_2 = 0$, $v = ?$

   By conservation of linear momentum,

   $$(m_1 + m_2)\ v = m_1u_1 + m_2u_2$$

   or $(60 + 100)\ v = 60 \times 5 + 100 \times 0$

   or $v = \frac{60 \times 5}{160} = \frac{15}{8} = 1.88\ \text{ms}^{-1}$.
10. Change in the momentum of one bullet

    $$= m(v - u) = 2 \times 10^{-3}\ (500 - 0) = 10\ \text{kg ms}^{-1}$$

    Change in momentum of 10 bullets

    $$= 10 \times 10 = 100\ \text{kg ms}^{-1}$$

    Force required to hold the gun

    $$= \frac{\text{Change in momentum}}{\text{Time taken}} = \frac{100\ \text{kg ms}^{-1}}{1\text{s}}$$

    $= 100$ N.

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5.19 EXAMPLES OF VARIABLE-MASS SITUATION : ROCKET PROPULSION *

42. *(a) Briefly describe the propulsion of a rocket. (b) Derive the expression for the velocity of a rocket at any time, after being fired. (c) What is meant by burnt out speed of a rocket ? (d) Also obtain an expression for the thrust on a rocket.*

Rocket propulsion. The propulsion of a rocket is an example of momentum conservation. In a rocket, gases at high temperature and pressure, are produced by the combustion of fuel. They escape with a large constant velocity through a nozzle. The large backward momentum of the gases imparts an equal forward momentum to the rocket. But due to the decrease in the mass of the rocket fuel system, the acceleration of the rocket keeps on increasing.

Expression for the velocity gained by a rocket.

Suppose at time $t = 0$, we have

m_0 = initial mass of the rocket including that of the fuel

v_0 = initial velocity of the rocket

Suppose at time $t = t$, we have

m = mass of the rocket left

v = velocity acquired by the rocket

As the gases are escaping from rocket, we must have

$$m < m_0 \quad \text{and} \quad v > v_0$$

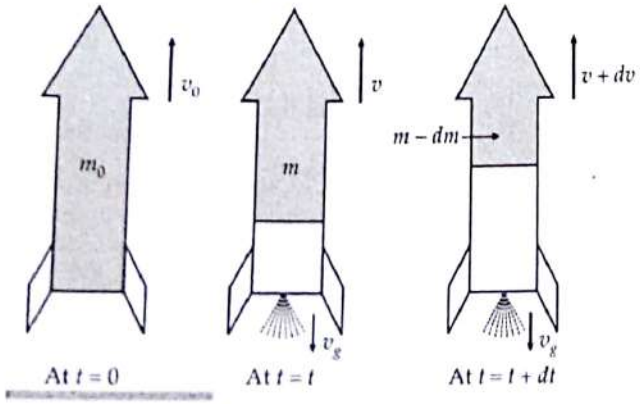

Fig. 5.24 Rocket propulsion.

Suppose in a small interval of time dt,

dm = a small decrease in mass of the rocket

= mass of the exhaust gases that escape

dv = corresponding small increase in velocity of the rocket.

v_g = velocity of exhaust gases *w.r.t.* the earth

Using the principle of conservation of linear momentum, we get

$$mv = (m - dm)(v + dv) + dm(-v_g) \quad ...(1)$$

Here v_g is taken negative because gases escape downwards as the rocket moves upwards.

From (1), we have

$$mv = mv + m(dv) - (dm)\,v - dm\,dv - (dm)\,v_g$$

As dm and dv both are small, their product term can be neglected.

$$\therefore \quad m\,dv = dm(v + v_g) \quad ...(2)$$

If u is relative velocity of burnt gases *w.r.t.* rocket, then

u = Velocity of burnt gases *w.r.t.* the earth

–Velocity of the rocket *w.r.t.* the earth

$$= -v_g - v = -(v_g + v)$$

or $\quad v + v_g = -u$

The negative sign shows the downward direction of motion of the gases.

$$\therefore \quad m\,dv = -u\,dm$$

$$dv = -u\frac{dm}{m} \quad ...(3)$$

Integrating both sides, we get

$$\int_{v_0}^{v} dv = -u\int_{m_0}^{m}\frac{dm}{m}$$

$$[v]_{v_0}^{v} = -u\,[\log_e m]_{m_0}^{m}$$

or $\quad v - v_0 = -u\,[\log_e m - \log_e m_0]$

$$= -u\log_e\left(\frac{m}{m_0}\right) = +u\log_e\left(\frac{m_0}{m}\right)$$

or $\quad v = v_0 + u\log_e\left(\frac{m_0}{m}\right) \quad ...(4)$

This gives the velocity of the rocket at any time t, when its mass is m.

At time $t = 0$, $v_0 = 0$, so

$$v = u\log_e\left(\frac{m_0}{m}\right) \quad ...(5)$$

Thus *in the absence of any external force, the instantaneous velocity of the rocket is proportional to*

(i) the exhaust speed of the burnt gases.

(ii) natural logarithm of the ratio of initial mass of the rocket to its mass at the instant.

Burnt out speed of the rocket. *The speed acquired by the rocket when whole of its fuel gets burnt is called the burnt out speed of the rocket.* If m_r is residual mass of the rocket at that instant, then from equation (4), we get

$$v_b = v_0 + u\log_e\left(\frac{m_0}{m_r}\right)$$

Obviously, v_b *is the maximum velocity that the rocket can acquire.*

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Thrust on the rocket. *It is the force with which the rocket moves upwards.*

Dividing both sides of equation (3) by small time interval dt, we get

$$m\frac{dv}{dt} = -u\frac{dm}{dt} \qquad ...(6)$$

As $dv/dt = a$, acceleration of the rocket at time t.

$\therefore m\,dv/dt = ma = F$ = thrust on the rocket at time t.

From (6), $F = -u\,dm/dt$

Thus the upthrust on a rocket at any instant is equal to the product of the exhaust speed of the burnt gases and the rate of consumption of fuel at that instant.

Examples Based on Rocket Propulsion

Formulae Used

1. Resultant force on the rocket

F = upthrust on the rocket − weight of the rocket

$$= u\frac{dm}{dt} - mg$$

2. Acceleration of the rocket after time t

$$a = \left[\frac{u}{m_0 - t\dfrac{dm}{dt}}\right]\frac{dm}{dt} - g$$

3. Velocity of the rocket after time t,

$$v = v_0 + u\log_e\frac{m_0}{m} - gt$$

If the effect of gravity is neglected, then

$$F = u\frac{dm}{dt}$$

$$a = \left[\frac{u}{m_0 - t\dfrac{dm}{dt}}\right]\frac{dm}{dt}\,;\; v = v_0 + u\log_e\frac{m_0}{m}$$

4. Burn-out speed of the rocket,

$$v_b = v_0 + u\log_e\frac{m}{m_r}$$

Here :

u = Velocity of exhaust gases

v_0 = Initial velocity of the rocket

v = Velocity of the rocket at any instant t

m_0 = Initial mass of the rocket

m = Final mass of the rocket

m_r = Mass of the empty rocket

dm/dt = Rate of ejection of fuel

Units Used

Velocities u, v_0, v and v_b are in ms^{-1} and masses m_0, m and m_r are in kg.

Example 25. Fuel is consumed at the rate of 100 kg s^{-1} in a rocket. The exhaust gases are ejected at speed of 4.5×10^4 ms^{-1}. What is thrust experienced by the rocket?

Solution. Here

$$\frac{dm}{dt} = 100\text{ kg s}^{-1},\; u = 4.5\times10^4\text{ ms}^{-1}$$

Thrust, $F = u\dfrac{dm}{dt} = 4.5\times10^4\times100 = \mathbf{4.5\times10^6\ N.}$

Example 26. A rocket of initial mass 6,000 kg ejects mass at a constant rate of 16 kg s^{-1} with constant relative speed of 11 kms^{-1}. What is acceleration of the rocket a minute after the blast ? Neglect gravity. **[NCERT]**

Solution. Here $m_0 = 6{,}000$ kg,

$$u = 11\text{ km s}^{-1} = 11{,}000\text{ ms}^{-1},$$

$$t = 1\text{ min} = 60\text{ s},\; \frac{dm}{dt} = 16\text{ kgs}^{-1}$$

Acceleration,

$$a = \left[\frac{u}{m_0 - t\dfrac{dm}{dt}}\right]\frac{dm}{dt}$$

$$= \left[\frac{11000}{6000 - 60\times16}\right]\times16 = \mathbf{34.92\ ms^{-2}.}$$

Example 27. The mass of a rocket is 500 kg and relative velocity of gases ejecting out is 250 ms^{-1} with respect to the rocket. Determine the rate of burning of the fuel in order to give the rocket an initial acceleration of 20 ms^{-2} in the upward direction.

Solution. Initial acceleration, $a = \dfrac{u}{m_0}\dfrac{dm}{dt} - g$

$$\therefore \qquad 20 = \frac{250}{500}\frac{dm}{dt} - 9.8$$

or $$\frac{dm}{dt} = 2\,(20 + 9.8) = \mathbf{59.6\ kgs^{-1}.}$$

Example 28. A rocket is set for vertical firing. If the exhaust speed is 1200 ms^{-1}, how much gas must be ejected per second to supply the thrust needed (i) to overcome the weight of the rocket, (ii) to give to the rocket an initial vertical upward acceleration of 19.6 ms^{-2} ? Given mass of rocket = 6000 kg.

Solution. (*i*) Here $u = 1200\text{ ms}^{-1}$, $m = 6000$ kg,

$$\frac{dm}{dt} = ?$$

Given : Thrust = Weight of rocket

$$\therefore \qquad u\frac{dm}{dt} = mg$$

or $$\frac{dm}{dt} = \frac{mg}{u} = \frac{6000\times9.8}{1200} = \mathbf{49\ kg\ s^{-1}.}$$

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(ii) Here $u = 1200\ \text{ms}^{-1}$, $m = m_0 = 6000\ \text{kg}$, $t = 0$, $a = 29.6\ \text{ms}^{-2}$

As $$a = \left[\frac{u}{m_0 - t\frac{dm}{dt}}\right]\frac{dm}{dt} - g$$

$$\therefore \quad 29.6 = \left[\frac{1200}{6000 - 0 \times \frac{dm}{dt}}\right]\frac{dm}{dt} - 9.8$$

or $$29.6 + 9.8 = \frac{1200}{6000} \times \frac{dm}{dt}$$

or $$\frac{dm}{dt} = \frac{39.4 \times 6000}{1200} = \mathbf{197\ kg\ s^{-1}}.$$

EXAMPLE 29. If the maximum possible exhaust velocity of a rocket be 2 kms^{-1}, calculate the ratio m_0/m for it if it is to acquire the escape velocity of 11.2 kms^{-1} after starting from rest.

Solution. As the rocket starts from rest, velocity acquired by it is

$$v = u \log_e \frac{m_0}{m} = 2.303\, u \log_{10} \frac{m_0}{m}$$

Given $v = 11.2\ \text{km s}^{-1}$, $u = 2\ \text{km s}^{-1}$

$$\therefore \quad 11.2 = 2.303 \times 2 \times \log_{10} \frac{m_0}{m}$$

or $$\log_{10} \frac{m_0}{m} = \frac{11.2}{2.303 \times 2} = 2.432$$

or $$\frac{m_0}{m} = \text{antilog}\,(2.432) = \mathbf{270.4}$$

EXAMPLE 30. A rocket motor consumes 100 kg of fuel per second, exhausting it with a speed of $5 \times 10^3\ ms^{-1}$. (i) What force is exerted on the rocket ? (ii) What will be the velocity of the rocket at the instant its mass is reduced (1/20)th of its initial mass, its initial velocity being zero. Neglect gravity.

Solution. Here $\frac{dm}{dt} = 100\ \text{kg s}^{-1}$,

$$u = 5 \times 10^3\ \text{ms}^{-1},\ v_0 = 0,\ \frac{m}{m_0} = \frac{1}{20}$$

(i) Thrust $= u\frac{dm}{dt} = 5 \times 10^3 \times 100 = \mathbf{5 \times 10^5\ N}.$

(ii) $v = v_0 + u \log_e \frac{m_0}{m} = 0 + 5 \times 10^3 \log_e 20$

$= 5 \times 10^3 \times 2.303 \log_{10} 20$

$= 5 \times 10^3 \times 2.303 \times 1.301$

$= \mathbf{14.98 \times 10^3\ ms^{-1}}.$

EXAMPLE 31. A balloon with mass M is descending down with an acceleration a, where $a < g$. What mass m of its contents must be removed so that it starts moving up with acceleration a ?

Solution. As shown in Fig. 5.25, a retarding force F acts on the balloon in the vertically upward direction and its weight Mg acts in the vertically downward direction. Net force acting on the balloon in the vertically downward direction

$= Mg - F$

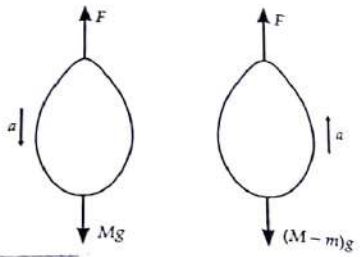

Fig. 5.25

As the balloon descends down with acceleration a,

so $$Mg - F = Ma \qquad ...(i)$$

When mass m is removed from the balloon, its net weight $= (M - m)g$.

Now the balloon moves with acceleration a, therefore

$$F - (M - m)g = (M - m)a$$

or $Mg - Ma - (M - m)g = (M - m)a$ [Using (i)]

or $Mg - Ma - Mg + mg = Ma - ma$

or $$m(g + a) = 2Ma$$

or $$m = \frac{2Ma}{g + a}.$$

PROBLEMS FOR PRACTICE

1. A rocket with a lift mass of 3.5×10^4 kg is blasted upwards with an acceleration of 10 ms^{-2}. Calculate the initial thrust of the blast. [AIEEE 03]
(**Ans.** 7.0×10^5 N)

2. Fuel is consumed at the rate of 50 g per second in a rocket. The exhaust gases are rejected at the rate of $5 \times 10^5\ cms^{-1}$. What is the thrust experienced by the rocket ? (**Ans.** 250 N)

3. Calculate the ratio m/m_0 for a rocket to attain the escape velocity of 11.2 kms^{-1} after starting from rest, when maximum exhaust velocity of the gases is 1.6 kms^{-1}. (**Ans.** 1096)

4. In the first second of its flight, a rocket ejects 1/60 of its mass with a relative velocity of 2073 ms^{-1}. What is the initial acceleration of the rocket ?
(**Ans.** 24.75 ms^{-2})

5. A rocket motor consumes 100 kg of fuel per second, exhausting it with a speed of $6 \times 10^3\ ms^{-1}$. (i) What

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thrust is exerted on the rocket ? (*ii*) What will be the velocity of the rocket at the instant its mass is reduced to (1/40)th of its initial mass, its initial velocity being zero ? Neglect gravity.

[Ans. (*i*) 6×10^5 N (*ii*) 22.13×10^3 ms^{-1}]

6. A rocket fired from the earth's surface ejects 1% of its mass at a speed of 2000 ms^{-1} in the first second. Find the average acceleration of the rocket in the first second. (Ans. 20 ms^{-2})

7. A rocket is going upwards with accelerated motion. A man sitting in it feels his weight increased by 5 times his own weight. If the mass of the rocket including that of the man is 1.0×10^4 kg, how much force is being applied by rocket engine ? Given $g = 10$ ms^{-2}. (Ans. 5×10^5 N)

8. A balloon of mass m is rising up with an acceleration a. Show that the fraction of weight of the balloon that must be detached in order to double its acceleration is $[ma/(2a + g)]$, assuming the upthrust of air to remain the same.

HINTS

3. $v = u \log_e \dfrac{m_0}{m} = 2.303\, u \log_{10} \dfrac{m_0}{m}$

$$\log_{10} \frac{m}{m_0} = \frac{v}{2.303 \times u}$$

$$= \frac{11.2 \text{ kms}^{-1}}{2.303 \times 1.6 \text{ kms}^{-1}} = 3.0400$$

$$\therefore \quad \frac{m}{m_0} = \text{antilog } (3.0400) = \mathbf{1096.}$$

4. Acceleration at $t = 0$ is

$$a = \frac{u}{m_0}\frac{dm}{dt} - g = \frac{2073}{m_0} \times \frac{1/m_0}{1} - 9.8$$

$$= 34.55 - 9.8 = \mathbf{24.75 \text{ ms}^{-2}}.$$

6. $F = u\dfrac{dm}{dt}$

Here $dm = \dfrac{m_0}{100}$, $dt = 1$ s, $u = 2000$ ms^{-1}

$$\therefore \quad F = 2000 \times \frac{m_0/100}{1} = 20\, m_0$$

or $\quad a = \dfrac{F}{m_0} = \mathbf{20 \text{ ms}^{-2}}.$

7. As the weight of man increases by 5 times, so acceleration of the rocket,

$$a = 5g = 5 \times 10 = 50 \text{ ms}^{-2}$$

Force applied by rocket engine is

$$F = ma = 1.0 \times 10^4 \times 50 = \mathbf{5 \times 10^5 \text{ N}}.$$

8. Let F be the upthrust of the air. As the balloon rises with acceleration a, so

$$F = mg + ma \qquad ...(1)$$

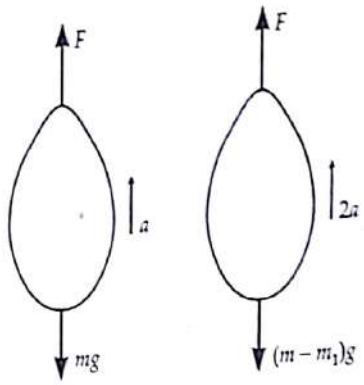

Fig. 5.26

Suppose mass m_1 is removed from the balloon so that the acceleration becomes $2a$. Then

$$F = (m - m_1)\, g + (m - m_1)\, 2a \qquad ...(2)$$

From (1) and (2), we get,

$$(m - m_1)\, g + (m - m_1)\, 2a = m(g + a)$$

or $$m - m_1 = \frac{m(g + a)}{(g + 2a)}$$

or $$m_1 = m - \frac{m(g + a)}{(g + 2a)} = \frac{ma}{g + 2a}.$$

5.20 EQUILIBRIUM OF CONCURRENT FORCES

43. *What are concurrent forces ? Obtain the condition for the equilibrium of a number of concurrent forces.*

Equilibrium of concurrent forces. *Forces acting at the same point on a body are called concurrent forces. When a number of forces act on a body at the same point and the net unbalanced force is zero, the body will continue in its state of rest or of uniform motion along a straight line and is said to be in equilibrium.*

Consider three concurrent forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ acting at the same point O of a body, as shown in Fig. 5.27(*a*). By parallelogram law,

Resultant of $\vec{F}_1$ and $\vec{F}_2 = \vec{F}_1 + \vec{F}_2$

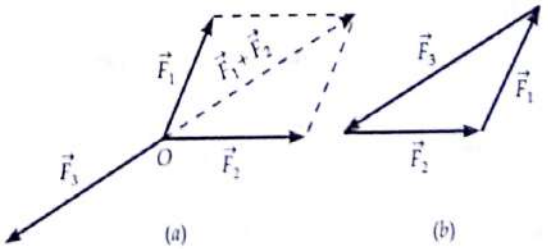

Fig. 5.27 Equilibrium under concurrent forces.

If the third force $\vec{F}_3$ acts on the body such that $\vec{F}_3 = -(\vec{F}_1 + \vec{F}_2)$, then the body will be in equilibrium.

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i.e., $\vec{F_3} = -(\vec{F_1} + \vec{F_2})$

or $\vec{F_1} + \vec{F_2} + \vec{F_3} = \vec{0}$

As shown in Fig. 5.27(*b*), these three forces in equilibrium can be represented by the sides of a triangle taken in the same order.

Thus *the condition for the equilibrium of a number of forces acting at the same point is that the vector sum of all these forces is equal to zero.*

i.e., $\vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4} + ... + \vec{F_n} = \vec{0}$

In general, particle is in equilibrium under the action of n forces if these forces can be represented by the sides of closed n-side polygon taken in the same order.

44. *State and prove Lami's theorem.*

Lami's theorem. Fig. 5.28(*a*) shows a particle O under the equilibrium of three concurrent forces $\vec{F_1}$, $\vec{F_2}$ and $\vec{F_3}$. Let α be angle between $\vec{F_2}$ and $\vec{F_3}$, β between $\vec{F_3}$ and $\vec{F_1}$; and γ between $\vec{F_1}$ and $\vec{F_2}$.

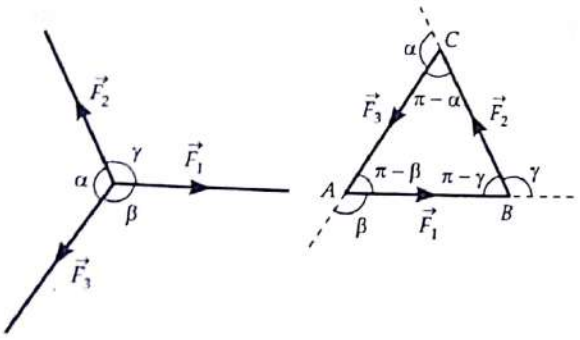

Fig. 5.28 Lami's theorem

As shown in Fig. 5.28(*b*), the forces $\vec{F_1}$, $\vec{F_2}$ and $\vec{F_3}$ can be represented by the sides of ΔABC, taken in the same order.

Applying law of sines to ΔABC, we get

$$\frac{F_1}{\sin(\pi - \alpha)} = \frac{F_2}{\sin(\pi - \beta)} = \frac{F_3}{\sin(\pi - \gamma)}$$

or $$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

This is ***Lami's theorem*** which states that *if three forces acting on a particle keep it in equilibrium, then each force is proportional to the sine of the angle between the other two forces.*

Examples based on Equilibrium of Concurrent Forces

FORMULAE USED

1. A number of forces acting at the same point are called concurrent forces.
2. A number of concurrent forces are said to be in equilibrium if their resultant is zero.
$$\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} + ... + \vec{F_n} = \vec{0}$$
3. If $\vec{F_1}$, $\vec{F_2}$ and $\vec{F_3}$ are three concurrent forces in equilibrium
   (*i*) $\vec{F_1} + \vec{F_2} + \vec{F_3} = \vec{0}$
   (*ii*) $\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$ (Lami's theorem)

UNITS USED

All forces are in newton.

EXAMPLE 32. *A mass of 6 kg is suspended by a rope of length 2 m from a ceiling. A force of 50 N in the horizontal direction is applied at the midpoint of the rope as shown in Fig. 5.29. What is the angle the rope makes with the vertical in equilibrium ? Take $g = 10\ ms^{-2}$. Neglect mass of the rope.* [NCERT]

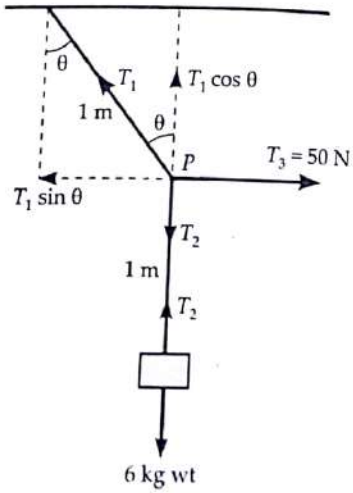

Fig. 5.29

Solution. As shown in Fig. 5.29, there are three forces acting on the midpoint P of the rope. Suppose the rope makes an angle θ with the vertical in equilibrium. Resolving the forces horizontally and vertically, we get

$$T_1 \sin\theta = T_3 = 50 \text{ N} \quad ...(i)$$

$$T_1 \cos\theta = T_2 = 6 \text{ kg wt} = 60 \text{ N} \quad ...(ii)$$

Dividing (*i*) by (*ii*), we get,

$$\tan\theta = \frac{5}{6} \text{ or } \theta = \tan^{-1}\left(\frac{5}{6}\right) = 39.8°$$

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EXAMPLE 33. Determine the tensions T_1 and T_2 in the strings shown in Fig. 5.30(a).

Solution. As shown in Fig. 5.30(*b*), resolve the tension T_1 along horizontal and vertical directions. As the body is in equilibrium,

$$T_1 \sin 60° = 4 \text{ kg wt} = 4 \times 9.8 \text{ N} \quad ...(i)$$

$$T_1 \cos 60° = T_2 \quad ...(ii)$$

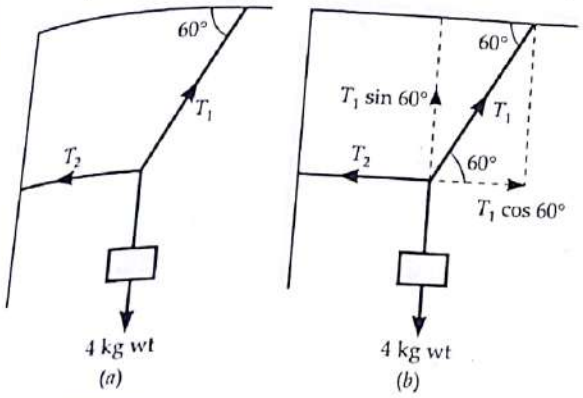

Fig. 5.30

From (*i*), $T_1 = \dfrac{4 \times 9.8}{\sin 60°} = \dfrac{4 \times 9.8 \times 2}{\sqrt{3}} = \mathbf{45.26\ N}$

From (*ii*), $T_2 = T_1 \cos 60° = 45.26 \times 0.5 = \mathbf{22.63\ N.}$

EXAMPLE 34. A train is moving along a horizontal track. A pendulum suspended from the roof makes an angle of 4° with the vertical. Obtain the acceleration of the train. Take $g = 10\ ms^{-2}$.

Solution. Fig. 5.31 shows the equilibrium position of a pendulum suspended in a train which is moving towards right with acceleration *a* along a horizontal track.

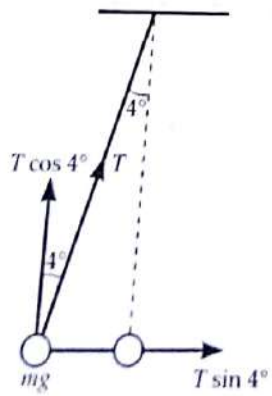

Fig. 5.31

Two forces acting on the bob are :

(*i*) Weight mg acting vertically downwards,

(*ii*) Tension T acting along the string.

Resolving T along horizontal and vertical directions, we find that

$$T \cos 4° = mg \quad ...(i)$$

As the acceleration *a* of the train and hence that of the pendulum is responsible for tension $T \sin 4°$, so

$$T \sin 4° = ma \quad ...(ii)$$

Dividing (*ii*) by (*i*), we get

$$\tan 4° = \frac{a}{g}$$

or $$a = g \tan 4° = 10 \times 0.07 = \mathbf{0.7\ ms^{-2}}.$$

EXAMPLE 35. A ball of mass 1 kg hangs in equilibrium from two strings OA and OB as shown in Fig. 5.32. What are the tensions in strings OA and OB ? Take $g = 10\ ms^{-2}$.

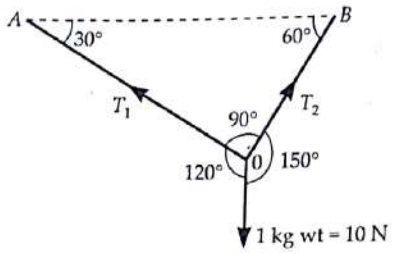

Fig. 5.32

Solution. Various forces acting at the point *O* are as shown in Fig. 5.32. The three forces are in equilibrium. Using Lami's theorem,

$$\frac{T_1}{\sin 150°} = \frac{T_2}{\sin 120°} = \frac{10}{\sin 90°}$$

or $$\frac{T_1}{\sin 30°} = \frac{T_2}{\sin 60°} = \frac{10}{1}$$

$$\therefore \quad T_1 = 10 \sin 30° = 10 \times 0.5 = \mathbf{5\ N.}$$

and $$T_2 = 10 \sin 60° = 10 \times \frac{\sqrt{3}}{2} = \mathbf{5\sqrt{3}\ N.}$$

EXAMPLE 36. A body of mass m is suspended by two strings making angles α and β with the horizontal as shown in Fig. 5.33. Find the tensions in the strings.

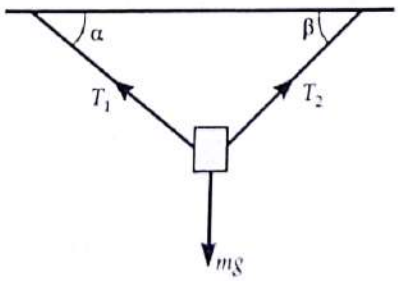

Fig. 5.33

Solution. The free-body diagram is shown in Fig. 5.34. As the body is in equilibrium, the various forces must add to zero. Taking horizontal components,

$$T_1 \cos \alpha = T_2 \cos \beta \quad \text{or} \quad T_2 = T_1 \frac{\cos \alpha}{\cos \beta}$$

Taking vertical components,

$$T_1 \sin \alpha + T_2 \sin \beta = mg$$

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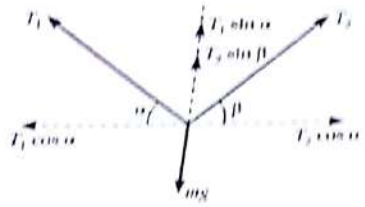

Fig. 5.34

$$\text{or} \quad T_1 \sin\alpha + T_1 \frac{\cos\alpha}{\cos\beta} \sin\beta = mg$$

$$\text{or} \quad T_1 = \frac{mg}{\sin\alpha + \dfrac{\cos\alpha}{\cos\beta}\sin\beta} = \frac{mg\cos\beta}{\sin\alpha\cos\beta + \cos\alpha\sin\beta}$$

$$\text{or} \quad T_1 = \frac{mg\cos\beta}{\sin(\alpha+\beta)}$$

$$\text{and} \quad T_2 = T_1 \frac{\cos\alpha}{\cos\beta} = \frac{mg\cos\beta}{\sin(\alpha+\beta)} \cdot \frac{\cos\alpha}{\cos\beta} = \frac{mg\cos\alpha}{\sin(\alpha+\beta)}.$$

EXAMPLE 37. A uniform rope of length L, resting on a frictionless horizontal surface is pulled at one end by a force F. What is the tension in the rope at a distance l from the end where the force is applied ?

Solution. Let M be the mass of uniform rope of length L. Then

$$\text{Mass per unit length of rope} = \frac{M}{L}$$

$$\text{Acceleration in the rope} = \frac{F}{M}$$

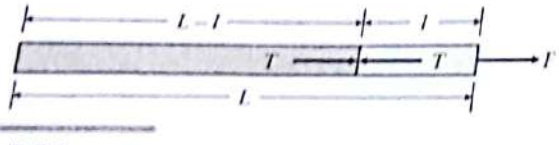

Fig. 5.35

Let T be the tension in the rope at a distance l from the end where the force F is applied.

Mass of length $(L - l)$ of the rope is

$$M' = \frac{M}{L}(L - l)$$

As tension T is the only force on the length $(L - l)$ of the rope, so

$$T = M' \times \frac{F}{M} = \frac{M}{L}(L - l) \times \frac{F}{M} = \left(1 - \frac{l}{L}\right)F.$$

✕ PROBLEMS FOR PRACTICE

1. *A mass of 10 kg is suspended vertically by a rope of length 2 m from a ceiling. A force of 60 N is applied at the middle point of the rope in the horizontal direction, as shown in Fig. 5.36. Calculate the angle the rope makes with the vertical. Neglect the mass of the rope and take $g = 10\ ms^{-2}$.* **(Ans. 31°)**

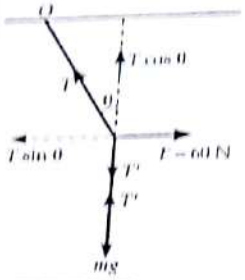

Fig. 5.36

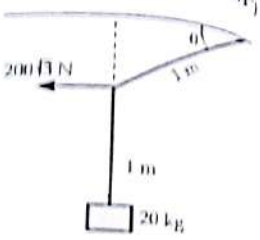

Fig. 5.37

2. *A mass of 20 kg is suspended by a rope of length 2 m from a ceiling. A force of 173.2 N in the horizontal direction is applied at the midpoint of the rope as shown in Fig. 5.37. What is the angle the rope makes with the horizontal in equilibrium ? Take $g = 10\ ms^{-2}$. Neglect mass of the rope.* **(Ans. 30°)**
3. *A body of weight 200 N is suspended with the help of strings as shown in Fig. 5.38. Find the tensions T_1 and T_2.* **(Ans. $T_1 = 146.4$ N, $T_2 = 179.3$ N)**

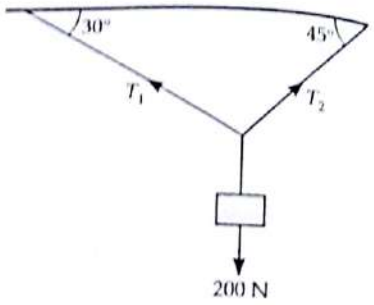

Fig. 5.38

✕ HINTS

1. For equilibrium of the body,

$$T\sin\theta = F = 60\ \text{N} \quad \ldots(i)$$

$$T\cos\theta = T' = mg \quad \ldots(ii)$$

Dividing (*i*) by (*ii*), we get

$$\frac{T\sin\theta}{T\cos\theta} = \frac{60}{mg} = \frac{60}{10 \times 10} = 0.6$$

or $\tan\theta = 0.6 \ \therefore \ \theta = 31°$.

3. The free-body diagram is shown in Fig. 5.39.

Resolution of forces along horizontal direction gives

$$T_1\cos 30° = T_2\cos 45°$$

$$T_1 \times \frac{\sqrt{3}}{2} = T_2 \times \frac{1}{\sqrt{2}} \quad \text{or} \quad T_2 = \sqrt{\frac{3}{2}}\,T_1$$

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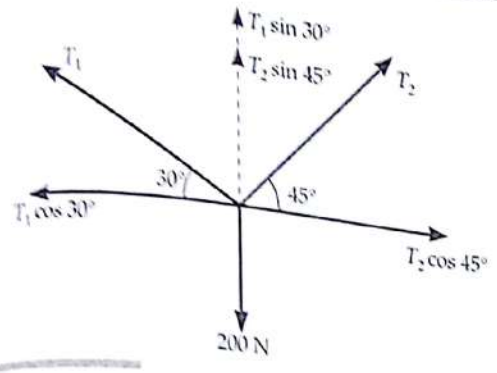

Fig. 5.39

Resolution of forces along vertical direction gives

$$T_1 \sin 30° + T_2 \sin 45° = 200 \text{ N}$$

$$T_1 \times \frac{1}{2} + \sqrt{\frac{3}{2}}\, T_1 \times \frac{1}{\sqrt{2}} = 200$$

or $$T_1 (1 + \sqrt{3}) = 400$$

$$T_1 = \frac{400}{2.732} = 146.4 \text{ N};$$

$$T_2 = \sqrt{\frac{3}{2}} \times 146.4 = 179.3 \text{ N}.$$

5.21 MOTION OF CONNECTED BODIES

45. *Two masses M and m are connected at the two ends of an inextensible string. The string passes over a smooth frictionless pulley. Calculate the acceleration of the masses and the tension in the string. Given $M > m$.*

Connected motion. Let a be the acceleration with which the heavier mass M moves downwards and the lighter mass m moves upwards. Let T be the tension in the string. According to Newton's second law, the resultant downward force on mass M is

$$Ma = Mg - T \quad ...(i)$$

Resultant upward force on mass m is

$$ma = T - mg \quad ...(ii)$$

Adding equations (i) and (ii), we get

$$a(M + m) = (M - m)\, g$$

or $$a = \frac{M - m}{M + m} \cdot g$$

Obviously, $a < g$ *i.e.*, the acceleration a of the two connected bodies is less than the acceleration due to gravity.

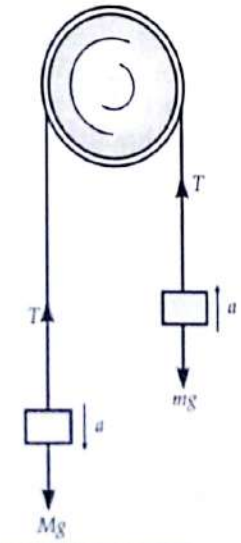

Fig. 5.40

Dividing (i) by (ii), we get

$$\frac{Ma}{ma} = \frac{Mg - T}{T - mg} \quad \text{or} \quad \frac{M}{m} = \frac{Mg - T}{T - mg}$$

or $$MT - Mmg = Mmg - mT$$

or $$T(M + m) = 2\, Mmg$$

or $$T = \frac{2\, Mm}{M + m} g.$$

46. *What is meant by a free-body diagram ? What is its use ?*

Solving problems in mechanics : Use of free-body diagram. When a number of bodies are connected together by strings, rods, etc, it is convenient to consider each body separately and to write equation of motion for each body by taking into account all the forces acting on it and then equating the net force acting on the body to its mass times the acceleration produced.

A diagram for each body of the system showing all the forces exerted on the body by the remaining parts of the system is called free-body diagram.

The equations of motion obtained for different bodies can be solved to determine unknown quantities.

Examples based on Motion of Connected Bodies

CONCEPTS USED

1. When a number of bodies are connected together by strings, rods, etc., it is convenient to draw a free body diagram for each body separately by showing all the forces acting on it.
2. Equation of motion for each body is written by equating the net force acting on the body to its mass times the acceleration produced.

Units Used

All forces are in newton (N)

EXAMPLE 38. *A pull of 15 N is applied to a rope attached to a block of mass 7 kg lying on a smooth horizontal surface. The mass of the rope is 0.5 kg. What is the force exerted by the block on the rope ?*

Solution. The situation is shown in Fig. 5.41. If acceleration a is produced in the block on applying a force of 15 N, then

$$(7 + 0.5)\, a = 15 \quad \text{or} \quad a = \frac{15}{7.5} = 2 \text{ ms}^{-2}$$

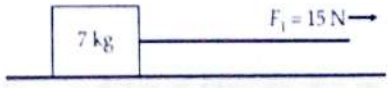

Fig. 5.41

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As shown in Fig. 5.42, let F_2 be the force exerted by the block on the rope and F_2' be reaction of the rope on the block. According to Newton's third law of motion,

$$F_2 = F_2'$$

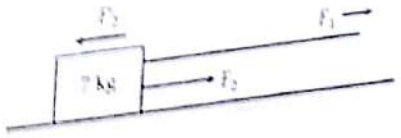

Fig. 5.42

As F_2' is the only force acting on the block which has an acceleration of 2 ms^{-2}, so

$$F_2 = F_2' = 7 \times 2 = 14 \text{ N}.$$

EXAMPLE 39. A wooden block of mass 2 kg rests on a soft horizontal floor. When an iron cylinder of mass 25 kg is placed on top of the block, the floor yields steadily and the block and the cylinder together go down with an acceleration of 0.1 ms^{-2}. What is the action of the block on the floor (a) before and (b) after the floor yields ? Take $g = 10\ ms^{-2}$. [NCERT]

Solution. (*a*) In Fig. 5.43(*a*), the block is at rest. Its free-body diagram [Fig. 5.43(*b*)] shows two forces on the block :

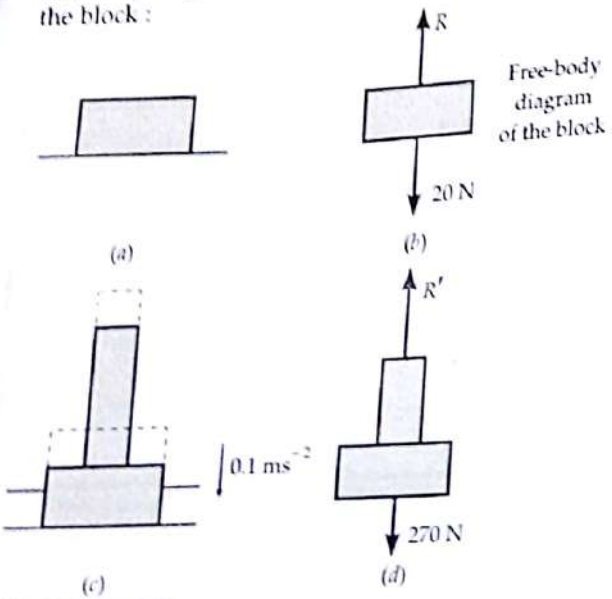

Fig. 5.43

(*i*) Force of gravitational attraction of the earth

= Weight of block

$= mg = 2 \times 10 = 20$ N

(*ii*) Normal reaction R of the floor on the block. By the first law, the net force on the block is zero, so $R = 20$ N

By the third law, the action of the block *i.e.*, the force exerted by the block on the floor.

= **20 N, vertically downwards.**

(*b*) In Fig. 5.43(*c*), the system (block + cylinder) accelerates downwards with 0.1 ms^{-2} due to the yielding of the floor. The free-body diagram [Fig. 5.43(*d*)] shows two forces on system :

(*i*) Force of gravity due to the earth

= Weight of block + system

$= (25 + 2) \times 10 = 270$ N

(*ii*) Normal reaction R' of the floor.

Applying second law to the system, we get

$$270 - R' = (25 + 2) \times 0.1$$

or $$R' = 270 - 2.7 = 267.3 \text{ N}$$

By the third law, action of the system on the floor

= **267.3 N vertically downwards.**

EXAMPLE 40. Two blocks of masses m_1 and m_2 in contact lie on a horizontal smooth surface, as shown in Fig. 5.44. The blocks are pushed by a force F. If the two blocks are always in contact, what is the force at their common interface ?

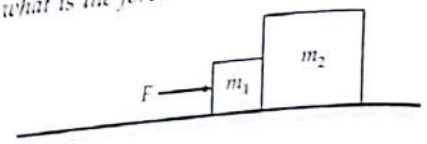

Fig. 5.44

Solution. From Newton's second law, the common acceleration produced in the system will be

$$a = \frac{F}{m_1 + m_2}$$

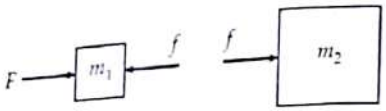

Fig. 5.45

If block of mass m_1 exerts force f on block of mass m_2, then the force of reaction on block of mass m_1 will be equal and opposite to f. These forces are shown in the free body diagrams of Fig. 5.45. As the block of mass m_2 has acceleration a, so

$$f = m_2 a = \frac{m_2 F}{m_1 + m_2}.$$

EXAMPLE 41. As shown in Fig. 5.46, three blocks connected together lie on a horizontal frictionless table and pulled to the right with a force $F = 50$ N. If $m_1 = 5$ kg, $m_2 = 10$ kg and $m_3 = 15$ kg, find the tensions T_1 and T_2.

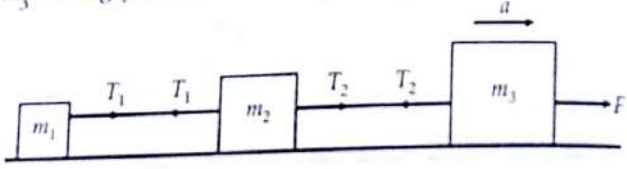

Fig. 5.46

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Solution. All the blocks move with common acceleration a under the force $F = 50$ N.

$$\therefore \quad F = (m_1 + m_2 + m_3)\, a$$

or
$$a = \frac{F}{m_1 + m_2 + m_3} = \frac{50}{5 + 10 + 15} = \frac{5}{3}\ \text{ms}^{-2}$$

To determine T_1. Refer to the free-body diagram for m_1 shown in Fig. 5.47. Clearly, the tension T_1 produces acceleration a in mass m_1.

$$\therefore \quad T_1 = m_1\, a = 5 \times \frac{5}{3} = \frac{25}{3} = 8.33\ \text{N}.$$

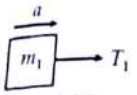

Fig. 5.47

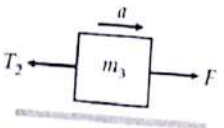

Fig. 5.48

To determine T_2. Refer to the free-body diagram for m_3 shown in Fig. 5.48. Force F acts towards right and tension T_2 acts towards left.

$$\therefore \quad F - T_2 = m_3\, a \ \text{ or } \ 50 - T_2 = 15 \times \frac{5}{3} \ \text{ or } \ T_2 = 25\ \text{N}.$$

EXAMPLE 42. The masses m_1, m_2 and m_3 of the three bodies shown in Fig. 5.49 are 5, 2 and 3 kg respectively. Calculate the values of the tensions T_1, T_2 and T_3 when (i) the whole system is going upward with an acceleration of 2 ms^{-2} and (ii) the whole system is stationary. Given $g = 9.8\ ms^{-2}$.

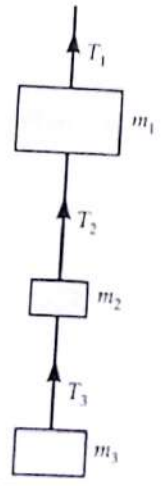

Fig. 5.49

Solution. (*i*) The three bodies together are moving upward with an acceleration of 2 ms^{-2}. The force pulling the system upward is T_1 and the downward force of gravity is $(m_1 + m_2 + m_3)\, g$.

According to Newton's second law,

$$T_1 - (m_1 + m_2 + m_3)\, g = (m_1 + m_2 + m_3)\, a$$

or
$$T_1 = (m_1 + m_2 + m_3)(a + g)$$
$$= (5 + 2 + 3)(2 + 9.8) = 10 \times 11.8$$
$$= \mathbf{118\ N}.$$

Similarly, for the motion of the system $m_2 + m_3$, we can write

$$T_2 = (m_2 + m_3)(a + g)$$
$$= (2 + 3)(2 + 9.8) = 5 \times 11.8$$
$$= \mathbf{59\ N}.$$

For the motion of body of mass m_3, we have

$$T_3 = m_3\,(a + g) = 3\,(2 + 9.8)$$
$$= \mathbf{35.4\ N}.$$

(*ii*) When the whole system is stationary, $a = 0$. From the above equations, we get

$$T_1 = (m_1 + m_2 + m_3)\, g = 10 \times 9.8 = \mathbf{98\ N}.$$

$$T_2 = (m_2 + m_3)\, g = 5 \times 9.8 = \mathbf{49\ N}.$$

$$T_3 = m_3\, g = 3 \times 9.8 = \mathbf{29.4\ N}.$$

EXAMPLE 43. A body m_1 of mass 10 kg is placed on a smooth horizontal table. It is connected to a string which passes over a frictionless pulley and carries at the other end, a body m_2 of mass 5 kg. What acceleration will be produced in the bodies when the nail fixed on the table is removed ? What will be the tension in the string during the motion of the bodies ? What when the bodies stop ? Take $g = 9.8\ N\ kg^{-1}$.

Solution. The situation is shown in Fig. 5.50. When the nail fixed on the table is removed, the system of two bodies moves with an acceleration a in the direction as shown.

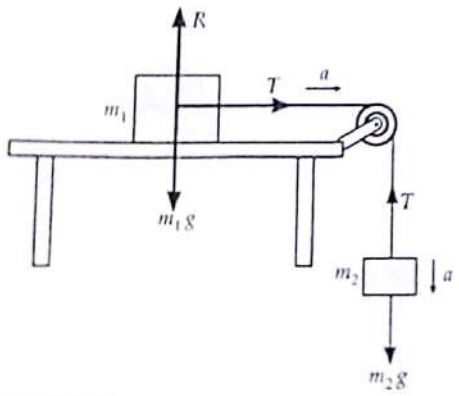

Fig. 5.50

From Newton's second law, we have

$$(m_1 + m_2)\, a = m_2\, g$$

$$a = \frac{m_2\, g}{m_1 + m_2} = \frac{5 \times 9.8}{(10 + 5)} = 3.27\ \text{ms}^{-2}$$

Also, $\quad T = m_1 \times a = 10 \times 3.27 = \mathbf{32.7\ N}$

When the bodies stop, acceleration, $a = 0$. Suppose the tension in the string becomes T'. As the net force on each body is zero, so for body m_2, we can write

$$T' = m_2\, g = 5 \times 9.8 = \mathbf{49\ N}.$$

EXAMPLE 44. A block of mass 100 kg is set into motion on a frictionless horizontal surface with the help of frictionless pulley and a rope system as shown in Fig. 5.51(a). What horizontal force should be applied to produce in the block an acceleration of 10 cms^{-2} ?

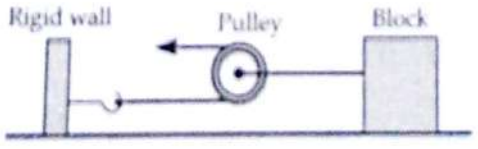

Fig. 5.51 (*a*)

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Solution. As shown in Fig. 5.51(*b*), when force *F* is applied at the end of the string, the tension in the lower

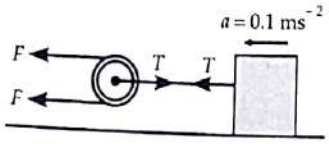

Fig. 5.51 (*b*)

part of the string is also *F*. If *T* is the tension in string connecting the pulley and the block, then from Newton's third law,

$$T = 2F$$

But $T = ma = 100 \times 0.1 = 10\ \text{N}$

$\therefore$ $2F = 10\,\text{N}$ or $F = 5\ \text{N}$.

EXAMPLE 45. In terms of masses m_1, m_2 and g, find the acceleration of both the blocks shown in Fig. 5.52. Neglect all friction and masses of the pulley.

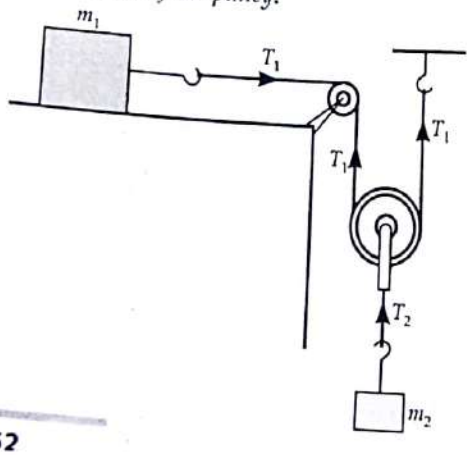

Fig. 5.52

Solution. As the mass m_1 moves towards right through distance x, the mass m_2 moves down through distance $x/2$. Clearly, if the acceleration of m_1 is a, then that of m_2 will be $a/2$.

Applying Newton's second law to the motions of m_1 and m_2, we have

$$T_1 = m_1 a$$

and $$m_2 g - T_2 = m_2 \cdot \frac{a}{2}$$

Also $$T_2 = 2T_1 = 2m_1 a$$

$\therefore$ $$m_2 g - 2m_1 a = m_2 \cdot \frac{a}{2}$$

or $$2m_2 g - 4m_1 a = m_2 a \quad \text{or} \quad 2m_2 g = (4m_1 + m_2)\, a$$

$\therefore$ Acceleration of $m_1 = a = \dfrac{2m_2 g}{4m_1 + m_2}$

Acceleration of $m_2 = \dfrac{a}{2} = \dfrac{m_2 g}{4m_1 + m_2}$.

EXAMPLE 46. Two identical point masses, each of mass m, are connected to one another by a massless string of length l. A constant force F is applied at the mid-point of the string. If l be the instantaneous distance between the two masses, find will be the acceleration of each mass ?

Solution. Fig. 5.53 shows the position of string at any instant after the application of a force F at the mid-point. It also shows the various forces acting on the two masses at any instant. If tension T in the string is resolved into horizontal and vertical components, then

$$F = 2T \sin\theta \qquad \ldots(i)$$

and $$Ma = T\cos\theta \qquad \ldots(ii)$$

where a is the acceleration of each mass.

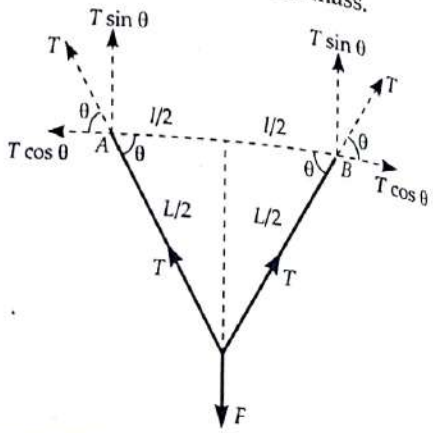

Fig. 5.53

Dividing (*ii*) by (*i*), we get

$$\frac{\cos\theta}{2\sin\theta} = \frac{Ma}{F}$$

or $$\cot\theta = \frac{2Ma}{F}$$

or $$\frac{l/2}{\sqrt{(L/2)^2 - (l/2)^2}} = \frac{2Ma}{F}$$

or $$\frac{2Ma}{F} = \frac{l}{\sqrt{L^2 - l^2}}$$

or $$a = \frac{F}{2M}\left(\frac{l}{\sqrt{L^2 - l^2}}\right).$$

EXAMPLE 47. Two blocks of masses 50 kg and 30 kg connected by a massless string pass over a light frictionless pulley and rest on two smooth planes inclined at angles 30° and 60° respectively with the horizontal. Determine the acceleration of the two blocks and the tension in the string. Take $g = 10\ ms^{-2}$.

Solution. Suppose the mass of 50 kg slides down with an acceleration a. The forces acting on the two

blocks are shown in Fig. 5.54.
two weights perpendicular to
balanced by the normal reacti

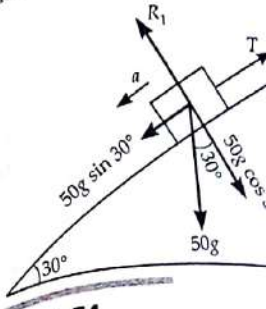

Fig. 5.54

The tension T of each
also the acceleration a of
$\therefore$ $50\,g\sin 30° - T$
and $T - 30\,g\sin 60°$
Adding (*i*) and (*ii*),
$(50\sin 30° - 30\sin 60$
or $a = \dfrac{(50 \times 0.5 - 30}{}$

The –ve sign indica
of sliding down, actu
block slides down a
inclined plane with a
From (*i*), $T = 50$
$= 50$
$= 50$

PROBLEMS

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2. Two masses
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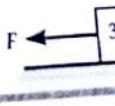

Fig. 5.55

3. Two boc
$m_2 = 150$
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blocks are shown in Fig. 5.54. The components of the two weights perpendicular to the inclined planes are balanced by the normal reactions R_1 and R_2.

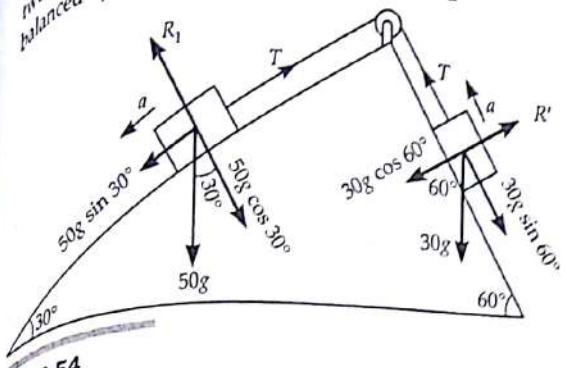

Fig. 5.54

The tension T of each part of the string is same and also the acceleration a of each block is same.

$$\therefore \quad 50\,g \sin 30° - T = 50\,a \qquad ...(i)$$

and $$T - 30\,g \sin 60° = 30\,a \qquad ...(ii)$$

Adding (*i*) and (*ii*), we get

$$(50 \sin 30° - 30 \sin 60°)\,g = (50 + 30)\,a$$

or $$a = \frac{(50 \times 0.5 - 30 \times 0.866) \times 10}{80} = -0.12 \text{ ms}^{-2}$$

The –ve sign indicates that the 50 kg block, instead of sliding down, actually slides up. Hence the 30 kg block slides down and that of 50 kg slides up the inclined plane with $a = 0.12 \text{ ms}^{-2}$.

From (*i*), $T = 50\,g \sin 30° - 50\,a$

$= 50\,(10 \times 0.5 - 0.12)$

$= 50 \times 4.88 = \mathbf{244}$ N.

✕ PROBLEMS FOR PRACTICE

1. A force of 9 N pulls a block of 4 kg through a rope of mass 0.5 kg. The block is resting on a smooth surface. What is the force of reaction exerted by the block on the rope ? (**Ans.** 8 N)

2. Two masses m_1 and m_2 are connected by a massless string as shown in Fig. 5.55. Find the value of tension in the string if a force of 200 N is applied on (*i*) m_1 and (*ii*) m_2. [**Ans.** (*i*) 125 N (*ii*) 75 N]

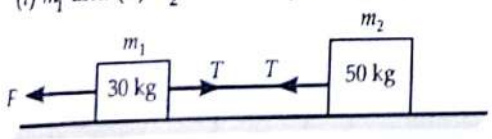

Fig. 5.55

3. Two bodies whose masses are $m_1 = 50$ kg and $m_2 = 150$ kg are tied by a light string and are placed on a frictionless horizontal surface. When m_1 is pulled by a force F, and acceleration of 5 ms^{-2} is produced in both the bodies. Calculate the value of F. What is the tension in the string ? (**Ans.** 1000 N, 750 N)

4. As shown in Fig. 5.56, three masses m, $3m$ and $5m$ connected together lie on a frictionless horizontal surface and pulled to the left by a force F. The

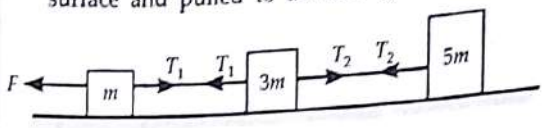

Fig. 5.56

tension T_1 in the first string is 24 N. Find (*i*) acceleration of the system, (*ii*) tension in the second string, and (*iii*) force F.

[**Ans.** (*i*) $a = 3/m$ (*ii*) $T_2 = 15$ N (*iii*) $F = 27$ N]

5. Three identical blocks, each having a mass m are pushed by a force F on a frictionless table as shown in Fig. 5.57. What is the acceleration of the blocks ?

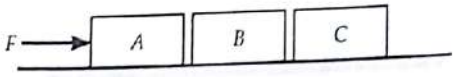

Fig. 5.57

What is the net force on the block A ? What force does A apply on B ? What force does B apply on C ? Show action-reaction pairs on the contact surfaces of the blocks. (**Ans.** $F/3$ m, $F/3$, $2F/3$, $F/3$)

6. Four blocks of the same mass m connected by cords are pulled by a force F on a smooth horizontal

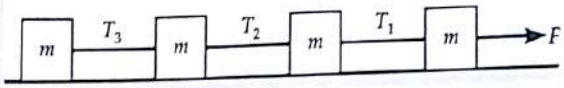

Fig. 5.58

surface, as shown in Fig. 5.58. Determine the tensions T_1, T_2 and T_3 in the cords.

$$\left(\text{Ans. } T_1 = \frac{3}{4}F,\ T_2 = \frac{1}{2}F,\ T_3 = \frac{1}{4}F\right)$$

7. In Fig. 5.59, find the acceleration a of the system and the tensions T_1 and T_2 in the strings. Assume that the table and the pulleys are frictionless and the strings are massless. Take $g = 9.8 \text{ ms}^{-2}$.

(**Ans.** 1.4 ms^{-2}, 22.4 N, 33.6 N)

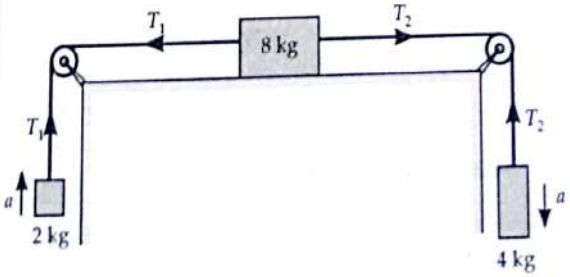

Fig. 5.59

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8. In the Atwood's machine [Fig. 5.60], the system starts from rest. What is the speed and distance moved by each mass at $t = 3$s ?

(Ans. 2.67 ms^{-1}, 4 m)

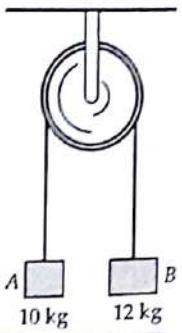

Fig. 5.60

9. Three bodies A, B and C, each of mass m are hanging on a string over a fixed pulley, as shown in Fig. 5.61. What are the tensions in the strings connecting bodies A to B and B to C ? (Ans. $\frac{4}{3}mg, \frac{2}{3}mg$)

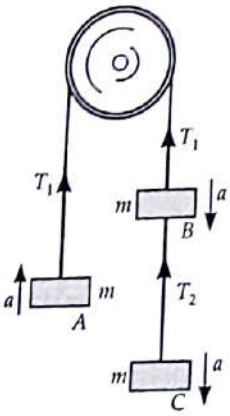

Fig. 5.61

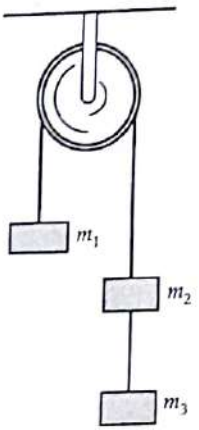

Fig. 5.62

10. In the arrangement shown in Fig. 5.62, show that the tension in the string between masses m_2 and m_3 is $T = \dfrac{2m_1 m_3 g}{m_1 + m_2 + m_3}$.

HINTS

4. (*i*) Tension T_1 of 24 N pulls the masses $(3m + 5m)$ with acceleration a.

$\therefore \quad 24 = (3m + 5m)\, a \quad \text{or} \quad a = 3/m.$

(*ii*) Tension T_2 pulls mass 5 m with acceleration $3/m$.

$\therefore \; T_2 = 5m \times \dfrac{3}{m} = 15$ N.

(*iii*) $\quad F = (m + 3m + 5m)\, a = 9m \times \dfrac{3}{m} = 27$ N.

5. Let a be the common acceleration. Then

$$F = 3m \times a \quad \text{or} \quad a = F/3m$$

Net force on block A will be

$$F_1 = m \times a = m \times \frac{F}{3m} = \frac{F}{3}$$

Force applied by A on B,

$$F_2 = (m_1 + m_2)\, a = 2m \times \frac{F}{3m} = \frac{2F}{3}$$

Force applied by B on C,

$$F_3 = m \times a = m \times \frac{F}{3m} = \frac{F}{3}$$

The action-reaction forces are shown in Fig. 5.63.

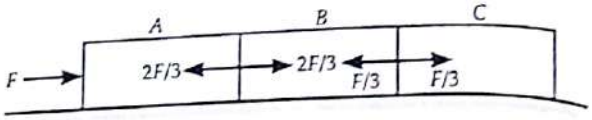

Fig. 5.63

6. Let a be the common acceleration of the system. Then

$$F = (m + m + m + m)\, a = 4ma \quad \text{or} \quad a = \frac{F}{4m}$$

Applying Newton's 2nd law separately for each block in Fig. 5.64, we get

$$F - T_1 = ma,\; T_1 - T_2 = ma,\; T_2 - T_3 = ma,\; T_3 = ma$$

On solving the above equations, we get

$$T_1 = \frac{3}{4}F, \quad T_2 = \frac{1}{2}F, \quad T_3 = \frac{1}{4}F$$

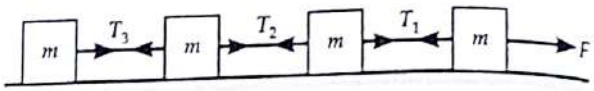

Fig. 5.64

7. Here $4g - T_2 = 4a$, $T_1 - 2g = 2a$ and $T_2 - T_1 = 8a$

On solving, $a = 1.4$ ms^{-2}, $T_1 = 22.4$ N, $T_2 = 33.6$ N.

8. Acceleration,

$$a = \frac{M - m}{M + m}\, g = \frac{12 - 10}{12 + 10} \times 9.8 = 0.89 \text{ ms}^{-2}$$

$$v = u + at = 0 + 0.89 \times 3 = 2.67 \text{ ms}^{-1}$$

$$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 0.89 \times 9 = 4 \text{ m.}$$

9. Here $T_1 - mg = ma$, $2mg - T_1 = 2ma$ and

$$mg - T_2 = ma$$

On solving, $a = \dfrac{2}{3}$, $T_1 = \dfrac{4}{3}mg$ and $T_2 = \dfrac{2}{3}mg$.

10. $\quad m_1 a = T_1 - m_1 g \quad \ldots(i)$

$$(m_2 + m_3)\, a = (m_2 + m_3)\, g - T_1 \quad \ldots(ii)$$

$$m_3 a = m_3 g - T \quad \ldots(iii)$$

Adding (*i*) and (*ii*), get the value of a and put in (*iii*).

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5.22 FRICTION

47. *What is friction ? Explain it with an example.*

Friction. *Whenever a body moves or tends to move over the surface of another body, a force comes into play which acts parallel to the surface of contact and opposes the relative motion. This opposing force is called friction.*

Example. Consider a wooden block placed on a horizontal surface. Give it a gentle push. The block slides through a small distance and then comes to rest. According to Newton's second law, a retarding force must be acting on the block. This opposing force is called friction. As shown in Fig. 5.65, the force of friction always acts tangential to the surfaces in contact and in a direction opposite to the direction of (relative) motion of the body.

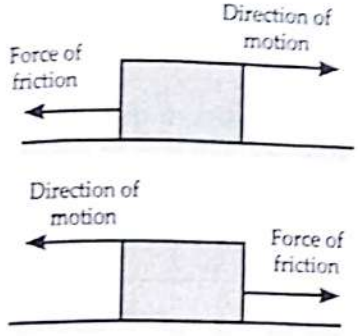

Fig. 5.65 Force of friction always opposes relative motion.

48. *How is friction a component of the contact force ?*

Friction as the component of contact force. When two bodies are placed in contact, attractive forces act between their particles at the surface of contact. As a result, each body exerts a contact force on the other. These mutual contact forces are equal and opposite, obeying Newton's third law. The component of the contact force F normal to the contact surface is called *normal force* or *normal reaction* N. *The component parallel to the contact surface is called friction.*

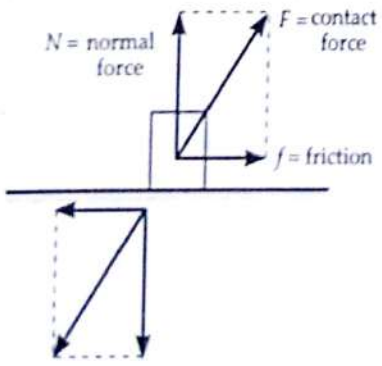

Fig. 5.66 Friction as a component of contact force.

49. *Explain the origin of the force of friction.*

Origin of friction. The force of friction is due to the atomic or molecular forces of attraction between the two surfaces at the points of actual contacts. Fig. 5.67 shows two surfaces in contact, as seen through a powerful microscope. Due to the surface irregularities,

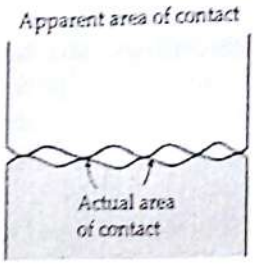

Fig. 5.67 Cause of sliding friction.

the actual area of contact is much smaller than the apparent area of contact. The pressure at the points of contacts is very large. Molecular bonds are formed at these points. When one body is pulled over the other, the bonds break, the material is deformed and new bonds are formed. The local deformation sends vibration waves into the bodies. These vibrations finally damp out and energy appears as heat. Hence a force is needed to start or maintain the motion.

5.23 STATIC, LIMITING AND KINETIC FRICTIONS

50. *With the help of a suitable example, explain the terms static friction, limiting friction and kinetic friction. Show that static friction is a self-adjusting force.*

Static, limiting and kinetic frictions : static friction is a self-adjusting force. Consider a wooden block placed over a horizontal table. Apply a small force F on it [Fig. 5.68(*a*)]. The block does not move. The force of friction f comes into action which balances the applied force F.

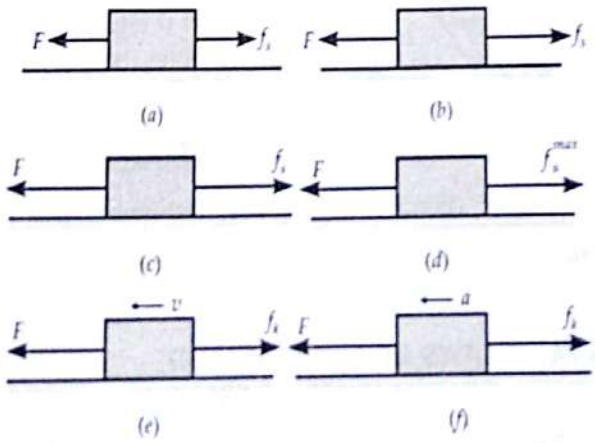

Fig. 5.68 Types of friction.

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The force of friction which comes into play between two bodies before one body actually starts moving over the other is called **static friction** (f_s).

Static friction opposes the impending motion *i.e.,* the motion that would take place (but does not take place) under the applied force, if friction were absent.

As the applied force on the block is increased, the static friction f_s also increases [Fig. 5.68(*b*) and (*c*)] to balance the applied force and the block does not move. Once the applied force is increased beyond a certain limit, the block just begins to move. At this stage static friction is maximum [Fig. 5.68(*d*)].

The maximum force of static friction (f_s^{max}) *which comes into play when a body just starts moving over the surface of another body is called limiting friction.* Clearly $f_s \le f_s^{max}$.

Once the motion has begun, the force of friction decreases. A smaller force is now necessary to maintain uniform motion.

The force of friction which comes into play when a body is in a state of steady motion over the surface of another body is called kinetic or dynamic friction (f_k) [Fig. 5.68(*e*)]. When $F = f_k$, the body moves with a constant velocity v. The kinetic friction opposes the actual relative motion.

When the applied force becomes greater than the kinetic friction, the block accelerates with acceleration equal to $(F - f_k)/m$ [Fig. 5.68(*f*)].

Fig. 5.69 shows the variation of the force of friction f with the applied force F. Obviously, as the applied force F increases, the static friction f_s increases accordingly to balance it. This shows that *static friction is a self adjusting force.* The kinetic friction is always less than the limiting friction f_s^{max} *i.e.,* $f_k < f_s^{max}$.

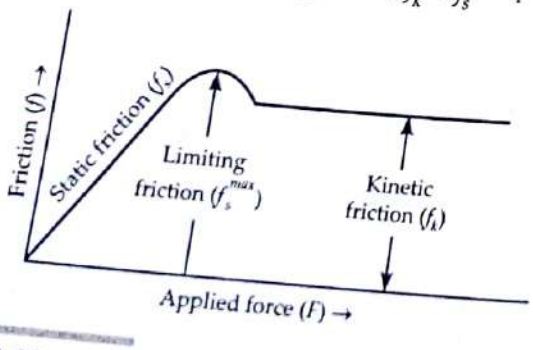

Fig. 5.69 Variation of the force of friction with the applied force.

5.24 LAWS OF FRICTION AND COEFFICIENTS OF FRICTION

51. *State the laws of limiting friction. Define coefficient of limiting friction.*

Laws of limiting friction. From experimental studies, the limiting friction is found to obey the following laws:

(*i*) The limiting friction depends on the nature of the surfaces in contact and their state of polish.

(*ii*) The limiting friction acts tangential to the two surfaces in contact and in a direction opposite to the direction of motion of the body.

(*iii*) The value of limiting friction is independent of the area of the surface in contact so long as the normal reaction remains the same.

(*iv*) The limiting friction (f_s^{max}) is directly proportional to the normal reaction R between the two surfaces.

i.e., $f_s^{max} \propto R$ or $f_s^{max} = \mu_s R$

or $$\mu_s = \frac{f_s^{max}}{R} = \frac{\text{Limiting friction}}{\text{Normal reaction}}$$

The proportionality constant μ_s is called **coefficient of static friction.** *It is defined as the ratio of limiting friction to the normal reaction.*

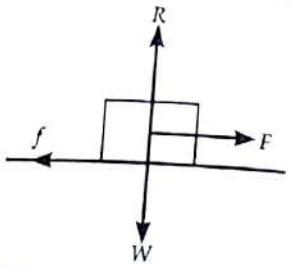

Fig. 5.70

52. *State the laws of kinetic friction. Define coefficient of kinetic friction. Is it less than or greater than the coefficient of static friction ?*

Laws of kinetic friction. These may be stated as follows :

(*i*) The kinetic friction opposes the relative motion and has a constant value, depending on the nature of the two surfaces in contact.

(*ii*) The value of kinetic friction f_k is independent of the area of contact so long as the normal reaction remains the same.

(*iii*) The kinetic friction does not depend on velocity, provided the velocity is neither too large nor too small.

(*iv*) The value of kinetic friction f_k is directly proportional to the normal reaction R between the two surfaces.

i.e., $f_k \propto R$ or $f_k = \mu_k R$

or $$\mu_k = \frac{f_k}{R} = \frac{\text{Kinetic friction}}{\text{Normal reaction}}$$

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The proportionality constant μ_k is called **coefficient of kinetic friction**. *It is defined as the ratio of kinetic friction to the normal section*

As $f_k < f_s^{max}$ or $\mu_k R < \mu_s R$ $\therefore \mu_k < \mu_s$.

Thus the coefficient of kinetic friction is less than the coefficient of static friction.

5.25 ANGLE OF FRICTION

53. *Define angle of friction. Deduce its relation with coefficient of friction.*

Angle of friction. *The angle of friction may be defined as the angle which the resultant of the limiting friction and the normal reaction makes with the normal reaction.*

Relation between angle of friction and coefficient of friction. In Fig. 5.71, W is the weight of the body, R is the normal reaction, f_s^{max} is the limiting friction, P is the applied force and $\vec{OC}$ is the resultant of f_s^{max} and R. The angle θ between the normal reaction R and the resultant $\vec{OC}$ is, by definition, the angle of friction.

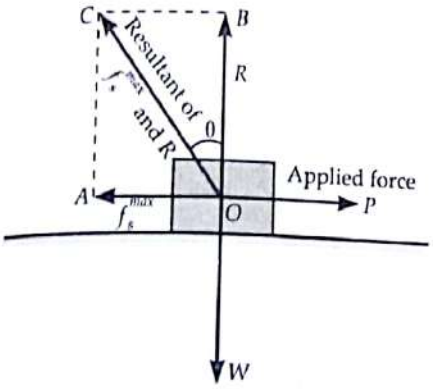

Fig. 5.71 Angle of friction.

$$\therefore \quad \tan\theta = \frac{BC}{OB} = \frac{OA}{OB} = \frac{f_s^{max}}{R}$$

But $\dfrac{f_s^{max}}{R} = \mu_s$ = Coefficient of static friction

$$\therefore \quad \tan\theta = \mu_s$$

Thus *the coefficient of static friction is equal to the tangent of the angle of friction.*

5.26 ANGLE OF REPOSE

54. *Define angle of repose. Deduce its relation with coefficient of static friction ?*

Angle of repose. *It is the minimum angle that an inclined plane makes with the horizontal when a body placed on it just begins to slide down.*

Relation between angle of repose and coefficient of friction. As shown in Fig. 5.72, consider a body of mass m placed on an inclined plane. The angle of inclination ϕ of the inclined plane is so adjusted that a body placed on it just begins to slide down. Thus ϕ is the angle of repose.

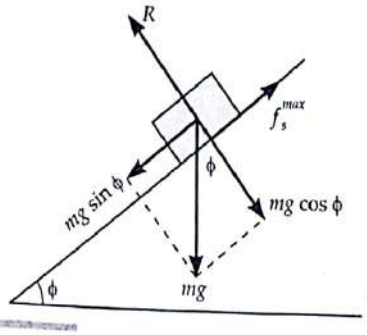

Fig. 5.72 Angle of repose.

Various forces acting on the body are :

(*i*) Weight mg of the body acting vertically downwards.

(*ii*) The limiting friction f_s^{max} in upward direction along the inclined plane. It balances the component $mg \sin\phi$ of the weight mg along the inclined plane. Thus

$$f_s^{max} = mg \sin\phi \qquad ...(1)$$

(*iii*) The normal reaction R perpendicular to the inclined plane. It balances the component $mg \cos\phi$ of the weight mg perpendicular to the inclined plane. Thus

$$R = mg \cos\phi \qquad ...(2)$$

Dividing equation (1) by (2), we get

$$\frac{f_s^{max}}{R} = \frac{mg \sin\phi}{mg \cos\phi}$$

or $$\mu_s = \tan\phi$$

Thus *the coefficient of static friction is equal to the tangent of the angle of repose.*

As $\mu_s = \tan\theta = \tan\phi$ $\therefore \theta = \phi$

Thus *the angle of repose is equal to the angle of friction.*

5.27 SLIDING AND ROLLING FRICTIONS

55. *What are the two types of kinetic friction ? Which one of them is smaller than the other ?*

Types of kinetic friction. Kinetic friction is of two types :

(*i*) ***Sliding friction.*** *The force of friction that comes into play when a body slides over the surface of another body is called sliding friction.* When a wooden block is pulled or pushed over a horizontal surface, sliding friction comes into play.

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(ii) Rolling friction. The force of friction that comes into play when a body rolls over the surface of another body is called rolling friction. When a wheel rolls over a road, rolling friction comes into play.

For the same magnitude of normal reaction, *rolling friction is always much smaller than the sliding friction.*

56. *Why is friction greatly reduced when a body rolls over a surface ? What is the cause of rolling friction ?*

Rolling friction is smaller than sliding friction. When a wheel rolls without slipping over a horizontal plane, the surfaces at contact do not rub each other. The relative velocity of the point of contact of the wheel with respect to the plane is zero, if there is no slipping. There is no sliding or static friction in such an ideal situation. We need to overcome rolling friction only which is much smaller than sliding friction. For this reason, wheel has been considered as one of the greatest inventions.

Cause of rolling friction. Consider a wheel rolling along a road. As the wheel rolls, it exerts a large pressure (weight/area) due to its small area. This causes a slight depression of the road below and a small elevation or

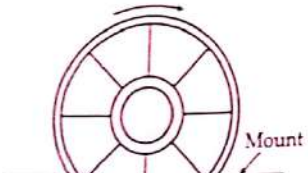

Fig. 5.73 Cause of rolling friction.

mount in front of it. In addition to this, the rolling wheel has to continuously detach itself from the surface on which it rolls. This is opposed by the adhesive force between the two surfaces in contact. On account of both of these factors, a force originates which retards the rolling motion. This retarding force is known as rolling friction.

57. *On what factors does the rolling friction depend ? Write an expression for the coefficient of rolling friction.*

Laws of rolling friction. Experiments show that

(i) Rolling friction is directly proportional to the normal reaction i.e.,

$$f_r \propto R$$

(ii) Rolling friction is inversely proportional to the radius of the rolling cylinder or wheel i.e.,

$$f_r \propto \frac{1}{r}$$

Combining the two laws, we get

$$f_r \propto \frac{R}{r} \quad \text{or} \quad f_r = \mu_r \frac{R}{r}$$

Here μ_r is the *coefficient of rolling friction.* Unlike μ_s or μ_k (which is a pure ratio and has no dimensions), μ_r has the dimensions of length and its SI unit is metre. The above equation is applicable only when there is *rolling without slipping.*

Examples based on Coefficient of Friction and Angle of Friction

FORMULAE USED

1. Coefficient of limiting friction $= \dfrac{\text{Limiting friction}}{\text{Normal reaction}}$

   or $\mu_s = \dfrac{f_s^{max}}{R}$ or $f_s^{max} = \mu_s R$

2. Coefficient of kinetic friction $= \dfrac{\text{Kinetic friction}}{\text{Normal reaction}}$

   or $\mu_k = \dfrac{f_k}{R}$ or $f_k = \mu_k R$

3. For a body placed on horizontal surface, $R = mg$
   $\therefore f_s^{max} = \mu_s . mg$ and $f_k = \mu_k . mg$
4. Statatic friction, $f_s \le f_s^{max}$ or $f_s \le \mu_s R$
5. Kinetic friction, $f_k < f_s^{max}$.
6. If θ is the angle of friction, then $\mu_s = \tan\theta$
7. If ϕ is the angle of repose, then $\mu_s = \tan\phi$
8. Angle of repose = Angle of friction *i.e.*, $\theta = \phi$
9. For a body moving on a rough horizontal surface with retardation a, $\mu = \dfrac{f}{R} = \dfrac{ma}{mg} = \dfrac{a}{g}$
10. $f_r = \mu_r . \dfrac{R}{r}$ and $\mu_r < \mu_k < \mu_s$

    where μ_r is the coefficient of rolling friction, f_r is the rolling friction and r is the radius of the rolling body.

UNITS USED

Force of friction f and normal reaction R are in newton (N), coefficient of friction μ has no units.

EXAMPLE 48. A block of mass 2 kg is placed on the floor. The coefficient of static friction is 0.4. A force of friction of 2.5 N is applied on the block as shown in Fig. 5.74. Calculate the force of friction between the block and the floor. [REC 95]

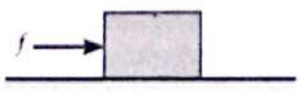

Fig. 5.74

Solution. Here $m = 2$ kg, μ

The value of limiting fric

$f_s^{max} = \mu_s R = \mu_s . mg$ =

As the applied force o
limiting friction (7.84 N), so
In this situation,

Force of friction = Appl

EXAMPLE 49. A block of horizontal table and a tensio
8 N before the block begins to
shown in Fig. 5.75. A force o
constant speed once it has
coefficient of static and kine

Fig. 5.75

Solution. For static
relations for vertical
forces :

$R - mg = 0$ o

and $T - f_s^{max} = 0$ o

$\therefore$ Coefficient stat

$\mu_s = \dfrac{f_s^m}{}$

For kinetic frictio
relations :

$R - mg = 0$

and $T - f_k = 0$

$\therefore$ Coefficient of

$\mu_k =$

EXAMPLE 50. A f block of wood weigh
Calculate the coeffi

Solution. He

Coefficient o

μ

As tan

$\therefore$

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Solution. Here $m = 2$ kg, $\mu_s = 0.4$, $g = 9.8\ ms^{-2}$

The value of limiting friction,

$$f_s^{max} = \mu_s R = \mu_s . mg = 0.4 \times 2 \times 9.8 = 7.84\ N$$

As the applied force of 2.5 N is less than the limiting friction (7.84 N), so the block does not move. In this situation,

Force of friction = Applied force = **2.5 N.**

EXAMPLE 49. A block of weight 20 N is placed on a horizontal table and a tension T, which can be increased to 8 N before the block begins to slide, is applied at the block as shown in Fig. 5.75. A force of 4 N keeps the block moving at constant speed once it has been set in motion. Find the coefficient of static and kinetic friction.

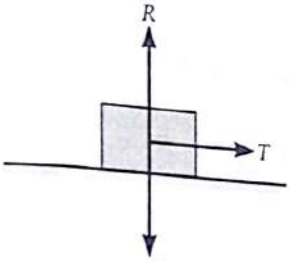

Fig. 5.75

Solution. *For static friction :* We have the following relations for vertical and horizontal component of forces :

$$R - mg = 0 \quad \text{or} \quad R = mg = 20\ N$$

and $\quad T - f_s^{max} = 0 \quad \text{or} \quad f_s^{max} = T = 8\ N$

∴ Coefficient static friction,

$$\mu_s = \frac{f_s^{max}}{R} = \frac{8}{20} = \mathbf{0.40.}$$

For kinetic friction : We can write the following relations :

$$R - mg = 0 \quad \text{or} \quad R = mg = 20\ N$$

and $\quad T - f_k = 0 \quad \text{or} \quad f_k = T = 4\ N$

∴ Coefficient of kinetic friction,

$$\mu_k = \frac{f_k}{R} = \frac{4}{20} = \mathbf{0.20.}$$

EXAMPLE 50. A force of 49 N is just sufficient to pull a block of wood weighing 10 kg on a rough horizontal surface. Calculate the coefficient of friction and angle of friction.

Solution. Here P = applied force = 49 N,

$$m = 10\ kg,\ g = 9.8\ ms^{-2}$$

Coefficient of friction,

$$\mu = \frac{f}{R} = \frac{P}{mg} = \frac{49}{10 \times 9.8} = \mathbf{0.5}$$

As $\quad \tan\theta = \mu = 0.5$

$$\therefore \quad \theta = \tan^{-1}(0.5) = \mathbf{26°34'}.$$

EXAMPLE 51. A cubical block rests on an inclined plane of $\mu = 1/\sqrt{3}$, determine the angle of inclination when the block just slides down the inclined plane.

Solution. When the block just slides down the inclined plane, the angle of inclination is equal to the angle of repose (α).

$$\therefore \quad \tan\alpha = \mu = \frac{1}{\sqrt{3}} \quad \text{or} \quad \alpha = \mathbf{30°}.$$

EXAMPLE 52. A mass of 4 kg rests on a horizontal plane. The plane is gradually inclined until at an angle $\theta = 15°$ with the horizontal, the mass just begins to slide. What is the coefficient of static friction between the block and the surface ? **[NCERT]**

Solution. Here $\theta = 15°$ is the angle of repose.

∴ Coefficient of friction, $\mu = \tan\theta = \tan 15° = \mathbf{0.27.}$

EXAMPLE 53. A body rolled on ice with a velocity of 8 ms^{-1} comes to rest after travelling 4 m. Compute the coefficient of friction. Given $g = 9.8\ ms^{-2}$.

Solution. Here $u = 8\ ms^{-1}$, $v = 0$, $s = 4$ m, $a = ?$

As $\quad v^2 - u^2 = 2as$

$$\therefore \quad 0^2 - 8^2 = 2a \times 4 \quad \text{or} \quad a = -\frac{64}{8} = -8\ ms^{-2}$$

Negative sign indicates retardation.

Now $\quad \mu = \dfrac{f}{R} = \dfrac{ma}{mg} = \dfrac{a}{g} = \dfrac{8}{9.8} = \mathbf{0.8164.}$

EXAMPLE 54. The coefficient of friction between the ground and the wheels of a car moving on a horizontal road is 0.5. If the car starts from rest, what is the minimum distance in which it can acquire a speed of 72 kmh^{-1} ? Take $g = 10\ ms^{-2}$.

Solution. Here $\mu = 0.5$, $u = 0$,

$$v = 72\ kmh^{-1} = 72 \times \frac{5}{18} = 20\ ms^{-1}$$

As $\quad \mu = \dfrac{f}{R} = \dfrac{ma}{mg} = \dfrac{a}{g}$

$$\therefore \quad a = \mu g = 0.5 \times 10 = 5\ ms^{-2}$$

As $\quad v^2 - u^2 = 2as$

$$\therefore \quad (20)^2 - 0^2 = 2 \times 5 \times s \quad \text{or} \quad s = \frac{400}{10} = \mathbf{40\ m.}$$

EXAMPLE 55. Determine the maximum acceleration of the train in which a box lying on its floor will remain stationary, given that the coefficient of static friction between the box and the train's floor is 0.15. Take $g = 10\ ms^{-2}$. **[NCERT]**

Solution. As the acceleration of the box is due to static friction, so

$$ma = f \le \mu_s R = \mu_s mg$$

$$\therefore \quad a \le \mu_s g$$

or $\quad a_{max} = \mu_s g = 0.15 \times 10 = \mathbf{1.5\ ms^{-2}.}$

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EXAMPLE 56. A bullet of mass 0.01 kg is fired horizontally into a 4 kg wooden block at rest on a horizontal surface. The coefficient of the kinetic friction between the block and the surface is 0.25. The bullet gets embedded in the block and the combination moves 20 m before coming to rest. With what speed did the bullet strike the block ?

Solution. Mass of block, $M = 4$ kg

Mass of bullet, $m = 0.01$ kg

After the bullet gets embedded in the block, the force of kinetic friction is

$$f_k = \mu_k R = \mu_k (M + m) g$$

If the kinetic friction produces retardation a in the system, then

$$f_k = (M + m) a$$

$$\therefore (M + m) a = \mu_k (M + m) g$$

or $$a = \mu_k \times g = 0.25 \times 9.8 = 2.45 \text{ ms}^{-2}$$

After the bullet enters the block, suppose the system attains velocity V. Now the system comes to rest after covering a distance, $s = 20$ m, as shown in Fig. 5.76.

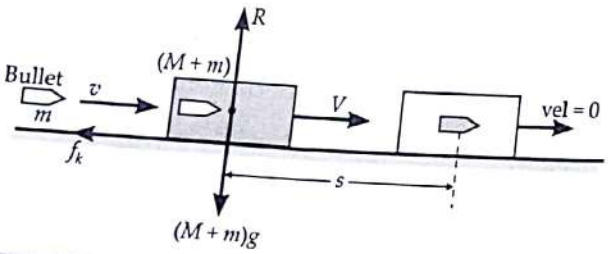

Fig. 5.76

As $v^2 - u^2 = 2as$

$$\therefore \quad 0 - V^2 = 2 \times (-2.45) \times 20$$

or $$V = \sqrt{98} = 9.8995 \text{ ms}^{-1}$$

If v is the velocity with which the bullet struck the block, then applying the law of conservation of momentum, we get

$$mv = (M + m) V$$

or $$v = \frac{M + m}{m} . V = \frac{4 + 0.01}{0.01} \times 9.8995$$

$$= 401 \times 9.8995 = \mathbf{3969.7 \text{ ms}^{-1}}.$$

EXAMPLE 57. What is the acceleration of the block and the trolley system shown in Fig. 5.77, if the coefficient of kinetic friction between the trolley and the surface is 0.04 ? What is the tension in the string ? Take $g = 10$ ms $^{-2}$. Neglect the mass of the string. **[NCERT ; Central Schools 14]**

Solution. As the block and the trolley are connected together by a string of fixed length, both will have same acceleration a.

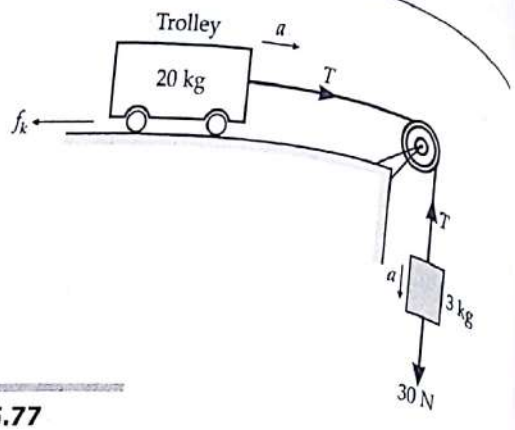

Fig. 5.77

Applying Newton's second law to the motion of the block,

$$30 - T = 3a \quad \ldots(i)$$

Applying second law to the motion of the trolley,

$$T - f_k = 20\, a$$

But $f_k = \mu_k R = \mu_k mg = 0.04 \times 20 \times 10 = 8$ N

$$\therefore \quad T - 8 = 20\, a \quad \ldots(ii)$$

Adding (*i*) and (*ii*), we get

$$22 = 23\, a \quad \text{or} \quad a = \frac{22}{23} = \mathbf{0.96 \text{ ms}^{-2}}$$

From (*i*), $30 - T = 3 \times 0.96$

or $$T = 30 - 2.88 = \mathbf{27.12 \text{ N}}.$$

EXAMPLE 58. A block A of mass 4 kg is placed on another block B of mass 5 kg, and the block B rests on a smooth horizontal table. For sliding the block A on B, a horizontal force of 12 N is required to be applied on it. How much maximum horizontal force can be applied on B so that both A and B move together ? Also find out the acceleration produced by this force. **[Central Schools 14]**

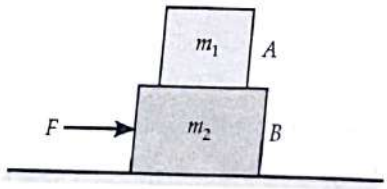

Fig. 5.78

Solution. Here $m_1 = 4$ kg, $m_2 = 5$ kg

Force applied on block $A = 12$ N

This force must atleast be equal to the kinetic friction applied on A by B.

$$\therefore \quad 12 = f_k = \mu_k R = \mu_k m_1 g$$

or $$12 = \mu_k \times 4g$$

or $$\mu_k = \frac{12}{4g} = \frac{3}{g}$$

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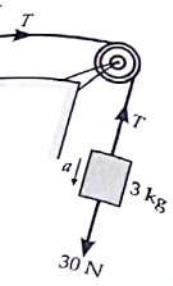

the motion of ...(i)
f the trolley, 8 N ...(ii)
s^{-2}
other ooth ntal ich A n l

The block B is on a smooth surface. Hence to move A and B together, the (maximum) force F that can be applied on B is equal to the frictional forces applied on A by B and applied on B by A.

$$F = \mu_k m_1 g + \mu_k m_2 g = \mu_k (m_1 + m_2) g$$
$$= \frac{3}{8}(4 + 5) g = \mathbf{27\ N}$$

As this force moves both the blocks together on a smooth table, so the acceleration produced is

$$a = \frac{F}{m_1 + m_2} = \frac{27}{4 + 5} = \mathbf{3\ ms^{-2}}.$$

EXAMPLE 59. An engine of 100 H.P. draws a train of mass 200 metric ton with a velocity of 36 kmh^{-1}. Find the coefficient of friction.

Solution. Power of engine,

$$P = 100\ H.P. = 100 \times 746 = 74600\ W$$

Velocity, $v = 36\ \text{kmh}^{-1} = 10\ \text{ms}^{-1}$

If the frictional force overcome by the engine is F, then

$$P = F \times v \quad \text{or} \quad F = \frac{P}{v} = \frac{74600}{10} = 7460\ N$$

Normal reaction,

$$R = mg = 200 \times 1000 \times 9.8\ N$$

Coefficient of friction,

$$\mu = \frac{F}{R} = \frac{7460}{200 \times 1000 \times 9.8} = \mathbf{0.0038.}$$

✕ PROBLEMS FOR PRACTICE

1. A block of mass 1 kg lies on a horizontal surface in a truck. The coefficient of static friction between the block and the surface is 0.6. If the acceleration of the truck is 5 ms^{-2}, calculate the frictional force acting on the block. *[IIT 84]* **(Ans.** 5 N)
2. A body weighing 20 kg just slides down a rough inclined plane that rises 5 m in every 13 m. What is the coefficient of friction ? **(Ans.** 0.4167)
3. A scooter weighs 120 kg f. Brakes are applied so that wheels stop rolling and start skidding. Find the force of friction if the coefficient of friction is 0.4. **(Ans.** 48 kg f)
4. An automobile is moving on a horizontal road with a speed v. If the coefficient of friction between the tyres and road is μ, show that the shortest distance in which the automobile can be stopped is $v^2 / 2\mu g$.
5. Find the distance travelled by a body before coming to rest, if it is moving with a speed of 10 ms^{-1} and the coefficient of friction between the ground the body is 0.4. **(Ans.** 12.75 m)
6. A motor car running at the rate of 7 ms^{-1} can be stopped by applying brakes in 10 m. Show that total resistance to the motion, when brakes are on, is one fourth of the weight of the car.
7. Find the power of an engine which can maintain a speed of 50 ms^{-1} for a train of mass 3×10^5 kg on a rough line. The coefficient of friction is 0.05. Take $g = 10\ \text{ms}^{-2}$. **(Ans.** 7500 kW)
8. A train weighing 1000 quintals is running on a level road with a uniform speed of 72 km h^{-1}. If the frictional resistance amounts to 50 g wt per quintal, find power in watt. Take $g = 9.8\ \text{ms}^{-2}$. **(Ans.** 9800 W)
9. An automobile of mass m starts from rest and accelerates at a maximum rate possible without slipping on a road with $\mu_s = 0.5$. If only the rear wheels are driven and half the weight of the automobile is supported on these wheels, how much time is required to reach a speed of 100 kmh^{-1}. **(Ans.** 11.3 s)
10. A suitcase is gently dropped on a conveyor belt moving at 3 ms^{-1}. If the coefficient of friction between the belt and the suitcase is 0.5, how far will the suitcase move on the belt before coming to rest ? **(Ans.** 0.92 m)
11. A truck moving at 72 kmh^{-1} carries a steel girder which rests on its wooden floor. What is the minimum time in which the truck can come to stop without the girder moving forward ? Coefficient of static friction between steel and wood is 0.5. **(Ans.** 4.08 s)
12. A bullet of mass 10 g is fired horizontally into a 5 kg wooden block, at rest on a horizontal surface. The coefficient of kinetic friction between the block and the surface is 0.1. Calculate speed of the bullet striking the block, if the combination moves 20 m before coming to rest. **(Ans.** 3136.26 ms^{-1})
13. In Fig. 5.79, the masses of A and B are 10 kg and 5 kg respectively. Calculate the minimum mass of C which may stop A from slipping. Coefficient of static friction between A and the table is 0.2. **(Ans.** 15 kg)

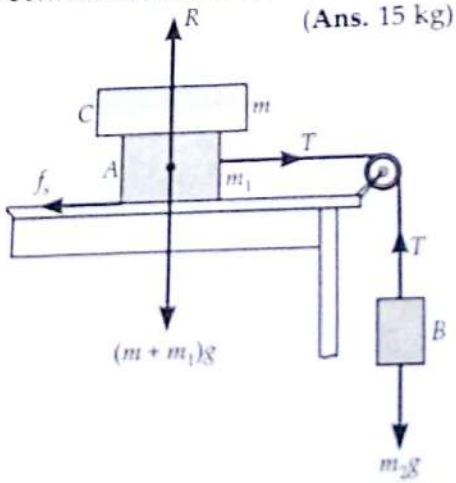

Fig. 5.79

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HINTS

1. Limiting friction, $f = \mu mg = 0.6 \times 1 \times 9.8 = 5.88$ N
   Applied force, $F = ma = 1 \times 5 = 5$ N
   As $F < f$, so force of friction = **5 N.**

2. Here $\sin\theta = \frac{5}{13}$ $\therefore \cos\theta = \sqrt{1-\left(\frac{5}{13}\right)^2} = \frac{12}{13}$
   $$\mu = \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{5}{12} = 0.4167.$$

3. Here R = Weight of scooter = 120 kg f, $\mu = 0.4$
   $\therefore$ $f = \mu R = 0.4 \times 120 = 48$ **kg f.**

4. Here $u = v$, $v = 0$, $a = -\mu g$, $s = ?$
   As $v^2 - u^2 = 2as$ $\therefore$ $0 - v^2 = 2(-\mu g)s$
   or $s = \frac{v^2}{2g}$.

5. Here $a = \mu g = 0.4 \times 9.8 = 3.92$ ms^{-2}
   Now $u = 10$ ms^{-1}, $v = 0$, $s = ?$, $a = -3.92$ ms^{-2}
   $\therefore$ $0^2 - 10^2 = 2 \times (-3.92) \times s$ or $s = 12.75$ **m.**

6. Here $u = 7$ ms^{-1}, $v = 0$, $s = 10$ m
   As $v^2 - u^2 = 2as$ $\therefore$ $0 - 7^2 = 2a \times 10$
   or $a = -2.45$ ms^{-2} $= -\frac{9.8}{4}$ ms^{-2} $= -\frac{g}{4}$
   Total resistance to motion
   $= -ma = \frac{1}{4} \times mg = \frac{1}{4} \times$ weight of the car.

7. Here $v = 50$ ms^{-1}, $\mu = 0.05$, $m = 3 \times 10^5$ kg
   Friction,
   $f = \mu R = \mu mg = 0.05 \times 3 \times 10^5 \times 10 = 1.5 \times 10^5$ N
   Power,
   $P = f \times v = 1.5 \times 10^5 \times 50 = 75 \times 10^5$ W = **7500 kW.**

8. Here $m = 1000$ quintals, $v = 72$ kmh^{-1} $= 20$ ms^{-1}
   Total frictional resistance,
   $f = 50 \times 1000$ g wt $= 50$ kg wt $= 50 \times 9.8$ N
   Power, $P = f \times v = 50 \times 9.8 \times 20 = 9800$ **W.**

9. Normal load on rear wheel
   $= \frac{1}{2} Mg = R$ (Normal reaction)
   Maximum sustainable frictional force
   $f_s^{max} = \mu_s R = \frac{1}{2} \mu_s Mg$
   Acceleration of the automobile
   $= \frac{F}{M} = \frac{f_s^{max}}{M} = \frac{1}{2} \mu_s g$
   As $v = u + at = 0 + at$
   $\therefore$ $t = \frac{v}{a} = \frac{2v}{\mu_s g} = \frac{2 \times 100 \times 5}{0.5 \times 9.8 \times 18} =$ **11.3 s.**

11. Here $u = 3$ ms^{-1}, $v = 0$, $a = -\mu g = -0.5 \times 9.8$ ms^{-2}
    As $v^2 - u^2 = 2as$ $\therefore$ $0 - 3^2 = -2 \times 0.5 \times 9.8 \times s$
    or $s = \frac{9}{9.8} = 0.92$ **m.**

12. Here $u = 72$ kmh^{-1} $= 20$ ms^{-1}, $\mu_s = 0.5$
    $f_s^{max} = \mu_s R = \mu_s mg = 0.5\, mg$
    Retardation $= \frac{f_s^{max}}{m} = \frac{0.5\, mg}{m} = 0.05\, g$
    As $v = u + at$ $\therefore$ $0 = 20 - 0.5\, gt$
    or $t = \frac{20}{0.5\, g} = \frac{20}{0.5 \times 9.8} = 4.08$ **s.**

13. Net force on each body is zero.
    For A : $R = (m + m_1) g$
    and $T = f_s = \mu R = \mu (m + m_1) g$
    For B : $T = m_2 g$ or $\mu (m + m_1) g = m_2 g$
    $\therefore$ $m = \frac{m_2 - \mu m_1}{\mu} = \frac{m_2}{\mu} - m_1$
    $= \frac{5}{0.2} - 10 = 25 - 10 =$ **15 kg.**

5.28 WORK DONE AGAINST FRICTION

58. *Find an expression for the work done in sliding a body over a rough horizontal surface.*

Work done in sliding a body over a horizontal surface. Consider a body of weight mg resting on a rough horizontal surface, as shown in Fig. 5.80. The weight mg is balanced by the normal reaction R of the horizontal surface.

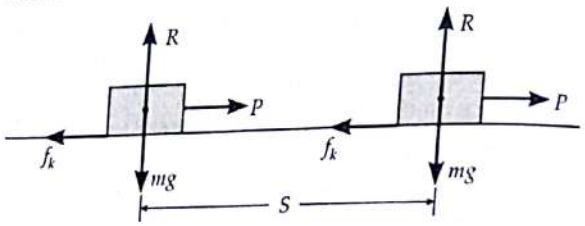

Fig. 5.80 Work done in sliding a body over horizontal surface.

Suppose a force P is applied horizontally so that the body just begins to slide. Let f_k be the kinetic friction.

Work done against friction in moving the body through distance S will be

$$W = f_k \times S$$

But $f_k = \mu_k R = \mu_k . mg$

$\therefore$ $W = \mu_k . mg\, S.$

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59. *Find an expression for the work done against friction when a body is made to slide up an inclined plane.*

Work done in moving a body up an inclined plane. Suppose a body of weight mg is placed on an inclined plane, as shown Fig. 5.81. Let θ be the angle of inclination. A force P is applied on the body so that it just begins to slide up the inclined plane.

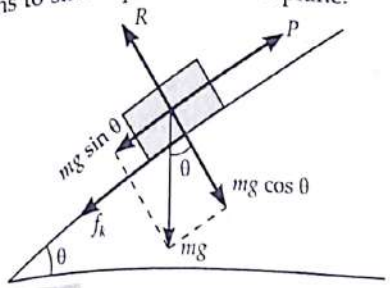

Fig. 5.81 Work done in moving a body up an inclined plane

The weight mg of the body has two components :

(*i*) $mg \cos \theta$ perpendicular to the inclined plane. It balances the normal reaction R. Thus

$$R = mg \cos \theta$$

(*ii*) $mg \sin \theta$ down the inclined plane.

If f_k is the kinetic friction, then the force needed to be applied upwards to just move the body up the inclined plane must be

$$P = mg \sin \theta + f_k$$

But $f_k = \mu_k R = \mu_k mg \cos \theta$

$\therefore$ $P = mg \sin \theta + \mu_k mg \cos \theta$

$= mg (\sin \theta + \mu_k \cos \theta)$

Work done in pulling the body through distance S up the inclined plane is

$$W = P \times S = mg (\sin \theta + \mu_k \cos \theta) S.$$

60. *Find an expression for the work done against friction when a body is made to slide down an inclined plane ?*

Work done in moving a body down an inclined plane. Suppose a body of weight mg is placed on an inclined plane, as shown in Fig. 5.82. Suppose the angle of inclination θ be less than the angle of repose. A force P is applied to just slide the body down the inclined plane.

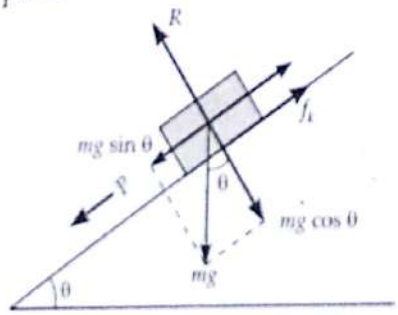

Fig. 5.82 Work done in moving a body down an inclined plane.

The weight mg of the body has two components :

(*i*) $mg \cos \theta$ perpendicular to the inclined plane. It balances the normal reaction R. Thus

$$R = mg \cos \theta$$

(*ii*) $mg \sin \theta$ down the inclined plane.

The force of friction f_k acts up the inclined plane. The applied force needed to just move the body down the inclined plane must be

$$P = f_k - mg \sin \theta$$

But $f_k = \mu_k R = \mu_k mg \cos \theta$

$\therefore$ $P = \mu_k mg \cos \theta - mg \sin \theta$

$= mg (\mu_k \cos \theta - \sin \theta)$

The work done in sliding the body through distance S down the inclined plane is

$$W = P \times S = mg (\mu_k \cos \theta - \sin \theta) S.$$

5.29 ACCELERATION OF A BODY SLIDING DOWN A ROUGH INCLINED PLANE

61. *Derive an expression for the acceleration of a body sliding down a rough inclined plane.*

Acceleration of a body sliding down an inclined plane. As shown in Fig. 5.83, consider a body of weight mg placed on an inclined plane. Suppose the angle of inclination θ be greater than the angle of repose. Let a be the acceleration with which the body slides down the inclined plane.

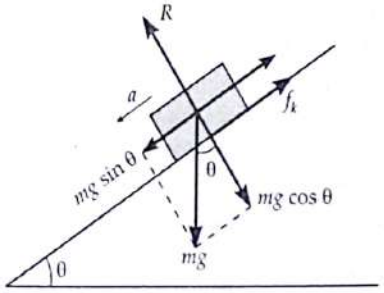

Fig. 5.83 Acceleration of a body down an inclined plane.

The weight mg has two rectangular components :

(*i*) $mg \cos \theta$ perpendicular to the inclined plane. It balances the normal reaction R. Thus

$$R = mg \cos \theta$$

(*ii*) $mg \sin \theta$ down the inclined plane.

If f_k is the kinetic friction, then the net force acting down the plane is

$$F = mg \sin \theta - f_k$$

But $f_k = \mu_k R = \mu_k mg \cos \theta$

$\therefore$ $ma = mg \sin \theta - \mu_k mg \cos \theta$

Hence $a = g (\sin \theta - \mu_k \cos \theta)$.

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Examples based on Motion along Rough Inclined Plane

Formulae Used

1. For a body placed on an inclined plane of inclination θ,
   Normal reaction, $R = mg \cos\theta$
   Friction, $f = \mu R = \mu\, mg \cos\theta$
2. When a body moves down an inclined plane without any acceleration, net downward force needed is
   $F = mg \sin\theta - f = mg (\sin\theta - \mu \cos\theta)$
   Work done, $W = Fs = mg (\sin\theta - \mu \cos\theta)\, s$
3. When a body moves up an inclined plane without acceleration, net upward force needed is
   $F = mg \sin\theta + f = mg (\sin\theta + \mu \cos\theta)$
   $W = mg (\sin\theta + \mu \cos\theta)\, s$
4. When a body moves up an inclined plane, with acceleration a, net upward force needed is
   $F = ma + mg \sin\theta + f$
   $= m(a + g \sin\theta + \mu g \cos\theta)$
   $W = m(a + g \sin\theta + \mu\, g \cos\theta)\, s$

Units Used

Forces F, f, R and mg are in newton, angle θ in degree, distance s in metre, work done W in joule, coefficient of friction μ has no units.

Example 60. A box of mass 4 kg is placed on a wooden plank of length 1.5 m which is lying on the ground. The plank is lifted from one end along its length so that it becomes inclined. It is noted that when the vertical height of the top end of the plank from the ground becomes 0.5 m, the box begins to slide. Find the coefficient of friction between the box and the plank.

Solution. Here $m = 4$ kg, $AB = 1.5$ m, $BC = 0.5$ m

Various forces acting on the box are as shown in Fig. 5.84. In equilibrium,

$$R = mg \cos\theta$$

and $$f = mg \sin\theta$$

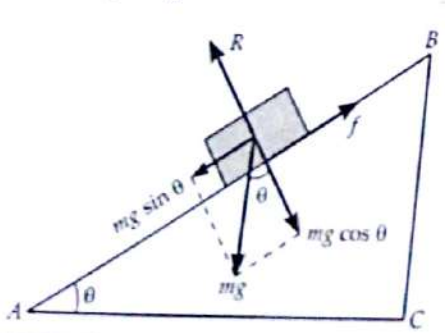


Fig. 5.84

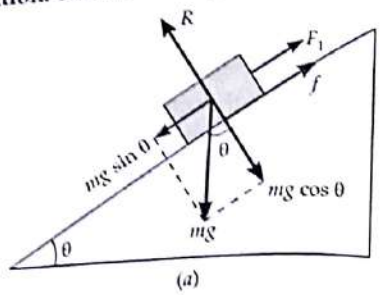
$$\therefore \quad \mu = \frac{f}{R} = \frac{mg \sin\theta}{mg \cos\theta} = \tan\theta = \frac{BC}{AC}$$

$$= \frac{BC}{\sqrt{AB^2 - BC^2}} = \frac{0.5}{\sqrt{(1.5)^2 - (0.5)^2}}$$

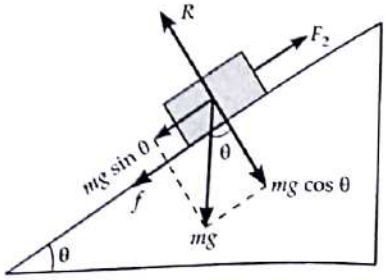
$$= \frac{0.5}{\sqrt{2}} = 0.7 \times 0.5 = 0.35.$$

Example 61. A mass of 200 kg is resting on a rough inclined plane of 30°. If the coefficient of friction is $1/\sqrt{3}$, find the least and the greatest forces acting parallel to the plane to keep the mass in equilibrium. [Central Schools 08]

Solution. Here $m = 200$ kg, $\theta = 30°$, $\mu = 1/\sqrt{3}$



(a)



(b)

Fig. 5.85

From the above figures, force of friction is

$$f = \mu R = \mu\, mg \cos\theta = \frac{1}{\sqrt{3}} \times 200 \times 9.8 \cos 30°$$

$$= \frac{1}{\sqrt{3}} \times 200 \times 9.8 \times \frac{\sqrt{3}}{2} = 980 \text{ N}$$

Component of the weight acting down the inclined plane

$$= mg \sin\theta = 200 \times 9.8 \sin 30°$$

$$= 200 \times 9.8 \times 0.5 = 980 \text{ N}$$

(*i*) From Fig. 5.85(*a*), the least force required to prevent the mass from sliding down (when friction f acts upwards) is

$$F_1 = mg \sin\theta - f = 980 - 980 = 0.$$

(*ii*) From Fig. 5.85(*b*), the greatest force required to prevent the mass from sliding up (when friction f acts downwards) is

$$F_2 = mg \sin\theta + f = 980 + 980 = \mathbf{1960\ N}.$$

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EXAMPLE 62. A box of mass 4 kg rests upon an inclined plane. The inclination of the plane to the horizontal is gradually increased. It is found that when the slope of the plane is 1 in 3, the box starts sliding down the plane. Given $g = 9.8\ ms^{-2}$.

(i) Find the coefficient of friction between the box and the plane.

(ii) What force applied to the box parallel to the plane will just make it move up the plane ?

Solution. Here $m = 4$ kg, $\sin\theta = \frac{1}{3}$, $g = 9.8\ ms^{-2}$

(i) Various forces acting on the box are shown in Fig. 5.86. When the box just begins to slide, the forces are in equilibrium.

$\therefore \quad f = mg\sin\theta$ and $R = mg\cos\theta$

$$\mu = \frac{f}{R} = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta$$

$$= \frac{1}{\sqrt{3^2 - 1^2}} = \frac{1}{\sqrt{8}} = \frac{\sqrt{2}}{4} = 0.35.$$

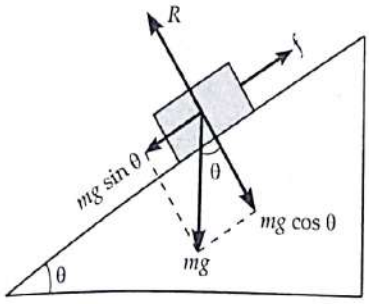

Fig. 5.86

(ii) When the block moves up the inclined plane, friction f acts down the plane. So minimum force needed to just move the box up the inclined plane is

$$F = mg\sin\theta + f = mg\sin\theta + \mu R$$

$$= mg(\sin\theta + \mu\cos\theta) \qquad [\because R = mg\cos\theta]$$

$$= 4 \times 9.8\left(\frac{1}{3} + \frac{1}{\sqrt{8}} \cdot \frac{\sqrt{8}}{3}\right) = 4 \times 9.8 \times \frac{2}{3} = \mathbf{26.13\ N.}$$

EXAMPLE 63. A block of metal of mass 50 g when placed over an inclined plane at an angle of 15° slides down without acceleration. If the inclination is increased by 15°, what would be the acceleration of the block ?

Solution. Here $m = 50\ g = 0.05$ kg

Angle of repose,

$$\alpha = 15°$$

$\therefore \quad \mu = \tan\alpha = \tan 15° = 0.2679$

New angle of inclination $= 15 + 15 = 30°$

Let a be the downward acceleration produced in the block.

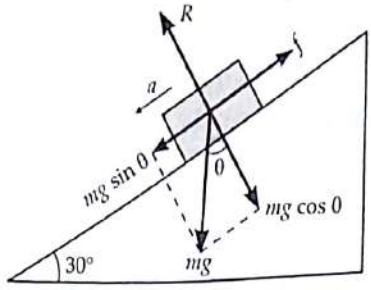

Fig. 5.87

Net downward force on the block is

$$F = mg\sin\theta - f$$

$$ma = mg\sin\theta - \mu\, mg\cos\theta \qquad [\because f = \mu R = \mu\, mg\cos\theta\ ]$$

$\therefore \quad a = g(\sin\theta - \mu\cos\theta)$

$$= 9.8(\sin 30° - 0.2679\cos 30°)$$

$$= 9.8(0.5 - 0.2679 \times 0.866)$$

$$= 9.8 \times 0.2680 = \mathbf{2.6\ ms^{-2}.}$$

EXAMPLE 64. When an automobile moving with a speed of 36 kmh^{-1} reaches an upward inclined road of angle 30°, its engine is switched off. If the coefficient of friction involved is 0.1, how much distance will the automobile move before coming to rest ? Take $g = 10\ ms^{-2}$.

Solution. As shown in Fig. 5.88, when a body moves up an inclined plane, force of friction f acts down the plane. So the force against which work is needed to be done is

$$F = mg\sin\theta + f$$

$$= mg\sin\theta + \mu\, mg\cos\theta = mg(\sin\theta + \mu\cos\theta)$$

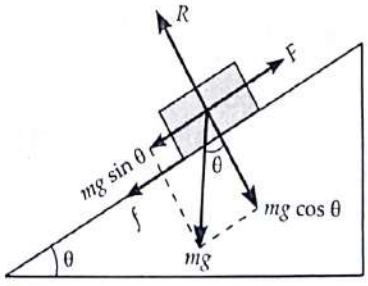

Fig. 5.88

If m is the mass of the automobile, retardation produced in it will be

$$a = \frac{F}{m} = g(\sin\theta + \mu\cos\theta)$$

$$= 10(\sin 30° + 0.1\cos 30°)$$

$$= 10(0.5 + 0.1 \times 0.866) = 5.866\ ms^{-2}$$

Now $\quad u = 36\ kmh^{-1} = 10\ ms^{-1}$,

$v = 0$, $a = -5.866\ ms^{-2}$, $s = ?$

As $\quad v^2 - u^2 = 2as$

$\therefore \quad 0^2 - 10^2 = 2 \times (-5.866)\, s \quad$ or $\quad s = \mathbf{8.52\ m.}$

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EXAMPLE 65. Find the force required to move a train of 2000 quintals up an incline of 1 in 50, with an acceleration of 2 ms^{-2}, the force of friction being 0.5 newton per quintal.

Solution. Here $m = 2000$ quintals $= 2 \times 10^5$ kg,

$$\sin\theta = \frac{1}{50},\ a = 2 \text{ ms}^{-2}.$$

Force of friction = 0.5 newton per quintal

$\therefore$ Total force of friction $= 0.5 \times 2000 = 1{,}000$ N

Force required against gravity in moving the train up the inclined plane

$$= mg \sin\theta = 2 \times 10^5 \times 9.8 \times \frac{1}{50} = 39{,}200 \text{ N}$$

Force required to produce an acceleration of 2 ms^{-2}

$$= ma = 2 \times 10^5 \times 2 = 400{,}000 \text{ N}$$

$\therefore$ Total force required

$$= 1{,}000 + 39{,}200 + 400{,}000 = \mathbf{440{,}200\ N.}$$

EXAMPLE 66. An engine of mass 6.5 metric ton is going upon incline of 5 in 13 at the rate of 9 kmh^{-1}. Calculate the power of the engine if $\mu = \frac{1}{12}$ and $g = 9.8$ ms^{-2}.

Solution. Here

$m = 6.5$ metric ton $= 6500$ kg, $g = 9.8$ ms^{-2},

$$v = 9 \text{ kmh}^{-1} = 9 \times \frac{5}{18} = 2.5 \text{ ms}^{-1},\ \sin\theta = \frac{5}{13}$$

$$\cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - \frac{25}{169}} = \frac{12}{13}$$

Total force required against which the engine needs to work

$$F = mg \sin\theta + f = mg \sin\theta + \mu\, mg \cos\theta$$

$$= mg(\sin\theta + \mu \cos\theta)$$

$$= 6500 \times 9.8\left[\frac{5}{13} + \frac{1}{12} \times \frac{12}{13}\right] = 29400 \text{ N}$$

Power of the engine

$$= Fv = 29400 \times 2.5 = 73500 \text{ W} = \mathbf{73.5\ kW.}$$

EXAMPLE 67. A body of mass m is released from the top of a rough inclined plane as shown in Fig. 5.89. If the force of friction be f, then prove that the body will reach the bottom with a velocity given by $v = \sqrt{\frac{2}{m}(mgh - fl)}$

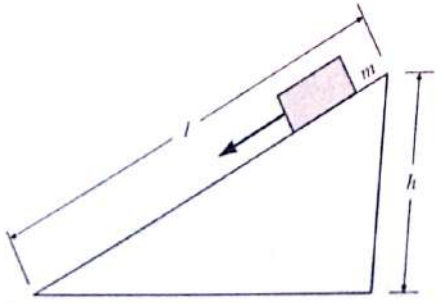

Fig. 5.89

Solution. P.E. lost by the body in reaching the bottom $= mgh$

K.E. gained by the body in reaching the bottom

$$= \frac{1}{2} mv^2$$

$\therefore$ Net loss in energy $= mgh - \frac{1}{2} mv^2$

Work done against friction $= f\,l$

Hence $mgh - \frac{1}{2} mv^2 = f\,l$

$$\frac{1}{2} mv^2 = mgh - f\,l$$

or

$$v = \sqrt{\frac{2}{m}(mgh - f\,l)}.$$

or

EXAMPLE 68. Two blocks of mass 2 kg and 5 kg are connected by an ideal string passing over a pulley. The block of mass 2 kg is free to slide on a surface inclined at an angle of 30° with the horizontal whereas 5 kg block hangs freely. Find the acceleration of the system and the tension in the string. Given $\mu = 0.30$.

Solution. The situation is shown in Fig. 5.90.

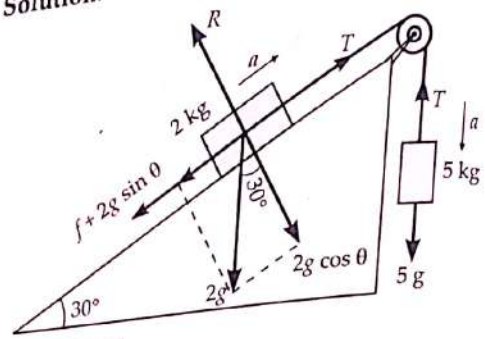

Fig. 5.90

Let a be the acceleration of the system in the direction as shown and T the tension in the string.

Equations of motion for 5 kg and 2 kg blocks can respectively be written as

$$5g - T = 5a \quad \ldots(1)$$

and $$T - 2g \sin\theta - f = 2a \quad \ldots(2)$$

where f is the limiting friction and is given by

$$f = \mu R = \mu\, mg \cos\theta$$

$$= 0.3 \times 2g \cos 30°$$

Adding (1) and (2), we get

$$5g - 2g \sin\theta - f = 7a$$

or $$5g - 2g \sin 30° - 0.6\, g \cos 30° = 7a$$

or $$g(5 - 2 \times 0.5 - 0.6 \times 0.866) = 7a$$

or $$a = \frac{9.8 \times 3.4804}{7} = 4.87 \text{ ms}^{-2}$$

From (1),

$$T = 5g - 5a = 5(9.8 - 4.87) = 5 \times 4.93 = \mathbf{24.65\ N.}$$

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EXAMPLE 69. A truck tows a trailer of mass 1200 kg at a speed of 10 ms^{-1} on a level road. The tension in the coupling is 1000 N. (i) What is the power expended on the trailer ? (ii) Find the tension in the coupling when the truck ascends a road having an inclination of 1 in 6. Assume that the frictional resistance of the incline is the same as that on the level road.

Solution. On the level road, force applied by the truck is equal to the friction overcome.

$$\therefore \quad f = 1000 \text{ N}$$

Speed of truck, $v = 10 \text{ ms}^{-1}$

Power expanded on the trailor,

$$P = fv = 1000 \times 10 = \mathbf{10^4 \text{ W}}.$$

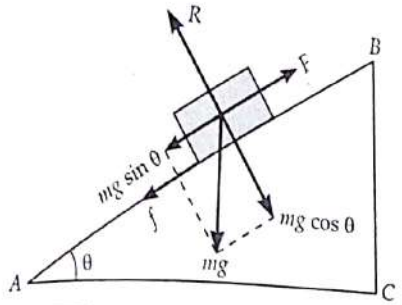

Fig. 5.91

As shown in Fig. 5.91, when the truck ascends a road of inclination 1 in 6 (*i.e.* $\sin\theta = 1/6$), it overcomes not only the force of friction f but also the component $mg \sin\theta$ of the weight of the trailor. So the tension in the coupling is

$$F = f + mg \sin\theta = 1000 + 1200 \times 9.8 \times \frac{1}{6}$$

$$= 1000 + 1960 = \mathbf{2960 \text{ N}}.$$

✕ PROBLEMS FOR PRACTICE

1. A block of mass 2 kg rests on a plane inclined at an angle of 30° with the horizontal. The coefficient of friction between the block and the surface is 0.7. What will be the frictional force acting on the block ? **[IIT 80]**

   (Ans. 11.9 N)

2. A block of mass 10 kg is sliding on a surface inclined at an angle of 30° with the horizontal. If the coefficient of friction between the block and the surface is 0.5, find the acceleration produced in the block. **(Ans. 0.657 ms^{-2})**

3. Find the force required to move a train of mass 10^5 kg up an incline 1 in 50 with an acceleration of 2 ms^{-2}. Coefficient of friction between the train and the rails is 0.005. Take $g = 10 \text{ ms}^{-2}$.

   (Ans. 2.25×10^5 N)

4. A block slides down an incline of 30° with an acceleration equal to $g/4$. Find the coefficient of kinetic friction. **[Central Schools 07]**

   (Ans. $1/2\sqrt{3}$)

5. A 10 kg block slides without acceleration down a rough inclined plane making an angle of 20° with the horizontal. Calculate the acceleration when the inclination of the plane is increased to 30° and the work done over a distance of 1.2 m. Take $g = 9.8 \text{ ms}^{-2}$. **(Ans. 1.8 ms^{-2}, 21.6 J)**

6. A railway engine weighing 40 metric ton is travelling along a level track at a speed of 54 kmh^{-1}. What additional power is required to maintain the same speed up an incline rising 1 in 49. Given $\mu = 0.1$, $g = 9.8 \text{ ms}^{-2}$. **(Ans. 120 kW)**

7. A metal block of mass 0.5 kg is placed on a plane inclined to the horizontal at an angle of 30°. If the coefficient of friction is 0.2, what force must be applied (*i*) to just prevent the block from sliding down the inclined plane (*ii*) to just move the block up the inclined plane and (*iii*) to move it up the inclined plane with an acceleration of 20 cms^{-2} ?

   [Ans. (*i*) 1.6 N (*ii*) 3.299 N (*iii*) 3.399 N]

8. A block '*A*' of mass 14 kg moves along an inclined plane that makes an angle of 30° with the horizontal [Fig. 5.92]. Block *A* is connected to another block *B* of mass 14 kg by a taut massless string that runs around a frictionless, massless pulley. The block *B* moves downward with constant velocity. What is (*i*) the magnitude of the frictional force and (*ii*) the coefficient of kinetic friction ?

   [Ans. (*i*) 68.6 N (*ii*) 0.58]

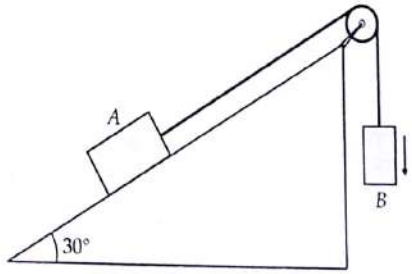

Fig. 5.92

9. A wooden block of mass 100 kg rests on a flat wooden floor, the coefficient of friction between the two being 0.4. The block is pulled by a rope making an angle of 30° with the horizontal. What is the minimum tension along the rope that just makes the block sliding ? **(Ans. 367.7 N)**

✕ HINTS

1. Here $f = \mu R = \mu\, mg \cos\theta = 0.7 \times 2 \times 9.8 \cos 30°$

   $= 0.7 \times 9.8 \times 0.866 = \mathbf{11.9 \text{ N}}.$

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2. Acceleration,

$$a = g(\sin\theta - \mu_k \cos\theta) = 9.8(\sin 30° - 0.5\cos 30°)$$
$$= 9.8(0.5 - 0.5 \times 0.866) = \mathbf{0.657\ ms^{-2}}$$

3. Here $m = 10^5$ kg, $a = 2\ ms^{-2}$, $\mu = 0.005$, $g = 10\ ms^{-2}$

$$\sin\theta = 1/50$$
$$\therefore \quad \cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{1}{2500}} = \sqrt{\frac{2499}{2500}} = 1$$
$$F = ma + mg(\sin\theta + \mu\cos\theta)$$
$$= 10^5 \times 2 + 10^5 \times 10\left(\frac{1}{50} + 0.005 \times 1\right)$$
$$= 2 \times 10^5 + 0.25 \times 10^5 = \mathbf{2.25 \times 10^5\ N.}$$

4. Here $mg\sin\theta - f_k = ma$ or $mg\sin 30° - f_k = mg/4$

$$\therefore \quad f_k = mg/2 - mg/4 = mg/4$$
and $$R = mg\cos\theta = mg\cos 30° = mg(\sqrt{3}/2)$$
Hence $$\mu_k = \frac{f_k}{R} = \frac{mg/4}{mg(\sqrt{3}/2)} = \frac{1}{2\sqrt{3}}.$$

5. Here $\mu = \tan 20° = 0.3647$

When the block slides down the inclined plane,

$$a = g(\sin\theta - \mu\cos\theta)$$
$$= 9.8(\sin 30° - 0.3647\cos 30°)$$
$$= 9.8(0.5 - 0.3647 \times 0.866) = \mathbf{1.8\ ms^{-2}}.$$
$$W = Fs = mas = 10 \times 1.8 \times 1.2 = \mathbf{21.6\ J}.$$

6. Here $m = 40$ metric ton $= 40 \times 10^3$ kg,

$$v = 54\ kmh^{-1} = 15\ ms^{-1}$$
$$\sin\theta = \frac{1}{49},\ \mu = 0.1,\ g = 9.8\ ms^{-2}$$
$$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\left(\frac{1}{49}\right)^2} = \sqrt{\frac{2400}{2401}} = 1$$

Power required on level track, $P_1 = f \times v = \mu mg \times v$

Power required up the incline,

$$P_2 = (mg\sin\theta + \mu mg\cos\theta)v$$

Additional power required

$$= P_2 - P_1 = [\mu\ mg\sin\theta + \mu\ mg\cos\theta - \mu\ mg]v$$

or $$P = (mg\sin\theta + \mu mg \times 1 - \mu\ mg)\ v$$
$$= mg\sin\theta \times v = 40 \times 10^3 \times 9.8 \times \frac{1}{49} \times 15$$
$$= 120 \times 10^3\ W = \mathbf{120\ kW.}$$

Here $\theta = 30°$, $\mu = 0.2$, $m = 0.5$ kg

(i) Force needed to just prevent the block from sliding down the inclined plane,

$$F_1 = mg\sin\theta - f = mg\sin\theta - \mu\ mg\cos\theta$$
$$[\because f = \mu R = \mu mg\cos\theta]$$
$$= mg(\sin\theta - \mu\cos\theta)$$
$$= 0.5 \times 9.8(\sin 30° - 0.2\cos 30°) = \mathbf{1.6\ N.}$$

Force needed to just move the block up the inclined plane,

$$F_2 = mg(\sin\theta + \mu\cos\theta)$$
$$= 0.5 \times 9.8(\sin 30° + 0.2 \times \cos 30°)$$
$$= \mathbf{3.299\ N.}$$

(iii) Force needed to move the block up the inclined plane with an acceleration of $20\ cms^{-2}$

$$F_3 = F_2 + ma = 3.299 + 0.5 \times 0.20$$
$$= 3.299 + 0.1 = \mathbf{3.399\ N.}$$

8. (i) Various forces acting on the bodies A and B are shown in Fig. 5.93.

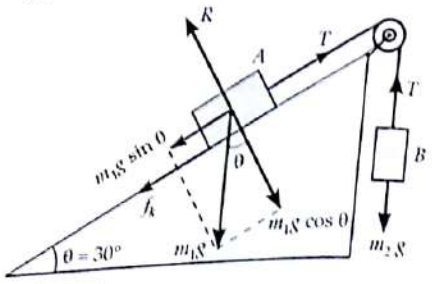

Fig. 5.93

Let T be the tension in the chord. As the block B moves downward with a constant velocity, so net force on it is zero.

$$\therefore \quad T - m_2 g = 0 \quad \text{or} \quad T_2 = m_2 g \qquad \ldots(i)$$

Net force on block A is also zero.

$$\therefore \quad T - f_k - m_1 g\sin\theta = 0$$
or $$f_k = T - m_1 g\sin\theta = m_2 g - m_1 g\sin 30°$$
$$= 14 \times 9.8 - 14 \times 9.8 \times \frac{1}{2} = \mathbf{68.6\ N.}$$

(ii) $$\mu_k = \frac{f_k}{R} = \frac{68.6}{m_1 g\cos 30°} = \frac{68.6 \times 2}{14 \times 9.8 \times \sqrt{3}} = \mathbf{0.58.}$$

9. Various forces acting on the wooden block are as shown in Fig. 5.94. Let T be the minimum tension that just makes the block sliding. When the block just slides,

$$R = mg - T\sin 30°$$
and $$T\cos 30° = f = \mu R = \mu\ (mg - T\sin 30°)$$
$$\therefore \quad T(\cos 30° + \mu\sin 30°) = \mu\ mg$$
or $$T = \frac{\mu\ mg}{\cos 30° + \mu\sin 30°}$$
$$= \frac{0.4 \times 100 \times 9.8}{0.866 + 0.4 \times 0.5} = \frac{392}{1.060} = \mathbf{367.7\ N.}$$

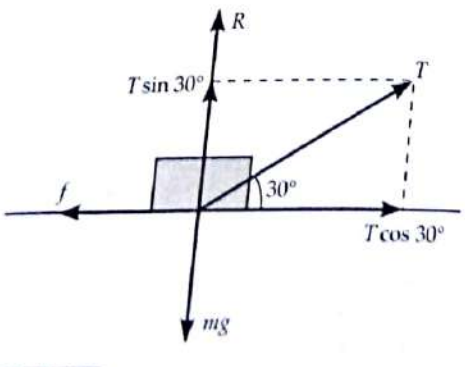

Fig. 5.94

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5.30 FRICTION IS A NECESSARY EVIL

62. *Friction is a necessary evil. Explain.*

Friction is a necessity (*Advantages of friction*) :

(*i*) It is due to friction between the ground and the feet that we are able to walk.

(*ii*) The brakes of a vehicle cannot work without friction.

(*iii*) Various parts of a machine are able to rotate because of friction between belt and pulley.

(*iv*) The tyres of vehicle are made rough to increase friction.

(*v*) Nails and screws join various wooden parts due to friction.

(*vi*) In the absence of friction, it will not be possible to write on a paper with a pen or pencil.

Friction is an evil (*Disadvantages of friction*) :

(*i*) Wear and tear of the machinery is due to friction.

(*ii*) A large amount of power is wasted in overcoming friction and the efficiency of the machines decreases considerably.

(*iii*) Excessive friction between rotating parts of a machine produces enough heat and causes damage to the machinery.

As friction has both advantages and disadvantages, we can say that *friction is a necessary evil.*

5.31 METHODS OF CHANGING FRICTION

63. *Describe the various methods by which friction between two surfaces can be reduced.*

Methods of reducing friction. Friction between two surfaces can be reduced by any of the following methods :

(*i*) **By polishing.** By making the surfaces in contact highly polished and smooth, the bumps and depressions get minimised and so friction is reduced considerably.

(*ii*) **Lubrication.** A lubricant is a substance (solid, liquid or gas) which forms a thin layer between two surfaces in contact. It fills depressions present in the surfaces of contact and hence reduces friction.

(*iii*) **Streamlining.** Friction due to air is considerably reduced by streamlining the shape of the body (sharp in front) moving through air. For example, jets, aeroplanes, fast moving cars, etc., have streamline shape.

(*iv*) **By using ball-bearings.** A ball bearing consists of two coaxial cylinders, between which steel balls are placed. The inner cylinder is fitted on the axle and the outer cylinder is fitted to the wheel. When the axle rotates in clockwise direction (say), the steel balls rotate in the anticlockwise direction making the outer cylinder and the wheel to rotate in anticlockwise direction. As the sliding friction between the cylinders gets converted into rolling friction, so friction is considerably reduced.

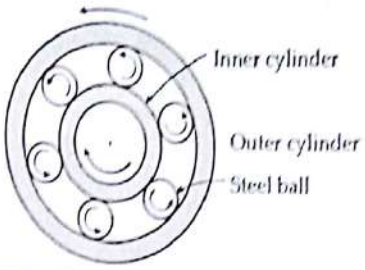

Fig. 5.95 Ball bearing.

(*v*) **By using antifriction alloys.** It has been found that when steel slides over an alloy such as bronze or brass, the friction is much less than when steel slides over steel. Thus friction is also reduced by lining the moving parts with alloys called antifriction alloys.

(*vi*) **By using air cushion.** Friction can be considerably reduced by maintaining a thin cushion of compressed air between solid surfaces in relative motion.

64. *Describe some methods of increasing friction.*

Methods of increasing friction. Some of the methods are as follows :

(*i*) **Treading of tyres** is done to increase friction between the road and the tyres. Moreover, synthetic rubber is preferred over the natural rubber in the manufacture of tyres because of its larger coefficient of friction with the road.

(*ii*) **Sand is thrown on tracks covered with snow.** This increases the force of friction between the wheels and the track and the driving becomes safer.

(*iii*) **On a rainy day, we throw some sand on the slippery ground.** This increases the friction between our feet and the ground. This reduces the chances of slipping.

5.32 PULLING A LAWN ROLLER IS EASIER THAN TO PUSH IT

65. *Why is it easier to pull a lawn roller than to push it ? Explain.*

It is easier to pull a body than to push it. As shown in Fig. 5.96, suppose a force F is applied to pull a lawn roller of weight W. The force F has two rectangular components :

(*i*) Horizontal component $F \cos \theta$ helps to move the roller forward.

(*ii*) Vertical component $F \sin \theta$ acts in the upward direction.

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If R is the normal reaction, then

$R + F \sin\theta = W$ [Equating the vertical components]

$R = W - F \sin\theta$ ...(1)

or

Force of kinetic friction,

$f_k = \mu_k R = \mu_k (W - F \sin\theta)$

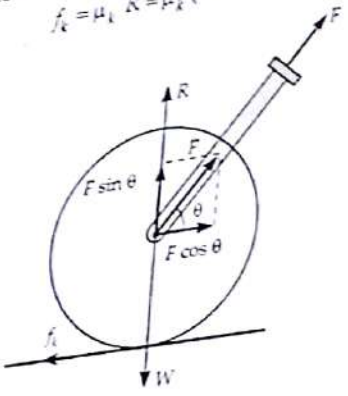

Fig. 5.96 Pulling a roller.

As shown in Fig. 5.97, if a force F is applied to push a roller of weight W, then the normal reaction is

$R' = W + F \sin\theta$

Force of kinetic friction,

$f_k' = \mu_k R' = \mu_k (W + F \sin\theta)$ ...(2)

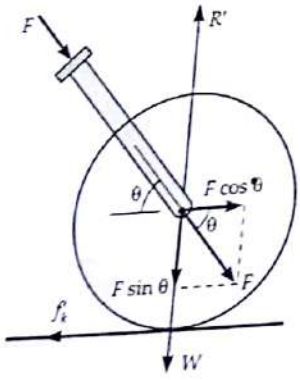

Fig. 5.97 Pushing a roller.

Comparing (1) and (2), we find that $f_k' > f_k$

i.e., the force of friction is more in case of push than in case of pull. So it is easier to pull a body than to push it.

5.33 CENTRIPETAL FORCE

66. *What is centripetal force ? Write an expression for it. Give some examples of centripetal force.*

Centripetal force. *A force required to make a body move along a circular path with uniform speed is called centripetal force.* It always acts along the radius and towards the centre of the circular path.

As shown in Fig. 5.98, such a force continuously deflects a particle from its straight line path to make it move along a circle.

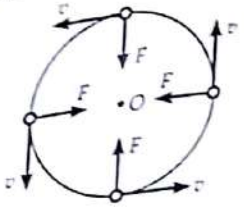

Fig. 5.98 Centripetal force.

Expression for centripetal force. When a body is in uniform circular motion, its velocity changes continuously due to change in the direction of motion. Hence it undergoes an acceleration which acts radially inwards. It is called centripetal acceleration and is given by

$$a = \frac{v^2}{r} = r\omega^2 \qquad [\because v = r\omega]$$

where v and ω are the linear and angular speeds of the body and r is the radius of the circular path. According to Newton's second law, the centripetal force required to move a body of mass m along a circular path of radius r is given by

F = mass × centripetal acceleration

or $$F = \frac{mv^2}{r} = mr\omega^2$$

Examples of centripetal force :

(*i*) For a stone rotated in a circle, the tension in the string provides the centripetal force.

(*ii*) The centripetal force for the motion of the planet around the sun is provided by the gravitational force exerted by the sun on the planet.

(*iii*) For a car taking a circular turn on a horizontal road, the centripetal force is provided by the force of friction between the tyres and the road.

(*iv*) For an electron revolving around the nucleus, the centripetal force is provided by the electrostatic attraction between the electron and the nucleus.

For Your Knowledge

▲ Centripetal force is not a new kind of force. The material forces such as tension, gravitational force, electrical force, friction, etc., may act as the centripetal force in any circular motion.

▲ Centripetal force is the name given to any force that provides radial inward acceleration to a body in circular motion.

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Examples based on Centripetal Force

FORMULAE USED

1. For a body moving along a horizontal circular path, centripetal force is

$$F = \frac{mv^2}{r} = mr\omega^2 = mr(2\pi\nu)^2 = mr\left(\frac{2\pi}{T}\right)^2$$

2. Centrifugal force is equal to centripetal force in magnitude but acts away from the centre

UNITS USED

Force F is in newton, velocity v in ms^{-1}, angular frequency ω in rad s^{-1}.

EXAMPLE 70. A string breaks under a load of 4.8 kg. A mass of 0.5 kg is attached to one end of the string 2 m long and is rotated in a horizontal circle. Calculate the greatest number of revolutions that the mass can make without breaking the string.

Solution. Here $m = 0.5$ kg, $r = 2$ m, $g = 9.8\ ms^{-2}$,

The maximum tension that the string can withstand,

$$F = 4.8 \text{ kg wt} = 4.8 \times 9.8 \text{ N}$$

Let the maximum number of revolutions per second $= \nu$

Now $$F = mr\omega^2 = mr(2\pi\nu)^2 = 4\pi^2 mr\nu^2$$

or $$\nu^2 = \frac{F}{4\pi^2 mr} = \frac{4.8 \times 9.8}{4 \times 9.87 \times 0.5 \times 2} = 1.215$$

or $$\nu = \sqrt{1.215} = 1.102 \text{ rps}$$

$$= 1.102 \times 60 = \mathbf{66.13\ rpm}.$$

EXAMPLE 71. A ball of mass 0.1 kg is suspended by a string 30 cm long. Keeping the string always taut, the ball describes a horizontal circle of radius 15 cm. Calculate the angular speed. **[Delhi 06]**

Solution. The situation is shown in Fig. 5.99.

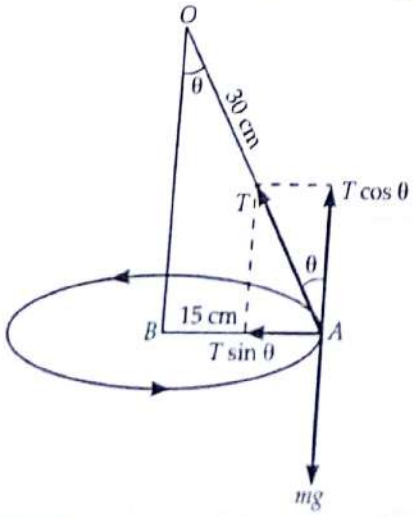

Fig. 5.99

Here $m = 0.1$ kg, $r = 15$ cm $= 0.15$ m

Let the string make angle θ with the vertical. Then

$$\sin\theta = \frac{AB}{OA} = \frac{15}{30} = \frac{1}{2} \quad \text{or} \quad \theta = 30°.$$

Let T be the tension in the string. Its vertical component balances the weight mg while the horizontal component $T\sin\theta$ provides the centripetal force.

$\therefore$ $$T\cos\theta = mg \qquad ...(i)$$

and $$T\sin\theta = mr\omega^2 \qquad ...(ii)$$

Dividing (ii) by (i), we get

$$\frac{T\sin\theta}{T\cos\theta} = \frac{mr\omega^2}{mg}$$

or $$\tan\theta = \frac{r\omega^2}{g}$$

or $$\omega = \sqrt{\frac{g\tan\theta}{r}} = \sqrt{\frac{9.8 \times \tan 30°}{0.15}}$$

$$= \mathbf{6.14\ rad\ s^{-1}}.$$

PROBLEMS FOR PRACTICE

1. A particle of mass 21 g attached to a string of 70 cm length is whirled round in a horizontal circle. If the period of revolution is 2 s, find the tension in the string. **(Ans.** 14520 dyne)
2. A stone of mass 4 kg is attached to a string of 10 m length and is whirled in a horizontal circle. The string can withstand a maximum tension of 160 N. Calculate the maximum velocity of revolution that can be given to the stone without breaking the string. **(Ans.** 20 ms^{-1})
3. A 100 g weight is tied at the end of a string and whirled around in a horizontal circle of radius 15 cm at the rate of 3 revolutions per second. What is the tension in the string ? **(Ans.** 5.334 N)
4. A gramophone disc rotates at 60 rpm. A coin of mass 18 g is placed at a distance of 8 cm from the centre. Calculate the centrifugal force on the coin. Take $\pi^2 = 9.87$. **(Ans.** 4105.92 dyne)
5. A ball of mass 100 g is suspended by a string 40 cm long. Keeping the string taut, the ball describes a horizontal circle of radius 10 cm. Find the angular speed. **(Ans.** 1.59 rad s^{-1})

HINTS

1. Here $m = 21$ g, $r = 70$ cm, $T = 2$ s

$\therefore$ Tension $= mr\omega^2 = mr\left(\frac{2\pi}{T}\right)^2$

$$= 21 \times 70 \times \left(\frac{2 \times 22}{2 \times 7}\right)^2 = \mathbf{14520\ dyne}.$$

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2. Here $m = 4$ kg, $r = 10$ m, $T_{max} = 160$ N, $v = ?$

As $T_{max} = \frac{mv_{max}^2}{r}$ or $v_{max}^2 = \frac{T_{max} \times r}{m}$

$\therefore \quad v_{max} = \sqrt{\frac{T_{max} \times r}{m}} = \sqrt{\frac{160 \times 10}{4}} = 20 \text{ ms}^{-1}$.

3. Here $m = 0.1$ kg, $r = 0.15$ m, $\nu = 3$ rps

$\omega = 2\pi\nu = 2\pi \times 3 = 6\pi$ rads^{-1}

$T = mr\omega^2 = 0.3 \times 0.15 \times (6\pi)^2 = 5.334$ N.

5.34 CIRCULAR MOTION OF A CAR ON A LEVEL ROAD

67. *Briefly explain how is a vehicle able to go round a level curved track. Determine the maximum speed with which the vehicle can negotiate this curved track safely.*

Car on a circular level road. When a car negotiates a curved level road, the force of friction between the road and the tyres provides the centripetal force required to keep the car in motion around the curve.

As shown in Fig. 5.100, consider a car of weight mg going around a circular level road of radius r with constant speed v. While taking the turn, the tyres of the car tend to leave the road and go away from the centre of the curve. The forces of friction f_1 and f_2 act inward at the inner and the outer tyres respectively. If R_1 and R_2 are the reactions of the ground at the inner and outer tyres, then

$$f_1 = \mu R_1 \quad \text{and} \quad f_2 = \mu R_2$$

where μ is the coefficient of friction between the tyres and the road. As the total normal reaction balances the weight of the car, therefore $R_1 + R_2 = mg$

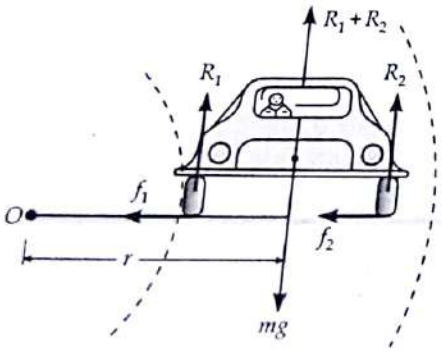

Fig. 5.100 Motion of a car on a circular level road.

Total force of friction

$$= f_1 + f_2 = \mu (R_1 + R_2) = \mu . mg$$

For the car to stay on the road, the maximum force of friction must be equal to or greater than the centripetal force *i.e.*,

$$\mu . mg \geq \frac{mv^2}{r} \quad \text{or} \quad \frac{v^2}{rg} \leq \mu \quad \text{or} \quad v^2 \leq \mu rg$$

The maximum speed with which the car can turn safely is

$$v_{max} = \sqrt{\mu rg}$$

If the speed exceeds v_{max}, the car will skid and go off the road in a circle of radius greater than r. This is because the maximum available friction is insufficient to provide the necessary centripetal force. Clearly, the maximum safe speed v_{max} depends on the radius r of the curved path and the coefficient of friction μ between the road and the tyres.

5.35 CIRCULAR MOTION OF A CAR ON A BANKED ROAD

68. *What do you mean by banking of a curved road? Determine the angle of banking so as to minimise the wear and tear of the tyres of a car negotiating a banked curve.*

Banking of the curved road. The large amount of friction between the tyres and the road produces considerable wear and tear of the tyres. To avoid this, the curved road is given an inclination sloping upwards towards the outer circumference. This reduces wearing out of the tyres because the horizontal component of the normal reaction provides the necessary centripetal force.

The system of raising the outer edge of a curved road above its inner edge is called ***banking of the curved road.*** *The angle through which the outer edge of the curved road is raised above the inner edge is called* ***angle of banking.***

Circular motion of a car on a banked road. As shown in Fig. 5.101, consider a car of weight mg going along a curved path of radius r with speed v on a road banked at an angle θ. The forces acting on the vehicle are

1. Weight mg acting vertically downwards.
2. Normal reaction R of the road acting at an angle θ with the vertical.
3. Force of friction f acting downwards along the inclined plane.

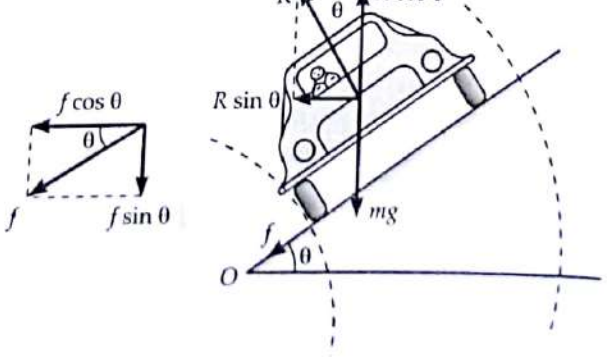

Fig. 5.101 Circular motion of a car on a banked road.

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Equating the forces along horizontal and vertical directions respectively, we get

$$R \sin\theta + f \cos\theta = \frac{mv^2}{r} \quad ...(1)$$

$$mg + f \sin\theta = R \cos\theta, \text{ where } f = \mu R$$

or $$R \cos\theta - f \sin\theta = mg \quad ...(2)$$

Dividing equation (1) by equation (2), we get

$$\frac{R \sin\theta + f \cos\theta}{R \cos\theta - f \sin\theta} = \frac{v^2}{rg}$$

Dividing numerator and denominator of L.H.S. by $R \cos\theta$, we get

$$\frac{\tan\theta + \frac{f}{R}}{1 - \frac{f}{R}\tan\theta} = \frac{v^2}{rg}$$

or $$\frac{\tan\theta + \mu}{1 - \mu\tan\theta} = \frac{v^2}{rg} \qquad \left[\because \mu = \frac{f}{R}\right]$$

or $$v^2 = rg\left[\frac{\mu + \tan\theta}{1 - \mu\tan\theta}\right] \text{ or } v = \sqrt{rg \cdot \frac{\mu + \tan\theta}{1 - \mu\tan\theta}}$$

Special case. When there is no friction between the road and the tyres, $\mu = 0$, so that the safe limit for maximum speed is $v = \sqrt{rg\tan\theta}$.

The angle of banking θ for minimum wear and tear of tyres is given by

or $$\tan\theta = \frac{v^2}{rg} \quad \text{or} \quad \theta = \tan^{-1}\left(\frac{v^2}{rg}\right).$$

5.36 BENDING OF A CYCLIST

69. Why does a cyclist lean inward when moving along a curved path ? Determine the angle through which a cyclist bends from the vertical while negotiating a curve.

Bending of a cyclist. When a cyclist goes round a curved path, a centripetal force is required. The force of friction between the tyres and the road is too small to provide the necessary centripetal force. That is a *why a cyclist going round a curve leans inward because then the horizontal component of the normal reaction provides the necessary centripetal force.*

Suppose the cyclist bends inwards through an angle θ from the vertical. The horizontal component $R \sin\theta$ of the normal reaction R provides the centripetal force.

$$\therefore \quad R \sin\theta = \frac{mv^2}{r} \quad ...(1)$$

The vertical component $R \cos\theta$ of the normal reaction balances the weight mg of the cycle and the cyclist.

$$\therefore \quad R \cos\theta = mg \quad ...(2)$$

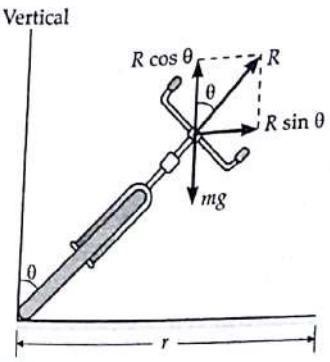

Fig. 5.102 Bending of a cyclist.

Dividing equation (1) by equation (2), we get

$$\frac{R \sin\theta}{R \cos\theta} = \frac{mv^2/r}{mg} \text{ or } \tan\theta = \frac{v^2}{rg} \text{ or } \theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

The angle θ through which the cyclist bends would be greater, if

(*i*) the radius of the curved path is small or the curved turn is sharp.

(*ii*) the speed of the cyclist is large.

Examples based on Banking of Roads and Bending of a Cyclist

FORMULAE USED

1. *A vehicle taking a circular turn on a level road.* If μ is the coefficient of friction between tyres and road, then the maximum velocity with which the vehicle can safely take a circular turn of radius r is given by

$$v_{max} = \sqrt{\mu rg}$$

2. *Banking of tracks (roads).* The maximum velocity with which a vehicle (in the absence of friction) can negotiate a circular road of radius r and banked at an angle θ is given by

$$v = \sqrt{rg\tan\theta}$$

When the frictional forces are also taken into account, the maximum safe velocity is given by

$$v_{max} = \sqrt{rg\left(\frac{\mu + \tan\theta}{1 - \mu\tan\theta}\right)}$$

3. *Bending of a cyclist.* In order to take a circular turn of radius r with speed v, the cyclist should bend himself through an angle θ from the vertical such that

$$\tan\theta = \frac{v^2}{rg}$$

UNITS USED

Velocity v is in ms^{-1}, radius r in metre, angle θ in degrees and coefficient of friction μ has no units.

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EXAMPLE 72. A bend in a level road has a radius of 100 metres. Find the maximum speed which a car turning this bend may have without skidding, if the coefficient of friction between the tyres and road is 0.8 [NCERT]

Solution. Here $r = 100$ m, $\mu = 0.8$, $g = 9.8$ ms^{-2}

The car will not skid if the force of friction provides the necessary centripetal force.

$$\therefore \quad f = \mu mg = \frac{mv_{max}^2}{r}$$

or $$v_{max} = \sqrt{\mu rg} = \sqrt{0.8 \times 100 \times 9.8} = 28 \text{ ms}^{-1}.$$

EXAMPLE 73. A car of mass 1500 kg is moving with a speed of 12.5 ms^{-1} on a circular path of radius 20 m on a level road. What should be the frictional force between the car and the road so that the car does not slip ? What should be the value of the coefficient of friction to attain this force ?

Solution. Here $m = 1500$ kg, $v = 12.5$ ms^{-1}, $r = 20$ m

Frictional force = Required centripetal force

or $$f = \frac{mv^2}{r} = \frac{1500 \times 12.5 \times 12.5}{20}$$

$$= 1.172 \times 10^4 \text{ N}$$

But $f = \mu R = \mu mg$

∴ Coefficient of friction,

$$\mu = \frac{f}{mg} = \frac{1.172 \times 10^4}{1500 \times 9.8} = 0.8.$$

EXAMPLE 74. A train has to negotiate a curve of 400 m. By how much should the outer rail be raised with respect to the inner rail for a speed of 48 kmh^{-1} ? The distance between the rails is 1 m [NCERT]

Solution. Here

$$v = 48 \text{ kmh}^{-1} = \frac{48 \times 1000}{3600} = \frac{40}{3} \text{ ms}^{-1}$$

$r = 400$ m, $l = 1$ m

Let h be the height through which the outer rail must be raised with respect to the inner rail, as shown in Fig. 5.103. If θ is the angle of banking, then

$$\sin\theta = \frac{h}{l}$$

For a small value of θ,

$$\sin\theta \simeq \tan\theta$$

$$\frac{h}{l} = \frac{v^2}{rg}$$

$$h = \frac{v^2 l}{rg}$$

$$= \frac{(40/3)^2 \times 1}{400 \times 9.8} = 0.0454 \text{ m}.$$

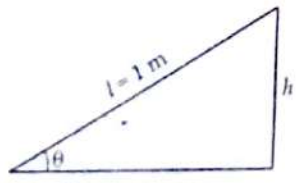

Fig. 5.103

EXAMPLE 75. Calculate the angle through which a cyclist bends from the vertical when he crosses a circular path of 34.3 m in circumference in $\sqrt{22}$ s. Take $g = 9.8$ ms^{-2}.

Solution. Radius of circular path,

$$r = \frac{\text{Circumference}}{2\pi} = \frac{34.3 \times 7}{2 \times 22} \text{ m}$$

Velocity, $$v = \frac{\text{Length of path}}{\text{Time taken}} = \frac{34.3}{\sqrt{22}} \text{ ms}^{-1}$$

$$\tan\theta = \frac{v^2}{rg} = \left(\frac{34.3}{\sqrt{22}}\right)^2 \times \frac{2 \times 22}{34.3 \times 7} \times \frac{1}{9.8} = 1$$

∴ Angle through which the cyclist bends from the vertical, $\theta = 45°$.

EXAMPLE 76. A cyclist speeding at 18 km h^{-1} on a level road takes a sharp circular turn of radius 3 m without reducing the speed and without bending towards the centre of the circular path. The coefficient of static friction between the tyres and the road is 0.1. Will the cyclist slip while taking the turn ? [NCERT]

Solution. On a level road, force of friction alone provides centripetal force for going along the circular turn.

As $$\tan\theta = \mu = \frac{v^2}{rg}$$

Therefore, the maximum safe speed is given by

$$v_{max} = \sqrt{\mu rg} = \sqrt{0.1 \times 3 \times 9.8} = \sqrt{2.94} = 1.715 \text{ ms}^{-1}$$

Actual speed, $v = 18$ kmh$^{-1} = 5$ ms^{-1}

As the actual speed is greater than the maximum safe speed, so the cyclist will *slip* while taking the turn.

EXAMPLE 77. A circular race track of radius 300 m is banked at an angle of 15°. If the coefficient of friction between the wheels of a race car and the road is 0.2 , what is the

(i) optimum speed of the race car to avoid wear and tear on its tyres, and

(ii) maximum permissible speed to avoid slipping ? [NCERT]

Solution. Given $r = 300$ m, $\theta = 15°$, $\mu_s = 0.2$

(*i*) The optimum speed of the race car will be

$$v_0 = \sqrt{rg \tan\theta} = \sqrt{300 \times 9.8 \times \tan 15°}$$

$$= \sqrt{300 \times 9.8 \times 0.2679} = 28.1 \text{ ms}^{-1}.$$

(*ii*) The maximum permissible speed of race car will be

$$v_{max} = \sqrt{rg \frac{\mu_s + \tan\theta}{1 - \mu_s \tan\theta}}$$

$$= \sqrt{300 \times 9.8 \times \frac{0.2 + 0.2679}{1 - 0.2 \times 0.2679}}$$

$$= 38.1 \text{ ms}^{-1}.$$

EXAMPLE 78. A car is sp curving round with a radius between the wheels and the ro gravity of the car from the distance between the whe maximum safe velocity for th Will the vehicle skid or topp

Solution. Here $r = 60$

Distance of any wheel through CG of the car,

$$b = \frac{0.8}{2} = 0$$

As $$\tan\theta = \frac{v^2}{rg}$$

For skidding,

$$\tan\theta = \mu$$

$$\therefore \quad v_s = \sqrt{\mu rg}$$

For toppling,

$$\tan\theta = \frac{b}{h}$$

$$\therefore \quad v_t = \sqrt{\frac{b}{}}$$

Thus the maxim negotiate the curve exceeds this value, $v = 28$ ms^{-1}.

EXAMPLE 79. An el railway train by a f 19°48′, when the trai of 200 m radius. Fi

Solution. Var shown in Fig. 5. length and perpe

Fig. 5.104

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EXAMPLE 78. A car is speeding on a horizontal road curving round with a radius 60 m. The coefficient of friction between the wheels and the road is 0.5. The height of centre of gravity of the car from the road level is 0.3 m and the distance between the wheels is 0.8 m. Calculate the maximum safe velocity for the vehicle to negotiate the curve. Will the vehicle skid or topple if this velocity is exceeded ?

Solution. Here $r = 60$ m, $\mu = 0.5$, $h = 0.3$ m

Distance of any wheel from the vertical line passing through CG of the car,

$$b = \frac{0.8}{2} = 0.4 \text{ m}$$

As $\tan\theta = \dfrac{v^2}{rg}$ $\quad\therefore\quad v = \sqrt{rg\tan\theta}$

For skidding,

$$\tan\theta = \mu$$

$$\therefore \quad v_s = \sqrt{\mu rg} = \sqrt{0.5 \times 60 \times 9.8} = \mathbf{17.15\ ms^{-1}}.$$

For toppling,

$$\tan\theta = \frac{b}{h}$$

$$\therefore \quad v_t = \sqrt{\frac{brg}{h}} = \sqrt{\frac{0.4 \times 60 \times 9.8}{0.3}} = \mathbf{28\ ms^{-1}}.$$

Thus the maximum safe velocity for the car to negotiate the curve is 17.15 ms^{-1}. When the velocity exceeds this value, the car skids until it topples at $v = 28\ ms^{-1}$.

EXAMPLE 79. An electric bulb suspended from the roof of a railway train by a flexible wire shifts through an angle of 19°48', when the train goes horizontally round a curved path of 200 m radius. Find the speed of the train.

Solution. Various forces acting on the bulb are shown in Fig. 5.104. Resolving the forces along the length and perpendicular to the wire, we get

$$mg\sin\theta = \frac{mv^2}{r}\cdot\cos\theta$$

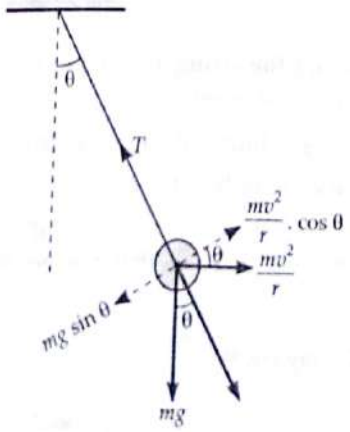

Fig. 5.104

or $\quad \tan\theta = \dfrac{v^2}{rg}$

or $\quad v = \sqrt{rg\tan\theta} = \sqrt{200 \times 9.8 \times \tan 19°48'}$

$$= \sqrt{200 \times 9.8 \times 0.3600} = \sqrt{705.6}$$

$$= \mathbf{26.56\ ms^{-1}}.$$

PROBLEMS FOR PRACTICE

1. Find the maximum speed at which a car can take turn round a curve of 30 m radius on a level road if the coefficient of friction between the tyres and the road is 0.4. Take $g = 10\ ms^{-2}$. **[IIT 86, Roorkee 86]**
(Ans. 11 ms^{-1})
2. What should be the coefficient of friction between the tyres and the road, when a car travelling at 60 km h^{-1} makes a level turn of radius 40 m ?
(Ans. 0.71)
3. The mass of a bicycle rider along with the bicycle is 100 kg. He wants to cross over a circular turn of radius 100 m with a speed of 10 ms^{-1}. If the coefficient of friction between the tyres and the road is 0.6, will the rider be able to cross the turn ? Take $g = 10\ ms^{-2}$. **(Ans. Yes)**
4. A cyclist riding at a speed of $14\sqrt{3}\ ms^{-1}$ takes a turn around a circular road of radius $20\sqrt{3}$ m. What is the inclination to the vertical ? **(Ans. 60°)**
5. A cyclist speeding at 6 ms^{-1} in a circle of 18 m radius makes an angle θ with the vertical. Calculate θ. Also determine the minimum possible value of the coefficient of friction between the tyres and the ground. **(Ans. 11°32', 0.2041)**
6. A motor cyclist goes round a circular race course of diameter 320 m at 144 km h^{-1}. How far from the vertical must he lean inwards to keep his balance ? Take $g = 10\ ms^{-2}$. **(Ans. 45°)**
7. An aeroplane travelling at a speed of 500 kmh^{-1} tilts at an angle of 30° as it makes a turn. What is the radius of the curve ? **(Ans. 3.41×10^3 m)**
8. For traffic moving at 60 kmh^{-1}, if the radius of the curve is 0.1 km, what is the correct angle of banking of the road ? Take $g = 10\ ms^{-2}$. **(Ans. 15.5°)**
9. A railway carriage has its CG at a height of 1 m above the rails which are 1 m apart. Calculate the maximum safe speed at which it can travel round an unbanked curve of radius 80 m. **(Ans. 19.8 ms^{-1})**
10. A curve in a road forms an arc of radius 800 m. If the road is 19.6 m wide and outer edge is 1 m higher than the inner edge, calculate the speed for which it is banked. **(Ans. 20 ms^{-1})**

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11. A train has to negotiate a curve of radius 400 m. By how much should the outer rail be raised with respect to the inner rail for a speed of 48 kmh^{-1}? The distance between the rails is 1 m. (Ans. 0.0454 m)

12. A 2000 kg car has to go over a turn whose radius is 750 m and the angle of slope is 5°. The coefficient of friction between the wheels and the road is 0.5. What should be the maximum speed of the car so that it may go over the turn without slipping ? (Ans. 67.2 ms^{-1})

✕ HINTS

2. Here $v = 60 \text{ kmh}^{-1} = \frac{60 \times 5}{18} = \frac{50}{3} \text{ ms}^{-1}$, $r = 40$ m

$$\therefore \quad \mu = \frac{v^2}{rg} = \frac{2500}{9 \times 40 \times 9.8} = 0.71.$$

3. Required centripetal force

$$= \frac{mv^2}{r} = \frac{100 \times 10 \times 10}{100} = 100 \text{ N}$$

Available frictional force

$$= \mu mg = 0.6 \times 100 \times 10 = 600 \text{ N}$$

As the available frictional force is greater than the required centripetal force, so the rider will be able to cross the turn.

5. Here $v = 6 \text{ ms}^{-1}$, $r = 18$ m, $g = 9.8 \text{ ms}^{-2}$

$$\therefore \tan\theta = \frac{v^2}{rg} = \frac{6 \times 6}{18 \times 9.8} = 0.2041 \text{ or } \theta = 11°32'.$$

Also $\mu = \tan\theta = \frac{v^2}{rg} = 0.2041.$

8. Here $v = 60 \text{ kmh}^{-1} = \frac{50}{3} \text{ ms}^{-1}$,

$r = 0.1 \text{ km} = 100 \text{ m}$, $g = 10 \text{ ms}^{-2}$

$$\tan\theta = \frac{v^2}{rg} = \frac{2500}{9 \times 100 \times 10} = \frac{5}{18} \quad \therefore \theta = 15.5°.$$

9. Here $r = 80$ m, $h = 1$ m, $b = \frac{1}{2} = 0.5$ m, $g = 9.8 \text{ ms}^{-2}$

$$\tan\theta = \frac{v^2}{rg}$$

For maximum safe speed, $\tan\theta = \frac{b}{h}$

$$\therefore \quad \frac{v^2}{rg} = \frac{b}{h}$$

or $$v = \sqrt{\frac{brg}{h}} = \sqrt{\frac{0.5 \times 80 \times 9.8}{1}} = \sqrt{392} = 19.8 \text{ ms}^{-1}.$$

For small θ, $\tan\theta = \sin\theta$ or $\frac{v^2}{rg} = \frac{h}{l}$

$$\therefore \quad v = \sqrt{\frac{hrg}{l}} = \sqrt{\frac{1 \times 800 \times 9.8}{19.6}} = \sqrt{400} = 20 \text{ ms}^{-1}.$$

From the above problem, we have

$$h = \frac{v^2 l}{rg} = \left(\frac{40}{3}\right)^2 \times \frac{1}{400 \times 9.8} = 0.0454 \text{ m}.$$

5.37 MOTION IN A VERTICAL CIRCLE

70. A body tied to one end of a string is made to revolve in a vertical circle. Derive the expression for the velocity of the body and tension in the string at any point. Hence find (a) tension at the bottom and the top of the circle (b) minimum velocity at the lowest point so that it is just able to loop the loop and (c) the minimum velocity at the top.

Motion in a vertical circle. Consider a body of mass m tied to the one end of a string and rotating in a vertical circle of radius r, as shown in Fig. 5.105.

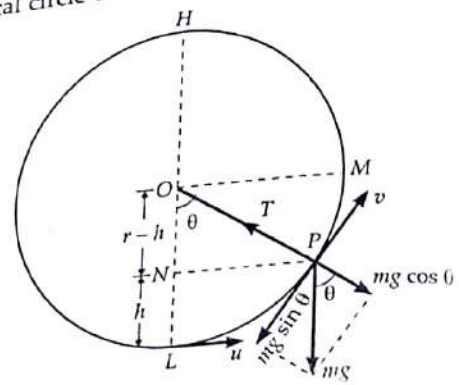

Fig. 5.105 Motion in a vertical circle.

Velocity at any point. Suppose the body passes through lowest point L with velocity u and through any point P with velocity v. In moving from L to P, it has moved up through a vertical height $LN = h$.

According to the law of conservation of energy,

(K.E. + P.E.) at L = (K.E. + P.E.) at P

$$\frac{1}{2}mu^2 + 0 = \frac{1}{2}mv^2 + mgh$$

or $$u^2 = v^2 + 2gh$$

or $$v^2 = u^2 - 2gh \qquad ...(1)$$

or $$v = \sqrt{u^2 - 2gh}$$

This equation gives velocity v of the body at any point.

Tension along the string at any point. The forces acting on the body at point P are

(*i*) Weight mg acting vertically downwards.

(*ii*) Tension T along the string.

The component $mg\cos\theta$ of the weight along the string acts opposite to T, so that the net centripetal force is

$$T - mg\cos\theta = \frac{mv^2}{r}$$

or $$T = mg\cos\theta + \frac{mv^2}{r} \qquad ...(2)$$

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From right angled ΔOPN,

$$\cos\theta = \frac{ON}{OP} = \frac{r-h}{r} \qquad ...(3)$$

Using equations (1) and (3), equation (2) becomes

$$T = mg\frac{r-h}{r} + \frac{m}{r}(u^2 - 2gh)$$

$$= \frac{m}{r}(gr - gh + u^2 - 2gh)$$

or $$T = \frac{m}{r}(u^2 + gr - 3gh) \qquad ...(4)$$

This equation gives tension along the string at any point of the circle.

(*a*) **Tension in the string at the bottom and the top.** *At the lowest point L, $h = 0$,* hence the tension in the string is

$$T_L = \frac{m}{r}[u^2 + gr] \qquad ...(5)$$

At the highest point H, $h = 2r$, hence the tension in the string is

$$T_H = \frac{m}{r}(u^2 + gr - 6gr)$$

$$= \frac{m}{r}(u^2 - 5gr) \qquad ...(6)$$

Now $T_L - T_H = \frac{m}{r}[(u^2 + gr) - (u^2 - 5gr)] = 6mg$

Thus *the difference in tensions at the lowest and the highest points is equal to six times the weight of the revolving body.*

(*b*) **Minimum velocity of projection at the lowest point for looping the loop.** The body will be able to cross the highest point H without any slackening of the string if T_H is positive *i.e.*,

$$T_H \geq 0 \quad \text{or} \quad \frac{m}{r}(u^2 - 5gr) \geq 0$$

or $$u^2 \geq 5gr \quad \text{or} \quad u \geq \sqrt{5gr}$$

Hence $\sqrt{5gr}$ is the minimum velocity which the body must possess at the bottom of the circle so as to go round the circle completely *i.e.*, for looping the loop.

(*c*) **Minimum velocity at the top.** If V is the velocity which the body possesses at highest point H in just the case of no slackening of the string, then

$$V^2 = u^2 - 2g.2r$$

$[\because h = 2r$, at the highest point$]$

$$= 5gr - 4gr \qquad [\because u = \sqrt{5gr}]$$

or $$V = \sqrt{gr}$$

This gives the minimum or *critical velocity* at the highest point.

71. *For a body in motion in a vertical circle, deduce the conditions for (i) oscillation of the body about the lowest point and (ii) the body to leave the circular path.*

Condition for oscillation. If $u < \sqrt{5gr}$, the body will either oscillate about the lowest point L or will leave its circular path.

Suppose the velocity of the body becomes zero at height h_1. As

$$v^2 = u^2 - 2gh$$

$$\therefore \quad 0^2 = u^2 - 2gh_1 \quad \text{or} \quad h_1 = \frac{u^2}{2g}$$

Suppose the tension in the string becomes zero at height h_2. As

$$T = \frac{m}{r}(u^2 + gr - 3gh)$$

$$\therefore \quad 0 = \frac{m}{r}(u^2 + gr - 3g\,h_2) \quad \text{or} \quad h_2 = \frac{u^2 + gr}{3g}$$

The body will oscillate if its velocity becomes zero earlier than the tension, *i.e.*,

$$h_1 < h_2 \qquad \text{or} \qquad \frac{u^2}{2g} < \frac{u^2 + gr}{3g}$$

or $3u^2 < 2u^2 + 2gr$ or $u^2 < 2gr$ or $u \leq \sqrt{2gr}$

Hence *the body will oscillate about the lowest point of the vertical circle if $u < \sqrt{2gr}$. If $u = \sqrt{2gr}$, the path of oscillation will be a semicircle.*

Condition for the body to leave the circle. For this to happen, the tension in the string should become zero earlier than the velocity *i.e.*,

$$h_2 < h_1$$

or $$\frac{u^2 + gr}{3g} < \frac{u^2}{2g} \quad \text{or} \quad u > \sqrt{2gr}$$

Hence *if $\sqrt{2gr} < v_L < \sqrt{5gr}$, the body leaves the circle somewhere between M and H, follows a parabolic path for a short while and then rejoins the circular path.*

For Your Knowledge

- For a body looping the loop, the following conditions hold good :

  $v_L \geq \sqrt{gr}$, $v_M \geq \sqrt{3gr}$, $v_H \geq \sqrt{gr}$

  and $T_L \geq \sqrt{6mg}$, $T_M \geq \sqrt{3mg}$, $T_H \geq 0$.
- Even if a body is projected with a minimum velocity of $\sqrt{5gr}$ from the lowest point, its velocity at the highest point will be $\sqrt{gr}$ and not zero.
- The body will oscillate about the lowest point of the vertical circle if $v < \sqrt{2gr}$.
- For the body to leave the circle, $\sqrt{2gr} < v_L < \sqrt{5gr}$.
- $T_L - T_H = 6 \times$ Weight of the body.

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Examples based on Motion in a Vertical Circle

Formulae Used

1. Velocity of the body at any point at a height h from the lowest point,
$$v = \sqrt{u^2 - 2gh}$$
2. Tension in the string at any point,
$$T = \frac{m}{r}(u^2 - 3gh + gr)$$
3. Tension at the lowest point,
$$T_L = \frac{m}{r}(u^2 + gr)$$
4. Tension at the highest point,
$$T_H = \frac{m}{r}(u^2 - 5gr)$$
5. Difference in tensions at the highest and lowest points,
$$T_L - T_H = 6\,mg$$
6. Minimum velocity at the lowest point for looping the vertical loop,
$$v_L = \sqrt{5gr}$$
7. Velocity at the highest point for looping the loop,
$$v_H = \sqrt{gr}$$

Units Used

Radius r and height h are in metre, velocities u and v are in ms^{-1}, tensions T, T_L and T_H are in newton.

Example 80. One end of a string of length 1.5 m is tied to a stone of mass 0.4 kg and the other end to a small pivot on a smooth vertical board. What is the minimum speed of the stone required at its lower-most point so that the string does not slack at any point in its motion along the vertical circle ? [NCERT]

Solution. Here $m = 0.4$ kg, $r = 1.5$ m, $g = 9.8\ ms^{-2}$

The minimum speed required at the lowermost point so that the string does not slack is given by

$$v_{min} = \sqrt{5gr} = \sqrt{5 \times 9.8 \times 1.5} = \mathbf{8.57\ ms^{-1}}$$

Example 81. A body weighing 0.4 kg is whirled in a vertical circle making 2 revolutions per second. If the radius of the circle is 1.2 m find the tension in the string, when body is (i) at the bottom of the circle, (ii) at the top of the circle. [NCERT]

Solution. Here $m = 0.4$ kg, $r = 1.2$ m, $\nu = 2$ rps

Angular speed, $\omega = 2\pi\nu = 2\pi \times 2 = 4\pi$ rad s^{-1}

(i) When body is at bottom of the circle. Let T be tension in the string. Then

$T_1 - mg$ = Centripetal force $= mr\omega^2$

or $T_1 = m(r\omega^2 + g) = 0.4\,[1.2 \times (4\pi)^2 + 9.8]$
$= 0.4\,[1.2 \times 16 \times 9.87 + 9.8]$
$= 0.4\,[189.5 + 9.8] = \mathbf{79.32\ N.}$

(ii) When the body is at the top of the circle. Let T_2 be the tension in the string. Then

$T_2 + mg$ = Centripetal force $= mr\omega^2$

or $T_2 = m(r\omega^2 - g) = 0.4\,[1.2 \times (4\pi)^2 - 9.8]$
$= 0.4\,[189.5 - 9.8] = \mathbf{71.88\ N.}$

Example 82. A bucket containing water is tied to one end of a rope of length 2.5 m and rotated about the other end in a vertical circle in such a way that the water in it does not spill. What is the minimum velocity of the bucket at which this happens and how many rotations per minute is it making then ? Take $g = 10\ ms^{-2}$.

Solution. Water in the bucket will not spill if the centripetal force is equal to the weight of water.

$$\frac{mv^2}{r} = mg \quad \text{or} \quad v = \sqrt{rg}$$

But $r = 2.5$ m, $g = 10\ ms^{-2}$

$\therefore$ $v = \sqrt{2.5 \times 10} = \mathbf{5\ ms^{-1}}$.

Angular speed, $\omega = \frac{v}{r} = \frac{5}{2.5} = 2$ rads^{-1}

Frequency of rotation, $\nu = \frac{\omega}{2\pi} = \frac{2}{2\pi}$ rps $= \frac{60}{\pi}$ **rpm.**

Example 83. A stone of mass 0.3 g is tied to one end of a string 0.8 m long and rotated in a vertical circle. At what speed of the ball will the tension in the string be zero at the highest point of the circle ? What would be the tension at the lowest point in this case ? Given $g = 9.8\ ms^{-2}$.

Solution. When tension is zero at the highest point, speed is minimum. It is given by

$$v_H = \sqrt{gr} = \sqrt{9.8 \times 0.8} = \mathbf{2.8\ ms^{-1}}$$

As $T_L - T_H = 6\,mg$

$\therefore$ $T_L = 6 \times 0.3 \times 9.8 - 0 = \mathbf{17.64\ N.}$

Example 84. In a circus, the diameter of globe of death is 20 m. From what minimum height must a cyclist start in order to go round the globe successfully ?

Solution. When the cyclist rolls down the incline,

Loss in P.E. = Gain in K.E.

$$mgh = \frac{1}{2}mv^2$$

$\therefore$ Velocity gained, $v = \sqrt{2gh}$

For looping the loop, the minimum velocity at the lowest point should be $\sqrt{5gr}$.

$\therefore$ $\sqrt{2gh} = \sqrt{5gr}$

or $h = \frac{5}{2}r = \frac{5}{2} \times 10 = \mathbf{25\ m.}$

[$\because$ Diameter $= 20$ m]

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Example 5.3. *An aeroplane flying in the sky dives with a speed of 36 kmh^{-1} in a vertical circle of radius 200 m. The weight of the pilot sitting in it is 75 kg. Calculate the force with which the pilot presses his seat when the aeroplane is (i) at the highest position and (ii) at the lowest position of the circle. Take $g = 10$ ms^{-2}.*

Solution. As shown in Fig. 5.106, let R_1 and R_2 be the normal reactions at highest and lowest positions of the vertical circle.

Fig. 5.106

(i) At the highest position. Here the net force $R_1 + mg$ provides the centripetal force mv^2/r.

$$\therefore \quad R_1 + mg = \frac{mv^2}{r}$$

or
$$R_1 = m\left(\frac{v^2}{r} - g\right)$$

But $m = 75$ kg, $v = 36$ kmh$^{-1} = 100$ ms^{-1},

$r = 200$ m, $g = 10$ ms^{-2}

$$\therefore \quad R_1 = 75\left[\frac{100 \times 100}{200} - 10\right]$$

$= 3000$ N $= 300$ kg wt.

(ii) At the lowest position. Here the net force $R_2 - mg$ provides the centripetal force mv^2/r.

$$\therefore \quad R_2 - mg = \frac{mv^2}{r}$$

or
$$R_2 = m\left[\frac{v^2}{r} + g\right] = 75\left[\frac{100 \times 100}{200} + 10\right]$$

$= 4500$ N $= 450$ kg wt.

✕ PROBLEMS FOR PRACTICE

1. A body weighing 0.5 kg tied to a string is projected with a velocity of 10 ms^{-1}. The body starts whirling in a vertical circle. If the radius of circle is 0.8 m, find the tension in the string when the body is (*i*) at the top of the circle and (*ii*) at the bottom of the circle. **[Ans.** (*i*) 3.8 N (*ii*) 67.4 N]

2. A child revolves a stone of mass 0.5 kg tied to the end of a string of length 40 cm in a vertical circle. The speed of the stone at the lowest point of the circle is 3 ms^{-1}. Calculate the tension in the string at this point. **[Ans.** 16.15 N]

3. A stone is tied to a weightless string and revolved in a vertical circle of radius 5 m. (*i*) What should be the minimum speed of the stone at the highest point of the circle so that the string does not slack ? (*ii*) What should be speed of the stone at the lowest point in this situation ? Take $g = 9.8$ ms^{-2}.

[Ans. (*i*) 7 ms^{-1} (*ii*) $7\sqrt{5}$ ms^{-1}]

4. The railway bridge over a canal is in the form of an arc of a circle of radius 20 m. What is the minimum speed with which a car can cross the bridge without leaving contact with the ground at the highest point ? Take $g = 9.8$ ms^{-2}. **(Ans.** 14 ms^{-1})

5. An aeroplane describes a vertical circle when looping. Find the radius of the greatest possible loop if the velocity of the aeroplane at the lowest point of its path is 50 ms^{-1}. Take $g = 10$ ms^{-2}.

(Ans. 50 m)

6. A weightless thread can bear tension upto 3.7 kg wt. A stone of mass 500 g is tied to it and revolves in a circular path of radius 4 m in vertical plane. If $g = 10$ ms^{-2}, then what will be the maximum angular velocity of the stone ? **(Ans.** 4 rad s^{-1})

✕ HINTS

1. Here $m = 0.5$ kg, $u = 10$ ms^{-1},
$r = 0.8$ m, $g = 9.8$ ms^{-2}

(*i*) $T_H = \frac{m}{r}(u^2 - 5gr) = \frac{0.5}{0.8}(10^2 - 5 \times 9.8 \times 0.8)$

$= 3.8$ N.

(*ii*) $T_L = \frac{m}{r}(u^2 - gr) = \frac{0.5}{0.8}(10^2 - 9.8 \times 0.8)$

$= 67.4$ N.

2. From Fig. 5.107,

$$T - mg = \frac{mv^2}{r}$$

or
$$T = m\left(\frac{v^2}{r} + g\right)$$

$$= 0.5 \times \left[\frac{3^2}{0.40} + 9.8\right]$$

$= 16.15$ N.

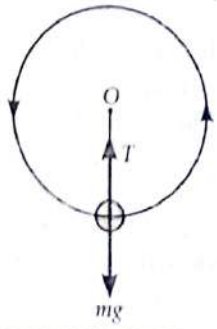

Fig. 5.107

6. Here $T_{max} = 3.7$ kg wt $= 3.7 \times 10 = 37$ N,

$m = 500$ g $= 0.5$ kg, $r = 4$ m

As $T_{max} = \frac{mv^2}{r} + mg$

$$\therefore \quad \frac{mv^2}{r} = T_{max} - mg = 37 - 0.5 \times 10 = 32$$

or
$$v^2 = \frac{32 \times r}{m} = \frac{32 \times 4}{0.5} = 256 \quad \text{or} \quad v = 16 \text{ ms}^{-1}$$

Hence $\omega = \frac{v}{r} = \frac{16}{4} = 4$ rads^{-1}.

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5.38 INERTIAL AND NON-INERTIAL FRAMES OF REFERENCE

71. *What are inertial frames of reference ? Give their important features and examples.*

Inertial frame of reference. *A frame of reference in which Newton's first law of motion holds good is called an inertial frame of reference.* In such a frame if no force acts on a body, it continues to be at rest or in uniform motion. So it is called an inertial frame. A frame of reference moving with a constant velocity with respect to an inertial frame of reference is also an inertial frame. Acceleration of particles in inertial frames is caused by real forces only.

The important features of inertial frames are as follows :

(*i*) All the fundamental laws of physics have been formulated in respect of inertial frame of reference.

(*ii*) All the fundamental laws of physics can be expressed so as to have the same mathematical form in all inertial frames of reference.

(*iii*) The mechanical and optical experiments performed in an inertial frame in any direction will always yield the same results.

Examples : (*i*) A frame of reference at rest w.r.t. fixed star is an inertial frame of reference.

(*ii*) Earth can be approximately taken as an inertial frame for measurements taken on its surface.

72. *What are non-inertial frames of reference ? Give examples. What are fictitious or pseudo forces ?*

Non-inertial frame of reference. *A frame of reference which is accelerating with respect to an inertial frame of reference is called non-inertial frame of reference.* Newton's laws cannot be applied to the non-inertial frames of reference.

For example, a bus moving along a circular track is an example of accelerated or non-inertial frame of reference. In such a frame, a body experiences an acceleration even if no external force acts on it. When the moving bus turns round a curve, the passengers are thrown away from the centre. This is due to centrifugal force. *Such a force which does not really act on particles but appears due to acceleration of the frame is called a fictitious or pseudo force.*

Very Short Answer Conceptual Problems

Problem 1. Why do bodies of small mass require small initial effort to bring them into motion ?

Solution. The inertia of a body is proportional to its mass. Smaller the mass, lesser is the opposition to the change of state of motion.

Problem 2. Can a body in linear motion be in equilibrium ?

Solution. Yes, provided the vector sum of the forces acting upon the body is zero.

Problem 3. A body is acted upon by a number of external forces. Can it remain at rest ? *[Himachal 05]*

Solution. If the vector sum of all the external forces is zero, then the body will remain at rest.

Problem 4. If the net force acting on the body be zero, will the body remain necessarily in rest position ? Explain your answer. *[Himachal 06]*

Solution. No, the body may be in the state of unifrom motion along a straight line even when the net force acting on the body is zero.

Problem 5. If a force is acting on a moving body perpendicular to the direction of motion, then what will be its effect on the speed and direction of the body ?

Solution. There will be no change in the speed of the body but the direction will change continuously.

Problem 6. Why do the passangers fall in backward direction when a bus suddenly starts moving from the rest position ? *[Delhi 96 ; Himachal 07C]*

Solution. When the bus suddenly starts moving, the lower part of the passenger's body begins to move along with the bus while the upper part tend to remain at rest due to inertia of rest. That is why a passenger standing or sitting loosely in a bus falls backward when the bus suddenly starts moving.

Problem 7. Why are passengers thrown forward from their seats when a speeding bus stops suddenly ? *[Himachal 09 ; Delhi 09]*

Solution. This is due to inertia of motion. When the speeding bus stops suddenly, lower part of the body in contact with the seat stops. The upper part of the body of the passengers tends to maintain its uniform motion. Hence the passengers are thrown forward.

Problem 8. A man jumping out of a moving train falls with his head forward. Why ?

Solution. As the man jumps out from a moving train, his feet suddenly come to rest on touching the ground while the upper part of his body continues to move forward. That is why he falls with his head forward. In order to save himself, he should run through some distance in the forward direction.

Problem 9. Why do the blades of an electric fan continue to rotate for some time after the current is switched off ?

Solution. This is due to inertia of motion that the blades of an electric fan continue to rotate for some time even after the fan has been switched off.

Problem 10. A stone when throw
dow smashes the windowpane to
from the gun passes through making

Solution. Due to its small speed
contact with the windowpane for
transfers its motion to the pane an
But the particles of windowpane n
to share the fast motion of the
undisturbed.

Problem 11. We beat a blank
dust particles. Why ?

Solution. When we beat bla
into motion. But the dust parti
due to inertia and get detache

Problem 12. If you jerk
under a book very quickly, the

Solution. The book conti
inertia.

Problem 13. Why fruits
its branches are shaken ?

Solution. Before shakin
When branches are shaken
fruits tend to remain at res
fruits get detached from

Problem 14. Why ar
taking a jump ?

Solution. An athle
before taking a jump
him in his muscular e

Problem 15. A pa
pushes it from withi

Solution. No, in
in a system.

Problem 16. If
it comes back to t

Solution. Bot
motion, the ball
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period, the ball
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Problem 17
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Problem 10. A stone when thrown on a glass window smashes the windowpane to pieces, but a bullet from the gun passes through making a clean hole. Why ?

Solution. Due to its small speed, the stone remains in contact with the windowpane for a longer duration. It transfers its motion to the pane and breaks it into pieces. But the particles of windowpane near the hole are unable to share the fast motion of the bullet and so remain undisturbed.

Problem 11. We beat a blanket with stick to remove dust particles. Why ? *[Himachal 04, 07, 09]*

Solution. When we beat blanket with a stick, it comes into motion. But the dust particles continue to be at rest due to inertia and get detached from the blanket.

Problem 12. If you jerk a piece of a paper placed under a book very quickly, the book will not move. Why ?

Solution. The book continues in its state of rest due to inertia.

Problem 13. Why fruits fall down from a tree, when its branches are shaken ?

Solution. Before shaking the branches, fruits are at rest. When branches are shaken, they come in motion while the fruits tend to remain at rest due to inertia of rest. As a result fruits get detached from the branches and fall down.

Problem 14. Why an athlete runs some steps before taking a jump ? *[Himachal 09]*

Solution. An athlete always runs for some distance before taking a jump so that inertia of motion may help him in his muscular efforts to take a longer jump.

Problem 15. A passenger sitting in a carriage at rest pushes it from within. Will the carriage move ?

Solution. No, internal forces cannot produce motion in a system.

Problem 16. If a ball is thrown up in a moving train, it comes back to the person's hands. Why ?

Solution. Both during its upward and downward motion, the ball continues to move (inertia of motion) with the same horizontal velocity as the train. In this period, the ball covers the same horizontal distance as the train and so it comes back to the thrower's hands.

Problem 17. Why are the passengers thrown outwards when a car in which they are travelling suddenly takes a circular turn ? *[Himachal 06]*

Solution. This is because the passengers tend to maintain their direction of motion (inertia of direction) while the direction of car changes when it takes the turn.

Problem 18. Why are the wheels of vehicles are provided with mudguards ? *[Himachal 05C, 06]*

Solution. When the wheel rotates at a high speed, the mud sticking to the wheel flies off tangentially, this is due to inertia of direction. In order that the flying mud does not spoil the clothes of passersby, the wheels are provided with mudguards.

Problem 19. An astronaut accidentally gets thrown out of his small spaceship accelerating in inter-stellar space at a constant rate of 100 ms $^{-2}$. What is the acceleration of the astronaut the instant after he is outside the spaceship ? *[NCERT]*

Solution. Assuming that there are no nearby stars to exert gravitational force and the small spaceship exerts negligible gravitational force on the astronaut, then moment he gets out of the ship, there is no external force on him. By the first law of motion, the acceleration of the astronaut is zero.

Problem 20. Why is Newton's first law of motion also called law of inertia ?

Solution. According to Newton's first law of motion, a body by itself cannot change its state of rest or of uniform in a straight line. This inability of a body is called inertia. That is why Newton's first law of motion is also called law of inertia.

Problem 21. A thief jumps from the upper storey of a house with a load on his back. What is the force of the load on his back when the thief is in air ?

Solution. Zero. This is because a condition of weightlessness exists during a free fall.

$$W = m(g - a) = m(g - g) = 0.$$

Problem 22. A soda-water bottle is falling freely. Will the bubbles of the gas rise in the water of the bottle ?

Solution. Bubbles will not rise in water. The water in the freely-falling bottle is in the state of weightlessness. Consequently, the pressure in water does not increase with depth. No upthrust acts on the bubbles and they do not rise.

Problem 23. A bird is sitting on the floor of a wire cage and the cage is in the hand of a boy. The bird starts flying in the cage. Will the boy experience any change in the weight of the cage ?

Solution. The air inside the wire cage is in free contact with the atmospheric air. When the bird starts flying inside the cage, the weight of the bird is no more experienced and the cage will appear lighter than before.

Problem 24. The distance travelled by a body is directly proportional to time. Is any external force acting on it ?

Solution. As $s \propto t$ or $s = kt$

$$\therefore \quad v = \frac{ds}{dt} = k, \qquad a = \frac{dv}{dt} = 0$$

i.e., the body is moving with a uniform velocity and no external force is acting on it.

Problem 25. Chinawares are wrapped in straw paper before packing. Why ?

Solution. The straw paper between the chinaware increases the time of experiencing the jerk during transportation. Hence, they strike against each other with less force and are less likely to be damaged.

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Problem 26. Why we are hurt less when we jump on a muddy floor in comparison to a hard floor ?

Solution. When we jump on a muddy floor, the floor is carried in the direction of the jump and the time interval Δt for which force acts is increased. This decreases rate of change of momentum and hence the force of reaction. Therefore we are hurt less.

Problem 27. Why are shockers used in cars, scooters and motorcycles ?

Solution. In the event of jerk or jump, the time for which the force acts increases. Since the product of force and time is to remain constant in a given situation, therefore the force decreases.

Problem 28. Why buffers are provided between the bogies of a railway train ?

Solution. Due to buffer spring, the time of impact between the bogies increases, and the force acting between the bogies (I = Impulse / time) decreases. Consequently, passengers sitting inside the bogies do not experience strong jerks. For the same reason shockers are provided in all vehicles.

Problem 29. Why is it necessary to bend knees while jumping from greater height ?

Solution. During the jump, our feet at once come to rest and for this smaller time (F = Impulse / time) a large force acts on feet. If we bend the knees slowly, the value of time of impact increases and less force acts on our feet. So we get less hurt.

Problem 30. A brick can be pushed gently on the smooth floor by applying force with our foot. But if we kick the brick, the foot is hurt. Why ?

Solution. When the brick is kicked, the time for which force is impressed is short and hence rate of change of momentum of the brick is large. The brick in turn, due to reaction applies large force on the foot. This force may hurt our foot.

Problem 31. Why is it that when a man jumps down from a height of several storeys into a stretched trapaulin, he receives no injury ?

Solution. When the man jumps, the trapaulin gets depressed at the place of impact. This increases the time of impact (Δt). As a result, the trapaulin exerts a very small force (F = impulse/time) on the man, and he receives no injury.

Problem 32. According to Newton's third law, every force is accompanied by equal and opposite force. How can anything move then ?

Solution. Though action and reaction act simultaneously, but they act on different bodies. This makes the motion possible.

Problem 33. Why is it difficult to drive a nail into a wooden block without supporting it ?

Solution. When we hit the nail with a hammer, the nail and unsupported block together move forward as a single system. There is no force of reaction. When the block is rested against a support, the reaction of the support holds the block in position and the nail is driven into the block.

Problem 34. Why it is difficult to climb up a greesy pole?

Solution. When a person climbs up a pole, he presses the pole downward with his feet and the pole, in turn, pushes the person upwards with an equal force. If the pole is greasy, its surface becomes slippery and the person is not able to press it. As there is no action, there will be no reaction. Hence it becomes difficult for the person to climb up.

Problem 35. In a tug of war, the team that pushes harder against the ground wins. Why ?

Solution. The team that pushes harder against ground gets greater reactional force and this leads them to win.

Problem 36. Can a sailboat be propelled by air blown at the sails from a fan attached to the boat ?

Solution. No. When the fan pushes the sail by blowing air, the air also pushes the fan in the opposite direction. As the fan is also a part of the boat, the vector sum of the momenta of the fan and the boat is zero. The boat will move only if some external agency applies force on it.

Problem 37. A man is at rest in the middle of a pond on perfectly frictionless ice. How can he get himself to the shore ?

Solution. If the man throws away his shirt in a direction opposite to the desired direction of motion, he can get himself to the shore.

Problem 38. Two bodies of masses M and m are allowed to fall from the same height. If the air-resistance be same for each body, will the two bodies reach the earth simultaneously ?

Solution. No. Let the air-resistance on each body be F.

Net downward force on body of mass $M = Mg - F$

Acceleration of body of mass M, $a = \dfrac{Mg - F}{M} = g - \dfrac{F}{M}$

Similarly acceleration of body of mass m, $a' = g - \dfrac{F}{m}$

As $M > m \quad \therefore \quad a > a'$

Thus the acceleration of the larger mass is greater, it will reach the earth earlier.

Problem 39. There is some water in a beaker placed on the pan of a spring balance. If we dip our finger in this water without touching the bottom of the beaker, then what would be the effect on the reading of the balance ?

Solution. The reading will increase. The water will exert upthrust on the finger and the finger will exert an equal force of reaction on water in the downward direction *i.e.*, on the bottom of the beaker.

Problem 40. Two boys having the same mass are standing on ice-skates at some distance apart on a frictionless surface. A rope is fastened around the body of a boy, the other end of which is in the hand of

What would happen if the second boy

Solution. The two boys will move to with the same velocity so that their com is still zero (conservation of momentu

Problem 41. A retarding force is motor car. If the speed of the motor much more distance will it cover bef the same retarding force ?

Solution. Work done against ret = Loss in

In first case, $F \times s = \frac{1}{2} mv^2$

In second case, $F \times s' = \frac{1}{2} m(2$

From (*i*) and (*ii*), $s' = 4s$

Thus the motor car will cove longer than before.

Problem 42. Why does a rifl on firing a bullet ?

Solution. Before firing, both at rest and their total momentu bullet gains a large momentum To conserve momentum, the rifl in the opposite direction. So the

Problem 43. Why is it advi one's shoulder when it is be

Solution. The recoiling g the gun is held tightly agains the gun will constitute one large and the recoil velocity

Problem 44. When a bo also moves up to meet it. noticeable. Why ?

Solution. We know t applied force and mass o massive body, therefore Hence the motion of the

Problem 45. A mete before it reaches the ear momentum ?

Solution. Since the gravity, therefore its conserved before and

Problem 46. Why strongly as a light rif

Solution. Recoil

$$\vec{V} = -\frac{m}{M}$$

the recoil of the ligh

m 47. W orce ?

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the other end of which is in the hand of the second boy. What would happen if the second boy pulls the rope ?

Solution. *The two boys will move towards each other with the same velocity so that their combined momentum is still zero (conservation of momentum).*

Problem 41. A retarding force is applied to stop a motor car. If the speed of the motor car is doubled, how much more distance will it cover before stopping under the same retarding force ?

Solution. Work done against retarding force

= Loss in K.E.

In first case, $F \times s = \frac{1}{2} mv^2$ ...(*i*)

In second case, $F \times s' = \frac{1}{2} m(2v)^2$ ...(*ii*)

From (*i*) *and* (*ii*), $s' = 4s$

Thus the motor car will cover a distance four times longer than before.

Problem 42. Why does a rifle give a backward kick on firing a bullet ?

Solution. *Before firing, both the bullet and the rifle are at rest and their total momentum is zero. After firing, the bullet gains a large momentum in the forward direction. To conserve momentum, the rifle gains an equal momentum in the opposite direction. So the rifle gives a backward kick.*

Problem 43. Why is it advisable to hold a gun tight to one's shoulder when it is being fired ? *[Himachal 08]*

Solution. *The recoiling gun can hurt the shoulder. If the gun is held tightly against the shoulder, the body and the gun will constitute one system. Total mass becomes large and the recoil velocity becomes small.*

Problem 44. When a body falls to the earth, the earth also moves up to meet it. But the earth's motion is not noticeable. Why ?

Solution. *We know that acceleration is the ratio of applied force and mass of the body. Since the earth is a massive body, therefore its acceleration is very small. Hence the motion of the earth is not noticeable.*

Problem 45. A meteorite burns in the atmosphere before it reaches the earth's surface. What happens to its momentum ?

Solution. *Since the meteorite moves under the force of gravity, therefore its momentum changes but remains conserved before and after burning.*

Problem 46. Why does a heavy rifle not kick as strongly as a light rifle using the same cartridge ?

Solution. *Recoil velocity of a rifle,*

$$\vec{V} = -\frac{m}{M}\vec{v} \qquad i.e., \qquad V \propto \frac{1}{M}$$

Thus the recoil velocity of the heavy rifle is smaller than that of the light rifle, so it does not kick strongly.

Problem 47. Why is static friction called a self-adjusting force ?

Solution. As the applied force increases, the static friction also increases and becomes equal to the applied force. That is why static friction is called a self-adjusting force.

Problem 48. Can we get off a frictionless horizontal surface by jumping ?

Solution. No, because a frictionless surface is unable to provide reaction which is necessary for jumping.

Problem 49. Automobile tyres are generally provided with irregular projections over their surfaces. Why ?

Solution. Irregular projections increase the friction between the rubber tyres and the road. This provides a firm grip between the tyres and the road and prevents slipping.

Problem 50. Carts with rubber tyres are easier to fly than those with iron wheels. Why ?

Solution. The coefficient of friction between rubber tyres and road is much smaller than that between iron wheels and the road.

Problem 51. Sand is thrown on tracks covered with snow. Why ? *[Himachal 09]*

Solution. When tracks are covered with snow, there is considerable reduction of frictional force. So, the driving is not safe. When sand is thrown on the snow-covered tracks, the frictional force increases. So, safe driving is possible.

Problem 52. Why are wheels of automobiles made circular ?

Solution. Circular wheels roll on the road and rolling friction comes into play during the motion of automobile. Rolling friction is less than the sliding friction. It is due to this reason that wheels of automobiles are made circular.

Problem 53. Proper inflation of tyres of vehicles saves fuel. Why ?

Solution. When the tyre is properly inflated, the area of contact between the tyre and the ground is reduced. This reduces rolling friction. Consequently, the automobile covers greater distance for the same quantity of fuel consumed.

Problem 54. It is difficult to move a cycle along a road with its brakes on. Explain ? *[Himachal 07C]*

Solution. When the cycle is moved with brakes on, the wheels can only skid. So, the friction is sliding in nature. Since the sliding friction is greater than rolling friction, therefore, it is difficult to move a cycle with its brakes on.

Problem 55. A large size brake on bicycle is as effective as small one. Comment.

Solution. Action of brakes is based upon friction. But the friction is independent of the area of surfaces in contact so long as the normal reaction remains the same. Hence, large size brakes and normal size brakes will be equally effective if the material of brakes remains unchanged.

Problem 56. Why is it difficult to put a cycle into motion than to maintain its motion ? *[Chandigarh 02]*

Solution. To put a cycle into motion, one needs to overcome limiting friction while to maintain its motion,

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one needs to overcome kinetic friction. Limiting friction is greater than the kinetic friction. So it is difficult to put a cycle into motion than to maintain its motion.

Problem 57. How does friction help us in walking ?

Solution. Due to friction, we are able to push the ground backward during walking. The reaction of the ground helps us to move forward.

Problem 58. Why do we slip on a rainy day ?

[Himachal 05C, 06 ; Delhi 10]

Solution. On a rainy day, the wet ground becomes very smooth. The friction between our feet and the ground is greatly reduced. It causes us to slip.

Problem 59. Why is it difficult to walk on sand ?

Solution. Because of its yielding nature, sand cannot exert as much forward thrust as hard ground does.

Problem 60. Why frictional force gets increased when a surface is polished beyond a certain limit ?

Solution. When surfaces are highly polished, the area of contact between them increases. As a result of this, a large number of atoms and molecules lying on both the surfaces start exerting strong attractive forces on each other and therefore frictional force increases.

Problem 61. When a person walks on a rough surface, the frictional force exerted by the surface on the person is opposite to direction of his motion. State whether it true or false ?

Solution. False. When a person walks, he pushes the ground backward with his foot. The tendency of the foot when it is in the contact of the earth is to move backward. Hence the force of friction acts in the forward direction, *i.e.*, in a direction in which the man walks.

Problem 62. Why has a horse to pull a cart harder during the first few steps of his motion ?

Or

A horse has to apply more force to start a cart than to keep it moving. Why ? *[Himachal 07C ; Delhi 09]*

Solution. During the first few steps of his motion, the horse has to work against the limiting friction and once the cart starts moving, the horse has to work against kinetic friction which is less than limiting friction.

Problem 63. Is it unreasonable to expect the coefficient of friction to exceed unity ?

Solution. No. The coefficient is less than unity for normal plane surfaces. But when the surfaces are so irregular that they have sharp minute projections and cavities on them, then the coefficient of friction may exceed unity.

Problem 64. Is friction a non-conservative force ?

Solution. Yes. When the direction of motion of a body reverses, the direction of friction is also reversed. Work has to be done against friction both during forward and return journey *i.e.*, work done against friction along a closed path is not zero. So friction is a non-conservative force.

Problem 65. For uniform circular motion, does the direction of centripetal force depend on the sense of rotation ?

Solution. No. Whether a body revolves clockwise or anticlockwise, the centripetal force always acts along the radius towards the centre of the circle.

Problem 66. A stone tied at the end of a string is whirled in a circle. When the string breaks, the stone flies away tangentially ? Explain why. *[Meghalaya 19]*

Solution. The instantaneous velocity of the stone moving round the circle is along the tangent to the circular path. When the string breaks, the centripetal force vanishes. Due to the inertia of motion, the stone flies away tangentially.

Problem 67. Why does a child in a merry-go-round press the side of his seat radially outward ?

Solution. In accordance with Newton's third law, the seat will press the child inward, providing the necessary centripetal force.

Problem 68. Why does skidding takes place generally on a rainy day along a curved path ?

Solution. The force of friction between the road and the tyres of the vehicle may not be sufficient to provide the necessary centripetal force.

Problem 69. A car is taking a sudden turn to the left. A passenger in the front seat finds himself sliding towards the door. Explain, indicating the forces acting on the passenger and the car at this instant.

Solution. The passenger in the front seat slides towards the door *i.e.*, away from the circular turn. This is because of the centrifugal force acting on the passenger.

Problem 70. The outer rail of a curved railway track is generally raised over the inner. Why ?

Solution. When the outer rail of a curved railway track is raised over the inner, the horizontal component of the normal reaction of the rails provides the necessary centripetal force for the train to enable it move along the curved path.

Problem 71. A motor cyclist is going in a vertical circle. What is the necessary condition so that he may not fall down ?

Solution. The necessary condition that the motor cyclist may not fall down is $mv^2 / r \geq mg$ *i.e.*, $v \geq \sqrt{rg}$ at the highest point and $v \geq \sqrt{5gr}$ at the lowest point.

Problem 72. A bucket containing water is rotated in a vertical circle. Explain, why the water does not fall.

Solution. For its revolution in a vertical circle, water in the bucket needs a centripetal force. The weight of the water due to which water can fall is used up in providing the necessary centripetal force and the water does not fall.

Problem 73. Why does a pilot looping a vertical loop not fall down even at the highest point ?

Solution. At the highest point of the vertical loop, the weight of the pilot due to which he can fall is used up in providing him the necessary centripetal force.

Problem 74. What is the maxi... at the lowest position for os... pendulum of length L. What can ... the velocity exceeds this value ?

Solution. The maximum p... lowest point for oscillation of si... When velocity exceeds $\sqrt{3gL}$ but ... leaves the vertical circle. W... completes the vertical circle.

Problem 75. A cricket player... the ball safely. Explain, why ? *[Hima...*

Solution. The impulse is e... force exerted by the ball and the... the hands, the time of catch... exerted on the hands become... not hurt the cricketer.

Problem 76. It is easy to c... a cricket ball even both are ... Why ? *[Hi...*

Solution. Due to its smal... table tennis ball is much sm... ball of same velocity. Less ... table tennis ball than the cr... catch the table tennis ball t...

Problem 77. Why does ... riding along a curved roa...

Solution. A cyclist be... horizontal component of ... ground provides the neces... along the curved road.

Problem 78. The mo... described by $y = ut + 1/$... the particle.

Solution. We have

Velocity,

Acceleration,

Force,

Thus the given e... particle under accele... position coordinate ir...

Problem 79. A par... force of $F = -kx$ w...

Short Answe...

Problem 1. Why ... appear to be contra...

Solution. In our ... when we push a b... covers some dista...

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Problem 74. What is the maximum possible velocity at the lowest position for oscillation of a simple pendulum of length L. What can happen to the motion if the velocity exceeds this value ?

Solution. The maximum possible velocity at the lowest point for oscillation of simple pendulum $= \sqrt{3gL}$. When velocity exceeds $\sqrt{3gL}$ but is less than $\sqrt{5gL}$, the bob leaves the vertical circle. When $v = \sqrt{5gL}$, the bob completes the vertical circle.

Problem 75. A cricket player lowers his hands to catch the ball safely. Explain, why ?

[Himachal 04, 05C, 07C, 09, 09C]

Solution. The impulse is equal to the product of the force exerted by the ball and the time of catch. By lowering the hands, the time of catch increases. Then the force exerted on the hands becomes much smaller and it does not hurt the cricketer.

Problem 76. It is easy to catch a table tennis ball than a cricket ball even both are moving with same velocity. Why ? *[Himachal 09 ; Central Schools 05]*

Solution. Due to its small mass, the momentum of the table tennis ball is much smaller than that of the cricket ball of same velocity. Less force is required to stop the table tennis ball than the cricket ball. Hence it is easy to catch the table tennis ball than the cricket ball.

Problem 77. Why does a cyclist bend inwards while riding along a curved road ? *[C.S. 05 ; Himachal 07C]*

Solution. A cyclist bends inwards because then the horizontal component of the normal reaction of the ground provides the necessary centripetal force for going along the curved road.

Problem 78. The motion of a particle of mass m is described by $y = ut + 1/2\, gt^2$. Find the force acting on the particle. *[NCERT]*

Solution. We have $y = ut + 1/2gt^2$

Velocity, $v = \dfrac{dy}{dt} = u + gt$

Acceleration, $a = \dfrac{dv}{dt} = g$

Force, $F = ma = mg$

Thus the given equation describes the motion of a particle under acceleration due to gravity and y is the position coordinate in the direction of g.

Problem 79. A particle of mass 0.3 kg is subjected to a force of $F = -kx$ with $k = 15\ \text{Nm}^{-1}$. What will be its initial acceleration, if it is released from a point 20 cm away from the origin ? *[AIEEE 05]*

Solution. Here

$$k = 15\ \text{Nm}^{-1},\ x = 20\ \text{cm} = 0.20\ \text{m},\ m = 0.3\ \text{kg}$$

$$F = -kx = 15 \times 0.20 = -3\ \text{N}$$

Acceleration, $a = \dfrac{F}{m} = \dfrac{-3}{0.3} = -10\ \text{ms}^{-2}$.

Problem 80. A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m. A force P is applied at the free end of the rope. Find the force exerted by the rope on the block. *[AIEEE 03 ; Delhi 13]*

Solution. If a is the acceleration produced, then

$$P = (M + m)\, a \quad \text{or} \quad a = \frac{P}{M + m}$$

Force exerted by the rope on the block

$$F = Ma = \frac{MP}{M + m}.$$

Problem 81. Three forces F_1, F_2 and F_3 are acting on a particle of mass m, such that F_2 and F_3 are mutually perpendicular and under their effect, the particle remains stationary. What will be the acceleration of the particle, if the force F_1 is removed ? *[AIEEE 02]*

Solution. As the particle remains stationary under the action of the three forces, so $\vec{F_1} + \vec{F_2} + \vec{F_3} = 0$

When force $\vec{F_1}$ is removed, the net force left is

$$\vec{F_2} + \vec{F_3} = -\vec{F_1}$$

Acceleration, $a = -\dfrac{\vec{F_1}}{m}$.

Problem 82. A spring balance is attached to the ceiling of a lift. A man hangs his bag on the spring and the spring reads 49 N, when the lift is stationary. What will be the reading of the spring balance, if the lift moves downward with an acceleration of 5 ms^{-2}. *[AIEEE 03]*

Solution. *When the lift is stationary.* The reaction of the spring is equal to weight of the bag.

$$\therefore \quad R = mg = 49\ \text{N} \quad \text{or} \quad m = \frac{49}{g} = \frac{49}{9.8} = 5\ \text{kg}$$

When the lift moves downward. The reaction is

$$R' = m(g - a) = 5(9.8 - 5) = \mathbf{24\ N}.$$

Short Answer Conceptual Problems

Problem 1. Why does Newton's first law of motion appear to be contradicted in our day-to-day life ?

Solution. In our day-to-day observations, we find that when we push a block lying on the surface of a table, it covers some distance before it stops. At first sight, it seems to contradict Newton's first law of motion. However, the motion of the block is being opposed by the force of friction between the block and the table and also by air resistance. In the absence of such opposing forces, the block would continue to move on its own.

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Problem 2. Explain, why

(a) The passengers are thrown forward from their seats, when a speeding bus stops suddenly.

(b) Does a cricketer move his hand backwards while holding a catch ?

(c) Is the boat pushed away when a man jumps out of the boat ? *[Delhi 08]*

Solution. (*a*) Refer to the solution of Problem 7 on page 5.62.

(*b*) Refer to the answer of NCERT Exercise 5.23(*d*) on page 5.79.

(*c*) Initially, the total momentum of the boat and the man is zero. As the man jumps out of the boat, he gains momentum in the forward direction. To conserve momentum, the boat also gains an equal and opposite momentum. So the boat moves away from the shore.

Problem 3. Explain : (a) Why are ball bearings used in machinery ? (b) Why does a horse have to apply more force to start a cart than to keep it moving ? (c) What is the need for banking the tracks ? *[Delhi 09]*

Solution. (*a*) By using ball bearings between the moving parts of a machinery, the sliding friction gets converted into rolling friction. The rolling friction is much smaller than sliding friction. This reduces power dissipation.

(*b*) Refer to the solution of Problem 62 on page 5.66.

(*c*) When the circular track is banked, the horizontal component of the normal reaction of the road provides the necessary centripetal force for the vehicle to move it along the curved path. This reduces wear and tear of the tyres.

Problem 4. The speed of driving a car safely depends upon the range of headlight. Explain.

Solution. By the range of the headlight of a car, we mean the maximum distance(*s*) upto which an obstacle on the road can be seen by the driver in the darkness. The driver has to stop the car before it reaches the obstacle. The retarding force acting on the car is constant. If, on applying brakes, the ratardation in the car is a, then to stop the car within distance s, the speed v of the car should be less than $\sqrt{2as}$. Thus the speed of the car depends on the range (s) of the headlight.

Problem 5. A bird is sitting on the floor of a closed glass cage and the cage is in the hand of a girl. Will the girl experience any change in the weight of the cage when the bird (*i*) starts flying in the cage with a constant velocity (*ii*) flies upwards with acceleration (*iii*) flies downwards with acceleration ?

Solution. In a closed cage, the inside air is bound with the cage.

(*i*) As the acceleration is zero, there is no change in the weight of the cage.

(*ii*) In this case, the reaction R is given by

$$R - Mg = Ma \quad \text{or} \quad R = M(g + a)$$

Thus the cage will appear heavier than before.

(*iii*) In this case, the reaction R is given by

$$Mg - R = Ma \quad \text{or} \quad R = M(g - a)$$

Thus the cage will appear lighter than before.

Problem 6. A man stands in a lift going downward with uniform velocity. He experiences a loss of weight at the start but not when lift is in uniform motion. Explain why ?

Solution. The apparent weight is given by

$$R = W - ma = mg - ma \quad \text{or} \quad R = m(g - a)$$

Since lift is in acceleration in the start, $a \neq 0$, so $R < mg$. When lift comes in uniform motion, acceleration ceases ($a = 0$) and man experiences his own weight.

Problem 7. Aeroplanes having wings fly at low altitudes while jet planes fly at high altitudes. Why ?

Solution. The wings of the aeroplane push the external air backward and the aeroplane moves forward by the reaction of the pushed air. At lower altitude the air is dense and so the aeroplane receives sufficient reactional force to move forward. In the jet plane the external air is sucked into the plane and compressed. Hence for the air to be dense is not only unnecessary but also undesirable. The reason is that jet planes fly with very large velocity. If air is dense, then due to the air-friction the plane will become very hot. Therefore, jet planes fly at high altitude where air-density is very small.

Problem 8. A person (mass m) is hanging from a rope fastened to a stationary balloon (mass M). If the person climbs along the rope, then with what velocity the balloon will move and in what direction ? The velocity of the person relative to the rope is v.

Solution. The initial momentum of the person and balloon is zero. Hence when the person climbs, the balloon will come down with such a velocity u that the total momentum be still zero. The velocity of the person relative to earth is $(v - u)$. Hence

$$m(v - u) - Mu = 0 \quad \text{or} \quad u = \frac{mv}{M + m}.$$

Problem 9. A rope is hanging from a tree, as shown in Fig. 5.108. Bananas are fastened to the higher end of the rope, and a monkey is hanging along the rope near its lower end. The monkey climbs along the rope, will it be able to eat the bananas ?

Solution. No. As the monkey climbs up, the bananas also move up through equal distance so that the momentum is conserved. Here the branch of the tree acts as a pulley which reverses the direction of momentum. When the monkey and the bananas both move up with the same velocity, the combined momentum remains zero.

Fig. 5.108

Problem 10. A long rope is hanging, ... pulley. Two monkeys of equal weights ... the opposite ends of the rope. One of t... more rapidly relative to rope. Which mo... first at the top ? The pulley is frictionless ... massless and inextensible.

Solution. There is no external fo... provide momentum to any monkey ... themselves give equal momenta to eac... the rope). Therefore, two monkeys will ... at the same rate relative to the earth. A... equal, they will reach the top simultan...

Problem 11. When a ball is th... momentum first increases and then ... conservation of momentum violated ...

Solution. No. The combined mo... and the earth is conserved. The ball ... the same force as the earth attracts th... moves upward, its momentum decr... direction but simultaneously the mo... increases in the upward directio... Similarly, when the ball falls d... increases in the downward directio... the momentum of the earth incr... direction at the same rate.

Problem 12. A disc of mass m ... stiff spring is attached to it and i... end of the spring is attached a d... What minimum force should b... disc to press the spring such tha... off the table when the exte... removed ?

Solution. The minimum ap... When a force mg is applied ver... upper disc, the lower disc gets ... with a force equal to mg. The ... reaction equal to mg. When the ... removed, this force of reactio...

Problem 13. Can a sing... nature ? Give reason.

Solution. No. According ... motion, to every action the ... opposite reaction. So the f... When we talk of a single f... only one aspect of the mutu...

Problem 14. A body is ... transparent cabin falling ... Describe the motion of ... observer (*i*) sitting in the c...

Solution. (*i*) The body ...

(*ii*) The body will appe...

Problem 15. A ball is ... ceiling of a motor car. ...

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Problem 10. A long rope is hanging, passing over a pulley. Two monkeys of equal weights climb up from the opposite ends of the rope. One of them climbs up more rapidly relative to rope. Which monkey will reach first at the top ? The pulley is frictionless and the rope is massless and inextensible.

Solution. There is no external force which may provide momentum to any monkey. The monkeys themselves give equal momenta to each other (through the rope). Therefore, two monkeys will climb up the rope at the same rate relative to the earth. As their masses are equal, they will reach the top simultaneously.

Problem 11. When a ball is thrown upward, its momentum first increases and then decreases. Is the conservation of momentum violated in this process ?

Solution. No. The combined momentum of the ball and the earth is conserved. The ball attracts the earth by the same force as the earth attracts the ball. When the ball moves upward, its momentum decreases in the upward direction but simultaneously the momentum of the earth increases in the upward direction at the same rate. Similarly, when the ball falls down, its momentum increases in the downward direction but simultaneously the momentum of the earth increases in the upward direction at the same rate.

Problem 12. A disc of mass m is placed on a table. A stiff spring is attached to it and is vertical. To the other end of the spring is attached a disc of negligible mass. What minimum force should be applied to the upper disc to press the spring such that the lower disc is lifted off the table when the external force is suddenly removed ?

Solution. The minimum applied force should be mg. When a force mg is applied vertically downwards on the upper disc, the lower disc gets pressed against the floor with a force equal to mg. The floors exerts an upward reaction equal to mg. When the external force is suddenly removed, this force of reaction lifts up the lower disc.

Problem 13. Can a single isolated force exist in nature ? Give reason.

Solution. No. According to Newton's third law of motion, to every action there is always an equal and opposite reaction. So the forces always exist in pairs. When we talk of a single force, we are just considering only one aspect of the mutual interaction.

Problem 14. A body is dropped from the ceiling of a transparent cabin falling freely towards the earth. Describe the motion of the body as observed by an observer (*i*) sitting in the cabin (*ii*) standing on the earth.

Solution. (*i*) The body will appear stationary in air.

(*ii*) The body will appear falling freely under gravity.

Problem 15. A ball is suspended by a cord from the ceiling of a motor car. What will be the effect on the position of the ball if (*i*) the car is moving with uniform velocity (*ii*) the car is moving with accelerated motion and (*iii*) the car is turning towards right ?

Solution.

(*i*) The ball will remain suspended vertically.

(*ii*) The ball will move in backward direction.

(*iii*) The ball will move towards left.

Problem 16. A block of mass M is supported by a cord C from a rigid support, and another cord D is attached to the bottom of the block. If you give a sudden jerk to D, it will break. But if you pull on D steadily, cord C will break. Why ? *[Delhi 13]*

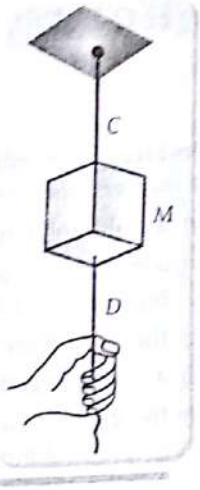

Fig. 5.109

Solution. When a sudden jerk is given to D, the upper portion of the system is not able to share the force in short time and the block tends to remain at rest (inertia of rest). So the cord D breaks.

When the cord D is pulled steadily, the force gets sufficient time to reach the cord C which ultimately breaks.

Problem 17. A vessel containing water is given a constant acceleration a towards the right, along a horizontal path. Which of the following figures correctly represents the surface of the liquid ?

Solution. Fig. 5.110(*c*) is correct. This is because the liquid experiences a backward force.

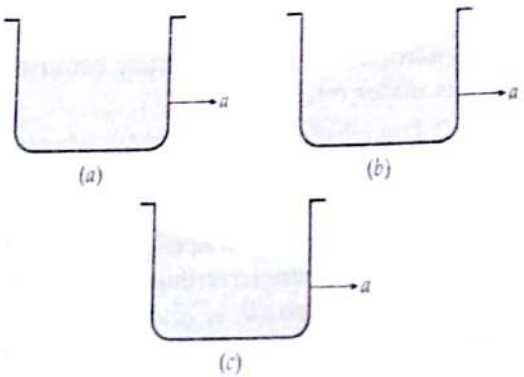

Fig. 5.110

Problem 18. When brakes are applied in a bicycle, the brake shoes apply frictional force on the wheels. The forces are internal for the system (bicycle). For retardation, external forces must act on the system. The force of friction at the road surface remains the same before and after the brakes are applied ($f = \mu R$). How does the bicycle stop ?

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Solution. When brakes are applied, the wheels are prevented from rolling. During rolling, the friction at the road surface does not cause retardation. When rolling is prevented, the wheels have to slip on the road. The friction of the road now causes retardation.

Problem 19. A light string passing over a smooth pulley connects two blocks of masses m_1 and m_2 (vertically). If the acceleration of the system is $g/8$, find the ratio of the two masses. *[AIEEE 02]*

Solution. As $a = \dfrac{m_1 - m_2}{m_1 + m_2} g$ $\therefore$ $\dfrac{g}{8} = \dfrac{m_1 - m_2}{m_1 + m_2} g$

or $\dfrac{m_1 - m_2}{m_1 + m_2} = \dfrac{1}{8}$

or $\dfrac{(m_1 + m_2) + (m_1 - m_2)}{(m_1 + m_2) - (m_1 - m_2)} = \dfrac{8 + 1}{8 - 1}$

or $\dfrac{m_1}{m_2} = \dfrac{9}{7} = 9 : 7.$

HOTS

Problems on Higher Order Thinking Skills

Problem 1. *For ordinary terrestrial experiments, which of the observers below are inertial and which non-inertial :*

(a) a child revolving in a "giant wheel",

(b) a driver in a sports car moving with a constant high speed of 200 km/h on a straight road,

(c) the pilot of an aeroplane which is taking off,

(d) a cyclist negotiating a sharp turn,

(e) the guard of a train which is slowing down to stop at a station ?

Solution. *(a)* The child is *non-inertial observer*, because the revolving giant wheel has an accelerated motion.

(b) The driver is an *inertial observer*, because the sports car is moving with a constant speed in a straight line.

(c) The pilot is *non-inertial observer*, because the aeroplane accelerates while taking off.

(d) The cyclist is *non-inertial observer*, because cycle has an accelerated motion at the sharp turn.

(e) The guard is *non-inertial observer*, because the train is under retardation.

Problem 2. *One often comes across the following type of statement concerning circular motion : 'A particle moving uniformly along a circle experiences a force directed towards the centre (centripetal force) and an equal and opposite force directed away from the centre (centrifugal force). The two forces together keep the particle in equilibrium'. Explain what is wrong with this statement.* **[NCERT]**

Solution. The statement is correct relative to a non-inertial frame rotating with the particle. For example, consider an observer moving with the same acceleration $(= v^2 / r)$ as that of the particle. For him, the particle remains at rest. The centripetal force equals the centrifugal force. Centrifugal force is not a real force. It arises only because of the non-inertial nature of the observer himself.

The given statement is wrong relative to an inertial frame e.g., the laboratory frame. For an inertial observer (stationary observer), the particle in circular motion is not in equilibrium, it has a centripetal acceleration. There is no force such as centrifugal force.

Problem 3. *Two particles of masses m_1 and m_2 in projectile motion have velocities $\vec{v}_1$ and $\vec{v}_2$ respectively at time $t = 0$. They collide at time t_0. Their velocities become $\vec{v}_1'$ and $\vec{v}_2'$ at time $2t_0$ while still moving in air. Find the value of $|(m_1\vec{v}_1' + m_2\vec{v}_2') - (m_1\vec{v}_1 + m_2\vec{v}_2)|$.* **[IIT Screening 01]**

Solution. As there is no external force in the horizontal direction, the momentum is changed in the vertical direction only by the gravitational force in time 0 to $2t_0$. Change in momentum = external force × time interval

$$\therefore \; |(m_1\vec{v}_1' + m_2\vec{v}_2') - (m_1\vec{v}_1 + m_2\vec{v}_2)|$$

$$= (m_1 + m_2)\, g \times (2t_0 - 0) = 2(m_1 + m_2)\, g\, t_0.$$

Problem 4. *Two bodies A and B, each of mass m, are connected together by a massless spring. A force F acts on the mass B as shown in Fig. 5.111. At the instant shown, the body A has an acceleration a, what is the acceleration of B ?* **[CPMT 93]**

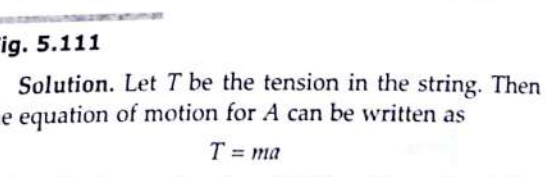

Fig. 5.111

Solution. Let T be the tension in the string. Then the equation of motion for A can be written as

$$T = ma$$

Let a' be the acceleration of B. Then for motion of B,

$$ma' = F - T = F - ma$$

or $$a' = \frac{F}{m} - a.$$

Problem 5. *A piece of uniform s*
that its free end just touches horizo
now the upper end of the string is re
instant during the falling of the stri
surface is three times the weight o
lying on the surface.

Solution. Let m be the mass
string. Each element of the string fa
at the end is zero. When the str
distance y, its velocity v at that

$$v^2 - 0^2 = 2gy$$

or $$v = \sqrt{2gy}$$

After this instant, the length of the string that falls in small time dt is

$$dy = v\,dt$$

Mass of length dy of the string

$$= m\,dy = mv\,dt$$

Momentum transferred to the table in small time dt is

$$dp = (mv\,dt)\,.\,v = m$$

$\therefore$ Force exerted on the tabl

$$F_1 = \frac{dp}{dt} = mv^2 = m$$

Since a length y of the str
table which also exerts force o

F_2 = weight of len

Total force exerted on the

$$F = F_1 + F_2 = (2\ my)\,g +$$

$= 3 \times$ Weight of the par

Problem 6. *An explosion blo*
Two pieces whose masses are 20
right angles to each other with
12 ms^{-1} respectively. If third pie
25 ms^{-1}, then calculate the mas
the direction of flight of this piec

Solution. As shown in Fig
explodes at point O. The 100 k
and 200 kg piece along OB. T
moves along OC making an a

Before explosion, momer
direction is zero.

By conservation of mome

$$100 \times 12 - mv \cos\theta = 0$$

or $$1200 = mv$$

$\therefore$ $$m \cos\theta = \frac{12}{v}$$

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Problem 5. *A piece of uniform string hangs vertically so that its free end just touches horizontal surface of a table. If now the upper end of the string is released, show that at any instant during the falling of the string, the total force on the surface is three times the weight of that part of the string lying on the surface.* **[Roorkee 96]**

Solution. Let m be the mass per unit length of the string. Each element of the string falls freely. Initial velocity at the end is zero. When the string has fallen through distance y, its velocity v at that instant is given by

$$v^2 - 0^2 = 2gy$$

or $$v = \sqrt{2gy}$$

After this instant, the length of the string that falls in small time dt is

$$dy = v\,dt$$

Mass of length dy of the string

$$= m dy = mv dt$$

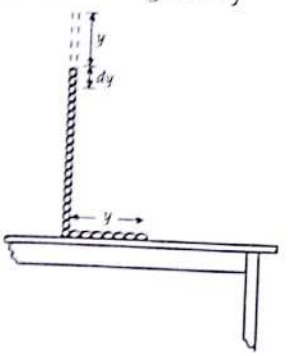

Fig. 5.112

Momentum transferred to the table in small time dt is

$$dp = (mv dt).v = mv^2\,dt$$

∴ Force exerted on the table,

$$F_1 = \frac{dp}{dt} = mv^2 = m(2gy) = (2my)g$$

Since a length y of the string already lies on the table which also exerts force on the table given by

$$F_2 = \text{weight of length } y = (my)g$$

Total force exerted on the table is

$$F = F_1 + F_2 = (2my)g + (my)g = 3(my)g$$

$= 3 \times$ Weight of the part of the string on the table.

Problem 6. *An explosion blows a rock into three pieces. Two pieces whose masses are 200 kg and 100 kg go off at right angles to each other with a velocity of 8 ms^{-1} and 12 ms^{-1} respectively. If third piece flies off with a velocity of 25 ms^{-1}, then calculate the mass of this piece and indicate the direction of flight of this piece in a diagram.*

Solution. As shown in Fig. 5.113, suppose the rock explodes at point O. The 100 kg piece moves along OA and 200 kg piece along OB. The third piece of mass m moves along OC making an angle θ with OA.

Before explosion, momentum of the rock in any direction is zero.

By conservation of momentum along OA,

$$100 \times 12 - mv\cos\theta = 0$$

or $$1200 = mv\cos\theta$$

$$\therefore \quad m\cos\theta = \frac{1200}{v} = \frac{1200}{25} = 48 \qquad ...(i)$$

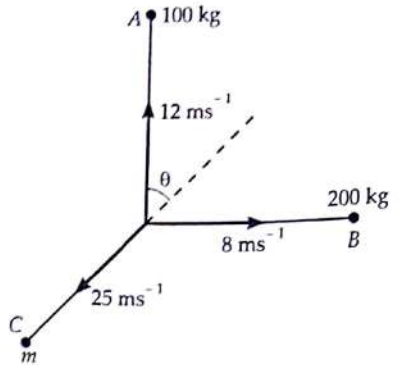

Fig. 5.113

By conservation of momentum along OB,

$$200 \times 8 - mv\sin\theta = 0$$

or $$1600 = mv\sin\theta$$

or $$m\sin\theta = \frac{1600}{v} = \frac{1600}{25} = 64 \qquad ...(ii)$$

Squaring and adding (*i*) and (*ii*), we get

$$m^2 = 48^2 + 64^2 = 6400 \quad \therefore \quad m = 80 \text{ kg}$$

Dividing (*ii*) by (*i*), we get,

$$\tan\theta = \frac{64}{48} = \frac{4}{3} \quad \therefore \quad \theta = \tan^{-1}\left(\frac{4}{3}\right) = 53°.$$

Problem 7. *A block of mass 0.1 kg is held against a wall by applying a horizontal force of 5 N on the block. If the coefficient of friction between the block and the wall is 0.5, what is the magnitude of the frictional force acting on the block ?* **[IIT 94]**

Solution. In Fig. 5.114, the block presses the wall with a hori- zontal force, $F = 5$ N. The wall exerts a force of reaction R on the block. The weight mg acts vertically downwards and static friction f_s acts vertically upward.

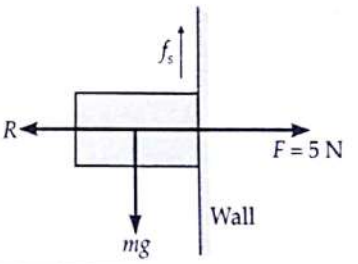

Fig. 5.114

In equilibrium,

$$R = 5 \text{ N}$$

and $$f_s = mg$$

$$= 0.1 \times 9.8 = \mathbf{0.98\ N.}$$

Problem 8. *A body of mass 2 kg is being dragged with a uniform velocity of 2 ms^{-2} on a rough horizontal plane. The coefficient of friction between the body and the surface is 0.2. Calculate the amount of heat generated per second. Take $g = 9.8\ ms^{-2}$ and $J = 4.2\ J\ cal^{-1}$.* **[IIT 80]**

Solution. Here $m = 2$ kg, $u = 2\ ms^{-1}$, $\mu = 0.2$

Force of friction,

$$f = \mu R = \mu mg = 0.2 \times 2 \times 9.8 = 3.92 \text{ N}$$

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Distance moved per second, $s = ut = 2 \times 1 = 2$ m

Work done per second, $W = f \cdot s = 3.92 \times 2 = 7.84$ J

Heat produced,

$$H = \frac{W}{J} = \frac{7.84}{42} = 1.87 \text{ cal.}$$

Problem 9. *An aeroplane requires for take off a speed of 80 kmh^{-1}, the run on the ground being 100 m. The mass of the aeroplane is 10^4 kg and the coefficient of friction between the plane and the ground is 0.2. Assume that the plane accelerates uniformly during the take off. What is the maximum force required by the engine of the plane for take off ?* [IIT 77]

Solution. Here $u = 0$, $s = 100$ m,

$$v = 80 \text{ kmh}^{-1} = 80 \times \frac{5}{18} = \frac{200}{9} \text{ ms}^{-1}$$

As $v^2 - u^2 = 2as$

$$\therefore \quad \left(\frac{200}{9}\right)^2 - 0 = 2a \times 100$$

or $$a = \frac{40000}{81 \times 200} = \frac{200}{81} \text{ ms}^{-2}$$

Force required to produce acceleration a,

$$F_1 = ma = 10^4 \times \frac{200}{81} = 2.47 \times 10^4 \text{ N}$$

Force required to overcome friction,

$$F_2 = \mu R = \mu mg = 0.2 \times 10^4 \times 9.8 = 1.96 \times 10^4 \text{ N}$$

Maximum force required by the engine for take off,

$$F = F_1 + F_2 = 2.47 \times 10^4 + 1.96 \times 10^4 = 4.43 \times 10^4 \text{ N.}$$

Problem 10. *A body of mass m rests on a horizontal floor with which it has a coefficient of static friction μ. It is desired to make the body move by applying the minimum possible force F. Find the magnitude of F and the direction in which it has to be applied.*

Solution. As shown in Fig. 5.115, suppose the force F is applied at an angle θ with the horizontal.

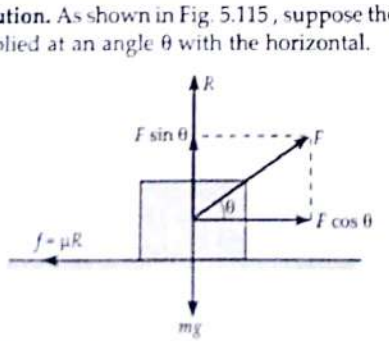

Fig. 5.115

For horizontal equilibrium,

$$F \cos \theta = \mu R \quad \ldots(1)$$

For vertical equilibrium,

$$R + F \sin \theta = mg$$

or $$R = mg - F \sin \theta$$

Substituting this value of R in (1), we get $\ldots(2)$

$$F \cos \theta = \mu (mg - F \sin \theta)$$

$$= \mu mg - \mu F \sin \theta$$

or $$F (\cos \theta + \mu \sin \theta) = \mu mg$$

or $$F = \frac{\mu mg}{\cos \theta + \mu \sin \theta} \quad \ldots(3)$$

For F to be minimum, the denominator $(\cos \theta + \mu \sin \theta)$ should be maximum.

$$\therefore \quad \frac{d}{d\theta}(\cos \theta + \mu \sin \theta) = 0$$

$$- \sin \theta + \mu \cos \theta = 0$$

or $$\tan \theta = \mu$$

or

Then $$\sin \theta = \frac{\mu}{\sqrt{1+\mu^2}} \text{ and } \cos \theta = \frac{1}{\sqrt{1+\mu^2}}$$

Hence $$F_{min} = \frac{\mu mg}{\frac{1}{\sqrt{1+\mu^2}} + \frac{\mu^2}{\sqrt{1+\mu^2}}} = \frac{\mu mg}{\sqrt{1+\mu^2}}.$$

Angle which the force makes with the horizontal,

$$\theta = \tan^{-1} \mu.$$

Problem 11. *The pulleys and strings shown in Fig. 5.116 are smooth and of negligible mass. For the system to remain in equilibrium, what should be the angle θ ?* [IIT Screening 01]

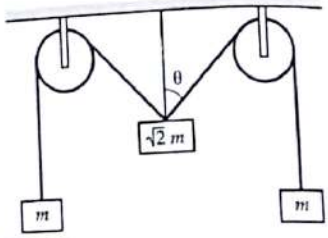

Fig. 5.116

Solution. The free body diagrams of the masses $\sqrt{2}m$ and m are shown in Fig. 5.117.

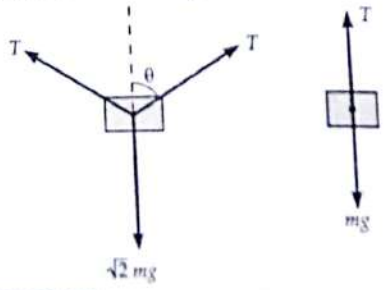

Fig. 5.117

For the equilibrium of mass m,

$$mg = T$$

For the equilibrium of mass $\sqrt{2}m$,

$$\sqrt{2}mg = 2T \cos \theta$$

$$\sqrt{2} \times T = 2T \cos \theta$$

or $$\cos \theta = \frac{1}{\sqrt{2}} \text{ or } \theta = 45$$

or

Problem 12. *A string of negligible ma clamped pulley of mass m supports a bloc shown in Fig. 5.118. Find the force exerted the clamp.*

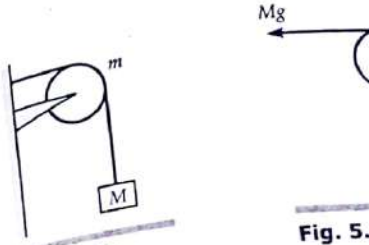

Fig. 5.118

Solution. The free body diagra shown in Fig. 5.119.

For the equilibrium of the pul

F_x = Horizontal componen by the clamp on the p

F_y = Vertical component o by the clamp on the p

The net force exerted on the

$$F = \sqrt{F_x^2 + F_y^2} = g\sqrt{(M$$

Problem 13. *An insect crawls very slowly (see Fig. 5.120). between the insect and the surfa the centre of the hemispherical su angle α with the vertical, find t of α.*

Fig. 5.120

Solution. Refer to Fig components of the forces

$F \cos \alpha$

or $\tan \alpha$

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For the equilibrium of mass m,

$$mg = T$$

For the equilibrium of mass $\sqrt{2}m$,

$$\sqrt{2}mg = 2T\cos\theta$$

$$\sqrt{2} \times T = 2T\cos\theta$$

or $$\cos\theta = \frac{1}{\sqrt{2}} \quad \text{or} \quad \theta = 45°.$$

Problem 12. *A string of negligible mass going over a clamped pulley of mass m supports a block of mass M as shown in Fig. 5.118. Find the force exerted on the pulley by the clamp.* **[IIT Screening 01]**

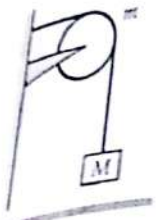

Fig. 5.118

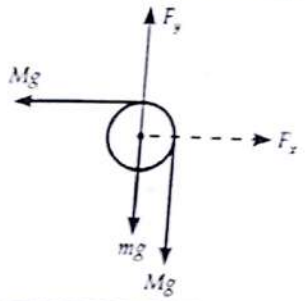

Fig. 5.119

Solution. The free body diagram for the pulley is shown in Fig. 5.119.

For the equilibrium of the pulley,

F_x = Horizontal component of the force by the clamp on the pulley = Mg

F_y = Vertical component of the force by the clamp on the pulley = $(M + m)\,g$

The net force exerted on the pulley by the clamp,

$$F = \sqrt{F_x^2 + F_y^2} = g\sqrt{(M+m)^2 + M^2}.$$

Problem 13. *An insect crawls up a hemispherical surface very slowly (see Fig. 5.120). The coefficient of friction between the insect and the surface is 1/3. If the line joining the centre of the hemispherical surface to the insect makes an angle α with the vertical, find the maximum possible value of α.* **[IIT Screening 01]**

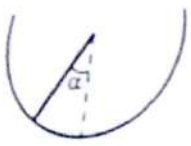

Fig. 5.120

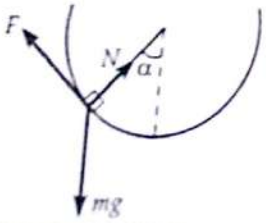

Fig. 5.121

Solution. Refer to Fig. 5.121. Balancing horizontal components of the forces, we get,

$$F\cos\alpha = N\sin\alpha$$

or $$\tan\alpha = F/N.$$

For α to be maximum, F must be maximum, i.e.,

$$F = \mu N$$

$$\therefore \quad \tan\alpha = \mu = 1/3 \quad \text{or} \quad \cot\alpha = 3.$$

Problem 14. *What is the maximum value of the force F such that the block shown in the arrangement of Fig. 5.122 does not move ?* **[IIT Screening 03]**

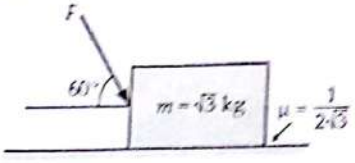

Fig. 5.122

Solution. The free body diagram for the block is shown in Fig. 5.123.

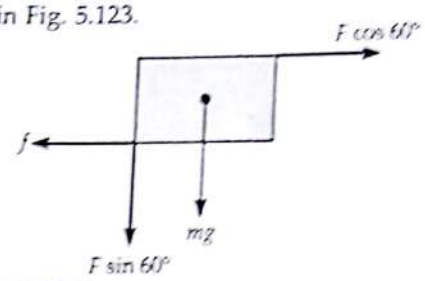

Fig. 5.123

For no motion of the block,

$$F\cos 60° \le f$$

or $$F\cos 60° \le \mu N$$

or $$F\cos 60° \le \mu\,(mg + F\sin 60°)$$

or $$\frac{F}{2} \le \frac{1}{2\sqrt{3}}\left(\sqrt{3} \times g + \frac{F\sqrt{3}}{2}\right)$$

or $$\frac{F}{2} \le g$$

$$\therefore \quad F_{max} = 2g = 2 \times 10 = 20\ \text{N}.$$

Problem 15. *The driver of a truck travelling with a velocity v suddenly notices a brick wall in front of him at a distance d. Is it better for him to apply brakes or to make a circular turn without applying brakes in order to just avoid crashing into the wall ? Explain.* **[Delhi 05 ; Central Schools 08]**

Solution. Suppose F_B force is required in applying brakes to stop the truck in distance d. Then

$$F_B \times d = \frac{1}{2}mv^2 \quad \text{or} \quad F_B = \frac{mv^2}{2d}$$

Suppose F_T force is required in taking a turn of radius d. Then

$$F_T = \frac{mv^2}{d} = 2F_B \quad \text{or} \quad F_B = \frac{1}{2}F_T$$

Clearly, it is better to apply brakes than to take circular turn.

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Problem 16. *A smooth block is released at rest on a 45° incline and then slides a distance d. If the time taken to slide on rough incline is n times as large as that to slide than on a smooth incline, find the coefficient of friction.* **[AIEEE 05]**

Solution. When there is no friction, the block slides down the inclined plane with acceleration,

$$a = g \sin\theta$$

When there is friction, the downward acceleration of the block is

$$a' = g(\sin\theta - \mu\cos\theta)$$

As the block slides a distance d in each case, so

$$d = \frac{1}{2}at^2 = \frac{1}{2}a't'^2$$

or $$\frac{a}{a'} = \frac{t'^2}{t^2} = \frac{(nt)^2}{t^2} = n^2$$

or $$\frac{g\sin\theta}{g(\sin\theta - \mu\cos\theta)} = n^2$$

or $$\sin\theta = n^2\sin\theta - \mu n^2\cos\theta$$

or $$\mu n^2\cos\theta = (n^2-1)\sin\theta$$

or $$\mu = \left(1 - \frac{1}{n^2}\right)\tan\theta = \left(1 - \frac{1}{n^2}\right)\tan 45°$$

or $$\mu = 1 - \frac{1}{n^2}.$$

Problem 17. *A block of wood of mass 3 kg is resting on the surface of a rough inclined surface, inclined at an angle θ as shown in Fig. 5.124.*

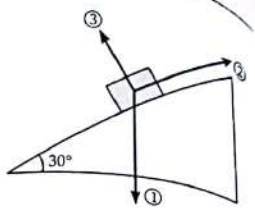

Fig. 5.124

(a) Name the forces (1, 2, 3).

(b) If the coefficient of static friction is 0.2, calculate the value of all the three forces.

(use $g = 10\,m/s^2$) **[Central Schools 08]**

Solution. (a) Force 1 = Weight of block = mg

Force 2 = Upward applied force for keeping block at rest.

Force 3 = Normal reaction R.

(b) Here $\mu = 0.2$, $m = 3$ kg, $g = 10\,ms^{-2}$, $\theta = 30°$.

$\therefore$ Force $1 = mg = 3\times10 = \mathbf{30\ N}$.

Force 3 = Normal reaction $= mg\cos\theta$

$= 3\times10\times\cos30° =$

Now $mg\sin\theta = 3\times10\times\sin$

Friction, $f = \mu R = 0.2\times1$ N

$\therefore$ Force $2 = mg\sin\theta + f$

$= 15 + 3\sqrt{3} = \mathbf{20.2\ N}$.

Guidelines to NCERT Exercises

5.1. *Give the magnitude and direction of the net force acting on*

(a) a drop of rain falling down with a constant speed,

(b) a cork of mass 10 g floating on water, **[Delhi 13]**

(c) a kite skilfully held stationary in the sky,

(d) a car moving with a constant velocity of 30 km/h on a rough road, **[Delhi 12]**

(e) a high-speed electron in space far from all gravitating objects, and free of electric and magnetic fields.

Ans.

(a) As the rain drop is falling down with a constant speed, no net force is acting on it. The weight of the drop is balanced by the upthrust and viscosity of air.

(b) Zero, because the weight of the cork is balanced by the upthrust exerted by water.

(c) As the kite is held stationary, no net force acts on it. The force exerted by air on the kite is balanced by the tension produced in the string.

(d) As the car is moving with a constant velocity, no net force acts it. The force exerted by the engine is balanced by the friction due to rough road.

(e) As no field (gravitational/electric/magnetic) is acting on the electron, the net force on it is zero.

5.2. *A pebble of mass 0.05 kg is thrown vertically upwards. Give the direction and magnitude of the net force on the pebble*

(i) during its upward motion.

(ii) during its downward motion.

(iii) at the highest point where it is momentarily at rest.

Do your answers alter if the pebble were thrown at an angle of say 45° with the horizontal direction ? Take $g = 10\ ms^{-2}$

Ans. Here $m = 0.05$ kg, $g = 10\ ms^{-2}$

(i) Net force on the pebble $= mg$

$= 0.05\times10 = \mathbf{0.5\ N}$, vertically downwards

(ii) Net force on the pebble $= mg$

$= 0.05\times10 = \mathbf{0.5\ N}$, vertically downwards

(iii) Net force on the pebble

$= mg = 0.05\times10 = \mathbf{0.5\ N}$, vertically downwards

The answers will not alter if the pebble were thrown at an angle of 45° with the horizontal because the horizontal component of velocity remains constant.

5.3. *Give the magnitude and direction of the net force acting on a stone of 0.1 kg,*

(i) just after it is dropped from the window of a stationary train.

(ii) just after it is dropped from the window of a train running at a constant velocity of 36 kmh^{-1}.

(iii) just after it is dropped from the window of a train accelerating with 1 ms^{-2}.

(iv) lying on the floor of a train which is accelerating with 1 ms^{-2}, the stone being at rest relative to the train. Neglect air resistance throughout, and take $g = 10\ m$

Ans. Here $m = 0.1$ kg, $g = 10\ ms^{-2}$

(i) When the stone is just dropped from the window of a stationary train,

$F = mg = 0.1\times10 = \mathbf{1\ N}$, vertically down

(ii) When the stone is dropped from the window of a train running at a constant velocity, no force on the stone due to the motion of the train

$\therefore$ $F = mg = \mathbf{1\ N}$, vertically downwards.

(iii) In the train accelerating with 1 ms^{-2}, the stone experiences an additional force,

$F' = ma = 0.1\times1 = 0.1$ N, along horiz

As the stone is dropped, the force F' no on the stone and so net force on the sto

$F = mg = \mathbf{1\ N}$, vertically downwar

(iv) Here weight of the stone is balanced b reaction of the floor.

Acceleration of the stone

= Acceleration of the train =

$\therefore$ $F = ma = 0.1\times1 = \mathbf{0.1\ N}$, along

5.4. *One end of a string of length l is conn of mass m and the other to a small peg on a table. If the particle moves in a circle with sp on the particle (directed towards the centre)*

(i) T, (ii) $T - \frac{mv^2}{l}$, (iii) T +

T is the tension in the string. Choose th

Ans. Alternative (i) is correct. The net directed towards the centre is T. Thus pr centripetal force to the particle movi

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Ans. Here $m = 0.05$ kg, $g = 10$ ms^{-2}

(*i*) Net force on the pebble $= mg$
$= 0.05 \times 10 = \mathbf{0.5\ N}$, vertically downwards

(*ii*) Net force on the pebble $= mg$
$= 0.05 \times 10 = \mathbf{0.5\ N}$, vertically downwards

(*iii*) Net force on the pebble
$= mg = 0.05 \times 10 = \mathbf{0.5\ N}$, vertically downwards

The answers will not alter if the pebble were thrown at an angle of 45° with the horizontal because the horizontal component of velocity remains constant.

5.3. *Give the magnitude and direction of the net force acting on a stone of 0.1 kg,*

(*i*) *just after it is dropped from the window of a stationary train.*

(*ii*) *just after it is dropped from the window of a train running at a constant velocity of 36 kmh*$^{-1}$.

(*iii*) *just after it is dropped from the window of a train accelerating with 1 ms*$^{-2}$.

(*iv*) *lying on the floor of a train which is accelerating with 1 ms*$^{-2}$, *the stone being at rest relative to the train. Neglect air resistance throughout, and take* $g = 10$ *ms*$^{-2}$.

Ans. Here $m = 0.1$ kg, $g = 10$ ms^{-2}

(*i*) When the stone is just dropped from the window of a stationary train,
$F = mg = 0.1 \times 10 = \mathbf{1\ N}$, vertically downwards

(*ii*) When the stone is dropped from the window of a train running at a constant velocity, no force acts on the stone due to the motion of the train.
$\therefore$ $F = mg = \mathbf{1\ N}$, vertically downwards.

(*iii*) In the train accelerating with 1 ms^{-2}, the stone experiences an additional force,
$F' = ma = 0.1 \times 1 = 0.1$ N, along horizontal.
As the stone is dropped, the force F' no longer acts on the stone and so net force on the stone is
$F = mg = \mathbf{1\ N}$, vertically downwards

(*iv*) Here weight of the stone is balanced by the normal reaction of the floor.
Acceleration of the stone
= Acceleration of the train = 1 ms^{-2}
$\therefore$ $F = ma = 0.1 \times 1 = \mathbf{0.1\ N}$, along horizontal.

5.4. *One end of a string of length l is connected to a particle of mass m and the other to a small peg on a smooth horizontal table. If the particle moves in a circle with speed v the net force on the particle (directed towards the centre) is :*

(*i*) T, (*ii*) $T - \frac{mv^2}{l}$, (*iii*) $T + \frac{mv^2}{l}$, (*iv*) 0.

T is the tension in the string. Choose the correct alternative.

Ans. Alternative (*i*) is correct. The net force on the particle directed towards the centre is T. This provides the necessary centripetal force to the particle moving in the circle.

5.5. *A constant retarding force of 50 N is applied to a body of mass 20 kg moving initially with a speed of 15 ms*$^{-1}$. *How long does the body take to stop ?*

Ans. Here $F = -50$ N, $m = 20$ kg, $u = 15$ ms^{-1}, $v = 0$

As $F = ma$

$\therefore$ $a = \frac{F}{m} = \frac{-50}{20} = -2.5$ ms^{-2}

Also, $v = u + at$ $\therefore$ $0 = 15 - 2.5 \times t$

or $t = 6$ s.

5.6. *A constant force acting on a body of mass 3 kg changes its speed from 2 ms*$^{-1}$ *to 3.5 ms*$^{-1}$ *in 25 s. The direction of motion of the force remains unchanged. What is the magnitude and the direction of the force ?* **[Delhi 06]**

Ans. Here $m = 3$ kg, $u = 2$ ms^{-1}, $v = 3.5$ ms^{-1}, $t = 25$ s

As $v = u + at$

$\therefore$ $3.5 = 2 + a \times 25$

or $a = \frac{3.5 - 2}{25} = 0.06$ ms^{-2}

Force, $F = ma = 3 \times 0.06 = \mathbf{0.18\ N}$

As the applied force increases the speed of the body, it acts in the direction of motion of the body.

5.7. *A body of mass 5 kg is acted upon by two perpendicular forces of 8 N and 6 N. Give the magnitude and direction of the acceleration of the body.*

Ans. As shown in Fig. 5.125,

$F_1 = 8$ N, $F_2 = 6$ N, $m = 5$ kg

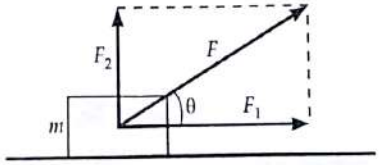

Fig. 5.125

The magnitude of the resultant force,

$$F = \sqrt{F_1^2 + F_2^2} = \sqrt{8^2 + 6^2} = 10\ \text{N}$$

The magnitude of the acceleration produced,

$$a = \frac{F}{m} = \frac{10}{5} = \mathbf{2\ ms^{-2}}.$$

If the force F makes angle θ with F_1, then

$$\cos\theta = \frac{F_1}{F} = \frac{8}{10} = 0.8$$

$\therefore$ $\theta = \cos^{-1}(0.8) = \mathbf{36.87°}$,

with the 8 N force.

5.8. *The driver of a three wheeler moving with a speed of 36 kmh*$^{-1}$ *sees a child standing in the middle of the road and brings his vehicle to rest in 4 s just in time to save the child.*

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What is the average retarding force on the vehicle ? The mass of the three-wheeler is 400 kg and the mass of the driver is 65 kg.

Ans. Here $u = 36$ kmh$^{-1} = 10$ ms^{-1}, $v = 0$,

$t = 4$ s, $m = 400 + 65 = 465$ kg

As $v = u + at$

$\therefore \quad 0 = 10 + a \times 4 \quad$ or $\quad a = -2.5$ ms^{-2}

Magnitude of the retarding force on the vehicle is

$$F = ma = 465 \times 2.5 = \mathbf{1162.5\ N}.$$

5.9. *A rocket with a lift off mass 20,000 kg is blasted upwards with an initial acceleration of 5 ms^{-2}. Calculate the initial thrust of the blast.*

Ans. Initial thrust = upthrust required to impart acceleration a + upthrust required to overcome gravitational pull

$$= ma + mg = m(a + g)$$
$$= 20{,}000\,(5 + 9.8) = 20{,}000 \times 14.8\ N$$
$$= \mathbf{2.96 \times 10^5\ N}.$$

5.10. *A body of mass 0.4 kg moving with a constant speed of 10 ms^{-1} to the north is subjected to a constant force of 8 N directed towards the south for 30 s. Take the instant the force is applied to be $t = 0$, the position of the body at that time to be $x = 0$ and predict its position at $t = -5$ s, 25 s and 100 s.*

Ans. We take south to north as the positive direction. Then $u = +10$ ms^{-1} (due north), $F = -8$ N (due south), $t = 30$ s, $m = 0.4$ kg

$$a = \frac{F}{m} = \frac{-8}{0.4} = -20\ \text{ms}^{-2}$$

(i) At $t = -5$s, no force acts on the particle.

$\therefore \quad x = ut = 10 \times (-5) = \mathbf{-50\ m}.$

(ii) At $t = 25$ s, the position of the particle will be

$$x = ut + \frac{1}{2}at^2 = 10 \times 25 - \frac{1}{2} \times 20 \times (25)^2$$
$$= 250 - 6250 = -6000\ \text{m} = \mathbf{-6\ km}.$$

(iii) At $t = 100$ s, there is no force because force stops acting after $t = 30$ s.

$\therefore$ Distance covered during first 30 s is

$$x_1 = ut + \frac{1}{2}at^2 = 10 \times 30 - \frac{1}{2} \times 20 \times (30)^2$$
$$= -8700\ \text{m}$$

Velocity acquired at $t = 30$ s will be

$$v = u + at = 10 - 20 \times 30 = -590\ \text{ms}^{-1}$$

Distance covered in next 70 s with constant velocity of -590 ms^{-1} is

$$x_2 = vt = -590 \times 70 = -41300\ \text{m}$$

$\therefore$ Position of the particle at $t = 100$ s is

$$x_1 + x_2 = -8700 - 41300$$
$$= -50{,}000\ \text{m} = \mathbf{-50\ km}.$$

5.11. *A truck starts from rest and accelerates uniformly with 2.0 ms^{-2}. At $t = 10$ s, a stone is dropped by a person standing on the top of the truck (6 m high from the ground). What are the (i) velocity, and (ii) acceleration of the stone at $t = 11$ s ? Neglect air resistance. Take $g = 10$ ms^{-2}*

Ans. Here $u = 0$, $a = 2$ ms^{-2}, $g = 10$ ms^{-2}, $t = 10$ s.

(i) During first 10 s, the horizontal component of the velocity is

$$v_x = u + at = 0 + 2 \times 10 = 20\ \text{ms}^{-1}$$

From 10 s to 11 s (i.e., for 1 s), the vertical component of the velocity is

$$v_y = u + gt = 0 + 10 \times 1 = 10\ \text{ms}^{-1}$$

$\therefore$ Resultant velocity,

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(20)^2 + (10)^2} = 10\sqrt{5} = 22.4\ \text{ms}^{-1}$$

The direction of resultant velocity with the horizontal is given by

$$\tan\theta = \frac{v_y}{v_x} = \frac{10}{20} = \frac{1}{2} \quad \text{or} \quad \theta = \tan^{-1}(1/2) = 26.7^\circ.$$

(ii) As there is no horizontal acceleration, the only acceleration is vertical.

$\therefore$ Vertically downward acceleration $= g = 10$ ms^{-2}.

5.12 *A bob of mass 0.1 kg hung from the ceiling of a room by a string 2 m long is set into oscillation. The speed of the bob at its mean position is 1 ms^{-1}. What is the trajectory of the bob if the string is cut when the bob is (a) at one of its extreme positions, (b) at its mean position ?*

Ans.

(a) At the extreme position the speed of bob is zero. The bob is momentarily at rest. If the string is cut, the bob will fall vertically downwards.

(b) At the mean position, the bob has a horizontal velocity. If the string is cut, it will fall along a parabolic path under the effect of gravity.

5.13. *A man weighs 70 kg. He stands on a weighing machine in a lift, which is moving*

(i) upwards with a uniform speed of 10 ms^{-1}.

(ii) downwards with a uniform acceleration of 5 ms^{-2}.

(iii) upwards with a uniform acceleration of 5 ms^{-2}.

What would be the readings on the scale in each case ? What would be the reading, if the lift mechanism failed and it came down freely under gravity ? **[Delhi 05C ; Central Schools 08]**

Ans. The apparent weight measured by the weighing machine is the measure of the reaction R exerted on the man due to the lift.

(i) When the lift moves upward with uniform velocity, reaction of the lift is equal to the weight of the man.

$\therefore$ Apparent weight,

$$R = mg = 70 \times 9.8 = \mathbf{686\ N = 70\ kg\ wt}$$

(ii) When the lift moves downwards with uniform acceleration,

$$a = 5\ \text{ms}^{-2}$$

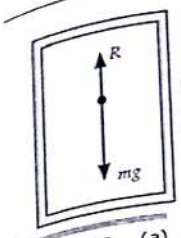

Fig. 5.126 (a)

Resultant downward force,

$$F = mg - R \quad \text{or} \quad ma = m$$

$\therefore$ Apparent weight,

$$R = m(g - a) = 70\,(9.8 - 5)$$
$$= 70 \times 4.8 = \mathbf{336\ N = 34.29}$$

(iii) When the lift moves upwards acceleration, $a = 5$ ms^{-2}

Resultant upward force,

$$F = R - mg \quad \text{or} \quad ma = R$$

$\therefore$ Apparent weight,

$$R = m(g + a) = 70\,(9.8 + 5$$
$$= 70 \times 14.8 = 1036\ \text{N} = 1$$

When the lift falls freely under grav

$\therefore$ Apparent weight, $R = m(g - a) =$

This is the condition of weightlessn

5.14. *Fig. 5.127 shows the position-tim of mass 4 kg. What is the (i) force acting $t < 0, t >> 4$ s, $0 < t < 4$ s ? (ii) impulse a Assume that the motion is one dimensiona*

Fig. 5.127

Ans. *(i)* For $t < 0$ and $t > 4$ s, the p is not changing *i.e.* the particle is a acting on the particle during these i

For $0 < t < 4$ s, the position continuously changing. As the po straight line, the motion of the p acceleration, $a = 0$. Hence no forc during this interval also.

(ii) Before $t = 0$, the particle is a

After $t = 0$, the particle has a c

$$v = \text{Slope of } OA = \frac{3}{4}\ \text{ms}^{-}$$

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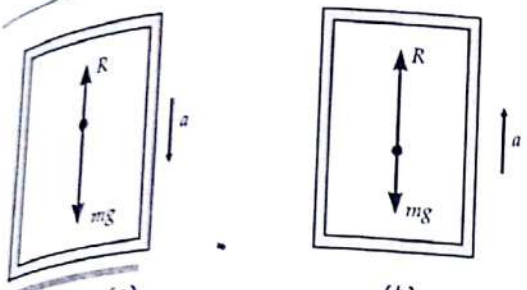

Fig. 5.126

Resultant downward force,

$$F = mg - R \quad \text{or} \quad ma = mg - R$$

∴ Apparent weight,

$$R = m(g - a) = 70(9.8 - 5)$$

$$= 70 \times 4.8 = \mathbf{336\ N = 34.29\ kg\ wt.}$$

(iii) When the lift moves upwards with uniform acceleration, $a = 5\ \text{ms}^{-2}$

Resultant upward force,

$$F = R - mg \quad \text{or} \quad ma = R - mg$$

∴ Apparent weight,

$$R = m(g + a) = 70(9.8 + 5)$$

$$= 70 \times 14.8 = 1036\ \text{N} = \mathbf{105.7\ kg\ wt.}$$

When the lift falls freely under gravity, $a = g$

∴ Apparent weight, $R = m(g - a) = m(g - g) = 0$

This is the condition of weightlessness.

5.14. *Fig. 5.127 shows the position-time graph of a particle of mass 4 kg. What is the (i) force acting on the particle for $t < 0, t >> 4$ s, $0 < t < 4$ s ? (ii) impulse at $t = 0$ and $t = 4$ s ? Assume that the motion is one dimensional.* **[Delhi 09]**

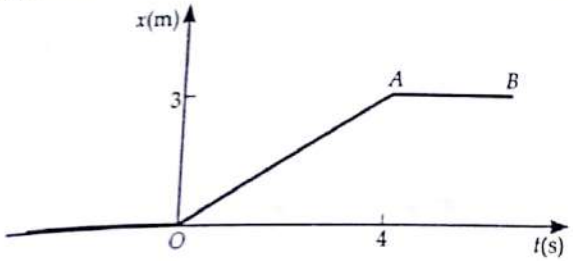

Fig. 5.127

Ans. (i) For $t < 0$ and $t > 4$ s, the position of the particle is not changing *i.e.* the particle is at rest. So no force is acting on the particle during these intervals.

For $0 < t < 4$ s, the position of the particle is continuously changing. As the position-time graph is a straight line, the motion of the particle is uniform, so acceleration, $a = 0$. Hence no force acts on the particle during this interval also.

(ii) Before $t = 0$, the particle is at rest, so $u = 0$

After $t = 0$, the particle has a constant velocity,

$$v = \text{Slope of } OA = \frac{3}{4}\ \text{ms}^{-1}$$

∴ At $t = 0$,

Impulse = Change in momentum

$$= m(v - u) = 4\left(\frac{3}{4} - 0\right) = \mathbf{3\ kg\ ms^{-1}}.$$

Before $t = 4$ s, the particle has a constant velocity,

$$u = \text{Slope of } OA = \frac{3}{4}\ \text{ms}^{-1}$$

After $t = 4$ s, the particle is at rest, so $v = 0$

At $t = 4$ s,

$$\text{Impulse} = m(v - u) = 4\left(0 - \frac{3}{4}\right) = \mathbf{-3\ kg\ ms^{-1}}.$$

5.15. *Two bodies of masses 10 kg and 20 kg respectively kept on a smooth, horizontal surface are tied to the ends of a light string. A horizontal force $F = 600$ N is applied to (i) B (ii) A along the direction of string. What is the tension in the string in each case ?* **[Central Schools 04, 09]**

Ans. Here $F = 600$ N, $m_1 = 10$ kg, $m_2 = 20$ kg

Let T be the tension in the string and a be the acceleration produced in the system, in the direction of applied force F. Then

$$a = \frac{F}{m_1 + m_2} = \frac{600}{10 + 20} = 20\ \text{ms}^{-2}.$$

(i) Suppose the pull F is applied on the body B of mass 20 kg, as shown in Fig. 5.128(*a*).

Let T_1 be the tension in the string. As T_1 is the only force acting on mass 10 kg, so

$$T_1 = m_1\ a = 10 \times 20 = \mathbf{200\ N}.$$

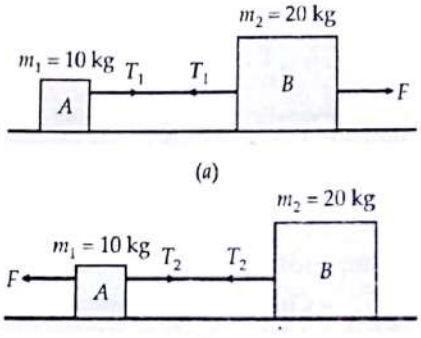

Fig. 5.128

(ii) When the pull F is applied on body A of mass 10 kg [Fig. 5.128(*b*)], tension in the string will be

$$T_2 = m_2\ a = 20 \times 20 = \mathbf{400\ N}.$$

Clearly, the tension depends on which mass end the pull is applied.

5.16. *Two masses 8 kg and 12 kg are connected at the two ends of a light inextensible string that goes over a frictionless pulley. Find the acceleration of the masses and the tension in the string when the masses are released.* **[Delhi 10]**

Ans. Here $m = 8$ kg, $M = 12$ kg, $g = 10\ \text{ms}^{-2}$

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From the derivation of connected motion in Q.45 on page 5.31, we have

$$a = \frac{M-m}{M+m} \cdot g = \frac{12-8}{12+8} \times 10 = 2 \text{ ms}^{-2}$$

$$T = \frac{2Mm}{M+m} \cdot g = \frac{2 \times 12 \times 8}{12+8} \times 10 = 96 \text{ N.}$$

5.17. *A nucleus is at rest in the laboratory frame of reference. Show that if it disintegrates into two smaller nuclei, the products must move in opposite directions.*

Ans. Let M be the mass of the nucleus at rest. Suppose it disintegrates into two smaller nuclei of masses m_1 and m_2 which move with velocities v_1 and v_2 respectively.

$\therefore$ Momentum before disintegration $= M \times 0 = 0$

Momentum after disintegration $= m_1 v_1 + m_2 v_2$

According to the law of conservation of momentum,

$$m_1 v_1 + m_2 v_2 = 0 \quad \text{or} \quad v_2 = -\frac{m_1}{m_2} \cdot v_1$$

As masses m_1 and m_2 cannot be negative, the above equation shows that v_1 and v_2 must have opposite signs *i.e.* the two products must move in opposite directions.

5.18. *Two billiard balls each of mass 0.05 kg moving in opposite directions with speed of 6 ms^{-1} collide and rebound with the same speed. What is the impulse imparted to each ball by the other ?*

Ans. Fig. 5.129(*a*) and 5.129(*b*) show the situations of the two balls before and after the collision.

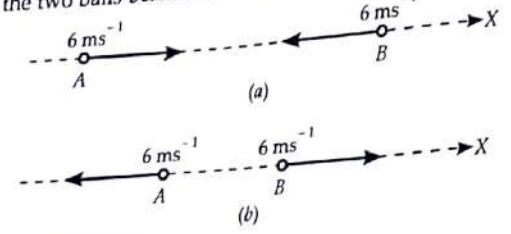

Fig. 5.129

Impulse imparted to one ball by the other

= Change in momentum.

For ball A : p_i = Momentum before collision

$= 0.05 \times 6 = 0.3 \text{ kg ms}^{-1}$

p_f = Momentum after collision

$= 0.05 \times (-6) = -0.3 \text{ kg ms}^{-1}$.

$\therefore$ Impulse imparted to ball A due to ball B

$= p_f - p_i = -0.3 - 0.3 = \mathbf{-0.6 \text{ kg ms}^{-1}}$.

For ball B : p_i = Momentum before collision

$= 0.05 \times (-6) = -0.3 \text{ kg ms}^{-1}$

p_f = Momentum after collision

$= 0.05 \times (6) = 0.3 \text{ kg ms}^{-1}$

$\therefore$ Impulse imparted to ball B due to ball A

$= p_f - p_i = 0.3 - (-0.3) = \mathbf{0.6 \text{ kg ms}^{-1}}$.

5.19. *A shell of mass 0.02 kg is fired by a gun of mass 100 kg. If the muzzle speed of the shell is 80 ms^{-1}, what is the recoil speed of the gun ?* **[Central Schools '11]**

Ans. Mass of shell, $m = 0.02$ kg

Mass of gun, $M = 100$ kg

Speed of shell, $v = 80 \text{ ms}^{-1}$

Let V be the recoil speed of the gun. According to the law of conservation of momentum,

Initial momentum = Final momentum

$$0 = mv + MV$$

or $$V = -\frac{mv}{M} = -\frac{0.02 \times 80}{100} = -0.016 \text{ ms}^{-1}.$$

$\therefore$ Negative sign indicates that the gun moves backward as the bullet moves forward.

5.20. *A batsman deflects a ball by an angle of 45° without changing its initial speed which is equal to 54 kmh^{-1}. What is the impulse imparted to the ball ? Mass of the ball is 0.15 kg.*

Ans. Speed of the ball $= 54 \text{ kmh}^{-1} = 15 \text{ ms}^{-1}$

Let $\vec{v_1}$ and $\vec{v_2}$ be the velocities of the ball before and after deflection.

As the speed of the ball remains unchanged even after deflection, so

$$|\vec{v_1}| = |\vec{v_2}| = 15 \text{ ms}^{-1}$$

In Fig. 5.130, $\vec{AO} = \vec{v_1}$ and $\vec{OB} = \vec{v_2}$. Clearly, the change in velocity of the ball is

$$\vec{v_2} - \vec{v_1} = \vec{v_2} + (-\vec{v_1}) = \vec{OB} + \vec{BC} = \vec{OC} = \vec{v} \text{ (say)}$$

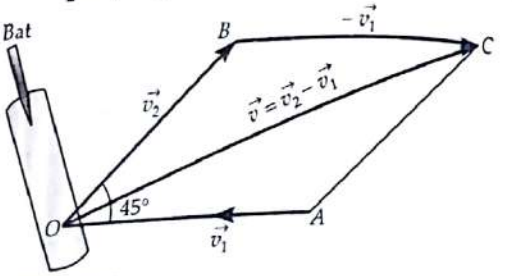

Fig. 5.130

Then $$v = \sqrt{v_1^2 + v_2^2 + 2v_1 v_2 \cos 45°}$$

$$= \sqrt{(15)^2 + (15)^2 + 2 \times 15 \times 15 \times (1/\sqrt{2})}$$

$$= \sqrt{225 + 225 + 225\sqrt{2}} = 27.72 \text{ ms}^{-1}$$

Impulse imparted to the ball

= Mass × Change in velocity of the ball

$= 0.15 \times 27.72 = 4.16 \text{ kg ms}^{-1}$

Impulse is imparted along $\vec{v}$. As the velocity $\vec{v}$ is the resultant of two velocities $-\vec{v_1}$, and $\vec{v_2}$, which have equal

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magnitude, so $\vec{v}$ equally divides the angle between $\vec{v}_1$ and $\vec{v}_2$ *i.e.* impulse is directed along the bisector of initial and final directions.

5.21. *A stone of 0.25 kg tied to the end of a string is whirled round in a circle of radius 1.5 m with a speed of 40 rev./min in a horizontal plane. What is the tension in the string ? What is the maximum speed with which the stone can be whirled around if the string can withstand a maximum tension of 200 N ?*

[Chandigarh 04 ; Central Schools 14]

Ans. (*i*) Here $m = 0.25$ kg, $r = 1.5$ m

$$\nu = 40 \text{ rev min}^{-1} = 40 \text{ rev } (60\text{ s})^{-1} = \frac{2}{3} \text{ rps}$$

$$\omega = 2\pi\nu = 2\pi \times \frac{2}{3} = \frac{4\pi}{3} \text{ rad s}^{-1}$$

Tension in the string = Centripetal force

or $$T = mr\,\omega^2 = 0.25 \times 1.5 \times \left(\frac{4\pi}{3}\right)^2 = \mathbf{6.6\ N.}$$

(*i*) Given $T_{max} = 200$ N

As $$\frac{mv_{max}^2}{r} = T_{max}$$

$$\therefore \quad v_{max} = \sqrt{\frac{T_{max} \times r}{m}} = \sqrt{\frac{200 \times 1.5}{0.25}} = \mathbf{34.6\ ms^{-1}.}$$

5.22. *A stone tied to the end of string is whirled round in a circle in a horizontal plane. If the speed of the stone is increased beyond the maximum permissible value, and the string breaks suddenly, which of the following correctly describes the trajectory of the stone after the string breaks :*

(*a*) *the stone jerks radially outwards,*

(*b*) *the stone flies off tangentially from the instant the string breaks,*

(*c*) *the stone flies off at an angle with the tangent whose magnitude depends on the speed of the particle ?*

Ans. The alternative (*b*) is correct. When the string breaks, the stone flies off tangentially from the instant, the string breaks. This is because the velocity at any point is directed along the tangent at that point.

5.23. *Explain why*

(*a*) *a horse cannot pull a cart and run in empty space,*

(*b*) *passengers are thrown forward from their seats when a speeding bus stops suddenly,*

(*c*) *it is easier to pull a lawn mower than to push it,*

(*d*) *a cricketer moves his hands backwards while holding a catch.* **[Chandigarh 09; Central Schools 13, 14]**

Ans. (*a*) For pulling a cart or for running, the horse pushes the earth with its feet and reaction of the earth makes it move in the forward direction. Since in empty space there is no reaction force, therefore the horse cannot run in empty space.

(*b*) Refer to solution of Problem 7 on page 5.62.

(*c*) Refer to Figs. 5.96 and 5.97. A lawn mower is pulled or pushed by applying force at an angle. The vertical component of the applied force reduces the effective weight of the mower in case of pull and increases the effective weight in case of push. Consequently, the normal reaction and hence the force of friction is less in case of pull than that in push. Hence it is easier to pull a lawn mower than to push it.

(*d*) When the ball is caught, the impulse received by the hands is equal to the product of the force exerted by the ball and the time taken to complete the catch. By moving the hands backwards, the cricketer increases the time of catch. The force exerted on his hands becomes much smaller and it does not hurt him.

5.24. *Fig. 5.131 below shows the position-time graph of a particle of mass 0.04 kg. Suggest a suitable physical context for this motion. What is the time between two consecutive impulses received by the particle ? What is the magnitude of each impulse ?*

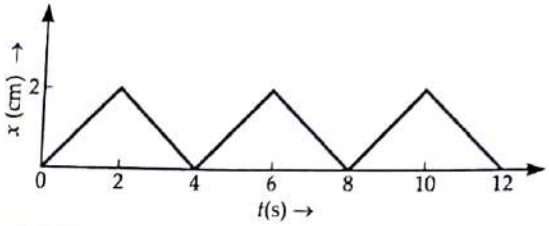

Fig. 5.131

Ans. Fig. 5.131 shows that (*i*) the direction of motion of the particle changes after every 2 s and (*ii*) in both directions, the particle moves with a uniform speed.

Before $t = 2$ s, velocity of the particle,

$$u = \text{Slope of } x\text{-}t \text{ graph} = \frac{(2-0)\text{ cm}}{(2-0)\text{ s}} = 1\text{ cms}^{-1} = 0.01\text{ ms}^{-1}$$

After $t = 2$ s, velocity of the particle,

$$v = \frac{(0-2)\text{ cm}}{(4-2)\text{ s}} = -1\text{ cms}^{-1} = -0.01\text{ ms}^{-1}$$

Mass of particle, $m = 0.04$ kg.

At $t = 2$ s, magnitude of impulse

$$= \text{Change in momentum} = m(u - v)$$
$$= 0.04\,[0.01 - (-0.01)]\text{ kg ms}^{-1}$$
$$= \mathbf{8 \times 10^{-4}\ kg\ ms^{-1}.}$$

The given x-t graph may represent the repeated rebounding of a particle between two walls situated at $x = 0$ and $x = 2$ cm. The particle receives an impulse of 8×10^{-4} kg ms^{-1} after every 2 s.

5.25. *Fig. 5.132 shows a man standing stationary with respect to a horizontal conveyor belt that is accelerating with 1 ms^{-2}. What is the net force on the man ? If the coefficient of static friction between the man's shoes and the belt is 0.2 , upto what acceleration of the belt can the man continue to be stationary relative to the belt ? Mass of the man = 65 kg.*

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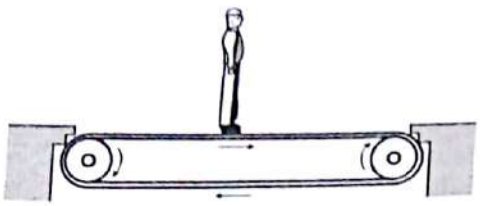

Fig. 5.132

Ans. As the man is standing stationary w.r.t. the belt,

∴ Acceleration of the man

= Acceleration of the belt = $a = 1\ ms^{-2}$

Mass of the man, $m = 65$ kg

Net force on the man = $ma = 65 \times 1 = \mathbf{65\ N}$.

Given coefficient of friction, $\mu = 0.2$

∴ Limiting friction, $f = \mu R = \mu\ mg$

If the man remains stationary w.r.t. the maximum acceleration a' of the belt, then

$$ma' = f = \mu mg$$

$$\therefore \quad a' = \mu\ g = 0.2 \times 9.8 = \mathbf{1.96\ ms^{-2}}.$$

5.26. *A stone of mass m tied to the end of a string is revolved in a vertical circle of radius R. The net forces at the lowest and highest points of the circle directed vertically downwards are :*

Lowest Point	Highest Point
(i) $mg - T_1$	$mg + T_2$
(ii) $mg + T_1$	$mg - T_2$
(iii) $mg + T_1 - (mv_1^2)/R$	$mg - T_2 + (mv_2^2)/R$
(iv) $mg - T_1 - (mv_1^2)/R$	$mg + T_2 + (mv_2^2)/R$

Here T_1, T_2 (and v_1, v_2) denote the tension in the string (and the speed of the stone) at the lowest and the highest point respectively. Choose the correct alternative.

Ans. The alternative (*i*) is correct because the net force at the lowest point L is $F_L = mg - T_1$, and the net force at highest point H is $F_H = mg + T_2$.

5.27. *A helicopter of mass 1000 kg rises with a vertical acceleration of 15 ms^{-2}. The crew and the passengers weigh 300 kg. Give the magnitude and direction of*

(i) force on the floor by the crew and passengers.

(ii) action of the rotor of the helicopter on the surrounding air.

(iii) force on the helicopter due to the surrounding air.

Take $g = 10\ ms^{-2}$. **[Delhi 03]**

Ans. Mass of helicopter, $M = 1000$ kg

Mass of the crew and passengers, $m = 300$ kg

Vertically upward acceleration, $a = 15\ ms^{-2}$

(*i*) Force on the floor by the crew and passengers,

F = Apparent weight = $m(g + a)$

$= 300(10 + 15) = \mathbf{7500\ N}$, vertically downwards.

(*ii*) Action of the rotor of the helicopter on the surrounding air

= Apparent weight of the helicopter, crew and passengers

$= (M + m)(g + a) = (1000 + 300)(10 + 15)$

$= \mathbf{32500\ N}$, vertically downwards.

(*iii*) Force on the helicopter due to the surrounding air is equal and opposite to the action of the rotor of the helicopter on the surrounding air.

∴ Force on surrounding air

$= \mathbf{32500\ N}$, vertically upwards.

5.28. *A stream of water flowing horizontally with a speed of 15 ms^{-1} gushes out of a tube of cross-sectional area $10^{-2}\ m^2$, and hits at a vertical wall nearby. What is the force exerted on the wall by the impact of water, assuming it does not rebound ?*

Ans. Here $u = 15\ ms^{-1}$, $v = 0$, $t = 1$ s, $A = 10^{-2}\ m^2$

Density of water = $1000\ kgm^{-3}$

m = Mass of water gushed out per second

$$= \frac{\text{Volume} \times \text{density}}{\text{Time}} = \frac{\text{Area} \times \text{distance} \times \text{density}}{\text{Time}}$$

= Area × velocity × density

$= Aup = 10^{-2} \times 15 \times 1000 = 150$ kg

Force exerted by the wall on water,

$$F = ma = m\left(\frac{v - u}{t}\right) = 150 \times \frac{0 - 15}{1} = -2250\ \text{N}$$

Force exerted on the wall by the impact of water,

$$F' = -F = \mathbf{2250\ N}.$$

5.29. *Ten one-rupee coins are put on top of each other on a table. Each coin has a mass m kg. Give the magnitude and direction of*

(i) the force on the 7th coin (counted from the bottom) due to all the coins on its top.

(ii) the force on the 7th coin by the eighth coin.

(iii) the reaction of the 6th coin on the 7th coin.

[Central Schools 09]

Ans. (*i*) Force on the 7th coin

= Force due to 3 coins on its top = $3mg$.

(*ii*) Force on the 7th coin by the 8th coin

= Masses of 8th, 9th and 10th coins × g = $3mg$

(*iii*) Reaction of the 6th coin on the 7th coin

= Force on the 6th coin due to 7th coin = $4mg$.

5.30. *An aircraft executes a horizontal loop at a speed of 720 kmh^{-1} with its wings banked at 15°. What is the radius of the loop ?* **[Central Schools 07]**

Ans. Here $\theta = 15°$, $g = 9.8\ ms^{-2}$

$$v = 720\ \text{kmh}^{-1} = \frac{720 \times 5}{18} = 200\ \text{ms}^{-1}$$

As $\quad \tan\theta = \dfrac{v^2}{rg}$

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$$\therefore \quad r = \frac{v^2}{g \tan\theta} = \frac{200 \times 200}{9.8 \times \tan 15^\circ}$$

$$= \frac{200 \times 200}{9.8 \times 0.2679} = 15.24 \times 10^3 \text{ m} = \mathbf{15.24 \text{ km}}.$$

5.31. *A train runs along an unbanked circular track of radius 30 m at a speed of 54 kmh^{-1}. The mass of the train is 10^6 kg. What provides the centripetal force required for this purpose ? The engine or the rails ? The outer or the inner rail ? Which rail will wear out faster, the outer or the inner rail ? What is the angle of banking required to prevent wearing out of the rails ?*

Ans. Here

$r = 30$ m, $v = 54$ kmh$^{-1} = 15$ ms^{-1}, $m = 10^6$ kg

The centripetal force required for the purpose is provided by the lateral thrust by the outer rail on the flanges of the wheels. By Newton's third law of motion, the train exerts an equal and opposite thrust on the outer rail causing its wear and tear.

$$\tan\theta = \frac{v^2}{rg} = \frac{(15)^2}{30 \times 9.8} = 0.7653$$

$\therefore$ Angle of banking, $\theta = \mathbf{37.4^\circ}$.

5.32. *A block of mass 25 kg is raised by a 50 kg man in two different ways as shown in Fig. 5.133. What is the action on the floor by the man in the two cases ? If the floor yields to a normal force of 700 N, which mode should the man adopt to lift the block without the floor yielding ?*

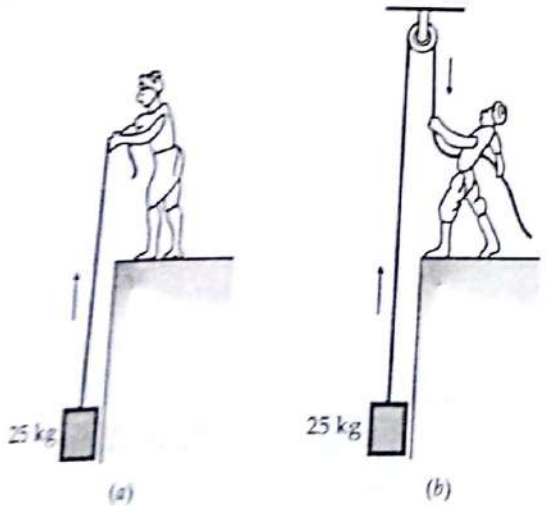

Fig. 5.133

Ans. In mode (*a*), the man applies force equal to 25 kg wt in the upward direction. According to Newton's third law of motion, there will be a downward force of reaction on the floor.

$\therefore$ Total action on the floor by the man

= 50 kg wt + 25 kg wt = 75 kg wt

= 75 × 9.8 N = **735 N.**

In mode (*b*), the man applies a downward force equal to 25 kg wt. According to Newton's third law, the reaction will be in the upward direction.

$\therefore$ Total action on the floor by the man

= 50 kg wt − 25 kg wt = 25 kg wt = 25 × 9.8 N = **245 N.**

As the floor yields to a downward force of 700 N, so the man should adopt mode (*b*).

5.33. *A monkey of mass 40 kg climbes on a rope which can stand a maximum tension of 600 N [Fig. 5.134]. In which of the following cases will the rope break : the monkey*

(*i*) *climbs up with an acceleration of 6 ms^{-2}*

(*ii*) *climbs down with an acceleration of 4 ms^{-2}*

(*iii*) *climbs up with a uniform speed of 5 ms^{-1}*

(*iv*) *falls down the rope nearly freely under gravity ?*

Take $g = 10$ ms^{-2}. Ignore the mass of the rope.

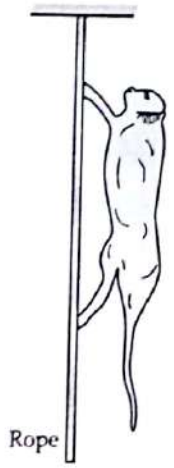

Fig. 5.134

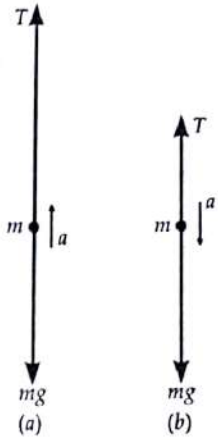

Fig. 5.135

Ans. (*i*) When the monkey climbs up with an acceleration $a = 6$ ms^{-2}, the tension T in the string must be greater than the weight of the monkey [Fig. 5.135(*a*)],

$$T - mg = ma$$

or $$T = m(g + a) = 40(10 + 6) = \mathbf{640 \text{ N}}.$$

(*ii*) When the monkey climbs down with an acce- leration, $a = 4$ ms^{-2} [Fig. 5.135(*b*)],

$$mg - T = ma$$

or $$T = m(g - a) = 40(10 - 4) = \mathbf{240 \text{ N}}.$$

(*iii*) When the monkey climbs up with uniform speed,

$$T = mg = 40 \times 10 = \mathbf{400 \text{ N}}.$$

(*iv*) When the monkey falls down the rope nearly freely, $a = g$

$$\therefore \quad T = m(g - a) = m(g - g) = \mathbf{0}.$$

As the tension in the rope in case (*i*) is greater than the maximum permissible tension (600 N), so the rope will break in case (*i*) only.

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5.34. *Two bodies A and B of masses 5 kg and 10 kg in contact with each other rest on a table against a rigid partition. The coefficient of friction between the bodies and the table is 0.15. A force of 200 N is applied horizontally at A. What are (i) the reaction of the partition (ii) the action-reaction forces between A and B ? What happens when the partition is removed ? Does answers to (ii) change, when the bodies are in motion ? Ignore difference between μ_s and μ_k.*

Ans. Mass of body A,

$$m_A = 5 \text{ kg}$$

Mass of body B,

$$m_B = 10 \text{ kg}$$

Coefficient of friction,

$$\mu = 0.15$$

Applied force, $P = 200$ N

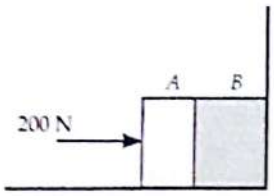

Fig. 5.136

(*i*) Force of limiting friction is

$$f = \mu R = \mu (m_1 + \mu_2) g$$
$$= 0.15 \times (5 + 10) \times 9.8 = 22.05 \text{ N}$$
(towards left)

When a force of 200 N is applied, the net force exerted on the partition is

$$P' = P - f = 200 - 22.05 = 177.95 \text{ N}$$
(towards right)

Reaction of the partition = **177.95 N** (towards left)

(*ii*) Force of limiting friction on body A is

$$f_1 = \mu m_1 g = 0.15 \times 5 \times 9.8 = 7.35 \text{ N}$$

Net force exerted by body A on body B is

$$P_1 = P - f_1 = 200 - 7.35 = \mathbf{192.65 \text{ N}}$$
(towards right)

Reaction of body B on A = **192.65 N** (towards left)

When the partition is removed. The system of the two bodies moves under the action of the net force,

$$P' = 177.95 \text{ N}$$

Acceleration produced in the system,

$$a = \frac{P'}{m_1 + m_2} = \frac{177.95}{5 + 10} = \mathbf{11.86 \text{ ms}^{-2}}$$

Force producing motion in the body A

$$= m_1 a = 5 \times 11.86 = 59.3 \text{ N}$$

Net force exerted by A on B after the removal of partition

$$= P_1 - 59.3 = 192.65 - 59.3 = \mathbf{133.35 \text{ N}}$$
(towards right)

Reaction of the body B on A = 133.5 N
(towards left)

5.35. *A block of mass 15 kg is placed on a long trolley. The coefficient of friction between the block and the trolley is 0.18. The trolley accelerates from rest with 0.5 ms^{-2} for 20 s and then moves with uniform velocity. Discuss the motion of the block as viewed by (i) a stationary observer on the ground (ii) an observer moving with the trolley.*

Ans. Mass of block, $m = 15$ kg

$$\mu_s = 0.18, \ a = 0.5 \text{ ms}^{-2}, \ t = 20 \text{ s}$$

Maximum value of static friction,

$$f_{ms} = \mu_s R = \mu_s mg = 0.18 \times 15 \times 9.8 = 26.46 \text{ N}$$

Force acting on the block during the accelerated motion,

$$F = ma = 15 \times 0.5 = 7.5 \text{ N}$$

As $f_{ms} > F$, so the block does not move. It remains at rest w.r.t. the trolley, even when it is accelerated. When the trolley moves with uniform velocity, acceleration is zero and hence no force is acting on the trolley.

(*i*) The stationary observer will see the accelerated and the uniform motions.

(*ii*) When the observer is in the trolley, he is in an accelerated or non-inertial frame. The laws of motion are not applicable. But during uniform motion he will see that the block is at rest w.r.t. him.

5.36. *The rear side of a truck is open and a box of 40 kg mass is placed 5 m away from the open end as shown in Fig. 5.137. The coefficient of friction between the box and the surface below it is 0.15. On a straight road, the truck starts from rest and accelerates with 2 ms^{-2}. At what distance from the starting point does the box fall off the truck ? Ignore the mass of the box.*

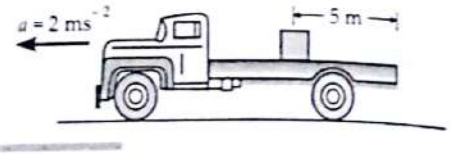

Fig. 5.137

Ans. Mass of box, $m = 40$ kg

Acceleration of truck,

$$a = 2 \text{ ms}^{-2}$$

Distance of the box from the rear end,

$$s = 5 \text{ m}$$

Coefficient of friction,

$$\mu = 0.15$$

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As the box is in an accelerated frame, it experiences a backward force,

$$F = ma$$

Motion of the box is opposed by the frictional force,

$$f = \mu R = \mu\ mg$$

∴ Net force on the box in the backward direction is

$$F' = F - f = ma - \mu\ mg = m(a - \mu\ g)$$

$$= 40(2 - 0.15 \times 9.8) = \mathbf{21.2\ N.}$$

Acceleration produced in the box in the backward direction,

$$a' = \frac{F'}{m} = \frac{21.2}{40} = 0.53\ \text{ms}^{-2}$$

If the box takes time t to fall off the truck, then

$$s = ut + \frac{1}{2}a't^2 \text{ or } 5 = 0 \times t + \frac{1}{2} \times 0.53 \times t^2$$

or $$t^2 = \frac{5 \times 2}{0.53} = \frac{10}{0.53}$$

The distance covered by the truck accelerating at $2\ \text{ms}^{-2}$ during this time is

$$s' = \frac{1}{2}at^2 = \frac{1}{2} \times 2 \times \frac{10}{0.53} = \mathbf{18.57\ m.}$$

5.37. *A disc revolves with a speed of $33\frac{1}{3}$ rev min^{-1}, and has a radius of 15 cm. Two coins are placed at 4 cm and 14 cm away from the centre of the disc. If the coefficient of friction between the coins and the disc is 0.15, which of the two coins will revolve with the disc ? Take $g = 9.8\ \text{ms}^{-2}$.*

Ans. Here $\nu = 33\frac{1}{3}\ \text{rpm} = \frac{100}{3}\ \text{rpm} = \frac{100}{3 \times 60}\ \text{rps}$

$$\omega = 2\pi\nu = 2 \times \frac{22}{7} \times \frac{100}{3} \times \frac{1}{60} = \frac{220}{63}\ \text{rad s}^{-1}$$

$$r = 15\ \text{cm},\ \mu = 0.15$$

The coin will revolve with the disc if the force of friction is enough to provide the necessary centripetal force,

i.e. $$mr\omega^2 \le \mu\ mg \quad \text{or} \quad r \le \frac{\mu\ g}{\omega^2}$$

Now $$\frac{\mu g}{\omega^2} = \frac{0.5 \times 9.8}{\left(\frac{220}{63}\right)^2} = 0.12\ \text{m} = 12\ \text{cm}$$

Thus the coin placed at a distance of 4 cm from the centre of the disc will revolve with the disc.

5.38. *You may have seen in a circus a motorcyclist driving in vertical loops inside a 'death-well' (a hollow spherical chamber with holes, so that the spectators can watch from outside). Explain clearly why the motorcyclist does not drop down when he is at the uppermost point, with no support from below. What is the minimum speed required to perform a vertical loop if the radius of the chamber is 25 m ?*

Ans. At the highest point of the death-well, the normal reaction R of the walls of the chamber acts downwards. The centripetal force is provided by his weight mg and the normal reaction R.

$$\therefore \quad \frac{mv^2}{r} = R + mg$$

The motorcyclist does not fall down due to the balancing of these forces. For minimum speed, at the highest point, $R = 0$, so that

$$\frac{mv_{\min}^2}{r} = mg$$

or $$v_{\min} = \sqrt{rg} = \sqrt{25 \times 9.8} = \mathbf{15.65\ ms^{-1}}.$$

5.39. *A 70 kg man stands in contact against the wall of a cylindrical drum of radius 3 m rotating about its vertical axis with 200 rev/min. The coefficient of friction between the wall and his clothing is 0.15. What is the minimum rotational speed of the cylinder to enable the man to remain stuck to the wall (without falling) when the floor is suddenly removed ?*

[Central Schools 14]

Ans. Here $r = 3\ \text{m}, \mu = 0.15,$

$$\nu = 200\ \text{rpm} = \frac{200}{60}\ \text{rps}$$

$$\omega = 2\pi\nu = 2 \times \frac{22}{7} \times \frac{200}{60} = \frac{400}{7}\ \text{rad s}^{-1}$$

The horizontal reaction R of the wall on the man provides the necessary centripetal force.

$$R = \frac{mv^2}{r} = mr\omega^2 \qquad [\because v = r\omega]$$

The frictional force f acting vertically upwards balances the weight of the man. The man will remain struck to the wall after the floor is removed, if

$$f \le \mu R$$

or $$mg \le \mu mr\omega^2 \qquad [\because f = mg]$$

or $$g \le \mu r\omega^2 \text{ or } \omega^2 \ge \frac{g}{\mu r}$$

The minimum rotational speed of the cylinder is

$$\omega_{\min} = \sqrt{\frac{g}{\mu r}} = \sqrt{\frac{9.8}{0.15 \times 3}} = \sqrt{21.78} = \mathbf{4.7\ rad\ s^{-1}}.$$

5.40. *A thin circular loop of radius R rotates about its vertical diameter with an angular frequency ω. Show that a small bead on the wire remains at its lowermost point for $\omega \le \sqrt{\frac{g}{R}}$. What is the angle made by the radius vector joining the centre to the bead with the vertical downward direction for $\omega = \sqrt{\frac{2g}{R}}$? Neglect friction.*

Ans. Fig. 5.138 shows the free-body diagram of the bead when the radius vector joining the centre to the bead makes an angle θ with the vertical downward direction. The normal reaction here is equal to the centrifugal force,

i.e., $$N = \frac{mv^2}{R} = m\omega^2\ R$$

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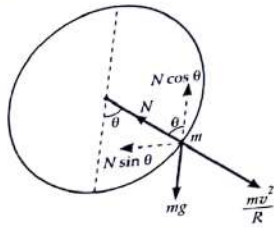

Fig. 5.138

Also $mg = N\cos\theta = m\omega^2 R\cos\theta$

$\therefore \quad \omega^2 = \frac{g}{R\cos\theta}$ or $\omega = \sqrt{\frac{g}{R\cos\theta}}$

For the bead to remain in the lowermost position, $\theta = 0°$ and $\cos\theta = 1$ Hence

$$\omega = \sqrt{\frac{g}{R}}$$

Thus the bead will remain in the lowermost position, if

$$\omega \le \sqrt{\frac{g}{R}}.$$

For $\omega = \sqrt{\frac{2g}{R}}$, we have

$$\cos\theta = \frac{g}{\omega^2 R} = \frac{g}{\frac{2g}{R}\cdot R} = \frac{1}{2}$$

$\therefore \quad \theta = 60°.$

Text Based Exercises

Type A : Very Short Answer Questions

1 Mark Each

1. Is any external force required to keep a body in uniform motion ?
2. State Aristotelian law of motion. What is flaw in this law ?
3. State Galileo's law of inertia.
4. Define the term inertia.
5. What gives the measure of inertia ?
6. Define inertial mass of a body.
7. Name the scientist who first gave the laws of motion.
8. State Newton's first law of motion. **[Delhi 03C ; Central Schools 07]**
9. A piece of cork is floating on water. What is the net force acting on it ?
10. How does Newton's first law of motion define force ?
11. Which law of motion gives the measure of force ?
12. Name the scientist who first introduced the concept of momentum.
13. Write the dimensions of momentum.
14. What are the SI and CGS units of momentum ?
15. What is the ratio of SI to the CGS units of momentum ?
16. State Newton's Second law of motion. **[Delhi 03 ; Central Schools 07]**
17. Define one newton force. **[Central Schools 13]**
18. Define one dyne.
19. Write the relation between newton and dyne.
20. Is it correct to say that the second law of motion is the real law of motion ?
21. Write the second law of motion in vector form.
22. Show that if the force acting on a particle is zero, its momentum will remain unchanged. **[BIT Ranchi 97]**
23. The two ends of a spring balance are pulled each by a force of 10 kg wt. What will be the reading of the spring balance ?
24. A person is standing on a weighing machine placed near a door. What would be the effect on the reading of the machine if a person (*i*) presses the edge of the door upward (*ii*) pulls downwards ?
25. What are the conditions for maximum and minimum pull of a lift on a supporting cable.
26. A lift is going up with an acceleration 2 *g*. A man is inside the lift and his mass is *m*. What will be the reaction of the floor on the man ?
27. Which of Newton's laws of motion is involved in rocket propulsion ?
28. A person sitting in the compartment of a train moving with uniform speed throws a ball in the upward direction (*i*) What path of the ball will appear to him ? (*ii*) What to a person standing outside ?
29. State the principle of conservation of linear momentum. **[Kerala 01 ; Delhi 08]**
30. Will the momentum remain conserved if some external force acts on the system ?
31. What is the principle of working of a rocke
32. Vehicles stop on applying brakes. Does thi menon violate the principle of conserv momentum ?
33. A bomb explodes in mid-air into tw fragments. What is the relation between th of motion of the two fragments ?
34. The total momentum of the univers constant. Is this statement true ?
35. Can a rocket operate in free space ?
36. Is impulse a scalar or vector quantity ? unit.
37. The dimensional formula of impuls Name another physical quantity havi dimensions.
38. A bus weighing 900 kg is at rest on t What is the linear momentum of the
39. What is dry friction ?
40. What is wet friction ?
41. What is normal reaction ?
42. Define limiting friction.
43. Define coefficient of limiting frictio
44. What is the unit of coefficient of li
45. Which is greater μ_s or μ_k ?
46. Write the relation between coef and angle of friction ?
47. What is the relation between an angle of repose ?
48. Is rolling friction more than slid
49. On what factors does the co depend ?
50. Which is greatest out of sta friction and kinetic friction ?
51. What is the angle between instantaneous velocity of a rough surface ?
52. A body is just sliding down to its own weight. What is angle of inclination and ang
53. A body slides down an inclin Indicate the directions of th reaction of the plane on th
54. Give an example of the us
55. Does the force of friction contact ?
56. What is the angle of frict in contact if the coefficie

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31. What is the principle of working of a rocket ? [Delhi 10]

32. Vehicles stop on applying brakes. Does this phenomenon violate the principle of conservation of momentum ?

33. A bomb explodes in mid-air into two equal fragments. What is the relation between the direction of motion of the two fragments ?

34. The total momentum of the universe remains constant. Is this statement true ?

35. Can a rocket operate in free space ?

36. Is impulse a scalar or vector quantity ? Write its SI unit.

37. The dimensional formula of impulse is MLT^{-1}. Name another physical quantity having the same dimensions. [Delhi 96]

38. A bus weighing 900 kg is at rest on the bus stand. What is the linear momentum of the bus ? [Himachal 02]

39. What is dry friction ?

40. What is wet friction ?

41. What is normal reaction ?

42. Define limiting friction. [Himachal 09]

43. Define coefficient of limiting friction.

44. What is the unit of coefficient of limiting friction. [Himachal 01]

45. Which is greater μ_s or μ_k ?

46. Write the relation between coefficient of friction and angle of friction ?

47. What is the relation between angle of friction and angle of repose ?

48. Is rolling friction more than sliding friction ?

49. On what factors does the coefficient of friction depend ?

50. Which is greatest out of static friction, limiting friction and kinetic friction ?

51. What is the angle between frictional force and instantaneous velocity of a body moving over a rough surface ?

52. A body is just sliding down an inclined plane due to its own weight. What is the relation between angle of inclination and angle of repose ?

53. A body slides down an inclined plane having friction. Indicate the directions of the frictional force and the reaction of the plane on the body. [Manipur 99]

54. Give an example of the use of friction.

55. Does the force of friction depend on the area of contact ?

56. What is the angle of friction between two surfaces in contact if the coefficient of friction is $1/\sqrt{3}$? [Himachal 07C]

57. Define angle of friction. [Himachal 01C ; Central Schools 07]

58. It is easier to roll a barrel than to pull it along the road. Why ? [Himachal 07]

59. Why are rockets given conical shape ?

60. Arrange μ_s, μ_k and μ_r in ascending order.

61. A body is moving along a circular path such that its speed is always constant. Should there be a force acting on the body ? [Himachal 02]

62. What provides the centripetal force to a satellite revolving around the earth ? [Punjab 91]

63. What furnishes the centripetal force for the electrons to go round the nucleus ?

64. Can a centripetal force produce rotation ?

65. Which is real among the centripetal and centripetal forces ?

66. What provides the centripetal force to a car taking a turn on a level road ? [Himachal 06]

67. Does the angle of banking depend on the mass of the vehicle ?

68. When a body moves in a circular path, which thing experiences a centrifugal force ?

69. A body is travelling with a constant speed on the circumference of a circle. Of the quantities (*i*) linear velocity (*ii*) linear acceleration (*iii*) acceleration towards the centre and (*iv*) centripetal force, which remains constant ?

70. A circus man starts down an inclined plane on his scooter. At the end of the inclined plane, there is a vertical circular arch. If he is to safely negotiate the arch, what must be his velocity at the end of the plane ?

71. How many newtons make one kg wt. ? [Central Schools 04, 12]

72. Define impulse. [Central Schools 05 ; Himachal 09C]

73. What is difference between mN and nm ? [Himachal 03]

74. From which Newton's laws of motion, the definition of force comes ? [Himachal 03]

75. Give and state the SI unit of force. [Himachal 03]

76. What is friction ? [Himachal 04, 09]

77. What is the unit of coefficient of limiting friction ? [Himachal 01]

78. What happens to coefficient of friction, when weight of body is doubled ? [Chandigarh 03]

79. What is the need of banking a circular road ? [Himachal 04 ; Delhi 09]

80. Three forces start acting simultaneously on a particle moving with a velocity $\vec{v}$. These forces are represented in magnitude and direction by three sides of a triangle taken in the same order. What now will be the velocity of the particle ? [AIEEE 03]

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81. Action and reaction forces do not balance each other. Why ? [Himachal 06 ; Delhi 12]

Or

Action and reaction are equal and opposite. Why cannot they cancel each other ? [Central Schools 12]

82. If the net external force acting on a body is zero, then the body at rest continues to remain at rest and a body in motion continues to move with uniform motion. What is the name given to this property of the body ? [Delhi 08]

83. A tennis ball of mass 'm' strikes the massive wall with vel 'v' and traces the same path. Calculate the change in momentum. [Central Schools 08, 09]

84. What will be the maximum velocity with which a vehicle can negotatie a turn of radius r safely, when the coefficient of friction between the tyres and the road is μ ? [Himachal 07]

85. Athlete runs a certain distance before long jump. Name the law that explains it. [Central Schools 07]

86. Two objects of masses 2 kg and 4 kg are having the same magnitude of momentum. Find the ratio of their speeds. [Central Schools 13]

Answers

1. No.
2. Aristotelian law of motion states that an external force is required to keep a body in motion. This law is wrong. In practice, a force is required to counter the opposing force of friction.
3. A body moving in a straight line with a certain speed will continue moving in the same straight line with the same speed in the absence of any external force.
4. Inertia is the inherent property of a material body by virtue of which it cannot change, by itself, its state of rest or of uniform motion in a straight line.
5. Mass of a body gives the measure of its inertia.
6. Inertial mass of a body is equal to the force required to produce unit acceleration in the body.
7. Isaac Newton.
8. According to Newton's first law of motion, every body continues in its state of rest or of uniform motion, unless an external force is applied to change that state.
9. Zero, because the weight of the piece of cork is balanced by the upthrust of water on it.
10. Refer answer to Q. 11 on page 5.4.
11. Newton's second law of motion.
12. Isaac Newton.
13. $[MLT^{-1}]$.
14. The SI unit of momentum is kg ms^{-1} and CGS unit of momentum is g cms^{-1}.
15. 10^5.
16. Newton's second law of motion states that the rate of change of momentum of a body is directly proportional to the applied force and the change in momentum takes place in the direction of the applied force.
17. One newton is defined as the force which produces an acceleration of 1 ms^{-2} in a body of mass 1 kg.
18. One dyne is defined as the force which produces an acceleration of 1 cms^{-2} in a body of mass 1 gram.
19. 1 newton = 10^5 dyne.
20. Yes, because both the first and third law of motions are contained in the second law of motion.
21. $\vec{F} = m\vec{a}$.
22. According to Newton's second law,

$$\vec{F} = \frac{d\vec{p}}{dt}.$$

As $\vec{F} = 0 \quad \therefore \frac{d\vec{p}}{dt} = 0$ or $\vec{p}$ = constant.

23. The reading of the spring balance will be 10 kg wt.
24. (*i*) The reading in the weighing machine will increase.

(*ii*) The reading in the weighing machine will decrease.

25. When the lift is falling freely, the pull of the cable is minimum (zero). When the lift is moving up with some acceleration, then the pull is maximum.
26. Let R be the reaction, then

$$R - mg = 2mg \quad \therefore \quad R = 3mg.$$

27. Newton's third law of motion.
28. (*i*) Vertically downwards.

(*ii*) Parabolic to the person standing outside.

29. If no external force acts on a system, the momentum of the system remains constant.
30. No, it is true only for an isolated system.
31. Law of conservation of linear momentum.
32. No, it does not violate the law of conservation of momentum because an external force is acting on the system.
33. The two fragments will fly off in exactly opposite directions.
34. Yes. As no external force can be applied on the universe, so its total momentum remains constant.
35. Yes.
36. Impulse is a vector quantity. Its SI unit is Ns.

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37. Linear momentum has same dimensions as the impulse.
38. Zero.
39. The force of friction between solid surfaces in contact is called dry friction.
40. The force of friction between a solid surface and a liquid surface is called wet friction.
41. It is the force of reaction on a body due to the surface on which it is placed. It acts perpendicular to the surface of contact.
42. The maximum force of static friction which comes into play when a body just starts moving over the surface of another body is called limiting friction.
43. The coefficient of limiting friction is defined as the ratio of limiting friction to the normal reaction.
44. The coefficient of limiting friction has no units.
45. $\mu_s > \mu_k$.
46. Coefficient of friction = Tangent of the angle of friction or $\mu = \tan\theta$.
47. Angle of friction = Angle of repose.
48. No, rolling friction is less than sliding friction.
49. The coefficient of friction depends on (*i*) nature of the surfaces in contact and (*ii*) nature of motion.
50. Limiting friction is greatest.
51. As the force of friction always opposes the relative motion, so the angle is 180°.
52. Angle of inclination = Angle of repose.
53. See Fig. 5.81 on page 5.45.
54. The friction between our feet and the ground helps us to move forward.
55. No, force of friction does not depend on the area of contact.
56. $\mu = \tan\theta = 1/\sqrt{3} \quad \therefore \quad \theta = 30°$.
57. Angle of friction is the angle which the resultant of limiting friction and the normal reaction makes with the normal reaction.
58. This is because rolling friction is less than the sliding friction.
59. Conical shape of the rockets reduces atmospheric friction.
60. $\mu_r < \mu_k < \mu_s$.
61. Yes, a centripetal force must be acting on the body.
62. Gravitational force of attraction on the satellite due to the earth.
63. The electrostatic attraction on the electrons due to the nucleus.
64. No. A centripetal force can move a body along a circular path but cannot produce rotational motion.
65. Centripetal force.
66. The force of friction between the tyres and the road provides the necessary centripetal force to the car for taking a turn on a level road.
67. No, the angle of banking does not depend on the mass of the vehicle.
68. The agency forcing the body to move in a circular path experiences the centrifugal force.
69. No quantity remains constant because the directions of all these vectors change from instant to instant. However, the magnitudes of all these quantities remain constant.
70. $v = \sqrt{5gr}$, where r is the radius of the circular arch.
71. 1 kg wt = 9.8 N.
72. Impulse is the total effect of a large force which acts for a short time to produce a finite change in momentum.

    Impulse = Force × its time duration

    = Total change in momentum.
73. 1 mN = 1 millinewton = 10^{-3} N

    1 nm = 1 nanometre = 10^{-9} m
74. First law of motion.
75. The SI unit of force is newton. One newton is the force which produces an acceleration of 1 ms^{-2} in a body of mass 1 kg.
76. Friction is the opposing force which comes into play between the surfaces in contact, when one body moves or tends to move over the surface of another body.
77. The coefficient of limiting friction is the ratio of two forces, so it has no unit.
78. The coefficient of friction remains unchanged.
79. When the circular road is banked, the horizontal component of the normal reaction of the road provides the necessary centripetal force for the vehicle to move it along the curved path.
80. The resultant of the three forces is zero. The velocity $\vec{v}$ of the particle will remain unaffected.
81. As the action and reaction forces act on different bodies, they cannot cancel each other.
82. Inertia.
83. Change in momentum $= m(-v) - mv = -2mv$.
84. The maximum safe velocity, $v = \sqrt{\mu r g}$.
85. Law of inertia.
86. $m_1 v_1 = m_2 v_2 \Rightarrow v_1 : v_2 = m_2 : m_1 = 4:2 = 2:1$

Type B : Short Answer Questions

2 or 3 Marks Each

1. What is inertia ? Discuss its types giving one example in each case.
2. Define momentum. Is it a scalar or vector quantity ? Give its units and dimensions.

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3. State Newton's second law of motion. Hence, derive the relation $\vec{F} = m\vec{a}$, where the symbols have their usual meanings. [Himachal 09C ; Delhi 96]
4. State Newton's second law of motion. Show that it gives a measure of force. Hence define 1 N force. [Himachal 04, 05C, 07C]
5. Show that Newton's second law of motion is the real law of motion. [Delhi 98 ; Chandigarh 03 ; Himachal 07C, 08, 09]
6. State the Newton's second law of motion and deduce the Newton's first law from it. [Delhi 09]
7. How can you show that Newton's third law of motion follows from the Newton's second law of motion ?
8. What is force ? What are its absolute and gravitational units ? How are these related to each other ? [Himachal 03, 09]
9. State Newton's Second law of motion. Derive a mathematical formula to calculate force if mass remains constant. Prove that it contains Newton's first law. [Central Schools 03]
10. State Newton's third law of motion. Derive the law of conservation of linear momentum from it. [Manipur 97 ; Himachal 03, 05, 07C]
11. (*i*) State principle of conservation of momentum.
(*ii*) A bullet of mass '*m*' is fired from a gun of mass '*M*' with a horizontal velocity '*V*'. Calculate the recoil velocity '*v*' of the gun. [Delhi 97]
12. State the principle of conservation of linear momentum. Explain, why the gun recoils when a bullet is fired from it. [Himachal 02, 07C]
13. Derive the law of conservation of momentum from Newton's second law of motion. [C. Schools 13]
14. Derive Newton's third law of motion from the law of conservation of momentum.
15. State law of conservation of momentum and prove it by using third law of motion. [Chandigarh 02 ; Himachal 09C]
16. Name a varying mass system. Derive an expression for velocity of propulsion of a rocket at any instant. [Chandigarh 03]
17. Two masses *M* and *m* are connected at the two ends of an inextensible string. The string passes over a smooth frictionless pulley. Obtain the acceleration of the masses and the tension in the string. Given $M > m$. [Delhi 12, 14 ; Central Schools 12, 14]
18. Define the terms momentum and impulse. State and prove impulse-momentum theorem. [Himachal 06, 09C]
19. Define the term impulse. Give its SI unit. Prove that impulse of a force is equal to the change in momentum. [Himachal 06, 07C, 08]
20. What are concurrent forces ? Obtain a condition for the equilibrium of three concurrent forces. [Himachal 06, 09C]
21. What do you mean by equilibrium of concurrent forces. Prove that under the action of three concurrent forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$, a body will be in equilibrium, when $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$. [Himachal 01, 05C]
22. What are inertial and non-inertial frames of reference ? Give an example of each.
23. What is friction ? Explain the cause of sliding friction. [Himachal 08]
24. What is limiting friction ? State the laws of limiting friction. [Himachal 07 ; Central Schools 12, 14]
25. State the laws of kinetic friction. Define coefficient of kinetic friction.
26. Define angle of friction. Show that the tangent of the angle of friction is equal to the coefficient of static friction.
27. Define coefficient of friction and angle of friction and hence derive a relation between them. [Chandigarh 08]
28. Define angle of repose. Deduce its relation with coefficient of static friction. [Himachal 04, 07]
29. Explain why it is easier to pull a lawn roller than to push it ? [Central Schools 05 ; Delhi 14]
30. Graphically show that static friction is a self adjusting force. [Delhi 13]
31. Friction is a necessary evil. Explain. [Himachal 05, 06, 09C]
32. Describe any three methods of reducing friction. [Kerala 01 ; Himachal 04, 08]
33. Define force of friction. How does the use of ball bearing reduce friction ? [Delhi 05]
34. Distinguish between static friction, limiting friction and kinetic friction. How do they vary with the applied force, explain by diagram. [Chandigarh 04]
35. Find an expression for the work done in sliding a body over a rough horizontal surface.
36. Derive an expression for the work done when a body is made to slide up a rough inclined plane.
37. Derive an expression for the work done when a body slides down a rough inclined plane.
38. Obtain an expression for the acceleration of a body sliding down a rough inclined plane. [Delhi 13 ; Central Schools 14]
39. State two advantages of friction in daily life. [Delhi 96]
40. Define the term coefficient of limiting friction between the surfaces. A body of mass 10 kg is placed on an inclined surface of angle 30°. If the coefficient of limiting friction is $1/\sqrt{3}$, find the force required to just push the b
plane. The force is being ap
inclined surface.
41. When a horse pulls a cart, ac
third law, the cart also pulls th
and opposite force. What cau
horse, cart and the system as a
a diagram.
42. Why does a cyclist lean inward
curve ? Explain with a diagra
[Delhi 99,
43. Obtain an expression for the a
will have to make with the ver
circular turn. [For successful n
[Chandigarh 07
44. Find an expression for the m
circular motion of a car in a circu
of radius '*R*'. The coefficient
between the car tyres and th
surfaces is μ_s.
45. Show various forces on the ve
rough inclined circular turn. He
mum safe velocity of the vehicle
negotiate the circular turn. [Ce

Answers

1. Refer answer to Q. 4 on page 5.2
2. Refer answer to Q. 7 on page 5.3
3. Refer answer to Q. 16 on page 5.
4. Refer answer to Q. 16 on page 5.
5. Refer answer to Q. 36 on page 5.
6. Refer answer to Q.36 on page 5.1
7. Refer answer to Q.36 on page 5.1
8. Refer answer to Q. 18 on page 5.6
9. Refer answer to Q. 16 on page 5.5
Q. 36 on page 5.17.
10. Refer answer to Q. 39 on page 5.2
11. (*i*) Refer answer to Q. 38 on page
(*ii*) Refer answer to Q. 41 on page
12. Refer answer to Q. 38 on page 5.20
Q. 41 on page 5.21.
13. Refer answer to Q. 38 on page 5.20
14. Refer answer to Q. 40 on page 5.20
15. Refer answer to Q. 39 on page 5.20
16. Refer answer to Q. 42 on page 5.24
17. Refer answer to Q. 45 on page 5.31
18. Refer answer to Q 28 on page 5.11.
19. Refer answer to Q. 28 on page 5.11.
20. Refer answer to Q 43 on page 5.27.
21. Refer answer to Q. 43 on page 5.27.
22. Refer answer to Q. 71 on page 5.62.

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required to just push the body up the inclined plane. The force is being applied parallel to the inclined surface. [Delhi 02]

41. When a horse pulls a cart, according to Newton's third law, the cart also pulls the horse with an equal and opposite force. What causes the motion of the horse, cart and the system as a whole ? Explain with a diagram. [Central Schools 08]

42. Why does a cyclist lean inwards while negotiating a curve ? Explain with a diagram. [Delhi 99, 02 ; Himachal 02, 09C]

43. Obtain an expression for the angle which a cyclist will have to make with the vertical, while taking a circular turn. [For successful negotiation]. [Chandigarh 07 ; Central Schools 14]

44. Find an expression for the maximum speed of circular motion of a car in a circular horizontal track of radius 'R'. The coefficient of static friction between the car tyres and the road along the surfaces is μ_s. [Delhi 03]

45. Show various forces on the vehicle negotiating a rough inclined circular turn. Hence find the maximum safe velocity of the vehicle with which it can negotiate the circular turn. [Central Schools 04, 14]

46. With the help of suitable diagram, obtain an expression for the maximum speed with which a vehicle can safely negotiate a curved road banked at an angle θ. The coefficient of fiction between the wheels and road is μ. [Delhi 06, 13 ; Central Schools 12]

47. State Newton's second and third laws of motion. Explain how these laws lead to an important consequence – the law of conservation of angular momentum. [Delhi 03C]

48. A block of wood of mass m rests on a rough horizontal plane. The plane is gradually inclined at an angle θ with the horizontal until the block just begins to slide. Find the coefficient of static friction between the block and the plane. [Delhi 03C]

49. A small body tied to one end of the string is whirled in a vertical circle.
   (*i*) Represent the forces on a diagram when the string makes an angle θ with initial position.
   (*ii*) Find the tension and velocity at the highest and lowest point respectively. [Delhi 05]

50. What do you mean by ball bearings ? How sliding friction is converted into rolling friction ? [Himachal 09]

Answers

1. Refer answer to Q. 4 on page 5.2.
2. Refer answer to Q. 7 on page 5.3.
3. Refer answer to Q. 16 on page 5.5.
4. Refer answer to Q. 16 on page 5.5.
5. Refer answer to Q. 36 on page 5.17.
6. Refer answer to Q.36 on page 5.17.
7. Refer answer to Q.36 on page 5.17.
8. Refer answer to Q. 18 on page 5.6.
9. Refer answer to Q. 16 on page 5.5 and Q. 36 on page 5.17.
10. Refer answer to Q. 39 on page 5.20.
11. (*i*) Refer answer to Q. 38 on page 5.20.
    (*ii*) Refer answer to Q. 41 on page 5.21.
12. Refer answer to Q. 38 on page 5.20 and Q. 41 on page 5.21.
13. Refer answer to Q. 38 on page 5.20.
14. Refer answer to Q. 40 on page 5.20.
15. Refer answer to Q. 39 on page 5.20.
16. Refer answer to Q. 42 on page 5.24.
17. Refer answer to Q. 45 on page 5.31.
18. Refer answer to Q.28 on page 5.11.
19. Refer answer to Q. 28 on page 5.11.
20. Refer answer to Q.43 on page 5.27.
21. Refer answer to Q. 43 on page 5.27.
22. Refer answer to Q. 71 on page 5.62.
23. Refer answer to Q. 47 on page 5.37 and Q. 49 on page 5.37.
24. Refer answer to Q. 51 on page 5.38.
25. Refer answer to Q. 52 on page 5.38.
26. Refer answer to Q. 53 on page 5.39.
27. Refer answer to Q.53 on page 5.39.
28. Refer answer to Q. 54 on page 5.39.
29. Refer answer to Q. 65 on page 5.51.
30. Refer answer to Q.65 on page 5.51.
31. Refer answer to Q. 62 on page 5.51.
32. Refer answer to Q. 63 on page 5.51.
33. Refer answer to Q. 63 on page 5.51.
34. Refer answer to Q. 50 on page 5.37.
35. Refer answer to Q. 58 on page 5.44.
36. Refer answer to Q. 59 on page 5.45.
37. Refer answer to Q. 60 on page 5.45.
38. Refer answer to Q. 61 on page 5.45.
39. Refer answer to Q. 62 on page 5.51.
40. The coefficient of limiting friction is defined as the ratio of limiting friction to the normal reaction.

$$F = mg(\sin\theta + \mu\cos\theta)$$

$$= 10 \times 9.8\left(\sin 30^\circ + \frac{1}{\sqrt{3}}\cos 30^\circ\right) = 98\left(\frac{1}{2} + \frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{2}\right)$$

$$= 98 \text{ N}.$$

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41. Refer answer to Q. 35 on page 5.16.
42. Refer answer to Q. 69 on page 5.55.
43. Refer answer to Q.60 on page 5.45.
44. Refer answer to Q. 68 on page 5.54.
45. Refer answer to Q. 68 on page 5.54.
46. Refer answer to Q. 68 on page 5.54.
47. Refer answer to Q. 38 on page 5.20 and Q. 5.39 on page 5.20.
48. Refer answer to Q. 54 on page 5.39.
49. Refer answer to Q. 70 on page 5.58.
50. Refer answer to Q. 63 (*iv*) page 5.51.

Type C : Long Answer Questions

5 Marks Each

1. Prove that the velocity of a rocket at any instant when its mass is m is given by $v = v_0 + u \log_e \frac{m_0}{m}$

   where, v_0 and m_0 are the velocity and mass of the rocket at $t = 0$ and u is the velocity of exhaust gases relative to the rocket. **[Himachal 02 ; Delhi 06]**

2. (*a*) A person of mass m is standing in a lift. Find his apparent weight when the lift is : (*i*) moving upward with uniform acceleration a. (*ii*) moving downward with uniform acceleration $a(< g)$. (*iii*) falls freely.

   (*b*) Explain why, it is easier to pull a lawn mover than to push it. **[Delhi 12, 14]**

3. (*a*) Obtain an expression for the centripetal force required to make a body of mass m, moving with a velocity v around a circular path of radius r.

   (*b*) Find an expression for velocity of recoil of gun. **[Chandigarh 07]**

4. Define the terms static friction, limiting friction and kinetic friction. Draw the graph between friction and applied force on any object and show static friction, limiting friction and kinetic friction in graph.

   Using graph show that static friction is a self-adjusting force. **[Delhi 12 ; Central Schools 14]**

5. What do you understand by friction ? Discuss about static friction, limiting friction, kinetic friction and rolling friction. Show how the force of friction varies with the applied friction F. **[Delhi 08, 10]**

6. Define angle of friction and angle of repose. Show that both are numerically equal. **[Himachal 02, 03]**

7. Explain the terms : friction and limiting friction. State the laws of limiting friction. Give some methods for reducing friction. **[Himachal 02]**

8. What is meant by banking of roads ? What is the need for banking a road ? Obtain an expression for the maximum speed with which a vehicle can safely negotiate a curved road banked at an angle θ. The coefficient of friction between the wheels and the road is μ. **[Delhi 08, 10]**

9. Derive an expression for velocity of a car on a banked circular road having coefficient of friction μ. Hence write the expression for optimum velocity. **[Chandigarh 08]**

10. (*a*) Why are circular roads banked ? Deduce an expression for the angle of banking.

    (*b*) A 1000 kg car rounds a curve on a flat road of radius 50 m at a speed of 50 km h^{-1} (14 ms^{-1}). Will the car make the turn or will it skid if the coefficient of friction is 0.60 ? Justify. **[BIT Ranchi 99]**

11. What are centripetal and centrifugal forces ? A body attached to a string of length l describes a vertical circle such that it is just able to cross the highest point. Find the minimum velocity at the bottom of the circle. **[Meghalaya 99]**

Answers

1. Refer answer to Q. 42 on page 5.24.
2. (*a*) Refer answer to Q. 37 on page 5.17.

   (*b*) Refer answer to Exercise 5.23(*c*) on page 5.79.
3. (*a*) Refer answer to Q. 66 on page 5.52.

   (*b*) Refer to answer to Q 41(*i*) on page 5.21.
4. Refer answer to Q. 50 on page 5.37.
5. Refer answer to Q. 50 on page 5.37.
6. Refer answer to Q. 53 and Q. 54 on page 5.39.
7. Refer answer to Q. 51 on page 5.38 and Q. 63 on page 5.51.
8. Refer answer to Q.68 on page 5.54.
9. Refer answer to Q.68 on page 5.54.
10. (*a*) Refer answer to Q. 68 on page 5.54.

    (*b*) Required centripetal force

    $$= \frac{mv^2}{r} = \frac{1000 \times (14)^2}{50} = 3920 \text{ N}$$

    Force of friction

    $$= \mu mg = 0.60 \times 1000 \times 9.8 = 5880 \text{ N}$$

    As the available force of friction is greater than the required centripetal force, so the car will not skid.
11. Refer answer to Q. 70 on page 5.58.

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Competition Section

Laws of Motion

GLIMPSES

1. **Force.** It may be defined as an agency (a push or pull) which changes or tends to change the state of rest or of uniform motion or the direction of motion of a body. Force is a vector quantity.

2. **Inertia.** It is the inherent property of a material body by virtue of which it cannot change, by itself, its state of rest or of uniform motion in a straight line. Inertia is of *three* types (*i*) inertia of rest (*ii*) inertia of motion and (*iii*) inertia of direction.

3. **Mass as the measure of inertia.** If a body has more mass, it has more inertia *i.e.*, it is more difficult to change its state of rest or of uniform motion.

4. **Momentum.** It is the quantity of motion in a body. It is equal to the product of mass m and velocity v of the body.

$$\text{Momentum, } p = mv \text{ or } \vec{p} = m\vec{v}$$

Momentum is a vector quantity having the direction of velocity $\vec{v}$. Its SI unit is kg ms^{-1} and CGS unit in g cms^{-1}.

5. **Newton's first law of motion.** It states that every body continues in its state of rest or of uniform motion along a straight line, unless an external force is applied to change that state. This law defines force.

6. **Newton's second law of motion.** It states that the rate of change of momentum of a body is directly proportional to the applied force and the change in momentum takes place in the direction of the applied force. This law gives a measure of the force.

Mathematically,

$$F = \frac{dp}{dt} = \frac{d}{dt}(mv) = m\frac{dv}{dt} = ma = m\left(\frac{v-u}{t}\right)$$

(*i*) When m is constant and v changes,

$$F = m\frac{dv}{dt}$$

(*ii*) When v is constant and m changes,

$$F = v\frac{dm}{dt}$$

In component form, Newton's second law may be expressed as

$$F_x = \frac{dp_x}{dt} \; ; \; F_y = \frac{dp_y}{dt} \; ; \; F_z = \frac{dp_z}{dt}.$$

7. **Newton's third law of motion.** It states that to every action, there is an equal and opposite reaction. Mathematically,

$$\vec{F}_{BA} = -\vec{F}_{AB}$$

Forces of action and reaction never cancel out because they act on different bodies.

8. **Absolute units of force.** The SI unit of force is *newton* (N) and CGS unit is *dyne* (dyn).

$$1 \text{ newton (N)} = 1\,\text{kg} \times 1\,\text{ms}^{-2} = 1\,\text{kg ms}^{-2}$$
$$1 \text{ dyne (dyn)} = 1\,\text{g} \times 1\,\text{cms}^{-2} = 1\,\text{g cms}^{-2}$$
$$1\,\text{N} = 10^5 \text{ dyne.}$$

9. **Gravitational units of force.** The SI unit of force is *kilogram weight* (kg wt) or *kilogram force* (kg f) and the CGS unit is *gram weight* (g wt) or gram force (g f).

$$1\,\text{kg wt} = 1\,\text{kg f} = 1\,\text{kg} \times 9.8\,\text{ms}^{-2} = 9.8\,\text{N}$$
$$1\,\text{g wt} = 1\,\text{g f} = 1\,\text{g} \times 980\,\text{cms}^{-2} = 980\,\text{dyn}$$

10. **Impulse of a force.** Impulse is the total effect of a large force which acts for a short time to produce a finite change in momentum. It is defined as the product of the force and the time for which it acts and equal to the total change in momentum.

Impulse = Force × time duration
= Total change in momentum.

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Impulse is a vector quantity, denoted by $\vec{I}$.

$$\vec{I} = \int_{t_1}^{t_2} \vec{F}\,.\,dt$$

= Area under the force-time (F-t) graph

or $$\vec{I} = \vec{F}_{av} \times t = \vec{p}_2 - \vec{p}_1 = m(\vec{v} - \vec{u})$$

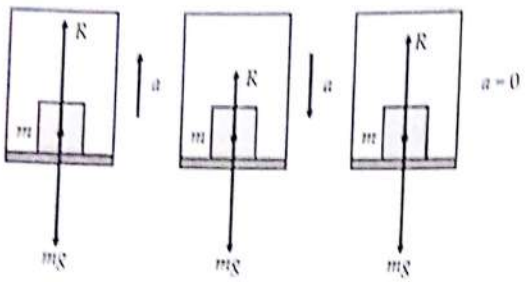
The SI unit of impulse is Ns or kg ms^{-1} and CGS unit is dyn s or g cms^{-1}.

11. Apparent weight of body in a lift.

(*i*) When a lift moves upwards with uniform acceleration a, apparent weight of a body in the lift increases.

From Fig. (*a*),

$$R - mg = ma \quad \text{or} \quad R = m(g + a)$$



(*ii*) When a lift moves downwards with acceleration a, the apparent weight of a body in the lift decreases. From Fig. (*b*),

$$mg - R = ma \quad \text{or} \quad R = m(g - a).$$

(*iii*) When a lift is at rest or moves with uniform velocity, $a = 0$, the apparent weight of the body is equal to its true weight.

From Fig. (*c*),

$$R = mg$$

(*iv*) When a lift falls freely, ($a = g$) the apparent weight of a body in the lift becomes zero.

$$R = m(g - g) = 0$$

12. Law of conservation of linear momentum.

(*i*) In the absence of any external force, vector sum of the linear momenta of a system of particles remains constant.

$$\vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \ldots + \vec{p}_n = \text{constant}$$

or $$m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \ldots + m_n\vec{v}_n = \text{constant}$$

(*ii*) When two bodies collide,

Total momentum before collision

= Total momentum after collision

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

(*iii*) When a bullet of mass m is fired with velocity v from a gun of mass M, the gun recoils with velocity V.

Momentum of gun = − Momentum of bullet

$$MV = -mv$$

Recoil velocity of gun,

$$V = -\frac{mv}{M}.$$

13. Rocket propulsion. It is an example of momentum conservation in which the large backward momentum of the ejected gases imparts an equal forward momentum to the rocket. Due to the decrease in mass of the rocket-fuel system, the acceleration of the rocket keeps on increasing. Let

u = velocity of exhaust gases

v_0, v = initial velocity and velocity of the rocket at any instant t

m_0, m, m_e = initial mass, mass of the rocket at any instant t and mass of empty rocket

$\frac{dm}{dt}$ = rate of ejection of fuel.

Thrust on rocket : $$F = -u\frac{dm}{dt}$$

Acceleration of rocket : $$a = \left[\frac{u}{m_e - t\dfrac{dm}{dt}}\right]\frac{dm}{dt}$$

Velocity of rocket : $$v = v_0 + u\log_e\frac{m_0}{m}$$

Burnt-out speed of rocket : $$v_b = v_0 + u\log_e\frac{m_0}{m_e}.$$

14. Concurrent forces. The forces acting at the same point of a body are called concurrent forces.

15. Equilibrium of concurrent forces. A number of concurrent forces acting on a body are said to be in equilibrium if their vector sum is zero or if these forces can be completely represented by the sides of a closed polygon taken in the same order.

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots + \vec{F}_n = 0$$

16. Lami's theorem. It states that if three forces acting on a particle keep it in equilibrium, then each force is proportional to the sine of the angle between other two forces. If α, β, γ be the angles between $\vec{F}_2$ and $\vec{F}_3$; $\vec{F}_3$ and $\vec{F}_1$; $\vec{F}_1$ and $\vec{F}_2$ respectively, then according to Lami's theorem :

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}.$$

17. Free-body diagram. A diagram for each body of the system showing all the forces exerted on the body by the remaining parts of the system is called free-body diagram.

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SECTION

18. **Motion of connected bodies.** Suppose two bodies of masses M and m ($M > m$) are tied at the ends of an inextensible string passing over a frictionless pulley. Then

Acceleration of the masses, $a = \frac{M-m}{M+m} \cdot g$

Tension in the string, $T = \frac{2Mm}{M+m} \cdot g$

Clearly, $a < g$.

19. **Friction.** Whenever a body moves or tends to move over the surface of another body, a force comes into play which acts parallel to the surface of contact and opposes the relative motion. This opposing force is called friction.

20. **Static friction.** The force of friction which comes into play between two bodies before one body actually starts moving over the other is called static friction (f_s). Static friction is a self-adjusting force.

21. **Limiting friction.** The maximum force of static friction which comes into play when a body just starts moving over the surface of another body is called limiting friction (f_s^{max}).

22. **Kinetic friction.** The force of friction which comes into play when a body is in a steady motion over the surface of another body is called kinetic or dynamic friction (f_k). kinetic friction is less than limiting friction.

23. **Laws of limiting friction :**

(*i*) The force of limiting friction depends upon the nature of the two surfaces in contact and their state of roughness.

(*ii*) The force of limiting friction acts tangential to the two surfaces in contact and in a direction opposite to that of the applied force.

(*iii*) The force of limiting friction between any two surfaces is independent of the shape or area of the surfaces in contact so long as the normal reaction remains the same.

(*iv*) The force of limiting friction between two given surfaces is directly proportional to the normal reaction between the two surfaces.

$$f \propto R \quad \text{or} \quad f = \mu_s R$$

where the constant of proportionality μ_s is called the coefficient of limiting friction.

24. **Coefficient of limiting friction.** It is the ratio of limiting friction to the normal reaction.

$$\mu_s = \frac{f_s^{max}}{R} = \frac{\text{Limiting friction}}{\text{Normal reaction}}.$$

25. **Coefficient of kinetic friction.** It is the ratio of kinetic friction (f_k) to the normal reaction.

$$\mu_k = \frac{f_k}{R} = \frac{\text{Kinetic friction}}{\text{Normal reaction}}$$

As $f_k < f_s^{max}$ or $\mu_k R < \mu_s R$ $\therefore \mu_k < \mu_s$.

26. **Angle of friction.** It is the angle which the resultant of the limiting friction and the normal reaction makes with the normal reaction. If θ is the angle of friction, then $\tan\theta = \mu_s$.

27. **Angle of repose.** It is the minimum angle that an inclined plane makes with the horizontal when a body placed on it just begins to slide down. If ϕ is the angle of repose, then $\tan\phi = \mu_s$.

28. **Motion along a rough horizontal surface.** If a body of mass m is moved over a rough horizontal surface through distance s, then

Force of friction, $f = \mu R = \mu mg$

Retardation produced, $a = \frac{f}{m} = \mu g$

Work done against friction, $W = f \times s = \mu mg s$

Power $= f \times v = \mu mg v$.

29. **Motion along a rough inclined plane.**

(*i*) When a body moves down an inclined plane with uniform velocity ($a = 0$), net downward force needed is

$$F = mg \sin\theta - f = mg(\sin\theta - \mu\cos\theta)$$

Work done, $W = F_s = mg(\sin\theta - \mu\cos\theta)\, s$

(*ii*) When a body moves up an inclined plane with uniform velocity ($a = 0$), net upward force needed is

$$F = mg \sin\theta + f = mg(\sin\theta + \mu\cos\theta)$$

$$W = Fs = mg(\sin\theta + \mu\cos\theta)\, s$$

(*iii*) When a body moves up an inclined plane with acceleration a, net upward force needed is

$$F = ma + mg\sin\theta + f$$

$$= m(a + g\sin\theta + \mu g\cos\theta)$$

$$W = m(a + g\sin\theta + \mu g\cos\theta)\, s.$$

30. **Acceleration of body sliding down a rough inclined plane.** When the angle of inclination is greater than the angle of repose, the acceleration produced is

$$a = g(\sin\theta - \mu_k\cos\theta).$$

31. **Centripetal force.** It is the force required to make a body move along a circular path with a uniform speed. It always acts along the radius and towards the centre of the circular path. The centripetal force required to move a body of mass m along a circular path of radius r with speed v is given by

$$F = \frac{mv^2}{r} = mr\omega^2 = mr(2\pi\nu)^2 = mr\left(\frac{2\pi}{T}\right)^2$$

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32. **Centrifugal force.** It is a fictitious force acting radially outwards on a particle moving in a circle and is equal in magnitude to the centripetal force.

33. **A vehicle taking circular turn on a level road.** If μ is the coefficient of friction between tyres and road, then the maximum velocity with which the vehicle can safely take a circular turn of radius r is given by

$$v = \sqrt{\mu r g}$$

34. **Banking of tracks (roads).** The maximum angle with which a vehicle (in the absence of friction) can negotiate a circular turn of radius r and banked at an angle θ is given by

$$v = \sqrt{rg \tan\theta}$$

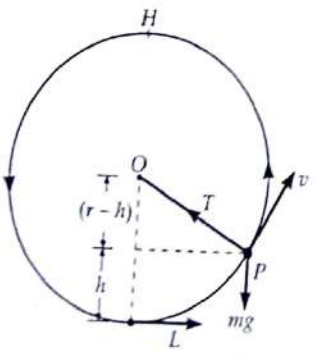
When the frictional forces are also taken into account, the maximum safe velocity is given by

$$v = \sqrt{rg\left(\frac{\mu + \tan\theta}{1 - \mu \tan\theta}\right)}$$

35. **Bending of a cyclist.** In order to take a circular turn of radius r with speed v, the cyclist should bend himself through an angle θ from the vertical such that

$$\tan\theta = \frac{v^2}{rg}$$

36. **Motion in a vertical circle.** As shown in the figure, consider a body of mass m tied at the end of a string and rotating in a vertical circle of radius r.



Then

(i) Velocity at any point P at a height h from its lowest point L,

$$v = \sqrt{u^2 - 2gh}$$

(ii) Tension in the string at any point P,

$$T = \frac{m}{r}(u^2 - 3gh + gr)$$

(iii) Tension at the lowest point ($h = 0$),

$$T_L = \frac{m}{r}(u^2 + gr)$$

(iv) Tension at the highest point H ($h = 2r$),

$$T_H = \frac{m}{r}(u^2 - 5gr)$$

(v) Difference in tensions at the highest and lowest points,

$$T_L - T_H = 6mg$$

(vi) Minimum velocity at the lowest point L for looping the loop,

$$v_L = \sqrt{5gr}$$

(vii) Velocity at the highest point for looping the loop,

$$v_H = \sqrt{gr}.$$

37. **Inertial frame of reference.** An inertial frame of reference is one in which Newton's first law of motion holds good. All frames moving with uniform motion relative to an inertial frame are also inertial.

38. **Non-inertial frame of reference.** A frame of reference which is accelerating with respect to an inertial frame of reference is called a non-inertial frame of reference. In a non-inertial frame, the second law of motion $\vec{F} = m\vec{a}'$ is not valid. Instead, it takes the form

$$\vec{F} - m\vec{a} = m\vec{a}' \quad \text{or} \quad \vec{F} + \vec{F}_p = m\vec{a}'$$

where $\vec{a}$ is the acceleration of the non-inertial frame relative to any inertial frame and $\vec{a}'$ is the acceleration of the body relative to the non-inertial frame. The force $\vec{F}_p = -m\vec{a}$ is an example of pseudo force.

IIT Entrance Exam

Multiple Choice Questions with One Correct Answer

1. A ship of mass 3×10^7 kg initially at rest is pulled by a force of 5×10^4 N through a distance of 3 m. Assuming that the resistance due to water is negligible, the speed of the ship is

(a) 1.5 m/s (b) 60 m/s
(c) 0.1 m/s (d) 5 m/s [IIT 80]

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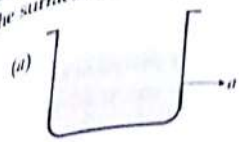
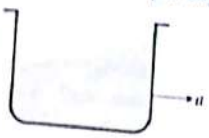
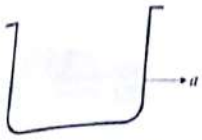
2. A vessel containing water is given a constant acceleration a towards the right, along a straight horizontal path. Which of the following figures represents the surface of the liquid ? [IIT 80]

(a) 

(b) 

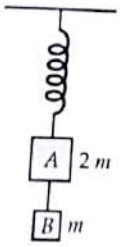
(c) 

3. Two particles of masses m_1 and m_2 in projectile motion have velocities $\vec{v}_1 < \vec{v}_2$ respectively at time $t = 0$. They collide at time t_0. Their velocities become $\vec{v}_1'$ and $\vec{v}_2'$ at time $2t_0$, while still moving in air. The value of $|(m_1\vec{v}_1' + m_2\vec{v}_2') - (m_1\vec{v}_1 + m_2\vec{v}_2)|$ is

(a) $(m_1 + m_2)gt_0$ (b) $2(m_1 + m_2)gt_0$

(c) $\frac{1}{2}(m_1 + m_2)gt_0$ (d) zero [IIT 01]

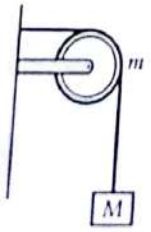
4. Two blocks A and B of masses $2m$ and m, respectively are connected by a massless and inextensible string. The whole system is suspended by a massless spring as shown in the figure. The magnitudes of acceleration of A and B, immediately after the string is cut, are respectively



(a) $g, g/2$ (b) $g/2, g$

(c) g, g (d) $g/2, g/2$ [IIT 06]

5. A string of negligible mass going over a clamped pulley of mass m supports a block of mass M as shown in the figure. The force on the pulley by the clamp is given by



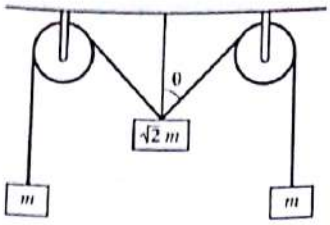
(a) $\sqrt{2}Mg$

(b) $\sqrt{2}mg$

(c) $\sqrt{(M+m)^2 + m^2}g$

(d) $\sqrt{(M+m)^2 + M^2}g$ [IIT 01]

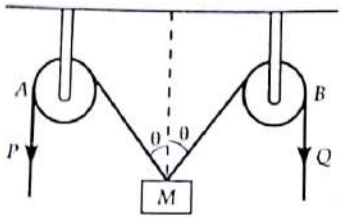
6. The pulleys and strings shown in the figure are smooth and of negligible mass. For the system to remain in equilibrium, the angle θ should be



(a) 0° (b) 30°

(c) 45° (d) 60° [IIT 01]

7. In the arrangement, shown in the figure, the ends P and Q of an unstretchable string move downwards with uniform speed u. Pulleys A and B are fixed. Mass M moves upwards with a speed



(a) $2u\cos\theta$ (b) $u/\cos\theta$

(c) $2u/\cos\theta$ (d) $u\cos\theta$ [IIT 82]

8. A particle moves in the X-Y plane under the influence of a force such that its linear momentum is $\vec{p}(t) = A[\hat{i}\cos(kt) - \hat{j}\sin(kt)]$, where A and k are constants. The angle between the force and the momentum is

(a) 0° (b) 30°

(c) 45° (d) 90° [IIT 07]

9. When a bicycle is in motion, the force of friction exerted by ground on the two wheels is such that, it acts

(a) in the backward direction on the front wheel and in the forward direction on the rear wheel.

(b) in the forward direction on the front wheel and in the backward direction on the rear wheel.

(c) in the backward direction on both the front and the rear wheels.

(d) in the forward direction on both the front and the rear wheels. [IIT 90]

10. A block of mass 2 kg rests on a rough inclined plane making an angle 30° with the horizontal. The coefficient of static friction between the block and the plane is 0.7. The frictional force on the block is

(a) 9.8 N (b) $0.7 \times 9.8\sqrt{3}$ N

(c) $98\sqrt{3}$ (d) 0.7×9.8 N [IIT 80]

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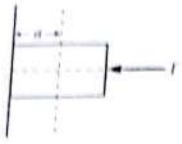
11. A block of mass 0.1 kg is held against a wall applying a horizontal force of 5 N on the block. If the coefficient of friction between the block and the wall is 0.5, the magnitude of the frictional force acting on the block is

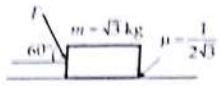
(a) 2.5 N (b) 0.98 N
(c) 4.9 N (d) 0.49 N [HT 94]

12. A block of mass m is at rest under the action of force F against a wall as shown in the figure. Which of the following statements is incorrect ?



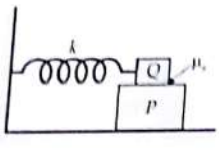
(a) $F = mg$; where f is frictional force
(b) $F = N$; where N is normal force
(c) F will not produce torque
(d) N will not produce torque. [HT 05]

13. What is the maximum value of the force F such that the block shown in the arrangement, does not move ?



(a) 20 N (b) 10 N
(c) 12 N (d) 15 N [HT 03]

14. A block P of mass m is placed on a horizontal frictionless plane. A second block of same mass m is placed on it and is connected to a spring of spring constant k. The two blocks are pulled by distance A. Block Q oscillates without slipping. What is maximum value of frictional force between the two blocks ?

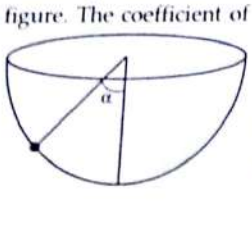


(a) $kA/2$ (b) kA
(c) $\mu_s mg$ (d) zero. [HT 80]

15. A weight W rests on a rough horizontal plane. If the angle of friction be θ, the least force that will move the body along the plane will be

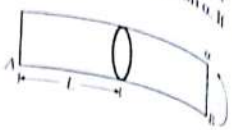
(a) $W\cos\theta$ (b) $W\tan\theta$
(c) $W\cot\theta$ (d) $W\sin\theta$ [HT 87]

16. An insect crawls up a hemispherical surface very slowly as shown in the figure. The coefficient of friction between the surface and the insect is 1/3. If the line joining the centre of the hemispherical surface to the insect makes an angle α with the vertical, the maximum possible value of α is given by



(a) $\cot\alpha = 3$ (b) $\tan\alpha = 3$
(c) $\sec\alpha = 3$ (d) $\operatorname{cosec}\alpha = 3$ [HT 91]

17. A long horizontal rod had a bead which can slide along its length and initially placed at a distance L from one end A of the rod. The rod is set in angular motion about A with constant angular acceleration α. If the coefficient of friction between the rod and the bead is μ, and gravity is neglected, then the time after which the bead starts slipping is

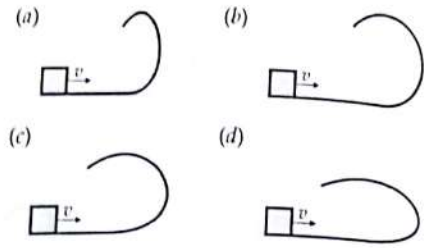


(a) $\sqrt{\mu/\alpha}$ (b) $\mu/\sqrt{\alpha}$
(c) $1/\sqrt{\mu\alpha}$ (d) infinitesimal [HT 96]

18. A car is moving in a circular horizontal track of radius 10 m with a constant speed of 10 m/s. A plumb bob is suspended from the roof of the car by a light rigid rod. The angle made by the rod with the track ($g = 10\ m/s^2$) is

(a) zero (b) 30°
(c) 45° (d) 60° [HT 92]

19. A small block is shot into each of the four tracks as shown below. Each of the tracks rises to the same height. The speed with which the block enters the track is the same in all cases. At the highest point of the track, the normal reaction is maximum in



[HT 01]

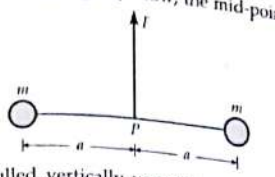
20. The driver of a car suddenly sees a broad wall in front of him. He should

(a) brake sharply (b) turn sharply
(c) (a) and (b) both (d) none of the above. [HT 87]

21. A stone tied to a string of length L is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time, the stone is at its lowest position and has a speed u. The magnitude of the change in its velocity as it reaches a position where the string is horizontal, is

(a) $\sqrt{u^2 - 2gL}$ (b) $\sqrt{2gL}$
(c) $\sqrt{u^2 - gL}$ (d) $\sqrt{2(u^2 - gL)}$ [HT 98 ; CBSE

22. Two articles of mass m each are tied at the ends of a light string of length $2a$. The whole system is placed on a frictionless horizontal surface with the string just tight so that each mass is at a distance a from the centre P (as shown in the figure). Now, the mid-point of the string is pulled vertically upwards with a small but constant force F. As a result, the particles move towards each other on the surface. The magnitude of acceleration, when the separation between them becomes $2x$ is



(a) $\frac{F}{2m}\frac{a}{\sqrt{a^2 - x^2}}$ (b) $\frac{F}{2m}\frac{x}{\sqrt{a^2 - x^2}}$
(c) $\frac{F}{2m}\frac{x}{a}$ (d) $\frac{F}{2m}\frac{\sqrt{a^2 - x^2}}{x}$

23. A block of base 10 cm × 10 cm and height 15 cm is kept on an inclined plane. The coefficient of friction between them is $\sqrt{3}$. The inclination θ of this inclined plane from the horizontal plane is gradually increased from 0°. Then

(a) at $\theta = 30°$, the block will start sliding down the plane, the block will remain at rest on the plane upto certain θ and then it will topple
(b) The block will remain at rest on the plane upto certain angle θ and then it will topple
(c) at $\theta = 60°$, the block will start sliding down the plane and continue to do so at higher
(d) at $\theta = 60°$, the block will start sliding down the plane and further increasing θ, it will topple at certain θ.

24. A block of mass m is on an inclined plane of angle θ. The coefficient of friction between the block and the plane is μ and $\tan\theta > \mu$. The block is held stationary by applying a force P parallel to the plane. The direction of force pointing up the plane is taken to be positive. As P is varied from $P_1 = mg(\sin$

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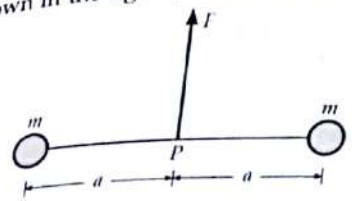
the change in its velocity as it reaches a position, where the string is horizontal, is

(a) $\sqrt{u^2-2gL}$ (b) $\sqrt{2gL}$

(c) $\sqrt{u^2-gL}$ (d) $\sqrt{2(u^2-gL)}$

[IIT 98 ; CBSE PMT 03]

22. Two articles of mass m each are tied at the ends of a light string of length $2a$. The whole system is kept on a frictionless horizontal surface with the string held tight so that each mass is at a distance a from the centre P (as shown in the figure). Now, the mid-point of the



string is pulled vertically upwards with a small but constant force F. As a result, the particles move towards each other on the surface. The magnitude of acceleration, when the separation between them becomes $2x$ is

(a) $\frac{F}{2m}\frac{a}{\sqrt{a^2-x^2}}$ (b) $\frac{F}{2m}\frac{x}{\sqrt{a^2-x^2}}$

(c) $\frac{F}{2m}\frac{x}{a}$ (d) $\frac{F}{2m}\frac{\sqrt{a^2-x^2}}{x}$ [IIT 07]

23. A block of base 10 cm × 10 cm and height 15 cm is kept on an inclined plane. The coefficient of friction between them is $\sqrt{3}$. The inclination θ of this inclined plane from the horizontal plane is gradually increased from 0°. Then

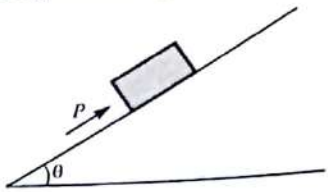
(a) at $\theta = 30°$, the block will start sliding down the plane, the block will remain at rest on the plane upto certain θ and then it will topple

(b) The block will remain at rest on the plane upto certain angle θ and then it will topple

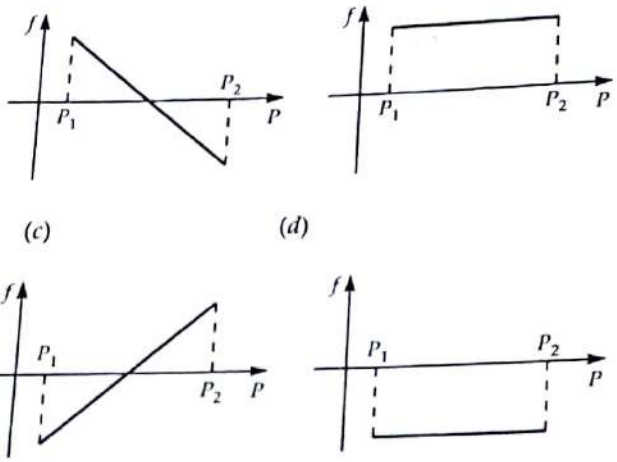
(c) at $\theta = 60°$, the block will start sliding down the plane and continue to do so at higher angles

(d) at $\theta = 60°$, the block will start sliding down the plane and further increasing θ, it will topple at certain θ. [IIT 09]

24. A block of mass m is on an inclined plane of angle θ. The coefficient of friction between the block and the plane is μ and $\tan\theta > \mu$. The block is held stationary by applying a force P parallel to the plane. The direction of force pointing up the plane is taken to be positive. As P is varied from $P_1 = mg(\sin\theta - \mu\cos\theta)$ to $P_2 = mg(\sin\theta + \mu\cos\theta)$, the frictional force f versus P graph will look like



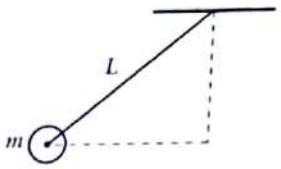
(a) (b)



(c) (d)

[IIT 2010]

25. A ball of mass (m) 0.5 kg is attached to the end of a string having length (L) 0.5 m. The ball is rotated on a horizontal circular path about vertical axis. The

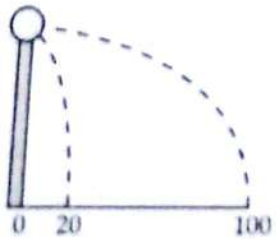


maximum tension the string can bear is 324 N. The maximum possible value of angular velocity of ball (in radian/s) is

(a) 9 (b) 18

(c) 27 (d) 36 [IIT 2011]

26. A ball of mass 0.2 kg rests on a vertical post of height 5 m. A bullet of mass 0.01 kg travelling with a



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velocity v m/s in a horizontal direction, hits the centre of the ball. After the collision, the ball and bullet travel independently. The ball hits the ground at a distance 20 m and the bullet at a distance of 100 m from the foot of the post. The initial velocity of the bullet is

(a) 250 m/s (b) $250\sqrt{2}$ m/s

(c) 400 m/s (d) 500 m/s [IIT 2011]

MULTIPLE CHOICE QUESTIONS WITH ONE OR MORE THAN ONE CORRECT ANSWER

27. A reference frame attached to the earth

(a) is an inertial frame by definition.

(b) cannot be an inertial frame because the earth is revolving round the sun.

(c) is an inertial frame because Newton's laws are applicable in this frame.

(d) cannot be an inertial frame because the earth is rotating about is own axis. [IIT 80]

28. The balls, having linear momenta $\vec{p_1} = p\hat{i}$ and $\vec{p_2} = -p\hat{i}$, undergo a collision in free space. There is no external force acting on the balls. Let $\vec{p_1'}$ and $\vec{p_2'}$ be their final momenta. The following option(s) is (are) *Not Allowed* for any non-zero value of $p, a_1, a_2, b_1, b_2, c_1$ and c_2.

(a) $\vec{p_1'} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$; $\vec{p_2'} = a_1\hat{i} + b_2\hat{j}$

(a) $\vec{p_1'} = c_1\hat{k}$; $\vec{p_2'} = c_2\hat{k}$

(c) $\vec{p_1'} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$; $\vec{p_2'} = a_2\hat{i} + b_2\hat{j} - c_1\hat{k}$

(d) $\vec{p_1'} = a_1\hat{i} + b_1\hat{j}$; $\vec{p_2'} = a_2\hat{i} + b_1\hat{j}$ [IIT 08]

REASONING TYPE

Instructions. Each question contains statement – 1 (assertion) and statement – 2 (reason). Of these statements mark correct choice if

(a) statements – 1 and 2 are true and statement – 2 is a correct explanation for statement – 1.

(b) statements – 1 and 2 are true and statement – 2 is not a correct explanation for statement – 1

(c) statement – 1 is true, statement – 2 is false

(d) statement – 1 is false, statement – 2 is true.

29. **Statement – 1.** A block of mass m starts moving rough horizontal surface with a velocity v. It stops o friction between the block and the surface after moving through a certain distance. The surface is now tilted to an angle of 30° with the horizontal and the same block is made to go up on the surface with the same initial velocity v. The decrease in the mechanical energy in the second situation is smaller than that in the first situation.

Statement – 2. The coefficient of friction between the block and the surface decreases with the increase in the angle of inclination. [IIT 07]

30. **Statement – 1.** A cloth covers a table. Some dishes are kept on it. The cloth can be pulled out without dislodging the dishes from the table.

Statement – 2. For every action, there is an equal and opposite reaction. [IIT 07]

31. **Statement – 1.** It is easier to pull a heavy object than to push it on a level ground.

Statement – 2. The magnitude of frictional force depends on the nature of the two surfaces in contact.

(a) Statement – 1 is True ; Statement – 2 is True ; Statement – 2 is a correct explanation for Statement – 1.

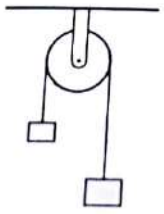
(b) Statement – 1 is True ; Statement – 2 is true ; Statement – 2 is not a correct explanation for Statement – 1.

(c) Statement – 1 is True ; Statement – 2 is false.

(d) Statement – 1 is False ; Statement – 2 is true. [IIT 08]

INTEGER ANSWER TYPE

32. A light inextensible string that goes over a smooth fixed pulley as shown in the figure connects



two blocks of masses 0.36 kg and 0.72 kg. Taking $g = 10\ m/s^2$, find the work done (in joules) by the string on the block of mass 0.36 kg during the first second after the system is released from rest. [IIT 09]

33. A block is moving on an inclined plane making an angle 45° with the horizontal and the coefficient of friction is μ. The force required to just push it up the inclined plane is 3 times the force required to just prevent it from sliding down. If we define $N = 10\mu$, then find N. [IIT 2011]

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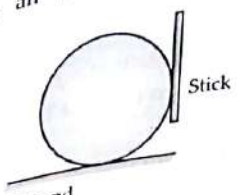
34. A boy is pushing a ring of mass 2 kg and radius 0.5 m with a stick as shown in the figure. The stick applies a force of 2 N on the ring and rolls it without slipping with an acceleration of 0.3 m/s^2. The coefficient of friction between the ground and the ring is large enough that rolling always occurs and the coefficient of friction between the stick and the ring is (P/10). Find the value of P. [IIT 2011]



35. A train is moving along a straight line with a constant acceleration a. A boy standing in the train throws a ball forward with a speed of 10 m/s, at an angle of 60° to the horizontal. The boy has to move forward by 1.15 m inside the train to catch the ball back at the initial height. Find the acceleration of the train, in m/s^2. [IIT 2011]

Answers and Explanations

1. (c) $a = \frac{F}{m} = \frac{5\times10^4}{3\times10^7}$

$= \frac{5}{3}\times10^{-3}\ ms^{-2}$

Speed attained by the ship,

$v = \sqrt{2as} = \sqrt{2\times\frac{5}{3}\times10^{-3}\times3} = \sqrt{10^{-2}}$

$= 10^{-1} = \mathbf{0.1\ ms^{-1}}$.

2. (c) This is because the liquid experiences a backward force of reaction.

3. (b) At $t=0$, the momentum of the two-particle system is

$$\vec{p_i} = m_1\vec{v_1} + m_2\vec{v_2}$$

The collision between two particles does not affect the momentum of the system.

An external force $(m_1+m_2)g$ acts on the system. Impulse of this force in time $t=0$ to $t=2t_0$ is $(m_1+m_2)g\times2t_0$

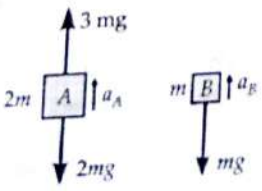
Change in momentum = Impulse

$$|(m_1\vec{v_1}+m_1\vec{v_2})-(m_1\vec{v_1}+m_2\vec{v_2})| = 2(m_1+m_2)gt_0$$

4. (b) Before the string is cut, the upward stretching force on the spring is

$$kx = 3mg$$

After the string is cut, the free body diagrams for the two blocks can be shown as in adjacent figure.



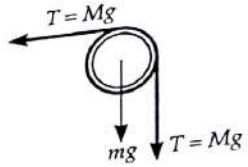
For block A: $2ma_A = 3mg - 2mg$

or $a_A = g/2$

For block B: $ma_B = mg$

or $a_B = g$.

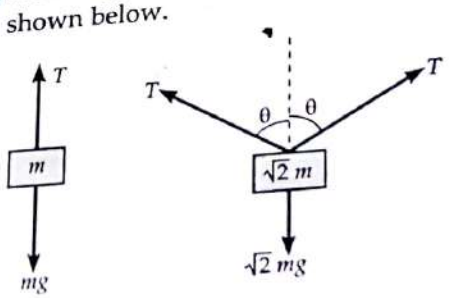
5. (d) For the block of mass M to be in equilibrium, the tension in the string, $T = Mg$. Free body diagram of pulley is shown. The force on the pulley by the clamp must balance the resultant of forces shown in the figure. This force has magnitude,



$$F = \sqrt{(Mg+mg)^2+(Mg)^2}$$

$$= \sqrt{(M+m)^2+M^2}\,g.$$

6. (c) Free body diagram for bodies of masses m and $\sqrt{2}m$ are shown below.



For equilibrium of mass m,

$$T = mg \qquad ...(i)$$

For equilibrium of mass $\sqrt{2}m$,

$$2T\cos\theta = \sqrt{2}mg \qquad ...(ii)$$

Dividing (ii) by (i), we get

$$\cos\theta = \frac{1}{\sqrt{2}} \quad \therefore \quad \theta = \mathbf{45°}.$$

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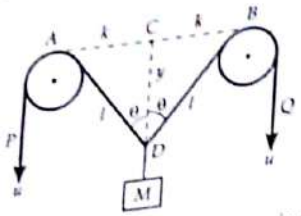
7. (*b*)



In right angled ΔACD, $AC = k$ remains constant. When ends P and Q move downwards, l and y change in accordance with the equation

$$l^2 = k^2 + y^2$$

Differentiating w.r.t. time t, we get

$$2l\frac{dl}{dt} = 0 + 2y\frac{dy}{dt}$$

or $$2l \times u = 2y \times v_M$$

$$\therefore \quad v_M = \frac{l}{y} \times u = \frac{u}{\cos\theta}.$$

8. (*d*) Given $\vec{p}(t) = A[\hat{i}\cos(kt) - \hat{j}\sin(kt)]$

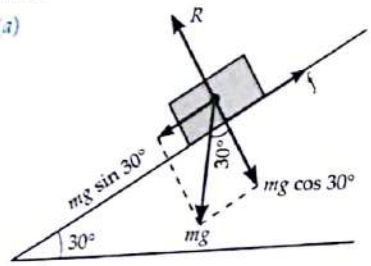
$$\therefore \quad \vec{F} = \frac{d\vec{p}}{dt} = Ak[-\hat{i}\sin(kt) - \hat{j}\cos(kt)]$$

$$\vec{F}.\vec{p} = -\sin(kt)\cos(kt) + \sin(kt)\cos(kt)$$

or $$\vec{F}.\vec{p} = 0 \quad \Rightarrow \quad \vec{F} \perp \vec{p}$$

9. (*a*) When a bicycle is in motion, the force of friction acts in the backward direction on the front wheel and in the forward direction on the rear wheel.

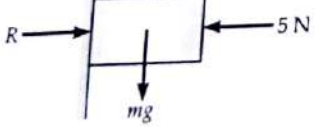
10. (*a*)



For equilibrium of the block,

$$f = mg\sin 30° = 2 \times 9.8 \times \frac{1}{2} = \mathbf{9.8\ N}.$$

11. (*b*) Different forces acting on the block are shown in the figure.



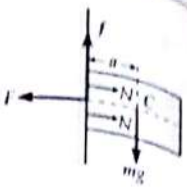
In equilibrium,

$$R = 5\ \text{N}$$

Force of friction,

$$f = \mu R = mg = 0.1 \times 9.8\ \text{N} = \mathbf{0.98\ N}.$$

12. (*d*) We consider the equilibrium of coplanar forces.



Horizontally, $F = N$,

where N is the normal force.

Vertically, $f = mg$,

where f is the force of friction.

Torque due to F is zero as its line of action passes through the centre C.

As the body is in equilibrium,

$$\vec{\tau}_f + \vec{\tau}_N = 0$$

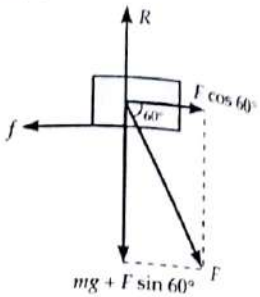
Now $$\vec{\tau}_f \neq 0.$$

$$\therefore \quad \vec{\tau}_N \neq 0$$

Therefore, N does not produce torque, is wrong.

The only correct option is (*d*).

13. (*a*) For vertical equilibrium of the block,



$$R = mg + F\sin 60°$$

$$= \sqrt{3}g + \frac{\sqrt{3}}{2}F$$

For no motion of the block,

$$f \geq F\cos 60°$$

or $$\mu R \geq F\cos 60°$$

or $$\frac{1}{2\sqrt{3}}\left(\sqrt{3}g + \frac{\sqrt{3}}{2}F\right) \geq \frac{F}{2}$$

or $$g \geq \frac{F}{2}$$

or $$F \leq 2g \quad \text{or} \quad \mathbf{F_{max} = 20\ N}.$$

14. (*a*) The block Q oscillates but does not slip on P. This indicates acceleration is same both for Q and P. A force of friction acts between the two blocks but the horizontal plane is frictionless. The system P-Q oscillates with angular frequency,

$$\omega = \sqrt{\frac{k}{m+m}} = \sqrt{\frac{k}{2m}}$$

Maximum acceleration of the system will be

$$a_{max} = \omega^2 A = \frac{kA}{2m}$$

This acceleration is provided to the lower block by the force of friction.

$$\therefore \quad f_{max} = ma_{max} = m\frac{kA}{2m} = \frac{kA}{2}.$$

15. (*b*) The least fo
on a horizontal plane
$f = \mu R$
$f = W$
or

16. (*a*) For equilibr
$R = mg$
$f = mg$

But $f = \mu R$

$\therefore \mu mg\cos\alpha = mg$

or $\cot\alpha = \frac{1}{\mu}$

17. (*a*) Linear acc
Force of reaction
$R = m$
After time t, an
$\omega = \alpha$
$\therefore$ Centripetal ac
$= \omega$
Limiting force
$= \mu$
For slipping,
$\mu mL\alpha = n$
or $t =$

18. (*c*)
$T\sin\theta =$
(centri
$T\cos\theta =$
$\therefore \quad \tan\theta =$
$=$
or $\theta =$

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15. *(b)* The least force required to move weight W on a horizontal plane is equal to the force of friction,

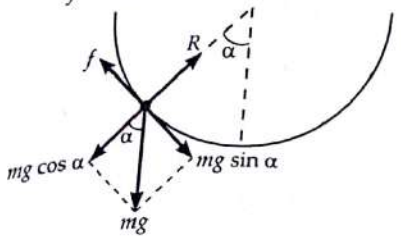
$$f = \mu R = \mu mg$$

$$f = W\tan\theta \quad [mg = W, \mu = \tan\theta]$$

16. *(a)* For equilibrium,

$$R = mg\cos\alpha$$

$$f = mg\sin\alpha$$



But $$f = \mu R = \mu\, mg\cos\alpha$$

$$\therefore \mu\, mg\cos\alpha = mg\sin\alpha$$

or $$\cot\alpha = \frac{1}{\mu} = \frac{1}{1/3} = 3.$$

17. *(a)* Linear acceleration of the bead, $a = L\alpha$

Force of reaction on the bead due to thread,

$$R = ma = mL\alpha$$

After time t, angular velocity of the bead,

$$\omega = \alpha t$$

$\therefore$ Centripetal acceleration of the bead

$$= \omega^2 L = \alpha^2 t^2 L$$

Limiting force of friction

$$= \mu R = \mu mL\alpha$$

For slipping,

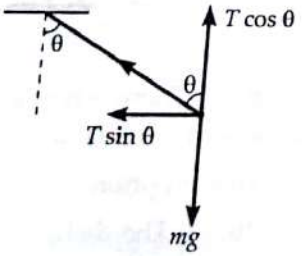
$$\mu mL\alpha = m\alpha^2 t^2 L$$

or $$t = \sqrt{\frac{\mu}{\alpha}}.$$

18. *(c)*

$$T\sin\theta = \frac{mv^2}{R}$$

(centripetal force)



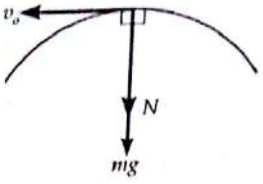
$$T\cos\theta = mg$$

$$\therefore \quad \tan\theta = \frac{v^2}{Rg}$$

$$= \frac{(10)^2}{10\times 10} = 1$$

or $$\theta = 45°.$$

19. *(a)* As the block rises to the same height in all the four cases, by conservation of energy, speed of the block (say, v_0) at the highest point will be same in all the four cases.



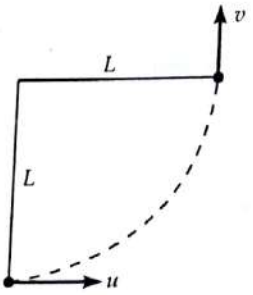
If R is the radius of curvature of the track and N the normal reaction, then the centripetal force will be

$$N + mg = \frac{mv_0^2}{R}$$

Clearly, N will be maximum when R is minimum. This occurs when the track is most sharply curved.

20. *(a)* Refer to the solution of Problem 15 on Page 5.73.

21. *(d)* By conservation of energy,



$$\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = mgL$$

or $$v^2 = u^2 - 2gL$$

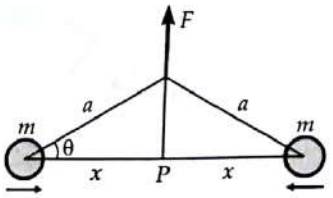
Change in velocity,

$$\Delta\vec{v} = \vec{v}_f - \vec{v}_i$$

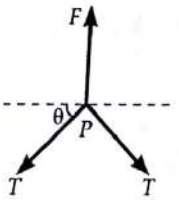
$$= v\hat{j} - u\hat{i}$$

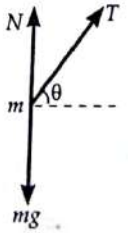
$$\therefore \quad |\Delta\vec{v}| = \sqrt{v^2 + u^2} = \sqrt{u^2 - 2gL + u^2} = \sqrt{2(u^2 - gL)}.$$

22. *(b)* The situation is shown in the figure. The two particles move towards each other. At any instant, the separation between them is $2x$.



Let T be the tension in the string. Then the forces acting at point P and on either mass m will be as shown in the figures.





For equilibrium of point P,

$$2T\sin\theta = F$$

For equilibrium of each mass m,

$$N + T\sin\theta - mg = 0$$

and $$T\cos\theta = ma_m$$

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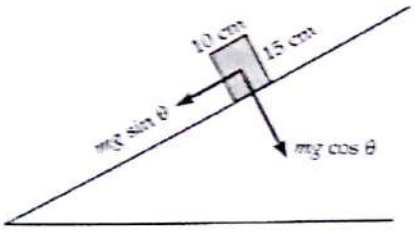
$$\therefore \quad \frac{ma_n}{F} = \frac{T\cos\theta}{2T\sin\theta} = \frac{1}{2}\cot\theta$$

or $$a_n = \frac{F}{2m}\cot\theta = \frac{F}{2m}\cdot\frac{x}{\sqrt{a^2-x^2}}$$

23. (b) Block will topple at an angle θ given by

$$mg\sin\theta \times \frac{15}{2} = mg\cos\theta \times 5$$

or $$\tan\theta = \frac{2}{3} \quad \Rightarrow \quad \theta < 60°$$



Sliding will occur at the angle of repose given by

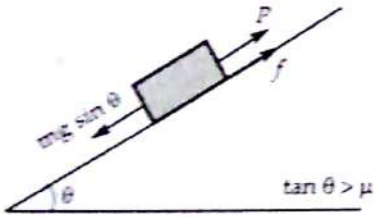
$$\tan\theta = \mu = \sqrt{3} \quad i.e., \quad \theta = 60°$$

Hence the block will topple before sliding and option (b) is correct.

24. (a) (i) When $P < mg\sin\theta$

$$P - f = mg\sin\theta$$

$$f = mg\sin\theta - P > 0, \text{ acting upwards}$$

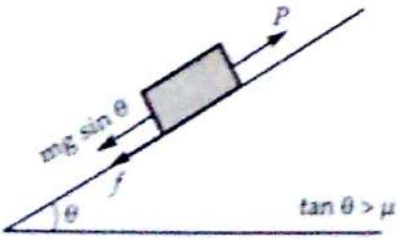


(ii) When $P = mg\sin\theta$, $f = 0$

(i) When $P > mg\sin\theta$

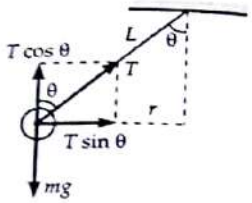
$$f + mg\sin\theta = P$$

$$f = P - mg\sin\theta < 0, \text{ acting downwards}$$



The horizontal component of the tension T e necessary centripetal force.

$\sin\theta = mr\omega^2$



But $$\frac{r}{L} = \sin\theta \quad \Rightarrow \quad r = L\sin\theta$$

$$\therefore \quad T\sin\theta = m(L\sin\theta)\omega^2$$

or $$\omega = \sqrt{\frac{T}{mL}} = \sqrt{\frac{324}{0.5\times 0.5}} = \frac{18}{0.5}$$

$$= \mathbf{36\ rad/s}$$

26. (d) $$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 5}{10}} = 1\text{ s}$$

By conservation of linear momentum,

$$v \times 0.01 = 0.2 \times 20 + 0.01 \times 100$$

$$\Rightarrow \quad v = \mathbf{500\ m/s}$$

27. (b) and (d) The rotating or revolving earth is an accelerated frame of reference. So it cannot be an inertial frame. Hence options (b) and (d) are correct.

28. (a), (d) Net linear momentum before collision,

$$\vec{p_i} = \vec{p_1} + \vec{p_2} = p\hat{i} - p\hat{i} = 0$$

As no external force is acting on the balls, so net linear momentum is conserved.

$$\vec{p_f} = \vec{p_1'} + \vec{p_2'} = 0$$

In option (a),

$$\vec{p_1'} + \vec{p_2'} = (a_1 + a_2)\hat{i} + (b_1 + b_2)\hat{j} + c_1\hat{k} \neq 0$$

[$\because$ c_1 is non-zero]

In option (d),

$$\vec{p_1'} + \vec{p_2'} = (a_1 + a_2)\hat{i} + 2b_1\hat{j} \neq 0$$

[$\because$ b_1 is non-zero]

29. (c) There is nothing wrong in statement – 1. But statement – 2 is false.

Hence option (c) is correct.

30. (b) The dishes will remain on the table due to inertia of rest and not due to action-reaction forces. Statement – 2 is not a correct explanation of statement – 1. Hence option (b) is correct.

31. (b) Normal contact force is greater in pushing than in pulling.

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32. | 0 | 0 | 0 | 8 |

$$T = \frac{2m_1m_2}{m_1+m_2}\cdot g$$

$$= \frac{2\times 0.72\times 0.36}{0.72+0.36}\times 10 = 4.8 \text{ N}$$

$$a = \frac{m_1-m_2}{m_1+m_2}\cdot g$$

$$= \frac{0.72-0.36}{0.72+0.36}\times 10 = \frac{10}{3} \text{ ms}^{-2}$$

$$s = ut+\frac{1}{2}at^2$$

$$= 0+\frac{1}{2}\times\frac{10}{3}(1)^2 = \frac{10}{3} \text{ m}$$

$$W = T.s = 4.8\times\frac{10}{6} = 8 \text{ J}.$$

33. | 0 | 0 | 0 | 5 |

Force required to just push the block up the inclined plane

= 3× Force required just to prevent the block from sliding down

or $(mg\sin+\mu mg\ \cos) = 3(mg\sin\theta-\mu mg\cos\theta)$

$\Rightarrow$ $1+\mu = 3(1-\mu)$

$\Rightarrow$ $\mu = 0.5$

$\Rightarrow$ $N = 10\mu = 5$

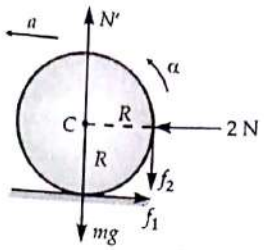
34. | 0 | 0 | 0 | 4 |

The force of the stick (2 N) tends to slide the point of contact of the ring with the ground. But the force of friction f_1 presents it from slipping and produces a torque which starts rolling the ring. Also, a force a friction f_2 acts between the ring and the stick.

In horizontal direction,

$$F_{net} = ma \quad \text{or} \quad 2-f_1 = 2\times 0.3$$

$\therefore$ $f_1 = 1.4$ N



Applying $\tau = I\alpha$ about C, we get

$$(f_2-f_1)R = I\alpha = I\frac{a}{R} \quad [\because \text{For rolling } a = R\alpha]$$

$\therefore$ $[1.4-\mu\times 2]\times 0.5 = 2\times(0.5)^2\times\frac{0.3}{0.5}$ $[\because I = MR^2]$

$\therefore$ $\mu = 0.4$

Given $\mu = \frac{P}{10}$ $\therefore$ $P = 4$

35. | 0 | 0 | 0 | 5 |

Time of flight of the ball is given by

$$t = \frac{2u\sin\theta}{g}$$

$$= \frac{2\times 10\sin 60^\circ}{10} = \sqrt{3} \text{ s}$$

The acceleration of the boy due to the pseudo force is negative.

$\therefore$ $s_x = u_xt+\frac{1}{2}a_xt^2$

$$1.15 = 5\times t-\frac{1}{2}a\times t^2$$

$$1.15 = 5\times\sqrt{3}-\frac{3}{2}a$$

$$\frac{3a}{2} = 5\times 1.73-1.15 = 8.65-1.15$$

$$\frac{3a}{2} = 7.5$$

or $a = \frac{15}{3} = 5 \text{ m/s}^2$

AIEEE

1. A rocket with a lift-off mass 3.5×10^4 kg is blasted upwards with an initial acceleration of 10 ms^{-2}. Then the initial thrust blast is

(a) 3.5×10^5 N (b) 7.0×10^5 N

(c) 14.0×10^5 N (d) 1.75×10^5 N [AIEEE 03]

2. A block of mass m is connected to another block of mass M by a massless spring of spring constant k. The blocks are kept on a smooth horizontal plane. Initially the blocks are at rest and the spring is unstretched. Then, a constant force F starts acting on

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the block of mass M to pull it. Find the force on block of mass m.

(a) $\frac{mF}{M}$ (b) $\frac{(M+m)F}{m}$

(c) $\frac{mF}{(M+m)}$ (d) $\frac{MF}{(M+m)}$ [AIEEE 07]

3. A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m. If a force P is applied at the free end of the rope, the force exerted by the rope on the block is

(a) $\frac{Pm}{(M+m)}$ (b) $\frac{Pm}{(M-m)}$

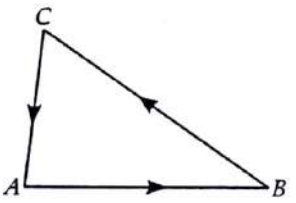
(c) P (d) $\frac{PM}{(M+m)}$ [AIEEE 03]

4. A particle of mass 0.3 kg is subjected to a force $F=-kx$ with $k=15$ Nm^{-1}. What will be its initial acceleration, if it is released from a point 20 cm away from the origin ?

(a) 3 m s^{-2} (b) 15 m s^{-2}

(c) 5 m s^{-2} (d) 10 m s^{-2} [AIEEE 05]

5. Three forces start acting simultaneously on a particle moving with velocity $\vec{v}$. These forces are



represented in magnitude and direction by the three sides of a triangle ABC as shown in the figure. The particle will now move with velocity

(a) greater than $\vec{v}$

(b) $|\vec{v}|$ in the direction of the largest force BC

(c) $\vec{v}$, remaining unchanged

(d) less than $\vec{v}$. [AIEEE 03]

6. When forces F_1, F_2 and F_3 are acting on a particle of mass m such that F_2 and F_3 are mutually perpendicular, then the particle remains stationary. If the force F_1 is now removed, then the acceleration of the particle is

(a) F_1/m (b) F_2F_3/mF

(c) $(F_2-F_3)/m$ (d) F_2/m [AIEEE 02]

7. A ball of mass 0.2 kg is thrown vertically upwards by applying a force by hand. If the hand moves 0.2 m while applying the force and the ball goes upto 2 m height further, find the magnitude of the force. Take $g=10\,\text{ms}^{-2}$.

(a) 20 N (b) 22 N

(c) 4 N (d) 16 N [AIEEE 04]

8. A player caught a cricket ball of mass 150 g moving at a rate of 20 ms^{-1}. If the ball catching process is completed in 0.1 s, the force on the blow exerted by the ball on the hand of the player is equal to

(a) 30 N (b) 300 N

(c) 150 N (d) 3 N [AIEEE 06]

9. A machine gun fires a bullet of mass 40 g with a velocity 1,200 ms^{-1}. The man holding it, can exert a maximum force of 144 N on the gun. How many bullets can he fire per second at the most ?

(a) One (b) Four

(c) Two (d) Three [AIEEE 04]

10. A body of mass 3.513 kg is moving along the x-axis with a speed of 5.00 ms^{-1}. The magnitude of its momentum is

(a) 17.6 kg ms^{-1} (b) 17.565 kg ms^{-1}

(c) 17.56 kg ms^{-1} (d) 17.57 kg ms^{-1} [AIEEE 08]

11. A lift is moving down with acceleration a. A man in the lift drops a ball inside the lift. The acceleration of the ball as observed by the man in the lift and a man standing stationary on the ground are respectively.

(a) g, g (b) $g-a, g-a$

(c) $g-a, g$ (d) a, g [AIEEE 02]

12. A light spring balance hangs from the hook of the other light spring balance and a block of mass M kg hangs from the former one. Then, the true statement about the scale reading is

(a) Both the scales read M kg each

(b) The scale of the lower one reads M kg and of the upper one zero.

(c) The reading of the two scales can be anything but the sum of the readings will be M kg

(d) Both the scales read $M/2$ kg. [AIEEE 03]

13. A spring balance is attached to the ceiling of a lift. A man hangs his bag on the spring and the spring balance reads 49 N, when the lift is stationary. If the lift moves downward with an acceleration of 5 ms^{-2}, the reading of the spring balance will be

(a) 24 N (b) 74 N

(c) 15 N (d) 49 N [AIEEE 03]

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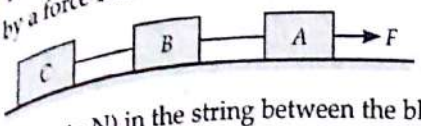
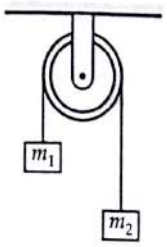
14. Three identical blocks of masses $m = 2$ kg are drawn by a force 10.2 N on a frictionless surface. What is the tension (in N) in the string between the blocks B and C?



(a) 9.2 (b) 8
(c) 3.4 (d) 9.8 [AIEEE 02]

15. Two masses $m_1 = 5$ kg and $m_2 = 4.8$ kg tied to a string are hanging over a light frictionless pulley. What is the acceleration of the masses, when left free to move ? Given : $g = 9.8\ ms^{-2}$.

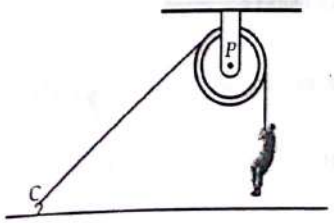


(a) 0.2 ms^{-2}
(b) 9.8 ms^{-2}
(c) 5 ms^{-2}
(d) 4.8 ms^{-2} [AIEEE 04]

16. A light string passing over a smooth light pulley connects two blocks of masses m_1 and m_2 (vertically). If the acceleration of the system is $g/8$, then the ratio of the masses m_2 / m_1 is

(a) 8 : 1 (b) 9 : 7
(c) 4 : 3 (d) 5 : 3 [AIEEE 02]

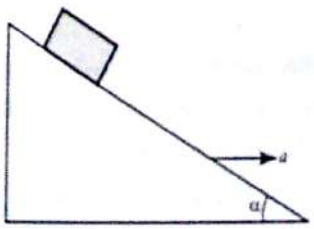
17. One end of massless rope, which passes over a massless and frictionless pulley P is tied to a hook C, while the other end is free. Maximum tension that the



rope can bear is 640 N. With what value of maximum safe acceleration (in ms^{-2}) can a man of 40 kg climb on the rope ? Take $g = 10\ ms^{-2}$.

(a) 16 (b) 12
(c) 8 (d) 6 [AIEEE 02]

18. A block is kept on a frictionless inclined surface with angle of inclination α. The incline is given an acceleration a to keep the block stationary. Then a is equal to



(a) $g / \tan\alpha$ (b) $g\,\text{cosec}\,\alpha$
(c) g (d) $g \tan\alpha$ [AIEEE 05]

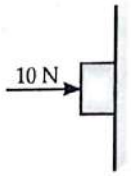
19. A marble block of mass 2 kg lying on ice, when given a velocity of 6 ms^{-1}, is stopped by friction in 10 s. Then the coefficient of friction is

(a) 0.02 (b) 0.03
(c) 0.06 (d) 0.01 [AIEEE 03]

20. A block rests on a rough inclined plane making an angle of 30° with the horizontal. The coefficient of static friction between the block and the plane is 0.8. If the frictional force on the block is 10 N, the mass of the block (in kg) ($g = 10\,ms^{-2}$) is

(a) 2.0 (b) 4.0
(c) 1.6 (d) 2.5 [AIEEE 04]

21. A horizontal force of 10 N is necessary to just hold a block stationary against a wall. The coefficient



of friction between the block and the wall is 0.2. The weight of the block is

(a) 20 N (b) 50 N
(c) 100 N (d) 2 N [AIEEE 03]

22. The upper half of an inclined plane with inclination θ is perfectly smooth, while the lower half is rough. A body starting from rest at the top will again come to rest at the bottom, if the coefficient of friction for the lower half is given by

(a) $2\tan\theta$ (b) $\tan\theta$
(c) $2\sin\theta$ (d) $2\cos\theta$ [AIEEE 05]

23. A smooth block is released at rest on a 45° incline and then slides a distance d. The time taken to slide is n times as much to slide on rough incline than on a smooth incline. The coefficient of friction is

(a) $\mu_k = 1 - \frac{1}{n^2}$ (b) $\mu_k = \sqrt{1 - \frac{1}{n^2}}$
(c) $\mu_s = 1 - \frac{1}{n^2}$ (d) $\mu_s = \sqrt{1 - \frac{1}{n^2}}$ [AIEEE 05]

24. A particle is acted upon by a force of constant magnitude which is always perpendicular to the

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velocity of the particle. The motion of the particle takes place in a plane.

It follows that

(a) its velocity is constant

(b) its acceleration is constant

(c) its kinetic energy is constant

(d) it moves in a straight line. [AIEEE 04]

25. An annular ring with inner and outer radii R_1 and R_2 is rolling without slipping with a uniform angular speed. The ratio of the forces F_1 / F_2 experienced by two identical particles situated on the inner and outer parts of the ring is

(a) $\frac{R_2}{R_1}$ (b) $\left(\frac{R_1}{R_2}\right)^2$

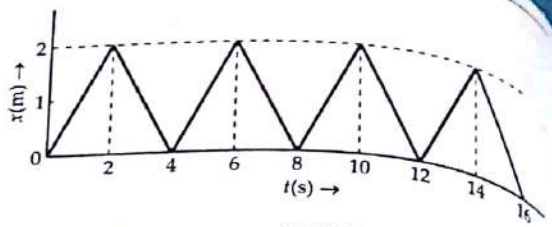
(c) 1 (d) $\frac{R_1}{R_2}$ [AIEEE 05]

26. The maximum velocity (in ms^{-1}) with which a car driver must traverse a flat curve of radius 150 m and coefficient of friction 0.6 to avoid skidding is

(a) 60 (b) 30

(c) 15 (d) 25 [AIEEE 02]

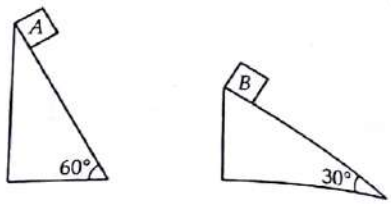
27. The figure shows the position-time (x-t) graph of one-dimensional motion of a body of mass 0.4 kg. The magnitude of each impulse is



(a) 0.2 Ns (b) 0.4 Ns

(c) 0.8 Ns (d) 1.6 Ns [AIEEE 2011]

28. Two fixed frictionless inclined planes making angles 30° and 60° with the vertical are shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect of B ?



(a) 4.9 ms^{-2} in vertical direction

(b) 4.9 ms^{-2} in horizontal direction

(c) 9.8 ms^{-2} in vertical direction

(d) zero [AIEEE 2010]

Answers and Explanations

1. (b) Initial thrust $= ma + mg$

$$= m(a+g) = 3.5 \times 10^4 (10+10)\ \text{N}$$
$$= 7.0 \times 10^5\ \text{N}.$$

2. (c) Accelerating of the system, $a = \frac{F}{M+m}$

Force on block of mass m, $F' = ma = \frac{mF}{m+m}$.

3. (d) Refer to the solution of Problem 80 on page 5.67.

4. (d) Refer to the solution of Problem 79 on page 5.67.

5. (c) As the three forces are represented by the three sides of a triangle taken in the same order, their resultant is zero. Hence the velocity of the particle will remain unchanged.

6. (a) Refer to the solution of Problem 81 on page 5.67.

7. (b) For motion of the ball, just after the throwing process :

$$v = 0,\ s = 2\ \text{m},\ a = -g = -10\ \text{ms}^{-2}$$

As $v^2 - u^2 = 2as$

$\therefore$ $0^2 - u^2 = 2 \times (-10) \times 2$

or $u = \sqrt{40}\ \text{ms}^{-1}$

For motion of the ball during the throwing process :

$$u = 0,\ v = \sqrt{40}\ \text{ms}^{-1},\ s = 0.2\ \text{m}$$

As $v^2 - u^2 = 2as$

$\therefore$ $40 - 0^2 = 2a \times 0.2$

or $a = \frac{40}{0.4} = 100\ \text{ms}^{-2}$

Force applied on the ball produces acceleration a and also overcomes the weight of the ball.

$\therefore$ $F = ma + mg = m(a+g)$

$$= 0.2(100+10) = \mathbf{22}\ \text{N}.$$

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8. (c) Impulse = Change in momentum

$$F \times t = m(v-u)$$

$$\text{or} \quad F = \frac{m(v-u)}{t} = \frac{0.150(0-20)}{0.1}$$

$$\text{or} \quad = -30 \text{ N}.$$

9. (d) Refer to the solution of Example 13 on page 5.13.

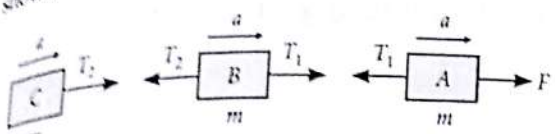
10. (b) $p = mv = 3.513 \times 500 = 17.565 \text{ kg ms}^{-1}$.

11. (c) When the ball dropped, its acceleration is g as is observed by a man standing stationary on the ground. Now the man inside the lift has downward acceleration a, the acceleration of the ball as observed by this man will be $g - a$.

12. (d) As the suspended spring balance is lighter one, both the scales will read M kg each.

13. (a) Refer to the solution of Problem 82 on page 5.67.

14. (c) Free body diagrams for the three blocks are shown below :



Applying Newton's second law,

$$F - T_1 = ma$$

$$T_1 - T_2 = ma$$

$$T_2 = ma$$

Adding the three equations, we get

$$F = 3ma$$

$$\text{or} \quad a = \frac{F}{3m}$$

$$\therefore \quad T_2 = ma = \frac{F}{3}$$

$$= \frac{10.2 \text{ N}}{3} = \mathbf{3.4 \text{ N}}.$$

15. (a) $$a = \frac{(m_1 - m_2)g}{m_1 + m_2} = \frac{(5-4.8) \times 9.8}{5 + 4.8}$$

$$= \mathbf{0.2 \text{ ms}^{-2}}.$$

16. (b) Refer to the solution of Problem 18 on page 5.69.

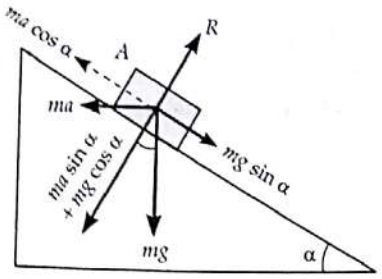
17. (d) When the man climbs up the rope with acceleration a, the tension T in the rope must be greater than his weight mg.

$$T - mg = ma$$

$$\text{or} \quad a = \frac{T}{m} - g = \frac{640}{40} - 10$$

$$= 16 - 10 = \mathbf{6 \text{ ms}^{-2}}.$$

18. (d) When the incline is given an acceleration a towards the right, the block receives a reaction ma towards left.



For the block to remain in stationary,

$$ma\cos\alpha = mg\sin\alpha$$

$$\text{or} \quad a = \frac{g\sin\alpha}{\cos\alpha} = g\tan\alpha.$$

19. (c) Here $u = 6 \text{ ms}^{-1}$, $v = 0$, $t = 10$ s

$$\therefore \quad a = \frac{v-u}{t}$$

$$= \frac{0-6}{10} = -0.6 \text{ ms}^{-2}$$

$$\mu = \frac{f}{R} = \frac{ma}{mg} = \frac{a}{g}$$

$$= \frac{0.6}{10} = \mathbf{0.06}.$$

20. (a) As the block is at rest, the component of its weight along the inclined plane balances the force of friction.

$$\therefore \quad mg\sin\theta = f$$

$$\text{or} \quad m = \frac{f}{g\sin\theta} = \frac{10}{10 \times \sin 30^\circ} = \mathbf{2.0 \text{ kg}}.$$

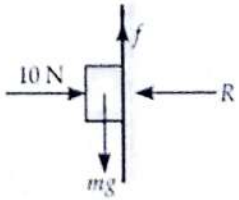
21. (d) For the block to remain stationary,

$$R = 10 \text{ N}, \; f = mg$$

But $f = \mu R$

$$\therefore \quad mg = \mu R = 0.2 \times 10$$

$$= \mathbf{2 \text{ N}}.$$



22. (a) As the initial and final speeds of the block are zero,

Work done by gravity = Work done by friction

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$$mg\sin\theta \times l = f \times \frac{l}{2}$$

or $$mg\sin\theta \times l = \mu mg\cos\theta \times \frac{l}{2}$$

or $$\mu = 2\tan\theta$$

23. (c) Refer to the solution of Problem 16 on page 5.74.

24. (c) It is a case of uniform circular motion in which velocity and acceleration vectors change due to change in direction. As the magnitude of velocity remains constant, the kinetic energy is constant.

25. (d) m is the mass of each particle and ω is angular speed of the annular ring, then

$$\frac{F_1}{F_2} = \frac{mR_1\omega^2}{mR_2\omega^2} = \frac{R_1}{R_2}.$$

26. (b) $v_{max} = \sqrt{\mu rg} = \sqrt{0.6 \times 150 \times 10}$

$$= \sqrt{900} = 30 \text{ ms}^{-1}.$$

27. (c) Velocity before $t = 2$ s,

$$u = \frac{2-0}{2-0} = 1 \text{ ms}^{-1}$$

Velocity after $t = 2$ s,

$$v = \frac{0-2}{4-2} = -1 \text{ ms}^{-1}$$

Impulse $= m(v-u) = 0.4(-1-1) = -0.8$ Ns

$\therefore$ Impulse $| = 0.8$ Ns

28. (a) If a is the acceleration along the inclined plane, then

$mg\sin\theta = ma \Rightarrow a = g\sin\theta$

$\therefore$ Vertical component of acceleration $= g\sin^2\theta$

Relative vertical acceleration of A with respect to B

$$= g(\sin^2 60° - \sin^2 30°) = g\left(\frac{3}{4} - \frac{1}{4}\right)$$

$$= 9.8 \times \frac{1}{2} = 4.9 \text{ ms}^{-2}, \text{ in vertical direction}$$

DCE and G.G.S. Indraprastha University Engineering Entrance Exam

(Similar Questions)

1. The dimensions of action are

(a) $[M^2LT^{-3}]$ (b) $[MLT^{-1}]$

(c) $[MLT^{-2}]$ (d) $[ML^2T^{-1}]$ [DCE 97]

2. If two forces of 5 N each are acting along X- and Y- axes, then the magnitude and direction of resultant is

(a) $5\sqrt{2}$, $\pi/3$ (b) $5\sqrt{2}$, $\pi/4$

(c) $-5\sqrt{2}$, $\pi/3$ (d) $-5\sqrt{2}$, $\pi/4$ [DCE 04]

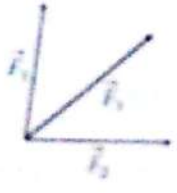
3. If two forces are acting at a point such that the magnitude of each force is 2 N and the magnitude of their resultant is also 2 N, then the angle between the two forces is

(a) 120° (b) 60°

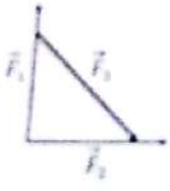
(c) 90° (d) 0° [DCE 04]

4. Which of the four arrangements in the figure correctly shows the vector addition of two forces $\vec{F}_1$ and $\vec{F}_2$ to yield the third force $\vec{F}_3$? [IPUEE 05]

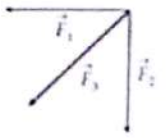
(a)



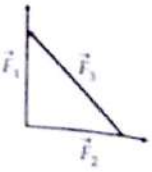
(b)



(c)



(d)



5. A body of mass 8 kg is moved by a force $F = 3x$ N, where x is the distance covered. Initial position is $x = 2$ m and the final position is $x = 10$ m. The initial speed is zero. The final speed is

(a) 6 m/s (b) 12 m/s

(c) 18 m/s (d) 14 m/s [IPUEE 04]

6. A body of mass 5 kg starts from the origin with an initial velocity $\vec{u} = (30\hat{i} + 40\hat{j})$ m/s. If a constant force $(-6\hat{i} - 5\hat{j})$ N acts on the body, the time in which the y-component of the velocity becomes zero is

(a) 5 s (b) 20 s

(c) 40 s (d) 80 s [DCE 05]

7. A student unable to answer a question on Newton's laws of motion attempts to pull himself up by tugging on his hair.

He will not succeed

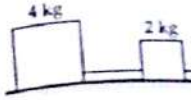
(a) as the force exerted is s

(b) the frictional force whi

(c) Newton's law of iner living beings

(d) as the force applied is

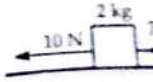
8. In the arrangement sh are light and inextensible. blocks are placed is smooth. each block ?



(a) 8 m/s^2

(c) 2 m/s^2

9. Three blocks of ma connected to each other placed on a frictionless s The system is pulled by T_1 is



(a) 1 N

(c) 8 N

10. Two masses m_1 which passes over a f $m_1 = 10$ kg, $m_2 = 6$ kg,

(a) 20 m/s^2

(c) 2.5 m/s^2

11. The adjace stretched net. Sec 10 N.

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He will not succeed

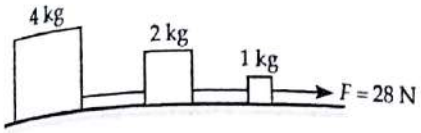
(a) as the force exerted is small

(b) the frictional force while gripping, is small

(c) Newton's law of inertia is not applicable to living beings

(d) as the force applied is internal to the system. [IPUEE 06]

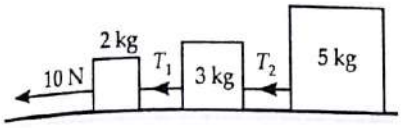
8. In the arrangement shown in figure, the strings are light and inextensible. The surface over which blocks are placed is smooth. What is the acceleration of each block ?



(a) 8 m/s^2 (b) 4 m/s^2

(c) 2 m/s^2 (d) 14 m/s^2 [DCE 07]

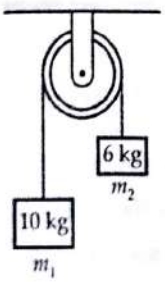
9. Three blocks of masses 2 kg, 3 kg and 5 kg are connected to each other with light string and are then placed on a frictionless surface as shown in the figure. The system is pulled by a force $F = 10$ N, then tension T_1 is



(a) 1 N (b) 5 N

(c) 8 N (d) 10 N [IPUEE 04]

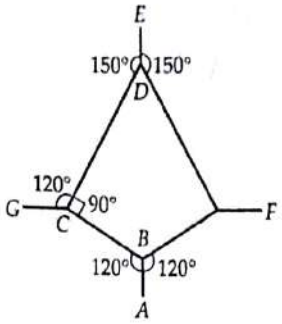
10. Two masses m_1 and m_2 are attached to a string which passes over a frictionless smooth pulley. When $m_1 = 10$ kg, $m_2 = 6$ kg, the acceleration of masses is



(a) 20 m/s^2 (b) 5 m/s^2

(c) 2.5 m/s^2 (d) 10 m/s^2 [IPUEE 04]

11. The adjacent figure is the part of a horizontally stretched net. Section AB is stretched with a force of 10 N.



The tensions in the sections BC and BF are

(a) 10 N, 11 N (b) 10 N, 6 N

(c) 10 N, 10 N

(d) cannot be calculated due to insufficient data [IPUEE 06]

12. A plumb line is suspended from a ceiling of a car moving with horizontal acceleration of a. What will be the angle of inclination with vertical ?

(a) $\tan^{-1}\left(\frac{a}{g}\right)$ (b) $\tan^{-1}\left(\frac{g}{a}\right)$

(c) $\cos^{-1}\left(\frac{a}{g}\right)$ (d) $\cos^{-1}\left(\frac{g}{a}\right)$ [IPUEE 05]

13. Dimensions of impulse are same as that of

(a) force (b) momentum

(c) energy (d) acceleration [DCE 06]

14. Momentum is closely related to

(a) force (b) impulse

(c) velocity (d) kinetic energy [DCE 97]

15. A particle is moving in a circle with uniform speed v. In moving from a point to another diametrically opposite point,

(a) the momentum changes by mv

(b) the momentum changes by $2mv$

(c) the kinetic energy changes by $(\frac{1}{2})\, mv^2$

(d) the kinetic energy changes by mv^2 [IPUEE 05]

16. A bullet of mass 10 g is fired from a gun of mass 1 kg. If the recoil velocity is 5 m/s, the velocity of the muzzle is

(a) 0.05 m/s (b) 5 m/s

(c) 50 m/s (d) 500 m/s [IPUEE 04]

17. A person is sitting in a lift accelerating upwards. Measured weight of person will be

(a) less than actual weight

(b) equal to actual weight

(c) more than actual weight

(d) zero. [DCE 04]

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18. A lift is falling under gravity, what is the time period of a pendulum attached to its ceiling ?

(a) zero (b) infinite
(c) 1 s (d) 2 s [DCE 08]

19. According to the special theory of relativity, which of the following has same value in all inertial frames ?

(a) Mass of an object (b) Length of an object
(c) Velocity of sound (d) Velocity of light. [IIPUEE 05]

20. What should be the velocity of an object so that its mass becomes twice of its rest mass ?

(a) 3×10^8 m/s (b) 2.59×10^8 m/s
(c) 4×10^8 m/s (d) none of the above. [DCE 04]

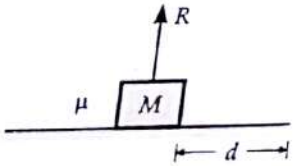
21. The velocity of a body of rest mass m_0 is $\frac{\sqrt{3}}{2}c$ (where c is the velocity of light in vacuum). Then mass of this body is

(a) $(\sqrt{3}/2)m_0$ (b) $(1/2)m_0$
(c) $2m_0$ (d) $(2/\sqrt{3})m_0$ [IIPUEE 04]

22. A body of mass 2 kg is kept stationary by pressing to a vertical wall by a force of 100 N. The coefficient of friction between wall and body is 0.3. Then the frictional force is equal to

(a) 6 N (b) 20 N
(c) 600 N (d) 700 N [IIPUEE 05]

23. If reaction is R and coefficient of friction is μ, what is work done against friction in moving a body by distance d ?



(a) $\frac{\mu Rd}{4}$ (b) $2\mu Rd$
(c) μRd (d) $\mu Rd/2$ [DCE 07]

24. An ice cube is kept on an inclined plane of angle 30°. Coefficient of kinetic friction between block and inclined plane is $\frac{1}{\sqrt{3}}$. What is acceleration of block ?

(a) zero (b) 2 m/s^2
(c) 1.5 m/s^2 (d) 5 m/s^2 [DCE 06]

25. A small object placed on a rotating horizontal rn table just slips when it is placed at a distance of 4 cm from the axis of rotation. If the angular velocity of the turn-table is doubled the object slips when its distance from the axis of rotation is

(a) 1 cm (b) 2 cm
(c) 4 m (d) 8 cm [IIPUEE 05]

26. A particle is moving in a vertical circle. The tensions in the string when passing through two positions at angles 30° and 60° from vertical (lowest positions) are T_1 and T_2 respectively. Then

(a) $T_1 = T_2$ (b) $T_2 > T_1$
(c) $T_1 > T_2$
(d) tension in the string always remains the same. [IIPUEE 04]

27. Which of the following statements is true ?

(a) Velocity of light is constant in all media
(b) Velocity of light in vacuum is maximum
(c) Velocity of light is same in all reference frames
(d) Laws of nature have identical form in all reference frames [IIPUEE 04]

28. For ordinary terrestrial experiments, the observer in an inertial frame in the following cases is

(a) a child revolving in a giant wheel
(b) a driver in a sports car moving with a constant high speed of 200 kmh^{-1} on a straight rod
(c) the pilot of an aeroplane which is taking off
(d) a cyclist negotiating a sharp curve. [IIPUEE 07]

29. A man of mass 60 kg and a boy of mass 30 kg are standing together on frictionless ice surface. If they push each other apart, the man moves away with a speed of 0.4 m/s relative to ice. After 5 s they will be away from each other at a distance of

(a) 9.0 m (b) 3.0 m
(c) 6.0 m (d) 30 m [DCE 08]

30. A batsman deflects a ball by an angle of 45° without changing its initial speed which is equal to 50 kmh^{-1}. Mass of the ball is 0.15 kg. What is the impulse imparted to the ball ?

(a) 4.2 kg ms^{-1} in the direction of the final velocity
(b) 4.2 kg ms^{-1} in the direction of the initial velocity
(c) 4.2 kg ms^{-1} in the direction opposite the initial velocity
(d) 4.2 kg ms^{-1} in the direction along the bisector of the initial and final directions of the velocity. [DCE 08]

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31. A satellite in a force-free space sweeps stationary interplanetary dust at the rate $\frac{dM}{dt} = \beta v$. The acceleration of the satellite is

(a) $-\beta v^2 / M$ (b) $-\beta v^2 / 2M$

(c) $-\beta v^2$ (d) $-M\beta / v^2$.

[IDCE 08]

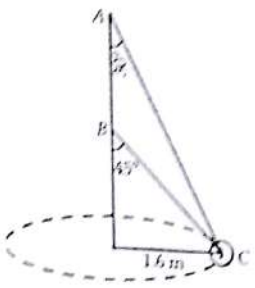
32. A car when passes through a bridge exerts a force on it which is equal to

(a) $Mg + \frac{Mv^2}{r}$ (b) $\frac{Mv^2}{r}$

(c) $Mg - \frac{Mv^2}{r}$ (d) none of these.

[IDCE 08]

33. Two wires AC and BC are tied to a small sphere C of mass 5 kg, which revolved at a constant speed v in the horizontal circle of radius 1.6 m. The minimum value of v is



(a) 3.01 ms^{-1} (b) 4.01 ms^{-1}

(c) 8.2 ms^{-1} (d) 3.96 ms^{-1}

[IDCE 08]

34. A lift is moving upward with increasing speed with acceleration a.

The apparent weight will be

(a) less than the actual weight

(b) more than the actual weight and have a fixed value.

(c) more than the actual weight which increases as long as velocity increases.

(d) zero.

35. A block of mass M at the end of a string is whirled round a vertical circle of radius R. The critical speed of the block at the top of the swing is

(a) $(R/g)^{1/2}$ (b) g/R

(c) M/Rg (d) $(Rg)^{1/2}$

[IDCE 09]

Answers and Explanations

1. (c) [Action] = [Force] = [MLT^{-2}]

2. (b) $R = \sqrt{5^2 + 5^2} = 5\sqrt{2}$

$$\tan\phi = \frac{5\sin 90^\circ}{5 + 5\cos 90^\circ} = \frac{5}{5} = 1$$

$$\therefore \quad \phi = \frac{\pi}{4}.$$

3. (b) $R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$$2^2 = 2^2 + 2^2 + 2 \times 2 \times 2\cos\theta$$

$$\cos\theta = -\frac{1}{2} \text{ or } \theta = \mathbf{120^\circ}.$$

4. (a) Only in option (a), $\vec{F}_1$ and $\vec{F}_2$ can add up in same order to give $\vec{F}_3$ in opposite order.

5. (a) $F = ma = m\frac{dv}{dt} = m\frac{dv}{dx}\cdot\frac{dx}{dt}$

$$F\,dx = mv\,dv \qquad \left[\frac{dx}{dt} = v\right]$$

$$3x\,dx = 8v\,dv$$

$$3\int_2^{10} x\,dx = 8\int_0^v v\,dv$$

$$\frac{3}{2}[x^2]_2^{10} = \frac{8}{2}[v^2]_0^v$$

$$\frac{3}{2}[100 - 4] = 4[v^2 - 0]$$

or $$\frac{3}{2} \times \frac{94}{4} = v^2 \quad \text{or} \quad v = \mathbf{6\ m/s}.$$

6. (c) $\vec{u} = (30\hat{i} + 40\hat{j})$ m/s

$\therefore$ $u_y = 40$ m/s

$$\vec{F} = (-6\hat{i} - 5\hat{j})\text{ N}$$

$\therefore$ $F_y = -5$ N

$$a_y = \frac{F_y}{m} = \frac{-5}{5} = -1\text{ m/s}^2$$

$$v_y = 0$$

But $v_y = u_y + a_y t$

$\therefore$ $0 = 40 - 1 \times t$ or $t = \mathbf{40\ s}$.

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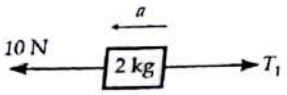
7. *(d)* The force applied by the student into himself is an internal force. According to Newton's laws, only an external force can change the state of motion of an object.

8. *(b)* $a = \dfrac{F}{m_1 + m_2 + m_3} = \dfrac{28\,\text{N}}{(4+2+1)\,\text{kg}} = 4\ \text{ms}^{-2}.$

9. *(c)* $a = \dfrac{10}{2+3+5} = 1\ \text{ms}^{-2}$



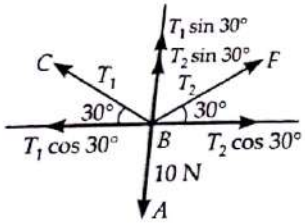
By Newton's second law,

$$2a = 10 - T_1$$
$$T_1 = 10 - 2a$$
$$= 10 - 2 \times 1 = 8\ \text{N}.$$

10. *(c)* $a = \dfrac{m_1 - m_2}{m_1 + m_2} g$

$$g = \frac{10-6}{10+6} \times 10$$
$$= 2.5\ \text{ms}^{-2}.$$

11. *(d)* Let T_1 and T_2 be the tensions in sections BC and BF respectively.



$$T_1 \cos 30° = T_2 \cos 30°$$

$$\Rightarrow \quad T_1 = T_2$$

Also, $T_1 \sin 30° + T_2 \sin 30° = 10$

$$2T_1 \sin 30° = 10$$
$$2T_1 \times \frac{1}{2} = 10$$

or $$T_1 = T_2 = 10\ \text{N}.$$

12. *(a)* Refer to the solution of Problem 10 on page 5.113.

13. *(b)* [Impulse] = [Change in momentum]

$$= [MLT^{-1}]$$

14. *(b)* Impulse = Change in momentum

15. *(b)* Change in momentum

$$= mv - (-mv) = 2mv.$$

16. *(d)* $mv = MV$

$$v = \frac{MV}{m} = \frac{1 \times 5}{0.01} = 500\ \text{ms}^{-1}$$

17. *(c)* Measured weight $= m(g + a)$. It is more than actual weight (mg).

18. *(b)* In a lift falling under gravity, $g = 0$

$$T = 2\pi\sqrt{\frac{l}{g}} = \infty$$

19. *(d)* Velocity of light is same in all inertial frames.

20. *(b)* According to theory of relativity,

$$m = \frac{m_0}{\sqrt{1 - \dfrac{v^2}{c^2}}}$$

$$2m_0 = \frac{m_0}{\sqrt{1 - \dfrac{v^2}{c^2}}}$$

$$1 - \frac{v^2}{c^2} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\frac{v^2}{c^2} = 1 - \frac{1}{4} = \frac{3}{4}$$

$$v = \frac{\sqrt{3}}{2} c = \frac{1.732 \times 3 \times 10^8}{2}\ \text{ms}^{-1}$$

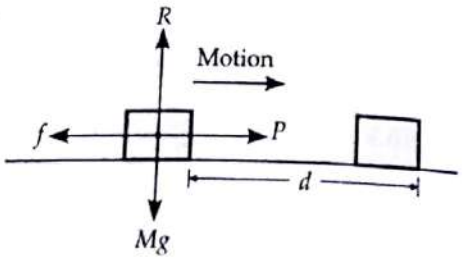
$$= 2.59 \times 10^8\ \text{ms}^{-1}.$$

21. *(c)* $m = \dfrac{m_0}{\sqrt{1 - \dfrac{v^2}{c^2}}} = \dfrac{m_0}{\sqrt{1 - \dfrac{3}{4}c^2 \cdot \dfrac{1}{c^2}}}$

$$= 2m_0.$$

22. *(b)* $f = mg = 2 \times 10 = 20\ \text{N}.$

23. *(c)*



To move the block with acceleration, the applied force

$$P = f = \mu R$$

$\therefore$ $$W = P \times d = \mu R d.$$

24. *(a)* According to Newton's second law,

$$ma = f_k - mg\sin\theta = \mu_k R - mg\sin\theta$$
$$= \mu_k \cdot mg\cos\theta - mg\sin\theta$$

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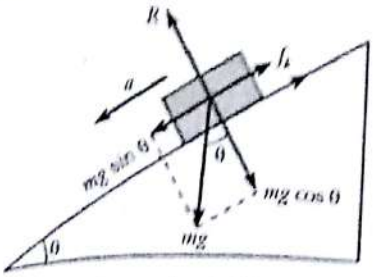


$$a = g(\mu_k \cos\theta - \sin\theta)$$

$$\therefore \quad = g\left(\frac{1}{\sqrt{3}}\cos 30° - \sin 30°\right)$$

$$= g\left(\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{2}\right) = 0.$$

25. *(a)* The object will slip if centripetal force ≥ force of friction

$$mr\omega^2 \geq \mu mg$$

$$r\omega^2 \geq \mu g$$

$$r\omega^2 = \text{constant}$$

$$\frac{r_1}{r_2} = \left(\frac{\omega_2}{\omega_1}\right)^2$$

$$\frac{4\text{ cm}}{r_2} = \left(\frac{2\omega}{\omega}\right)^2 \quad \text{or} \quad r_2 = \mathbf{1\text{ cm}}.$$

26. *(b)* $T = \dfrac{mv^2}{r} + mg\cos\theta$

$$\therefore \quad T \propto \cos\theta$$

$$\frac{T_1}{T_2} = \frac{\cos 30°}{\cos 60°} = \frac{\sqrt{3}/2}{1/2} = \frac{\sqrt{3}}{1} \quad \text{or} \quad T_1 > T_2.$$

27. *(b)* Velocity of light is maximum in vacuum.

28. *(c)* An inertial frame is a non-accelerated frame. Hence only option *(b)* is correct.

29. *(c)* The man and the boy move in opposite directions.

Momentum of man = – Momentum of boy

$$60 \times 0.4 = -30 \times v$$

$\therefore$ Velocity of the boy,

$$v = -0.8\text{ ms}^{-1}$$

Relative velocity $= 0.4 + 0.8 = 1.2\text{ ms}^{-1}$

Distance between man and boy after 5 s

$$= 1.2 \times 5 = \mathbf{6.0\text{ m}}.$$

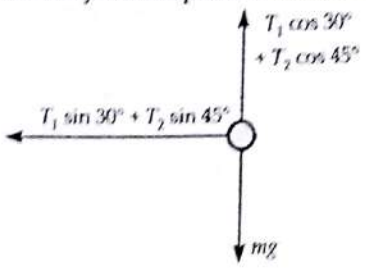
30. *(d)* Refer answer to NCERT Exercise 5.20 on page 5.78.

31. *(a)* Thrust, $F = -v\dfrac{dM}{dt} = -v.\beta v = -\beta v^2$

Acceleration, $a = \dfrac{F}{M} = -\dfrac{\beta v^2}{M}$.

32. *(c)* $F = Mg - \dfrac{Mv^2}{r}$.

33. *(d)* Free body for the sphere is shown in the figure.



$$T_1\cos 30° + T_2\cos 45° = mg \qquad ...(1)$$

$$T_1\sin 30° + T_2\sin 45° = \frac{mv^2}{r} \qquad ...(2)$$

On solving (1) and (2), we get

$$T_1 = \frac{mg - \dfrac{mv^2}{r}}{\dfrac{(\sqrt{3}-1)}{2}}$$

But $T_1 > 0$

$$\therefore \quad mg > \frac{mv^2}{r} \quad \text{or} \quad v < \sqrt{rg}$$

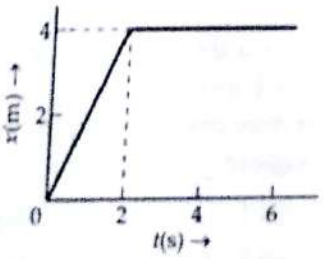
$$\therefore \quad v_{max} = \sqrt{rg} = \sqrt{1.6 \times 9.8} = \mathbf{3.9\text{ ms}^{-1}}.$$

34. *(b)* $R = m(g + a)$.

35. *(d)* At the top of the swing, $v_c = \sqrt{Rg}$.

AIIMS Entrance Exam

1. In the figure given, the position-time graph of a particle of mass 0.1 kg is shown.



The impulse at $t = 2$ s is

(a) 0.2 kg ms^{-1} (b) –02 kg ms^{-1} (c) 0.1 kg ms^{-1} (d) – 0.4 kg ms^{-1} [AIIMS 05]

2. A gun fires a bullet of mass 50 g with a velocity of 30 ms^{-1}. Because of this, the gun is pushed back with a velocity of 1 ms^{-1}. The mass of the gun is

(a) 5.5 kg (b) 3.5 kg (c) 1.5 kg (d) 0.5 kg [AIIMS 2K]

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3. A body of mass M moving with velocity V explodes into two equal parts. If one comes to rest and the other body moves with velocity v, what would be the value of v ?

(a) v (b) $v/\sqrt{2}$
(c) $4v$ (d) $2v$ [AIIMS]

4. Rocket engines lift a rocket from the earth surface, because hot gases with high velocity

(a) push against the air
(b) push against the earth
(c) react against the rocket and push it up
(d) heat up the air which lifts the rocket. [AIIMS 98]

5. The motion of a rocket is based on the principle of conservation of

(a) linear momentum (b) angular momentum
(c) kinetic energy (d) mass [AIIMS 95]

6. A bullet is fired from a rifle. If the rifle recoils freely, then the kinetic energy of the rifle is

(a) less than that of the bullet
(b) more than that of the bullet
(c) same as that of the bullet
(d) equal or less than that of the bullet. [AIIMS 90]

7. A person is standing in an elevator. In which situation he finds his weight less ?

(a) When the elevator moves upward with constant acceleration
(b) When the elevator moves downward with constant acceleration
(c) When the elevator moves upward with uniform velocity
(d) When the elevator moves downward with uniform velocity. [AIIMS 05]

8. If a person with a spring balance and a body hanging from it goes up and up in an aeroplane, then the reading of the weight of the body as indicated by the spring balance, will

(a) go on increasing (b) go on decreasing
(c) first increase and then decrease
(d) remain the same. [AIIMS 98]

9. A man of mass 60 kg records his weight on a weighing machine placed inside a lift. The ratio of the weights of man recorded when lift is ascending up with a uniform speed of 2 m/s to when it is descending down with a uniform speed of 4 m/s will be

(a) 0.5 (b) 1
(c) 2 (d) none of the above.
[AIIMS 07]

10. A simple pendulum is set up in a trolley which is moving with an acceleration a on a horizontal plane. Then the thread of the pendulum in the mean position makes an angle with the vertical equal to

(a) $\tan^{-1}(a/g)$ in the forward direction
(b) $\tan^{-1}(a/g)$ in the backward direction
(c) $\tan^{-1}(g/a)$ in the forward direction
(d) $\tan^{-1}(g/a)$ in the backward direction. [AIIMS 00]

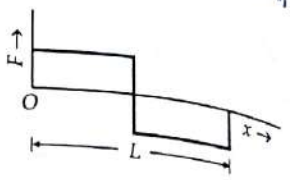
11. The forces, which meet at one point but their liens of action do not lie in one plane, are called

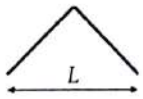
(a) coplanar concurrent forces
(b) coplanar non-concurrent forces
(c) non-coplanar concurrent forces
(d) non-coplanar non-concurrent forces. [AIIMS 96]

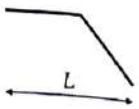
12. Which of the following is non-conservative force ?

(a) Interatomic force (b) Gravitational force
(c) Electrostatic force (d) Viscous force [AIIMS 96]

13. A person used force (F), shown in figure to move a load with constant velocity on a surface. Identify the correct surface profile (A).



(a)


(b)


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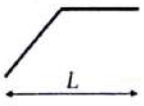
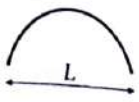


(d)


[AIIMS 06]

14. A particle revolves round a circular path. The acceleration of the particle is inversely proportional to

(a) radius (b) velocity
(c) mass of particle (d) both (b) and (c)
[AIIMS 94]

15. If the radii of circular paths of two particles of same masses are in the ratio of 1 : 2, then to have a constant centripetal force, their velocities should be in a ratio of

(a) $1:\sqrt{2}$ (b) $\sqrt{2}:1$
(c) 4 : 1 (d) 1 : 4 [AIIMS 96]

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16. A stone tied to a string is rotated with a uniform speed in a vertical plane. If mass of the stone is m, length of the string is r and linear speed of the stone is v, then tension in the string, when the stone is at its lowest point, (g = acceleration due to gravity) is

(a) mg (b) $\frac{mv^2}{r}$

(c) $\frac{mv^2}{r} - mg$ (d) $\frac{mv^2}{r} + mg$ [AIIMS 01]

17. A body is allowed to slide down a frictionless track freely under gravity. The track ends in a semicircular shaped part of diameter D. What should be the height (minimum) from which the body must fall, so that it completes the circle ?

(a) $4D/5$ (b) $5D/4$

(c) D (d) $2D$ [AIIMS 2K]

18. If the normal force is doubled, then coefficient of friction is

(a) halved (b) tripled

(c) doubled (d) not changed [AIIMS 94]

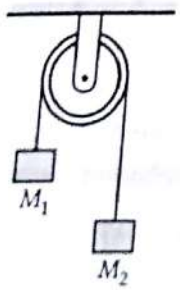
19. A body of mass m is placed on a rough surface with coefficient of friction μ inclined at θ. If the mass is in equilibrium, then

(a) $\theta = \tan^{-1}\mu$ (b) $\theta = \tan^{-1}\frac{1}{\mu}$

(c) $\theta = \tan^{-1}\frac{m}{\mu}$ (d) $\theta = \tan^{-1}\frac{\mu}{m}$

[AIIMS 96, 99, DPMT 97]

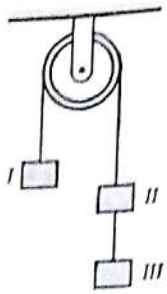
20. Two masses $M_1 = 5$ kg and $M_2 = 10$ kg are connected at the ends of an inextensible string passing over a frictionless pulley as shown. When the masses are released, then the acceleration of the masses will be



(a) g (b) $g/2$

(c) $g/3$ (d) $g/4$ [AIIMS 2010]

21. Three equal weights of 3 kg each are hanging on a string passing over a frictionless pulley as shown in the figure. The tension in the string between masses II and III will be (Take $g = 10\,m/s^2$)



(a) 5 N (b) 6 N

(c) 10 N (d) 20 N [AIIMS 2009]

22. A block of mass m is pulled along a horizontal surface by applying a force at an angle θ with the horizontal. If the block travels with a uniform velocity and has a displacement d and the coefficient of friction is μ, then the work done by the applied force is

(a) $\frac{\mu mgd}{\cos\theta + \mu\sin\theta}$ (b) $\frac{\mu mgd\cos\theta}{\cos\theta + \mu\sin\theta}$

(c) $\frac{\mu mgd\sin\theta}{\cos\theta + \mu\sin\theta}$ (d) $\frac{\mu mgd\cos\theta}{\cos\theta - \mu\sin\theta}$

[AIIMS 2010]

ASSERTIONS AND REASONS

Directions. In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as

(a) If both assertion and reason are true and reason is the correct explanation of the assertion.

(b) If both assertion and reason are true but reason is not correct explanation of the assertion.

(c) If assertion is true, but reason is false.

(d) If both assertion and reason are false.

23. **Assertion.** A rocket moves forward by pushing the surrounding air backwards.

Reason. It derives the necessary thrust to move forward, according to Newton's third law of motion. [AIIMS 01]

24. **Assertion.** Frictional forces are conservative forces.

Reason. Potential energy can be associated with frictional forces. [AIIMS 05]

25. **Assertion.** The driver in a vehicle moving with a constant speed on a straight road is a non-inertial frame of reference.

Reason. A reference frame, in which Newton's laws of motion are applicable, is non-inertial. [AIIMS 04]

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26. **Assertion.** A man in a closed cabin, which is falling freely, does not experience gravity.

Reason. Inertial and gravitational masses have equivalence. [AIIMS 06]

27. **Assertion.** It is difficult to move a cycle along the road with its brakes on.

Reason. Sliding friction is greater than rolling friction. [AIIMS 02]

28. **Assertion.** On a rainy day, it is difficult to drive a car or bus at high speed.

Reason. The value of coefficient of friction is lowered due to wetting of the surface. [AIIMS 95, 99]

29. **Assertion.** Use of ball bearings between two moving parts of a machine is a common practice.

Reason. Ball bearings reduce vibrations and provide good stability. [AIIMS 06]

30. **Assertion.** During turning, a cyclist leans towards the centre of the curve; while a man sitting in the car leans outwards of the curve.

Reason. An acceleration is acting towards the centre of the curve. [AIIMS 97]

31. **Assertion.** The apparent weight of a body in an elevator moving with some downward acceleration is less than the actual weight of a body.

Reason. The part of the weight is spent in producing downward acceleration, when body is in elevator. [AIIMS 2010]

32. **Assertion.** Impulsive force is large and acts for a short time.

Reason. Finite change in momentum should be produced by the force. [AIIMS 2006]

33. **Assertion.** A horse has to pull a cart harder during the first few steps of his motion.

Reason. The first few few steps are always difficult. [AIIMS 2010]

Answers and Explanations

1. (*b*) Before $t = 2$ s, the particle has a constant velocity,

$$u = \frac{4-0}{2-0} = 2 \text{ ms}^{-1}$$

After $t = 2$, the particle is at rest, so $v = 0$.

Impulse $= m(v-u) = 0.1(0-2)$

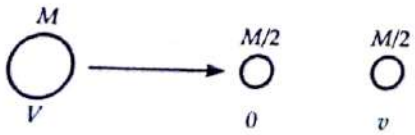
$= -0.2 \text{ kg ms}^{-1}$.

2. (*c*) By conservation of momentum

$$MV = mv$$

$$M = \frac{mv}{V} = \frac{0.05 \times 30}{1} = 1.5 \text{ kg}.$$

3. (*d*)



By conservation of momentum,

$$MV = \frac{M}{2} \times 0 + \frac{M}{2} \times v$$

or $v = 2V$.

4. (*c*) Hot gases with high velocity react against the rocket and push it up.

5. (*a*) A rocket works on the principle of conservation of linear momentum.

6. (*a*) By conservation of momentum,

$$MV = mv$$

or $$V = \frac{mv}{M}$$

Kinetic energy of the rifle is

$$K_r = \frac{1}{2}MV^2 = \frac{1}{2}M\frac{m^2v^2}{M^2}$$

$$= \frac{m}{M} \cdot \frac{1}{2}mv^2 = \frac{m}{M}K_b$$

As $m < M$, $K_r < K_b$

Thus the K.E. of the rifle is less than that of the bullet.

7. (*b*) When the elevator moves downward with acceleration a, the apparent weight,

$$W' = m(g-a) < W(=mg)$$

8. (*b*) As the person goes up above the earth's surface, the value of 'g' decreases. Hence the reading in the spring balance will go on decreasing.

9. (*b*) Uniform speed does not affect the weight of man.

$$\text{Required ratio} = \frac{60 \text{ kg wt}}{60 \text{ kg wt}} = 1.$$

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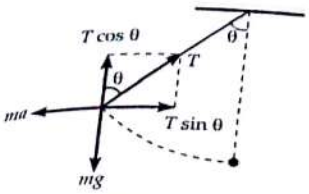
10. (b)



A backward force equal to ma acts on the bob. In the equilibrium position,

$$T\sin\theta = ma$$

$$T\cos\theta = mg$$

$$\therefore \quad \tan\theta = \frac{T\sin\theta}{T\cos\theta} = \frac{ma}{mg} = \frac{a}{g}$$

or $$\theta = \tan^{-1}\left(\frac{a}{g}\right)$$

11. (c) Non-coplanar and concurrent forces.

12. (d) Viscous force is a non-conservative force.

13. (c) Initial force is applied to overcome static friction.

Applied force > Limiting friction

$$F > \mu_s R$$

or

Body has constant acceleration and its velocity increases proportionately with time. Then suddenly the force is decreased to just balance the kinetic friction ($\mu_k R$). Then the body continues to move with the constant maximum velocity attained by it earlier.

14. (a) $a = \dfrac{v^2}{r}$ *i.e.,* $a \propto \dfrac{1}{r}$.

15. (a) Centripetal force, $F = \dfrac{mv^2}{r}$

$$\therefore \quad v = \sqrt{\frac{Fr}{m}}$$

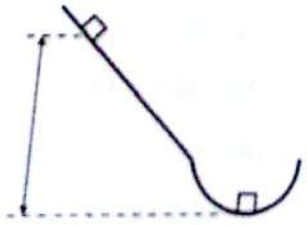
For same F and m, $v \propto \sqrt{r}$

$$\therefore \quad \frac{v_1}{v_2} = \sqrt{\frac{r_1}{r_2}} = \sqrt{\frac{1}{2}} = \mathbf{1 : \sqrt{2}}.$$

16. (d) $T_L = \dfrac{mv^2}{r} + mg.$

For derivation, refer answer to Q. 70 on page 5.58.

17. (b) The body will complete the vertical circle if its minimum speed at the bottom is $\sqrt{5gR}$, where R is the radius of the circular track. If the body falls through height h,

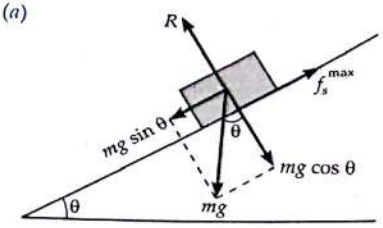


then $$\sqrt{2gh} = \sqrt{5g\frac{D}{2}}$$

or $$h = \frac{5D}{4}$$

18. (c) Coefficient of friction is independent of normal force.

19. (a)



For equilibrium of the body,

$$mg\sin\theta = f_s^{max}$$

$$mg\cos\theta = R$$

$$\therefore \quad \frac{mg\sin\theta}{mg\cos\theta} = \frac{f_s^{max}}{R}$$

or $$\tan\theta = \mu$$

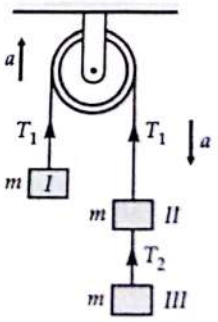
or $$\theta = \tan^{-1}(\mu).$$

20. (c) $$a = \frac{M_2 - M_1}{M_2 + M_1}g = \frac{10-5}{10+5}\times g = \frac{g}{3}$$

21. (d) $$T_1 - mg = ma$$

$$2mg - T_1 = 2ma$$

$$mg - T_2 = ma$$



On solving, $$T_2 = \frac{2}{3}mg = \frac{2}{3}\times 3\times 10$$

$$= \mathbf{20\ N}$$

22. (b) As the block moves with a uniform velocity, the resultant force on it is zero.

$$R + F\sin\theta = mg$$

or $$R = mg - F\sin\theta$$

$$\therefore \quad f = \mu R = \mu(mg - F\sin\theta)$$

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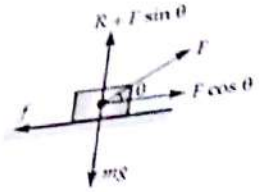

Also, $F\cos\theta = f$

or $F\cos\theta = \mu(mg - F\sin\theta)$

or $F(\cos\theta + \mu\sin\theta) = \mu mg$

or $F = \dfrac{\mu mg}{\cos\theta + \mu\sin\theta}$

$\therefore$ $W = Fd\cos\theta = \dfrac{\mu mgd\cos\theta}{\cos\theta + \mu\sin\theta}$

23. (*a*) Both the assertion and reason are true.

24. (*d*) Both the assertion and reason are false.

25. (*c*) The assertion is true and reason is false.

26. (*c*) Both the assertion and reason are true but reason is not correct explanation of the assertion.

27. (*a*) With brakes on, a cycle can slide and not roll. Both the assertion and reason are true.

28. (*a*) Both assertion and reason are true. Due to wetting, friction decreases between tyres and road. This may lead to skidding of the vehicle.

29. (*c*) The assertion is true but the reason is false. The ball bearings reduce friction by converting sliding friction into rolling friction.

30. (*c*) The assertion is true but reason is false. The cyclist leans inward to get necessary centripetal force. So an acceleration acts towards the centre of the curve. The man sitting in the car leans outwards due to the centrifugal force acting on him. The acceleration acts on him away from the centre of the curve.

31. (*b*) Assertion is true but the reason is not its correct explanation.

Apparent weight $= m(g - a)$.

32. (*a*) Assertion is true, the reason is its correct explanation.

33. (*c*) During the first few steps of his motion, the horse has to work against the limiting friction and once the cart starts moving, the horse has to work against kinetic friction which is less than the limiting friction.

CBSE PMT Prelims and Final Exams

1. A 10 N force is applied on a body to produce in it an acceleration of 1 m/s^2.
The mass of the body is
(*a*) 15 kg (*b*) 20 kg
(*c*) 10 kg (*d*) 5 kg [CBSE PMT 96]

2. A cricketer catches a ball of mass 150 g in 0.1 s moving with speed 20 m/s. Then he experiences a force of
(*a*) 300 N (*b*) 30 N
(*c*) 3 N (*d*) 0.3 N [CBSE PMT 01]

3. A force of 6 N acts on a body at rest and of mass 1 kg. During this time, the body attains a velocity of 30 m/s. The time for which the force acts on the body is
(*a*) 7 seconds (*b*) 5 seconds
(*c*) 10 seconds (*d*) 8 seconds [CBSE PMT 97]

4. Physical independence of force is a consequence of
(*a*) third law of motion
(*b*) second law of motion
(*c*) first law of motion
(*d*) all of these laws [CBSE PMT 91]

5. A force vector applied on a mass is represented as $\vec{F} = 6\hat{i} - 8\hat{j} + 10\hat{k}$ and accelerates with 1 m/s^2. What will be the mass of the body ?
(*a*) 10 kg (*b*) 20 kg
(*c*) $10\sqrt{2}$ kg (*d*) $2\sqrt{10}$ kg [CBSE PMT 96, 09]

6. A ball is dropped from a spacecraft revolving the earth at a height of 120 km. What will happen to the ball ?
(*a*) It will continue to move with the same speed along the original orbit of spacecraft.
(*b*) It will move with the same speed, tangentially to the spacecraft.
(*c*) It will fall down to the earth gradually.
(*d*) It will go very far in the space. [CBSE PMT 96]

7. An object of mass 3 kg is at rest. Now a force of $\vec{F} = 6t^2\hat{i} + 4t\hat{j}$ is applied on the object. Then velocity of object at $t = 3$ is
(*a*) $18\hat{i} + 3\hat{j}$ (*b*) $18\hat{i} + 6\hat{j}$
(*c*) $3\hat{i} + 18\hat{j}$ (*d*) $18\hat{i} + 4\hat{j}$ [CBSE PMT 02]

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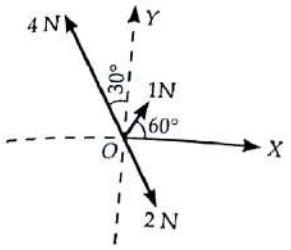
8. Sand is being dropped from a conveyer belt at the rate of M kg/s. The force necessary to keep the belt moving with a constant velocity of v m/s will be

(a) Mv newton (b) $2Mv$ newton

(c) $\frac{Mv}{2}$ newton (d) zero [CBSE PMT 08]

9. Three forces acting on a body are shown in the figure. To have the resultant force only along the



y-direction, the magnitude of the minimum additional force needed along OX is

(a) 0.5 N (b) 1.5 N

(c) $\sqrt{3}/4$ N (d) $\sqrt{3}$ N [CBSE PMT 08]

10. A particle of mass m is projected with velocity v making an angle of 45° with the horizontal. When the particle lands on the level ground, the magnitude of the change in its momentum will be

(a) $2mv$ (b) $mv/\sqrt{2}$

(c) $\sqrt{2}mv$ (d) zero [CBSE PMT 08]

11. A particle of mass m is moving a uniform velocity v_1. It is given an impulse such that its velocity becomes v_2. The impulse is equal to

(a) $m[|v_2| - |v_1|]$ (b) $\frac{1}{2}m[v_2^2 - v_1^2]$

(c) $m[v_1 + v_2]$ (d) $m[v_2 - v_1]$ [CBSE PMT 90]

12. A bullet is fired from a gun. The force on the bullet is given by

$$F = 600 - 2 \times 10^5 t$$

where F is in newton and t in seconds. The force on the bullet becomes zero as soon as it leaves the barrel. What is the average impulse imparted to the bullet ?

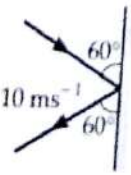
(a) 9 Ns (b) zero

(c) 1.8 Ns (d) 0.9 Ns [CBSE PMT 98]

13. A body of mass 3 kg hits a wall at an angle of 60° and returns at the same angle. The impact time was 0.2 sec. The force exerted on the wall

(a) $150\sqrt{3}$ N (b) $50\sqrt{3}$ N

(c) 100 N (d) $75\sqrt{3}$ N [CBSE PMT 2K]



14. A 0.5 kg ball moving with a speed of 12 m/s strikes a hard wall at an angle of 30° with the wall. It is reflected with the same speed at the same angle. If the ball is in contact with the wall for 0.25 second, the average force acting on the wall is

(a) 96 N (b) 48 N

(c) 24 N (d) 12 N [CBSE PMT 06]

15. A lift of mass 1000 kg is moving with an acceleration of 1 m/s^2 in upward direction, then the tension developed in the string which is connected to lift is

(a) 9800 N (b) 10,800 N

(c) 11,000 N (d) 10,000 N [CBSE PMT 02]

16. A mass of 1 kg is suspended by a thread. It is

(i) lifted up with an acceleration 4.9 m/s^2

(ii) lowered with an acceleration 4.9 m/s^2

The ratio of the tensions is

(a) 1 : 3 (b) 1 : 2

(c) 3 : 1 (d) 2 : 1 [CBSE PMT 98]

17. A man weighs 80 kg. He stands on a weighing scale in a lift which is moving upwards with a uniform acceleration of 5 m/s^2. What would be the reading on the scale ? ($g = 10$ m/s^2)

(a) zero (b) 400 N

(c) 800 N (d) 1200 N [CBSE PMT 03]

18. A monkey of mass 20 kg is holding a vertical rope. The rope will not break when a mass of 25 kg is suspended from it but will break if the mass exceeds 25 kg. What is the maximum acceleration with which the monkey can climb up along the rope ? ($g = 10$ m/s^2)

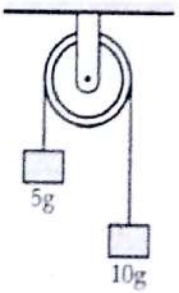
(a) 5 m/s^2 (b) 10 m/s^2

(c) 25 m/s^2 (d) 2.5 m/s^2 [CBSE PMT 03]

19. A monkey is descending from the branch of a tree with constant acceleration. If the breaking strength is 75% of the weight of the monkey, the minimum acceleration with which monkey can slide down without breaking branch is

(a) g (b) $\frac{3g}{4}$ (c) $\frac{g}{4}$ (d) $\frac{g}{2}$ [CBSE PMT 93]

20. Two masses as shown in the figure are suspended from a massless pulley. The acceleration of the system when masses are left free is



(a) $\frac{2g}{3}$ (b) $\frac{g}{3}$

(c) $\frac{g}{9}$ (d) $\frac{g}{7}$ [CBSE PMT 2K]

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21. A man fires a bullet of mass 200 g at a speed of 5 m/s. The gun is of one kg mass. By what velocity the gun rebounds backward ?

(a) 1 m/s (b) 0.01 m/s
(c) 0.1 m/s (d) 10 m/s [CBSE PMT 96]

22. A 1 kg stationary bomb is exploded in three parts having mass 1 : 1 : 3 respectively. Parts having same mass move in perpendicular direction with velocity 30 m/s, then the velocity of bigger part will be

(a) $10\sqrt{2}$ m/sec (b) $\frac{10}{\sqrt{2}}$ m/sec
(c) $15\sqrt{2}$ m/sec (d) $\frac{15}{\sqrt{2}}$ m/sec [CBSE PMT 01]

23. A body of mass 5 kg explodes at rest into three fragments with masses in the ratio 1 : 1 : 3. The fragments with equal masses fly in mutually perpendicular directions with speeds of 21 m/s. The velocity of heaviest fragment in m/s will be

(a) $7\sqrt{2}$ (b) $5\sqrt{2}$
(c) $3\sqrt{2}$ (d) $\sqrt{2}$ [CBSE PMT 89]

24. In a rocket, fuel burns at the rate of 1 kg/s. This fuel is ejected from the rocket with a velocity of 60 km/s. This exerts a force on the rocket equal to

(a) 6000 N (b) 60000 N
(c) 60 N (d) 600 N [CBSE PMT 94]

25. If the force on a rocket, moving with a velocity of 300 m/s is 210 N, then the rate of combustion of the fuel is

(a) 0.07 kg/s (b) 1.4 kg/s
(c) 0.7 kg/s (d) 10.7 kg/s [CBSE PMT 99]

26. A 600 kg rocket is set for a vertical firing. If the exhaust speed is 1000 ms^{-1}, the mass of the gas ejected per second to supply the thrust needed to overcome the weight of rocket is

(a) 117.6 kgs^{-1} (b) 58.6 kgs^{-1}
(c) 6 kgs^{-1} (d) 76.4 kgs^{-1} [CBSE PMT 90]

27. A 5000 kg rocket is set for vertical firing. The exhaust speed is 800 ms^{-1}. To give an inertial upward acceleration of 20 ms^{-2}, the amount of gas ejected per second to supply the needed thrust will be ($g = 10 ms^{-2}$)

(a) 185.5 kgs^{-1} (b) 187.5 kgs^{-1}
(c) 127.5 kgs^{-1} (d) 137.5 kgs^{-1} [CBSE PMT 98]

28. A block of mass 10 kg is placed on rough horizontal surface having coefficient of friction $\mu = 0.5$. If a horizontal force of 100 N is acting on it, then acceleration of the block will be

(a) 10 m/s^2 (b) 5 m/s^2
(c) 15 m/s^2 (d) 0.5 m/s^2 [CBSE PMT 02]

29. On the horizontal surface of a truck a block of mass 1 kg is placed ($\mu = 0.6$) and truck is moving with acceleration 5 m/sec^2, then the frictional force on the block will be

(a) 5 N (b) 6 N
(c) 5.88 N (d) 8 N [CBSE PMT 01]

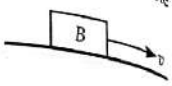
30. Consider a car moving along a straight horizontal road with a speed of 72 km/h. If the coefficient of static friction between the tyres and the road is 0.5, the shortest distance in which the car can be stopped (taking $g = 10 m/s^2$) is

(a) 30 m (b) 40 m
(c) 72 m (d) 20 m [CBSE PMT 92]

31. A heavy uniform chain lies on horizontal table top. If the coefficient of friction between the chain and the table surface is 0.25, then the maximum fraction of the length of the chain that can hang over one edge of the table is

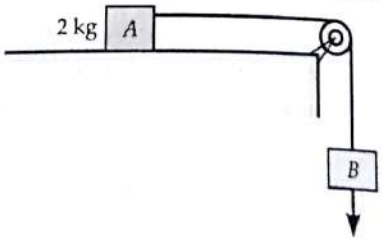
(a) 20% (b) 25%
(c) 35% (d) 15% [CBSE PMT 91]

32. A block B is pushed momentarily along a horizontal surface with an initial velocity v. If μ is the coefficient of sliding friction between B and the surface, block B will come to rest after a time



(a) $g\mu / v$ (b) g / v
(c) v / g (d) $v / (g\mu)$ [CBSE PMT 07]

33. The coefficient of static friction, μ_s, between block A of mass 2 kg and the table as shown in the figure is 0.2. What would be the maximum mass value

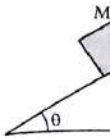


of block B so that the two blocks do not move ? The string and the pulley are assumed to be smooth and massless. ($g = 10 m/s^2$)

(a) 2.0 kg (b) 4.0 kg
(c) 0.2 kg (d) 0.4 kg [CBSE PMT 04]

34. A block has been pla
the slope angle θ, block slid
speed. The coefficient of
(a) sin θ
(c) g

35. A mass M is plac
resting on a surface with



released, the acceleration
M remains at rest is a,
(a) a is applied to t
(b) a is applied to t
(c) a is applied to t
(d) a is applied to

36. A block of mas
of inclination θ. Th
horizontally so that
wedge. The force exe
is acceleration due t
(a) mg cos θ
(c) mg

37. Starting fro
inclined plane in tw
the same distance in
of friction between
(a) 0.80
(c) 0.25

38. When mill
due to
(a) centripetal
(c) frictional f

39. A 500 kg
with a velocity
(a) 1000 N
(c) 250 N

40. A ball o
string of length
The string will

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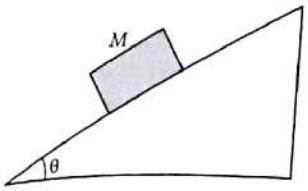
34. A block has been placed on an inclined plane with the slope angle θ, block slides down the plane at constant speed. The coefficient of kinetic friction is equal to
(a) $\sin\theta$ (b) $\cos\theta$
(c) S (d) $\tan\theta$ [CBSE PMT 93]

35. A mass M is placed on a very smooth wedge resting on a surface without friction. Once the mass is



released, the acceleration to be given to the wedge so that M remains at rest is a, where
(a) a is applied to the left and $a = g\tan\theta$
(b) a is applied to the right and $a = g\tan\theta$
(c) a is applied to the left and $a = g\sin\theta$
(d) a is applied to the left and $a = g\cos\theta$. [CBSE PMT 98]

36. A block of mass m is placed on a smooth wedge of inclination θ. The whole system is accelerated horizontally so that the block does not slip on the wedge. The force exerted by the wedge on the block (g is acceleration due to gravity) will be
(a) $mg\cos\theta$ (b) $mg\sin\theta$
(c) mg (d) $mg/\cos\theta$ [CBSE PMT 04]

37. Starting from rest, a body slides down a 45° inclined plane in twice the time it takes to slide down the same distance in the absence of friction. The coefficient of friction between the body and the inclined plane is
(a) 0.80 (b) 0.75
(c) 0.25 (d) 0.33 [CBSE PMT 88]

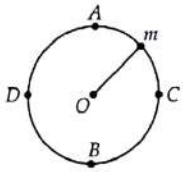
38. When milk is churned, cream gets separated due to
(a) centripetal force (b) centrifugal force
(c) frictional force (d) gravitational force [CBSE PMT 04]

39. A 500 kg car takes a round turn of radius 50 m with a velocity of 36 km/hr. The centripetal force is
(a) 1000 N (b) 750 N
(c) 250 N (d) 1200 N m/s [CBSE PMT 99]

40. A ball of mass 0.25 kg attached to the end of a string of length 1.96 m is moving in a horizontal circle. The string will break if the tension is more than 25 N. What is the maximum speed with which the ball can be moved ?
(a) 5 m/s (b) 3 m/s
(c) 14 m/s (d) 3.92 m/s [CBSE PMT 98]

41. A tube of length L is filled completely with an incompressible liquid of mass M and closed at both the ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity ω. The force exerted by the liquid at the other end is
(a) $\frac{ML^2\omega^2}{2}$ (b) $\frac{ML\omega^2}{2}$
(c) $\frac{ML^2\omega}{2}$ (d) $ML\omega^2$. [CBSE PMT 06]

42. As shown in the figure at point O, a mass m is performing vertical circular motion. The average velocity of the particle is increased. Then at which point will the string break ?



(a) A (b) B
(c) C (d) D [CBSE PMT 2K]

43. An explosion blows a rock into three parts. Two parts go off at right angles to each other. These two are, 1 kg first part moving with a velocity of 12 ms^{-1} and 2 kg second part moving with a velocity of 8 ms^{-1}. If the third part flies off with a velocity of 4 ms^{-1}, its mass would be
(a) 3 kg (b) 5 kg
(c) 7 kg (d) 17 kg. [CBSE PMT 09]

44. The mass of a lift is 2000 kg. When the tension in the cable is 28000 N, then its acceleration is
(a) 14 ms^{-2} upwards
(b) 30 ms^{-2} downwards
(c) 4 ms^{-2} upwards
(d) 4 ms^{-2} downwards. [CBSE PMT 09]

45. A mass m moving horizontally (along the x-axis) with velocity v collides and sticks to a mass of $3m$ moving vertically upward (along the y-axis) with velocity $2v$.

The final velocity of the combination is
(a) $\frac{2}{3}v\hat{i}+\frac{1}{3}v\hat{j}$ (b) $\frac{3}{2}v\hat{i}+\frac{1}{4}v\hat{j}$
(c) $\frac{1}{4}v\hat{i}+\frac{3}{2}v\hat{j}$ (d) $\frac{1}{3}v\hat{i}+\frac{2}{3}v\hat{j}$ [CBSE Final 2011]

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46. A body of mass M hits normally a rigid wall with velocity V and bounces back with the same velocity. The impulse experienced by the body is

(a) MV (b) $1.5\,MV$

(c) $2\,MV$ (d) zero [CBSE Pre 2011]

47. A man of 50 kg mass is standing in a gravity free space at a height of 10 m above the floor. He throws a stone of 0.5 kg mass downwards with a speed 2 m/s. When the stone reaches the floor, the distance of the man above the floor will be

(a) 9.9 m (b) 10.1 m

(c) 10 m (d) 20 m [CBSE Pre 2010]

48. A person of mass 60 kg is inside a lift of mass 940 kg and presses the button on control panel. The lift starts moving upwards with an acceleration 1.0 m / s^2. If $g = 10$ ms^{-2}, the tension in the supporting cable is

(a) 8600 N (b) 9680 N

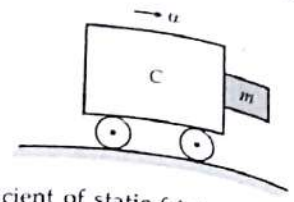
(c) 11000 N (d) 1200 N [CBSE Pre 2011]

49. A conveyor belt is moving at a constant speed of 2 m/s. A box is gently dropped on it. The coefficient of friction between them is $\mu = 0.5$. The distance that the box will move relative to belt before coming to rest on it, taking $g = 10$ ms^{-2} is

(a) zero (b) 0.4 m

(c) 1.2 m (d) 0.6 m [CBSE Final 2011]

50. A block of mass m is in contact with the cart C as shown in the figure.



The coefficient of static friction between the block and the cart is μ. The acceleration α of the cart that will prevent the block from falling satisfies

(a) $\alpha > \frac{mg}{\mu}$ (b) $\alpha > \frac{g}{\mu m}$

(c) $\alpha \geq \frac{g}{\mu}$ (d) $\alpha < \frac{g}{\mu}$ [CBSE Pre 2010]

51. A gramophone record is revolving with an angular velocity ω. A coin is placed at a distance r from the centre of the record. The static coefficient of friction is μ. The coin will revolve with the record if

(a) $r = \mu g\omega^2$ (b) $r < \frac{\omega^2}{\mu g}$

(c) $r \leq \frac{\mu g}{\omega^2}$ (d) $r \geq \frac{\mu g}{\omega^2}$ [CBSE Pre 2010]

Answers and Explanations

1. (c) $m = \frac{F}{a} = \frac{10\,\text{N}}{1\,\text{ms}^{-2}} = 10$ **kg.**

2. (b) $F = m\left(\frac{v-u}{t}\right) = \frac{150}{1000}\left(\frac{0-20}{0.1}\right) = -30$ N.

Force experienced by the cricketer = **30 N.**

3. (b) $a = \frac{F}{m} = \frac{6\,\text{N}}{1\,\text{kg}} = 6\ \text{ms}^{-2}$

$t = \frac{v-u}{a} = \frac{30-0}{6} = \mathbf{5\ s.}$

4. (c) Newton's first law of motion is related to the physical independence of force.

5. (c) $\vec{F} = 6\hat{i} - 8\hat{j} + 10\hat{k}$

$F = |\vec{F}| = \sqrt{36+64+100} = 10\sqrt{2}$ N

$m = \frac{F}{a} = \frac{10\sqrt{2}}{1\,\text{ms}^{-2}} = \mathbf{10\sqrt{2}\ kg.}$

6. (a) As no external force acts on the ball, it will continue to move with the same speed along the original orbit of the spacecraft.

7. (b) $\vec{a} = \frac{\vec{F}}{m} = \frac{1}{3}(6t^2\hat{i} + 4t\hat{j}) = \frac{d\vec{v}}{dt}$

$d\vec{v} = \frac{1}{3}(6t^2\hat{i} + 4tj)dt$

Velocity at $t = 3$ s will be

$\vec{v} = \int d\vec{v} = \frac{1}{3}\int_0^3 (6t^2\hat{i} + 4t\hat{j})\,dt$

$= \frac{1}{3}\left|2t^3\hat{i} + 2t^2\hat{j}\right|_0^3 = \frac{2}{3}(3^3\hat{i} + 3^2\hat{j})$

$= \mathbf{(18\hat{i} + 6\hat{j})\ ms^{-1}}.$

8. (a) $F = \frac{dp}{dt} = \frac{d}{dt}(mv) = v\frac{dm}{dt}$

$= v \times M = Mv$ newton.

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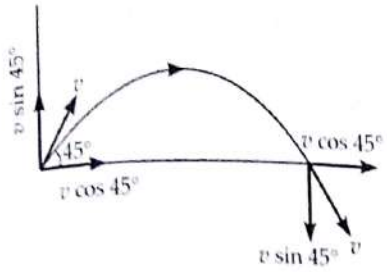
9. (*a*) The resultant force will be along *y*-direction if the resultant of the components along *x*-direction is zero. If F is the force needed along OX, then

$$F + 1\cos 60° + 2\cos 60° = 4\cos 60°$$

$$F = (4 - 1 - 2)\cos 60°$$

or $$= \frac{1}{2} = \mathbf{0.5\ N}.$$

10. (*c*) When the particle lands on the level ground, its momentum along horizontal direction does not change.



But vertical component of velocity gets reversed.

Change in momentum in vertical direction

$$= mv\sin 45° - (-mv\sin 45°)$$

$$= 2mv \times \frac{1}{\sqrt{2}} = \sqrt{2}\,mv.$$

11. (*d*) Impulse = Change in momentum = $m(v_2 - v_1)$.

12. (*d*) When the force on the bullet becomes zero,

$$600 - 2 \times 10^5 t = 0$$

$$t = 3 \times 10^{-3}\ \text{s}$$

or

$$\text{Impulse, } J = \int_0^t F\,dt$$

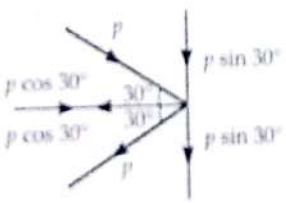
$$= \int_0^t (600 - 2 \times 10^5 t)\,dt = 600t - 2 \times 10^5 \frac{t^2}{2}$$

$$= 600 \times 3 \times 10^{-3} - 10^5 \times (3 \times 10^{-3})^2$$

$$= 1.8 - 0.9 = \mathbf{0.9\ Ns}.$$

13. (*a*) Change in momentum along vertical direction

$$= p\sin 30° - p\sin 30° = 0$$



Change in momentum along horizontal direction

$$= -p\cos 30° - p\cos 30°$$

$$= -2p\cos 30° = -2mv\cos 30°$$

$$= -2 \times 3 \times 10 \times \frac{\sqrt{3}}{2} = -30\sqrt{3}\ \text{kg ms}^{-1}$$

Force exerted on the wall

$$= \frac{\text{Change in momentum}}{\text{Time of impact}}$$

$$= \frac{30\sqrt{3}}{0.2} = \mathbf{150\sqrt{3}\ N}.$$

14. (*c*) Proceeding as in the above problem,

$$F = \frac{2mv\cos(90° - 30°)}{t}$$

$$= \frac{2 \times 0.5 \times 12 \times \cos 60°}{t} = \mathbf{24\ N}.$$

15. (*b*) For the lift moving upward with acceleration a,

$$T = m(g + a) = 1000(9.8 + 1)\ \text{N} = \mathbf{10800\ N}.$$

16. (*c*) For upward acceleration,

$$T_1 = m(g + a)$$

For downward acceleration,

$$T_2 = m(g - a)$$

$$\therefore \quad \frac{T_1}{T_2} = \frac{g + a}{g - a} = \frac{9.8 + 4.9}{9.8 - 4.9} = \frac{14.7}{4.9} = \mathbf{3:1}.$$

17. (*d*) As the lift accelerates upward, the apparent weight,

$$R = m(g + a) = 80(10 + 5) = \mathbf{1200\ N}.$$

18. (*d*) When the monkey climbs the rope with acceleration a,

$$T = m(g + a)$$

or $$Mg = mg + ma$$

$$25 \times 10 = 20 \times 10 + 20a$$

or $$a = \frac{50}{20} = \mathbf{2.5\ m/s^2}.$$

19. (*c*) When the monkey descends with acceleration a,

$$T = m(g - a)$$

$$\frac{75}{100}mg = m(g - a)$$

or $$a = g - \frac{3}{4}g = \frac{g}{4}.$$

20. (*b*) $$a = \frac{(m_1 - m_2)g}{m_1 + m_2} = \frac{(10 - 5) \times g}{(10 + 5)} = \frac{g}{3}.$$

21. (*a*) By conservation of momentum,

$$mv + MV = 0$$

$$\frac{200}{1000} \times 5 + 1 \times V = 0$$

or $$V = \mathbf{-1\ m/s}$$

22. (*a*) Refer to the solution of Example 23 on page 5.22.

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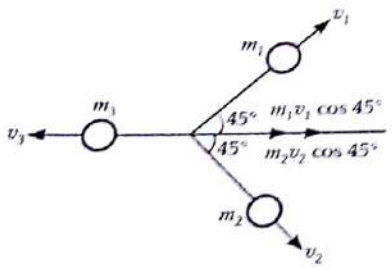
23. *(a)* Clearly, $m_1 = 1\ kg,\ m_2 = 1\ kg,\ m_3 = 3\ kg.$



By conservation of momentum along horizontal direction,

$$m_3v_3 = m_1v_1\cos 45° + m_2v_2\cos 45°$$

$$3v_3 = 1\times 21\times\frac{1}{\sqrt{2}} + 1\times 21\times\frac{1}{\sqrt{2}}$$

$$v_3 = \frac{7}{\sqrt{2}} + \frac{7}{\sqrt{2}}$$

$$= 7\sqrt{2}\ \mathbf{ms^{-1}}.$$

24. *(b)* $F = u\dfrac{dm}{dt} = 60\ \text{kms}^{-1}\times 1\ \text{kgs}^{-1}$

$$= 60000\ \text{kg ms}^{-2}$$

$$= \mathbf{60000\ N}.$$

25. *(c)* $F = u\dfrac{dm}{dt}$

$$210 = 300\times\frac{dm}{dt}$$

$$\frac{dm}{dt} = \frac{210}{300} = \mathbf{0.7\ kg\ s^{-1}}.$$

26. *(c)* Thrust needed to overcome the weight of the rocket,

$$F = mg = u\frac{dm}{dt}$$

$$\therefore \quad \frac{dm}{dt} = \frac{mg}{u} = \frac{600\times 10}{1000} = \mathbf{6\ kg\ s^{-1}}.$$

27. *(b)* Initial acceleration,

$$a = \frac{u}{m_0}\frac{dm}{dt} - g$$

$$20 = \frac{800}{5000}\times\frac{dm}{dt} - 10$$

$$\frac{dm}{dt} = \frac{30\times 50}{8} = \mathbf{187.5\ kg\ s^{-1}}.$$

28. *(b)* Net force = Applied force – Force of friction

$$F = 100 - \mu mg$$

$$= 100 - 0.5\times 10\times 10 = 50\ N$$

$$a = \frac{F}{m} = \frac{50}{10} = \mathbf{5\ ms^{-2}}.$$

29. *(a)* As the block is in the accelerated frame, it experiences a backward force,

$$F = ma = 1\times 5 = 5\ N$$

Limiting friction,

$$f_s^{max} = \mu mg = 0.6\times 1\times 10 = 6\ N.$$

As $\quad F < f_s^{max}$,

the block does not move.

Static friction = Backward force = 5 N.

30. *(b)* Retardation,

$$a = -\mu g = -0.5\times 10 = -5\ ms^{-2}$$

$$u = 72\ \text{km/h} = 20\ ms^{-1}$$

As $\quad v^2 - u^2 = 2as$

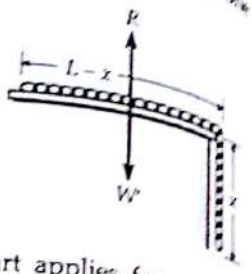
$$\therefore \quad 0^2 - 20^2 = 2(-5)s$$

or $\quad s = \mathbf{40\ m}.$

31. *(a)* Suppose length x of the chain hangs over the edge of the table.

Mass per unit length of the chain $= \dfrac{M}{L}$

Weight of the hanging part, $W = \dfrac{M}{L}xg$



Weight of the hanging part applies force on the remaining part of the chain. The chain will not slide further if the limiting friction balances the weight W.

$$\therefore \quad f_s^{max} = W = \frac{M}{L}xg$$

Normal reaction

= Weight of length $(L-x)$ of the chain

$$R = W' = \frac{M}{L}(L-x)g$$

Now $\quad f_s^{max} = \mu R$

or $\quad \dfrac{M}{L}xg = \mu\dfrac{M}{L}(L-x)g$

or $\quad x = \mu(L-x) \quad$ or $\quad x = \dfrac{\mu L}{1+\mu}$

Given $\mu = 0.25$

$$\therefore \quad \frac{x}{L}\times 100 = \frac{0.25}{1+0.25}\times 100 = \mathbf{20\%}.$$

32. *(d)* Retardation, $a = -\mu g$

As $\quad v = u + at$

$$0 = v - \mu gt \quad \text{or} \quad t = \frac{v}{\mu g}.$$

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33. (*d*) For the equilibrium of block *A* the force of friction must balance the tension in the string.

$$\mu m_A g = T$$

For the equilibrium of block *B* the tension in the string must be equal to its weight

$$m_B g = T$$

Hence $m_B g = \mu m_A g$

$$m_B = \mu m_A = 0.2 \times 2$$

or $= \mathbf{0.4\ kg}$.

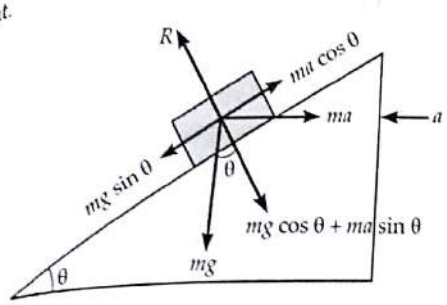
34. (*d*) As the block slides down the inclined plane with a constant speed, the angle of inclination is equal to the angle of repose.

μ = tangent of the angle of repose

$$\mu = \tan\theta$$

or

35. (*a*) When the wedge is given an acceleration *a* towards left, the block receives a reaction *ma* towards right.



For the block to remain at rest,

$$ma\cos\theta = mg\sin\theta$$

$$a = g\tan\theta$$

or

36. (*d*) Refer to the figure of the above problem.

$$ma\cos\theta = mg\sin\theta$$

$$a = g\tan\theta$$

Total reaction of the wedge on the block is

$$R = mg\cos\theta + ma\sin\theta$$

$$= mg\cos\theta + m\cdot\frac{g\sin\theta}{\cos\theta}\cdot\sin\theta$$

$$= \frac{mg(\cos^2\theta + \sin^2\theta)}{\cos\theta} = \frac{mg}{\cos\theta}$$

37. (*b*) Refer to the solution of Problem 16 on page 5.74.

$$\mu = 1 - \frac{1}{n^2}$$

In the given problem, $n = 2$

$$\therefore \quad \mu = 1 - \frac{1}{2^2} = \frac{3}{4} = \mathbf{0.75}.$$

38. (*b*) When milk is churned, cream gets separated due to centrifugal force.

39. (*a*) $v = 36\ \text{km h}^{-1} = 36 \times \frac{5}{18}\ \text{ms}^{-1} = 10\ \text{ms}^{-1}$

$$F = \frac{mv^2}{r}$$

$$= \frac{500 \times 10 \times 10}{50} = \mathbf{1000\ N}.$$

40. (*c*) $T = \frac{mv_{max}^2}{r}$

$$v_{max} = \sqrt{\frac{Tr}{m}} = \sqrt{\frac{25 \times 1.96}{0.25}} = \mathbf{14\ ms^{-1}}.$$

41. (*b*) The mass of the liquid acts at the centre of the tube.

Therefore, $r = \frac{L}{2}$.

Force exerted by the liquid at the other end

= Centrifugal force

$$= Mr\omega^2 = M\left(\frac{L}{2}\right)\omega^2 = \frac{ML\omega^2}{2}.$$

42. (*b*) The tension in the string will be maximum when the particle is at the lowest point *B*.

$$T_B = \text{Weight of particle} + \frac{mv^2}{R}$$

Hence the string will break at point *B* when the average velocity *v* is increased.

43. (*b*) $\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = \vec{0}$ or $\vec{p}_3 = -(\vec{p}_1 + \vec{p}_2)$

$$\therefore \quad m_3 v_3 = \sqrt{p_1^2 + p_2^2} \qquad [\because\ \vec{p}_1 \perp \vec{p}_2]$$

or $$m_3 = \frac{\sqrt{(1\times 12)^2 + (2\times 8)^2}}{4}$$

$$= \mathbf{5\ kg}.$$

44. (*c*) $T = m(g + a)$

or $$a = \frac{T - mg}{m} = \frac{28000 - 20000}{2000}$$

$$= \mathbf{4\ ms^{-2}, upwards}.$$

45. (*c*) For conservation of momentum along *x*-axis,

$$mv + 0 = 4mv_x' \quad \Rightarrow \quad v_x' = \frac{v}{4}$$

For conservation of momentum along *y*-axis,

$$3m \times 2v = 4mv_y' \quad \Rightarrow \quad v_y' = \frac{3}{2}v$$

$$\therefore \quad v' = \frac{v}{4}\hat{i} + \frac{3}{2}v\hat{j}$$

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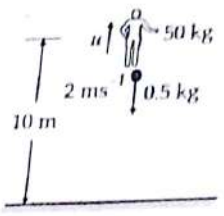
46. (*c*) Impulse $I = MV - (-MV) = 2MV$

47. (*b*) By conservation of momentum,

$$50u + 0.5 \times 2 = 0$$

$$u = -\frac{1}{50}\text{ms}^{-1}\left[\frac{1}{50}\text{ms}^{-1}\text{upward}\right]$$



Time taken by stone to reach the ground $= \frac{10}{2} = 5\text{s}$

Distance moved up by the man

$$= 5 \times \frac{1}{50} = 0.1\text{ m}$$

When the stone reaches the floor, the distance of the man above floor

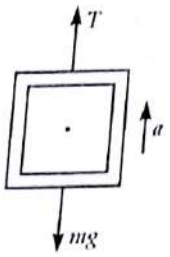
$$= 10 + 0.1 = \mathbf{10.1\ m}$$

48. (*c*) $T - mg = ma$

$$T = m(g + a)$$

$$= (940 + 60) \times (10 + 1)$$

$$= \mathbf{11000\ N}$$



49. (*b*) $u = 2\text{ m/s}$, $\mu = 0.5$, $g = 10\text{ ms}^{-2}$

$$a = \frac{F}{m} = \frac{\mu mg}{m}$$

$$= 0.5 \times 10 = 5\text{ m/s}^2$$

Retardation, $a = -5\text{ m/s}^2$

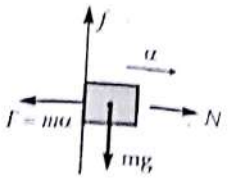
$$v^2 - u^2 = 2as$$

$$\therefore \quad 0^2 - 2^2 = 2(-5) \times s$$

or $$s = \mathbf{0.4\ m}$$

50. (*c*) Force of friction,

$$f = \mu N = \mu ma$$



To prevent the block from falling,

$$\mu ma \geq mg$$

or $$a \geq \frac{m}{g}$$

51. (*c*) The coin will revolve with the record if

Centripetal force ≤ Force of friction

$$mr\omega^2 \leq \mu mg$$

$$r \leq \frac{\mu g}{\omega^2}$$

Delhi PMT and VMMC Entrance Exam

[Similar Questions]

1. No force is required for
   (*a*) an object moving in straight line with constant velocity
   (*b*) an object moving in circular motion
   (*c*) an object moving with constant acceleration
   (*d*) an object moving in elliptical path. [DPMT 07]

2. Of the four basic forces, maximum and minimum range is for
   (*a*) gravitational, nuclear
   (*b*) weak, electromagnetic
   (*c*) gravitational, electromagnetic
   (*d*) electromagnetic, gravitational. [DPMT 07]

3. If a body is in equilibrium under a set of non-collinear forces, then the minimum number of forces has to be
   (*a*) four (*b*) two
   (*c*) three (*d*) five [DPMT 05]

4. If maximum and minimum values of the resultant of two forces acting at a point are 7 N and 3 N respectively, the smaller force is equal to
   (*a*) 4 N (*b*) 5 N
   (*c*) 3 N (*d*) 2 N [DPMT]

5. What is the momentum of a 100000 kg truck whose velocity is 20 m/s ?
   (*a*) 2×10^5 kg m/s (*b*) 1×10^5 kg m/s
   (*c*) 4 kg m/s (*d*) none of these. [DPMT 05]

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8. The force acting on a body of mass 10 kg is $(3t^2 + 4)$ N. If the body is initially at rest, then velocity at the end of 20 sec will be
(a) $5\sqrt{6}$ (b) $4\sqrt{2}$
(c) $10\sqrt{5}$ (d) $5\sqrt{3}$ [DPMT 98]

9. The mass of a ship is 2×10^7 kg. On applying a force of 2×10^5 N, it is displaced through 25 m. The speed of ship is
(a) 2.5 m/s (b) 3 m/s
(c) 4.7 m/s (d) 12.5 m/s [DPMT 98]

10. An army vehicle of mass 1000 kg is moving with a velocity of 40 m/s and is acted upon by a forward force of 8000 N due to the engine and a retarding force of 500 N due to friction. What will be its velocity after 10 s ?
(a) 5 m/s (b) 15 m/s
(c) 10 m/s (d) 20 m/s [DPMT 97]

11. A particle of mass 4 kg is acted upon by steady force of 4 N. Distance travelled by the particle in 4 sec is
(a) 16 m (b) 2 m
(c) 8 m (d) 4 m [VMMC 02]

12. If two balls, each of mass 0.06 kg, moving in opposite directions with speed 4 m/s, collide and rebound with same speed, then impulse imparted to each ball due to other is
(a) 0.48 kg m/s (b) 0.81 kg m/s
(c) 0.54 kg m/s (d) 0.92 kg m/s [DPMT 96]

13. A weight w is suspended from the midpoint of a rope, whose ends are at the same level. In order to make the rope perfectly horizontal, the force applied to each of its ends must be
(a) less than w (b) equal to w
(c) equal to $2w$ (d) infinitely large
[DPMT 96]

14. The force on a rocket moving with a velocity of 300 m/s is 210 N. Then the rate of combustion of the fuel will be
(a) 3.5 kg/sec (b) 2.1 kg/sec
(c) 1.4 kg/sec (d) 0.7 kg/sec
[DPMT 98 ; VMMC 04]

15. A man of mass 75 kg is standing in an elevator which is moving with an acceleration of 5 m/s^2 in upward direction. The apparent weight of the man will be (g = 10 m/s^2)
(a) 1125 N (b) 1375 N
(c) 1250 N (d) 1425 N [DPMT 98]

16. A boy of 50 kg is standing in a lift moving down with an acceleration 9.8 m/s^2. The apparent weight of the boy is
(a) zero (b) (50 × 9.8)N
(c) 50 N (d) $\frac{50}{9.8}$ N [DPMT 96]

17. A man is standing on a spring platform. Reading of spring balance is 60 kg wt. If man jumps outside from the platform, then reading of spring balance will
(a) increase (b) become zero
(c) remain same (d) first (a) and then (b)
[DPMT 95]

18. A lift accelerates upward, then decelerates and stops at higher floor. The apparent weight of the body in the later parts of its motion is
(a) more than actual weight
(b) less than actual weight
(c) no change
(d) equal to actual weight. [DPMT 07]

19. A force of 1000 N is applied on a body of mass 100 kg moving with a velocity of 5 m/s. How much time does it require to acquire a velocity of 25 m/s ?
(a) 2 sec (b) 6 sec
(c) 4 sec (d) 8 sec [DPMT]

20. A ball of mass 25 g, moving with a velocity of 2 ms^{-1} is stopped within 5 cm. The average resistance offered to the ball is
(a) 10 N (b) 5 N
(c) 2 N (d) 1 N [DPMT 05]

21. A 30 g bullet travelling initially at 500 m/s penetrates 12 cm into wooden block. The average force exerted will be
(a) 31250 N (b) 41250 N
(c) 31750 N (d) 3040 N
[DPMT 03 ; VMMC 05]

22. The effective acceleration of a body, when thrown upward with acceleration (a) is
(a) $(a + g)$ (b) $\sqrt{a^2 + g^2}$
(c) $(a - g)$ (d) $\sqrt{a^2 - g^2}$ [DPMT 93]

23. Three different objects of masses m_1, m_2 and m_3 are allowed to fall from rest and from the same point O along three different frictionless paths. The speeds of

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the three objects on reaching the ground will be in the ratio of

(a) $m_1 : m_2 : m_3$ (b) 1 : 1 : 1

(c) $\frac{1}{m_1} : \frac{1}{m_2} : \frac{1}{m_3}$ (d) $m_1 : 2m_2 : 3m_3$ [DPMT 92]

22. A ball strikes a bat with velocity v. The ball has mass m and after striking it retraces its path. What is the impulse imparted by the bat ?

(a) $3mv$ (b) mv

(c) zero (d) $2mv$. [DPMT 2K]

23. A bullet is fired from gun. The force on the bullet is given by $F = 600 - 2 \times 10^5 t$, where F is in newton and t in second. The force on the bullet becomes zero as soon as it leaves the barrel. What is the average impulse imparted to the bullet ?

(a) 1.8 Ns (b) zero

(c) 9 Ns (d) 0.9 Ns [VMMC 03]

24. Which of the following is the dimension of coefficient of friction ?

(a) $[MLT^{-2}]$ (b) $[M^0L^0T^0]$

(c) $[M^2LT^{-2}]$ (d) $[M^2LT]$ [DPMT 96, 07]

25. If μ_s is coefficient of static friction and μ_k is coefficient of kinetic friction, then

(a) there is no relation between μ_s and μ_k

(b) generally $\mu_s > \mu_k$ (c) generally, $\mu_s = \mu_k$

(d) generally $\mu_s < \mu_k$. [DPMT 07]

26. An iron block of sides 5 cm × 8 cm × 15 cm has to be pushed along the floor. The force required will be minimum when the surface in contact with ground is

(a) 8 cm × 15 cm surface

(b) 8 cm × 5 cm surface

(c) 5 cm × 15 cm surface

(d) force is the same for all surfaces [DPMT 97]

27. A truck and a car are moving with equal velocity. On applying brakes, both will stop after certain distance, then

(a) truck will cover less distance before stopping

(b) car will cover less distance before stopping

(c) both will cover equal distance

(d) none of these. [DPMT 04]

28. A lift is moving downwards with an acceleration equal to acceleration due to gravity. A body of mass m kept on the floor of the lift is pulled horizontally. If the coefficient of friction is μ, then the frictional resistance offered by the body is

(a) mg (b) μmg

(c) $2\mu mg$ (d) zero. [DPMT 05]

29. A cylinder of mass 10 kg is rolling on a plane with an initial velocity of 10 m/s. If coefficient of friction between surface and cylinder is 0.5, then before stopping, the cylinder will cover a distance of

(a) 10 m (b) 5 m

(c) 7.5 m (d) 2.5 m [DPMT 06]

30. A body of mass 10 kg lies on a rough horizontal surface. When a horizontal force of F newtons acts on it, it gets an acceleration of 5 m/s^2 and when the horizontal force is doubled, it gets an acceleration of 18 m/s^2. The coefficient of friction between the body and the horizontal surface (assume $g = 10$ m/s^2) is

(a) 0.2 (b) 0.6

(c) 0.4 (d) 0.8 [DPMT 07]

31. If a ladder weighing 250 N is placed against a smooth vertical wall having coefficient of friction between it and floor 0.3, then what is the maximum force of friction available at the point of contact between the ladder and the floor ?

(a) 75 N (b) 35 N

(c) 50 N (d) 25 N [DPMT 08]

32. A body of mass 2 kg begins to slide down a plane inclined at an angle of 45° with the horizontal. The force of friction will be

(a) 19.6 sin 45° (b) 9.8 sin 45°

(c) 19.6 cos 45° (d) 9.8 cos 45° [DPMT 09]

33. If the coefficient of friction of a surface is $\sqrt{3}$, then the angle of inclination of the plane to make a body on it just to slide is

(a) 30° (b) 60°

(c) 45° (d) 90° [DPMT 10]

34. A body of mass 100 g is sliding on an inclined plane with inclination of 30°. What is the frictional force experienced, if the coefficient of friction (μ) is 1/7

(a) $17 \times \sqrt{3}$ N (b) $17 \times 3 \times \frac{1}{\sqrt{2}}$ N

(c) $17 \times \sqrt{3} \times \frac{1}{2}$ N (d) $17 \times \sqrt{2} \times \frac{1}{\sqrt{3}}$ N [DPMT 93]

35. If the coeffici
45° is 0.5, then accel
is

(a) 4.9 m/s^2

(c) 9.8 m/s^2

36. A body tak
inclined plane of
plane is made r
coefficient of fricti

(a) $\frac{3}{4}\tan\theta$

(c) $\frac{1}{4}\tan\theta$

37. A block
horizontal distar
between their su
friction is

(a) 98 J

(c) 72 J

38. A perso
constant velocit
ball falls

(a) exactly i

(b) outside t

(c) in the ca

(d) in the ca

39. It is easi
road. This stat

(a) not pos

(c) false

40. Two ra
in circles of ra
such that each
The ratio of t
car is

(a) 1 : 1

(c) $m_1 : m_2$

41. If a
revolution
cycle is

(a) π m/s

(c) 4π m

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35. If the coefficient of friction of a plane inclined at 45° is 0.5, then acceleration of a body sliding freely on it is

(a) 4.9 m/s^2 (b) $\frac{9.8}{\sqrt{2}}$ m/s^2

(c) 9.8 m/s^2 (d) $\frac{9.8}{2\sqrt{2}}$ m/s^2 [DPMT 92]

36. A body takes time t to reach the bottom of an inclined plane of angle θ with the horizontal. If the plane is made rough, time taken now is $2t$. The coefficient of friction of the rough surface is

(a) $\frac{3}{4}\tan\theta$ (b) $\frac{2}{3}\tan\theta$

(c) $\frac{1}{4}\tan\theta$ (d) $\frac{1}{2}\tan\theta$ [DPMT 05]

37. A block of mass 50 kg just slides over a horizontal distance of 1 m. If coefficient of friction between their surfaces is 0.2, the work done against friction is

(a) 98 J (b) 56 J

(c) 72 J (d) 34 J [DPMT 95]

38. A person sitting in an open car moving at constant velocity throws a ball vertically into air, the ball falls

(a) exactly in the hand of thrower

(b) outside the car

(c) in the car ahead of the person

(d) in the car behind the person

[DPMT 92 ; VMMC 04]

39. It is easier to roll a barrel than to pull it along the road. This statement is

(a) not possible (b) uncertain

(c) false (d) true [DPMT 05]

40. Two racing cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2 respectively. Their speeds are such that each car makes a complete circle in same time t. The ratio of the angular speed of the first to the second car is

(a) 1 : 1 (b) $r_1 : r_2$

(c) $m_1 : m_2$ (d) $m_1 m_2 : r_1 r_2$ [DPMT 94]

41. If a cycle wheel of radius 4 m completes one revolution in two seconds, then acceleration of the cycle is

(a) π m/s^2 (b) $2\pi^2$ m/s^2

(c) 4π m/s^2 (d) $4\pi^2$ m/s^2 [DPMT 94]

42. A particle moves in a circular orbit under the action of a central attractive force inversely proportional to distance r. The speed of the particle is

(a) proportional to r^2 (b) independent of r

(c) proportional to r (d) proportional to $1/r$

[DPMT 03]

43. A stone of mass m tied to a string of length l is rotated in a circle with the other end of the string as the centre. The speed of the stone is v. If the string breaks, the stone will

(a) move towards the centre

(b) move away from the centre

(c) move along tangent

(d) stop [VMMC 06]

44. A body of mass 1 kg tied to one end of string is revolved in a horizontal circle of radius 0.1 m with a speed of 3 revolutions/sec, assuming the effect of gravity is negligible, then linear velocity, acceleration and tension in the string will be

(a) 2.88 m/s, 45.5 m/s^2, 45.5 N

(b) 1.88 m/s, 35.5 m/s^2, 35.5 N

(c) 3.88 m/s, 55.5 m/s^2, 55.5 N

(d) none of these. [DPMT 07]

45. For a car not to turn safely on a curved road

(a) speed is slow

(b) distance between tyres is large

(c) centre of gravity for car is low

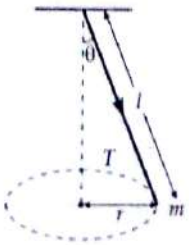
(d) low friction force. [DPMT 07]

46. The maximum speed with which a car can be driven round a curve of radius 18 m without skidding (when $g = 10\,m/s^2$ and the coefficient of friction between rubber tyres and the roadway is 0.2) is

(a) 36.0 km/h (b) 21.6 km/h

(c) 18.0 km/h (d) 14.4 km/h [DPMT 94]

47. A string of length l fixed at one end carries a mass 'm' at the other end. The string makes $\frac{2}{\pi}$ rev/s



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around the horizontal axis through the fixed end as shown in the figure. The tension in the string is

(a) 16 *ml* (b) 4 *ml*
(c) 8 *ml* (d) 2 *ml* [DPMT 04]

48. A car of mass 400 kg is pulling a coach of mass 300 kg with a force of 4500 N. If the coefficient of friction is 0.001, what is tension ?

(a) 2100 N (b) 2126 N
(c) 1929 N (d) 2750 N [DPMT 08]

49. Masses m and M on a pulley move 0.6 m in 4 s. What is the ratio m/M ?

(a) $\frac{55}{57}$ (b) $\frac{49}{50}$
(c) $\frac{57}{55}$ (d) $\frac{117}{113}$ [DPMT 08]

50. A block is lying static on the floor. The maximum value of static frictional force on the block is 10 N. If a horizontal force of 8 N is applied to the block, what will be the frictional force on the block ?

(a) 2 N (b) 18 N
(c) 8 N (d) 10 N [DPMT 09]

51. Two iron blocks of equal masses but with doule surface area slide down an inclined plane with friction coefficient μ. If the first block with surface area A experiences a friction force f, then the second block with surface area 2 A will experience a frictional force

(a) $f/2$ (b) f
(c) $2f$ (d) $4f$ [DPMT 09]

52. A rope of mass 0.1 kg is connected at the same height of two opposite walls. It is allowed to hang under its own weight. At the contact point between the rope and the wall, the rope makes an angle $\theta = 10°$ with respect to horizontal. The tension in the rope at its midpoint between the walls is

(a) 2.78 N (b) 2.56 N
(c) 2.82 N (d) 2.71 N [DPMT 09]

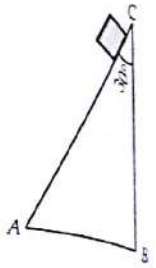
53. On the centre of frictionless table a small hole is made, through which a weightless string of length $2l$ is inserted. On the two ends of the string two balls of the same mass m are attached. Arrangement is made in such a way that half of the string is on the table top and half is hanging below. The ball on the table top is made to move in a circular path with a constant speed v. What is the centripetal acceleration of the moving ball ?

(a) mvl (b) g
(c) zero (d) $2mvl$ [DPMT 09]

54. An object is moving on a plane surface with uniform velocity 10 ms^{-1} in presence of a force 10 N. The frictional force between the object and the surface is

(a) 1 N (b) −10 N
(c) 10 N (d) 100 N [DPMT 2011]

55. A body of mass M starts sliding down on the inclined plane where the critical angle is $\angle ACB = 30°$ as shown in figure. The coefficient of kinetic friction will be



(a) $Mg/\sqrt{3}$ (b) $\sqrt{3}\,Mg$
(c) $\sqrt{3}$
(d) none of these [DPMT 2011]

56. In non-inertial frame, the second law of motion is written as

(a) $F = ma$ (b) $F = ma + F_p$
(c) $F = ma - F_p$ (d) $F = 2ma$

where F_p is a pseudo-force while a is the acceleration of the body relative to non-inertial frame. [DPMT 2011]

Answers and Explanations

1. (a) No force is required for an object moving in straight line with constant velocity or for non-accelerated motion.

2. (a) Gravitational force has a maximum range while nuclear force has a minimum range.

3. (c) A minimum of three non-collinear forces can be in equilibrium.

4. (d) $P + Q = 7$ N

and $P - Q = 3$ N

$\therefore$ $P = 5$ N, $Q = 2$ N

Thus smaller force is equal to 2 N.

5. (d) $p = mv = 100000 \times 20$

$= 2 \times 10^6$ kg m/s.

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6. (a) $\vec{v} = \vec{u} + \vec{a}t = 0 + \frac{\vec{F}}{m}.t$

$$= \left(\frac{2\hat{i} + \hat{j} - \hat{k}}{10}\right) \times 20$$

$$= 4\hat{i} + 2\hat{j} - 2\hat{k}$$

$$|\vec{v}| = \sqrt{16 + 4 + 4}$$

$$= \sqrt{24} = 2\sqrt{6} \text{ ms}^{-1}.$$

7. (a) $a = \frac{F}{m} = \frac{25 \times 10^5}{2 \times 10^7} = \frac{1}{8} \text{ ms}^{-2}$

$$v^2 - u^2 = 2as$$

$$v^2 - 0^2 = 2 \times \frac{1}{8} \times 25 = \frac{25}{4}$$

$$v = \frac{5}{2} = 2.5 \text{ ms}^{-1}.$$

8. (b) Net forward force = 1000 – 500 = 500 N

$$a = \frac{F}{m} = \frac{500}{1000} = \frac{1}{2} \text{ ms}^{-2}$$

$$v = u + at = 10 + \frac{1}{2} \times 10 = 15 \text{ ms}^{-1}.$$

9. (c) $a = \frac{F}{m} = \frac{4}{4} = 1 \text{ ms}^{-2}$

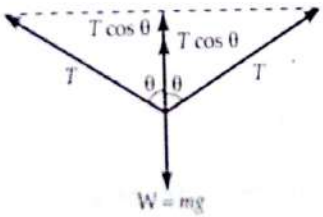
$$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 1 \times 4^2 = 8 \text{ m}.$$

10. (a) Impulse imparted = Change in momentum

$$= mv - (-mv) = 2mv$$

$$= 2 \times 0.06 \times 4 = \mathbf{0.48 \text{ kg m/s}}.$$

11. (d)



$$W = 2T\cos\theta$$

$$T = \frac{W}{2\cos\theta}$$

For string to be horizontal,

$$\theta \to 90^\circ \quad \text{and} \quad T \to \infty.$$

12. (d) $F = u\frac{dm}{dt}$

$$\therefore \quad 210 = 300 \times \frac{dm}{dt}$$

$$\therefore \quad \frac{dm}{dt} = \frac{210}{300} = \mathbf{0.7 \text{ kg/sec}}.$$

13. (a) Apparent weight

$$= m(g + a)$$

$$= 75(10 + 5) = \mathbf{1125 \text{ N}}.$$

14. (a) Apparent weight $= m(g - a)$

$$= 50(98 - 98) = 0$$

15. (d) To jump outside the platform, the man pushes the platform, so the reading of the spring balance first increases and then becomes zero.

16. (a) The apparent weight of the body is more than actual weight till the lift stops and then the weight becomes equal to mg.

17. (a) $a = \frac{1000}{100} = 10 \text{ ms}^{-2}$

$$t = \frac{v - u}{a} = \frac{25 - 5}{10} = \mathbf{2 \text{ s}}.$$

18. (d) $v^2 - u^2 = 2as$

$$0^2 - 2^2 = 2a \times 0.05$$

$$a = -\frac{2}{0.05} = 40 \text{ ms}^{-2}$$

Average resistance $= ma$

$$= \frac{25}{1000} \times 40 = \mathbf{1 \text{ N}}.$$

19. (a) $v^2 - u^2 = 2as$

$$0^2 - (500)^2 = 2a \times 0.12$$

$$a = -\frac{500 \times 500}{2 \times 0.12}$$

Average force $= ma = \frac{30}{1000} \times \frac{500 \times 500}{2 \times 0.12}$

$$= 31250 \text{ N}.$$

20. (c) Acceleration a acts upward while g acts of downwards

$\therefore$ Effective acceleration $= a - g$.

21. (b) $v^2 - 0^2 = 2gh$

$$v = \sqrt{2gh}$$

This velocity does not depend on mass.

22. (d) Impulse = Change in momentum

$$= mv - (-mv) = \mathbf{2mv}.$$

23. (d) Time t for which the force acts is given by

$$F = 600 - 2 \times 10^5 t = 0$$

$$t = \frac{600}{2 \times 10^5} = 3 \times 10^{-3} \text{ s}$$

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$$\text{Impulse} = \int_0^t F\,dt = \int_0^t (600 - 2\times10^5 t)\,dt$$

$$= [600t - 10^5 t^2]_0^{3\times10^{-3}}$$

$$= 600\times3\times10^{-3} - 10^5\times9\times10^{-6}$$

$$= 1.8 - 0.9 = \mathbf{0.9\ Ns}.$$

24. (*b*) Being the ratio of two forces, coefficient of friction is a dimensionless quantity.

25. (*b*) Generally, $\mu_s > \mu_k$.

26. (*d*) The force of friction does not depend on the area of contact between two surfaces.

27. (*b*) Being lighter than truck, the car has less kinetic energy. On applying brakes with the same force, the car will cover less distance before coming to rest.

28. (*d*) Apparent weight of the body in the lift

$$= m(g - g) = 0$$

Consequently, normal reaction and hence force friction are both zero.

29. (*a*) Work done against friction

= Initial K.E. of the cylinder

$$f.s = \frac{1}{2}mv^2$$

or $$\mu mg.s = \frac{1}{2}mv^2$$

$$\therefore \quad s = \frac{v^2}{2\mu g} = \frac{10\times10}{2\times0.5\times10} = \mathbf{10\ m}.$$

30. (*d*) If f is the force friction, then

$$F - f = m\times5$$

$$2F - f = m\times18$$

On solving,

$$f = 8\ \mathbf{m}$$

$$\mu = \frac{f}{R}$$

$$= \frac{8m}{mg} = \frac{8}{10} = \mathbf{0.8}.$$

31. (*a*) $f = \mu R = \mu mg$

$$= 0.3\times250 = \mathbf{75\ N}.$$

32. (*a*) When the body just slides down on inclined plane,

$$f = mg\sin\theta = 2\times9.8\sin45°$$

$$= \mathbf{19.6\ sin\ 45°}.$$

33. (*b*) If ϕ is the angle of repose, then

$$\tan\phi = \sqrt{3}$$

or $$\phi = \mathbf{60°}.$$

34. (*c*) $f = \mu R = \mu mg\cos\theta$

$$= 1.7\times\frac{100}{1000}\times10\times\cos30°$$

$$= \mathbf{1.7}\times\frac{\sqrt{3}}{2}\ \mathbf{N}.$$

35. (*d*) $a = g(\sin\theta - \mu\cos\theta)$

$$= 9.8(\sin45° - 0.4\cos45°)$$

$$= 9.8\left(\frac{1}{\sqrt{2}} - \frac{1}{2}\cdot\frac{1}{\sqrt{2}}\right)$$

$$= \frac{9.8}{2\sqrt{2}}\ \text{ms}^{-2}.$$

36. (*a*) When there is no friction, $a = g\sin\theta$

When there is friction, $a' = g(\sin\theta - \mu\cos\theta)$

If the length of the inclined plane is d, then

$$d = \frac{1}{2}at^2 = \frac{1}{2}a'(2t)^2$$

or $$a = 4a'$$

or $$g\sin\theta = 4g(\sin\theta - \mu\cos\theta)$$

$$\sin\theta = 4\sin\theta - 4\mu\cos\theta$$

$$4\mu\cos\theta = 3\sin\theta$$

$$\mu = \frac{3}{4}\tan\theta$$

37. (*a*) $W = f\times s = \mu mg\times s$

$$= 0.2\times50\times9.8\times1$$

$$= 98\ \text{J}$$

38. (*a*) Both the person and the ball share the horizontal velocity of the car. Hence the ball falls exactly in the hand of the thrower.

39. (*d*) As rolling friction is less than the sliding friction, it is easier to roll a barrel than to push it.

40. (*a*) $\omega = \frac{2\pi}{T}$

For both cars, period of revolution T is same, so their angular speed ω is also same.

41. (*d*) $\omega = \frac{2\pi}{T} = \frac{2\pi}{2\text{s}} = \pi\ \text{rad s}^{-1}$

$$a = r\omega^2 = 4m(\pi\ \text{rad s}^{-1})^2$$

$$= \mathbf{4\pi^2\ m/s^2}.$$

42. (*b*) Centripetal force,

$$\frac{mv^2}{r} \propto \frac{1}{r}$$

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$$\frac{mv^2}{r} = \frac{k}{r}$$

or $$v = \sqrt{\frac{k}{m}} = \text{constant}$$

or

Clearly, speed v is independent of r.

43. (*c*) When the string breaks, the centripetal force becomes zero. The stone moves along the tangent to the circular path *i.e.*, in the direction of velocity.

44. (*b*) Here $m = 1$ kg, $r = 0.1$ m, $\nu = 3$ rps

$$\omega = 2\pi\nu = 2\pi \times 3 = 6\pi \text{ rad s}^{-1}$$

Linear velocity,

$$v = r\omega = 0.1 \times 6 \times 3.14 = 1.88 \text{ ms}^{-1}$$

Linear acceleration,

$$a = r\omega^2 = 0.1 \times (6\pi)^2 = 35.5 \text{ ms}^{-2}.$$

Tension in the string,

$$T = mr\omega^2$$
$$= 1 \times 0.1 \times (6\pi)^2$$
$$= \mathbf{35.5\ N}.$$

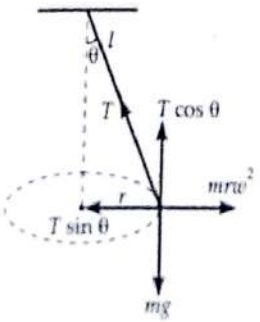
45. (*d*) If friction is low, the car will skid on the road.

46. (*b*) $v_{max} = \sqrt{\mu rg} = \sqrt{0.2 \times 18 \times 10} = \sqrt{36} = 6$ m/s

$$= 21.6 \text{ km/h}$$

47. (*c*) For equilibrium,

$$T\sin\theta = mr\omega^2$$
$$T \times \frac{r}{l} = mr\left(2\pi \cdot \frac{2}{\pi}\right)^2$$
$$T = \mathbf{16\ ml}.$$



48. (*c*) Friction,

$$f = \mu(m_1 + m_2)g$$
$$= 0.001(400 + 300)\,10 = 7 \text{ N}$$

For car :

$$4500 - T - 4 = 400 \times a$$

For coach : $$T - 3 = 300 \times a$$

$$\therefore \quad 4500 - 7 = 700a$$

or $$a = \frac{4493}{700} \text{ ms}^{-2}$$

Hence $$T = 300a + 3$$
$$= 300 \times \frac{4493}{700} + 3$$
$$= \mathbf{1929\ N}.$$

49. (*b*) $$a = \frac{M-m}{M+m} \cdot g$$

$s = 0.6$ m, $t = 4$ s

As $$s = ut + \frac{1}{2}at^2$$

$$\therefore \quad 0.6 = 0 + \frac{1}{2} \cdot \frac{M-m}{M+m} \cdot g \cdot 4^2$$

or $$0.6(M+m) = 80(M-m)$$

or $$0.6\left(1 + \frac{m}{M}\right) = 80\left(1 - \frac{m}{M}\right)$$

or $$80.6\frac{m}{M} = 79.4$$

or $$\frac{m}{M} = 0.985$$

or $$\frac{m}{M} = \frac{98.5}{100.0}$$
$$= \frac{\mathbf{49}}{\mathbf{50}}.$$

50. (*b*) In the stationary state,

Force of friction = Applied force

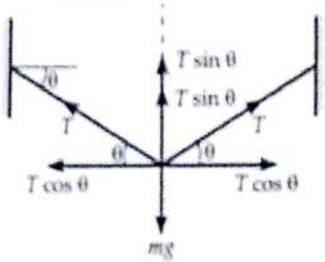
$$= \mathbf{8\ N}.$$

51. (*b*) Force of friction does not depend on the area of contact.

52. (*c*) $$\theta = 10° = \frac{\pi}{180} \times 10 \text{ rad}$$
$$= \frac{\pi}{18} \text{ rad}$$

$$2T\sin\theta = mg$$

or $$T = \frac{mg}{2\sin\theta}$$



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For small θ,

$$\sin\theta \simeq \theta$$

$$\therefore \quad T = \frac{0.1 \times 10}{2 \times (\pi/18)}$$

$$= 2.82 \text{ N}.$$

53. (*b*) For ball on the table,

$$\frac{mv^2}{l} = T$$

For the hanging ball,

$$mg = T$$

$$\therefore \quad \frac{mv^2}{l} = mg$$

or $$a_c = \frac{v^2}{l} = g.$$

54. (*b*) As the object is moving with a uniform velocity, the net force on it is zero.

55. (*c*) $\mu = \tan\angle BAC = \tan 60° = \sqrt{3}$

56. (*c*) A pseudo-force F_p appears on the body in the opposite direction of the accelerating non-inertial frame.

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CHAPTER 6

WORK, ENERGY AND POWER

6.1 WORK

1. *When is the work said to be done ? Give some examples.*

Work. *Work is said to be done whenever a force acts on a body and the body moves through some distance in the direction of the force.* Thus work is done on a body only if the following two conditions are satisfied :

(*i*) A force acts on the body.

(*ii*) The point of application of the force moves in the direction of the force.

Examples. Work is done when a horse pulls a cart, an engine pulls a train, a man goes up a hill, etc.

6.2 WORK DONE BY A CONSTANT FORCE

2. *How is work done measured when (i) the force acts along the direction of motion of the body and (ii) the force is inclined to the direction of motion of the body ?*

(*i*) Measurement of work done when the force acts along the direction of motion. As shown in Fig. 6.1, a force $\vec{F}$ displaces a body through a distance s parallel to the line of action of the force or in the direction of force.

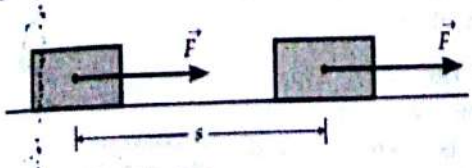

Fig. 6.1 Work done, when force and displacement are in same directions.

Work done = Force × distance moved in the direction of force

$$W = Fs$$

(*ii*) Measurement of work done when force and displacement are inclined to each other.

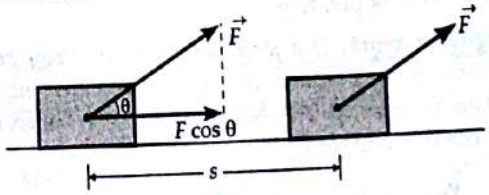

Fig. 6.2 Work done, when force is inclined to displacement.

Work done = Component of force in the direction of displacement × magnitude of displacement

$$W = F \cos\theta \times s$$

or $$W = Fs \cos\theta$$

or $$W = \vec{F} . \vec{s}$$

Thus work done is the dot product of force and displacement vectors. Hence work is a scalar quantity.

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6.3 NATURE OF WORK DONE IN DIFFERENT SITUATIONS

3. *What is meant by positive work, negative work and zero works ? Give examples of each type.*

Nature of work done. Although work done is a scalar quantity, yet its value may be positive, negative or zero, as discussed below.

Positive work. *If a force acting on a body has a component in the direction of the displacement, then the work done by the force is positive.* As shown in Fig. 6.3, when θ is acute, $\cos\theta$ is positive.

$\therefore$ $W = Fs\cos\theta$ = a positive value

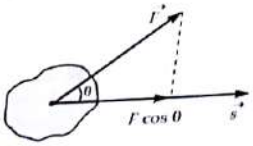


Fig. 6.3 Positive work ($\theta < 90°$).

Examples :

(i) When a body falls freely under gravity ($\theta = 0°$), the work done by the gravity is positive.

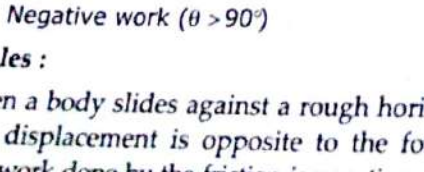
(ii) When a horse pulls a cart, the applied force and displacement are in the same direction, the work done by the horse is positive.

(iii) When a gas filled in a cylinder fitted with a movable piston is allowed to expand, the work done by the gas is positive, because the force due to gas pressure and displacement act in the same direction.

(iv) When a spring is stretched, both the stretching force and the displacement act in the same direction. So work done is positive.

Negative work. *If a force acting on the body has a component in the opposite direction of displacement, the work done is negative.* As shown in Fig. 6.4, when θ is *obtuse*, $\cos\theta$ is negative.

$\therefore$ $W = Fs\cos\theta$ = a negative value



. 6.4 Negative work ($\theta > 90°$)

Examples :

) When a body slides against a rough horizontal ce, its displacement is opposite to the force of . The work done by the friction is negative.

(ii) When brakes are applied to a moving vehicle, the work done by the braking force is negative. This is because the braking force and the displacement act in opposite directions.

(iii) When a body is lifted, the work done by the gravitational force is negative. This is because the gravitational force acts vertically downwards while the displacement is in the vertically upward direction.

(iv) When a positive charge is moved towards another positive charge, the work done by the force of repulsion (between them) is negative because displacement is in the opposition direction of repulsive force.

Zero work. *Work done by force is zero if the body gets displaced along a direction perpendicular to the direction of the applied force. Also, the work done is zero if $\vec{F}$ or $\vec{s}$ or both are zero.*

Examples :

(i) For a body moving in a circular path, the centripetal force and displacement are perpendicular to each other, as shown in Fig. 6.5. So the work done by *the centripetal force is zero.*

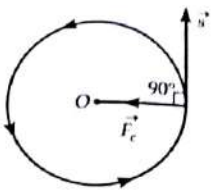


Fig. 6.5 Work done by centripetal force.

(ii) When a coolie walks on a horizontal platform with a load on his head, he applies a force on it in the upward direction equal to its weight. The displacement of the load is along the horizontal direction. Thus the angle between $\vec{F}$ and $\vec{s}$ is 90°. So $W = Fs\cos 90° = 0$ *i.e.*, the work done by the coolie on the load is zero.

(iii) As shown in Fig. 6.6, the tension in the string of a simple pendulum is always perpendicular to its displacement. So the work done by the tension is zero.

(iv) The work done in pushing an immovable stone is zero, because displacement of the stone is zero.

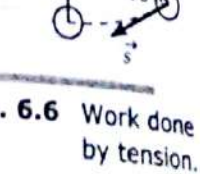


Fig. 6.6 Work done by tension.

6.4 DIMENSIONS AND UNITS

4. *Write the dimensional formula of*

Dimensions of work.

As work done = Force × distance

$[W] = MLT^{-2} \times L$

$\therefore$ $[W] = [ML^2T^{-2}]$

or This is the dimensional formula

5. *Define the various absolute units of work. Write relations betwe*

Units of work. As $W = Fs$

$\therefore$ 1 unit work = 1 unit force ×

Thus *unit work is the amount of force displaces a body through a direction of the force.*

A. Absolute units of work. *one absolute unit if an absolute unit through a unit distance in the dire*

(i) Joule. It is the absolute un after British physicist James Pre *One joule of work is said to be newton displaces a body through its own direction.*

1 joule = 1 newton ×

(ii) Erg. It is the absolu system. *One erg of work is sa dyne displaces a body through its own direction.*

1 erg = 1 dyne × 1

Relation between joul

1 joule = 1 newto

$= 10^5$ dyn

or 1 joule = 10^7 erg

B. Gravitational unit *one gravitational unit if a g body through a unit distar*

(i) Kilogram metre work in SI. *One kilogra when a force of one kilog one metre in its own di*

1 kg m = 1 kg

or 1 kg m = 9.8

(ii) Gram centim in CGS system. *One done when a force through one centimet*

1 g cm = 1

or 1 g cm =

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6.4 DIMENSIONS AND UNITS OF WORK

4. Write the dimensional formula of work.

Dimensions of work.

As work done = Force × distance

$$[W] = MLT^{-2} \times L$$

$$\therefore \quad [W] = [ML^2T^{-2}]$$

or This is the dimensional formula of work.

5. Define the various absolute and gravitational units of work. Write relations between them.

Units of work. As $W = Fs$

1 unit work = 1 unit force × 1 unit distance

∴ *Thus unit work is the amount of work done when a unit force displaces a body through a unit distance in the direction of the force.*

A. Absolute units of work. *Work done is said to be one absolute unit if an absolute unit of force displaces a body through a unit distance in the direction of the force.*

(*i*) **Joule.** It is the absolute unit of work in SI, named after British physicist *James Prescott Joule* (1811 – 1869). *One joule of work is said to be done when a force of one newton displaces a body through a distance of one metre in its own direction.*

1 joule = 1 newton × 1 metre or 1 J = 1 Nm

(*ii*) **Erg.** It is the absolute unit of work in CGS system. *One erg of work is said to be done if a force of one dyne displaces a body through a distance of one centimetre in its own direction.*

1 erg = 1 dyne × 1 cm

Relation between joule and erg.

1 joule = 1 newton × 1 metre

$= 10^5$ dyne × 10^2 cm = 10^7 dyne cm

or 1 joule = 10^7 erg

B. Gravitational units of work. *Work is said to be done one gravitational unit if a gravitational unit of force displaces a body through a unit distance in the direction of the force.*

(*i*) **Kilogram metre.** It is the gravitational unit of work in SI. *One kilogram metre of work is said to be done when a force of one kilogram weight displaces a body through one metre in its own direction.*

1 kg m = 1 kg wt × 1 m = 9.8 N × 1 m

or 1 kg m = 9.8 J

(*ii*) **Gram centimetre.** It is gravitational unit of force in CGS system. *One gram centimetre of work is said to be done when a force of one gram weight displaces a body through one centimetre in its own direction.*

1 g cm = 1 g wt × 1 cm = 980 dyne × 1 cm

or **1 g cm = 980 erg**

6.5 WORK DONE IN TERMS OF RECTANGULAR COMPONENTS

6. *Write an expression for work done in terms of rectangular components of force and displacement.*

Work done in terms of rectangular components. In terms of rectangular components, the force and displacement vectors can be written as

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$

and $$\vec{s} = s_x \hat{i} + s_y \hat{j} + s_z \hat{k}$$

$$\therefore \quad W = \vec{F} . \vec{s}$$

$$= (F_x \hat{i} + F_y \hat{j} + F_z \hat{k}) . (s_x \hat{i} + s_y \hat{j} + s_z \hat{k})$$

or $$W = F_x s_x + F_y s_y + F_z s_z .$$

Examples based on Work Done by a Constant Force

FORMULAE USED

1. $W = \vec{F} . \vec{s} = Fs \cos \theta$
2. If a body of mass m is raised through height h, then $W = mgh$
3. If a body moves up a plane inclined at an angle θ with a constant speed, then $W = mg \sin \theta \times s$

UNITS USED

In SI, force F is in newton, distance s in metre and work done W in joule. In CGS system, force F is in dyne, distance s in cm and work done W in erg.

CONVERSION USED

1 J = 10^7 erg

EXAMPLE 1. A gardener pushes a lawn roller through a distance of 20 m. If he applies a force of 20 kg wt in a direction inclined at 60° to the ground, find the work done by him. Take $g = 9.8\ ms^{-2}$.

Solution. Here $F = 20$ kg wt $= 20 \times 9.8$ N,

$s = 20$ m, $\theta = 60°$

$W = Fs \cos \theta = 20 \times 9.8 \times 20 \times \cos 60°$

$= 20 \times 9.8 \times 20 \times 0.5 =$ **1960 J.**

EXAMPLE 2. A person is holding a bucket by applying a force of 10 N. He moves a horizontal distance of 5 m and then climbs up a vertical distance of 10 m. Find the total work done by him.

Solution. For horizontal motion, the angle between force and displacement is 90°.

Here $F = 10$ N, $s = 5$ m, $\theta = 90°$

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Work done,
$$W_1 = Fs\cos\theta = 10 \times 5 \times \cos 90° = 0$$
For vertical motion, the angle between force and displacement is 0°.

Here $F = 10$ N, $s = 10$ m, $\theta = 0°$

Work done,
$$W_2 = 10 \times 10 \times \cos 0° = 100 \text{ J}$$
Total work done $= W_1 + W_2 = 0 + 100 = 100$ J.

EXAMPLE 3. A cyclist comes to a skidding stop in 10 m. During this process, the force on the cycle due to the road is 200 N and is directly opposed to the motion. (a) How much work does the road do on the cycle ? (b) How much work does the cycle do on the road ? **[NCERT]**

Solution. Work done on the cycle by the road is the work done by the stopping force of friction on the cycle due to the road

(a) The stopping force and the displacement make an angle of 180° with each other. Thus, work done by the road, or the work done by the stopping force is
$$W_r = Fs\cos\theta$$
$$= 200 \times 10 \times \cos 180° = -2000 \text{ J}.$$
It is this negative work that brings the cycle to a halt.

(b) From Newton's Third law, an equal and opposite force acts on the road due to the cycle. Its magnitude is 200 N. However, the road undergoes no displacement.

∴ Work done by the cycle on the road = **zero.**

EXAMPLE 4. A force $\vec{F} = \hat{i} + 5\hat{j} + 7\hat{k}$ acts on a particle and displaces it through $\vec{s} = 6\hat{i} + 9\hat{k}$. Calculate the work done if the force is in newton and displacement in metre. **[Delhi 2K]**

Solution. $W = \vec{F}.\vec{s}$
$$= (\hat{i} + 5\hat{j} + 7\hat{k}).(6\hat{i} + 0\hat{j} + 9\hat{k})$$
$$= 1 \times 6 + 5 \times 0 + 7 \times 9 = 69 \text{ J}.$$

EXAMPLE 5. A force $\vec{F} = -K(y\hat{i} + x\hat{j})$, where K is a positive constant, acts on a particle moving in the XY-plane. Starting from the origin, the particle is taken along the positive X-axis to a point (a, 0) and then parallel to the axis to the point (a, a). Calculate the total work done by the force on the particle. **[IIT 98]**

Solution. Position vector of point (a, 0),
$$\vec{r_1} = a\hat{i} + 0\hat{j}$$
Position vector of point (a, a),
$$\vec{r_2} = a\hat{i} + a\hat{j}$$
Displacement vector,
$$\vec{r} = \vec{r_2} - \vec{r_1} = (a\hat{i} + a\hat{j}) - (a\hat{i} + 0\hat{j}) = a\hat{j}$$
Also,
$$\vec{F} = -K(y\hat{i} + x\hat{j})$$
Work done,
$$W = \vec{F}.\vec{r} = -K(y\hat{i} + x\hat{j}).a\hat{j} = -Kax.$$
As $x = a$
So $W = -Ka^2$.

EXAMPLE 6. A uniform chain of length 2 m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain on the table ? Take $g = 10$ ms^{-2}. **[AIEEE 04]**

Solution. Mass of length 2 m of the chain
$$= 4 \text{ kg}$$
Mass of length 60 cm or 0.60 m of the chain
$$= \frac{4 \times 0.60}{2} = 1.2 \text{ kg}$$
Weight of the hanging part of the chain
$$= 1.2 \times 10 = 12 \text{ N}$$
As the centre of gravity of the hanging part lies at its mid-point, *i.e.*, 30 cm or 0.30 m below the edge of the table, so the work required in pulling the hanging part on the table is
$$W = \text{Force} \times \text{Distance} = 12 \times 0.30 = 3.6 \text{ J}.$$

EXAMPLE 7. Calculate the work done in raising a stone of mass 5 kg and specific gravity 3, lying at the bed of a lake through a height of 5 m.

Solution. Specific gravity of stone
$$= \frac{\text{Mass of stone in air}}{\text{Mass of an equal volume of water}}$$
or
$$3 = \frac{5 \text{ kg}}{\text{Mass of an equal volume of water}}$$
∴ Mass of an equal volume of water $= \frac{5}{3}$ kg

According to the Archimedes' principle, loss in the weight of stone when immersed in water
$$= \text{weight of water displaced} = \frac{5}{3} \text{ kg wt}$$
∴ Apparent weight of stone in the lake
$$= 5 - \frac{5}{3} = \frac{10}{3} \text{ kg wt}$$
Force required to lift the stone in the lake,
$$F = \frac{10}{3} \text{ kg wt} = \frac{10 \times 9.8}{3} \text{ N}$$
Work done in raising the stone through a height of 5 m,
$$W = F \times s = \frac{10 \times 9.8}{3} \times 5 = \mathbf{163.3 \text{ J}}.$$

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Example 5. A cluster of clouds at a height of 1000 m above the earth burst and enough rain fell to cover an area of 10^6 m^2 with a depth of 2 cm. How much work would have been done in raising water to the height of clouds ? Take $g = 9.8$ ms^{-2} and density of water $= 10^3$ kgm^{-3}.

Solution. Area $A = 10^6$ m^2

Depth $d = 2$ cm $= 2 \times 10^{-2}$ m

$\therefore$ Volume of water $= Ad = 10^6 \times 2 \times 10^{-2} = 2 \times 10^4$ m^3

Mass of water, m = Volume × density $= 2 \times 10^4 \times 10^3 = 2 \times 10^7$ kg

Force used in raising water to the height of clouds, F = Weight of water $= mg = 2 \times 10^7 \times 9.8$ N

Work done, $W = Fs = 2 \times 10^7 \times 9.8 \times 1000 = \mathbf{1.96 \times 10^{11}}$ **J**.

PROBLEMS FOR PRACTICE

1. What is the work done by a person in carrying a suitcase weighing 10 kg f on his head when he travels a distance of 5 m in the (*i*) vertical direction and (*ii*) horizontal direction ? Take $g = 9.8$ ms^{-2}.

[Ans. (*i*) 490 J (*ii*) Zero**]**

2. Calculate the amount of work done by a labourer who carries n bricks, each of mass m, to the roof of a house of height h by climbing up a ladder. **(Ans.** $nmgh$**)**

3. A man moves on a straight horizontal road with a block of mass 2 kg in his hand. If he covers a distance of 40 m with an acceleration of 0.5 ms^{-2}, find the work done by the man on the block during the motion. **(Ans.** 40 J**)**

4. A force $\vec{F} = (2\hat{i} - 6\hat{j})$ N is applied on a body, which is sliding over a floor. If the body is displaced through $(-3\hat{j})$ m, how much work is done by the force ? *[Himachal 09]* **(Ans.** 18 J**)**

5. Find the work done by force $\vec{F} = 2\hat{i} - 3\hat{j} + \hat{k}$ when its point of application moves from the point $A(1, 2, -3)$ to the point $B(2, 0, -5)$. **(Ans.** 6 units**)**

6. A particle is acted upon by constant forces $\vec{F_1} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{F_2} = -\hat{i} + 2\hat{j} - 3\hat{k}$, is displaced from the point $A(2, 1, 0)$ to the point $B(-3, -4, 2)$. Find the total work done by these forces. **(Ans.** 2 units**)**

7. A man weighing 50 kg f supports a body of 25 kg f on his head. What is the work done when he moves a distance of 20 m up an incline of 1 in 10 ? Take $g = 9.8$ ms^{-2}. **(Ans.** 1470 J**)**

HINTS

3. $W = Fs\cos 0° = ma\,s = 1 \times 2 \times 0.5 \times 40 = 40$ J.

4. $W = \vec{F} \cdot \vec{s} = (2\hat{i} - 6\hat{j}) \cdot (-3\hat{j}) = 18$ J.

6. Resultant force, $\vec{F} = \vec{F_1} + \vec{F_2}$

$= (2\hat{i} - 3\hat{j} + 4\hat{k}) + (-\hat{i} + 2\hat{j} - 3\hat{k}) = \hat{i} - \hat{j} + \hat{k}$

Displacement,

$\vec{s} = \vec{AB} = \vec{OB} - \vec{OA} = (-3\hat{i} - 4\hat{j} + 2\hat{k}) - (2\hat{i} + \hat{j})$

$= -5\hat{i} - 5\hat{j} + 2\hat{k}$

$W = \vec{F} \cdot \vec{s} = (\hat{i} - \hat{j} + \hat{k}) \cdot (-5\hat{i} - 5\hat{j} + 2\hat{k})$

$= 1 \times (-5) - 1 \times (-5) + 1 \times 2 = 2$ **units.**

7. Here $m = 50 + 25 = 75$ kg, $\sin\theta = \frac{1}{10}$,

$s = 10$ m, $g = 9.8$ ms^{-2}

Force needed to be applied against gravity $= mg\sin\theta$

$\therefore \quad W = Fs = mg\sin\theta \times s$

$= 75 \times 9.8 \times \frac{1}{10} \times 20 = \mathbf{1470}$ **J.**

6.6 WORK DONE BY A VARIABLE FORCE

7. *Explain how work done by a variable force may be measured.*

Work done by a variable force. Suppose a variable force F acts on a body along the fixed direction, say x-axis. The magnitude of the force F depends on x, as shown by force-displacement graph in Fig. 6.7(*a*). Let us calculate the work done when the body moves from the initial position x_i to the final position x_f under the force F.

The displacement can be divided into a large number of small equal displacements Δx. During a small displacement Δx, the force F can be assumed to be constant. Then the work done is

$W = F\,\Delta x$ = Area of rectangle *abcd*

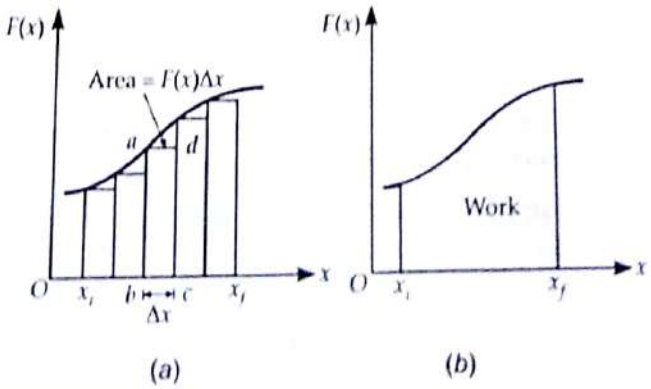

Fig. 6.7 Calculation of work done by a variable force $F(x)$ in moving a body from position x_i to x_f.

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Adding areas of all the rectangles in Fig. 6.7(a) we get the total work done as

$$W = \sum_{x_i}^{x_f} F\,\Delta x$$

= Sum of areas of all rectangles erected over all the small displacements

In the limit when $\Delta x \to 0$ the number of rectangles tends to be infinite but the above summation approaches a definite integral whose value is equal to the area under the curve. Thus the total work done is

$$W = \lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F\,\Delta x = \int_{x_i}^{x_f} F\,dx$$

= Area under the force-displacement curve

Hence, for a varying force the work done is equal to the definite integral of the force over the given displacement.

CBSE When the force varies both in magnitude and direction, the work done is given by

$$W = \int_{s_1}^{s_2} F\,ds\cos\theta = \int_{s_1}^{s_2} \vec{F}\cdot d\vec{s}$$

Examples based on Work done by a Variable Force

FORMULAE USED

1. $W = \sum_i \vec{F}_i \cdot \vec{s}_i$
2. $W = \int_{s_1}^{s_2} \vec{F}\cdot d\vec{s}$
3. W = Area under the force-displacement curve between the initial and final positions of the body.

UNITS USED

In SI, force F is in newton, distance s in metre and work done W in joule.

EXAMPLE 9. A 2 kg particle starts at the origin and moves along the positive x-axis. The net force acting on it measured at intervals of 1 m is : 27.9, 28.3, 30.9, 34.0, 34.5, 46.9, 48.2, 50.0, 63.5, 13.6, 12.2, 32.7, 46.6 and 27.0 (in newtons). What is the total work done on the particle in this interval ?

Solution. As the forces and displacements are in same direction, so

$$W = \sum_i F_i s_i = 27.9\times1 + 28.3\times1 + 30.9\times1 + 34.0\times1 + 46.9\times1 + 48.2\times1 + 50.0\times1 + 63.5\times1 + 13.6\times1 + 12.2\times1 + 46.6\times1 + 27.0\times1$$

$$= 496.3\ \text{J}.$$

EXAMPLE 10. A body moves from point A to B under the action of a force, varying in magnitude as shown in Fig. 6.8. Obtain the work done. Force is expressed in newton and displacement in metre.

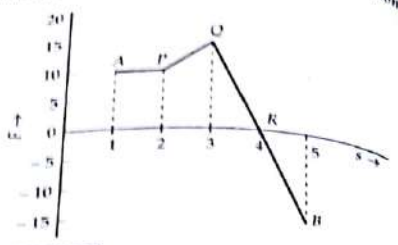

Fig. 6.8

Solution. Work done = Area under F-s curve

$W_{AB} = W_{12} + W_{23} + W_{34} + W_{45}$

= Area under AP + Area under PQ + Area under QR − Area above R_B

$$= 10\times1 + \tfrac{1}{2}(10+15)\times1 + \tfrac{1}{2}\times1\times15 - \tfrac{1}{2}\times1\times15$$

$$= 10 + 12.5 = \mathbf{22.5\ J}.$$

EXAMPLE 11. A woman pushes a trunk on railway platform which has a rough surface. She supplies a force of 100 N over a distance of 10 m. Thereafter she gets progressively tired and her applied force reduces linearly with distance to 50 N. The total distance by which the trunk has been moved is 20 m. Plot the force applied by the woman and the frictional force, which is 50 N. Calculate the work done by the two forces over 20 m. [NCERT]

Solution. Plots of force F applied by the woman and the opposing frictional force F are shown in Fig. 6.9.

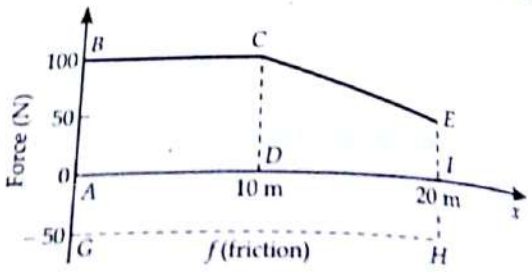

Fig. 6.9

Clearly at $x = 20$ m, $F = 50$ N

As the force of friction f [= 50 N] opposes the applied force F, so it has been shown on the negative side of the force-axis.

Work done by the force
W_T = Area of rectangle AI
$= 100\times10 + \frac{1}{2}(100+5$
Work done by the fric
W_f = Area of rectan
$= (-50)\times20 = -$

*EXAMPLE 12. A particle m
$x = 5$ m under the influence
Find the work done in the*

Solution. Work don
$x = 0$ to $x = 5$ m will be

$$W = \int_0^5 F\,dx = \int_0^5 (7$$

$$= \left[7x - \frac{2x^2}{2}\right.$$

$$= 35 - 25 + 1$$

PROBLEMS F

1. A force $F = (1$ X-direction, w Find the work ment from x =
2. A force F = X-direction, work done from $x = 0$ to
3. A body mo of a force s and distan work done

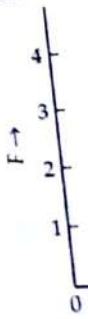

Fig. 6.10

4. The r time action metr the b

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Work done by the force F applied by the woman

W_f = Area of rectangle $ABCD$ + Area of trapezium $CEID$

$= 100 \times 10 + \frac{1}{2}(100 + 50) \times 10 = 1000 + 750 = \mathbf{1750\ J}.$

Work done by the frictional force,

W_f = Area of rectangle $AGHI$

$= (-50) \times 20 = \mathbf{-1000\ J}.$

Example 12. A particle moves along the X-axis from $x = 0$ to $x = 5$ m under the influence of a force given by $F = 7 - 2x + 3x^2$. Find the work done in the process. **[CBSE 94]**

Solution. Work done in moving the particle from $x = 0$ to $x = 5$ m will be

$$W = \int_0^5 F dx = \int_0^5 (7 - 2x + 3x^2)\, dx$$

$$= \left[7x - \frac{2x^2}{2} + \frac{3x^3}{3}\right]_0^5 = [7x - x^2 + x^3]_0^5$$

$$= 35 - 25 + 125 = \mathbf{135\ J}.$$

PROBLEMS FOR PRACTICE

1. A force $F = (15 + 0.50x)$ acts on a particle in the X-direction, where F is in newton and x in metre. Find the work done by this force during a displacement from $x = 0$ to $x = 2.0$ m. **(Ans. 31 J)**

2. A force $F = a + bx$ acts on a particle in the X-direction, where a and b are constants. Find the work done by this force during a displacement from $x = 0$ to $x = d$. **(Ans.** $\left(a + \frac{bd}{2}\right)d$ **)**

3. A body moves from a point A to B under the action of a force shown in Fig. 6.10. Force F is in newton and distance x in metre. What is the amount of work done ? **(Ans. 11.5 J)**

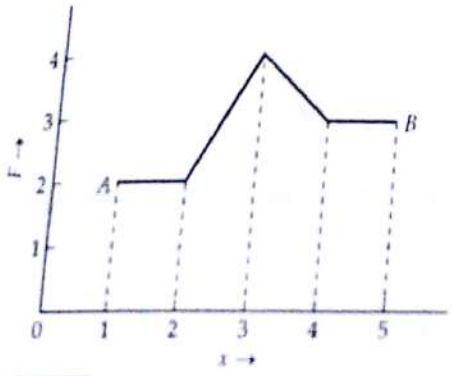

Fig. 6.10

4. The relation between the displacement x and the time t for a body of mass 2 kg moving under the action of a force is given by $x = t^3/3$, where x is in metre and t in second, calculate the work done by the body in first 2 seconds. **(Ans. 16 J)**

5. Fig. 6.11 shows the F-x graph. Here the force F is in newton and distance x in metre. What is the work done ? **(Ans. 10 J)**

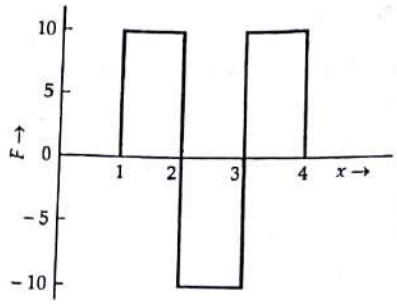

Fig. 6.11

6. Calculate work done in moving the object from $x = 2$ m to $x = 3$ m from the following graph :

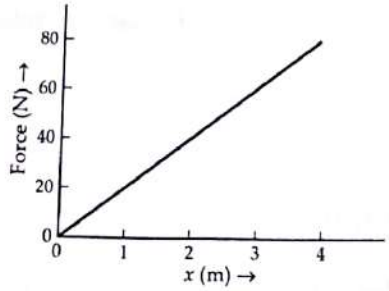

Fig. 6.12

[Central Schools 03] **(Ans. 50 J)**

HINTS

2. $dW = Fdx = (a + bx)\, dx$

Total work done in displacement from $x = 0$ to $x = d$ will be

$$W = \int dW = \int_0^d (a + bx)\, dx = \left[ax + \frac{bx^2}{2}\right]_0^d$$

$$= ad + \frac{bd^2}{2} = \left(a + \frac{bd}{2}\right)d.$$

6. At $x = 2$ m,

$F = 40$ N

At $x = 3$ m,

$F = 60$ N

Work done = Area under $F - x$ graph between $x = 2$ and $x = 3$ m

$$= \frac{1}{2}(40 + 60) \times (3 - 2)$$

$$= \frac{1}{2} \times 100 \times 1 = \mathbf{50\ J}.$$

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6.7 ENERGY

8. *Define the term energy. What are its units and dimensions ? Name the different forms of energy.*

Energy. *Energy of a body is defined as its capacity or ability to do work.* The energy of a body is measured by the amount of work the body can perform, therefore

(*i*) like work, energy is a scalar quantity.

(*ii*) the dimensional formula of energy is $[ML^2T^{-2}]$ *i.e.*, same as that of work.

(*iii*) energy is measured in the same units as the work.

The SI unit of energy is joule and the CGS unit is erg.

Energy has several forms : Mechanical energy, sound energy, heat energy, light energy, chemical energy, atomic energy, nuclear energy, electric energy, magnetic energy, solar energy, etc.

Table 6.1 *Some other units of work or energy.*

S.No.	Unit	Symbol	Value in SI
1.	erg	erg	10^{-7} J
2.	electron volt	eV	1.6×10^{-19} J
3.	calorie	cal	4.186 J
4.	kilowatt hour	kWh	3.6×10^{6} J

9. *What is mechanical energy ? What are its two forms ?*

Mechanical energy. *The energy produced by mechanical means is called mechanical energy.* It has two forms :

(*i*) Kinetic energy (*ii*) Potential energy.

6.8 KINETIC ENERGY

10. *What is kinetic energy ? Give some examples.*

Kinetic energy. *The energy possessed by a body by virtue of its motion is called its kinetic energy.* A moving object can do work. The amount of work that a moving object can do before coming to rest is equal to its kinetic energy.

Examples :

(*i*) A moving hammer drives a nail into the wood. Being in motion, it has kinetic energy or ability to do work.

(*ii*) A fast moving stone can break a window pane. The stone has kinetic energy due to its motion and so it can do work.

(*iii*) A bullet fired from a gun can pierce a target due to its kinetic energy.

(*iv*) The kinetic energy of air is used to run wind mills.

(*v*) The kinetic energy of a fast stream of water is used to run water mills.

11. *Derive an expression for the kinetic energy of a body of mass m moving with velocity v .*

Expression for kinetic energy. The kinetic energy of a body can be determined by calculating the amount of work required to bring the body into motion from its state of rest, as shown in Fig. 6.13.

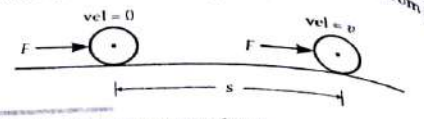

Fig. 6.13 Measurement of K.E.

Let m = mass of the body

$u = 0$ = initial velocity of the body

F = constant force applied on the body

a = acceleration produced in the body in the direction of force

v = final velocity of the body

s = distance covered by the body.

As $v^2 - u^2 = 2as$

$\therefore$ $v^2 - 0^2 = 2as$ or $a = \dfrac{v^2}{2s}$

As the force and displacement are in same direction, so the work done is

$$W = Fs = ma \,.\, s = m \,.\, \frac{v^2}{2s} \,.\, s = \frac{1}{2} mv^2$$

This work done appears as kinetic energy (K) of the body.

$\therefore$ $$K = \frac{1}{2} mv^2$$

Hence, *the kinetic energy of a body is equal to one-half the product of the mass of the body and the square of its velocity.*

12. *Derive an expression for the kinetic energy of a body by calculus method. Deduce its relation with linear momentum.*

Kinetic energy by calculus method. Consider a body of mass m initially at rest. A force $\vec{F}$ applied on the body produces a displacement $\vec{ds}$ in its own direction ($\theta = 0°$). The small work done is

$$dW = \vec{F} \,.\, \vec{ds} = Fds \cos 0° = Fds$$

According to Newton's second law of motion,

$$F = ma = m \frac{dv}{dt}$$

$\therefore$ $$dW = Fds = m \frac{dv}{dt} \,.\, ds = mv\, dv \qquad \left[\because \frac{ds}{dt} = v\right]$$

The total work done to increase its velocity from 0 to v is given by

$$W = \int dW = \int_0^v mv\, dv = m \int_0^v$$

This work done appears as kinetic energy of the body.

$\therefore$ $$K = \frac{1}{2} mv^2$$

Relation between K.E. and linear momentum,

$$p = mv$$

therefore $$K = \frac{1}{2} mv^2$$

or $$K = \frac{p^2}{2m}$$

6.9 WORK-ENE… A CONSTA…

13. State the wo… constant force.

Work-energy th… the net force acting o… in the kinetic energ…

Proof of W… Suppose a consta… produces accele… suppose the ve… We use the equ…

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Therefore,

or

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The total work done to increase its velocity from 0 to v is given by

$$W = \int dW = \int_0^v mv\,dv = m\int_0^v v\,dv = m\left[\frac{v^2}{2}\right]_0^v = \frac{1}{2}mv^2$$

This work done appears as the kinetic energy (K) of the body.

$$\therefore \quad K = \frac{1}{2}mv^2$$

Relation between K.E. and linear momentum. As linear momentum,

$$p = mv$$

$$\text{therefore} \quad K = \frac{1}{2}mv^2 = \frac{1}{2m}(m^2v^2) = \frac{1}{2m}(mv)^2$$

$$K = \frac{p^2}{2m} \quad \text{or} \quad p = \sqrt{2mK}$$

or

6.9 WORK-ENERGY THEOREM FOR A CONSTANT FORCE

13. *State the work-energy theorem. Prove it for a constant force.*

Work-energy theorem. *It states that the work done by the net force acting on a body is equal to the change produced in the kinetic energy of the body.*

Proof of W-E theorem for a constant force. Suppose a constant force F acting on a body of mass m produces acceleration a in it. After covering distance s, suppose the velocity of the body changes from u to v. We use the equation of motion,

$$v^2 - u^2 = 2as$$

Multiplying both sides by $\frac{1}{2}m$, we get

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mas$$

By Newton's second law, $ma = F$, the applied force. Therefore,

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = Fs = W$$

or

$$K_f - K_i = W$$

Change in K.E. of the body = Work done on the body by the net force.

This proves the work energy theorem for a constant force.

6.10 WORK-ENERGY THEOREM FOR A VARIABLE FORCE

14. *Prove the work-energy theorem for a variable force.*

Proof of W-E theorem for a variable force. Suppose a variable force $\vec{F}$ acts on a body of mass m and produces displacement $\vec{ds}$ in its own direction ($\theta = 0°$). The small work done is

$$dW = \vec{F} \cdot \vec{ds} = F\,ds\cos 0° = F\,ds$$

According to Newton's second law of motion,

$$F = ma = m\frac{dv}{dt}$$

$$\therefore \quad dW = m\frac{dv}{dt} \cdot ds$$

$$= mv\,dv \qquad \left[\because \frac{ds}{dt} = v\right]$$

If the applied force increases the velocity from u to v, then the total work done on the body will be

$$W = \int dW = \int_u^v mv\,dv = m\int_u^v v\,dv$$

$$= m\left[\frac{v^2}{2}\right]_u^v = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

or

$$W = K_f - K_i$$

= Change in K.E. of the body.

This proves the work-energy theorem for a variable force.

For Your Knowledge

- The work-energy theorem is not independent of Newton's second law. It may be viewed as scalar form of second law.
- The W-E theorem holds in all inertial frames. It can be extended to non-inertial frames provided we include the pseudo force in the calculation of the net force acting on the body under consideration.
- When force and displacement are in same direction, the kinetic energy of the body increases. The increase in K.E. is equal to the work done on the body.
- When force and displacement are oppositely directed, the kinetic energy of the body decreases. The decrease in K.E. is equal to the work done by the body against the retarding force.
- When a body moves along a circular path with uniform speed, there is no change in its kinetic energy. By W-E theorem, the work done by the centripetal force is zero.
- When K.E. increases, the work done is positive and when K.E. decreases, the work done is negative.
- In deriving the W-E theorem, it has been assumed that the work done by the force is effective only in changing the K.E. of the body. However, the work done on a body may also be stored as the P.E. of the body.

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Examples based on K.E. and W-E Theorem

Formulae Used

1. Kinetic energy, $K = \frac{1}{2} mu^2$
2. According to work-energy theorem,
$$W = K_f - K_i = \frac{1}{2} mv^2 - \frac{1}{2} mu^2$$

Units Used

Work done W, kinetic energies K_i and K_f are all in joule.

Conversions Used

$1 eV = 1.60 \times 10^{-19}$ J.

Example 13. A body of mass 4 kg initially at rest is subject to a force 16 N. What is the kinetic energy acquired by the body at the end of 10 s ? **[Himachal 07 ; Delhi 02]**

Solution. Here $m = 4$ kg, $F = 16$ N, $t = 10$ s

Acceleration,
$$a = \frac{F}{m} = \frac{16}{4} = 4 \text{ ms}^{-2}$$
$$\therefore \quad v = u + at = 0 + 4 \times 10 = 40 \text{ ms}^{-1}$$

The kinetic energy acquired by the body,
$$K = \frac{1}{2} mv^2 = \frac{1}{2} \times 4 \times (40)^2 = \mathbf{3200\ J}.$$

Example 14. A toy rocket of mass 0.1 kg has a small fuel of mass 0.02 kg which it burns out in 3 s. Starting from rest on a horizontal smooth track it gets a speed of 20 ms^{-1} after the fuel is burnt out. What is the approximate thrust of the rocket ? What is the energy content per unit mass of the fuel ? (Ignore the small mass variation of the rocket during fuel burning). **[NCERT]**

Solution. Here $m = 0.1$ kg, $u = 0$, $v = 20 \text{ ms}^{-1}$, $t = 3$ s

Thrust of the rocket
$$= ma = m\frac{v-u}{t} = 0.1 \times \frac{20-0}{3} = \frac{2}{3} \text{ N}$$

Kinetic energy gained by the rocket,
$$K = \frac{1}{2} mv^2 = \frac{1}{2} \times 0.1 \times (20)^2 = 20 \text{ J}$$

Energy content per unit mass of the fuel
$$= \frac{\text{Total energy}}{\text{Mass of the fuel}} = \frac{20 \text{ J}}{0.02 \text{ kg}} = \mathbf{1000\ J\ kg^{-1}}.$$

Example 15. A bullet weighing 10 g is fired with a velocity of 800 ms^{-1}. After passing through a mud wall 1 m thick, its velocity decreases to 100 ms^{-1}. Find the average resistance offered by the mud wall. **[NCERT]**

Solution. Mass of bullet, $m = 10$ g $= 0.01$ kg

Velocity of bullet before passing through mud wall,
$u = 800 \text{ ms}^{-1}$

Velocity of bullet after passing through mud wall,
$v = 100 \text{ ms}^{-1}$

Distance covered by the bullet,
$s = 1$ m

Let average resistance offered by the wall $= F$

According to work-energy theorem,

Work done by resistance offered by mud wall = Decrease in K.E.

or
$$Fs = \frac{1}{2} m(u^2 - v^2)$$
$$\therefore \quad F = \frac{m(u^2 - v^2)}{2s} = \frac{0.01 \times (800^2 - 100^2)}{2 \times 1} = 3150 \text{ N}.$$

Example 16. A shot travelling at the rate of 100 ms^{-1} is just able to pierce a plank 4 cm thick. What velocity is required to just pierce a plank 9 cm thick ?

Solution. Here $v_1 = 100 \text{ ms}^{-1}$, $s_1 = 4$ cm, $v_2 = ?$, $s_2 = 9$ cm.

K.E. lost by the shot = Work done against plank's resistance
$$\therefore \quad \frac{1}{2} mv_1^2 = F \times s_1 \quad \text{and} \quad \frac{1}{2} mv_2^2 = F \times s_2$$

On dividing,
$$\frac{v_2^2}{v_1^2} = \frac{s_2}{s_1} \quad \text{or} \quad \frac{v_2}{v_1} = \sqrt{\frac{s_2}{s_1}} = \sqrt{\frac{9}{4}} = \frac{3}{2}$$
$$\therefore \quad v_2 = \frac{3}{2} \times v_1 = \frac{3}{2} \times 100 = \mathbf{150\ ms^{-1}}.$$

Example 17. In a ballistics demonstration, a police officer fires a bullet of mass 50.0 g with speed 200 ms^{-1} on soft plywood of thickness 2.00 cm. The bullet emerges with only 10% of its initial kinetic energy. What is the emergent speed of the bullet ? **[NCERT]**

Solution. Here $m = 50.0$ g $= 0.05$ kg, $u = 200 \text{ ms}^{-1}$

Initial K.E. $= \frac{1}{2} mu^2 = \frac{1}{2} \times 0.05 \times (200)^2 = 1000$ J

Final K.E. $= 10\%$ of 1000 J $= \frac{10 \times 1000}{100} = 100$ J

or
$$\frac{1}{2} mv^2 = 100 \text{ J}$$
$$\therefore \quad v = \sqrt{\frac{2 \times 100}{m}} = \sqrt{\frac{2 \times 100}{0.05}} = \mathbf{63.2\ ms^{-1}}.$$

Clearly, the speed reduces nearly by 68% and not by 90% by which the K.E. reduces.

Example 18. It is well known that a raindrop or a small pebble falls under the influence of the downward gravitational force and the opposing resistive force. The latter is known to be proportional to the speed of the drop but is

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otherwise undetermined. Consider a drop of small pebble of mass 1.00 g falling from a cliff of height 1.00 km. It hits the ground with a speed of 50.0 ms^{-1}. What is the work done by the unknown resistive force ? **[NCERT]**

Solution. We assume that the pebble is initially at rest on the cliff.

$u = 0, \quad m = 1.00 \text{ g} = 10^{-3} \text{ kg},$

$\therefore \quad v = 50 \text{ ms}^{-1}, \quad h = 100 \text{ km} = 10^3 \text{ m}$

The change in K.E. of the pebble is

$$\Delta K = \tfrac{1}{2} mv^2 - \tfrac{1}{2} mu^2$$

$$= \tfrac{1}{2} \times 10^{-3} \times (50)^2 - 0 = 1.25 \text{ J}$$

Assuming that $g = 10 \text{ ms}^{-2}$ is constant, the work done by the gravitational force is

$$W_g = mgh = 10^{-3} \times 10 \times 10^3 = 10.0 \text{ J}$$

If W_r is the work done by the resistive force on the pebble, then from the work-energy theorem,

$$\Delta K = W_g + W_r$$

$$W_r = \Delta K - W_g = 1.25 - 10.0 = -8.75 \text{ J}.$$

or

EXAMPLE 19. A block of mass $m = 1$ kg, moving on a horizontal surface with speed $v_i = 2 \text{ ms}^{-1}$ enters a rough patch ranging from $x = 0.10$ m to $x = 2.01$ m. The retarding force F_r on the block in this range is inversely proportional to x over this range.

$$F_r = \frac{-k}{x} \qquad 0.1 < x < 2.01 \text{ m}$$

$$= 0 \quad \text{for } x < 0.1 \text{ m and } x > 2.01 \text{ m}$$

where $k = 0.5$ J. What is the final kinetic energy and speed v_f of the block as it crosses this patch ? **[NCERT]**

Solution. By work-energy theorem,

$$\Delta K = W_r \quad \text{or} \quad K_f - K_i = \int_{x_i}^{x_f} F_r \, dx$$

$$\therefore \quad K_f = K_i + \int_{0.1}^{2.01} \frac{(-k)}{x} dx$$

$$= \tfrac{1}{2} mv_i^2 - k \int_{0.1}^{2.01} \frac{1}{x} dx$$

$$= \tfrac{1}{2} \times 1 \times 2^2 - k \left[\ln x\right]_{0.1}^{2.01}$$

$$= 2 - 0.5 \left[\ln \frac{2.01}{0.1}\right] = 2 - 0.5 \ln 20.1$$

$$= 2 - 0.5 \times 2.303 \log 20.1$$

$$= 2 - 0.5 \times 2.303 \times 1.3032 = 2 - 1.5$$

$$= \mathbf{0.5 \text{ J}}.$$

Final speed, $v_f = \sqrt{\dfrac{2K_f}{m}} = \sqrt{\dfrac{2 \times 0.5}{1}} = \mathbf{1 \text{ ms}^{-1}}.$

EXAMPLE 20. Two identical 5 kg blocks are moving with same speed of 2 ms^{-1} towards each other along a frictionless horizontal surface. The two blocks collide, stick together and come to rest. Consider the two blocks as a system. Calculate work done by (i) external forces and (ii) internal forces. **[CBSE 91]**

Solution. As no external forces are acting on the system, so

$$\vec{F}_{ext} = 0 \qquad \therefore \quad W_{ext} = \vec{F}_{ext} \cdot \vec{s} = 0.$$

According to work-energy theorem,

Total work done = Change in K.E.

= Final K.E. – Initial K.E.

$$\text{or} \quad W_{ext} + W_{int} = 0 - \left(\tfrac{1}{2} mv^2 + \tfrac{1}{2} mv^2\right) = -mv^2$$

$$\text{or} \quad 0 + W_{int} = -5 \times (2)^2 = -20$$

$$\therefore \quad W_{int} = \mathbf{-20 \text{ J}}.$$

The negative sign indicates that internal forces of action and reaction act on the two blocks in a direction opposite to their motion.

EXAMPLE 21. If the linear momentum of a body increases by 20%, what will be the % increase in the kinetic energy of the body ? **[AFMC 97]**

Solution. Initial kinetic energy of the body,

$$K = \frac{1}{2} mv^2 = \frac{1}{2} \frac{m^2 v^2}{m} = \frac{p^2}{2m}$$

Increase in momentum = 20% of $p = \dfrac{20}{100} \times p = \dfrac{p}{5}$

Final momentum $= p + \dfrac{p}{5} = \dfrac{6p}{5}$

Final kinetic energy of the body,

$$K' = \frac{(6p/5)^2}{2m} = \frac{36}{25} \frac{p^2}{2m} = \frac{36}{25} K$$

Increase in kinetic energy

$$= K' - K = \frac{36}{25} K - K = \frac{11}{25} K$$

% Increase in K.E.

$$= \frac{K' - K}{K} \times 100 = \frac{\frac{11}{25} K}{K} \times 100 = \mathbf{44\%}.$$

EXAMPLE 22. If the kinetic energy of a body increases by 300%, by what % will the linear momentum of the body increase ? **[Delhi 95, 99]**

Solution. Initial kinetic energy,

$$K = \frac{1}{2} mv^2 = \frac{1}{2} \frac{m^2 v^2}{m} = \frac{p^2}{2m}$$

$\therefore$ Initial momentum, $p = \sqrt{2mK}$

Increase in kinetic energy = 300% of $K = 3K$

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Final kinetic energy,

$$K' = K + 3K = 4K$$

Final momentum,

$$p' = \sqrt{2mK'} = \sqrt{2m \times 4K} = 2\sqrt{2mK} = 2p$$

% Increase in momentum

$$= \frac{p' - p}{p} \times 100 = \frac{2p - p}{p} \times 100 = \mathbf{100\%}.$$

EXAMPLE 24. A body of mass 0.3 kg is taken up an inclined plane to length 10 m and height 5 m and then allowed to slide down to the bottom again. The coefficient of friction between the body and the plane is 0.15. What is the

(i) work done by the gravitational force over the round trip,

(ii) work done by the applied force over the upward journey,

(iii) work done by frictional force over the round trip,

(iv) kinetic energy of the body at the end of the trip ?

How is the answer to (iv) related to the first three answers ? [NCERT]

Solution. Here $m = 0.3$ kg, $l = 10$ m,

$$h = 5 \text{ m}, \quad \mu_k = 0.15$$

$$\sin\theta = \frac{h}{l} = \frac{5}{10} = \frac{1}{2} = 0.5$$

$$\therefore \quad \theta = 30°$$

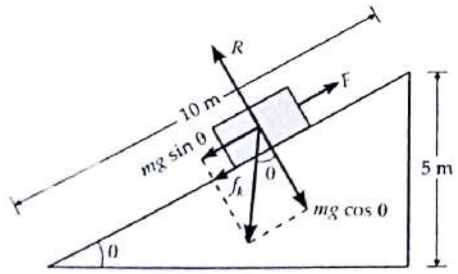

Fig. 6.14

(*i*) Work done by the gravitational force in moving the body up the inclined plane,

$$W = Fs = -mg \sin\theta \times l$$

$$= -0.3 \times 9.8 \times 0.5 \times 10 = -14.7 \text{ J}$$

Work done by the gravitational force in moving the body down the inclined plane,

$$W' = Fs = +mg \sin\theta \times l = +14.7 \text{ J}$$

Work done by the gravitational force over the round trip,

$$W_1 = W + W' = -14.7 + 14.7 = \mathbf{0}.$$

This is in conformity with the fact that the work done by a conservative force (*e.g.*, gravitational force) over one round trip is zero.

(*ii*) Force needed to move the body up the inclined plane,

$$F = mg \sin\theta + f_k = mg \sin\theta + \mu_k R$$

$$= mg \sin\theta + \mu_k mg \cos\theta$$

$$= mg (\sin\theta + \mu_k \cos\theta)$$

Work done by the applied force over the upward journey,

$$W_2 = F \times l = mg (\sin\theta + \mu_k \cos\theta) l$$

$$= 0.3 \times 9.8 (\sin 30° + 0.15 \cos 30°) \times 10$$

$$= 0.3 \times 9.8 (0.5 + 0.15 \times 0.866) \times 10 = 18.5 \text{ J}.$$

(*iii*) Work done by the frictional force over the round trip,

$$W_3 = -f_k (l + l) = -2 f_k \times l$$

$$= -2\mu_k mg \cos\theta \times l$$

$$= -2 \times 0.15 \times 0.3 \times 9.8 \times \cos 30° \times 10 = -7.6 \text{ J}$$

(*iv*) Kinetic energy of the body at the end of round trip

= Work done by the net force in moving the body down the inclined plane

$$= (mg \sin\theta - \mu_k mg \cos\theta) \times l$$

$$= mg (\sin\theta - \mu_k \cos\theta) l$$

$$= 0.3 \times 9.8 \times (0.5 - 0.13) \times 10 = \mathbf{10.9 \text{ J}}$$

From answers to (*i*), (*ii*) and (*iii*), net work done on the body

$$= 0 + 18.5 - 7.6 = 10.9 \text{ J} = \text{K.E. of the body}$$

Thus kinetic energy of the body is equal to the net work done on the body.

✕ PROBLEMS FOR PRACTICE

1. The momentum of a body of mass 5 kg is 500 kg ms^{-1}. Find its K.E. (Ans. 2.5×10^4 J)
2. A bullet of mass 20 g is found to pass two points 30 m apart in a time interval of 4 s. Calculate the kinetic energy of the bullet if it moves with constant speed. (Ans. 0.5625 J)
3. A body of mass 2 kg is resting on a rough horizontal surface. A force of 20 N is now applied to it for 10 s, parallel to the surface. If the coefficient of kinetic friction between the surfaces in contact is 0.2, calculate : (*a*) Work done by the applied force in 10 s. (*b*) Change in kinetic energy of the object in 10 s. Take $g = 10$ ms^{-2}. [Delhi 04] (Ans. 8000 J, 6400 J)
4. An electron and a proton are detected in a cosmic ray experiment, the electron with K.E. of 5 keV and the proton with K.E. of 50 keV. Find the ratio of their speeds. Given $m_e = 9.11 \times 10^{-31}$ kg and $m_p = 1.67 \times 10^{-27}$ kg. $\left(\text{Ans. } \frac{v_e}{v_p} = 4.28\right)$

5. A neutron of mass 1.67×10^{-27} kg is moving with speed of 7×10^5 ms^{-1}. Calculate (*i*) its kinetic energy and (*ii*) the average force it will exert in entering a body to a depth of 0.01 cm
[Ans. (*i*) 40.915×10^{-17} J (*ii*) 40.915 ×
6. A body of mass 1 kg is allowed to fall freely under gravity. Find the momentum and kinetic energy of the body 5 seconds after it starts falling. Take $g = 10$ ms^{-2}. (Ans. 50 kg m
7. Two bodies of masses 1 g and 16 g are moving with equal kinetic energies. Find the ratio of magnitudes of their linear momenta.
8. If the momentum of a body is increased by 100%, then what will be the percentage increase in the kinetic energy of the body ? [Cen
9. The kinetic energy of a body decreases by 19%. What is the percentage decrease in its momentum ?
10. A running man has half the kinetic energy that a boy of half his mass has. The man speeds up by 1.0 ms^{-1} and then has the same kinetic energy as the boy. What were the original speeds of the man and the boy ? (Ans. 2.41
11. While catching a cricket ball of mass 150 g moving with a velocity of 20 ms^{-1}, the fielder pulls his hands backwards through 20 cm. What is the average force done in catching the ball and what is the impulse exerted by the ball on the ha

✕ HINTS

1. K.E. $= \frac{p^2}{2m} = \frac{(500)^2}{2 \times 5} = 2.5 \times$
2. Speed, $v = \frac{\text{Distance}}{\text{Time}} = \frac{30}{}$

K.E. $= \frac{1}{2} \times 0.02 \times (7.$

3. Here $m = 2$ kg, $u = 0$, F

$f_k = \mu_k mg = 0.2 \times$

Net force, $F' = F -$

Acceleration, $a = \frac{F'}{m} =$

$s = ut + \frac{1}{2} at^2$

(*a*) Work done by

$W_1 =$

(*b*) Change in K.E

$W_2 =$

5. (*i*) K.E. $= \frac{1}{2} mv^2$

$= 40.91$

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6. A neutron of mass 1.67×10^{-27} kg is moving with a speed of 7×10^5 ms^{-1}. Calculate (*i*) its kinetic energy and (*ii*) the average force it will exert in entering a body to a depth of 0.01 cm.
[**Ans.** (*i*) 40.915×10^{-17} J (*ii*) 40.915×10^{-13} N]

6. A body of mass 1 kg is allowed to fall freely under gravity. Find the momentum and kinetic energy of the body 5 seconds after it starts falling. Take $g = 10$ ms^{-2}. (**Ans.** 50 kg ms^{-1}, 1250 J)

7. Two bodies of masses 1 g and 16 g are moving with equal kinetic energies. Find the ratio of the magnitudes of their linear momenta. (**Ans.** 1 : 4)

8. If the momentum of a body is increased by 50%, then what will be the percentage increase in the kinetic energy of the body ? *[Central Schools 10]*
(**Ans.** 125%)

9. The kinetic energy of a body decreases by 19%. What is the percentage decrease in its linear momentum ? (**Ans.** 10%)

10. A running man has half the kinetic energy that a boy of half his mass has. The man speeds up by 1.0 ms^{-1} and then has the same energy as the boy. What were the original speeds of the man and the boy ? (**Ans.** 2.414 ms^{-1}, 4.828 ms^{-1})

11. While catching a cricket ball of mass 200 g moving with a velocity of 20 ms^{-1}, the player draws his hands backwards through 20 cm. Find the work done in catching the ball and the average force exerted by the ball on the hand. (**Ans.** 40 J, 200 N)

HINTS

1. $K.E. = \dfrac{p^2}{2m} = \dfrac{(500)^2}{2 \times 5} = 2.5 \times 10^4$ J.

2. Speed, $v = \dfrac{\text{Distance}}{\text{Time}} = \dfrac{30\text{ m}}{4\text{ s}} = 7.5$ ms^{-1}

$$K.E. = \frac{1}{2} \times 0.02 \times (7.5)^2 = 0.5625 \text{ J}.$$

3. Here $m = 2$ kg, $u = 0$, $F = 2$ N, $\mu_k = 0.2$, $t = 10$ s

$$f_k = \mu_k mg = 0.2 \times 2 \times 10 = 4 \text{ N}$$

Net force, $F' = F - f_k = 20 - 4 = 16$ N

Acceleration, $a = \dfrac{F'}{m} = \dfrac{16}{2} = 8$ ms^{-2}

$$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 8 \times (10)^2 = 400 \text{ m}$$

(*a*) Work done by the applied force,

$$W_1 = Fs = 20 \times 400 = 8000 \text{ J}.$$

(*b*) Change in K.E. = Work done by the net force

$$W_2 = F's = 16 \times 400 = 6400 \text{ J}.$$

5. (*i*) $K.E. = \dfrac{1}{2}mv^2 = \dfrac{1}{2} \times 1.67 \times 10^{-27} \times (7 \times 10^5)^2$

$$= 40.915 \times 10^{-17} \text{ J}.$$

(*ii*) $F \times s = $ K.E. or $F \times \dfrac{0.01}{100} = 40.915 \times 10^{-17}$

or $F = \dfrac{40.915 \times 100 \times 10^{-17}}{0.01} = 40.915 \times 10^{-13}$ N.

7. Kinetic energy, $K = \dfrac{p^2}{2m}$ $\therefore$ $p = \sqrt{2mK}$ or $p \propto \sqrt{m}$

$$\therefore \quad \frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{1}{16}} = \frac{1}{4} = 1 : 4.$$

8. Proceed as in Example 21.

9. Initial kinetic energy, $K = \dfrac{p^2}{2m}$

Initial momentum, $p = \sqrt{2mK}$

Decrease in K.E. = 19% of $K = 0.19K$

Final K.E., $K' = K - 0.19K = 0.81K$

Final momentum,

$$p' = \sqrt{2mK'} = \sqrt{2m \times 0.81K} = 0.9\sqrt{2mK} = 0.9p$$

% Decrease in momentum

$$= \frac{p - p'}{p} \times 100 = \frac{p - 0.9p}{p} \times 100 = 10\%.$$

10. Let M be the mass of the man, $M/2$ that of the boy and V and v be their respective velocities. As the K.E. of the man is half the K.E. of the boy, so

$$\frac{1}{2}MV^2 = \frac{1}{2} \cdot \frac{1}{2}\left(\frac{M}{2}\right)v^2 \text{ or } v^2 = 4V^2 \text{ or } v = 2V$$

When the velocity of the man is increased to $(V + 1)$, their kinetic energies become equal.

$$\therefore \quad \frac{1}{2}M(V+1)^2 = \frac{1}{2}\left(\frac{M}{2}\right)v^2 = \frac{1}{4}M.(2V)^2$$

or $V^2 + 2V + 1 = 2V^2$ or $V^2 - 2V - 1 = 0$

$$\therefore \quad V = \sqrt{2} + 1 = 2.414 \text{ ms}^{-1},$$
$$v = 2(\sqrt{2} + 1) = 4.828 \text{ ms}^{-1}.$$

11. $W =$ Change in K.E. $= \dfrac{1}{2}mv^2 = \dfrac{1}{2} \times 0.2 \times 20 \times 20 = 40$ J

$$\therefore \quad F = \frac{W}{s} = \frac{40}{0.20} = 200 \text{ N}.$$

6.11 POTENTIAL ENERGY

15. *What is meant by potential energy ? Give some examples.*

Potential energy. *Potential energy is the energy stored in a body or a system by virtue of its position in a field of force or by its configuration.*

Potential energy is also called *mutual energy* or *energy of configuration*. It is measured by the amount of work that a body or system can do in passing from its present position or configuration to some standard position or configuration, called *zero position* or *zero configuration*.

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Examples of potential energy due to position :

(*i*) A body lying on the roof of a building has some potential energy. When allowed to fall down, it can do work.

(*ii*) The potential energy of water stored to great heights in dams is used to run turbines for generating hydroelectricity.

Examples of potential energy due to configuration :

(*i*) In a toy car, the wound spring has potential energy. As the spring is released, its potential energy changes into kinetic energy which moves the toy car.

(*ii*) A stretched bow possesses potential energy. As soon as it is released, it shoots the arrow in the forward direction with a large velocity. The potential energy of the stretched bow gets converted into the kinetic energy.

(*iii*) Due to the potential energy of the compressed spring in a loaded gun, the bullet is fired with a large velocity on firing the gun.

16. *Mention some common types of potential energy.*

Different types of potential energies. Three common types of potential energies are as follows :

(*i*) ***Gravitational potential energy.*** It is the potential energy associated with the state of separation of two bodies, which attract one another through the gravitational force.

(*ii*) ***Elastic potential energy.*** It is the potential energy associated with the state of compression or extension of an elastic (spring like) object.

(*iii*) ***Electrostatic potential energy.*** The energy due to the interaction between two electric charges is electrostatic potential energy.

6.12 GRAVITATIONAL POTENTIAL ENERGY

17. *Define gravitational potential energy. Derive an expression for the gravitational P.E. of a body of mass m ring at height h above the earth's surface.*

Gravitational potential ergy. *The gravitational ential energy of a body is energy possessed by the by virtue of its position e the surface of the earth.*

Expression for G.P.E. *ider a body of mass m on the surface of the as shown in Fig. 6.15. be the acceleration gravity at this place.*

Fig. 6.15 Computation of G.P.E.

For heights much smaller than the radius of the earth ($h << R_E$) the value g can be taken constant.

Force needed to lift the body up with zero acceleration,

F = Weight of the body = mg

Work done on the body in raising it through height h,

$$W = F \cdot h = mg \cdot h$$

This work done against gravity is stored as the gravitational potential energy (U) of the body.

$$\therefore \quad U = mg\,h$$

At the surface of the earth, $h = 0$

$\therefore$ Gravitational P.E. at the earth's surface = zero.

6.13 CONSERVATIVE AND NON-CONSERVATIVE FORCES

18. *What are conservative forces ? Explain.*

Conservative force. *A force is conservative if the work done by the force in displacing a particle from one point to another is independent of the path followed by the particle and depends only on the end points.*

Suppose a particle moves from point A to point B along either path 1 or path 2, as shown in Fig. 6.16 (*a*). If a conservative force F acts on the particle, then the work done on the particle is same along the two paths.

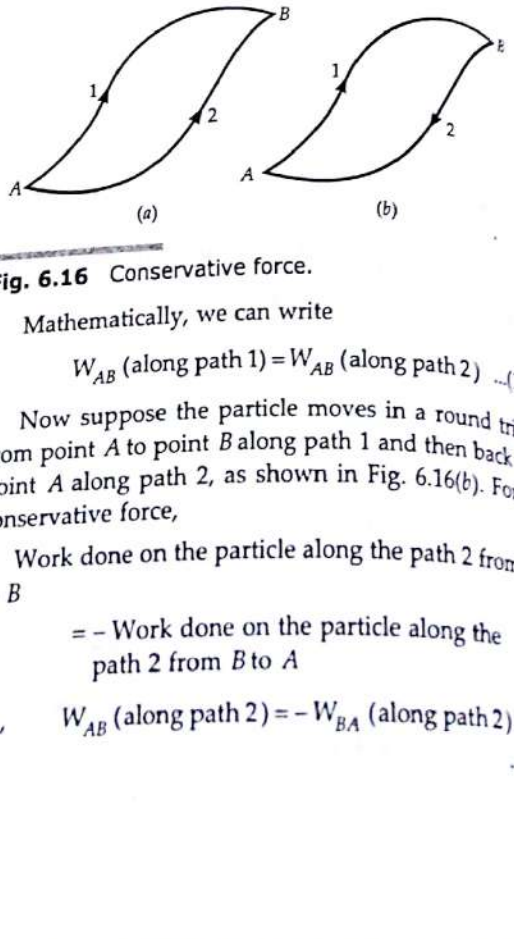

Fig. 6.16 Conservative force.

Mathematically, we can write

$$W_{AB}\,(\text{along path 1}) = W_{AB}\,(\text{along path 2}) \quad \ldots(1)$$

Now suppose the particle moves in a round trip, from point A to point B along path 1 and then back to point A along path 2, as shown in Fig. 6.16(*b*). For a conservative force,

Work done on the particle along the path 2 from A to B

= – Work done on the particle along the path 2 from B to A

i.e., $$W_{AB}\,(\text{along path 2}) = -\,W_{BA}\,(\text{along path 2}) \quad \ldots(2)$$

From (1) and (2), we have

$$W_{AB}\,(\text{along path 1}) = -\,W_{BA}\,(\text{along path 2})$$

or $$W_{AB}\,(\text{along path 1}) + W_{BA}\,(\text{along path 2}) = 0$$

or $$W_{\text{closed path}} = 0$$

Hence a force is conservative if the work done force in moving a particle around any closed path is

Examples. Gravitational force, electrostatic and elastic force of a spring are all conservative

19. *Explain qualitatively that how is the tional force a conservative force.*

Conservative nature of gravitational for shown in Fig. 6.17(*a*), suppose a body of m raised to a height h vertically upwards from to B. The work done against gravity is

$$W = mg \times AB = mg\,h$$

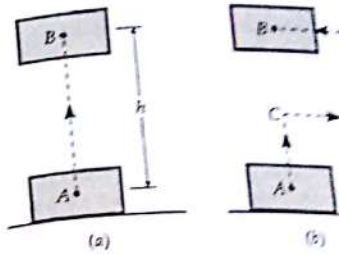

Fig. 6.17 Gravitational force as a cons

As shown in Fig. 6.17(*b*), now suppo taken from position A to B along the During the horizontal paths CD and gravity is perpendicular to the displac done is zero. Work is done only alon AC and DE. The total work done is

$$W = W_{AC} + W_{CD} + W_{DE}$$

$$= mg \times AC + 0 + mg \times$$

$$= mg\,(AC + DE) = mg$$

or $$W = mg\,h$$

Thus the work done in movin gravity is independent of the path only on the initial and final posi Hence gravitational force is a cons

(*ii*) Suppose a ball is thrown ve it rises, the gravitational force do it, decreasing its kinetic energy. the gravitational force does p increasing its kinetic energy. The point of projection with the sa

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From (1) and (2), we have

$$W_{AB} \text{ (along path 1)} = -W_{BA} \text{ (along path 2)}$$

or $$W_{AB} \text{ (along path 1)} + W_{BA} \text{ (along path 2)} = 0$$

or $$W_{\text{closed path}} = 0$$

Hence a force is conservative if the work done by the force in moving a particle around any closed path is zero.

Examples. Gravitational force, electrostatic force and elastic force of a spring are all conservative forces.

19. *Explain qualitatively that how is the gravitational force a conservative force.*

Conservative nature of gravitational force. (*i*) As shown in Fig. 6.17(*a*), suppose a body of mass m is raised to a height h vertically upwards from position A to B. The work done against gravity is

$$W = mg \times AB = mg\,h$$

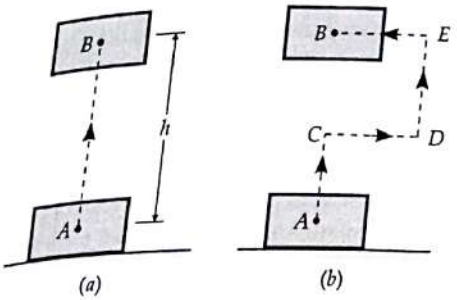

Fig. 6.17 Gravitational force as a conservative force.

As shown in Fig. 6.17(*b*), now suppose the body is taken from position A to B along the path $ACDEB$. During the horizontal paths CD and EB, the force of gravity is perpendicular to the displacement, so work done is zero. Work is done only along vertical paths AC and DE. The total work done is

$$W = W_{AC} + W_{CD} + W_{DE} + W_{EB}$$

$$= mg \times AC + 0 + mg \times DE + 0$$

$$= mg\,(AC + DE) = mg \times h$$

or $$W = mg\,h$$

Thus the work done in moving a body against gravity is independent of the path taken and depends only on the initial and final positions of the body. Hence gravitational force is a conservative force.

(*ii*) Suppose a ball is thrown vertically upward. As it rises, the gravitational force does negative work on it, decreasing its kinetic energy. As the ball descends, the gravitational force does positive work on it, increasing its kinetic energy. The ball falls back to the point of projection with the same velocity and K.E. with which it was thrown up. The net work done by the gravitational force on the ball during the round trip is zero. This again shows that the gravitational force is a conservative force.

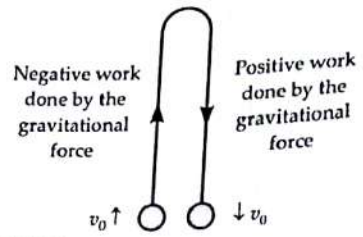

Fig. 16.18 Conservative nature of gravitational force.

20. *What is a non-conservative force ? Give examples.*

Non-conservative force. *If the amount of work done in moving an object against a force from one point to another depends on the path along which the body moves, then such a force is called a non-conservative force.* The work done in moving an object against a non-conservative force along a closed path is not zero.

Examples. Forces of friction and viscosity are non-conservative forces.

21. *Define potential energy in context with a conservative force. How can these two quantities be determined from each other ? Write the necessary relations.*

Potential energy in relation to conservative force. *The potential energy is the energy associated with the configuration of a system in which a conservative force acts.* When the conservative force $F(x)$ (for simplicity, in one dimension) does work W on a particle within the system, the change in potential energy ΔU of the system is equal to the negative of the work done by the conservative force, *i.e.*,

$$\Delta U = -W$$

But $$W = \int_{x_i}^{x_f} F(x)\,dx$$

$$\therefore \quad \Delta U = -\int_{x_i}^{x_f} F(x)\,dx$$

Differentiating the above equation, we get

$$\frac{dU(x)}{dx} = -F(x)$$

or $$F(x) = -\frac{dU(x)}{dx}$$

Hence *potential energy U may be defined as a function whose negative gradient gives the force. Conversely, we may define conservative force as a force which is equal to the negative gradient of the potential energy U.*

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22. *Show analytically that gravitational force is a conservative force.*

Gravitational force as a conservative force (quantitative approach). The gravitational potential energy of a body of mass m lying at height h above the ground is given by the function,

$$U(h) = mg\,h$$

The negative gradient of the potential energy U is given by

$$-\frac{dU(h)}{dh} = -\frac{d}{dh}(mg\,h) = -mg = F$$

i.e., the negative gradient of the potential energy is equal to the gravitational force. This proves the conservative nature of the gravitational force. The negative sign in the above equation indicates that the gravitational force acts in the downward direction.

Moreover, if a body of mass m is released from rest, from the top of a smooth (frictionless) inclined plane of height h, it gains speed at the bottom given by

$$v^2 - 0^2 = 2gh \quad \text{or} \quad v = \sqrt{2gh}$$

K.E. acquired by the body at the bottom of the inclined plane

$$= \frac{1}{2}mv^2 = \frac{1}{2}m \times 2gh = mg\,h$$

= Work done by the gravitational force

Clearly, the work done by the gravitational force does not depend on the angle of inclination of the inclined plane or the path of the falling body. It depends only on the initial and final positions. This again proves that the gravitational force is a conservative force.

6.14 PRINCIPLE OF CONSERVATION OF MECHANICAL ENERGY

23. *State and prove the principle of conservation of mechanical energy.*

Principle of conservation of mechanical energy. *This principle states that if only the conservative forces are doing work on a body, then its total mechanical energy (K.E. + P.E.) remains constant.*

Proof. Suppose that a body undergoes displacement Δx under the action of a conservative force $F(x)$. Then from work-energy theorem, the change in K.E. is

$$\Delta K = F(x)\,\Delta x$$

As the force is conservative the change in potential energy is given by

$$\Delta U = \text{Negative of the work done} = -F(x)\,\Delta x$$

Combining the above two equations, we get

$$\Delta K = -\Delta U$$

or $$\Delta K + \Delta U = 0 \quad \text{or} \quad \Delta(K + U) = 0$$

or $$K + U = \text{constant}$$

or $$K_i + U_i = K_f + U_f$$

Although, individually the kinetic energy K and potential energy U may change from one state of the system to another, but their sum or the total mechanical energy of the system remains constant under the conservative force.

24. *Mention the various properties of the conservative forces.*

Properties of the conservative forces :

(i) A force F is conservative if it can be defined from the scalar potential energy function $U(x)$ by the relation,

$$F(x) = -\frac{dU(x)}{dx}$$

(ii) The work done by a conservative force on an object is path independent and depends only on the end points.

$$W = \int_{x_i}^{x_f} F(x)\,dx = K_f - K_i = U_i - U_f$$

(iii) The work done by the conservative force is zero if the object moving around any closed path returns to its initial position.

$$W_{\text{closed path}} = \oint F(x)\,dx = 0$$

(iv) If only the conservative forces are acting on body, then its total mechanical energy is conserved.

6.15 CONSERVATION OF MECHANICAL ENERGY IN A FREELY FALLING BODY

25. *Show that the total mechanical energy of a freely falling body remains constant throughout its fall.*

Conservation of mechanical energy in case of a freely falling body. Consider a body of mass m lying at position A at a height h above the ground. As the body falls, its kinetic energy increases at the expense of potential energy.

At point A. The body is at rest.

K.E. of the body,

$$K_A = 0$$

P.E. of the body,

$$U_A = mg\,h$$

Total mechanical energy,

$$E_A = K_A + U_A = mg\,h$$

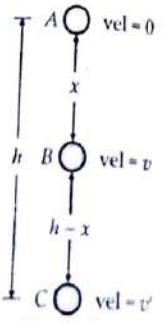

Fig. 6.19 Conservation of energy during a free fall.

At point B. Suppose the b
height x and reaches the point

$$v^2 - 0^2 = 2gx$$

$$v^2 = 2gx$$

or $$K_B = \frac{1}{2}mv^2 = \frac{1}{2}m$$

$$U_B = mg(h - x)$$

$$E_B = K_B + U_B =$$

At point C. Suppose th
point C on the ground wit
sidering motion from A to C

$$v'^2 - 0^2 = 2gh \quad \text{or}$$

$$K_C = \frac{1}{2}mv'^2 = \frac{1}{2}$$

$$U_C = mg \times 0 = 0$$

$$E_C = K_C + U_C =$$

Clearly, as the body falls,
increases by an equal am
mechanical energy remain
points. Thus total mechan
during free fall of a body. Fi
of K.E. and P.E. and the con
height.

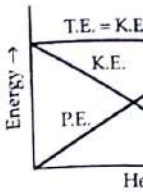

Fig. 6.20 Plots of K.E. a
of a body.

Examples

P.E. and Conse

FORMULAE USED

1. Gravitational P.E., U
2. For a conservative fo
3. $\Delta U = U_f - U_i = -W$
4. When work is done
only, mechanical en

$$K + U = \text{cons}$$

UNITS USED

Force is in newtor
energy K and poter

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At point B. Suppose the body falls freely through height x and reaches the point B with velocity v. Then

$$v^2 - 0^2 = 2gx \qquad [\text{Using } v^2 - u^2 = 2as]$$

$$v^2 = 2gx$$

or $$K_B = \frac{1}{2}mv^2 = \frac{1}{2}m \times 2gx = mgx$$

$$\therefore \quad U_B = mg(h - x)$$

$$E_B = K_B + U_B = mgx + mg(h - x) = mgh$$

At point C. Suppose the body finally reaches a point C on the ground with velocity v'. Then considering motion from A to C,

$$v'^2 - 0^2 = 2gh \quad \text{or} \quad v'^2 = 2gh$$

$$K_C = \frac{1}{2}mv'^2 = \frac{1}{2}m \times 2gh = mgh$$

$$\therefore \quad U_C = mg \times 0 = 0$$

$$E_C = K_C + U_C = mgh$$

Clearly, as the body falls, its P.E. decreases and K.E. increases by an equal amount. However, its total mechanical energy remains constant (= mgh) at all points. Thus total mechanical energy is conserved during free fall of a body. Fig. 6.20 shows the variation of K.E. and P.E. and the constancy of total energy with height.

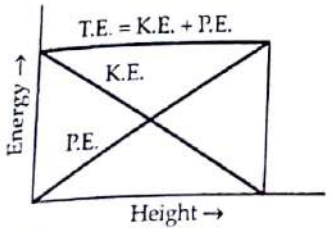

Fig. 6.20 Plots of K.E. and P.E. during free fall of a body.

Examples based on P.E. and Conservation of Energy

Formulae Used

1. Gravitational P.E., $U = mgh$
2. For a conservative force, $F = -\dfrac{dU}{dx}$
3. $\Delta U = U_f - U_i = -W = -\displaystyle\int_{x_i}^{x_f} F\,dx$
4. When work is done only by conservative forces only, mechanical energy is conserved.
   $K + U = \text{constant}$.

Units Used

Force is in newton and work done W, kinetic energy K and potential energy U are in joule.

Example 24. A vehicle of mass 15 quintal climbs up a hill 200 m high. It then moves on a level road with speed of 30 ms^{-1}. Calculate the potential energy gained by it and its total mechanical energy while running on the top of the hill.

Solution. Here $m = 15$ quintal $= 1500$ kg,

$g = 9.8\ ms^{-2}$, $h = 200$ m

P.E. gained,

$$U = mgh = 1500 \times 9.8 \times 200 = \mathbf{2.94 \times 10^6\ J}$$

When the vehicle runs on a level road with speed of 30 ms^{-1}, its K.E. is

$$K = \frac{1}{2}mv^2 = \frac{1}{2} \times 1500 \times (30)^2 = 0.675 \times 10^6\ J$$

Total mechanical energy,

$$E = K + U = (0.675 + 2.94) \times 10^6 = \mathbf{3.615 \times 10^6\ J}$$

Example 25. Calculate the velocity of the bob of a simple pendulum at its mean position if it is able to rise to a vertical height of 10 cm. Take $g = 9.8\ ms^{-2}$.

Solution. By conservation of energy,

K.E. of the bob at the mean position

= P.E. of the bob at the highest position

or $$\frac{1}{2}mv^2 = mgh$$

$$\therefore \quad v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 0.10} = \sqrt{1.96} = \mathbf{1.4\ ms^{-1}}.$$

Example 26. A girl of mass 40 kg sits in a swing formed by a rope of 6 m length. A person pulls the swing to a side so that the rope makes an angle of 60° with the vertical. What is the gain in potential energy of the girl ?

Solution. Here, $m = 40$ kg, $OA = OB = 6$ m

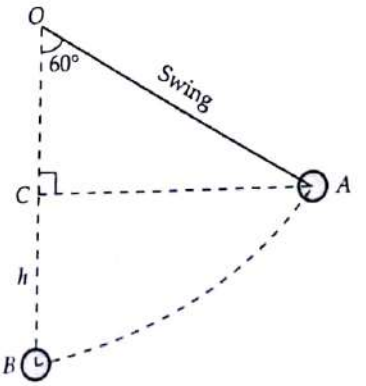

Fig. 6.21

From right ΔOCA,

$$\frac{OC}{OA} = \cos 60°$$

$$OC = OA \cos 60° = 6 \times \frac{1}{2} = 3\ \text{m}$$

Height through which the girl is raised,

$$h = CB = OB - OC = 6 - 3 = 3\ \text{m}$$

$\therefore$ P.E. gained by the girl

$$= mgh = 40 \times 9.8 \times 3 = \mathbf{1176\ J}.$$

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EXAMPLE 27. How high must a body be lifted to gain an amount of potential energy equal to the kinetic energy it has when moving at speed 20 ms^{-1} ? The value of acceleration due to gravity at a place is $g = 9.8\ ms^{-2}$. **[Delhi 97, 05]**

Solution. Here $mgh = \frac{1}{2}mv^2$

$$\therefore \quad h = \frac{v^2}{2g} = \frac{20 \times 20}{2 \times 9.8} = 20.2 \text{ m}.$$

EXAMPLE 28. The string of a pendulum is 2.0 m long. The bob is pulled sideways so that the string becomes horizontal and then the bob is released. What is the speed with which the bob arrives at the lowest point ? Assume that 10% of the initial energy is dissipated against air resistance, $g = 10\ ms^{-2}$.

Solution. Here $\frac{1}{2}mv^2 = 90\%$ of mgh

or $$\frac{1}{2}mv^2 = \frac{90}{100} \times mgh$$

$$\therefore \quad v = \sqrt{\frac{2 \times 90 \times g\,h}{100}} = \sqrt{\frac{2 \times 90 \times 10 \times 2}{100}} = 6 \text{ ms}^{-1}.$$

EXAMPLE 29. A ball bounces to 80% of its original height. What fraction of its mechanical energy is lost in each bounce ? Where does this energy go ? **[Delhi 97]**

Solution. Suppose the ball is dropped from height h.

Initial P.E. $= mgh$

P.E. after first bounce $= mg \times 80\%$ of $h = 0.80\ mgh$

P.E. lost in each bounce $= 0.20\ mgh$

Fraction of P.E. lost in each bounce

$$= \frac{0.20\ mgh}{mgh} = 0.20$$

This energy is lost in the form of heat, sound, etc.

EXAMPLE 30. A ball at rest is dropped from a height of 12 m. It loses 25% of its kinetic energy in striking the ground, find the height to which it bounces. How do you account for the loss in kinetic energy ?

Solution. K.E. gained by the ball in falling down

= P.E. lost by the ball in falling down $= mgh$

On bouncing upwards, the ball loses 25% of its kinetic energy and the remaining 75% changes back into potential energy. If the ball bounces to height h', then

$$mgh' = 75\% \text{ of } mgh$$

$$\therefore \quad mgh' = \frac{75}{100} \times mgh$$

or $$h' = 0.75\ h = 0.75 \times 12 = 9 \text{ m}.$$

The loss in kinetic energy of the ball occurs because part of it changes into sound and heat.

EXAMPLE 31. Fig. 6.22 shows a frictionless hemispherical bowl of radius R. A ball of mass m is pushed down the bowl from a point A. It just rises up to the edge of the bowl. Calculate the speed with which the ball is pushed down along the wall.

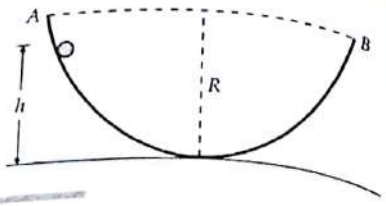

Fig. 6.22

Solution. Let the ball be pushed down along the wall with a speed v. According to law of conservation of energy,

Total energy at A = Total energy at B

or $$\frac{1}{2}mv^2 + mgh = 0 + mgR$$

or $$v^2 = 2g\,(R - h)$$

or $$v = \sqrt{2g\,(R - h)}.$$

EXAMPLE 32. A body of mass $M = 9.8$ kg with a small disc of mass $m = 0.2$ kg placed on its horizontal surface ab, rests on a smooth horizontal plane, as shown in Fig. 6.23. The disc can move freely along the smooth groove abc of mass M. To what height (relative to the initial position) will the disc rise after separating from the body of mass M when initial velocity $v = 5\ ms^{-1}$ is given to it in the horizontal direction ?

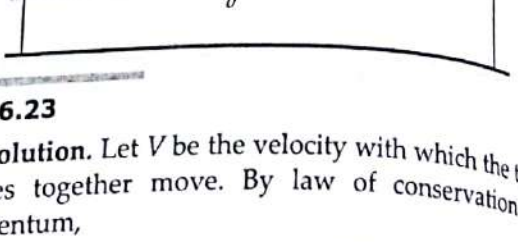

Fig. 6.23

Solution. Let V be the velocity with which the two bodies together move. By law of conservation of momentum,

$$(M + m)\,V = mv \quad \text{or} \quad V = \frac{mv}{M + m}$$

But $$m = 0.2 \text{ kg},\ M = 9.8 \text{ kg},\ v = 5 \text{ ms}^{-1}$$

$$\therefore \quad V = \frac{0.2 \times 5}{9.8 + 0.2} = 0.1 \text{ ms}^{-1}$$

K.E. lost by the disc

$$= \frac{1}{2}mv^2 - \frac{1}{2}(M + m)\,V^2$$

$$= \frac{1}{2} \times 0.2 \times 25 - \frac{1}{2}(9.8 + 0.2) \times 0.01$$

$$= 2.45 \text{ J}$$

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Suppose the disc rises to height h after separating from the body of mass M. Then

P.E. gained by the disc

$= mgh = 0.2 \times 9.8 \times h$ joule

By conservation of energy,

P.E gained = K.E. lost

$0.2 \times 9.8 \times h = 2.45$

$\therefore \quad h = \dfrac{2.45}{0.2 \times 9.8} = \mathbf{1.25\ m.}$

or

EXAMPLE 33. A bob of mass m is suspended by a light string of length L. It is imparted a horizontal velocity v_0 at the lowest point A such that it completes a semi-circular trajectory in the vertical plane with the string becoming slack only on reaching the topmost point, C. This is shown in Fig. 6.24. Obtain an expression for (i) v_0; (ii) the speeds at points B and C; (iii) the ratio of the kinetic energies (K_B / K_C) at B and C. Comment on the nature of the trajectory of the bob after it reaches the point C. [NCERT]

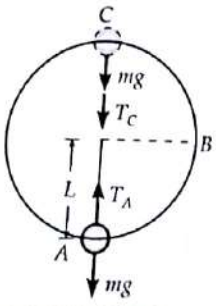

Fig. 6.24

Solution. (*i*) Two external forces act on the bob : gravity and tension (T) in the string. At the lowest point A, the potential energy of the system can be taken zero. So at point A,

Total mechanical energy = Kinetic energy

$$E = \frac{1}{2} mv_0^2 \qquad ...(1)$$

If T_A is the tension in the string at point A, then from Newton's second law,

$$T_A - mg = \frac{mv_0^2}{L} \qquad ...(2)$$

At the highest point C, the string slackens, so the tension T_C becomes zero. If v_C is the speed at point C, then by conservation of energy,

$$E = K + U \quad \text{or} \quad E = \frac{1}{2} mv_C^2 + 2\ mgL \qquad ...(3)$$

From Newton's second law,

$$mg = \frac{mv_C^2}{L} \qquad ...(4)$$

or
$$mv_C^2 = mgL \qquad ...(5)$$

Using (5) in (3),

$$E = \frac{1}{2} mgL + 2\ mgL = \frac{5}{2} mgL \qquad ...(6)$$

From equations (1) and (6), we get

$$\frac{5}{2} mgL = \frac{m}{2} v_0^2 \quad \text{or} \quad v_0 = \sqrt{5\,gL} \qquad ...(7)$$

(*ii*) From equation (4), we have

$$v_C = \sqrt{gL}$$

The total energy at B is

$$E = \frac{1}{2} mv_B^2 + mgL \qquad ...(8)$$

From equations (1) and (8), we get

$$\frac{1}{2} mv_B^2 + mgL = \frac{1}{2} mv_0^2$$

$$\frac{1}{2} mv_B^2 + mgL = \frac{1}{2} m \times 5\,gL \qquad \text{[using (7)]}$$

$$\therefore \quad v_B = \sqrt{3\,gL}$$

(*iii*) The ratio of the kinetic energies at B and C is

$$\frac{K_B}{K_C} = \frac{\frac{1}{2} mv_B^2}{\frac{1}{2} mv_C^2} = \frac{3}{1} = \mathbf{3:1.}$$

EXAMPLE 34. A ball falls under gravity from a height of 10 m with an initial downward velocity u. It collides with the ground, loses 50% of its energy in collision and then rises back to the same height. Find the initial velocity u. [IIT]

Solution. If m is the mass of the ball, then its total initial energy at height h

$$= \frac{1}{2} mu^2 + mgh$$

Energy after collision

$$= 50\% \text{ of } \left(\frac{1}{2} mu^2 + mgh\right) = \frac{1}{2}\left(\frac{1}{2} mu^2 + mgh\right)$$

As the ball rebounds to height h, so

$$\frac{1}{2}\left(\frac{1}{2} mu^2 + mgh\right) = mgh \quad \text{or} \quad \frac{1}{4} mu^2 = \frac{1}{2} mgh$$

or $\quad u = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 10} = \mathbf{14\ ms^{-1}.}$

PROBLEMS FOR PRACTICE

1. A stone of mass 0.4 kg is thrown vertically up with a speed of 9.8 ms^{-1}. Find the potential and kinetic energies after half second. (**Ans.** 14.386 J, 4.802 J)
2. A ball is thrown vertically up with a velocity of 20 ms^{-1}. At what height, will its K.E. be half its original value ? (**Ans.** 10.20 m)
3. 230 joules were spent in lifting a 10 kg weight to a height of 2 m. Calculate the acceleration with which it was raised. Take g = 10 ms^{-2}. (**Ans.** 1.5 ms^{-2})
4. Calculate the work done in lifting a 300 N weight to a height of 10 m with an acceleration 0.5 ms^{-2}. Take g = 10 ms^{-2}. (**Ans.** 3150 J)
5. A bullet of mass 10 g travels horizontally with speed of 100 ms^{-1} and is absorbed by a wooden block of mass 990 g suspended by a string. Find the vertical height through which the block rises. Take g = 10 ms^{-2}. (**Ans.** 5 cm)

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6. A simple pendulum of length 1 m has a wooden bob of mass 1 kg. It is struck by a bullet of mass 10^{-2} kg moving with a speed of 2×10^2 ms^{-1}. The bullet gets embedded into the bob. Obtain the height to which the bob rises before swinging back. Take $g = 10$ ms^{-2} [Chandigarh 04] (Ans. 0.2 m)

7. A 3.0 kg block, as shown in Fig. 6.25, has a speed of 2 ms^{-1} at A and 6 ms^{-1} at B. If the distance from A to B along the curve is 12 m, how large a frictional force acts on it ? Assuming the same friction, how far from B will it stop ? (Ans. 3.35 N, 24.5 m)

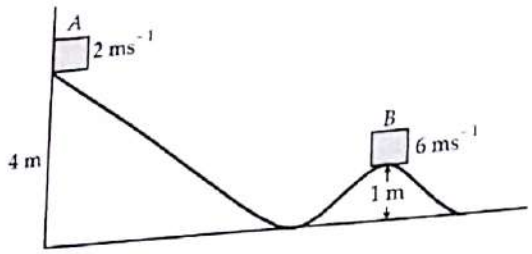

Fig. 6.25

HINTS

1. Here $m = 0.4$ kg, $u = 9.8$ ms^{-1}, $a = -g = -9.8$ ms^{-2}, $t = 1/2$ s

(i) $s = ut + \frac{1}{2}at^2 = 9.8 \times \frac{1}{2} - \frac{1}{2} \times 9.8 \times \frac{1}{2} \times \frac{1}{2} = 3.67$ m

$\therefore$ P.E. $= mgh = 0.4 \times 9.8 \times 3.67 =$ **14.386 J.**

(ii) $v = u + at = 9.8 - 9.8 \times \frac{1}{2} = 4.9$ ms^{-1}

$\therefore$ K.E. $= \frac{1}{2}mv^2 = \frac{1}{2} \times 0.4 \times 4.9 \times 4.9 =$ **4.802 J.**

2. Work done against gravitation = Change in K.E.

$$mgh = \frac{1}{2}mv^2 - \frac{1}{2} \times \frac{1}{2}mv^2 = \frac{1}{4}mv^2$$

$$\therefore \quad h = \frac{v^2}{4g} = \frac{20 \times 20}{4 \times 9.8} = \textbf{10.20 m.}$$

3. Here $W = mgh + mah = m(g + a)h$

$\therefore$ $230 = 10 \times (10 + a) \times 2$ or $10 + a = 11.5$

or $a =$ **1.5 ms^{-2}.**

4. Here $m = \frac{W}{g} = \frac{300 \text{ N}}{10 \text{ ms}^{-2}} = 30$ kg

$W = m(g + a)h = 30 \times (10 + 0.5) \times 10 =$ **3150 J.**

6. Momentum of the bullet

$= 10^{-2} \times 2 \times 10^2 = 2$ kg ms^{-1}

Let the combined velocity of the bob + bullet $= v$

Momentum of the bob + bullet

$= (10^{-2} + 1)v = 1.01v$

By conservation of momentum, $1.01v = 2$ kg ms^{-1}

or $v = \frac{2}{1.01} = 1.98$ ms^{-1}

By conservation of energy,

$\frac{1}{2}(M + m)v^2 = (M + m)gh$

or $h = \frac{v^2}{2g} = \frac{(1.98)^2}{2 \times 10} = 0.196 \text{ m} \approx 0.2 \text{ m.}$

7. Change in total energy of block

= Work done against friction

$\therefore$ $mg(h - x) + \frac{1}{2}m(v_A^2 - v_B^2) = f_s$

$3 \times 9.8 \times (4 - 1) + \frac{1}{2} \times 3 \times (2^2 - 6^2) = f \times 12$

$9 \times 9.8 - 3 \times 16 = f \times 12$

or $f = \frac{40.2}{12} =$ **3.35 N.**

Total energy at $B = 3 \times 9.8 \times 1 + \frac{1}{2} \times 3 \times 6^2$

$= 29.4 + 54 = 83.4$

This energy is used in doing work against friction.

$\therefore$ $f \times s' = 83.4$ or $s' = \frac{83.4}{f} = \frac{83.4}{3.35} = 24.5$ m.

6.16 POTENTIAL ENERGY OF A SPRING

26. *Show that the elastic force of a spring is a conservative force. Hence write an expression for the potential energy of an elastic stretched spring.*

Potential energy of a spring. Consider an elastic spring of negligibly small mass with its one end attached to a rigid support. Its other end is attached to a block of mass m which can slide over a smooth horizontal surface. The position $x = 0$ is the equilibrium position, as shown in Fig. 6.26(a). When the spring is stretched [Fig. 6.26(b)] or compressed [Fig. 6.26(c)] by pulling or pushing the block, a spring force F_s begins to act in the spring towards the equilibrium position.

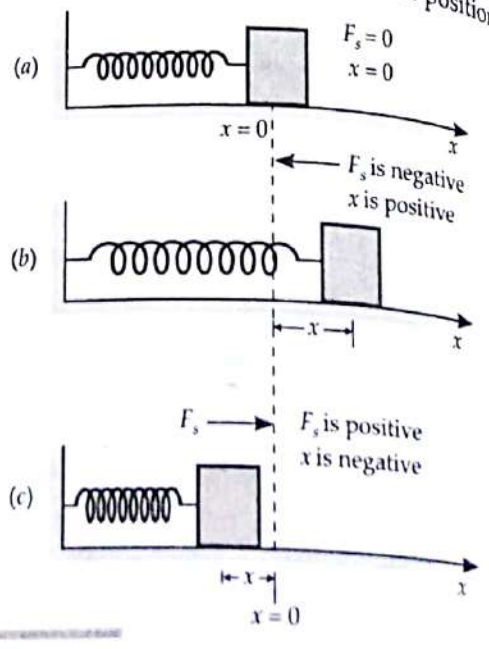

Fig. 6.26 Equilibrium, stretched and compressed states of an elastic spring.

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According to Hooke's law, the spring force F_s is proportional to the displacement of the block from the equilibrium position, i.e.,

$$F_s \propto x \quad \text{or} \quad F_s = -kx$$

The proportionality constant k is called *spring constant*. Its SI unit is Nm^{-1}. The spring is stiff if k is large and soft if k is small. The negative sign shows F_s acts in the opposite direction of x.

The work done by the spring force for the small extension dx is

$$dW_s = F_s\, dx = -kx\,dx$$

If the block is moved from an initial displacement x_i to the final displacement x_f, the work done by the spring force is

$$W_s = \int dW_s = -\int_{x_i}^{x_f} kx\,dx = -k\left[\frac{x^2}{2}\right]_{x_i}^{x_f}$$

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

or

If the block is pulled from x_i and allowed to return to x_i, then

$$W_s = -\int_{x_i}^{x_f} kx\,dx = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_i^2 = 0$$

The above discussion shows that

(i) The spring force is position dependent as is clear in Hooke's law : $F_s = -kx$,

(ii) The work done by the spring force depends on initial and final positions, and

(iii) The work done by the spring force in a cyclic process is zero.

Thus the spring force is a *conservative force*.

In order to pull the block outwards with a slow constant speed (quasi-static motion), an external force F equal and opposite to F_s has to be applied. The work done by the external force will be equal to the increase in P.E. of the spring and is given by

$$\Delta U = W = \frac{1}{2}k\,x_f^2 - \frac{1}{2}kx_i^2$$

If we take the potential energy $U(x)$ of the spring to be zero when the block is in equilibrium position, the P.E. of the spring for an extension x will be

$$U(x) - 0 = \frac{1}{2}kx^2 - 0 \quad \text{or} \quad U(x) = \frac{1}{2}kx^2$$

27. *Draw a plot of spring force F_s and displacement x. Hence find an expression for the P.E. of an elastic stretched spring.*

P.E. of an elastic stretched spring by graphical method. Fig. 6.27 shows the plot of spring force F_s versus displacement x of a block attached to the free end of the spring.

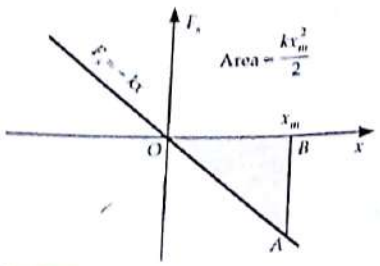

Fig. 6.27 Plot of F_s versus x.

According to Hooke's law, the spring force for an extension x_m is

$$F_s = -k\,x_m$$

The work done by the spring force for an extension x_m is

$$W_s = \text{Area of } \Delta OBA = \frac{1}{2}AB \times OB$$

$$= \frac{1}{2}F_s \times x_m = \frac{1}{2}(-k\,x_m) \times x_m = -\frac{1}{2}kx_m^2$$

In order to stretch the spring slowly, an external force F equal to and opposite to F_s has to be applied. So work done by the external force F is

$$W = -W_s = +\frac{1}{2}kx_m^2$$

This work done is stored as the P.E. of the spring.

$$\therefore \quad U = \frac{1}{2}kx_m^2$$

28. *Show that the total energy of the stretched spring remains conserved when it is released. Find the expression for the maximum speed.*

Conservation of energy in an elastic spring. If we stretch a spring to a distance x_m, its P.E. is $\frac{1}{2}kx_m^2$. When it is released, it begins to move under the spring force till it reaches the equilibrium position $x = 0$, where it has maximum velocity. All the P.E. is converted into K.E. Due to inertia of motion, the body overshoots the $x = 0$ position. Its velocity decreases until it momentarily stops at position $x = -x_m$, where all the K.E. is converted into P.E. The spring force again pulls the body towards the position $x = 0$. Thus the body keeps on oscillating. The total mechanical energy remains constant.

At the extreme positions. Here $x = \pm x_m$ and velocity $v = 0$.

$$K = \frac{1}{2}mv^2 = 0$$

$$U = \frac{1}{2}kx_m^2 = \text{a maximum value}$$

At any intermediate position x. For x between $-x_m$ to $+x_m$, the energy is partly kinetic and partly potential.

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Total energy = K.E. + P.E.

$$\frac{1}{2}kx_m^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$\therefore \quad K = \frac{1}{2}mv^2 = \frac{1}{2}k(x_m^2 - x^2)$$

Velocity, $v = \sqrt{\frac{k}{m}(x_m^2 - x^2)}$

At the equilibrium position. Here $x = 0$.

$$\therefore \quad U = \frac{1}{2}k(0)^2 = 0$$

$$K = \frac{1}{2}mv_m^2 = \frac{1}{2}kx_m^2$$

Maximum speed,

$$v_m = \sqrt{\frac{k}{m}}\,x_m.$$

The variations of K.E., P.E. and total energy with displacement x are shown in Fig. 6.28. As both K.E. and P.E. depend on x^2, their graphs are parabolic. Total mechanical energy $E = K + U$ remains constant, so its graph is a straight line parallel to displacement axis.

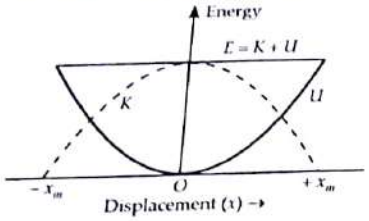

Fig. 6.28 Conservation of energy in an elastic spring.

For Your Knowledge

- The notion of potential energy applies to only those forces where the work done against the force gets *stored up* as energy by virtue of position or configuration of the body. When external constraints are removed, this energy appears as kinetic energy.
- The potential energy of a body subjected to a conservative force is uncertain upto a certain limit. This is because the point of zero potential energy is a matter of choice.
- For the gravitational P.E., the zero of potential energy is chosen to be the ground.
- For the spring potential energy $\frac{1}{2}kx^2$, the zero of the potential energy is the equilibrium position of the oscillating mass.
- Every mechanical force is not associated with a potential energy. The work done by friction over a closed path is not zero because no potential energy can be associated with friction.

Examples based on Potential Energy of a Spring

Formulae Used

1. According to Hooke's law, $F = -kx$
2. Force constant, $k = \frac{F}{x}$
3. Work done on a spring or P.E. of a spring stretched through distance x, $W = U = \frac{1}{2}kx^2$

Units Used

Force F is in newton, distance x in metre, potential energy U in joule and force constant k in Nm^{-1}.

EXAMPLE 35. Two springs have force constants k_1 and k_2 ($k_1 > k_2$). On which spring is more work done, if (i) they are stretched by the same force and (ii) they are stretched by the same amount ?

Solution. (*i*) Suppose the two springs get stretched by distances x_1 and x_2 by the same force F. Then

$$F = k_1 x_1 = k_2 x_2$$

$$\frac{W_1}{W_2} = \frac{\frac{1}{2}k_1 x_1^2}{\frac{1}{2}k_2 x_2^2} = \frac{k_1 x_1 \cdot x_1}{k_2 x_2 \cdot x_2} = \frac{F \cdot x_1}{F \cdot x_2} = \frac{x_1}{x_2} = \frac{k_2}{k_1}$$

As $k_1 > k_2 \;\therefore\; W_1 < W_2$ or $W_2 > W_1$.

(*ii*) Suppose the two springs are stretched by the same distance x. Then

$$\frac{W_1}{W_2} = \frac{\frac{1}{2}k_1 x^2}{\frac{1}{2}k_2 x^2} = \frac{k_1}{k_2}$$

As $k_1 > k_2 \;\therefore\; W_1 > W_2$.

EXAMPLE 36. The length of a steel wire increases by 0.5 cm when it is loaded with a weight of 5 kg. Calculate (i) force constant of the wire and (ii) work done in stretching the wire.

Solution. If a force F applied to a wire increases its length by x, then accordingly to Hooke's law,

$$F = kx$$

where k is force constant.

Given $\quad F = mg = 5 \times 10 = 50$ N,

$$x = 0.5 \text{ cm} = 0.5 \times 10^{-2} \text{ m}$$

$$\therefore \quad k = \frac{F}{x} = \frac{50}{0.5 \times 10^{-2}} = 1.0 \times 10^4 \text{ Nm}^{-1}.$$

(*ii*) Work done in stretching the wire,

$$W = \frac{1}{2}kx^2 = \frac{1}{2} \times 1.0 \times 10^4 \times (0.5 \times 10^{-2})^2 = 0.125 \text{ J}.$$

EXAMPLE 37. The potential energy of a spring when stretched through a distance x is 10 J. What is the amount of work done on the same spring to stretch it through an additional distance x ? [AFMC '91]

Solution. P.E. of the spring when stretched through a distance x,

$$U = \frac{1}{2}kx^2 = 10 \text{ J}$$

When x becomes $2x$, the potential energy

$$U' = \frac{1}{2}k(2x)^2 = 4 \times \frac{1}{2}kx^2 = 4 \times 10$$

$\therefore$ Work done $= U' - U = 40 - 10 = 30$ J.

EXAMPLE 38. To simulate car accidents, auto manufacturers study the collisions of moving cars with mounted springs of different spring constants. Consider a typical spring with a car of mass 1000 kg moving with a speed of 18 km/h on a smooth road and colliding with a horizontal spring of spring constant 6.25×10^3 Nm^{-1}. What is the maximum compression of the spring ?

Solution. Here $m = 1000$ kg, $k = 6.25 \times 10^3$ Nm^{-1}, $v = 18$ kmh$^{-1} = \frac{18 \times 5}{18}$ ms^{-1} $= 5$ ms^{-1}

At the maximum compression x_m, the kinetic energy of the car is converted entirely into the potential energy of the spring. Therefore,

Gain in P.E. of the spring = Loss in K.E. of the car

or $\quad \frac{1}{2}kx_m^2 = \frac{1}{2}mv^2$

or $\quad x_m^2 = \frac{mv^2}{k} = \frac{1000 \times 5 \times 5}{6.25 \times 10^3}$

$\therefore \quad x_m = 2.0$ m.

EXAMPLE 39. Consider Example 38 to be on a surface of friction, μ, to be 0.5 and calculate the maximum compression of the spring.

Solution. In presence of friction, the kinetic energy of the car is converted into the potential energy of the spring and the frictional force acting over the compression of the spring, as shown in Fig. 6.29.

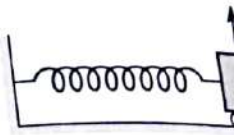

Fig. 6.29

The change in K.E. of the car $= \Delta K = K_f - K_i =$

The work done by the frictional force $= W = -\frac{1}{2}kx_m^2$

By work-energy theorem, $W = \Delta K$

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Solution. P.E. of the spring when stretched through a distance x,

$$U = \frac{1}{2} kx^2 = 10 \text{ J}$$

When x becomes $2x$, the potential energy will be

$$U' = \frac{1}{2} k (2x)^2 = 4 \times \frac{1}{2} kx^2 = 4 \times 10 = 40 \text{ J}$$

$\therefore$ Work done $= U' - U = 40 - 10 = 30$ **J.**

EXAMPLE 38. To simulate car accidents, auto manufacturers study the collisions of moving cars with mounted springs of different spring constants. Consider a typical simulation with a car of mass 1000 kg moving with a speed 18.0 kmh^{-1} on a smooth road and colliding with a horizontally mounted spring of spring constant 6.25×10^3 Nm^{-1}. What is the maximum compression of the spring ? [NCERT]

Solution. Here $m = 1000$ kg, $k = 6.25 \times 10^3$ Nm^{-1},

$$v = 18 \text{ kmh}^{-1} = \frac{18 \times 5}{18} \text{ ms}^{-1} = 5 \text{ ms}^{-1}$$

At the maximum compression x_m, the kinetic energy of the car is converted entirely into the potential energy of the spring. Therefore,

Gain in P.E. of the spring = Loss in K.E. of the car

$$\frac{1}{2} kx_m^2 = \frac{1}{2} mv^2$$

or

$$x_m^2 = \frac{mv^2}{k} = \frac{1000 \times 5 \times 5}{6.25 \times 10^3} = 4$$

or

$$\therefore \quad x_m = \mathbf{2.0 \text{ m}}.$$

EXAMPLE 39. Consider Example 38 taking the coefficient of friction, μ, to be 0.5 and calculate the maximum compression of the spring. [NCERT]

Solution. In presence of friction, both the spring force and the frictional force act so as to oppose the compression of the spring, as shown in Fig. 6.29.

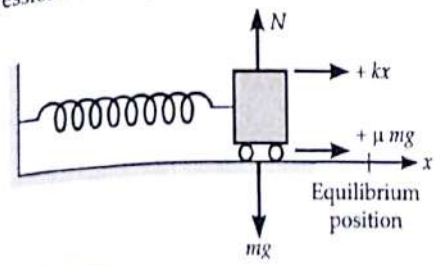

Fig. 6.29

The change in K.E. of the car is

$$\Delta K = K_f - K_i = 0 - \frac{1}{2} mv^2$$

The work done by the two opposing forces is

$$W = -\frac{1}{2} kx_m^2 - \mu mgx_m$$

By work-energy theorem,

$$W = \Delta K$$

$$\therefore \quad \frac{1}{2} kx_m^2 + \mu mg x_m = \frac{1}{2} mv^2$$

or

$$kx_m^2 + 2\mu mgx_m - mv^2 = 0$$

or

$$6.25 \times 10^3 x_m^2 + 2 \times 0.5 \times 1000 \times 10 x_m - 1000 (5)^2 = 0$$

or

$$5x_m^2 + 8x_m - 20 = 0$$

$$\therefore \quad x_m = \frac{-8 \pm \sqrt{64 + 400}}{10}$$

As x_m is positive, so

$$x_m = \frac{-8 + 21.54}{10} = \mathbf{1.354 \text{ m}}.$$

As expected, this value is less than the value obtained in the above example.

EXAMPLE 40. The spring shown in Fig. 6.30 has a force constant of 24 Nm^{-1}. The mass of the block attached to the spring is 4 kg. Initially the block is at rest and spring is unstretched. The horizontal surface is frictionless. If a constant horizontal force of 10 N is applied on the block, then what is the speed of the block when it has been moved through a distance of 0.5 m ?

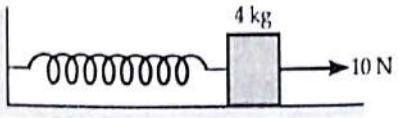

Fig. 6.30

Solution. Here $k = 24$ Nm^{-1}, $m = 4$ kg,

$x = 0.5$ m, $F = 24$ N

By the law of conservation of energy,

Work done on the spring

= Gain in K.E. + Gain in P.E.

or

$$Fx = \frac{1}{2} mv^2 + \frac{1}{2} kx^2$$

or

$$10 \times 0.5 = \frac{1}{2} \times 4 \times v^2 + \frac{1}{2} \times 24 \times (0.5)^2$$

or

$$5 = 2v^2 + 3 \quad \text{or} \quad v^2 = 1$$

$$\therefore \quad v = \mathbf{1 \text{ ms}^{-1}}.$$

EXAMPLE 41. A ball of mass m is dropped from a height h on a platform fixed at the top of a vertical spring, as shown in Fig. 6.31. The platform is depressed by a distance x. What is the spring constant k ? **[Central Schools 14]**

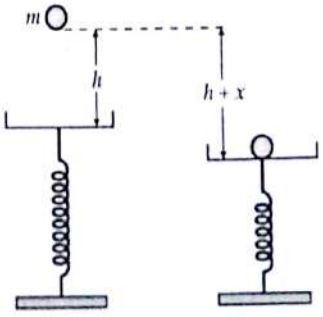

Fig. 6.31

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Solution. The ball falls through a total distance of $(h + x)$

$\therefore$ P.E. lost by the ball $= mg\,(h + x)$

Work done on the spring $= \frac{1}{2} kx^2$

By conservation of energy,

$$\frac{1}{2} kx^2 = mg\,(h + x)$$

$$\therefore \quad k = \frac{2\,mg\,(h + x)}{x^2}.$$

EXAMPLE 42. A block of mass 2 kg initially at rest is dropped from a height of 1 m into a vertical spring having force constant 490 Nm^{-1}. Calculate the maximum distance through which the spring will be compressed.

Solution. Here $m = 2$ kg, $h = 1$ m, $k = 490\ \text{Nm}^{-1}$

As shown in Fig. 6.31, let the spring be compressed through distance x. Then the block falls through a height $h + x$.

Get in P.E. of the spring = Loss in P.E. of the block

$$\frac{1}{2} kx^2 = mg\,(h + x)$$

or $$\frac{1}{2} \times 490 \times x^2 = 2 \times 9.8 \times (1 + x)$$

or $$12.5\,x^2 - x - 1 = 0$$

$$\therefore \quad x = \frac{1 \pm \sqrt{1 + 4 \times 12.5}}{2 \times 12.5} = \frac{1 \pm \sqrt{51}}{25} = 0.3256 \text{ m}.$$

EXAMPLE 43. Two blocks A and B are connected to each other as shown in Fig. 6.32. The string and spring is niassless and pulley frictionless. Block B slides over the horizontal top surface of stationary block C and the block A slides along the vertical side of C both with same uniform speed. The coefficient of friction between the blocks is 0.2 and the spring constant of spring is 1960 Nm^{-1}. If mass of block A is 2 kg, calculate (i) the mass of block B and (ii) energy stored in spring. [IIT 82]

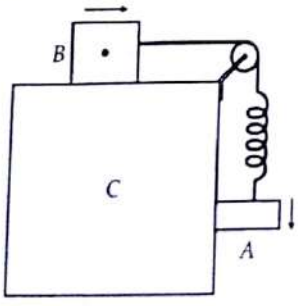

Fig. 6.32

Solution. Various forces acting on the blocks A and B are shown in Fig. 6.33.

Let mass of block $B = m$

Tension in the string $= T$

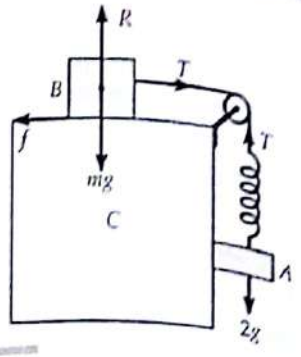

Fig. 6.33

For block A: $T = 2g$

For block B: $T = f = \mu R = \mu mg = 0.2 \times mg$ [$\because m = 2$ kg]

$\therefore \quad 0.2 \times mg = 2g \quad$ or $\quad m = \frac{2}{0.2} = 10$ kg

Also $\quad T = 2\,g = 2 \times 9.8 = 19.6$ N

Let x be the extension of the spring due to the tension T. Then $T = kx$

or $$x = \frac{T}{k} = \frac{19.6 \text{ N}}{1960 \text{ Nm}^{-1}} = 0.01 \text{ m}$$

Energy stored in the spring

$$U = \frac{1}{2} kx^2 = \frac{1}{2} \times 1960 \times (0.01)^2 = 0.098 \text{ J}.$$

PROBLEMS FOR PRACTICE

1. A spring gun has a spring constant of 18 N cm^{-1}. The spring is compressed 12 cm by a ball of mass 15 g. How much is the potential energy of the spring ? If the trigger is pulled, what will the velocity of the ball be ? (Ans. 57.6 J, 87.6 ms^{-1})
2. A solid of mass 2 kg moving with a velocity of 10 ms^{-1} strikes an ideal weightless spring and produces a compression of 25 cm in it. Calculate the force constant of the spring. (Ans. 3200 Nm^{-1})
3. A 16 kg block moving on a frictionless horizontal surface with a velocity of 5 ms^{-1} compresses an ideal spring and comes to rest. If the force constant of the spring be 100 Nm^{-1}, then how much is the spring compressed ? (Ans. 2.0 m)
4. A block of mass 2 kg is dropped from a height of 40 cm on a spring whose force-constant is 1960 Nm^{-1}. What will be the maximum distance x through which the spring is compressed ? (Ans. 10 cm)
5. A block of mass m, initially at rest, is dropped from a height h onto a spring whose force constant is k. Find the maximum distance x through which the spring will be compressed.

$$\left[\text{Ans. } x = \frac{1}{2}\left(\frac{2mg}{k} + \sqrt{\left(\frac{2mg}{k}\right)^2 + \frac{8\,mgh}{k}}\right)\right]$$

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6. An object is attached to a vertical spring and slowly lowered to its equilibrium position. This stretches the spring by a distance d. If the same object is attached to the same vertical spring but permitted to fall freely, through what distance does it stretch the spring ? **(Ans. 2 d)**

7. A massless platform is kept on a light elastic spring. When a sand particle of mass 0.1 kg is dropped on the pan from a height of 0.24 m, the particle strikes the pan, and the spring compresses by 0.01 m. From what height should the particle be dropped to cause a compression of 0.04 m ? **(Ans.** 3.96 m)

HINTS

1. Here $k = 80\ \text{Ncm}^{-1} = 80 \times 100\ \text{Nm}^{-1}$,

$x = 12\ \text{cm} = 0.12\ \text{m}$, $m = 15\ \text{g} = 15 \times 10^{-3}\ \text{kg}$

Potential energy,

$$U = \frac{1}{2} kx^2 = \frac{1}{2} \times 80 \times 100 \times (0.12)^2 = 57.6\ \text{J}.$$

Also, K.E. of ball = P.E. of the spring

or $\frac{1}{2} mv^2 = U = 57.6\ \text{J}$ or $v^2 = \frac{2 \times 57.6}{15 \times 10^{-3}} = 7680$

or $v = 87.6\ \text{ms}^{-1}$.

3. Here $\frac{1}{2} kx^2 = \frac{1}{2} mv^2$

$$\therefore \quad x = \sqrt{\frac{m}{k}} \cdot v = \sqrt{\frac{16}{100}} \times 5 = 2.0\ \text{m}.$$

4. Here $mg(h + x) = \frac{1}{2} kx^2$.

5. Here $mg(h + x) = \frac{1}{2} kx^2$ or $x^2 - \frac{2mg}{k} x - \frac{2mgh}{k} = 0$

On solving the quadratic equation, we get

$$x = \frac{1}{2}\left[\frac{2mg}{k} \pm \sqrt{\left(\frac{2mg}{k}\right)^2 + \frac{8mgh}{k}}\right]$$

6. If mg is weight of the object, then in equilibrium position,

$$kd = mg \quad \text{or} \quad k = \frac{mg}{d}$$

When the object falls freely, its gravitational P.E. is converted into elastic potential energy of the spring.

$$\therefore \quad mgx = \frac{1}{2} kx^2$$

$$\text{or} \quad mgx = \frac{1}{2}\left(\frac{mg}{d}\right) x^2$$

$$\text{or} \quad x = 2d.$$

7. Here $m = 0.1\ \text{kg}$, $h = 0.24\ \text{m}$, $x = 0.01\ \text{m}$,

$h' = ?$, $x' = 0.04\ \text{m}$

As the particle is dropped from a height h, it compresses the spring through distance x.

$\therefore$ Total loss in P.E. of the particle

= Total gain in P.E. of the spring.

$$\text{or} \quad mg(h + x) = \frac{1}{2} kx^2 \quad ...(i)$$

$$\text{and} \quad mg(h' + x') = \frac{1}{2} kx'^2 \quad ...(ii)$$

Dividing (*ii*) by (*i*), we get

$$\frac{h' + x'}{h + x} = \left(\frac{x'}{x}\right)^2$$

$$\text{or} \quad \frac{h' + 0.04}{0.24 + 0.01} = \left(\frac{0.04}{0.01}\right)^2$$

On solving, $h' = 3.96\ \text{m}$.

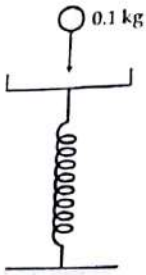

Fig. 6.34

6.17 DIFFERENT FORMS OF ENERGY

29. *Describe the various forms of energy.*

Different forms of energy. Energy can manifest itself in many forms. Some of these forms are as follows :

(*i*) ***Mechanical energy.*** The sum of the kinetic and potential energies in bulk is called mechanical energy. Kinetic energy is due to motion while the potential energy is due to position or configuration.

(*ii*) ***Internal energy.*** The molecules of a body vibrate with respect to one another. These molecules have kinetic energy due to their motion and potential energy due to their attractions and repulsions. The sum of the kinetic and potential energies of all the molecules is called the internal energy of the body. As the temperature of a body increases, its internal energy also increases.

(*iii*) ***Heat or thermal energy.*** A body possesses heat energy due to the random motion of its molecules. Heat energy is also related to the internal energy of the body. In winter, we generate heat by rubbing our hands against each other.

(*iv*) ***Chemical energy.*** A stable chemical compound has lesser energy than its constituent atoms, the difference being in the arrangement and motion of electrons in the compound. This difference is called chemical energy. A chemical reaction is just the rearrangement of atoms. If the total energy of the reactants is more than the products, heat is released and the reaction is *exothermic*. If the reverse is true, heat is absorbed and the reaction is *endothermic*. When 1 kg of coal is burnt, it releases 3×10^7 J of energy.

(*iv*) ***Electrical energy.*** Electric charges and currents attract or repel each other, *i.e.*, they exert forces on each other. Work has to be done to move charges with respect to one another. The energy associated with this work is called electrical or electromagnetic energy.

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(v) ***Nuclear energy.*** Neutrons and protons attract each other very strongly at distances of order 10^{-15} m, and bind together to form nuclei. The associated energy is called nuclear energy. Nuclear energy is released in the process of nuclear fission and nuclear fusion. In each of these processes, a part of the mass (called mass defect) is converted into energy in accordance with the Einstein's mass-energy relation.

6.18 EINSTEIN'S MASS-ENERGY EQUIVALENCE

30. *What is Einstein's mass-energy equivalence ? Mention some of its practical applications.*

Einstein's mass-energy equivalence. In 1905, Albert Einstein discovered that mass can be converted into energy and vice versa. He showed that mass and energy are equivalent and related by the relation

$$E = mc^2$$

where c the speed of light in vacuum is approximately 3×10^8 ms^{-1}. According to Einstein's mass-energy relation, if mass m disappears, an energy $E (= mc^2)$ appears in some form. Conversely, when energy E disappears, a mass $m (= E/c^2)$ appears.

Applications of mass-energy equivalence :

(i) ***Annihilation of matter.*** When an electron $({}_{-1}^{0}e)$ and a positron $({}_{1}^{0}e)$ come close to each other, they annihilate (destroy) each other forming two γ-rays (electromagnetic radiation) of total energy given by Einstein's mass-energy relation.

(ii) ***Pair production.*** When a γ-ray photon of energy 1.02 MeV passes close to a massive nucleus, it materialises into a pair of particles–an electron and a positron. Thus energy gets converted into matter.

(iii) ***Energy generation in the sun and stars.*** The energy generated in the sun and stars is due to the conversion of mass into energy.

6.19 PRINCIPLE OF CONSERVATION OF ENERGY

31. *State and explain the principle of conservation of energy.*

Principle of conservation of energy. When we throw a ball up, the chemical energy stored in our body gets transferred to the ball as its kinetic energy. As the ball moves up, it loses K.E. and gains P.E. At the highest point, all of its K.E. changes into P.E. As the ball falls on the ground, its P.E. changes into heat and sound. Although energy is being transformed from one form to another at every stage, yet its total amount remains the same.

This is the **principle of conservation of energy** which can be stated in a number of ways :

(i) *Energy can neither be created, nor destroyed. It may be transformed from one form to another.*

(ii) *The total energy of an isolated system remains constant.*

(iii) As the entire universe may be regarded as an isolated system, *the total energy of the universe is constant.* If one part of the universe loses energy, another part must gain an equal amount of energy.

For Your Knowledge

▲ In the principle of conservation of energy, we include mass into total energy, because mass can be converted into energy.

▲ The principle of conservation of energy cannot be proved mathematically, but is an empirical principle. The deductions made on the basis of this principle are found to be true.

Examples based on Mass-Energy Equivalence

FORMULAE USED

According to Einstein, energy equivalent of mass m is $E = mc^2$, where c = speed of light in free space $= 3 \times 10^8$ ms^{-1}

UNITS USED

Mass m is in kg and energy E in joule.

CONVERSIONS USED

$1\text{ eV} = 1.6 \times 10^{-19}$ J,

$1\text{ MeV} = 10^6\text{ eV} = 1.6 \times 10^{-13}$ J

1 amu = 931 MeV

EXAMPLE 44. *Express :*

(a) *The energy required to break one bond (10^{-20} J) in DNA in eV.*

(b) *The kinetic energy of an air molecule (10^{-21} J) in eV.*

(c) *The daily intake of a human adult (10^7 J) in kilocalories.*

Solution. (a) $1\text{ eV} = 1.6 \times 10^{-19}$ J

Energy required to break one bond in DNA

$$= 10^{-20}\text{ J} = \frac{10^{-20}}{1.6 \times 10^{-19}}\text{ eV} = 0.06\text{ eV}.$$

(b) $1\text{ eV} = 1.6 \times 10^{-19}$ J

Kinetic energy of an air molecule

$$= 10^{-21}\text{ J} = \frac{10^{-21}}{1.6 \times 10^{-19}} = 0.062\text{ eV}.$$

(c) 1 kcal = 4186 J

The average daily human consumption

$$= 10^{-21}\text{ J} = \frac{10^7}{4186}\text{ kcal} = 2389\text{ kcal} = \mathbf{2400\ kcal}.$$

EXAMPLE 45. *How much mass ... day in Tarapur nuclear power ...*

Solution. Power, $P = 10^7$ kW = ...

Time, $t = 1$ day = 24 ...

Energy produced per da...

$E = Pt = 10^{10}$ ...

$E = mc^2$

As $m = \frac{E}{c^2} = \frac{864}{(3 \times}$ ...

$= 9.6 \times 10$ ...

EXAMPLE 46. *If 1000 kg ... 100°C, calculate the increase ...*

Solution. Here $m = 1$ ...

$\theta = 1$ ...

Specific heat of water ...

Heat gained by water ...

$= ms\theta = 10^3 \times 1$ ...

Increase in mass,

$$\Delta m = \frac{4.2 \times 10^8}{c^2} = \frac{4}{(}$$

EXAMPLE 47. *Calculate the rest mass of an electr... electron, $m_0 = 9.1 \times 10^{-}$ ... speed of light, $c = 3 \times 10^8$ ...*

Solution. $E = m_0c^2$ ...

$= 81.9$ ...

$= \frac{81.1}{1.6}$ ...

EXAMPLE 48. *Estimate ... following nuclear fusio...*

${}_1^2H + {}_1^2H$ ...

Given mass of ${}_1^2H$ ... amu, mass of ... $= 1.661 \times 10^{-27}$ kg. Ex...

Solution. ${}_1^2H +$ ...

Total initial mas...

$= 2.$ ...

Total final mass ...

$= 3$ ...

Decrease in ma...

$\Delta m =$ ...

$=$ ...

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EXAMPLE 45. How much mass is converted into energy per day in Tarapur nuclear power plant operated at 10^7 kW ?

Solution. Power, $P = 10^7 \text{ kW} = 10^{10} \text{ W} = 10^{10} \text{ Js}^{-1}$

Time, $t = 1 \text{ day} = 24 \times 60 \times 60 \text{ s}$

Energy produced per day,

$$E = Pt = 10^{10} \times 24 \times 60 \times 60 = 864 \times 10^{12} \text{ J}$$

$$E = mc^2$$

As $$m = \frac{E}{c^2} = \frac{864 \times 10^{12}}{(3 \times 10^8)^2}$$

$\therefore$ $$= 9.6 \times 10^{-3} \text{ kg} = \mathbf{9.6 \text{ g}}.$$

EXAMPLE 46. If 1000 kg of water is heated from 0°C to 100°C, calculate the increase in the mass of water.

Solution. Here $m = 1000 \text{ kg} = 10^6 \text{ g}$,

$$\theta = 100 - 0 = 100^\circ \text{C}$$

Specific heat of water, $s = 1 \text{ cal g}^{-1} {}^\circ\text{C}^{-1}$

Heat gained by water

$$= ms\theta = 10^6 \times 1 \times 100 = 10^8 \text{ cal} = 4.2 \times 10^8 \text{ J}$$

[$\because$ 1 cal = 4.2 J]

Increase in mass,

$$\Delta m = \frac{4.2 \times 10^8}{c^2} = \frac{4.2 \times 10^8}{(3 \times 10^8)^2} = \mathbf{0.466 \times 10^{-8} \text{ kg}}.$$

EXAMPLE 47. Calculate the energy in MeV equivalent to the rest mass of an electron. Given that the rest mass of an electron, $m_0 = 9.1 \times 10^{-31}$ kg, 1 MeV $= 1.6 \times 10^{-13}$ J and speed of light, $c = 3 \times 10^8$ ms^{-1}.

Solution. $E = m_0 c^2 = 9.1 \times 10^{-31} \times (3 \times 10^8)^2$

$$= 81.9 \times 10^{-15} \text{ J}$$

$$= \frac{81.1 \times 10^{-15}}{1.6 \times 10^{-13}} = \mathbf{0.512 \text{ MeV}}.$$

EXAMPLE 48. Estimate the amount of energy released in the following nuclear fusion reaction :

$$_1^2H + {}_1^2H \rightarrow {}_2^3He + {}_0^1n$$

Given mass of $_1^2H = 2.0141$ amu, mass of $_2^3He = 3.0160$ amu, mass of $_0^1n = 1.0087$ amu and 1 amu $= 1.661 \times 10^{-27}$ kg. Express your answer in units of MeV.

Solution. $_1^2H + {}_1^2H \rightarrow {}_2^3He + {}_0^1n$

Total initial mass ($_1^2H + {}_1^2H$)

$$= 2.0141 + 2.0141 = 4.0282 \text{ amu}$$

Total final mass ($_2^3He + {}_0^1n$)

$$= 3.0160 + 1.0087 = 4.0247 \text{ amu}$$

Decrease in mass,

$$\Delta m = 4.0282 - 4.0247 = 0.0035 \text{ amu}$$

$$= 0.0035 \times 1.661 \times 10^{-27} \text{ kg}$$

$\therefore$ Energy released

$$= \Delta m \times c^2 = 0.0035 \times 1.661 \times 10^{-27} \times (3 \times 10^8)^2$$

$$= 5.232 \times 10^{-13} \text{ J} = \frac{5.232 \times 10^{-13}}{1.6 \times 10^{-13}} = \mathbf{3.27 \text{ MeV}}.$$

EXAMPLE 49. When slow neutrons are incident on a target containing $_{92}^{235}U$, a possible fission reaction is

$$_{92}^{235}U + {}_0^1n \rightarrow {}_{56}^{141}Ba + {}_{36}^{92}Kr + 3\ {}_0^1n$$

Estimate the amount of energy released using the following data :

Given, mass of $_{92}^{235}U = 235.04$ amu, mass of $_0^1n = 1.0087$ amu, mass of $_{56}^{141}Ba = 140.91$ amu, mass of $_{36}^{92}Kr = 91.926$ amu and energy equivalent to 1 amu = 931 MeV.

Solution. $_{92}^{235}U + {}_0^1n \rightarrow {}_{56}^{141}Ba + {}_{36}^{92}Kr + 3\ {}_0^1n$

Total initial mass ($_{92}^{235}U + {}_0^1n$)

$$= 235.04 + 1.0087 = 236.0487 \text{ amu}$$

Total final mass ($_{56}^{141}Ba + {}_{36}^{92}Kr + 3\ {}_0^1n$)

$$= 140.91 + 91.926 + 3 \times 1.0087$$

$$= 235.8621 \text{ amu}$$

Decrease in mass,

$$\Delta m = 236.0487 - 235.8621 = 0.1866 \text{ amu}$$

Energy released

$$= \Delta m \times 931 = 0.1866 \times 931 = \mathbf{173.725 \text{ MeV}}.$$

PROBLEMS FOR PRACTICE

1. About 4×10^9 kg of matter is converted into energy in the sun each second. What is the power output of the sun ? **(Ans.** 3.6×10^{27} W)
2. Show that energy equivalent to atomic mass unit equals nearly 933 MeV of energy. Given 1 atomic mass unit = 1.66×10^{-27} kg.
3. 500 kg of water is heated from 20° to 100°C. Calculate the increase in the mass of water. Given specific heat of water = 4.2×10^3 J kg^{-1} °C^{-1}.
   (Ans. 1.87×10^{-9} kg)
4. 1 mg of uranium is completely destroyed in an atomic bomb. How much energy is liberated ?
   [Himachal 03] (Ans. 9×10^{10} J)
5. An electron-positron pair annihilates at rest to produce γ-rays. Calculate the energy produced in MeV if the rest mass of electron is 9.1×10^{-31} kg.
   (Ans. 1.02 MeV)
6. If 10 g of mass is completely converted into energy, then find the corresponding energy produced.
   [Central Schools 14] (Ans. 9×10^{14} J)

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✕ HINTS

1. Here $m = 4 \times 10^9$ kg, $c = 3 \times 10^8$ ms^{-1}, $t = 1$ s

$$E = mc^2 = 4 \times 10^9 \times (3 \times 10^8)^2 = 3.6 \times 10^{27} \text{ J}$$

$$P = \frac{E}{t} = \frac{3.6 \times 10^{27} \text{ J}}{1 \text{ s}} = 3.6 \times 10^{27} \text{ W}.$$

2. Energy equivalent to 1 amu $= mc^2$

$$= 1.66 \times 10^{-27} \times (3 \times 10^8)^2 \text{ J} = 1.66 \times 9 \times 10^{-11} \text{ J}$$

$$= \frac{1.66 \times 9 \times 10^{-11}}{1.6 \times 10^{-13}} \text{ MeV} \quad [\because 1 \text{ MeV} = 1.6 \times 10^{-13} \text{ J}]$$

$$= 9.3375 \times 10^2 \text{ MeV} = \mathbf{933.75 \text{ MeV}}.$$

5. Rest mass of an electron or a positron $= 9.1 \times 10^{-31}$ kg

During the annihilation of electron-positron pair, the minimum amount of energy is released when the mass of electron-positron pair at rest is converted into energy.

∴ Minimum energy released

$$= 2 \times 9.1 \times 10^{-31} \times (3 \times 10^8)^2 \text{ J}$$

$$= \frac{2 \times 9.1 \times 9 \times 10^{-15}}{1.6 \times 10^{-13}} = \mathbf{1.02 \text{ MeV}}.$$

6.20 POWER

32. *Define the term power. Is it a scalar or vector quantity ? Give its dimensions and units.*

Power. *Power is defined as the rate of doing work.* If an agent does work W in time t, then its average power is given by

$$P_{av} = \frac{W}{t}$$

The shorter is the time taken by a person or a machine in performing a particular task, the larger is the power of that person or machine.

Power is a scalar quantity, because it is the ratio of two scalar quantities work (W) and time (t).

Dimensions of power.

$$[P] = \frac{[W]}{[t]} = \frac{[ML^2T^{-2}]}{[T]} = [ML^2T^{-3}]$$

Units of power. The SI unit of power is **watt** (W). *The power of an agent is one watt if it does work at the rate of 1 joule per second.*

$$1 \text{ watt} = \frac{1 \text{ joule}}{1 \text{ second}} \quad \text{or} \quad 1 \text{ W} = 1 \text{ Js}^{-1}$$

The bigger units of power are kilowatt (kW) and horse power (hp).

1 kilowatt = 1000 watt or $1 \text{ kW} = 10^3 \text{ W}$

1 horse power = 746 watt or 1 hp = 746 W.

33. *Define instantaneous power. Express it as the scalar product of force and velocity vectors.*

Instantaneous power. The power of an agent may not be constant during a time interval. The *instantaneous power is defined as the limiting value of the average power as the time interval approaches zero.* If ΔW work is done in a small time interval Δt, then the instantaneous power is given by

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$$

Power as dot product. The work done by a force $\vec{F}$ for a small displacement $\vec{dr}$ is given by

$$dW = \vec{F} \cdot \vec{dr}$$

So the instantaneous power can be expressed as

$$P = \frac{dW}{dt} = \vec{F} \cdot \frac{\vec{dr}}{dt}$$

But $\frac{\vec{dr}}{dt} = \vec{v}$, the instantaneous velocity

$$\therefore \quad P = \vec{F} \cdot \vec{v}$$

Thus the power of an agent at any instant is equal to the dot product of its force and velocity vectors at that instant.

34. *Of which physical quantity is the unit kilowatt hour ? Define one kilowatt hour. Express it in joules.*

Kilowatt hour (kWh). *Kilowatt hour* (kWh) or *Board of Trade* (B.O.T.) unit is the commercial unit of electrical energy. *One kilowatt hour is the electrical energy consumed by an appliance of 1000 watt in 1 hour.*

Relation between kWh and joule.

$$1 \text{ kWh} = 1 \text{ kW} \times 1 \text{ h} = 1000 \text{ W} \times 1 \text{ h}$$

$$= 1000 \text{ Js}^{-1} \times 3600 \text{ s}$$

or $1 \text{ kWh} = 3.6 \times 10^6 \text{ J}.$

Examples based on Power

FORMULAE USED

1. Power $= \frac{\text{Work}}{\text{Time}}$ or $P = \frac{W}{t}$

2. Also $P = \vec{F} \cdot \vec{v}$ When $\theta = 0°$, $P = Fv$

UNITS USED

Work W is in joule, force F in newton, time t in second, velocity v in ms^{-1}, power P in watt.

CONVERSIONS USED

1 kilowatt = 1000 watt or 1 kW = 1000 W

1 horsepower = 746 watt or 1 hp = 746 W.

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EXAMPLE 50. A man weighing 60 kg climbs up a staircase carrying a load of 20 kg on his head. The stair case has 20 steps each of height 0.2 m. If he takes 10 s to climb, find his power

Solution. Here $m = 60 + 20 = 80$ kg,

$h = 20 \times 0.2 = 4$ m

$g = 9.8\ ms^{-2}$, $t = 10$ s

$$P = \frac{W}{t} = \frac{mgh}{t} = \frac{80 \times 9.8 \times 4}{10} = \frac{3136}{10} = 313.6 \text{ W.}$$

EXAMPLE 51. A car of mass 2000 kg is lifted up a distance of 30 m by a crane in 1 min. A second crane does the same job in 2 min. Do the cranes consume the same or different amounts of fuel? What is the power supplied by each crane? Neglect power dissipation against friction. **[NCERT]**

Solution. Here $m = 2000$ kg, $s = 30$ m,

$t_1 = 1$ min $= 60$ s, $t_2 = 2$ min $= 120$ s

Work done by each crane,

$$W = Fs = mgs = 2000 \times 9.8 \times 30 = 5.88 \times 10^5 \text{ J}$$

As both the cranes do same amount of work, so both consume same amount of fuel.

Power supplied by first crane,

$$P_1 = \frac{W}{t_1} = \frac{5.88 \times 10^5}{60} = 9800 \text{ W.}$$

Power supplied by second crane,

$$P_2 = \frac{W}{t_2} = \frac{5.88 \times 10^5}{120} = 4900 \text{ W.}$$

EXAMPLE 52. The human heart discharges 75 ml of blood at each beat against a pressure of 0.1 m of Hg. Calculate the power of heart assuming that pulse frequency is 80 beats per minute. Density of Hg $= 13.6 \times 10^3\ kgm^{-3}$.

Solution. Volume of blood discharged per beat,

$$V = 75 \text{ ml} = 75 \times 10^{-6}\ m^{-3}$$

Pressure, $P = 0.1$ m of Hg

$$= 0.1 \times 13.6 \times 10^3 \times 9.8\ Nm^{-2} \quad [\because P = hpg]$$

Work done per beat $= PV$

Work done in 80 beats $= 80 \times PV$

Time, $t = 1$ min $= 60$ s

$$\text{Power} = \frac{\text{Work}}{\text{Time}} = \frac{80 \times PV}{t}$$

$$= \frac{80 \times 0.1 \times 13.6 \times 10^3 \times 9.8 \times 75 \times 10^{-6}}{60}$$

$$= 1.33 \text{ W.}$$

EXAMPLE 53. An electric motor is used to lift an elevator and its load (total mass = 1500 kg) to a height of 20 m. The time taken for the job is 20 s. What is the work done? What is the rate at which work is done? If the efficiency of the motor is 75%, at which rate is the energy supplied to the motor?

Solution. Here $m = 1500$ kg, $h = 20$ m,

$\eta = 75\%$, $t = 20$ s

Work done,

$$W = mgh = 1500 \times 9.8 \times 20 = 2.94 \times 10^5 \text{ J}$$

Rate of doing work

$$= \frac{W}{t} = \frac{2.94 \times 10^5}{20} = 1.47 \times 10^4 \text{ W.}$$

As $$\eta = \frac{\text{Output power}}{\text{Input power}}$$

$$\therefore \quad \frac{75}{100} = \frac{1.47 \times 10^4}{\text{Input power}}$$

Input power or the rate at which energy is supplied

$$= \frac{1.47 \times 10^4 \times 100}{75} = 1.96 \times 10^4 \text{ W.}$$

EXAMPLE 54. Calculate the horse power of a man who can chew ice at the rate of 30 g per minute. Given 1 hp = 746 W and $J = 4.2\ J\ cal^{-1}$.

Solution. Mass of ice chewed by man,

$$m = 30 \text{ g}$$

Latent heat of ice, $L = 80\ \text{cal}\ g^{-1}$

Heat required to melt ice,

$$H = mL = 30 \times 80 \text{ cal}$$

Work done, $W = JH = 4.2 \times 30 \times 80$ J

Time taken, $t = 1$ min $= 60$ s

Power, $$P = \frac{W}{t} = \frac{4.2 \times 30 \times 80}{60} = 168 \text{ W}$$

$$= \frac{168}{746} = 0.225 \text{ hp.}$$

EXAMPLE 55. A machine gun fires 60 bullets per minute with a velocity of $700\ ms^{-1}$. If each bullet has a mass of 50 g, find the power developed by the gun.

Solution. Mass of 60 bullets $= 60 \times 50 = 3000$ g $= 3$ kg

$v = 700\ ms^{-1}$, $t = 1$ min $= 60$ s

$$\text{Power} = \frac{W}{t} = \frac{\text{K.E.}}{t} = \frac{1}{2} \cdot \frac{mv^2}{t} = \frac{3 \times (700)^2}{2 \times 60}$$

$$= 12250 \text{ W.}$$

EXAMPLE 56. An elevator which can carry a maximum load of 1800 kg (elevator + passengers) is moving up with a constant speed of $2\ ms^{-1}$. The frictional force opposing the motion is 4000 N. Determine the minimum power delivered by the motor to the elevator in watts as well as in horse power. **[NCERT ; Delhi 03C]**

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Solution. The downward force on the elevator is

$$F = mg + F_f = 1800 \times 10 + 4000 = 22000 \text{ N}$$

The motor must supply enough power to balance this force. Hence,

$$P = Fv = 22000 \times 2 = 44000 \text{ W}$$

$$= \frac{44000}{746} \text{ hp} = \mathbf{59 \text{ hp}}.$$

EXAMPLE 57. A well 20 m deep and 3 m in diameter contains water to a depth of 14 metre. How long will a 5 hp engine take to empty it ?

Solution. Radius of the well, $r = \frac{3}{2}$ m

Area of cross-section of the well

$$= \pi r^2 = \frac{22}{7} \times \left(\frac{3}{2}\right)^2 = \frac{99}{14} \text{ m}^2$$

Volume of water in the well

= Area of cross-section × depth

$$= \frac{99}{14} \times 14 = 99 \text{ m}^3$$

$\therefore$ Mass of water in the well

= Volume × density $= 99 \times 10^3$ kg

As the well is emptied, the height through which water has to be raised by the engine changes from (20 – 14) m in the beginning to (20 – 0) m at the end.

$\therefore$ Average height raised $= \frac{6 + 20}{2} = 13$ m

Work required to empty the well,

$$W = mgh = 99 \times 10^3 \times 9.8 \times 13 = 12612600 \text{ J}$$

Power, $P = 5 \text{ hp} = 5 \times 746$ W

Required time, $t = \frac{W}{P} = \frac{12612600}{5 \times 746} = \mathbf{3381.6 \text{ s}}.$

EXAMPLE 58. The turbine pits at the Niagra falls are 50 m deep. The average horse power developed is 500. If the efficiency of the generator is 85%, how much water passes through the turbines per minute ? Take $g = 10$ ms^{-2}.

Solution. Useful power developed = 5000 hp

Efficiency = 85%

$\therefore$ Total power generated

$$= \frac{100}{85} \times 5000 \text{ hp} = \frac{100 \times 5000 \times 746}{85} \text{ W}$$

Total work done by the falling water in 1 min or 60 s,

$$W = Pt = \frac{100 \times 5000 \times 746}{85} \times 60 = 26.94 \times 10^7 \text{ J}$$

Now $mgh = W$

$\therefore$ $$m = \frac{W}{gh} = \frac{26.94 \times 10^7}{10 \times 50} = \mathbf{5.39 \times 10^5 \text{ kg}}.$$

EXAMPLE 59. A man cycles up a hill, whose slope is 1 in 20, with a velocity of 6.4 kmh^{-1} along the hill. The weight of the man and the cycle is 98 kg. What work per minute is he doing ? What is his horse power ?

Solution. Refer to Fig. 6.35. If the inclination of the hill with the horizontal is θ, then

$$\sin\theta = \frac{1}{20}$$

$$v = 6.4 \text{ kmh}^{-1} = \frac{6.4 \times 5}{18} \text{ ms}^{-1} = \frac{16}{9} \text{ ms}^{-1}$$

$m = 98$ kg, $t = 1$ min $= 60$ s

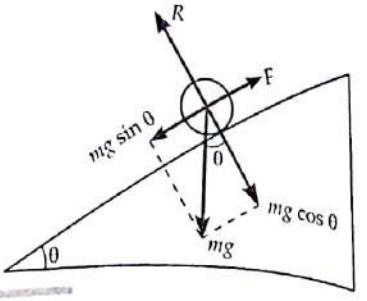

Fig. 6.35

As the velocity of the cyclist is uniform, so the only force he has to exert is against gravity. It is given by

$$F = mg \sin\theta$$

Power of the man,

$$P = Fv = mg \sin\theta \times v$$

$$= 98 \times 9.8 \times \frac{1}{20} \times \frac{16}{9} \text{ W}$$

$$= \frac{98 \times 9.6 \times 16}{746 \times 20 \times 9} \text{ hp} = \mathbf{0.144 \text{ hp}}$$

Work done per minute,

$$W = Pt = \frac{98 \times 9.8 \times 16}{20 \times 9} \times 60 = \mathbf{5122.1 \text{ J}}.$$

✕ PROBLEMS FOR PRACTICE

1. A lift is designed to carry a load of 4000 kg through 10 floors of a building averaging 6 m per floor in 10 seconds. Calculate the horse power of the lift. **(Ans.** 315.28 hp**)**
2. A machine can take out 1000 kg of mud per hour from a depth of 100 m. If efficiency of the machine is 0.9, calculate its power. **(Ans.** 302.47 W**)**
3. One coolie takes 1 min to raise a box through a height 2 m. Another takes 30 s for the same job and does the same amount of work. Which one of these two has a greater power and which one uses greater energy ?

(Ans. Second coolie has double power than first, both spend same amount of energy**)**

4. An engine of 4.9 kW p
from a well 50 m de
water in kilolitres wh
hour.
5. Water is pumped out
of a pump rated at 10
motor if 4200 kg of
minute. Take $g = 10$
6. A 30 m deep well is
engine evacuates i
power of the engin
4 m.
7. The human heart
minute through t
130 mm. The densi
is the horse power

8. A car of mass 100
rest to a velocit
Calculate (*i*) its
(*iii*) average powe
neglect friction.

[**Ans.** (*i*) 3 m

✕ HINTS

1. $P = \frac{W}{t} = \frac{mgh}{t} =$

$$= \frac{4000 \times 9.8 \times}{746}$$

2. Useful power =

$$\eta = \frac{\text{Useful p}}{\text{Actual p}}$$

$\therefore$ Actual po

3. As both the c
they spend t

$$W =$$

or $$\frac{P_2}{P_1} =$$

4. $W = Pt = m$

$\therefore$ 4.9×10^3

or

$\therefore$ Volume

5. Here $m =$

$t =$

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4. An engine of 4.9 kW power is used to pump water from a well 50 m deep. Calculate the quantity of water in kilolitres which it can pump out in one hour. **(Ans.** 36.0 kilo litre**)**

5. Water is pumped out of a well 10 m deep by means of a pump rated at 10 kW. Find the efficiency of the motor if 4200 kg of water is pumped out every minute. Take $g = 10\ ms^{-2}$. **(Ans.** 70%**)**

6. A 30 m deep well is having water upto 15 m. An engine evacuates it in one hour. Calculate the power of the engine if the diameter of the well is 4 m. **(Ans.** 11.55 kW**)**

7. The human heart forces 4000 cm^3 of blood per minute through the arteries under pressure of 130 mm. The density of blood is 1.03 g cm^{-3}. What is the horse power of the heart ?
(Ans. 1.17×10^{-4} hp**)**

8. A car of mass 1000 kg accelerates uniformly from rest to a velocity of 54 kh h^{-1} in 5 seconds. Calculate (*i*) its acceleration (*ii*) its gain in K.E. (*iii*) average power of the engine during this period, neglect friction. **[Chandigarh 03]**
[Ans. (*i*) 3 ms^{-2} (*ii*) 1.125×10^5 J (*iii*) 22500 W**]**

HINTS

1. $P = \frac{W}{t} = \frac{mgh}{t} = \frac{4000 \times 9.8 \times 10 \times 6}{10}$ W

$= \frac{4000 \times 9.8 \times 6}{746} =$ **315.28 hp.**

2. Useful power $= \frac{W}{t} = \frac{mgh}{t} = \frac{1000 \times 9.8 \times 100}{3600}$

$= 272.22$ W

$\eta = \frac{\text{Useful power}}{\text{Actual power}}$ or $0.9 = \frac{272.22}{\text{Actual power}}$

$\therefore$ Actual power $= \frac{272.22 \times 10}{9} =$ **302.47 W.**

3. As both the coolies do the same amount of work, so they spend the same amount of energy. Also

$W = P_1 t_1 = P_2 t_2$

or $\frac{P_2}{P_1} = \frac{t_1}{t_2} = \frac{1\ \text{min}}{20\ s} = 2$ or $P_2 = 2P_1$

4. $W = Pt = mgh$

$\therefore 4.9 \times 10^3 \times 3600 = m \times 9.8 \times 50$

or $m = 36{,}000$ kg

$\therefore$ Volume of water pumped out per hour
$= 36{,}000$ litre $=$ **36.0 kilo litre.**

5. Here $m = 4200$ kg, $h = 10$ m, $g = 10\ ms^{-2}$,
$t = 1$ min $= 60$ s

Output power
$= \frac{W}{t} = \frac{mgh}{t} = \frac{4200 \times 10 \times 10}{60} = 7000$ W $= 7$ kW

Efficiency,
$\eta = \frac{\text{Output power}}{\text{Input power}} = \frac{7}{10} = 0.7 = 70\%$.

6. Mass of water, $m =$ Volume $\times$ density $= \pi r^2 l \times \rho$
$= \pi (2)^2 \times 15 \times 1000$ kg

Mean $h = \frac{30 + 15}{2} = 22.5$ m

$P = \frac{W}{t} = \frac{mgh}{t}$

$= \frac{\pi \times (2)^2 \times 15 \times 1000 \times 9.8 \times 22.5}{3600\ s}$ J

$= 11549.99$ W $=$ **11.55 kW.**

7. Pressure $= 130$ mm of Hg $= 13$ cm of Hg
$= h\rho g = 13 \times 1.03 \times 980$ dyne cm^{-2}

Volume $= 4000\ cm^3$; Time, $t = 1$ min $= 60$ s

Power, $P = \frac{W}{t} = \frac{\text{Pressure} \times \text{volume}}{t}$

$= \frac{13 \times 1.03 \times 980 \times 4000}{60}$

$= 874813.33$ erg s^{-1}

$= 874813.33 \times 10^{-7}$ Js^{-1} or W

$= \frac{874813.33 \times 10^{-7}}{746}$ hp $=$ **1.17×10^4 hp.**

8. Here $m = 1000$ kg, $u = 0$, $v = 54$ $kmh^{-1} = 15\ ms^{-1}$, $t = 5$ s

(*i*) Acceleration, $a = \frac{v - u}{t} = \frac{15 - 0}{5} = 3\ ms^{-2}$.

(*ii*) Gain in K.E. $= \frac{1}{2} m (v^2 - u^2) = \frac{1}{2} \times 1000\ (15^2 - 0)$

$=$ **1.125×10^5 J.**

(*iii*) Power, $P = \frac{W}{t} = \frac{1.125 \times 10^5}{5} =$ **22500 W.**

6.21 COLLISIONS

35. *Define the terms : collision, elastic collision, inelastic collision, perfectly inelastic collision, super-elastic collision, head-on collision and oblique collision. Give important characteristics and examples of different types of collisions.*

Collision. *A collision is said to occur between two bodies, either if they physically collide against each other or if the path of one is affected by the force exerted by the other.* For a collision to take place, the actual physical contact is not necessary. In Rutherford's scattering experiment, α-particles get scattered due to the electrostatic

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repulsion between the α-particle and the nucleus from a distance. The α-particle is said to have suffered collision with the nucleus.

The collisions between particles are of following types :

1. **Elastic collision.** *If there is no loss of kinetic energy during a collision, it is called an elastic collision.*

Characteristics of elastic collisions :

(*i*) The momentum is conserved.
(*ii*) Total energy is conserved.
(*iii*) The kinetic energy is conserved.
(*iv*) Forces involved during the collision are conservative.
(*v*) The mechanical energy is not converted into heat, light, sound, etc.

Examples. Collision between subatomic particles, collision between glass balls, etc.

2. **Inelastic collision.** *If there is a loss of kinetic energy during a collision, it is called an inelastic collision.*

Characteristics of inelastic collisions :

(*i*) The momentum is conserved.
(*ii*) Total energy is conserved.
(*iii*) The kinetic energy is not conserved.
(*iv*) Some or all of the forces involved are non-conservative.
(*v*) A part of the mechanical energy is converted into heat, light, sound, etc.

Examples. Collision between two vehicles, collision between a ball and floor.

3. **Perfectly inelastic collision.** *If two bodies stick together after the collision and move as a single body with a common velocity, then the collision is said to be perfectly inelastic collision.* In such collisions, momentum is conserved, but the loss of kinetic energy is maximum.

Examples. Mud thrown on a wall and sticking to it, a man jumping into a moving trolley, a bullet fired into a wooden block and remaining embedded in it, etc.

4. **Superelastic or explosive collision.** In such a collision, *there is an increase in kinetic energy.* This occurs if there is a release of potential energy on an impact.

Examples. Bursting of a cracker when it hits the floor forcefully, the collision of a trolley with another may release a compressed spring and thereby releasing the energy stored in the spring.

5. **Head-on or one-dimensional collision.** *It is the collision in which the colliding bodies move along the same straight line path before and after the collision.*

Example. Collision between two railway compartments.

6. **Oblique or two-dimensional collision.** *If two bodies do not move along the same straight line path but lie in the same plane before and after the collision, the collision is said to be oblique or two-dimensional collision.*

Example. Collision between two carrom coins.

36. *Show that total linear momentum is conserved in all collisions.*

Conservation of linear momentum in a collision. Suppose two bodies 1 and 2 collide against each other. They exert mutual impulsive forces on each other during the collision time Δt. The changes produced in the momenta of the two bodies will be

$$\Delta \vec{p}_1 = \vec{F}_{12}\,\Delta t \quad \text{and} \quad \Delta \vec{p}_2 = \vec{F}_{21}\,\Delta t$$

where $\vec{F}_{12}$ = the force exerted on body 1 by body 2

and $\vec{F}_{21}$ = the force exerted on body 2 by body 1

According to Newton's third law,

$$\vec{F}_{12} = -\vec{F}_{21}$$

$$\therefore \quad \vec{F}_{12}\,\Delta t = -\vec{F}_{21}\,\Delta t$$

or $$\vec{F}_{12}\,\Delta t + \vec{F}_{21}\,\Delta t = 0$$

or $$\Delta \vec{p}_1 + \Delta \vec{p}_2 = 0$$

or $$\Delta(\vec{p}_1 + \vec{p}_2) = 0$$

or $$\vec{p}_1 + \vec{p}_2 = \text{constant}$$

Thus *linear momentum is conserved during a collision,* even though the forces vary in a complex manner during a collision.

For Your Knowledge

- Total linear momentum is conserved at each instant of every collision.
- Total energy is conserved in all collisions.
- The total kinetic energy may or may not be conserved during a collision.
- Even for an elastic collision, the kinetic energy conservation holds after the collision is over and does not hold at every instant of the collision.
- When two bodies collide ; they get deformed and may be momentarily at rest with respect to each other.
- The impact and deformation during a collision may convert part of the initial kinetic energy into heat and sound.

6.22 ELASTIC COLLISION IN ONE DIMENSION

37. *Prove that in an elastic one-dimensional collision between two bodies, the relative velocity of approach before collision is equal to the relative velocity of separation after the collision. Hence derive expressions*

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for the velocities of the two bodies in terms of their initial velocities before collision. Discuss the special cases also.

Elastic collision in one dimension. As shown in Fig. 6.36, consider two perfectly elastic bodies A and B of masses m_1 and m_2 moving along the same straight line with velocities u_1 and u_2 respectively. Let $u_1 > u_2$. After some time, the two bodies collide head-on and continue moving in the same direction with velocities v_1 and v_2 respectively. The two bodies will separate after the collision if $v_2 > v_1$.

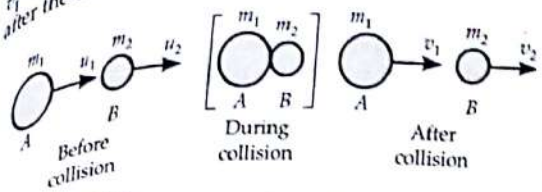

Fig. 6.36 Elastic collision in one dimension.

As linear momentum is conserved in any collision,

so $$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad ...(1)$$

$$m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$$

or $$m_1(u_1 - v_1) = m_2(v_2 - u_2) \quad ...(2)$$

or

Since K.E. is also conserved in an elastic collision,

so $$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$m_1u_1^2 - m_1v_1^2 = m_2v_2^2 - m_2u_2^2$$

or

or $$m_1(u_1 + v_1)(u_1 - v_1) = m_2(v_2 + u_2)(v_2 - u_2) \quad ...(3)$$

Dividing (3) by (2), we get

$$u_1 + v_1 = v_2 + u_2$$

$$u_1 - u_2 = v_2 - v_1 \quad ...(4)$$

or

or Relative velocity of A w.r.t. B before collision

= Relative velocity of B w.r.t. A after collision

or Relative velocity of approach

= Relative velocity of separation

Thus, *in an elastic one-dimensional collision, the relative velocity of approach before collision is equal to the relative velocity of separation after the collision.*

Velocities of the bodies after the collision. From equation (4), we get

$$v_2 = u_1 - u_2 + v_1$$

Putting this value of v_2 in equation (1), we get

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2(u_1 - u_2 + v_1)$$

$$= m_1v_1 + m_2u_1 - m_2u_2 + m_2v_1$$

or $$(m_1 - m_2)u_1 + 2m_2u_2 = (m_1 + m_2)v_1$$

or $$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2 \quad ...(5)$$

Interchanging the subscripts 1 and 2 in the above equation, we get

$$v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \left(\frac{2m_1}{m_1 + m_2}\right)u_1 \quad ...(6)$$

Equations (5) and (6) give the final velocities of the colliding bodies in terms of their initial velocities.

Special cases :

(*i*) When two bodies of equal masses collide. Let

$$m_1 = m_2 = m \text{ (say).}$$

From equation (5),

$$v_1 = \frac{2mu_2}{2m} = u_2$$

= velocity of body of mass m_2 before collision

From equation (6),

$$v_2 = \frac{2mu_1}{2m} = u_1$$

= velocity of body of mass m_1 before collision

Hence *when two bodies of equal masses suffer one dimensional elastic collision, their velocities get exchanged after the collision.*

(*ii*) When a body collides against a stationary body of equal mass. Here $m_1 = m_2 = m$ (say) and $u_2 = 0$.

From equation (5), $v_1 = 0$

From equation (6), $v_2 = u_1$

Hence *when an elastic body collides against another elastic body of equal mass, initially at rest, after the collision the first body comes to rest while second body moves with the initial velocity of the first.*

(*iii*) When a light body collides against a massive stationary body. Here $m_1 << m_2$ and $u_2 = 0$. Neglecting m_1 in equation (5), we get

$$v_1 = -\frac{m_2u_1}{m_2} = -u_1$$

From (6), $v_2 = 0$.

Hence *when a light body collides against a massive body at rest, the light body rebounds after the collision with an equal and opposite velocity while the massive body practically remains at rest.* A light ball on striking a wall rebounds almost with the same speed and the wall remains at rest.

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(iv) When a massive body collides against a light stationary body. Here $m_1 >> m_2$ and $u_2 = 0$.

Neglecting m_2 in equation (5), we get

$$v_1 = \frac{m_1 u_1}{m_1} = u_1$$

and $$v_2 = \frac{2 m_1 u_1}{m_1} = 2u_1$$

Hence when a massive body collides against a light body at rest, the velocity of the massive body remains almost unchanged while the light body starts moving with twice the velocity of the massive body.

6.23 PERFECTLY INELASTIC COLLISION IN ONE DIMENSION

38. *What is a perfectly inelastic collision ? Show that kinetic energy is invariably lost in such a collision.*

Perfectly inelastic collision. *When the two colliding bodies stick together and move as a single body with a common velocity after the collision, the collision is perfectly inelastic.*

As shown in Fig. 6.37, a body of mass m_1 moving with velocity u_1 collides head-on with another body of mass m_2 at rest. After the collision, the two bodies move together with a common velocity v.

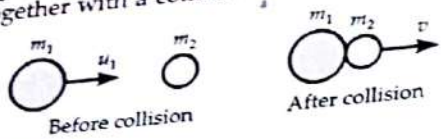

Fig. 6.37

As the linear momentum is conserved, so

$$m_1 u_1 + m_2 \times 0 = (m_1 + m_2) v$$

or $$v = \frac{m_1}{m_1 + m_2} u_1$$

The loss in kinetic energy on collision is

$$\Delta K = K_i - K_f = \frac{1}{2} m_1 u_1^2 - \frac{1}{2}(m_1 + m_2) v^2$$

$$= \frac{1}{2} m_1 u_1^2 - \frac{1}{2}(m_1 + m_2)\left[\frac{m_1}{m_1 + m_2} u_1\right]^2$$

$$= \frac{1}{2} m_1 u_1^2 - \frac{1}{2} \frac{m_1^2}{m_1 + m_2} u_1^2.$$

$$= \frac{1}{2} m_1 u_1^2 \left[1 - \frac{m_1}{m_1 + m_2}\right]$$

or $$\Delta K = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} u_1^2.$$

This is a positive quantity. The kinetic energy is lost mainly in the form of heat and sound.

Moreover,

$$\frac{K_f}{K_i} = \frac{\frac{1}{2}(m_1 + m_2) v^2}{\frac{1}{2} m_1 u_1^2} = \frac{m_1 + m_2}{m_1} \cdot \frac{v^2}{u_1^2}$$

$$= \frac{m_1 + m_2}{m_1} \left(\frac{m_1}{m_1 + m_2}\right)^2$$

or $$\frac{K_f}{K_i} = \frac{m_1}{m_1 + m_2}$$

which is < 1. This again shows that the kinetic energy after the collision is less than the kinetic energy before the collision.

If the target is massive, *i.e.*, $m_2 >> m_1$, then

$$\frac{K_f}{K_i} = 0 \qquad i.e., \quad K_f = 0$$

Hence when a light moving body collides against any massive body at rest and sticks to it, practically all of its kinetic energy is lost.

6.24 ELASTIC COLLISION IN TWO DIMENSIONS

39. *Discuss elastic collision in two dimensions. What are the conditions of (i) glancing collision (ii) head-on collision ? (iii) Show that two identical particles move at right angles to each other after elastic collision in two dimensions.*

Elastic collision in two dimensions. As shown in Fig. 6.38, suppose a particle of mass m_1 moving along X-axis with velocity u_1 collides with another particle of mass m_2 at rest. After the collision, let the two particles move with velocities v_1 and v_2, making angles θ_1 and θ_2 with X-axis.

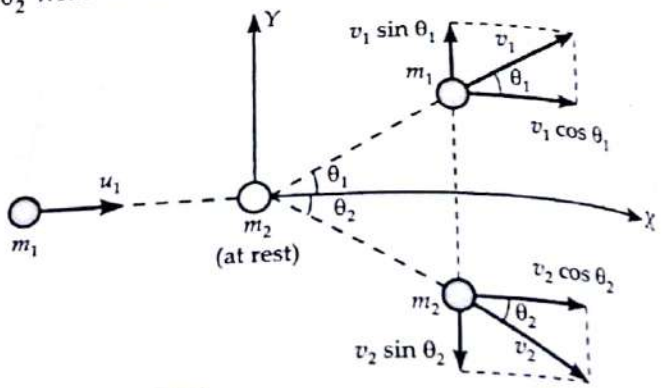

Fig. 6.38 Elastic collision in two dimensions.

After the collision, the rectangular components of the momentum of m_1 are

(*i*) $m_1 v_1 \cos \theta_1$, along +ve X-axis

(*ii*) $m_1 v_1 \sin \theta_1$, along +ve Y-axis

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After the collision, the rectangular components of the momentum of m_2 are

(i) $m_2 v_2 \cos\theta_2$, along +ve X-axis

(ii) $m_2 v_2 \sin\theta_2$, along –ve Y-axis

Applying the principle of conservation of momentum along X-axis,

$$m_1 u_1 = m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2 \quad ...(1)$$

The initial momentum of m_1 or m_2 along Y-axis is zero.

Applying the principle of conservation of momentum along Y-axis,

$$0 = m_1 v_1 \sin\theta_1 - m_2 v_2 \sin\theta_2 \quad ...(2)$$

As the K.E. is conserved in an elastic collision, so

$$\frac{1}{2} m_1 u_1^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \quad ...(3)$$

The four unknown quantities v_1, v_2, θ_1 and θ_2 cannot be calculated using the three equations (1), (2) and (3). By measuring one of the four unknowns, say θ_1, experimentally; the values of other three unknowns can be solved.

Special cases :

(i) **Glancing collision.** For such collisions, $\theta_1 \simeq 0°$ and $\theta_2 = 90°$.

From equations (1) and (2), we get

$$u_1 = v_1 \quad \text{and} \quad v_2 = 0$$

K.E. of the target particle $= \frac{1}{2} m_2 v_2^2 = 0$

Hence in a glancing collision, the incident particle does not lose any kinetic energy and is scattered almost undeflected.

(ii) **Head-on collision.** In such a collision, the target particle moves in the direction of the incident particle, i.e., $\theta_2 = 0°$. Then equations (1) and (2) take forms :

$$m_1 u_1 = m_1 v_1 \cos\theta_1 + m_2 v_2 \quad \text{and} \quad 0 = m_1 v_1 \sin\theta_1$$

Equation (3) for the kinetic energy remains unchanged.

(iii) **Elastic collision of two identical particles.** As the two particles are identical, so $m_1 = m_2 = m$ (say). By conservation of K.E. for elastic collision,

$$\frac{1}{2} m u_1^2 = \frac{1}{2} m v_1^2 + \frac{1}{2} m v_2^2 \quad \text{or} \quad u_1^2 = v_1^2 + v_2^2$$

By conservation of linear momentum,

$$m\vec{u_1} = m\vec{v_1} + m\vec{v_2} \quad \text{or} \quad \vec{u_1} = \vec{v_1} + \vec{v_2}$$

$$\therefore \quad \vec{u_1} \cdot \vec{u_1} = (\vec{v_1} + \vec{v_2}) \cdot (\vec{v_1} + \vec{v_2})$$

$$= \vec{v_1} \cdot \vec{v_1} + \vec{v_1} \cdot \vec{v_2} + \vec{v_2} \cdot \vec{v_1} + \vec{v_2} \cdot \vec{v_2}$$

or $$u_1^2 = v_1^2 + v_2^2 + 2\vec{v_1} \cdot \vec{v_2}$$

or $$u_1^2 = u_1^2 + 2\vec{v_1} \cdot \vec{v_2} \qquad [\because v_1^2 + v_2^2 = u_1^2]$$

or $$\vec{v_1} \cdot \vec{v_2} = 0.$$

This shows that the angle between $\vec{v_1}$ and $\vec{v_2}$ is 90°. Hence *two identical particles move at right angles to each other after elastic collision in two dimensions.*

6.25 COEFFICIENT OF RESTITUTION

40. *What is the coefficient of restitution ? What is its significance ?*

Coefficient of restitution or coefficient of resilience. Most of the real collisions are neither perfectly elastic nor perfectly plastic. They are *partially elastic collisions*, in which the K.E. reduces and so the speed of separation is less than the speed of approach.

The coefficient of restitution gives a measure of the degree of restitution of a collision and is defined as the ratio of the magnitude of relative velocity of separation after collision to the magnitude of relative velocity of approach before collision. It is given by

$$e = \frac{|v_1 - v_2|}{|u_1 - u_2|} = -\frac{v_1 - v_2}{u_1 - u_2}$$

The value of e depends on the materials of the colliding bodies. For two glass balls, $e = 0.95$ and for the lead balls, $e = 0.20$.

The coefficient of restitution can be used to distinguish between the different types of collisions as follows :

(i) *For a perfectly elastic collision,* $e = 1$ *i.e.,* relative velocity of separation is equal to the relative velocity of approach.

(ii) *For an inelastic collision,* $0 < e < 1$ *i.e.,* the relative velocity of separation is less than relative velocity of approach.

(iii) *For a perfectly inelastic collision,* $e = 0$ *i.e.,* the relative velocity of separation is zero. The two bodies move together with a common velocity.

(iv) *For a superelastic collision.* $e > 1$ *i.e.,* the kinetic energy increases.

Table 6.2 *Different types of collisions*

Collision	Kinetic energy	Coefficient of restitution	Main domain
Elastic	Conserved	$e = 1$	Between atomic particles
Inelastic	Not conserved	$0 < e < 1$	Between ordinary objects
Perfectly inelastic	Max. loss of K.E.	$e = 0$	During shooting
Super elastic	K.E. increases	$e > 1$	In explosions

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For Your Knowledge

- At each instant of the collision, the total kinetic energy and total linear momentum are both conserved in elastic as well as inelastic collisions.
- In an elastic collision, the kinetic energy conservation holds only after the collision is over. It does not hold during the short duration of actual collision.
- At the time of collision, the two colliding objects are deformed and may be momentarily at rest with respect to each other.
- When two equal masses suffer a glancing collision with one of them at rest, after the collision, the two masses move at right angles to each other.

Examples based on Collisions

Formulae Used

1. Linear momentum is conserved both in elastic and inelastic collisions.

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

2. Kinetic energy is conserved in elastic collisions.

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

3. In one-dimensional elastic collision, velocities after the collision are given by

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2} \cdot u_1 + \frac{2 m_2}{m_1 + m_2} \cdot u_2$$

$$v_2 = \frac{2m_1}{m_1 + m_2} \cdot u_1 + \frac{m_2 - m_1}{m_1 + m_2} \cdot u_2$$

4. Coefficient of restitution for a collision is given by

$$e = -\frac{v_1 - v_2}{u_1 - u_2} = \frac{|v_1 - v_2|}{|u_1 - u_2|}$$

5. For a ball rebounding from a floor, $e = \frac{v}{u}$

6. For an elastic collision (involving no loss of K.E.),

$$e = 1$$

7. For an inelastic collision (involving loss of K.E.),

$$e < 1$$

Units Used

Masses m_1, m_2 are in kg, velocities u_1, u_2, v_1, v_2 are in ms^{-1}, linear momenta in kg ms^{-1}, kinetic energy in joule and coefficient of restitution 'e' has no units.

Example 60. Two bodies of masses 5 kg and 3 kg moving in the same direction along the same straight line with velocities 5 ms^{-1} and 3 ms^{-1} respectively suffer one-dimensional elastic collision. Find their velocities after the collision.

Solution. Here $m_1 = 5$ kg, $u_1 = 5\ ms^{-1}$,
$m_2 = 3$ kg, $u_2 = 3\ ms^{-1}$

$$\therefore v_1 = \frac{m_1 - m_2}{m_1 + m_2} \cdot u_1 + \frac{2 m_2}{m_1 + m_2} \cdot u_2$$

$$= \frac{5-3}{5+3} \times 5 + \frac{2 \times 3}{5+3} \times 3$$

$$= \frac{2}{8} \times 5 + \frac{6}{8} \times 3 = \frac{5}{4} + \frac{9}{4} = \frac{14}{4} = 3.5\ ms^{-1}.$$

$$v_2 = \frac{2 m_1}{m_1 + m_2} \cdot u_1 + \frac{m_2 - m_1}{m_1 + m_2} \cdot u_2$$

$$= \frac{2 \times 5}{5+3} \times 5 + \frac{3-5}{5+3} \times 3$$

$$= \frac{50}{8} - \frac{6}{8} = \frac{44}{8} = 5.5\ ms^{-1}.$$

Example 61. A 10 kg ball and 20 kg ball approach each other with velocities 20 ms^{-1} and 10 ms^{-1} respectively. What are their velocities after collision if the collision is perfectly elastic ?

Solution. Here $m_1 = 10$ kg, $m_2 = 20$ kg,
$u_1 = 20\ ms^{-1}$, $u_2 = -10\ ms^{-1}$

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2} \cdot u_1 + \frac{2 m_2}{m_1 + m_2} \cdot u_2$$

$$= \frac{10-20}{10+20} \times 20 + \frac{2 \times 20}{10+20} \times (-10)$$

$$= -\frac{20}{3} - \frac{40}{3} = -\frac{60}{3} = -20\ ms^{-1}.$$

$$v_2 = \frac{2 m_1}{m_1 + m_2} \cdot u_1 + \frac{m_2 - m_1}{m_1 + m_2} \cdot u_2$$

$$= \frac{2 \times 10}{10+20} \times 20 + \frac{20-10}{10+20} \times (-10)$$

$$= \frac{40}{3} - \frac{10}{3} = \frac{30}{3} = 10\ ms^{-1}.$$

Example 62. Two ball bearings of mass m each moving in opposite directions with equal speeds v collide head on with each other. Predict the outcome of the collision, assuming it to be perfectly elastic. [NCERT]

Solution. Here $m_1 = m_2 = m$ (say), $u_1 = v$, $u_2 = -v$

As the collision is perfectly elastic, velocities after the collision will be

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2} \cdot u_1 + \frac{2 m_2}{m_1 + m_2} \cdot u_2$$

$$= \frac{m - m}{m + m} \cdot v + \frac{2m}{m+m}(-v)$$

$$= 0 - v = -v.$$

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$$v_2 = \frac{2m_1}{m_1 + m_2} \cdot u_1 + \frac{m_2 - m_1}{m_1 + m_2} \cdot u_2$$

$$= \frac{2m}{m + m} \cdot v + \frac{m - m}{m + m} \cdot (-v)$$

$$= v + 0 = v.$$

Thus the two balls bounce back with equal speeds after the collision.

EXAMPLE 63. A railway carriage of mass 9000 kg moving with a speed of 36 kmh^{-1} collides with a stationary carriage of the same mass. After the collision, the carriages get coupled and move together. What is their common speed after collision ? What type of collision is this ?

Solution. Here

$m_1 = 9000$ kg, $u_1 = 36$ kmh$^{-1} = 10$ ms^{-1}

$m_2 = 9000$ kg, $u_2 = 0$, $v_1 = v_2 = v = ?$

By conservation of momentum,

$$m_1u_1 + m_2u_2 = (m_1 + m_1)\, v$$

$$9000 \times 10 + 0 = (9000 + 9000)v$$

or $$v = \frac{90000}{18000} = 5 \text{ ms}^{-1}.$$

Total K.E. before collision

$$= \tfrac{1}{2} m_1 u_1^2 + \tfrac{1}{2} m_2 u_2^2$$

$$= \tfrac{1}{2} \times 9000 \times 10 \times 10 + 0 = 450000 \text{ J}$$

Total K.E. after collision $= \tfrac{1}{2}(m_1 + m_2) v^2$

$$= \tfrac{1}{2} \times 2 \times 9000 \times 5^2 = 225000 \text{ J}.$$

Thus total K.E. after collision < Total K.E. before collision.

Hence the collision is inelastic.

EXAMPLE 64. What percentage of kinetic energy of a moving particle is transferred to a stationary particle, when moving particle strikes with a stationary particle of mass (i) 9 times in mass (ii) equal in mass and (iii) 1/19th of its mass ?

Solution. For the moving particle, $m_1 = m$ (say),

initial velocity $= u_1$

For the stationary particle, $m_2 = xm$ (say),

initial velocity $= u_2 = 0$

As $$v_2 = \frac{2m_1}{m_1 + m_2} \cdot u_1 + \frac{m_2 - m_1}{m_1 + m_2} \cdot u_2$$

$$\therefore \quad v_2 = \frac{2m}{m + xm} \cdot u_1 + 0 = \frac{2u_1}{1 + x}$$

K.E. of the moving particle before collision,

$$K_1 = \tfrac{1}{2} m_1 u_1^2 = \tfrac{1}{2} mu_1^2$$

K.E. of the stationary particle after collision,

$$K_2 = \frac{1}{2} m_2 v_2^2 = \frac{1}{2} \cdot mx \cdot \frac{4u_1^2}{(1 + x)^2}$$

$$= \frac{4x}{(1 + x)^2} \cdot \frac{1}{2} mu_1^2 = \frac{4x}{(1 + x)^2} \cdot K_1$$

% of K.E. transferred

$$= \frac{K_2}{K_1} \times 100 = \frac{4xK_1}{(1 + x)^2} \times \frac{100}{K_1}$$

$$= \frac{4x}{(1 + x)^2} \times 100\ \%$$

(*i*) When moving particle strikes with a stationary particle 9 times in mass, $x = 9$

$$\% \text{ of K.E. transferred} = \frac{4 \times 9}{(1 + 9)^2} \times 100 = \mathbf{36\%}.$$

(*ii*) When moving particle strikes with a stationary particle of equal mass, $x = 1$

$$\% \text{ of K.E. transferred} = \frac{4 \times 1}{(1 + 1)^2} \times 100 = \mathbf{100\%}.$$

(*iii*) When moving particle strikes a stationary particle $\frac{1}{19}$ th of its mass, $x = \frac{1}{19}$.

$$\% \text{ of K.E. transferred} = \frac{4 \times (1/19)}{(1 + 1/19)^2} \times 100 = \mathbf{19\%}.$$

*EXAMPLE 65. **Slowing down of neutrons.** In a nuclear reactor a neutron of high speed (typically 10^7 ms^{-1}) must be slowed to 10^3 ms^{-1} so that it can have a high probability of interacting with isotope $^{235}_{92}U$ and causing it to fission. Show that a neutron can lose most of its kinetic energy in an elastic collision with a light nucleus like deuterium or carbon which has a mass of only a few times the neutron mass. The material making up the light nuclei, usually heavy water (D_2O) or graphite is called a moderator.* **[NCERT]**

Or

A body of mass M at rest is struck by a moving body of mass m. Prove that fraction of the initial K.E. of the mass m transferred to the struck body is $4\,mM/(m + M)^2$ in an elastic collision. **[Delhi 13]**

Solution. Here m_1 = mass of neutron = m

m_2 = mass of target nucleus = M

$u_1 = u$ and $u_2 = 0$

Now $$v_2 = \frac{2m_1}{m_1 + m_2} \cdot u_1 + \frac{m_2 - m_1}{m_1 + m_2} \cdot u_2$$

$$= \frac{2m}{m + M} \cdot u + 0 = \frac{2mu}{m + M}$$

Initial K.E. of mass m,

$$K_1 = \tfrac{1}{2} m_1 u_1^2 = \tfrac{1}{2} mu^2$$

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Final K.E. of mass M,

$$K_2 = \frac{1}{2} m_2 v_2^2 = \frac{1}{2} M \left(\frac{2mu}{m+M} \right)^2 = \frac{2\, Mm^2 u^2}{(m+M)^2}$$

Fraction of the Initial K.E. transferred,

$$f = \frac{K_2}{K_1} = \frac{2Mm^2 u^2}{(m+M)^2} \times \frac{2}{mu^2} = \frac{4mM}{(m+M)^2}.$$

(i) For deuterium, $M = 2m$, therefore

$$f = \frac{4m \times 2m}{(m+2m)^2} = \frac{8}{9} = 0.9$$

About 90% of the neutron's energy is transferred to deuterium.

(ii) For carbon, $M = 12\,m$, therefore

$$f = \frac{4m \times 12\ m}{(m+12m)^2} = 0.284$$

About 28.4 % of the neutron's energy is transferred to carbon.

EXAMPLE 66. A ball is dropped to the ground from a height of 2 m. The coefficient of restitution is 0.6. To what height will the ball rebound ?

Solution. As the ball falls to the ground, its potential energy mgh_1 changes into kinetic energy $\frac{1}{2} mv_1^2$.

$$\therefore \quad mgh_1 = \frac{1}{2} mv_1^2 \qquad ...(i)$$

After rebounding, its kinetic energy $\frac{1}{2} mv_2^2$ changes into potential energy mgh_2.

$$\therefore \quad mgh_2 = \frac{1}{2} mv_2^2 \qquad ...(ii)$$

Dividing (*ii*) by (*i*), we get $\frac{h_2}{h_1} = \left(\frac{v_2}{v_1} \right)^2$

But $h_1 = 2$ m, $e = \frac{v_2}{v_1} = 0.6$

$$\therefore \quad \frac{h_2}{2\text{ m}} = (0.6)^2 = 0.36 \quad \text{or} \quad h_2 = 0.72 \text{ m}.$$

EXAMPLE 67. A ball is dropped vertically from a height of 3.6 m. It rebounds from a horizontal surface to a height of 1.6 m. Find the coefficient of restitution of the material of the ball.

Solution. Here $h_1 = 3.6$ m, $h_2 = 1.6$ m

Velocity of the ball with which it reaches the horizontal surface,

$$u = \sqrt{2gh_1} = \sqrt{2 \times 9.8 \times 3.6} = 8.4 \text{ ms}^{-1}$$

Velocity of the ball with which it rebounds,

$$v = \sqrt{2gh_2} = \sqrt{2 \times 9.8 \times 1.6} = 5.6 \text{ ms}^{-1}$$

Coefficient of restitution, $e = \frac{v}{u} = \frac{5.6}{8.4} = 0.667.$

EXAMPLE 68. A ball is dropped from a height h. It rebounds from the ground a number of times. Given that the coefficient of restitution is e, to what height does it go after the n^{th} rebounding ?

Solution. Let v_0 be the velocity with which the ball strikes the ground first time and v_n the velocity after n^{th} rebounding. Then the coefficient of restitution will be

$$e = \frac{v_1}{v_0} = \frac{v_2}{v_1} = \frac{v_3}{v_2} = ... = \frac{v_n}{v_{n-1}}$$

$$\therefore \quad e^n = \frac{v_1}{v_0} \times \frac{v_2}{v_1} \times \frac{v_3}{v_2} \times .... \times \frac{v_n}{v_{n-1}} = \frac{v_n}{v_0}$$

But $v_0 = \sqrt{2gh}$ and $v_n = \sqrt{2gH}$

where H is the height to which the ball rises after n^{th} rebounding.

Hence $e^n = \frac{v_n}{v_0} = \sqrt{\frac{2gH}{2gh}} = \sqrt{\frac{H}{h}}$ or $H = he^{2n}$.

EXAMPLE 69. A sphere of mass m moving with a velocity u hits another stationary sphere of same mass. If e is the coefficient of restitution, what is the ratio of the velocities of two spheres after the collision ?

Solution. Here $u_1 = u$, $u_2 = 0$

$$\therefore \quad e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{v_2 - v_1}{u - 0}$$

or $$v_2 - v_1 = e\,u \qquad ...(i)$$

By the law of conservation of momentum,

$$mu + m \times 0 = mv_1 + mv_2$$

or $$v_1 + v_2 = u \qquad ...(ii)$$

Adding (*i*) and (*ii*),

$$2v_2 = u + eu = u(1 + e)$$

or $$v_2 = \frac{u(1+e)}{2}$$

Again, from (*ii*),

$$v_1 = u - v_2 = u - \frac{u(1+e)}{2} = \frac{u(1-e)}{2}$$

Hence $$\frac{v_2}{v_1} = \frac{1+e}{1-e}.$$

EXAMPLE 70. A ball moving with a speed of 9 ms^{-1} strikes an identical ball such that after the collision the direction of each ball makes an angle 30° with the original line of motion. Find the speeds of the two balls after the collision. Is the kinetic energy conserved in the collision process ?

Solution. Let m be the mass of each ball and v_1 and v_2 be their final velocities after collision.

Fig. 6.39

Here $u_1 = 9$ m

Initial mome

$$\therefore \quad m \times 9 +$$

or

or

By conservati

perpendicular to

$$m \times 0 + m \times 0 =$$

$$0 =$$

or $$v_1 - v_2 =$$

or $$v_1 =$$

or

From (1) and

$$v_1 =$$

Total K.E. bef

$$=$$

Total K.E. aft

$$=$$

$$=$$

i.e. Total K.E. afte

Hence K.E. is

EXAMPLE 71. A

plane hits an identi

If the collision is e

target ball if the sp

30 cm s^{-1}. Show th

to each other after

Solution. The

be the mass of ea

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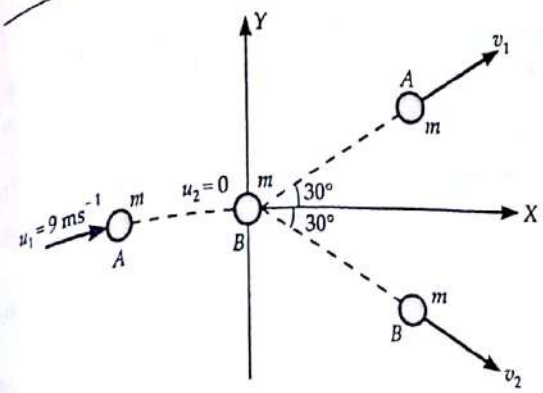

Fig. 6.39

Here $u_1 = 9 \text{ ms}^{-1}$, $u_2 = 0$

Initial momentum = Final momentum along the original line of motion

$$m \times 9 + m \times 0 = m v_1 \cos 30° + m v_2 \cos 30°$$

$\therefore$

$$m \times 9 = m \frac{\sqrt{3}}{2} (v_1 + v_2)$$

or

$$v_1 + v_2 = \frac{18}{\sqrt{3}} = \frac{6 \times 3}{\sqrt{3}} = 6\sqrt{3} \qquad ...(1)$$

or

By conservation of momentum along a direction perpendicular to the original line, we have

$$m \times 0 + m \times 0 = m v_1 \sin 30° - m v_2 \sin 30°$$

or

$$0 = m(v_1 - v_2) \times \frac{1}{2}$$

or

$$v_1 - v_2 = 0$$

or

$$v_1 = v_2 \qquad ...(2)$$

From (1) and (2), we have

$$v_1 = v_2 = 3\sqrt{3} \text{ ms}^{-1}.$$

Total K.E. before collision

$$= \frac{1}{2} \times m \times (9)^2 + 0 = \frac{81}{2} m = 40.5m$$

Total K.E. after collision

$$= \frac{1}{2} \times m \times (3\sqrt{3})^2 + \frac{1}{2} \times m \times (3 \times \sqrt{3})^2$$

$$= 27m$$

i.e. Total K.E. after collision < Total K.E. before collision

Hence K.E. is not conserved in the collision process.

EXAMPLE 71. A ball moving on a horizontal frictionless plane hits an identical ball at rest with a velocity of 0.5 ms^{-1}. If the collision is elastic, calculate the speed imparted to the target ball if the speed of the projectile after the collision is 30 cm s^{-1}. Show that the two balls will move at right angles to each other after the collision.

Solution. The situation is shown in Fig. 6.40. Let m be the mass of each ball.

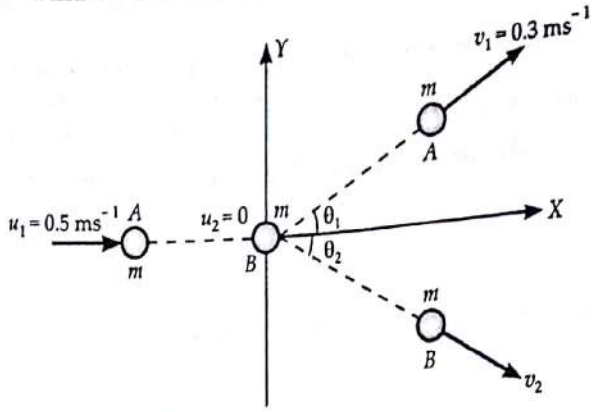

Fig. 6.40

As the collision is elastic, so K.E. is conserved.

$$\frac{1}{2} m u_1^2 + \frac{1}{2} m u_2^2 = \frac{1}{2} m v_1^2 + \frac{1}{2} m v_2^2$$

or

$$u_1^2 + u_2^2 = v_1^2 + v_2^2$$

or

$$(0.5)^2 + 0 = (0.3)^2 + v_2^2$$

or

$$v_2 = \mathbf{0.4 \text{ ms}^{-1}}.$$

Applying law of conservation of momentum along X-axis, we get

$$0.5\, m + 0 = 0.3\, m \cos \theta_1 + 0.4\, m \cos \theta_2$$

or

$$5 = 3 \cos \theta_1 + 4 \cos \theta_2$$

or

$$3 \cos \theta_1 = 5 - 4 \cos \theta_2 \qquad ...(1)$$

Again, applying law of conservation of momentum along Y-axis, we get

$$0.3\, m \sin \theta_1 = 0.4\, m \sin \theta_2$$

or

$$3 \sin \theta_1 = 4 \sin \theta_2 \qquad ...(2)$$

Squaring and adding equations (1) and (2), we get

$$9 (\cos^2 \theta_1 + \sin^2 \theta_1) = (5 - 4 \cos \theta_2)^2 + (4 \sin \theta_2)^2$$

or

$$9 = 16 (\cos^2 \theta_2 + \sin^2 \theta_2) + 25 - 40 \cos \theta_2$$

or

$$\cos \theta_2 = \frac{4}{5}$$

From (1), $\cos \theta_1 = \frac{1}{3}\left[5 - 4 \times \frac{4}{5} \right] = \frac{3}{5}$

Hence $\sin \theta_1 = \frac{4}{5}$ and $\sin \theta_2 = \frac{3}{5}$

Now $\sin (\theta_1 + \theta_2) = \sin \theta_1 \cos \theta_2 + \cos \theta_1 \sin \theta_2$

$$= \frac{4}{5} \times \frac{4}{5} + \frac{3}{5} \times \frac{3}{5} = 1$$

$\therefore$ $\theta_1 + \theta_2 = 90°$

Hence after the collision, the two balls will move off at right angles to each other.

EXAMPLE 72. Consider the collision depicted in Fig. 6.40 to be between two billiard balls with equal masses $m_1 = m_2$. The first ball is called the cue while the second ball is called the target. The billiard player wants to "sink" the target ball in a

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corner pocket, which is at an angle $\theta_2 = 37°$. Assume that the collision is elastic and that friction and rotational motion are not important. Obtain θ_1. [NCERT]

Solution. By conservation of momentum, we have

$$m\vec{u_1} + 0 = m\vec{v_1} + m\vec{v_2}$$

or $$\vec{u_1} = \vec{v_1} + \vec{v_2} \quad ...(i)$$

By conservation of energy, we have

$$\frac{1}{2}mu_1^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

or $$u_1^2 = v_1^2 + v_2^2 \quad ...(ii)$$

From (i), we have

$$\vec{u_1}\cdot\vec{u_1} = (\vec{v_1} + \vec{v_2})\cdot(\vec{v_1} + \vec{v_2})$$

$$= \vec{v_1}\cdot\vec{v_1} + \vec{v_1}\cdot\vec{v_2} + \vec{v_2}\cdot\vec{v_1} + \vec{v_2}\cdot\vec{v_2}$$

or $$u_1^2 = v_1^2 + v_2^2 + 2\vec{v_1}\cdot\vec{v_2}$$

or $$u_1^2 = u_1^2 + 2\vec{v_1}\cdot\vec{v_2} \quad \text{[using (ii)]}$$

or $$\vec{v_1}\cdot\vec{v_2} = 0$$

Thus the angle between $\vec{v_1}$ and $\vec{v_2}$ is 90°.

or $$\theta_1 + \theta_2 = 90°$$

$$\therefore \quad \theta_1 = 90° - \theta_2 = 90 - 37° = \mathbf{53°}.$$

EXAMPLE 7.3. A nucleus of radium ($^{226}_{88}Ra$) decays to $^{222}_{86}Rn$ by the emission of α-particle (^4_2He) of energy 4.8 MeV. If mass of $^{222}_{86}Rn$ = 222.0 amu and mass of ^4_2He = 4.003 amu, then calculate the recoil energy of the daughter nucleus $^{222}_{86}Rn$. [NCERT]

Solution. The nuclear decay may be represented as follows :

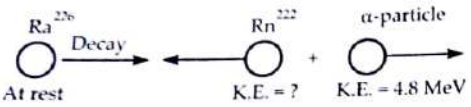

Fig. 6.41

The kinetic energy of a particle is given by

$$K = \frac{p^2}{2m} \quad \therefore \quad p = \sqrt{2mK}$$

As momentum is conserved in the absence of an external force, so

$$mK = \text{constant}$$

or $$m_{Rn}K_{Rn} = m_\alpha K_\alpha$$

or $$K_{Rn} = \frac{m_\alpha K_\alpha}{m_{Rn}}$$

$$= \frac{4.003 \times 4.8}{222} = \mathbf{0.0866\ MeV}.$$

EXAMPLE 7.4. The nucleus Fe^{57} emits a γ ray of energy 14.4 keV. If the mass of the nucleus is 56.935 amu, calculate the recoil energy of the nucleus.
Take 1 amu = 1.66×10^{-27} kg. [NCERT]

Solution. The nuclear decay may be represented as follows :

$$Fe^{57} \rightarrow Fe^{57} + h\nu \ (\gamma\text{ ray photon})$$

Excited state — Ground state

According to de-Broglie hypothesis, momentum of a photon of energy E is

$$p = \frac{E}{c} = \frac{14.4\ \text{keV}}{c} = \frac{14.4 \times 1.6 \times 10^{-16}\ \text{J}}{3 \times 10^8\ \text{ms}^{-1}}$$

$$= 7.68 \times 10^{-24}\ \text{kg ms}^{-1}.$$

By conservation of momentum, the momentum of daughter nucleus,

p = momentum of γ-ray photon

$$= 7.68 \times 10^{-24}\ \text{kg ms}^{-1}$$

The recoil energy of the nucleus will be

$$K = \frac{p^2}{2m} = \frac{(7.68 \times 10^{-24})^2}{2 \times 56.935 \times 1.66 \times 10^{-27}}$$

$$= 0.312 \times 10^{-21}\ \text{J} = \frac{0.312 \times 10^{-21}}{1.6 \times 10^{-16}}\ \text{keV}$$

$$= \mathbf{1.95 \times 10^{-6}\ keV}.$$

✕ PROBLEMS FOR PRACTICE

1. A vehicle of mass 30 quintals moving with a speed of 18 km h^{-1} collides with another vehicle of mass 90 quintals moving with a speed of 14.4 km h^{-1} in the opposite direction. What will be the velocity of each after the collision ?
(**Ans.** 30.6 km h^{-1}, 1.8 km h^{-1})

2. A ball of 0.1 kg makes an elastic head on collision with a ball of unknown mass that is initially at rest. If the 0.1 kg ball rebounds at one-third of its original speed, what is the mass of the other ball ?
(**Ans.** 0.2 kg)

3. A body of mass m strikes a stationary body of mass M and undergoes an elastic collision. After collision m has a speed one-third of its initial speed. What is the ratio M/m ?
(**Ans.** 1 : 2)

4. Two particles of masses 0.5 kg and 0.25 kg moving with velocities 4.0 ms^{-1} and $-3.0\ ms^{-1}$ collide head on in a perfectly inelastic collision. Find (*i*) the velocity of the composite particle after the collision and (*ii*) the kinetic energy lost in the collision.
[**Ans.** (*i*) 1.7 ms^{-1} (*ii*) 4.1 J]

5. What percentage of the K.E. of a moving particle is transferred to a stationary particle when it strikes the stationary particle of four times its mass ?

6. A neutron moving with a speed of 10⁶ ms⁻¹ makes a head-on collision with a nucleus of mass nearly 80. What is the fraction of energy retained by the nucleus ?

7. What percentage of kinetic energy of a moving particle is transferred to a stationary particle, when the particle strikes with a stationary particle (*i*) 19 times its mass (*ii*) equal in mass and (*iii*) 1/19 times its mass ? [Ans. (*i*) 72% (*ii*) 100% (*iii*) 10%]

8. Show that when a moving body collides with a stationary body of mass m or $\frac{1}{m}$ times its mass, then the moving body transfers $\frac{4m}{(1+m)^2}$ fraction of its kinetic energy to the stationary body.

9. A ball is dropped from a height of 2 m. Find the height upto which the ball will rebound, if the coefficient of restitution is 0.5.

10. A ball is dropped from a height h. If the coefficient of restitution is e, calculate the height to which the ball first rebounds ?

✕ HINTS

2. Here $m_1 = 0.1$ kg, $m_2 = ?$

$u_1 = u$ (say), $v_1 = -\frac{u}{3}$

Now $v_1 = \frac{m_1 - m_2}{m_1 + m_2}\cdot u_1 + \ldots$

$\therefore \quad -\frac{u}{3} = \frac{0.1 - m_2}{0.1 + m_2}\cdot u + \ldots$

or $-\frac{1}{3} = \frac{0.1 - m_2}{0.1 + m_2}$

or $-0.1 - m_2 = 0.3 - \ldots$

$\therefore \quad m_2 = 0.2$ k

8. **Case 1.** Mass of movin

initial velocity = u_1

Mass of stationary b

initial vel., $u_2 = 0$;

Very Short Ans

Problem 1(a). If a direction of motion o work done ?

Solution. The wo displacement in the

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5. What percentage of the K.E. of a moving particle is transferred to a stationary particle when it strikes the stationary particle of four times its mass ?
(**Ans.** 64%)

6. A neutron moving with a speed of 10^6 ms^{-1} suffers a head-on collision with a nucleus of mass number 80. What is the fraction of energy retained by the nucleus ? (**Ans.** 79/81)

7. What percentage of kinetic energy of a moving particle is transferred to a stationary particle, when moving particle strikes with a stationary particle of mass (*i*) 19 times its mass (*ii*) equal in mass and (*iii*) 1/9th of its mass ? [**Ans.** (*i*) 19% (*ii*) 100% (*ii*) 36%]

8. Show that when a moving body collides with stationary body of mass m or $1/m$ times its mass, then the moving body transfers $\frac{4m}{(1+m)^2}$ part of its kinetic energy to the stationary body.

9. A ball is dropped from a height of 3 m. What is the height upto which the ball will rebound ? The coefficient of restitution is 0.5. (**Ans.** 0.75 m)

10. A ball is dropped from a height h on to a floor. If the coefficient of restitution is e, calculate the height to which the ball first rebounds ? (**Ans.** $h e^2$)

✎ HINTS

2. Here $m_1 = 0.1$ kg, $m_2 = ?$

$u_1 = u$ (say), $v_1 = -\frac{u}{3}$, $u_2 = 0$

Now $v_1 = \frac{m_1 - m_2}{m_1 + m_2} \cdot u_1 + \frac{2m_2}{m_1 + m_2} \cdot u_2$

$\therefore \quad -\frac{u}{3} = \frac{0.1 - m_2}{0.1 + m_2} \cdot u + 0$

or $\quad -\frac{1}{3} = \frac{0.1 - m_2}{0.1 + m_2}$

or $\quad -0.1 - m_2 = 0.3 - 3m_2 \quad$ or $\quad 2m_2 = 0.4$

$\therefore \quad m_2 = 0.2$ kg.

8. **Case 1.** Mass of moving body $= m_1$

initial velocity $= u_1$

Mass of stationary body, $m_2 = mm_1$

initial vel., $u_2 = 0$;

Velocity of the stationary body after collision will be

$$v_2 = \frac{2m_1}{m_1 + m_2} \cdot u_1 + \frac{m_2 - m_1}{m_1 + m_2} \cdot u_2$$

$$= \frac{2m_1}{m_1 + mm_1} \cdot u_1 + 0 = \frac{2}{1+m} \cdot u_1$$

Fraction of the K.E. transferred to the stationary body

$$\frac{\frac{1}{2} m_2 v_2^2}{\frac{1}{2} m_1 u_1^2} = \frac{mm_1 \left(\frac{2}{1+m} \cdot u_1 \right)^2}{m_1 u_1^2} = \frac{4m}{(1+m)^2}$$

Case 2. Here $m_2 = \frac{m_1}{m}$

$$v_2 = \frac{2m_1}{m_1 + \frac{m_1}{m}} \cdot u_1 + 0 = \frac{2m_1}{m_1 \left(1 + \frac{1}{m}\right)} \cdot u_2$$

$$= \frac{2m}{1+m} \cdot u_1$$

Fraction of the K.E. transferred to the stationary body

$$\frac{\frac{1}{2} m_2 u_2^2}{\frac{1}{2} m_1 u_1^2} = \frac{\frac{m_1}{m} \left(\frac{2m}{1+m} u_1 \right)^2}{m_1 u_1^2} = \frac{4m}{(1+m)^2}$$

$\therefore$ In each case, K.E. of the stationary body after collision

$= \frac{4m}{(1+m)^2} \times$ K.E. of the moving body before collision.

10. The velocity with which the ball strikes the floor is

$$u = \sqrt{2gh}$$

If the ball rebounds with velocity v, then

$$e = \frac{v}{u} \quad \text{or} \quad v = eu = e\sqrt{2gh}$$

If the ball rebounds to height h', then

$$0^2 - v^2 = 2(-g)h'$$

or $\quad h' = \frac{v^2}{2g} = \frac{1}{2g} \times e^2 \cdot 2gh = he^2$.

Very Short Answer Conceptual Problems

Problem 1(a). If a force acts perpendicular to the direction of motion of a body, what is the amount of work done ?

Solution. The work done is zero because there is no displacement in the direction of force.

Problem 1(b). A body is moving at constant speed over a frictionless surface. What is the work done by the weight ?

Solution. Work done by the weight is zero, since the force and the displacement are at right angles to each other.

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Problem 2. The earth moving around the sun in a circular orbit is acted upon by a force and hence work must be done on the earth by the force. Do you agree by this statement ?

Solution. The statement is wrong. The gravitational force is a conservative force. So the work done by the gravitational force over every complete orbit of the earth is zero.

Problem 3. A body is moving along a circular path. How much work is done by the centripetal force ?

Or *[Himachal 05C, 08 ; Delhi 06]*

Why is the work done by centripetal force zero ?

[Central Schools 12]

Solution. For a body moving along a circular path, the centripetal force acts along the radius while the displacement is tangential *i.e.*, $\theta = 90°$, therefore $W = Fs \cos 90° = 0$.

Problem 4. A body of mass m is moving in a circle of radius r with a constant speed v. The force on the body is $\frac{mv^2}{r}$ and is directed towards the centre. What is the work done by this force in moving the body over half the circumference of the circle ?

Solution. Work done by the force is zero, because the direction of displacement of the body at every point is perpendicular to the direction of force acting on the body.

Problem 5. Is it possible that a body be in accelerated motion under a force acting on the body, yet no work is being done by the force ? Explain your answer giving a suitable example.

Solution. Yes, it is possible, when the force is perpendicular to the direction of motion. The moon revolves round the earth under the centripetal force of attraction of the earth, but earth does no work on the moon.

Problem 6. How much work is done by a coolie walking on a horizontal platform with a load on his head ?

Solution. Zero. In order to balance the load on his head, the coolie applies a force on it in the upward direction equal to its weight. His displacement is along the horizontal direction. Thus the angle between force $\vec{F}$ and displacement $\vec{s}$ is 90°. Therefore work done, $W = Fs \cos \theta = Fs \cos 90° = 0$.

Problem 7. A porter moving vertically up the stairs with a suitcase on his head does work. Why ?

Solution. The porter lifts the suitcase vertically to the upstairs. Force has to be applied on the suitcase against the force of gravitation. Hence the porter does work.

Problem 8. Does the work done in raising a suitcase on to a platform depend upon how fast it is raised up ?

[Himachal 08]

Solution. No. The work done depends on the force of gravity and the height through which the suitcase is raised. It does not depend on the time rate with which the suitcase is raised.

Problem 9. In a tug of war one team is slowly giving way to the other. What work is being done and by whom ?

Solution. The work is done by the winning team and is equal to the product of resultant force applied by the two teams and displacement that the losing team suffers.

Problem 10. A man rowing boat upstream is at rest with respect to the shore. Is he doing work ?

Solution. The boat is at rest with respect to the shore but it is moving upstream with respect to water. The man is doing work relative to the stream because he is applying force to produce relative motion between the boat and the stream. But he does no work relative to the shore as displacement relative to the shore is zero.

Problem 11. Mountain roads rarely go straight up but wind up gradually. Why ?

Solution. If the roads go straight up, the angle of slope θ would be large. In that case frictional force ($f = \mu\, mg \cos \theta$) would be less and the vehicles may slip. Moreover, while going up a large slope, a greater power would be required (as it will take less time).

Problem 12. What sort of energy is associated with a bird flying in air ?

Solution. A flying bird possesses both potential and kinetic energies because it is at a certain height above the ground and moving with certain velocity.

Problem 13. When an arrow is shot, where from the arrow will acquire its kinetic energy ?

Solution. A stretched bow possesses potential energy on account of a change in its shape. To shoot an arrow, the bow is released. The potential energy of the bow is converted into the kinetic energy of the arrow.

Problem 14. Can a body have energy without momentum ?

Solution. Yes, there is an internal energy in a body due to the thermal agitation of the particles of the body, while the vector sum of the momenta of the moving particles may be zero.

Problem 15. Can a body have momentum without energy ?

Solution. No, if a body has momentum, it must be in motion and consequently possess kinetic energy.

Problem 16. A light body and a heavy body have the same momentum. Which one will have greater kinetic energy ? *Or* *[Central Schools 14]*

Two bodies of unequal masses have same linear momentum, which one has greater kinetic energy ?

[Delhi 08]

Solution. Kinetic energy, $K = \frac{1}{2} mv^2 = \frac{(mv)^2}{2m} = \frac{p^2}{2m}$

For constant p, $\quad K \propto \frac{1}{m}$

Thus the lighter body has more kinetic energy than the heavier body.

Problem 17. A light body and a heavy body have the same kinetic energy. Which one will have the greater momentum ? *[Himachal 07, 08]*

Solution. Kinetic energy,

$$K = \frac{1}{2} mv^2 = \frac{(mv)^2}{2m} = \frac{p^2}{2m}$$

or $\quad p = \sqrt{2mK}$ *i.e.*, $p \propto \sqrt{m}$

Thus the heavier body has a greater momentum than the lighter one.

Problem 18. How does the kinetic energy of a body change if its momentum is doubled ?

[Himachal 04, 07C, 08]

Solution. $K = \frac{1}{2} mv^2 = \frac{(mv)^2}{2m} = \frac{p^2}{2m}$ *i.e.*, $K \propto p^2$

When the momentum of a body is doubled, its kinetic energy becomes four times the initial kinetic energy.

Problem 19. Two bodies of masses m_1 and m_2 have the same linear momentum. What is the ratio of their kinetic energies ?

Solution. If v_1 and v_2 are the velocities of two bodies having masses m_1 and m_2 respectively, then

$$m_1 v_1 = m_2 v_2 \quad \text{or} \quad m_1^2 v_1^2 = m_2^2 v_2^2$$

$$\text{or} \quad \frac{m_1 v_1^2}{m_2 v_2^2} = \frac{m_2}{m_1} \quad \text{or} \quad \frac{(m_1 v_1^2/2)}{(m_2 v_2^2/2)} = \frac{m_2}{m_1}$$

$$\text{or} \quad \frac{K_1}{K_2} = \frac{m_2}{m_1}$$

Problem 20. Two bodies of masses m_1 and m_2 ha equal kinetic energies. What is the ratio of their lin momenta ?

Solution. Given that $\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_2 v_2^2$

$$\text{or} \quad \frac{v_1}{v_2} = \sqrt{\frac{m_2}{m_1}}$$

$$\text{or} \quad \frac{m_1 v_1}{m_2 v_2} = \frac{m_1}{m_2} \sqrt{\frac{m_2}{m_1}} = \sqrt{\frac{m_1}{m_2}}$$

$$\text{or} \quad \frac{p_1}{p_2} = \frac{\sqrt{m_1}}{\sqrt{m_2}}$$

Problem 21. Can there be a situation in which E –

Solution. No. As $E = K + U$ or $K = E - U$. But energy K cannot be negative. So $E - U$ is never zero.

Problem 22. Can the overall energy of a negative ?

Solution. Yes. As $E = K + U$, when U is neg has magnitude greater than that of K, E is neg example, the energy of an electron bound to the an atom is negative.

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Problem 17. A light body and a heavy body have the same kinetic energy. Which one will have the greater momentum ? *[Himachal 02, 05]*

Solution. Kinetic energy,

$$K = \frac{1}{2} mv^2 = \frac{(mv)^2}{2m} = \frac{p^2}{2m}$$

or $$p = \sqrt{2mK} \quad i.e., \quad p \propto \sqrt{m}$$

Thus the heavier body has a greater momentum than the lighter one.

Problem 18. How does the kinetic energy of a body change if its momentum is doubled ?

[Himachal 04, 07C, 08]

Solution. $K = \frac{1}{2} mv^2 = \frac{(mv)^2}{2m} = \frac{p^2}{2m}$ *i.e.*, $K \propto p^2$

When the momentum of a body is doubled, its kinetic energy becomes four times the initial kinetic energy.

Problem 19. Two bodies of masses m_1 and m_2 have the same linear momentum. What is the ratio of their kinetic energies ?

Solution. If v_1 and v_2 are the velocities of two bodies having masses m_1 and m_2 respectively, then

$$m_1 v_1 = m_2 v_2 \quad \text{or} \quad m_1^2 v_1^2 = m_2^2 v_2^2$$

$$\text{or} \quad \frac{m_1 v_1^2}{m_2 v_2^2} = \frac{m_2}{m_1} \quad \text{or} \quad \frac{(m_1 v_1^2/2)}{(m_2 v_2^2/2)} = \frac{m_2}{m_1}$$

$$\text{or} \quad \frac{K_1}{K_2} = \frac{m_2}{m_1}$$

Problem 20. Two bodies of masses m_1 and m_2 have equal kinetic energies. What is the ratio of their linear momenta ?

Solution. Given that $\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_2 v_2^2$

$$\text{or} \quad \frac{v_1}{v_2} = \sqrt{\frac{m_2}{m_1}}$$

$$\text{or} \quad \frac{m_1 v_1}{m_2 v_2} = \frac{m_1}{m_2}\sqrt{\frac{m_2}{m_1}} = \sqrt{\frac{m_1}{m_2}}$$

$$\text{or} \quad \frac{p_1}{p_2} = \frac{\sqrt{m_1}}{\sqrt{m_2}}$$

Problem 21. Can there be a situation in which $E - U < 0$?

Solution. No. As $E = K + U$ or $K = E - U$. But kinetic energy K cannot be negative. So $E - U$ is never less than zero.

Problem 22. Can the overall energy of a body be negative ?

Solution. Yes. As $E = K + U$, when U is negative and has magnitude greater than that of K, E is negative. For example, the energy of an electron bound to the nucleus of an atom is negative.

Problem 23. Does the potential energy of a spring decrease or increase when it is compressed or stretched ?

Solution. The potential of the spring increases because work is done on it when it is compressed or stretched.

Problem 24. If a block attached to a spring (whose other end is rigidly fixed) is pulled up to distance x_0 and released, the amplitude of its motion cannot exceed $\pm x_0$. Why ?

Solution. If the amplitude of motion exceeds $\pm x_0$, then the potential energy would be greater than $\frac{1}{2} kx_0^2$ at $x > x_0$.

So its K.E. $= \frac{1}{2} kx_0^2 - \frac{1}{2} kx^2$ would be negative. This is impossible. Hence the block cannot go beyond $x = \pm x_0$.

Problem 25. Springs A and B are identical except that A is stiffer than B, *i.e.*, force constant $k_A > k_B$. In which spring is more work expended if they are stretched by the same amount ?

Solution. Work done in stretching a spring of force constant k through a distance x,

$$W = \frac{1}{2} kx^2$$

$$\therefore \quad \frac{W_A}{W_B} = \frac{(1/2) k_A x^2}{(1/2) k_B x^2} = \frac{k_A}{k_B}$$

As $k_A > k_B$, therefore, $W_A > W_B$.

Problem 26. Springs A and B are identical except that A is stiffer than B. In which spring is more work expended if they are stretched by the same force ?

Solution. As $F = kx$, so $x = \frac{F}{k}$

For same F, $W_A = \frac{1}{2} k_A x^2 = \frac{1}{2} k_A \left(\frac{F}{k_A}\right)^2 = \frac{F^2}{2k_A}$

and $$W_B = \frac{F^2}{2k_B}$$

$$\therefore \quad \frac{W_A}{W_B} = \frac{k_B}{k_A}$$

As $k_A > k_B$ therefore, $W_A < W_B$.

Problem 27. Which of the two : kilowatt hour or electron volt is a bigger unit of energy and by what factor ?

Solution. Kilowatt hour (kWh) is a bigger unit of energy.

$$\frac{1\,\text{kWh}}{1\,\text{eV}} = \frac{3.6 \times 10^6\ \text{J}}{1.6 \times 10^{-19}\ \text{J}} = 2.25 \times 10^{25}$$

Problem 28. Will water at the foot of the fall be at a different temperature from that at the top ? If yes, explain.

Solution. When water falls from a height, its P.E. is converted into kinetic energy and its velocity is maximum at the foot of the fall. The velocity is suddenly reduced to zero, and hence its kinetic energy gets converted into heat energy raising its temperature.

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Problem 29. Is it necessary that work done in the motion of a body over a closed loop is zero for every force in nature ? Why ? *[Central Schools 2003]*

Solution. No. The work done in the motion of body over a closed loop is zero only in case of a conservative force but it is not zero in case of a non-conservative force like friction.

Problem 30. When a constant force is applied to a body moving with constant acceleration, is the power of the force constant ? If not, how would force have to vary with speed for power to be constant ?

Solution. Power, $P = Fv$

As the body is moving with acceleration, v changes and so P also changes, F being constant. For P to be constant, Fv = a constant or ($F \propto 1/v$). Thus as v increases, F should decrease to keep P constant.

Problem 31. What are central forces ? Are they conservative in nature ?

Solution. Force between two objects is called a central force, if it acts between them along the line joining their centres. Electrostatic force between the charges and magnetic force between two poles are central forces and are conservative forces.

Problem 32. When is the exchange of energy maximum during an elastic collision ?

Solution. Energy exchange will be maximum if the two colliding bodies are of equal masses.

Problem 33. Is whole of the kinetic energy lost in any perfectly inelastic collision ?

Solution. No, only that much amount of kinetic energy is lost as is necessary for the conservation of momentum.

Problem 34. Can you associate potential energy with a non-conservative force ?

Solution. No, P.E. can be associated only with conservative force.

Problem 35. Two bodies moving towards each other collide and move away in opposite directions. There is some rise of temperature of the bodies in the process. Explain the reason for the rise of temperature and state what type of collision is it.

Solution. Involved collision of the process is inelastic because the bodies suffer loss of kinetic energy which appears in the form of heat energy raising the temperature of the bodies.

Problem 36. A spark is produced, when two stones are struck against each other. Why ? *[Central Schools 14]*

Solution. The work done in striking the two stones against each other gets converted into heat. This appears as a spark.

Short Answer Conceptual Problems

Problem 1. A lorry and a car with the same kinetic energy are brought to rest by the application of the brakes which provide equal retarding force. Which of them will come to rest in a shorter distance ?

Solution. By work-energy theorem,

Loss in K.E. of the vehicle

= Work done against retarding force

= Retarding force × distance travelled

$$\therefore \text{Distance travelled} = \frac{\text{Loss in K.E.}}{\text{Retarding force}}$$

As both the kinetic energy and retarding force are same, so both the lorry and the car would come to rest in the same distance.

Problem 2. A truck and a car are moving with the same kinetic energy on a straight road. Their engines are simultaneously switched off. Which one will stop at a lesser distance ?

Solution. By work-energy theorem,

Loss in K.E. of the vehicle

= Work done against the force of friction × distance

or $\text{K.E.} = f \times s = \mu R \times s = \mu mg s$

or $s = \dfrac{\text{K.E.}}{\mu mg}$

For constant K.E., $s \propto \dfrac{1}{m}$

As the truck has more mass than the car, so it will stop in a lesser distance than the car.

Problem 3. A rocket explodes in mid air. How does this affect (*a*) its total momentum and (*b*) its total kinetic energy ?

Solution. (*a*) Because no external force acts on the rocket, its total momentum remains unchanged.

(*b*) When the rocket explodes, its fragments receive additional kinetic energy from the explosion. The chemical energy of the fuel changes into kinetic energy. As a result of this, the total kinetic energy gets increased.

Problem 4. The velocity of an aeroplane is doubled. (*a*) What will happen to its momentum ? Will the momentum remain conserved ? (*b*) What will happen to its K.E. ? Will the energy remain conserved ?

Solution. (*a*) When the velocity of the aeroplane is doubled, its momentum also gets doubled. However, the combined momentum of aeroplane + air is conserve the momentum of the aeroplane increases, the mo tum of air also increases by an equal amount in opposite direction.

(*b*) The kinetic energy becomes four times. additional energy is obtained by burning of However, the total energy is still conserved.

Problem 5. A uniform rectangular parallelopip mass m having sides l, $2l$ and $4l$ is placed in turn on of its three sides on a horizontal surface. What i potential energy of the parallelopiped in each of positions ? Which position will be the most stable

Solution. When the parallelopiped is placed small, middle and large side, its centre of gravity w at heights $2l$, l and $l/2$ respectively above the horiz surface. If U_1, U_2 and U_3 are the potential energies three cases, then

$$U_1 = mg \times 2l = 2\,mgl$$

$$U_2 = mg \times l = mgl$$

$$U_3 = mg \times \frac{l}{2} = \frac{1}{2}\,mgl$$

When the parallelopiped rests on its large sid potential energy is minimum. So this position position of stable equilibrium.

Problem 6. What happens to the potential e when (*a*) two protons are brought close together (proton and one electron are brought close togethe

(*a*) The potential energy increases when two p are brought closer because work has to be done a the force of repulsion between them.

(*b*) The potential energy decreases when a proto an electron are brought closer because work is done force of attraction between them (origin of the force the system itself).

Problem 7. A particle is moving in a circular pa given radius with number of rotations per secc constant, (*ii*) increasing, (*iii*) decreasing. What ha to work done in the three cases ?

Solution. Work is equal to the change in energy of the system. (*i*) In case speed is constant, energy does not change, hence work done is (*ii*) when speed increases, K.E. increases, work is d the exerting force, (*iii*) when speed decreases decreases, work is done by the body on the force.

Problem 8. "Chemical, gravitational and r energies are nothing but potential energies for di types of forces in nature." Explain the statement with examples.

Solution. A system has potential energy, various objects are held at certain distance agains force, by virtue of their position or configu Chemical potential energy results from the ch

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combined momentum of aeroplane + air is conserved. As the momentum of the aeroplane increases, the momentum of air also increases by an equal amount in the opposite direction.

(ii) The kinetic energy becomes four times. The additional energy is obtained by burning of fuel. However, the total energy is still conserved.

Problem 5. A uniform rectangular parallelopiped of mass m having sides l, $2l$ and $4l$ is placed in turn on each of its three sides on a horizontal surface. What is the potential energy of the parallelopiped in each of these positions ? Which position will be the most stable ?

Solution. When the parallelopiped is placed on its small, middle and large side, its centre of gravity will lie at heights $2l$, l and $l/2$ respectively above the horizontal surface. If U_1, U_2 and U_3 are the potential energies in the three cases, then

$$U_1 = mg \times 2l = 2mgl$$

$$U_2 = mg \times l = mgl$$

$$U_3 = mg \times \frac{l}{2} = \frac{1}{2} mgl$$

When the parallelopiped rests on its large side, its potential energy is minimum. So this position is the position of stable equilibrium.

Problem 6. What happens to the potential energy when (a) two protons are brought close together (b) one proton and one electron are brought close together ?

(a) The potential energy increases when two protons are brought closer because work has to be done against the force of repulsion between them.

(b) The potential energy decreases when a proton and an electron are brought closer because work is done by the force of attraction between them (origin of the force being the system itself).

Problem 7. A particle is moving in a circular path of a given radius with number of rotations per second (i) constant, (ii) increasing, (iii) decreasing. What happens to work done in the three cases ?

Solution. Work is equal to the change in kinetic energy of the system. (i) In case speed is constant, kinetic energy does not change, hence work done is zero, (ii) when speed increases, K.E. increases, work is done by the exerting force, (iii) when speed decreases, K.E. decreases, work is done by the body on the force.

Problem 8. "Chemical, gravitational and nuclear energies are nothing but potential energies for different types of forces in nature." Explain the statement clearly with examples.

Solution. A system has potential energy, when various objects are held at certain distance against some force, by virtue of their position or configuration. Chemical potential energy results from the chemical bonding of atoms, gravitational potential energy results when objects are held at some distance against gravitational force while nuclear energy arises from the nuclear force between the nucleons.

Problem 9. In a thermal power station, coal is used for the generation of electricity. Mention how energy changes from one form to another before it is transformed into electrical energy ?

Solution. The heat energy produced due to combustion of coal converts water into steam. The heat energy of steam is converted into mechanical energy when it turns blades of a turbine. The mechanical energy so obtained is converted into electrical energy by the generators.

Problem 10. Explain how fast moving neutrons can be quickly slowed down by passing through water or heavy water.

Solution. Water and heavy water contain protons having approximately the same mass as that of a neutron. When fast moving neutrons collide with protons, the neutrons attain the low velocities of protons while the protons begin to move with the high velocities of neutrons.

Problem 11. Two ball bearings of mass m each moving in the opposite directions with equal speed v collide head on with each other. Predict the outcome of the collision, assuming it to be perfectly elastic.

Solution. Here $m_1 = m_2 = m$, $u_1 = v$, $u_2 = -v$

$$\therefore \quad v_1 = \frac{(m_1 - m_2)u_1 + 2m_2 u_2}{m_1 + m_2} = \frac{(m-m)v + 2m(-v)}{m+m} = -v$$

and

$$v_2 = \frac{(m_2 - m_1)u_2 + 2m_1 u_1}{m_1 + m_2} = \frac{(m-m)(-v) + 2mv}{m+m} = v$$

Hence after collision, the two ball bearings will move with same speeds but their directions of motion are reversed.

Problem 12. A body of mass m moving with speed v collides elastically head-on with another body of mass m initially at rest. Show that the moving body will come to a stop as a result of this collision. *[Delhi 04]*

Solution. By conservation of momentum,

$$m \times v + m \times 0 = mv_1 + mv_2$$

or $$v = v_1 + v_2$$

or $$v - v_1 = v_2 \qquad ...(1)$$

By conservation of kinetic energy,

$$\frac{1}{2} mv^2 + 0 = \frac{1}{2} mv_1^2 + \frac{1}{2} mv_2^2$$

or $$v^2 = v_1^2 + v_2^2$$

or $$(v + v_1)(v - v_1) = v_2^2 \qquad ...(2)$$

Dividing (2) by (1), we get

$$v + v_1 = v_2 \qquad ...(3)$$

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Solving (1) and (3),

$$v_1 = 0 \text{ and } v_2 = v$$

As a result of the collision, the moving body comes to a stop and the stationary body begins to move with the speed of the first body.

Problem 13. A uniform chain of length L and mass M is lying on a smooth table and one third of its length is hanging vertically down over the edge of the table. If g is acceleration due to gravity, calculate work required to pull the hanging part on the chain. *[IIT 85; MNREC 90]*

Solution. Weight of length L of the chain $= Mg$

$$\text{Weight of length } \frac{L}{3} \text{ of the chain} = \frac{1}{3} Mg$$

As the centre of gravity of the hanging part lies at its midpoint *i.e.* at a distance equal to $L/6$ below the edge of the table, so the work required to pull the hanging part on the table is

$$W = \text{Force} \times \text{distance} = \frac{1}{3} Mg \times \frac{L}{6} = \frac{MgL}{18}.$$

Problem 14. A body of mass M is moved along a straight line by a machine delivering a constant power P. Find the expression for the distance moved by the body in terms of M, P and t. *[AIEEE 03]*

Solution. Power, $P = Fv = ma \cdot v = m \frac{dv}{dt} \cdot v$

or $$v dv = \frac{P}{m} dt$$

Integrating both sides, we get

$$\int v dt = \int \frac{P}{m} dt = \frac{P}{m} \int dt \quad [\because P, m \text{ are constant}]$$

or $$\frac{v^2}{2} = \frac{P}{m} \cdot t$$

or $$v^2 = \frac{2P}{m} \cdot t$$

or $$v = \sqrt{\frac{2P}{m}} \cdot t^{1/2}$$

Thus $$v \propto t^{1/2}$$

Also $$v = \frac{dx}{dt} \quad \text{or} \quad dx = vdt = \sqrt{\frac{2P}{m}} \cdot t^{1/2} dt$$

Integrating both sides, we get

$$\int dx = \sqrt{\frac{2P}{m}} \int t^{1/2} dt$$

or $$x = \sqrt{\frac{2P}{m}} \cdot \frac{t^{3/2}}{3/2} = \frac{2}{3} \sqrt{\frac{2P}{m}} \cdot t^{3/2}$$

Thus $$x \propto t^{3/2}.$$

Problem 15. Why a metal ball rebounds better than a rubber ball ?

Solution. When a rubber ball hits a massive object, say, earth, the ball is distorted. A large amount of heat is generated in the ball by the rubbing of the rubber molecules against each other. This effect is essentially absent in a hard material. So, a metal ball would often lose less energy upon collision than would a rubber ball.

Problem 16. Explain, throwing mud on a wall is an example of perfectly inelastic collision.

Solution. When mud is thrown on a wall, it sticks to the wall. The kinetic energy of the mud is reduced to zero, and there is non-conservation of kinetic energy. Hence it is a case of perfectly inelastic collision.

Problem 17. Nuclear fission and fusion reactions are examples of conversion of mass into energy. Can we say that strictly speaking, mass is converted into energy even in an exothermic chemical reaction ?

Solution. Yes, the generation of heat in exothermic reaction involves the conversion of mass into energy. However, the amount of mass involved is much less than that in fission or fusion.

Problem 18. The energy released in fusion reaction of light nuclei is much less than the energy released in a fission reaction of a heavy nucleus. Why is then a hydrogen bomb (based on nuclear fusion) far more powerful than an atomic bomb (based on nuclear fission) ?

Solution. A hydrogen bomb (based on nuclear fusion) is more powerful than atom bomb (based on nuclear fission), because in case of fusion, the energy released per unit mass of the fuel is much larger than that in fission.

Problem 19. What is the minimum amount of energy released in the annihilation of an electron-positron pair?

Solution. The energy released in the annihilation is minimum when both electron and positron are at rest.

Hence, $$E_{min} = 2 m_0 c^2$$

(where, m_0 = rest mass of electron or positron)

$$E_{min} = 2 \times 9.1 \times 10^{-31} \times (3 \times 10^8)^2 \text{ J}$$

$$= \frac{2 \times 9.1 \times 10^{-31} \times (3 \times 10^8)^2}{1.6 \times 10^{-13}} \text{ MeV}$$

$$= \mathbf{1.02 \text{ MeV}}.$$

Problem 20. By convention, the forces which fall off to zero at large distances, the potential energy at infinity is taken to be zero. With this choice, is the potential energy positive for (*a*) electron-positron bound state (*b*) planet-satellite system and (*c*) electron-electron system ?

Solution.

(*a*) *Negative*, because of attraction between electron and positron.

(*b*) *Negative*, because of attraction between planet and satellite.

(*c*) *Positive*, because of repulsion between two electrons.

HOTS

Problem 1. *A locomotive of mass m st that its velocity varies according to the law α is a constant and s is the distance covere work done by all the forces acting on the lo the first t seconds after the beginning of m*

Solution. Velocity, $v = \alpha \sqrt{s} = \alpha s^{1}$

Acceleration,

$$a = \frac{dv}{dt} = \frac{1}{2} \alpha s^{-1/2} \cdot \frac{ds}{dt}$$

$$= \frac{1}{2} \alpha s^{-1/2} \cdot \alpha s^{1/2} =$$

Force, $$F = ma = \frac{1}{2} m\alpha^2$$

Distance covered by the locomoti

$$s = ut + \frac{1}{2} at^2 = 0 \times t + \frac{1}{2} \times$$

$\therefore$ Work done,

$$W = Fs = \frac{1}{2} m\alpha^2 \times \frac{1}{4} \alpha^2 t^2 =$$

Problem 2. *A 0.5 kg particle from $x = 5$ m to $x = 17.2$ m under $F(x)$,*

$$F = \frac{200}{2x + x}$$

Estimate (with at least 5% acc by this force during this displace

Solution.

$$W = \int_{x_i}^{x_f} F \, dx = \int_{5}^{17.2} \frac{200}{2x + x}$$

Put $t = x^2 + 2$, $dt = 2x dx$.

$$W = \frac{200}{2} \int \frac{dt}{(t-2)t}$$

$$= 100 \int \left[\frac{1}{2(t-2}$$

$$= 50 [\log_e (t-2)$$

$$= 50 \times 2.303 \Big| \text{lo}$$

$$= 50 \times 2.303 \text{ lo}$$

$$= 50 \times 2.303$$

$$= 50 \times 2.303$$

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HOTS

Problems on Higher Order Thinking Skills

Problem 1. *A locomotive of mass m starts moving so that its velocity varies according to the law* $v = \alpha\sqrt{s}$, *where* α *is a constant and s is the distance covered. Find the total work done by all the forces acting on the locomotive during the first t seconds after the beginning of motion.*

Solution. Velocity, $v = \alpha\sqrt{s} = \alpha s^{1/2}$

Acceleration,

$$a = \frac{dv}{dt} = \frac{1}{2}\alpha s^{-1/2}\cdot\frac{ds}{dt} = \frac{1}{4}\alpha s^{-1/2}\cdot v$$

$$= \frac{1}{2}\alpha s^{-1/2}\cdot\alpha s^{1/2} = \frac{1}{2}\alpha^2$$

Force, $F = ma = \frac{1}{2}m\alpha^2$

Distance covered by the locomotive in first t seconds,

$$s = ut + \frac{1}{2}at^2 = 0\times t + \frac{1}{2}\times\frac{1}{2}\alpha^2 t^2 = \frac{1}{4}\alpha^2 t^2$$

$\therefore$ Work done,

$$W = Fs = \frac{1}{2}m\alpha^2\times\frac{1}{4}\alpha^2 t^2 = \frac{1}{8}m\alpha^4 t^2.$$

Problem 2. *A 0.5 kg particle moves along the x-axis from* $x = 5$ *m to* $x = 17.2$ *m under the influence of a force* $F(x)$,

$$F = \frac{200}{2x + x^3}\ N$$

Estimate (with at least 5% accuracy) the total work done by this force during this displacement.

Solution.

$$W = \int_{x_i}^{x_f} F\,dx = \int_5^{17.2}\frac{200}{2x + x^3}\,dx = \int_5^{17.2}\frac{200}{x(x^2+2)}\,dx$$

Put $t = x^2 + 2$, $dt = 2x\,dx$. Then

$$W = \frac{200}{2}\int\frac{dt}{(t-2)\,t}$$

$$= 100\int\left[\frac{1}{2(t-2)} - \frac{1}{2t}\right]dt$$

$$= 50\left[\log_e(t-2) - \log_e t\right] = 50\left[\log_e\frac{t-2}{t}\right]$$

$$= 50\times 2.303\left|\log\frac{x^2}{x^2+2}\right|_5^{17.5}$$

$$= 50\times 2.303\log\frac{295.84}{297.84}\times\frac{27}{25}$$

$$= 50\times 2.303\log 1.07275$$

$$= 50\times 2.303\times 0.0305 = \mathbf{3.51\ J}$$

Problem 3. *The potential energy of a 2 kg particle free to move along the x-axis is given by*

$$U(x) = \left(\frac{x}{b}\right)^4 - 5\left(\frac{x}{b}\right)^2\ J$$

where $b = 1$ *m. Plot this potential, identifying the extremum points. Identify the regions where particle may be found and its maximum speed given that the total mechanical energy is (i) 36 J ; (ii) − 4 J.*

Solution. For $b = 1$, $U = x^4 - 5x^2$

$$\therefore\quad \frac{dU}{dx} = 4x^3 - 10x = 2x(2x^2 - 5)$$

For extremum points, $\frac{dU}{dx} = 0$

or $2x(\sqrt{2}x - \sqrt{5})(\sqrt{2}x + \sqrt{5}) = 0$

$\therefore\ x = 0, \sqrt{\frac{5}{2}}, -\sqrt{\frac{5}{2}}$ are the extremum points.

x	0	± 1	$\pm\sqrt{5/2}$	± 2	± 3
U	0	− 4	− 6.25	− 4	− 36

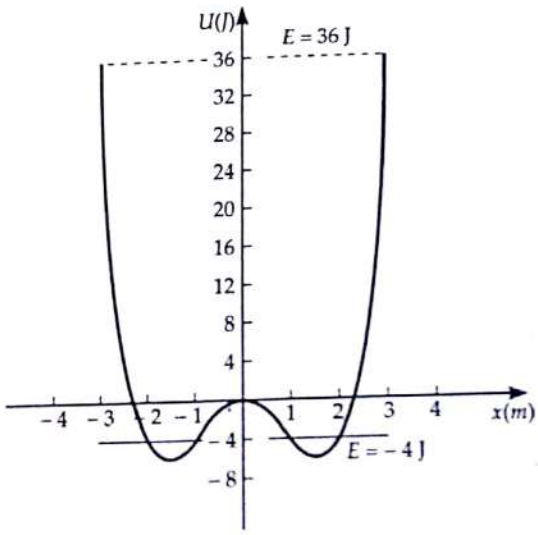

Fig. 6.42 Plot of potential energy U versus x

(*i*) Here $E = K + U = 36$ J. The particle may be found in the region in which its K.E. is positive. This region is $-3\text{ m} < x < 3\text{ m}$ because here $U_{max} = 0$ and $k_{min} = -36$ J (at $x = 0$)

At $x = \pm\sqrt{\frac{5}{2}}$, $U_{min} = -6.25$ J

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$$\therefore \quad K_{max} = E - U_{min} = 36 + 6.25 = 42.25 \text{ J}$$

or $$\frac{1}{2} m v_{max}^2 = 42.25$$

or $$v_{max} = \sqrt{\frac{2 \times 42.25}{m}} = \sqrt{\frac{2 \times 42.25}{2}} = 6.5 \text{ ms}^{-1}.$$

(ii) Here $E = K + U = -4$ J. Again for the positive K.E., the particle may be found in the regions

$$-2 \text{ m} < x < -1 \text{ m and } 1 \text{ m} < x < 2 \text{ m}$$

As $$K_{max} = E - U_{min}$$

or $$\frac{1}{2} m v_{max}^2 = -4 + 6.25 = 2.25$$

$$v_{max} = \sqrt{\frac{2 \times 2.25}{2}} = 1.5 \text{ ms}^{-1}.$$

Problem 4. *A particle of mass m is moving in a horizontal circle of radius r, under a centripetal force equal to $-(K/r^2)$, where K is constant. What is the total energy of the particle ?* **[IIT ; Delhi 10]**

Solution. As the particle is moving in horizontal circle, so

Centripetal force, $$F = \frac{mv^2}{r} = \frac{K}{r^2}$$

This gives $$mv^2 = \frac{K}{r}$$

$\therefore$ K.E. of the particle,

$$K = \frac{1}{2} mv^2 = \frac{K}{2r}$$

As $$F = -\frac{dU}{dr}$$

$\therefore$ Potential energy,

$$U = -\int_{\infty}^{r} F dr = -\int_{\infty}^{r} \left(-\frac{K}{r^2}\right) dr = K \int_{\infty}^{r} r^{-2} \, dr = -\frac{K}{r}$$

Total energy $= K + U = \frac{K}{2r} - \frac{K}{r} = -\frac{K}{2r}$.

Problem 5. *A small object of weight mg hangs from a string of length l, as shown in Fig. 6.43. A variable force F, which starts at zero and gradually increases is used to pull the object very slowly (so that equilibrium exists at all the times) until the string makes an angle θ with the vertical. Calculate the work done by the force F.* **[CPMT 93]**

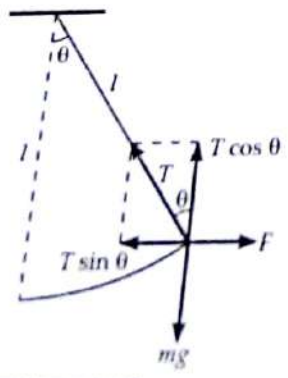

Fig. 6.43

Solution. Various forces acting on the small object are shown in the figure. As the object remains in equilibrium, so

$$F = T \sin \theta \quad \text{and} \quad mg = T \cos \theta$$

$\therefore$ $$\frac{F}{mg} = \frac{T \sin \theta}{T \cos \theta} = \tan \theta$$

or $$F = mg \tan \theta$$

Also, displacement of the object, $s = l\theta$

$\therefore$ $ds = l \, d\theta$

Work done by the force F,

$$W = \int_0^{\theta} \vec{F} \cdot d\vec{s} = \int_0^{\theta} F ds \cos \theta$$

$$= \int_0^{\theta} mg \tan \theta \times l \, d\theta \times \cos \theta$$

$$= mgl \int_0^{\theta} \sin \theta \, d\theta$$

$$= mgl \, [-\cos \theta]_0^{\theta} = mgl \, [-\cos \theta + \cos 0]$$

or $$W = mgl \, (1 - \cos \theta).$$

Problem 6. *A chain is held on a frictionless table with 1/n th of its length hanging over the edge. If the chain has a length l and a mass m, how much work is required to pull the hanging part back on the table ?* **[MNREC 90]**

Solution. Mass per unit length of the chain $= \frac{m}{l}$

Let length of the hanging part of the chain $= y$

Mass of the hanging part of the chain $= \frac{m}{l} \cdot y$

Force required to be applied = Weight of the hanging part

or $$F = \left[\frac{m}{l} y\right] g = \frac{mg}{l} \cdot y$$

The work done in pulling the chain through a small distance dy is

$$dW = -\frac{mg}{l} \cdot y \, dy$$

Here negative sign indicates that the weight and displacement are oppositely directed. Total work done in pulling the 1/n th length of the chain is

$$W = \int dW = -\frac{mg}{l} \int_{y = \frac{l}{n}}^{y = 0} y \, dy$$

$$= -\frac{mg}{l} \left[\frac{y^2}{2}\right]_{l/n}^{0}$$

$$= -\frac{mg}{2l} \left[0 - \frac{l^2}{n^2}\right] = \frac{mgl}{2n^2}.$$

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Problem 7. *A person decides to use his bath tub water to generate electric power to run a 40 W bulb. The bath tub is located at a height of 10 m from the ground and it holds 200 litres of water. He instals a water driven wheel generator on the ground. At what rate should the water drain from the bath tub to light the bulb ? How long can he keep the bulb on, if bath tub was full initially ? Efficiency of generator = 90%. Take $g = 98\ ms^{-2}$.* **[Roorkee 90]**

Solution. Let V be the volume of water flowing out per second from the tub which is kept at height h from the ground. Then,

Input power $= (V\rho)\, gh$ [$\because$ $V\rho$ = mass of water flowing per second]

Output power = 40 W

As efficiency, $\eta = \dfrac{\text{Output power}}{\text{Input power}}$

$$\therefore \quad \frac{90}{100} = \frac{40}{V\rho gh}$$

$$V = \frac{40 \times 100}{90\,\rho\, gh} = \frac{40 \times 100}{90 \times 1000 \times 9.8 \times 10}$$

or $$= 0.453 \times 10^{-3}\ \text{m}^3/\text{s} = 0.453\ \text{litre/s}$$

[$\because$ 1 litre $= 10^{-3}\ \text{m}^3$]

Time for which the bulb can be kept on

$$= \frac{200\ \text{litre}}{0.453\ \text{litre/s}} = \mathbf{441\ s.}$$

Problem 8. *A bullet of mass 0.01 kg and travelling at a speed of $500\ ms^{-1}$ strikes a block of mass 2 kg which is suspended by a string of length 5 m. The centre of gravity of the block is found to rise a vertical distance of 0.1 m. What is the speed of the bullet after it emerges from the block ? Take $g = 98\ ms^{-2}$.* **[Roorkee 90]**

Solution. In Fig. 6.44, $m = 0.01$ kg, $v = 500\ \text{ms}^{-1}$, $M = 2$ kg, $l = 5$ m, $h = 0.1$ m

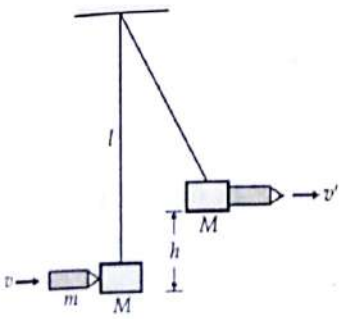

Fig. 6.44

Let V be the velocity gained by the block of mass M. Then by conservation of energy,

K.E. of the block = P.E. of the block

or $$\frac{1}{2} MV^2 = Mgh$$

or $$V = \sqrt{2gh} = \sqrt{2 \times 98 \times 0.1}$$

$$= 1.4\ \text{ms}^{-1}$$

Let v' be the speed of the bullet after it emerges from the block. Then by conservation of momentum, we have

$$mv + M \times 0 = MV + mv'$$

or $$0.01 \times 500 + 0 = 2 \times 1.4 + 0.01 \times v'$$

or $$v' = \mathbf{220\ ms^{-1}}.$$

Problem 9. *A pendulum bob of mass 10^{-2} kg is raised to a height of 5×10^{-2} m and then released. At the bottom of its swing, it picks up a mass of 10^{-3} kg. To what height will the combined mass rise ? Take $g = 10\ ms^{-2}$.* **[Roorkee 95]**

Solution. By conservation of energy,

P.E. of the bob at height h

= K.E. of bob at the bottom

$$Mgh = \frac{1}{2} Mv^2$$

or $$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 5 \times 10^{-2}}$$

$$= 1.0\ \text{ms}^{-1}$$

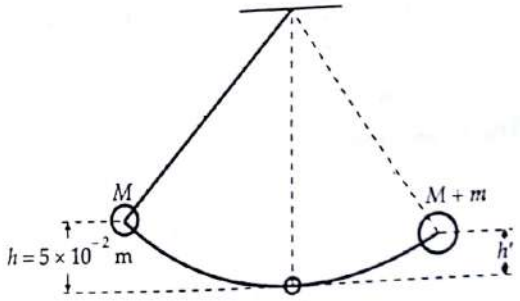

Fig. 6.45

When the bob picks up a mass $m = 10^{-3}$ kg, suppose its velocity changes to v'. By conservation of momentum, we have

$$Mv = (M + m)\, v'$$

$$v' = \frac{Mv}{M + m} = \frac{0.01 \times 1.0}{0.01 + 0.001} = 0.91\ \text{ms}^{-1}$$

Let the combined mass moving with velocity v' rise to height h'. Then

$$\frac{1}{2} (M + m)\, v'^2 = (M + m)\, gh'$$

or $$h' = \frac{1}{2} \frac{v'^2}{g} = \frac{1}{2} \times \frac{(0.91)^2}{10} = \mathbf{0.0414\ m.}$$

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Problem 10. *A particle of mass 1 g moving with a velocity* $\vec{v_1} = 3\hat{i} - 2\hat{j}$ *experiences a perfectly elastic collision with another particle of mass 2 g and velocity* $\vec{v_2} = 4\hat{j} - 6\hat{k}$. *Find the velocity of the particle formed (both* $\vec{v}$ *and* v*).* ($\vec{v_1}$ *and* $\vec{v_2}$ *are given in SI units).* **[Delhi 06]**

Solution. By conservation of momentum,

$$m_1\vec{v_1} + m_2\vec{v_2} = (m_1 + m_2)\vec{v}$$

or $\quad 1(3\hat{i} - 2\hat{j}) + 2(4\hat{j} - 6\hat{k}) = 3\vec{v}$

or $\quad 3\hat{i} + 6\hat{j} - 12\hat{k} = 3\vec{v}$

or $\quad \vec{v} = \hat{i} + 2\hat{j} - 4\hat{k}$

and $\quad v = |\vec{v}| = \sqrt{1 + 4 + 16} = 4.6 \text{ ms}^{-1}$.

Problem 11. *A ball, whose kinetic energy is* E, *is projected at an angle of 45° to the horizontal. What will be the kinetic energy of the ball at the highest point of its flight?* **[AIEEE 02]**

Solution. Suppose the ball is projected with velocity u. Then

$$E = \frac{1}{2}mu^2$$

At the highest point, the velocity of the ball will be

v = Horizontal component of the velocity of projection

$$= u\cos 45° = u/\sqrt{2}$$

The kinetic energy of the ball at the highest point,

$$E' = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{u}{\sqrt{2}}\right)^2$$

$$= \frac{1}{2} \times \frac{1}{2}mu^2 = \frac{1}{2}E$$

Problem 12. *A particle of mass m moves in a straight line with retardation proportional to its displacement. Find the expression for loss of kinetic energy for any displacement x.* **[AIEEE 04]**

Solution. Given $a \propto x$ or $a = -kx$ ...(1)

The negative sign is taken for retardation.

Now

$$a = \frac{dv}{dt} = \frac{dv}{dx}\cdot\frac{dx}{dt} = v\frac{dv}{dx} \quad ...(2)$$

From (1) and (2), we get

$$v\frac{dv}{dx} = -kx \quad \text{or} \quad v\,dv = -kx\,dx \quad ...(3)$$

Suppose the velocity of the particle changes from u to v during its displacement x. Then integrating (3) within these limits of velocity and displacement, we get

$$\int_u^v v\,dv = -k\int_0^x x\,dx$$

or $\quad \left[\frac{v^2}{2}\right]_u^v = -k\left[\frac{x^2}{2}\right]_0^x$

or $\quad \frac{1}{2}v^2 - \frac{1}{2}u^2 = -\frac{1}{2}kx^2$

or $\quad \frac{1}{2}u^2 - \frac{1}{2}v^2 = \frac{1}{2}kx^2$

$\therefore$ $\quad$ Loss in K.E. $= \frac{1}{2}mu^2 - \frac{1}{2}mv^2 = \frac{1}{2}mkx^2$.

Problem 13. *A body of mass m accelerates uniformly from rest to velocity* v_1 *in time* t_1. *Find the expression for the instantaneous power delivered to the body as a function of time.* **[AIEEE 04, 05]**

Solution. If a is the uniform acceleration of the body, then

$$v_1 = 0 + at_1 \quad \text{or} \quad a = v_1/t_1$$

The velocity of the body at instant t,

$$v = 0 + at = \frac{v_1}{t_1}\cdot t$$

Instantaneous power,

$$P = Fv = mav = m \times \frac{v_1}{t_1} \times \frac{v_1}{t_1}\cdot t = \frac{mv_1^2 t}{t_1^2}$$

Problem 14. *Show that coefficient of restitution for one dimensional elastic collision is equal to one.* **[Chandigarh 05]**

Solution. As momentum is conserved in any collision, so

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

or $\quad m_1(u_1 - v_1) = m_2(v_2 - u_2)$ ...(i)

Also, kinetic energy is conerved in an elastic collision, so

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

or $\quad m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$ ...(ii)

On dividing (*ii*) by (*i*), we get

$$u_1 + v_1 = v_2 + u_2$$

or $\quad v_2 - v_1 = u_1 - u_2$

Hence, the coefficient of restitution for one dimenional elastic collision will be

$$e = \frac{v_2 - v_1}{u_1 - u_2} = 1.$$

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Guidelines to NCERT Exercises

6.1. The sign of work done by a force on a body is important to understand. State carefully if the following quantities are positive or negative.

(i) Work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket.

(ii) Work done by gravitational force in the above case.

(iii) Work done by friction on a body sliding down an inclined plane.

(iv) Work done by an applied force on a body moving on a rough horizontal plane with uniform velocity.

(v) Work done by the resistive force of air on a vibrating pendulum in bringing it to rest.

Ans.

(i) Work done is *positive*, because the bucket moves in the direction of the force applied by the man.

(ii) Work done by gravitational force is *negative* because the bucket moves upwards while the gravitational force acts downwards.

(iii) Work done is *negative*, because force of friction acts on the body in the opposite direction of its motion.

(iv) Work done is *positive*, because the applied force acts in the direction of motion of the body.

(v) Work done is *negative*, because the resistive force of air acts in a direction opposite to the direction of motion of the vibrating pendulum.

6.2. A body of mass 2 kg initially at rest moves under the action of an applied horizontal force of 7 N on a table with coefficient of kinetic friction = 0.1. Compute the

(i) work done by the applied force in 10 s.

(ii) work done by the friction in 10 s.

(iii) work done by the net force on the body in 10 s, and

(iv) change in kinetic energy of the body in 10 s. Interpret your results.

Ans. Here $m = 2$ kg, $u = 0$, $F = 7$ N,

$$\mu_k = 0.1, \quad t = 10 \text{ s}$$

Force of friction,

$$f_k = \mu_k R = \mu_k mg = 0.1 \times 2 \times 9.8 = 1.96 \text{ N}$$

Net force with which the body moves,

$$F' = F - f_k = 7 - 1.96 = 5.04 \text{ N}$$

Acceleration,

$$a = \frac{F'}{m} = \frac{5.04}{2} = 2.52 \text{ ms}^{-2}$$

Distance,

$$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 2.52 \times (10)^2$$

$$= 126 \text{ m}$$

(i) Work done by the applied force,

$$W_1 = Fs = 7 \times 126 = 882 \text{ J}$$

(ii) Work done by the friction,

$$W_2 = -f_k \times s = -1.96 \times 126 = -246.9 \text{ J}$$

(iii) Work done by the net force,

$$W_3 = F' s = 5.04 \times 126 = 635 \text{ J}$$

(iv) Final velocity acquired by the body after 10 s,

$$v = u + at = 0 + 2.52 \times 10 = 25.2 \text{ ms}^{-1}$$

Change in K.E. of the body

$$= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2} \times 2 \times (25.2)^2 - 0 = 635 \text{ J}$$

Thus the change in K.E. of the body is equal to the work done by the net force on the body.

6.3. Given below (Fig. 6.46) are examples of some potential energy functions in one dimension. The total energy of the particle is indicated by a cross on the ordinate axis. In each case, specify the regions, if any, in which the particle cannot be found

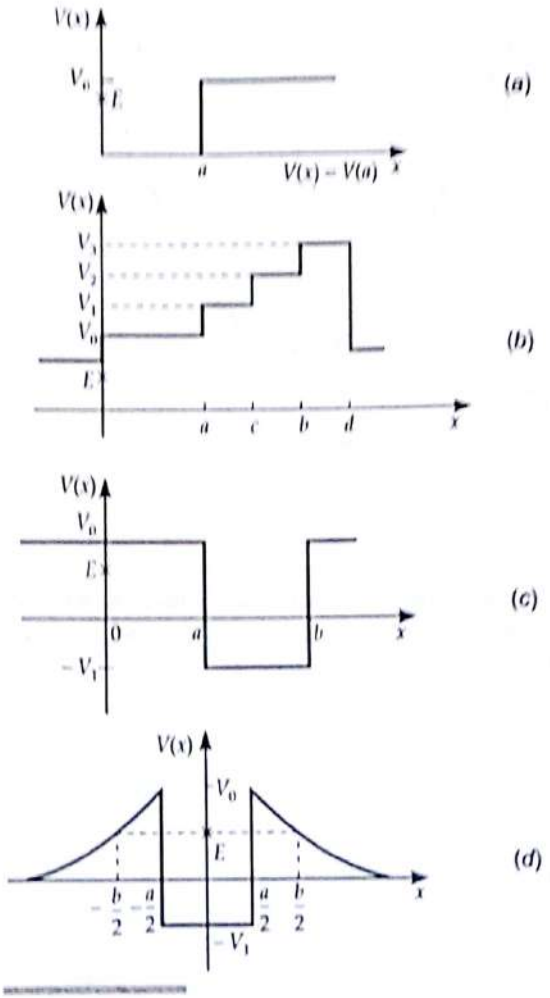

Fig. 6.46

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for the given energy. Also indicate the minimum total energy the particle must have in each case. Think of simple physical contexts for which these potential energy shapes are relevant.

Ans. Total energy, E = K.E. + P.E.

$\therefore$ K.E. = E − P.E.

The particle can exist in such a region in which its K.E. is positive.

(*a*) For $x > a$, P.E. $(V_0) > E$

$\therefore$ K.E. is negative. The particle cannot exist in the region $x > a$. Here $E_{min} = 0$.

(*b*) In every region of the graph, P.E. $(V) > E$.

$\therefore$ K.E. is negative. The particle cannot be found in any region. Here $E_{min} = -V_1$.

(*c*) For $x < a$ and $x > b$, P.E. $(V_0) > E$.

$\therefore$ K.E. is negative. The particle cannot be found in the region $x < a$ and $x > b$. Here $E_{min} = -V_1$.

(*d*) For $-\frac{b}{2} < x < -\frac{a}{2}$ and $\frac{a}{2} < x < \frac{b}{2}$, P.E. $(V) > E$.

$\therefore$ K.E. is negative. The particle cannot be present in these regions. Here $E_{min} = -V_1$.

6.4. *The potential energy function for a particle executing linear simple harmonic motion is given by $V(x) = kx^2/2$, where k is the force constant of the oscillator. For $k = 0.5$ Nm^{-1}, the graph of $V(x)$ versus x is shown in Fig. 6.47. Show that a particle of total energy 1 J moving under this potential must "turn back" when it reaches $x = \pm 2$ m.*

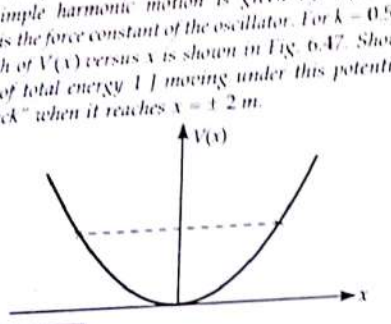

Fig. 6.47

Ans. At any instant, the energy of the oscillator is partly kinetic and partly potential. Its total energy is

$$E = K + V$$

or $$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

An oscillating particle turns back at the point where its instantaneous velocity is zero *i.e.*, the particle will turn back at such a point x where $v = 0$.

$\therefore$ $$E = 0 + \frac{1}{2}kx^2$$

But $E = 1$ J, $k = 0.5$ Nm^{-1}

$\therefore$ $$1 = \frac{1}{2} \times 0.5 \times x^2 \quad \text{or} \quad x^2 = 4$$

or $$x = \pm 2 \text{ m}.$$

6.5. *Answer the following :*

(*a*) *The casing of a rocket in flight burns up due to friction. At whose expense is the heat energy required for burning obtained ? The rocket or the atmosphere or both ?* **[Delhi 12]**

(*b*) *Comets move around the sun in highly elliptical orbits. The gravitational force on the comet due to the sun is not normal to the comet's velocity in general. Yet the work done by the gravitational force over every complete orbit of the comet is zero. Why ?*

(*c*) *An artificial satellite orbiting the earth in very thin atmosphere loses its energy gradually due to dissipation against atmospheric resistance, however small. How then does its speed increase progressively as it comes closer and closer to the earth ?*

(*d*) *In Fig. 6.48 (i) the man walks 2 m carrying a mass of 15 kg on his hands. In Fig. 6.48 (ii), he walks the same distance pulling the rope behind him. The rope goes over a pulley, and a mass of 15 kg hangs at its other end. In which case is the work done greater ?*

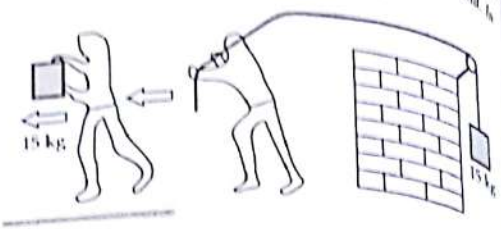

Fig. 6.48

Ans. (*a*) Heat energy required for the burning of the casing of a rocket in flight is obtained from the rocket itself. It is obtained at the expense of the mass of the rocket and its kinetic and potential energies.

(*b*) The gravitational force acting on the comet is a conservative force. The work done by a conservative over any path is equal to the negative of the change in P.E. Over a complete orbit of any shape, there is no change in P.E. of the comet. Hence no work is done by the gravitational force on the comet.

(*c*) As the satellite comes closer to the earth, its potential energy decreases. As the sum of kinetic and potential energy remains constant, the kinetic energy and velocity of the satellite increase. But the total energy of the satellite goes on decreasing due to the loss of energy against friction.

(*d*) In **case (*i*)**, no work is done against gravity because the displacement of 2 m (horizontal) and the weight of 15 kg (acting vertically downwards) are perpendicular to each other. Work is done only against friction.

In **case (*ii*)**, work has to be done against gravity ($= mg\,h = 15 \times 9.8 \times 2 = 294$ J) in addition to the work to be done against friction while moving a distance of 2 m. Thus the *work done in case (ii) is greater* than that in case (*i*).

6.6. *Underline the correct alternative :*

(*a*) *When a conservative force does positive work on a body the potential energy of the body increases / decreases / remains unaltered.*

(*b*) *Work done by a body against friction* ... *loss of its kinetic/potential energy.*

(*c*) *The rate of change of total momen* ... *particle system is proportional to the* ... *of the internal forces on the system.*

(*d*) *In an inelastic collision of two bo* ... *which do not change after the co* ... *kinetic energy/total linear momen* ... *the system of two bodies.*

Ans.

(*a*) The work done by a conservat ... the negative of the potential en ... done is positive, the potential en ...

(*b*) Friction always opposes mo ... work against friction at the ... energy. Work done by a body ag ... a loss of its kinetic energy.

(*c*) Internal forces in a many-p ... out in pairs and so they c ... momentum of the system. C ... can produce change in m ... change of momentum of a ... proportional to the external f ...

(*d*) In an elastic collision, th ... system decreases after th ... energy of the system and ... tum do not change after t ...

6.7. *State if each of the following ... Give reasons for your answer.*

(*a*) *In an elastic collision of tw ... energy of each body is cons ...*

(*b*) *Total energy of a system is ... what internal and external ...*

(*c*) *Work done in the motion ... zero for every force in na ...*

(*d*) *In an inelastic collision ... always less than the initi ...*

Ans.

(*a*) *False.* Total momentun ... system are conserved ...

(*b*) *False.* The external f ... change its energy.

(*c*) *False.* In case of a ... friction, the work i ... closed loop is not z ...

(*d*) *True.* In an elastic c ... of the system alwa ... of energy.

6.8. *Answer carefully, ...*

(*a*) *In an elastic collision ... energy conserved ... the balls (i.e., whe ...*

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(b) *Work done by a body against friction always results in a loss of its kinetic/potential energy.*

(c) *The rate of change of total momentum of a many-particle system is proportional to the external force/sum of the internal forces on the system.*

(d) *In an inelastic collision of two bodies, the quantities which do not change after the collision are the total kinetic energy/total linear momentum/total energy of the system of two bodies.* **[Central Schools 08]**

Ans.

(a) The work done by a conservative force is equal to the negative of the potential energy. *When the work done is positive, the potential energy decreases.*

(b) Friction always opposes motion. A body does work against friction at the expense of its kinetic energy. *Work done by a body against friction results in a loss of its kinetic energy.*

(c) Internal forces in a many-particle system cancel out in pairs and so they cannot change the net momentum of the system. Only the external forces can produce change in momentum. *The rate of change of momentum of a many-particle system is proportional to the external force on the system.*

(d) In an elastic collision, the kinetic energy of the system decreases after the collision but the total energy of the system and its total linear momentum do not change after the inelastic collision.

6.7. *State if each of the following statements is true or false. Give reasons for your answer.*

(a) *In an elastic collision of two bodies, the momentum and energy of each body is conserved.*

(b) *Total energy of a system is always conserved, no matter what internal and external forces on the body are present.*

(c) *Work done in the motion of a body over a closed loop is zero for every force in nature.*

(d) *In an inelastic collision, the final kinetic energy is always less than the initial kinetic energy of the system.*

Ans.

(a) *False.* Total momentum and total energy of the entire system are conserved and not of individual bodies.

(b) *False.* The external forces acting on a body may change its energy.

(c) *False.* In case of a non-conservative force like friction, the work in the motion of a body over a closed loop is not zero.

(d) *True.* In an elastic collision, a part of the initial K.E. of the system always changes into some other form of energy.

6.8. *Answer carefully, with reasons :*

(a) *In an elastic collision of two billiard balls, is the total kinetic energy conserved during the short time of collision of the balls (i.e., when they are in contact) ?* **[Delhi 12]**

(b) *Is the total linear momentum conserved during the short time of an elastic collision of two balls ?*

(c) *What are the answers to (a) and (b) for an inelastic collision ?*

(d) *If the potential energy of two billiard balls depends only on the separation distance between their centres, is the collision elastic or inelastic ? (Note, we are talking here of potential energy corresponding to the force during collision, not gravitational potential energy.)*

Ans.

(a) During the short time of collision when the balls are in contact, the kinetic energy of the balls gets converted into potential energy. In an elastic collision, though the kinetic energy before collision is equal to the kinetic energy after the collision but kinetic energy is not conserved during the short time of collision.

(b) Yes, the total linear momentum is conserved during the short time of an elastic collision of two balls.

(c) In an inelastic collision, the total K.E. is not conserved during collision as well as even after the collision. But the total linear momentum of the two balls is conserved.

(d) The collision is elastic because the forces involved are conservative.

6.9. *A body is initially at rest. It undergoes one-dimensional motion with constant acceleration. The power delivered to it at time t is proportional to*

(i) $t^{1/2}$ (ii) t (iii) $t^{3/2}$ (iv) t^2

Ans. Instantaneous velocity, $v = 0 + at = at$

Power, $P = Fv = mav = ma \times at = ma^2t$

As m and a are constant, so $P \propto t$

$\therefore$ *Alternative (ii) is correct.*

6.10. *A body is moving unidirectionally under the influence of a source of constant power. Its displacement in time t is proportional to*

(i) $t^{1/2}$ (ii) t (iii) $t^{3/2}$ (iv) t^2

Ans. By work-energy theorem,

$$W = P \times t = \frac{1}{2} mv^2$$

or $$v^2 = \frac{2\,Pt}{m}$$

$\therefore$ $$v = \frac{ds}{dt} = \left(\frac{2\,Pt}{m}\right)^{1/2}$$

On integration,

$$s = \left(\frac{2P}{m}\right)^{1/2} \frac{2}{3} t^{3/2}$$

$\therefore$ $$s \propto t^{3/2}$$

Hence, *alternative (iii) is correct.*

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6.11. *A body constrained to move along the Z-axis of a co-ordinate system is subject to a constant force* $\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$ *N, where* $\hat{i}, \hat{j}, \hat{k}$ *are unit vectors along the X-, Y-, and Z-axis of the system respectively. What is the work done by this force in moving the body a distance of* 4 *m along the Z-axis ?*

Ans. Here, $\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$ N

As the body moves a distance of 4 m along Z-axis, so

$\vec{s} = 4\hat{k}$ m.

$$\therefore \quad W = \vec{F} \cdot \vec{s} = (-\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (4\hat{k})$$

$$= (-\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (0\hat{i} + 0\hat{j} + 4\hat{k})$$

$$= -1 \times 0 + 2 \times 0 + 3 \times 4 = \mathbf{12\ J}.$$

6.12. *An electron and a proton are detected in a cosmic ray experiment, the first with kinetic energy* 10 *keV, and the second with* 100 *keV. Which is faster, the electron or the proton ? Obtain the ratio of their speeds.*

(Electron mass $= 9.11 \times 10^{-31}$ *kg,*

proton mass $= 1.67 \times 10^{-27}$ *kg,* $1\ eV = 1.60 \times 10^{-19}$ *J).*

[Delhi 08]

Ans. K.E. of the electron $= \frac{1}{2} m_e v_e^2 = 10$ keV

K.E. of the proton $= \frac{1}{2} m_p v_p^2 = 100$ keV

$$\therefore \quad \frac{\frac{1}{2} m_e v_e^2}{\frac{1}{2} m_p v_p^2} = \frac{10\ \text{keV}}{100\ \text{keV}} = \frac{1}{10}$$

$$\text{or} \quad \frac{9.11 \times 10^{-31} \times v_e^2}{1.67 \times 10^{-27} \times v_p^2} = \frac{1}{10}$$

$$\text{or} \quad \frac{v_e^2}{v_p^2} = \frac{1670}{9.11} = 183.3$$

$$\text{or} \quad \frac{v_e}{v_p} = \mathbf{13.53}$$

Thus the electron moves faster than the proton.

6.13. *A rain drop of radius* 2 *mm falls from a height of* 500 *m above the ground. It falls with decreasing acceleration (due to viscous resistance of the air) until at half its original height, it attains its maximum (terminal) speed and moves with uniform speed thereafter. What is the work done by the gravitational force on the drop in the first and second half of its journey ? What is the work done by the resistive force in the entire journey if its speed on reaching the ground is* 10 ms^{-1} *?*

Ans. Whether the rain drop falls with decreasing acceleration or with uniform speed, the work done by gravitational force on the drop remains same.

Here $r = 2$ mm $= 2 \times 10^{-3}$ m

Distance moved in each half journey,

$$h = \frac{500}{2} = 250 \text{ m}$$

Density of water,

$$\rho = 10^3 \text{ kgm}^{-3}$$

Mass of rain drop,

$$m = \text{Volume} \times \text{density} = \frac{4}{3} \pi r^3 \rho$$

$$= \frac{4}{3} \pi \times (2 \times 10^{-3})^3 \times 10^3 = \frac{32\pi}{3} \times 10^{-6} \text{ kg}$$

Work done by the gravitational force on the rain drop in each journey,

$$W = F \times s = mg \times h$$

$$= \frac{32\pi}{3} \times 10^{-6} \times 9.8 \times 250 = 0.082 \text{ J}.$$

For entire journey,

Work done by gravitational force + Work done by resistive force = Gain in K.E.

$$2 \times 0.082 + W_r = \frac{1}{2} mv^2$$

$$\text{or} \quad W_r = \frac{1}{2} \times \frac{32\pi \times 10^{-6} \times (10)^2}{3} - 0.164$$

$$= 0.0017 - 0.164 = -0.1623 \text{ J}.$$

6.14. *A molecule in a gas container hits a horizontal wall with speed* 200 ms^{-1} *and angle* 30° *with the normal, and rebounds with the same speed. Is momentum conserved in the collision ? Is the collision elastic or inelastic ?*

Ans. Momentum is always conserved, whether the collision is elastic or inelastic.

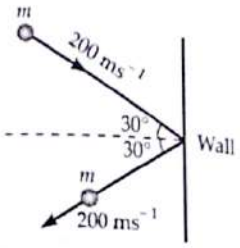

Fig. 6.49

As the wall is heavy, the molecule rebounding with its own speed does not produce any velocity in the wall. Let m be the mass of the molecule and M that of the wall.

K.E. before collision,

$$K_i = \frac{1}{2} m (200)^2 + \frac{1}{2} M (0)^2 = 2 \times 10^4 \text{ m}$$

K.E. after collision,

$$K_f = \frac{1}{2} m (200)^2 + \frac{1}{2} M (0)^2 = 2 \times 10^4 \text{ m}$$

$$\therefore \quad K_f = K_i$$

As the K.E. is conserved, the collision is elastic.

6.15. *A pump on the ground floor of a building can pump up water to fill a tank of volume* 30 m^3 *in* 15 *min. If the tank is* 40 *m above the ground, and the efficiency of the pump is* 30%, *how much electric power is consumed by the pump ?*

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Ans. Mass of water = Volume × density
$= 30 \times 1000 = 3 \times 10^4$ kg

$\therefore$ Output power $= \dfrac{\text{Work done}}{\text{Time}} = \dfrac{mgh}{t}$

$= \dfrac{3 \times 10^4 \times 9.8 \times 40}{15 \times 60} = \dfrac{39200}{3}$ W

As Efficiency $= \dfrac{\text{Output power}}{\text{Input power}} \times 100$

$\therefore$ Input power $= \dfrac{\text{Output power}}{\text{Efficiency}} \times 100$

$= \dfrac{39200}{3 \times 30} \times 100 = 43.6 \times 10^3$ W

$= \mathbf{43.6\ kW}$.

6.16. *Two identical ball bearings in contact with each other and resting on a frictionless table are hit head-on by another ball bearing of the same mass moving initially with a speed v. If the collision is elastic, which of the situations shown in Fig 6.50, is a possible result after collision ?*

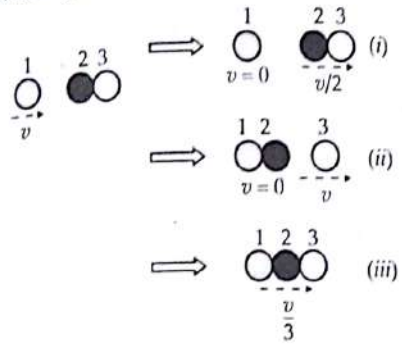

Fig. 6.50

Ans. The system consists of three identical ball bearings marked as 1, 2 and 3. Let m be the mass of each ball bearing.

Total kinetic energy of the system before collision

$$= \frac{1}{2}mv^2 + 0 + 0 = \frac{1}{2}mv^2$$

Case (*i*) K.E. of the system after collision

$$= 0 + \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 = \frac{1}{4}mv^2$$

Case (*ii*) K.E. of the system after collision

$$= 0 + \frac{1}{2}mv^2 = \frac{1}{2}mv^2$$

Case (*iii*) K.E. of the system after collision

$$= \frac{1}{2}(3m)\left(\frac{v}{3}\right)^2 = \left(\frac{1}{6}\right)mv^2$$

Because in an elastic collision, the kinetic energy of the system remains unchanged. Hence, *case (ii) is the only possible result of the collision.*

6.17. *The bob A of a pendulum released from 30° to the vertical hits another bob B of the same mass at rest on a table as shown in Fig. 6.51. How high does the bob A rise after the collision ? Neglect the size of the bobs and assume the collision to be elastic.*

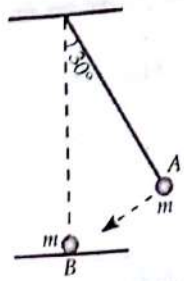

Fig. 6.51

Ans. When the bob A hits bob B on the table, it transfers its entire K.E. to the bob B because the collision is elastic. The bob A comes to rest at the location of B while the bob B begins to move with the velocity of A.

6.18. *The bob of a pendulum is released from a horizontal position A as shown. If the length of the pendulum is 1.5 m, what is the speed with which the bob arrives at the lowermost point B, given that it dissipates 5% of its initial energy against air resistance ?*

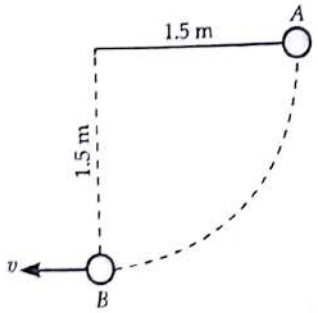

Fig. 6.52

Ans. Here $h = 1.5$ m, $v = ?$

P.E. of the bob at $A = mgh$

K.E. of the bob at $B = \dfrac{1}{2}mv^2$

As 5% of the P.E. is dissipated against air resistance, so

$$\frac{1}{2}mv^2 = 95\% \text{ of } mgh$$

or $$\frac{1}{2}mv^2 = \frac{95}{100} \times mgh$$

or $$v = \sqrt{\frac{2 \times 95 \times gh}{100}} = \sqrt{\frac{2 \times 95 \times 9.8 \times 1.5}{100}}$$

$$= \sqrt{27.93} = \mathbf{5.3\ ms^{-1}}.$$

6.19. *A trolley of mass 300 kg carrying a sandbag of 25 kg is moving uniformly with a speed of 27 km/h on a frictionless track. After a while, sand starts leaking out of a hole on the trolley's floor at the rate of 0.05 kg s^{-1}. What is the speed of the trolley after the entire sand bag is empty ?*

Ans. As the trolley carrying the sandbag is moving with uniform speed of 27 km/h, so no external force is acting on the trolley + sandbag system. When the sand leaks out, it does not cause any external force to act on the system. Hence the speed of the trolley remains unchanged even after the sandbag becomes empty.

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6.20. *A particle of mass 0.5 kg travels in a straight line with velocity* $v = ax^{3/2}$ *where* $a = 5\ m^{-1/2}\ s^{-1}$. *What is the work done by the net force during its displacement from* $x = 0$ *to* $x = 2\ m$.

Ans. Velocity, $v = ax^{3/2}$

$$\text{Acceleration} = \frac{dv}{dt} = \frac{3}{2}ax^{1/2}\frac{dx}{dt} = \frac{3}{2}ax^{1/2}.v$$

$$= \frac{3}{2}ax^{1/2}.ax^{3/2} = \frac{3}{2}a^2x^2.$$

Force, $F = m \times \text{acceleration} = \frac{3}{2}ma^2x^2$

Work done,

$$W = \int_0^2 F\,dx = \frac{3}{2}\int_0^2 ma^2x^2\,dx = \frac{3}{2}ma^2\left[\frac{x^3}{3}\right]_0^2$$

$$= \frac{3 \times 0.5 \times (5)^2}{2 \times 3}[2^3 - 0^3] = 50\ \text{J}.$$

6.21. *The blades of a windmill sweep out a circle of area A. (a) If the wind flows at a velocity v perpendicular to the circle, what is the mass of the air passing through it in time t ? (b) What is the kinetic energy of the air ? (c) Assume that the windmill converts 25% of the wind's energy into electrical energy, and that* $A = 30\ m^2$, $v = 36\ km/h$ *and the density of air is* $1.2\ kg\ m^{-3}$. *What is the electrical power produced ?*

Ans. (*a*) Volume of the air passing through the windmill in time t

= Area of circle × distance covered by wind in time t

$= A \times vt = Avt$

Mass of the air passing through the windmill in time t,

m = Density × volume $= \rho Avt$.

(*b*) Kinetic energy of the air is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\rho\, A\, vt \times v^2 = \frac{1}{2}\rho Av^3t.$$

(*c*) K.E. of air converted into electrical energy in time t

$$K' = 25\%\ \text{of}\ K = \frac{25}{100} \times \frac{1}{2}\rho Av^3t = \frac{1}{8}\rho\, Av^3t$$

Electrical power produced

$$= \frac{K'}{t} = \frac{1}{8}\rho\, a\, v^3 = \frac{1}{8} \times 1.2 \times 30 \times (10)^3$$

$$[\because\ v = 36\ \text{kmh}^{-1} = 10\ \text{ms}^{-1}]$$

$$= 4.5 \times 10^3\ \text{W} = \mathbf{4.5\ kW}.$$

6.22. *A person trying to lose weight (dieter) lifts a 10 kg mass 0.5 m, 1000 times. Assume that the potential energy lost each time she lowers the mass is dissipated. (a) How much work does she do against the gravitational force ? (b) Fat supplies* $3.8 \times 10^7\ J$ *of energy per kilogram which is converted to mechanical energy with a 20% efficiency rate. How much fat will the dieter use up ?*

Ans. (*a*) Here $m = 10\ \text{kg}$, $h = 0.5\ \text{m}$, $n = 1000$, $= 9.8\ \text{ms}^{-2}$

Work done against the gravitational force,

$$W = n \times mgh = 1000 \times 10 \times 9.8 \times 0.5$$

$$= \mathbf{49{,}000\ J}.$$

(*b*) Mechanical energy supplied by 1 kg of fat

$$= 20\%\ \text{of}\ 3.8 \times 10^7\ \text{J}$$

$$= \frac{20 \times 3.8 \times 10^7}{100} = 76 \times 10^5\ \text{J}$$

$\therefore$ Fat consumed for 76×10^5 J of energy = 1 kg

Fat consumed for 49,000 J of energy

$$= \frac{1 \times 49{,}000}{76 \times 10^5} = \mathbf{6.45 \times 10^{-3}\ kg}.$$

6.23. *A large family uses 8 kW of power. (a) Direct solar energy is incident on the horizontal surface at an average rate of 200 W per square meter. If 20% of this energy can be converted to useful electrical energy, how large an area is needed to supply 8 kW ? (b) Compare this area to that of the roof of a house constructed on a plot of size 20 m × 15 m with a permission to cover upto 70%.* **[Delhi 03]**

Ans. (*a*) Let the area needed to supply 8 kW = $A\ m^2$

Energy incident per unit area = 200 W

Energy incident on area $A = 200 \times A$ W

Energy converted into useful electrical energy

$= 20\%\ \text{of}\ 200 \times A = 40A$ W

But $40\ A$ W = 8 kW = 8000 W

or $$A = \frac{8000}{40} = \mathbf{200\ m^2}.$$

(*b*) Area of the roof of the given house,

A' = 70% of 20 m × 15 m

$$= \frac{70 \times 20 \times 15}{100} = 210\ \text{m}^2$$

$$\text{Required ratio} = \frac{A}{A'} = \frac{200}{210} = \mathbf{20 : 21}.$$

6.24. *A bullet of mass 0.012 kg and horizontal speed* $70\ ms^{-1}$ *strikes a block of wood of mass 0.4 kg and instantly comes to rest with respect to the block. The block is suspended from the ceiling by means of thin wires. Calculate the height to which the block rises. Also estimate the amount of heat produced in the block.* **[Chandigarh 07]**

Ans. Mass of bullet, $m = 0.012$ kg

Speed of bullet, $v = 70\ \text{ms}^{-1}$

Mass of block, $M = 0.4$ kg

If V is the velocity of the combination after collision, then from the law of conservation of momentum,

$$mv = (M + m)\,V$$

or $$V = \frac{mv}{M + m} = \frac{0.012 \times 70}{0.4 + 0.012}$$

$$= \frac{0.84}{0.412} = 2.04\ \text{ms}^{-1}$$

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Let h be the height through which the block rises. Then from the conservation of energy,

P.E. of the combination = K.E. of the combination

$$(M + m)\, gh = \frac{1}{2}(M + m)\, V^2$$

$$h = \frac{V^2}{2g} = \frac{(2.04)^2}{2 \times 9.8} = 0.212 \text{ m}$$

or

Amount of heat produced

= Loss in K.E. of the bullet

= Initial K.E. of the bullet

− K.E. of the combination

$$= \frac{1}{2} mv^2 - \frac{1}{2}(M + m)\, V^2$$

$$= \frac{1}{2} \times 0.012 \times (70)^2 - \frac{1}{2} \times 0.412 \times (2.04)^2$$

$$= 29.4 - 0.86 = \mathbf{28.54\ J}.$$

6.25. *Two inclined frictionless tracks, one gradual and the other steep meet at A from where two stones are allowed to slide down from rest, one on each track (Fig. 6.53). Will the stones reach the bottom at the same time ? Will they reach there with the same speed ? Explain. Given $\theta_1 = 30°$, $\theta_2 = 60°$ and $h = 10$ m, what are the speeds and times taken by the two stones ?*

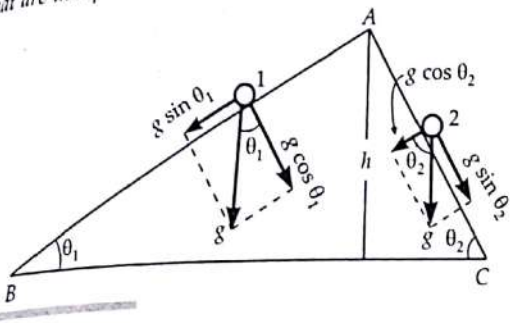

Fig. 6.53

Ans. Let a_1 be the acceleration of the stone 1 down the inclined track AB. Then

$$a_1 = g \sin \theta_1.$$

If the stone 1 takes time t_1 to slide down the track AB, then

$$AB = 0 + \frac{1}{2} a_1 t_1^2 \qquad [s = ut + \tfrac{1}{2} at^2]$$

$$\frac{h}{\sin \theta_1} = \frac{1}{2} g \sin \theta_1\, t_1^2$$

or $$t_1^2 = \frac{2h}{g \sin^2 \theta_1}$$

or $$t_1 = \frac{1}{\sin \theta_1} \sqrt{\frac{2h}{g}}$$

Similarly, for stone 2, we can write

$$t_2 = \frac{1}{\sin \theta_2} \sqrt{\frac{2h}{g}}$$

For both the stones, h is same.

As $\theta_1 < \theta_2$ $\quad \therefore \sin \theta_1 < \sin \theta_2$

Consequently, $t_1 > t_2$

Thus, the stone 2 on the steeper plane AC reaches the bottom earlier than stone 1.

As both the stones are initially at the same height h, so

P.E. at A = K.E. at B or C

$$mgh = \frac{1}{2} mv^2$$

$$v = \sqrt{2gh}$$

i.e., both the stones will reach the bottom with the same speed.

Given : $\theta_1 = 30°$, $\theta_2 = 60°$, $h = 10$ m, $g = 10\ \text{ms}^{-2}$

$$\therefore \quad t_1 = \frac{1}{\sin \theta_1} \sqrt{\frac{2h}{g}}$$

$$= \frac{1}{\sin 30°} \sqrt{\frac{2 \times 10}{10}} = 2\sqrt{2} \text{ s}.$$

$$t_2 = \frac{1}{\sin \theta_2} \sqrt{\frac{2h}{g}} = \frac{1}{\sin 60°} \sqrt{\frac{2 \times 10}{10}} = 2\sqrt{\frac{2}{3}} \text{ s}.$$

$$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 10} = \sqrt{2} \times 10$$

$$= 1.414 \times 10 = \mathbf{14.14\ ms^{-1}}.$$

6.26. *A 1 kg block situated on a rough incline is connected to a spring of spring constant 100 Nm^{-1} as shown in Fig. 6.54(a). The block is released from rest with the spring in the unstretched position. The block moves 10 cm down the incline before coming to rest. Find the coefficient of friction between the block and the incline. Assume that spring has negligible mass and the pulley is frictionless.*

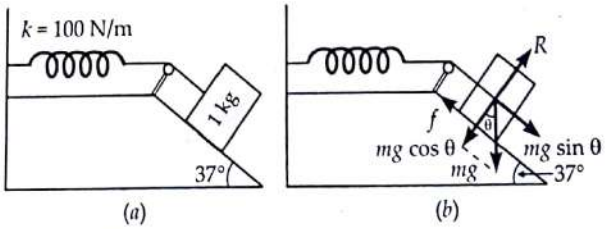

Fig. 6.54

Ans. Here $m = 1$ kg, $k = 100\ \text{Nm}^{-1}$, $g = 10\ \text{ms}^{-2}$

Clearly, from Fig. 6.54(*b*), we have

$$R = mg \cos \theta \ ; \quad f = \mu R = \mu\ mg \cos \theta$$

Net force on the block down the incline

$$= mg \sin \theta - f = mg \sin \theta - \mu\ mg \cos \theta$$

$$= mg\ (\sin \theta - \mu \cos \theta)$$

Distance moved, $x = 10$ cm $= 0.1$ m

In equilibrium

Work done = P.E. of stretched spring

$$mg\ (\sin \theta - \mu \cos \theta)\, x = \frac{1}{2} kx^2$$

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or $2\,mg\,(\sin 0 - \mu \cos 0) = kx$

or $2 \times 1 \times 10\,(\sin 37° - \mu \cos 37°) = 100 \times 0.1$

or $20\,(0.601 - \mu \times 0.798) = 10$

$\therefore \quad \mu = \mathbf{0.126.}$

6.27. *A bolt of mass 0.3 kg falls from the ceiling of an elevator moving down with a uniform speed of 7 ms^{-1}. It hits the floor of the elevator (length of the elevator 3 m) and does not rebound. What is the heat produced by the impact ? Would your answer be different, if the elevator were stationary ?*

Ans. As the elevator is moving down with a uniform speed ($a = 0$), so the value of g remains the same.

Here $m = 0.3$ kg, $h = 3$ m, $g = 9.8$ ms^{-2}

P.E. lost by the bolt $= mgh = 0.3 \times 9.8 \times 3 = 8.82$ J

As the bolt does not rebound, the energy is converted into heat.

$\therefore$ Heat produced = **8.82 J**

Even if the elevator were stationary, the same amount of heat would have produced because the value of g is same in all inertial frames of reference.

6.28. *A trolley of mass 200 kg moves with a uniform speed of 36 km/h on a frictionless track. A child of mass 20 kg runs on the trolley from one end to the other (10 m away) with a speed of 4 m s^{-1} relative to the trolley in a direction opposite to the trolley's motion, and jumps out of the trolley. What is the final speed of the trolley ? How much has the trolley moved from the time the child begins to run ?*

Ans. The child gives an impulse to the trolley at the start and then runs with a constant relative velocity of 4 ms^{-1} with respect to the trolley's new velocity.

Total initial momentum,

$$p_i = (m_1 + m_2)\,u_1 = (20 + 200) \times \frac{36 \times 5}{18} = 2200 \text{ kg ms}^{-1}$$

Let new velocity of the trolley = v_2

Child's velocity relative to the trolley in opposite direction = 4 ms^{-1}

$\therefore$ Child's actual velocity (relative to ground)

$$= v_2 - 4$$

Total final momentum,

$$p_f = m_1 v_1 + m_2 v_2 = 20\,(v_2 - 4) + 200\,v_2 = 220\,v_2 - 80$$

By conservation of linear momentum,

$$p_f = p_i$$

$$220\,v_2 - 80 = 2200$$

$$\therefore \quad v_2 = \frac{2280}{220} = \mathbf{10.36\ ms^{-1}}$$

Time taken by the child to cover length of the trolley

$$= \frac{10 \text{ m}}{4 \text{ ms}^{-1}} = 2.5 \text{ s}$$

Distance covered by the trolley in 2.5 s

$$= 10.36 \times 2.5 = \mathbf{25.9\ m.}$$

6.29. *Which of the following potential energy curves in Fig. 6.55 cannot possibly describe the elastic collision of two billiard balls ? Here r is the distance between centres of the balls.*

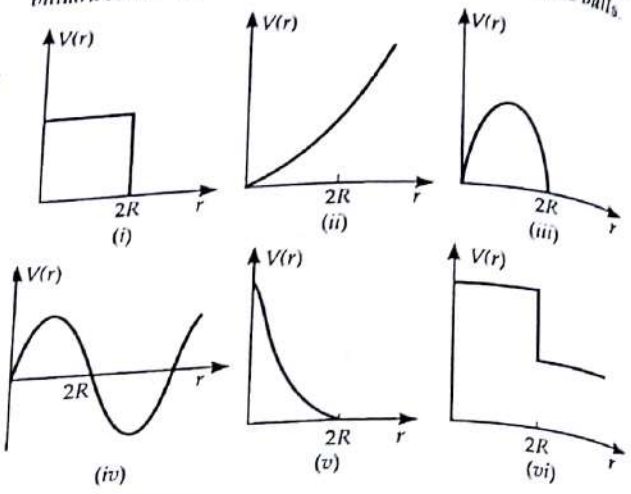

Fig. 6.55

Ans. During the short time of collision, the kinetic energy gets converted into potential energy. But the P.E. of a system of two masses varies inversely as the distance between them, *i.e.*, $V \propto 1/r$. Hence all the potential energy curves except the one shown in Fig. 6.55(*v*) cannot describe an elastic collision.

6.30. *Consider the decay of a free neutron at rest :*

$$n \rightarrow p + e^-$$

Show that the two-body decay of this type must necessarily give an electron of fixed energy and, therefore, cannot account for the observed continuous energy distribution in the β-decay of neutron or a nucleus (Fig. 6.56).

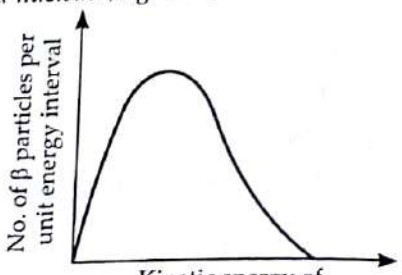

Fig. 6.56

Ans. If the decay of the neutron (inside the nucleus) into proton and electron is according to the given reaction, then the energy released in the decay must be carried by the electrons coming out of the nucleus. By mass-energy conservation, these electrons must have a definite value of energy. However, the given graph shows that the emitted electron can have any value of energy between zero and the maximum value. Hence the given decay mode cannot account for the observed continuous energy spectrum in the β-decay.

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Text Based Exercises

Type A : Very Short Answer Questions — 1 Mark Each

1. What do you mean by work done by a force ? **[Manipur 99]**
2. State the factors on which the work done by a force depends.
3. What should be the angle between the directions of force and displacement for maximum and minimum work ?
4. Give one such example in which the work done by the force acting upon a body is zero, although due to the applied force the body is displaced from its initial position.
5. State the conditions under which a force does no work.
6. What is the work done by a person in holding a 15 kg suitcase, while waiting for a bus for 15 minutes ? **[Himachal 05C]**
7. A person continues to push a rock for some time but fails to move it. What is the work done by it ? Explain.
8. What is the work done by the force of tension in the string of simple pendulum ?
9. Is work done a scalar or a vector quantity ?
10. What physical quantity does the area under the force-displacement curve represent ?
11. Moment of a force and work done by a force have same units. What is the difference between them ?
12. Name and define SI unit of work. **[Himachal 03, 05, 07, 08]**
13. How many joules make up one erg ?
14. What do you mean by gravitational unit of work ?
15. State gravitational unit of work in SI.
16. Find the number of joules in the gravitational unit of work in SI.
17. A man runs a distance on a level road. The same man ascends up a hill with the same velocity through the same distance. When does he do more work ?
18. Give an example in which a force does work on a body but fails to change its kinetic energy.
19. Does the work done depend on how fast or slow a body is moved ?
20. Has an object in motion ability to do work ?
21. Does a man standing at rest on a moving car possess kinetic energy ?
22. Does a single force acting on a particle necessarily change its K.E. and momentum ?
23. Can the K.E. of a body be negative ?
24. Can the P.E. of a body be negative ?
25. Out of joule, calorie, kilowatt and electron volt, which one is not the unit of energy ?
26. Is the mechanical energy always conserved ?
27. What kind of energy is stored in the spring of a watch ? **[Himachal 05C]**
28. When an air bubble rises in water, what happens to its potential energy ?
29. What kind of energy transformations takes place at a hydro-electric power house ?
30. A spring is cut into two halves. How is the spring constant of each half affected ?
31. Define power and its SI unit. **[Manipur 99 ; Delhi 09 ; Kerala 01]**
32. Is the unit watt second associated with energy or power ?
33. What are the dimensions of power ? Is it scalar or vector ? **[Delhi 05]**
34. How many watts are there in one horse power ? **[Himachal 05C, 08]**
35. Name the physical quantity which is expressed as force times velocity. Is it a scalar or vector quantity ? **[Delhi 98]**
36. When a planet revolves round the sun, when is its kinetic energy maximum ?
37. What is a conservative force ? **[Himachal 03]**
38. Which physical quantities are conserved in an elastic collision ?
39. What is that physical quantity which is conserved both in elastic and inelastic collisions ?
40. If two bodies stick together after collision, will the collision be elastic or inelastic ?
41. A bullet gets embedded into a block of wood, how does the loss of energy appear ?

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42. Give three examples of forces which are conservative in nature. [Himachal 05C]
43. Friction is non-conservative force. How ?
44. Name the process in which (*i*) momentum is conserved but K.E. is not conserved and (*ii*) momentum changes but K.E. dose not change.
45. What is Einstein's energy-mass equivalence relationship ? [Central Schools 14]
46. How much work a man will do in carrying a 10 kg load over his head horizontally on smooth surface for 10 metres ? [Delhi 96]
47. An aritificial satellite is at a height of 36,500 km above earth's surface. What is the work done by earth's gravitational force in keeping it in its orbit ? [Central Schools 08]
48. Give two examples from daily life where according to physics, work done is zero. [Central Schools 07]
49. What is the source of the kinetic energy of the falling rain drops ? [Central Schools 08 ; Delhi 10]
50. State work-energy theorem.
51. What is spring constant of a spring ? Give its unit. [Delhi
52. Calculate the number of joules in 1 kWh. [Himachal
53. Show that $1\,J = 10^7$ erg. [Himachal
54. How will the momentum of a body change if kinetic energy is doubled ? [Himachal 05C,
55. Draw a graph showing the variation of energy of an object thrown vertically upward by boy with respect to its height. [Central Schools
56. How does kinetic energy of a moving ball change if its speed is reduced to half of original speed. Take mass of ball = 4 kg and original speed = 20 m/s [Central Schools
57. What is the work done by centripetal force in moving a body through half cycle on the circular path of radius 35 m ? [Central Schools

Answers

1. Work is said to be done whenever a force acts on a body and the body moves in the direction of the force.
2. The work done by a force depends upon
   (*i*) the magnitude of force,
   (*ii*) the magnitude of displacement of the body,
   (*iii*) the angle between the force and the displacement.
3. (*i*) For maximum work, $\theta = 0°$.
   (*ii*) For minimum work, $\theta = 90°$.
4. The revolution of a satellite around the earth.
5. A force does no work, when
   (*i*) the displacement is zero, or
   (*ii*) the displacement is perpendicular to the force, or
   (*iii*) the conservative force moves a body over a closed path.
6. No work is done as displacement $\vec{s} = 0$ and so $W = Fs \cos\theta = 0$.
7. The work done by the person is zero. As $W = \vec{F} \cdot \vec{s}$ and the displacement $\vec{s}$ is zero, hence the work done is also zero.
8. Zero, because the tension in the string of a simple pendulum is always perpendicular to its displacement. So the work done by the tension is zero.
9. Scalar quantity, because work can be expressed as the scalar product of force and displacement vectors.
10. The area under the force-displacement curve represents the work done by the force over the given displacement.
11. Moment of force is a vector quantity while work done is a scalar quantity.
12. The SI unit of work is joule (J). One joule of work is said to be done when a force of one newton displaces a body through a distance of one metre in its own direction.
13. As 10^7 erg = 1 joule,
    therefore, $1\,\text{erg} = 10^{-7}$ joule.
14. Work done is said to be one gravitational unit if a force of one gravitational unit displaces a body through unit distance in its own direction.
15. In SI, the gravitational unit of work done is one kilogram metre. It is defined as the work done when a force of one kilogram weight displaces a body through a distance of one metre in its own direction.
16. 1 kg m = 1 kg wt × 1 m = 9.81 N × 1 m = 9.81 J.
17. More work is done in the second case.
18. When a body is pulled on a rough horizontal surface with constant velocity, work is done by the applied force on the body but K.E. of the body remains unchanged.
19. No. The work done is independent of time in which the work is completed.
20. Yes, because an object in motion possesses kinetic energy.

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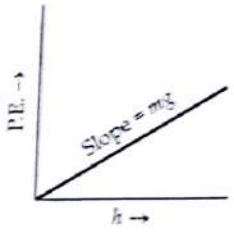
21. Yes, because the man forms part of the moving car and so he possesses kinetic energy.
22. When force and displacement are perpendicular to each other, $W = Fs\cos 90° = 0$ and so K.E. remains constant. But force changes the direction of velocity, so momentum changes.
23. No. As both m and v^2 are always positive, so K.E. $\left(=\frac{1}{2}mv^2\right)$ cannot be negative.
24. Yes, the potential energy is negative when forces involved are attractive.
25. Kilowatt, which is the unit of power and not of energy.
26. No. The mechanical energy is conserved only when conservative forces are involved and that too in an isolated system.
27. Potential energy.
28. Potential energy of air bubble decreases.
29. Potential energy of water changes into K.E. and then into electrical energy.
30. Spring constant of each half becomes twice the spring constant of the original spring.
31. Power is the rate of doing work. Its SI unit is watt. If an agency does work at the rate of 1 joule per second, its power is 1 watt.
32. Watt second is the unit of energy.
33. $[Power] = [ML^2T^{-3}]$. Power is a scalar quantity.
34. 1 Horse power = 746 watt.
35. The physical quantity is power. It is a scalar quantity.
36. When the planet is nearest to the sun.
37. A force is said to be conservative, if the amount of work done in moving an object against the force depends only on the initial and final positions of the object.
38. Momentum and kinetic energy.
39. Linear momentum is conserved both in elastic and inelastic collision.
40. Inelastic collision.
41. Kinetic energy of bullet gets converted into heat and sound energy.
42. Gravitational, magnetic and electrostatic forces.
43. Work done against friction in any round path is never zero, hence friction is non-conservative force.
44. (*i*) Inelastic collision, (*ii*) Uniform circular motion.
45. $E = mc^2$.
46. Zero, because force and displacement are perpendicular to each other.
47. Zero.
48. (*i*) Work done by the earth's gravitational force in keeping the moon in its orbit is zero.
    (*ii*) Work done by a man carrying a load on his head and walking along horizontal is zero.
49. Potential energy of the falling rain drops changes gradually into their kinetic energy.
50. According to work-energy theorem, the work done by the net force acting on a body is equal to the change produced in the kinetic energy of the body.
51. The spring constant of a string is the restoring force set up in the spring per unit extension. Its SI unit is Nm^{-1}.
52. $1\,kWh = 1\,kW \times 1\,h = 1000\,Js^{-1} \times 3600\,s = 3.6 \times 10^6\,J$.
53. $1\,J = 1\,N \times 1\,m = 10^5\,dyne \times 100\,cm = 10^7\,erg$.
54. $p = \sqrt{2mK}$. When the kinetic energy K is doubled, the momentum p increases by $\sqrt{2}$ times its initial value.
55. As P.E. $= mgh \Rightarrow$ P.E. $\propto h$. So the graph of P.E. versus height is a straight line as shown.



56. $K_i = \frac{1}{2} \times 4 \times 20^2 = 800\,J$

    $K_f = \frac{1}{2} \times 4 \times 10^2 = 200\,J$

    Decrease in K.E. $= 800 - 200 = 600\,J$.
57. Zero, because centripetal force $\perp$ direction of motion.

Type B : Short Answer Questions — 2 or 3 Marks Each

1. Define the term work. Calculate the work done by a constant force. Is work done a scalar or a vector quantity ? **[Himachal 06, 08]**

2. Give the expression for work done if angle between force $\vec{F}$ and displacement $\vec{S}$ is θ. Also find the dimensions and S.I. unit of work. **[Central Schools 07]**

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3. Define the term work and derive its SI unit. Write an expression for the kinetic energy of a body of mass m moving with a uniform speed v. [Delhi 03C]
4. How do we calculate work done by a force ? Write any two conditions under which work done by a force is zero. [Central Schools 03]
5. Define the term work. Show that work done is equal to the dot product of force and displacement vectors. [Himachal 04]
6. What is meant by positive work, negative work and zero work ? Give one example of each. [Himachal 04]
7. How do you calculate the work done by a variable force ? [Kerala 02]
8. What is the amount of work done by
   (*a*) a weight-lifter in holding a weight of 120 kg on his shoulder for 30 s, and
   (*b*) a locomotive against gravity, if it is travelling on a level plane ? [Delhi 02]
9. Define the absolute and gravitational units of work in SI and CGS systems.
10. What are conservative and non-conservative forces ? Give one example of each. [Central Schools 05, 14 ; Himachal 08]
11. Define kinetic energy. Derive an expression for the kinetic energy of a body moving with a uniform velocity. [Himachal 04, 07, 09C]
12. Define the term potential energy, and derive its dimensions. Write an expression for the gravitational potential energy of a body of mass m raised to a height h above the earth's surface. [Delhi 03C]
13. Derive an expression for the potential energy of an elastic stretched spring. [Himachal 05 ; Delhi 08]
14. An elastic spring of force constant k is compressed by an amount x. Show that its potential energy is $\frac{1}{2}kx^2$. [Himachal 09 ; Delhi 14 ; Central Schools 08]
15. Draw the graph of equation $F_s = -kx$, where F_s is the spring force and x is the displacement of block from equilibrium position. Using the graph, show that maximum work done by the spring at x_m is $W_s = -kx_m^2 / 2$ (k = spring constant). [Delhi 09 ; Central Schools 14]

6. Derive an expression for the potential energy stored in a system of a block attached to a massless spring, when the block is pulled from its equilbrium position. [Himachal 07]

. Derive an expression for the kinetic energy of a body of mass m moving with velocity v by calculus method.

18. Define power. Prove that $P = \vec{F} \cdot \vec{v}$, where the symbols have their usual meanings. [Himachal 09C]
19. State and prove work-energy theorem. [Chandigarh 02, 09 ; Himachal 07, 08 ; Delhi 04 ; Central Schools 14]
20. State work-energy theorem. Prove it for a variable force. [Delhi 09, 13]
21. Define potential energy and conservative force. Write two relations between them.
22. By what factor the velocity of a body should be increased so that its kinetic energy is increased by a factor of nine ? Justify your answer. [Delhi 96]
23. Two bodies A and B weighing 5 kg and 6 kg respectively have equal momenta. Which one has more kinetic energy ? [Delhi 96]
24. Define kinetic energy. Give its units and dimensional formula. [Himachal 09C]
25. Define potential energy. Give its units and dimensional formula. [Himachal 09C]
26. What is a conservative force ? Explain its various properties. [Himachal 08C]
27. What are conservative forces ? Show that gravitational force is a conservative force. [Himachal 06, 07]
28. Work done by a force is given by $W = \vec{F} \cdot \vec{S}$ where $\vec{F}$ is the force and $\vec{S}$ is the displacement. Show that:
   (*a*) Work done is also equal to change in K.E.
   (*b*) Work done is also equal to change in potential energy using this expression. [Central Schools 08]
29. Draw the variation of potential energy and kinetic energy of a block attached to a spring, which obeys Hooke's law. [Central Schools 08, 09]
30. Draw a graph showing the variation potential energy and kinetic energy with respect to height of a free fall under gravitational force. [Central Schools 08, 09]
31. State and prove the principle of conservation of mechanical energy. [Central Schools 07]
32. Show that the total mechanical energy of a body falling freely under gravity is conserved. Discuss it graphically also. [Central Schools 12 ; Delhi 12]
33. A particle of mass 'm' revolves in a horizontal circle of radius r under the influence of centripetal force $-K/r^2$, where K is a constant. Find total mechanical energy of the particle. [Delhi 95, 10]
34. Define work, power and energy and give their SI units. [Himachal 01, 02, 03, 04, 05C]

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35. Draw a plot of spring force versus displacement x. Hence find an expression for the P.E. of an elastic stretched spring.

36. Discuss the conservation of energy in an elastic spring. Hence write an expression for the maximum speed of a body of mass m oscillating at its one end.

37. What is meant by mass-energy equivalence ? Discuss its significance in physics.
[Chandigarh 03 ; Himachal 07]

38. What is the meaning of 'Collision' in physics ? Differentiate between elastic and inelastic collision. Give one example each.
[Central Schools 03 ; Himachal 07C]

39. Define the terms Elastic collision and Inelastic collision. What is the difference between an inelastic collision and a completely inelastic collision ?
[Delhi 03]

40. Define inelastic collision. Write its three important characteristics. [Central Schools 12]

41. Show that in case of one dimensional elastic collision of two bodies, the relative velocity of separation after the collision is equal to the relative velocity of approach before the collision.
[Himachal 06, 09C ; Chandigarh 02 ; Delhi 05 ; Central Schools 09]

42. Prove that bodies of identical masses exchange their velocities after head-on elastic collision.
[Himachal 03, 07, 09C ; Delhi 01]

43. Define elastic and inelastic collision. A lighter body collides with a much more massive body at rest. Prove that the direction of lighter body is reversed and massive body remains at rest. [Delhi 98]

44. Identify conservative and non-conservative forces given below and write one main difference between them on the basis of work done : frictional force, electrostatic force, gravitational force and viscous force. [Central Schools 13]

45. Show that there is a loss of kinetic energy during one dimensional inelastic collision. How will you account for this loss of energy ? [Central Schools 14]

Answers

1. Refer answer to Q.1 and Q.2 on page 6.1.
2. Refer answer to Q.2 on page 6.1 and Q.4 on page 6.3.
3. Refer to points 1, 3 and 7 of Glimpses.
4. Refer answer to Q. 2 on page 6.1.
   Work done is zero when $F = 0$ or $s = 0$ or $\theta = 90°$.
5. Refer answer to Q. 2 on page 6.1.
6. Refer answer to Q. 3 on page 6.2.
7. Refer answer to Q. 7 on page 6.5.
8. (a) Here $s = 0$, so $W = Fs \cos \theta = 0$.
   (b) Here $\theta = 90°$, so $W = Fs \cos \theta = 0$.
9. Refer answer to Q. 5 on page 6.3.
10. Refer answer to Q. 18 on page 6.14 and Q. 20 on page 6.15.
11. Refer answer to Q. 11 on page 6.8.
12. Refer answer to Q. 15 on page 6.13 and Q. 17 on page 6.14.
13. Refer answer to Q.27 on page 6.21.
14. Refer answer to Q. 27 on page 6.21.
15. Refer answer to Q.27 on page 6.21.
16. Refer answer to Q.27 on page 6.21.
17. Refer answer to Q. 12 on page 6.8.
18. Refer answer to Q. 32, 33 on page 6.28.
19. Refer answer to Q. 13 on page 6.9.
20. Refer answer to Q.14 on page 6.9.
21. Refer answer to Q. 21 on page 6.15.
22. $\frac{1}{2} mv'^2 = 9 \times \frac{1}{2} mv^2$

    $v' = 3v$

    Thus the velocity should be increased by a factor of three.
23. $\frac{K_A}{K_B} = \frac{m_B}{m_A} = \frac{6}{5}$

    Thus body A has more kinetic energy.
24. Refer to point 7 of Glimpses. Its SI unit is joule and dimensional formula is $[ML^2T^{-2}]$.
25. Refer to point 9 of Glimpses. Its SI unit is joule and dimensional formula is $[ML^2T^{-2}]$.
26. Refer answer to Q. 18 on page 6.14 and Q. 24 on page 6.16.
27. Refer answer to Q. 22 on page 6.16.
28. (a) Refer answer to Q.13 on page 6.9
    (b) Refer answer to Q.17 on page 6.14.
29. See Fig. 6.28 on page 6.22.
30. See Fig. 6.20 on page 6.17.
31. Refer answer to Q. 23 on page 6.16.
32. Refer answer to Q. 25 on page 6.16.
33. Refer to the solution of Problem 4 on page 6.48.
34. Refer to points 1 and 6 of Glimpses.
35. Refer answer to Q. 27 on page 6.21.
36. Refer answer to Q. 28 on page 6.21.

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37. Refer answer to Q. 30 on page 6.26.
38. Refer answer to Q. 35 on page 6.31.
39. Refer answer to Q. 35 on page 6.31.
40. Refer answer to Q. 35 on page 6.31.
41. Refer answer to Q. 37 on page 6.32.
42. Refer to solution of Example 62 on page 6.36.
43. Refer answer to Q. 37(iii) on page 6.33.
44. Refer answer to Q.39 on page 6.34.
44. Electrostatic and gravitational forces are conservative.
Frictional and viscous forces are non-conservative.
Work done against a conservative force is path independent while that done against a conservative force is path dependent.
45. Refer answer to Q. 38 on page 6.34.

Type C : Long Answer Questions

5 Marks Each

1. Obtain mathematically and graphically the work done by a variable force. **[Himachal 06]**
2. Define potential energy and kinetic energy. Derive expressions for potential and kinetic energies. **[Himachal 02, 08]**
3. State work-energy theorem. Prove it for a variable force. **[Delhi 13]**
4. State the law of conservation of mechanical energy. Show that the total mechanical energy of a body falling freely under gravity is conserved. Show it graphically. **[Meghalaya 2K ; Himachal 02, 03, 09C ; Delhi 10]**
5. Show that the elastic force of a spring is a conservative force. Hence write an expression for the potential energy of an elastic stretch of spring.
6. Define elastic collision and discuss it for two bodies in one dimension. Calculate the velocities of bodies after collision. Discuss special cases also. **[Chandigarh 02 ; Himachal 01, 02, 04]**
7. A large mass 'M' moving with a velocity 'v' collides head-on with a very small mass 'm' at rest. If the collision is elastic, obtain an expression for the energy lost by the large mass M (Take $M + m \approx M$). **[Delhi 97]**

Answers

1. Refer answer to Q.7 on page 6.5.
2. Refer answer to Q. 11 on page 6.8 and Q. 17 on page 6.14.
3. Refer answer to Q. 14 on page 6.9.
4. Refer answer to Q. 25 on page 6.16.
5. Refer answer to Q. 26 on page 6.20.
6. Refer answer to Q. 37 on page 6.33.
7. Here $m_1 = M$, $u_1 = v$, $m_2 = m$, $u_2 = 0$

Velocity of M after collision,

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2} u_1 + \frac{2m_2}{m_1 + m_2} \cdot u_2$$

$$= \frac{M - m}{M + m} \cdot v + \frac{2m}{M + m} \cdot 0$$

Kinetic energy lost by mass M

$$= \frac{1}{2} Mv^2 - \frac{1}{2} M \left(\frac{M - m}{M + m} v \right)^2$$

$$= \frac{1}{2} Mv^2 \left[1 - \left(\frac{M - m}{M + m} \right)^2 \right]$$

$$= \frac{1}{2} Mv^2 \cdot \frac{4Mm}{(M + m)^2}$$

$$= \frac{1}{2} Mv^2 \cdot \frac{4Mm}{M^2} \qquad [M + m \approx M]$$

$$= \mathbf{2mv^2}.$$

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Competition Section

Work, Energy and Power

GLIMPSES

1. **Work.** Work is said to be done wherever a force acts on a body and the body moves through some distance in the direction of the force. If the force $\vec{F}$ makes angle θ with the direction of displacement $\vec{s}$, then the work done is

$$W = \vec{F} \cdot \vec{s} = Fs\cos\theta$$

Work done is a scalar quantity. It can be positive or negative depending on θ is acute or obtuse. The work done by friction or viscous force on a moving body is negative.

Work done is a scalar quantity.

(*i*) If $\theta = 0°$, $W = Fs$ *i.e.*, work done is maximum.

(*ii*) If $\theta = 90°$, $W = 0$ *i.e.*, work done is zero.

2. **Work done against a variable force.** The work done by a variable force $\vec{F}$ in changing the displacement from $\vec{s_1}$ to $\vec{s_2}$ is

$$W = \int_{s_1}^{s_2} \vec{F} \cdot d\vec{s}$$

= Area under the force-displacement curve.

3. **Units of work.** (*i*) The SI unit of work is joule. One joule of work is said to be done when a force of one newton displaces a body through a distance of one metre in its own direction.

1 joule = 1 newton × 1 metre or 1 J = 1 Nm

(*ii*) The CGS unit is erg. One erg of work is said to be done if a force of one dyne displaces a body through a distance of one centimetre in its own direction.

1 erg = 1 dyne × 1 cm

1 joule = 10^7 erg

4. **Work done in terms of rectangular components.** If

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

and $\vec{s} = s_x\hat{i} + s_y\hat{j} + s_z\hat{k}$

then $W = F_xs_x + F_ys_y + F_zs_z$

5. **Newton's third law and work done.** For two bodies, the sum of the mutual forces exerted between the two bodies is zero from Newton's third law,

$$\vec{F}_{12} + \vec{F}_{21} = \vec{0}$$

But the sum of the work done by the two forces need not always cancel, *i.e.*,

$$W_{12} + W_{21} \neq 0.$$

However, it may sometimes be true.

6. **Energy.** It is defined as the ability of a body to do work. It is measured by the amount of work that a body can do. It is a scalar quantity. Like work, SI unit of energy is joule and the CGS unit is erg.

The unit of energy used at the atomic level is electron volt (eV).

$1\,\text{eV} = 1.6 \times 10^{-19}$ J

$1\,\text{keV} = 10^3\,\text{eV} = 1.6 \times 10^{-16}$ J

$1\,\text{MeV} = 10^6\,\text{eV} = 1.6 \times 10^{-13}$ J

7. **Kinetic energy.** It is the energy possessed by a body by virtue of its motion. The K.E. of a body of mass m moving with speed v is

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

8. **Work-energy theorem.** The work done by the net force acting on a body is equal to the change in kinetic energy of the body.

$$W = \text{Change in K.E.} = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

The *WE* theorem may be regarded as the scalar form of Newton's second law of motion.

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9. **Potential energy.** It is the energy possessed by a body by virtue of its position (in a field) or configuration (shape or size)

For a conservative force in one dimension, the potential energy functions $U(x)$ may be defined as

$$F(x) = -\frac{d\,U(x)}{d\,x}$$

or $$\Delta U = U_f - U_i = -\int_{x_i}^{x_f} F(x)\,dx$$

10. **Gravitational potential energy.** It is the energy possessed by a body by virtue of its position above the surface of the earth. The gravitational P.E. of a body of mass m at a height h above the earth's surface is $U = mgh$

11. **Potential energy of a spring.** According to Hooke's law, when a spring is stretched through distance x, the restoring force set up in the spring due to its elasticity is such that

$$F \propto x \quad \text{or} \quad F = -kx$$

where k is the force constant or spring constant of the spring. It is the restoring force set up in the spring per unit extension. Its unit is Nm^{-1}. The work done in stretching the spring through distance x will be

$$W = \int_0^x kx\,dx = \frac{1}{2}kx^2$$

This work done is stored as potential energy U of the spring.

$$\therefore \quad U = \frac{1}{2}kx^2$$

12. **Conservative force.** A force is conservative (i) if the work done by the force in displacing a particle from one point to another is independent of the path followed by the particle and (ii) if the work done by the force in moving a particle around any closed path is zero. Gravitational force, electrostatic force and elastic force of a spring are all conservative forces.

13. **Non-conservative force.** If the amount of work done in moving an object against a force from one point to another depends on the path along which the body moves, then such a force is called a non-conservative force. Forces of friction and viscosity are non-conservative forces.

14. **Mass-energy equivalence.** According to Einstein, mass can be converted into energy and energy into mass. A mass m is equivalent to energy E given by

$$E = mc^2$$

where c is the speed of light in vacuum and $c = 3 \times 10^8\ ms^{-1}$.

15. **Power.** It is the rate of doing work. It is a scalar quantity.

$$\text{Power} = \frac{\text{Work}}{\text{Time}} \quad \text{or} \quad P = \frac{W}{t}$$

Instantaneous power is given by

$$P = \frac{dW}{dt} = \frac{d}{dt}(\vec{F}\cdot\vec{s}) = \vec{F}\cdot\frac{d\vec{s}}{dt}$$

$$= \vec{F}\cdot\vec{v} = Fv\cos\theta$$

If $\theta = 0°$, then $P = Fv$.

16. **Units of power.** The SI unit of power is watt (W).

$$1\ \text{watt} = \frac{1\ \text{joule}}{1\ \text{second}} \quad \text{or} \quad 1\ W = 1\ J\,s^{-1}$$

The larger units of power are kilowatt (kW) and horse power (hp).

$$1\ kW = 1000\ W \quad \text{and} \quad 1\ hp = 746\ W$$

17. **Kilowatt hour.** It is the commercial unit of electrical energy. One kilowatt hour is the electrical energy consumed by an appliance of 1000 watt in 1 hour.

$$1\ kWh = 3.6 \times 10^6\ J$$

18. **Law of conservation of energy.** It states that energy can neither be created nor destroyed but can only be transformed from one form to another.

19. **Collision.** A collision between two bodies is said to occur if either they physically collide against each other or the path of the motion of one body is influenced by the other.

20. **Elastic collision.** If there is no loss of kinetic energy during a collision, it is called an elastic collision.

21. **Inelastic collision.** If there is a loss of kinetic energy during a collision, it is called an inelastic collision.

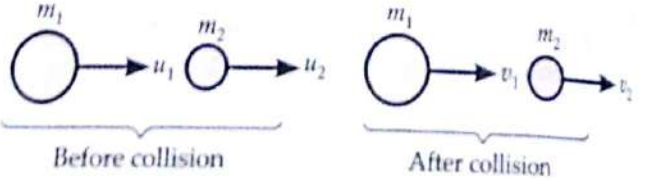
Linear momentum is conserved both in elastic and inelastic collisions.

22. **Perfectly inelastic collision.** It is the collision in which two bodies stick together after the collision.

23. **Head-on collision or one-dimensional collision.** It is a collision in which the colliding bodies move along the same straight line path before and after the collision.

24. **Oblique collision.** If the two bodies do not move along the same straight line path before and after the collision, the collision is said to be oblique collision.

25. **Velocities in one-dimensional elastic collision.** Suppose two bodies of masses m_1 and m_2 moving with velocities u_1 and u_2 ($u_1 > u_2$) in the same direction suffer head-on elastic collision. Let v_1 and v_2 be their velocities after collision.



Before collision After collision

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By the law of conservation of momentum,

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

As K.E. is conserved in an elastic collision, so

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

By solving the above two equations, it can be shown that in an elastic collision,

Velocity of approach = Velocity of separation

or $$u_1 - u_2 = v_2 - v_1$$

Also, $$v_1 = \frac{m_1 - m_2}{m_1 + m_2} \cdot u_1 + \frac{2m_2}{m_1 + m_2} \cdot u_2$$

and $$v_2 = \frac{2m_1}{m_1 + m_2} \cdot u_1 + \frac{m_2 - m_1}{m_1 + m_2} \cdot u_2$$

26. Coefficient of restitution. *The coefficient of restitution for a collision between two bodies is the ratio of the magnitude of their relative velocity after collision to the magnitude of their relative velocity before the collision.*

$$e = -\frac{v_1 - v_2}{u_1 - u_2} = \frac{|v_1 - v_2|}{|u_1 - u_2|}$$

For a perfectly elastic collision, $e = 1$ and for a perfectly inelastic collision, $e = 0$. Hence $0 \le e \le 1$.

For a ball rebounding from a floor,

$$e = \frac{v}{u}$$

where u and v are the magnitudes of the velocities of the ball before and after the collision respectively.

IIT Entrance Exam

MULTIPLE CHOICE QUESTIONS WITH ONE CORRECT ANSWER

1. Two masses of 1 g and 4 g are moving with equal kinetic energy. The ratio of the magnitudes of their momenta is

(a) 4 : 1 (b) $\sqrt{2}$: 1

(c) 1 : 2 (d) 1 : 16 [IIT 80]

2. If a machine is lubricated with oil

(a) the mechanical advantage of the machine increases

(b) the mechanical efficiency of the machine increases

(c) both its mechanical advantage and efficiency increase

(d) its efficiency increases, but its mechanical advantage decreases. [IIT 80]

3. A spring of force-constant k is cut into two pieces such that one piece is double the length of the other. Then the long piece will have a force-constant of

(a) (2/3) k (b) (3/2) k

(c) 3 k (d) 6 k [IIT 99]

4. An ideal spring with spring constant k is hung from the ceiling and a block of mass M is attached to its lower end. The mass is released with the spring initially unstretched. Then the maximum extension in the spring is

(a) $\frac{4Mg}{k}$ (b) $\frac{2Mg}{k}$

(c) $\frac{Mg}{k}$ (d) $\frac{Mg}{2k}$ [IIT 02]

5. A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time t as $a_c = k^2 r t^2$, where k is a constant. The power delivered to the particle by the force acting on it is

(a) $2\pi mk^2 r^2 t$ (b) $mk^2 r^2 t$

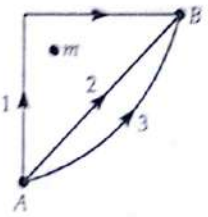
(c) $\frac{(mk^4 r^2 t^5)}{3}$ (d) zero [IIT 94]

6. A wind-powered generator converts wind energy into electrical energy. Assume that the generator converts a fixed fraction of the wind energy intercepted by its blades into electrical energy. For wind speed v, the electrical power output will be proportional to

(a) v (b) v^2

(c) v^3 (d) v^4 [IIT 2K]

7. If W_1, W_2 and W_3 represent the work done in moving a particle from A to B along three different paths 1, 2 and 3 respectively (as shown) in the gravitational field of a point mass m, find the correct relation between W_1, W_2 and W_3.



(a) $W_1 > W_2 > W_3$ (b) $W_1 = W_2 = W_3$

(c) $W_1 < W_2 < W_3$ (d) $W_2 > W_1 > W_3$ [IIT 03]

8. A particle is acted by a force $F = kx$, where k is a +ve constant. Its potential energy at $x = 0$ is zero. Which curve correctly represents the variation of potential energy of the block with respect to x? [IIT 04]

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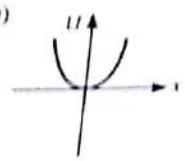
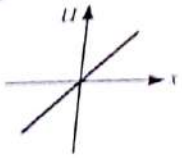
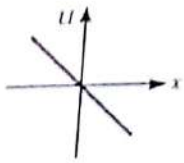
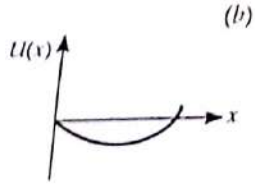
(a) 

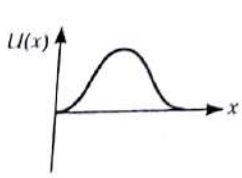
(b)

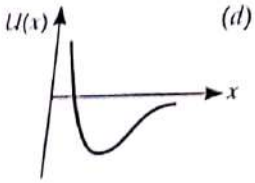
(c) 

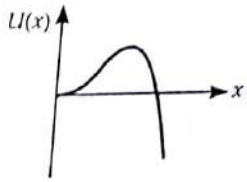
(d) 

9. A particle, which is constrained to move along the x-axis, is subjected to a force in the same direction which varies with the distance x of the particle from the origin as $F(x) = -kx + ax^3$. Here k and a are positive constants. For $x \geq 0$, the functional form of the potential energy $U(x)$ of the particle is [IIT 02]

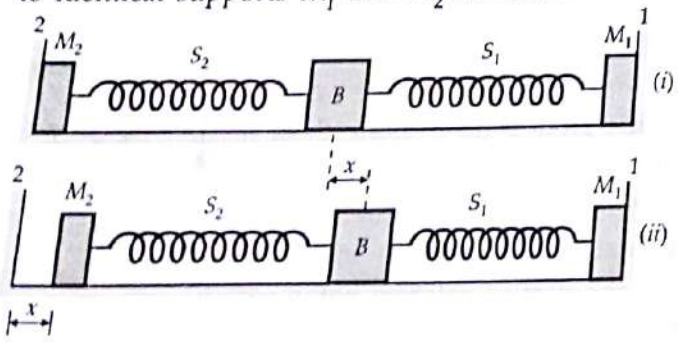
(a) 

(b) 

(c) 

(d) 

10. A block (B) is attached to two unstretched springs S_1 and S_2 with spring constants k and $4k$, respectively [see Fig. (i)]. The other ends are attached to identical supports M_1 and M_2 not attached to the



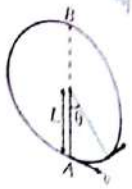
walls. The springs and supports have negligible mass. There is no friction anywhere. The block B is displaced towards wall 1 by a small distance x [see Fig. (ii)] and released. The block returns and moves a maximum distance y towards wall 2. Displacements x and y are measured with respect to the equilibrium position of the block B.

The ratio y/x is

(a) 4 (b) 2

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

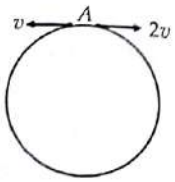
11. A bob of mass M is suspended by a massless string of length L. The horizontal velocity v at position A is just sufficient to make it reach the point B. The angle θ at which the speed of the bob is half of that at A, satisfies



(a) $\theta = \frac{\pi}{4}$ (b) $\frac{\pi}{4} < \theta < \frac{\pi}{2}$

(c) $\frac{\pi}{2} < \theta < \frac{3\pi}{4}$ (d) $\frac{3\pi}{4} < \theta < \pi$

12. Two small particles of equal masses start moving in opposite directions from a point A in a horizontal circular orbit. Their tangential velocities are v and $2v$, respectively, as shown in the figure. Between collisions, the particles move with constant speeds.

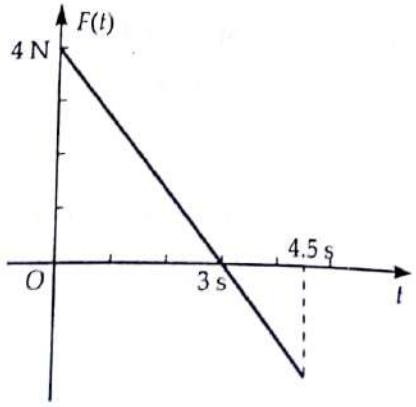


After making how many elastic collisions, other than that at A, these two particles will again reach the point A?

(a) 4 (b) 3

(c) 2 (d) 1

13. A block of mass 2 kg is free to move along the x-axis. It is at rest and from $t = 0$ onwards it is subjected to a time-dependent force $F(t)$ in the x-direction. The force $F(t)$ varies with t as shown in the figure. The kinetic energy of the block after 4.5 seconds is



(a) 4.50 J (b) 7.50 J

(c) 5.06 J (d) 14.06 J [IIT 2010]

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MULTIPLE CHOICE QUESTIONS WITH ONE OR MORE THAN ONE CORRECT ANSWER

14. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane. It follows that

(a) its velocity is constant

(b) its acceleration is constant

(c) its kinetic energy is constant

(d) it moves in a circular path. [IIT 87]

15. A force $F = -K(y\hat{i} + x\hat{j})$ (where K is a positive constant) acts on a particle moving in the xy-plane. Starting from the origin, the particle is taken along the positive x-axis to the point $(a, 0)$, and then parallel to the y-axis to the point (a, a). The total work done by the force F on the particle is

(a) $-2Ka^2$ (b) $2Ka^2$

(c) $-Ka^2$ (d) Ka^2 [IIT 98]

16. A body is moved along a straight line by a machine delivering constant power. The distance moved by the body in time t is proportional to

(a) $t^{1/2}$ (b) $t^{3/4}$

(c) $t^{3/2}$ (d) t^2 [IIT 84]

17. A uniform chain of length L and mass M is lying on a smooth table and one third of its length is hanging vertically down over the edge of the table. If g is acceleration due to gravity, the work required to pull the hanging part on to the table is

(a) MgL (b) $MgL/3$

(c) $MgL/9$ (d) $MgL/18$ [IIT 85]

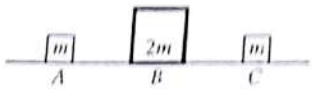
18. A small ball starts moving from A over a fixed track as shown in the figure. Surface AB has friction. From A to B the ball rolls without slipping. Surface BC is frictionless. K_A, K_B and K_C are kinetic energies of the ball at A, B and C respectively. Then

(a) $h_A > h_C$; $K_B > K_C$ (b) $h_A > h_C$; $K_C > K_A$

(c) $h_A = h_C$; $K_B = K_C$ (d) $h_A < h_C$; $K_B > K_C$ [IIT 06]

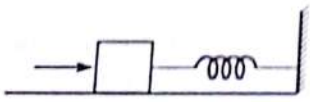
INTEGER ANSWER TYPE

19. Three objects A, B and C are kept in a straight line on a frictionless horizontal surface. These have masses m, $2m$ and m, respectively. The object A moves towards B with a speed 9 m/s and makes an elastic



collision with it. Therefore, B makes completely inelastic collision with C. All motions occur on the same straight line. Find the final speed (in m/s) of the object C. [IIT 09]

20. A block of mass 0.18 kg is attached to a spring of force-constant 2 N/m. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is un-stretched. An



impulse is given to the block as shown in the figure. The block slides a distance of 0.06 m and comes to rest for the first time. The initial velocity of the block in m/s is $v = N/10$. Find N [IIT 2011]

Answers and Explanations

1. (c) $p = \sqrt{2mK}$

For same K, $p \propto \sqrt{m}$

$$\frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{1}{4}} = \frac{1}{2} = 1:2.$$

2. (b) When a machine is lubricated with oil, energy wastage against friction decreases.

This increases the mechanical efficiency of the machine.

3. (b) For any spring, kl = constant

Length of longer piece $= \frac{2l}{3}$

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$\therefore \quad k' \times \frac{2l}{3} = kl \quad \text{or} \quad k' = \left(\frac{3}{2}\right)k$

4. (*b*) At the maximum extension x of the spring,
Increase in elastic P.E. of the spring
= Loss in P.E. by mass M

or $\quad \frac{1}{2}kx^2 = Mgx$

$\therefore \quad x = \frac{2Mg}{k}$.

5. (*b*) Given $a_c = k^2rt^2$

or $\quad \frac{v^2}{r} = k^2rt^2 \quad \text{or} \quad v = krt$

Tangential acceleration,

$$a_t = \frac{dv}{dt} = kr$$

Tangential force on the particle,

$$F_t = ma_t = mkr$$

Power is delivered only by the tangential force not by the radial force.

$\therefore \quad P = F_t.v = mkr \times krt = mk^2r^2t.$

6. (*e*) Force $= v\frac{dm}{dt} = v\frac{d}{dt}$ (volume × density)

$$= v\frac{d}{dt}(Ax \times \rho) = vA\rho\frac{dx}{dt}$$

$$= A\rho v^2 \qquad \left[\frac{dx}{dt} = v\right]$$

Power = Force × velocity

$$= A\rho v^2 \times v = A\rho v^3$$

$\therefore$ Power $\propto v^3$.

7. (*b*) Gravitational force is a conservative force. Work done against such a force does not depend on the path followed.

$\therefore \quad W_1 = W_2 = W_3.$

8. (*b*) For a conservative force,

$$F = -\frac{dU}{dt}$$

$$\therefore \quad \int_0^{U(x)} dU = -\int_0^x F\,dx = -\int_0^x kx\,dx$$

As $\quad U(0) = 0$

So, $\quad U(x) = -\frac{kx^2}{2}$

Thus, the U-x graph is a parabola, symmetric about -axis, lying below x-axis with its vertex at the origin. ence the correct option is (*b*).

9. (*d*) $\quad F = -\frac{dU}{dx}$

or $\quad dU = -F\,dx$

$$\therefore \quad U(x) = -\int_0^x (-kx + ax^3)dx$$

or $\quad U(x) = \frac{kx^2}{2} - \frac{ax^4}{4} = \frac{x^2}{2}\left(k - \frac{ax^2}{2}\right)$

Clearly, $U(x) > 0$ at $x = 0$ and $x < \sqrt{\frac{2k}{a}}$

For $x > \sqrt{\frac{2k}{a}}$, $U(x)$ will be negative.

At $x = 0$, $\quad F = -\frac{dU}{dx} = 0$

i.e., slope of U-x graph is zero at $x = 0$.

Hence, the most appropriate option is (*d*).

10. (*c*) By conservation of energy,

$$\frac{1}{2}kx^2 = \frac{1}{2}(4k)y^2$$

or $\quad \frac{y^2}{x^2} = \frac{1}{4} \quad \therefore \quad \frac{y}{x} = \frac{1}{2}$.

11. (*d*) By conservation of energy,

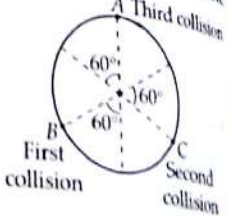
$$\frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{v}{2}\right)^2 + mgh$$

or $\quad \frac{1}{2}m.5gL = \frac{1}{2}m\frac{5gL}{4} + mgL(1 - \cos\theta)$

$$\frac{5}{2} = \frac{5}{8} + 1 - \cos\theta \quad \text{or} \quad \cos\theta = \frac{7}{8}$$

Hence $\quad \frac{3\pi}{4} < \theta < \pi.$

12. (*c*) After the collision the velocities of the two particles get exchanged. After the first collision the particles will be at position B and after the second collision the particles will be at position C. After this collision, the two particles will return to position A.



13. (*c*)

Area under F-t graph = Change in momentum

$$\therefore \quad \Delta p = \frac{1}{2} \times 4 \times 3 - \frac{1}{2} \times 1.5 \times \frac{4}{3} \times 1.5 = 4.5 \text{ kgms}^{-1}$$

Kinetic energy

$$= \frac{p^2}{2m} = \frac{4.5^2}{2 \times 2} = 5.06 \text{ J}.$$

COMPETITION

SECTION

14. (*c*), (*d*) For a particle in unifo
the directions of both velocity and
change continuously but their
unchanged. The kinetic energy is
(*c*) and (*d*) are correct options.

15. (*c*) Position vector of point

$$\vec{r}_1 = a\hat{i} + 0\hat{j}$$

Position vector of point (a,a),

$$\vec{r}_2 = a\hat{i} + a\hat{j}$$

Displacement vector,

$$\vec{r} = \vec{r}_2 - \vec{r}_1 = (a\hat{i} +$$

$$= a\hat{j}$$

Also, $\quad \vec{F} = -k(y\hat{i} + x\hat{j})$

$\therefore \quad W = \vec{F} \cdot \vec{r} = -k(y$

As $x = a$, so

$$W = -Ka^2.$$

16. (*c*) Refer answer to page 6.53.

17. (*d*) Refer to the so page 6.46.

18. (*a*), (*c*)

Total energy at A, E_A

Total energy at B, E_B

Total energy at C, E_C

By conservation of ene

$$E_A = E_B = E$$

Now, $\quad E_A = E_B \Rightarrow$

Consider $E_A = E_C$

or $\quad K_A + mgh_A = K_C +$

or $\quad h_A - h_C = \frac{K_C -}{m}$

For $\quad h_A > h_C$;

Also, $\quad h_A > h_C$ a

$\therefore$ Option (*a*) is cor

1. A force $\vec{F} = ($ particle which displa $\vec{r} = (2\hat{i} - \hat{j})m$. The w

(*a*) −7

(*c*) +10

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14. (c), (d) For a particle in uniform circular motion, the directions of both velocity and acceleration vectors change continuously but their magnitudes remain unchanged. The kinetic energy is not affected. Hence (c) and (d) are correct options.

15. (c) Position vector of point $(a, 0)$,

$$\vec{r_1} = a\hat{i} + 0\hat{j}$$

Position vector of point (a, a),

$$\vec{r_2} = a\hat{i} + a\hat{j}$$

Displacement vector,

$$\vec{r} = \vec{r_2} - \vec{r_1} = (a\hat{i} + a\hat{j}) - (a\hat{i} + 0\hat{j})$$
$$= a\hat{j}$$

Also, $\vec{F} = -k(y\hat{i} + x\hat{j})$

$\therefore$ $W = \vec{F} \cdot \vec{r} = -k(y\hat{i} + x\hat{j}) \cdot a\hat{j} = -kax$

As $x = a$, so

$$W = -Ka^2.$$

16. (c) Refer answer to NCERT Exercise 6.10 on page 6.53.

17. (d) Refer to the solution of Problem 13 on page 6.46.

18. (a), (c)

Total energy at A, $E_A = K_A + mgh_A$

Total energy at B, $E_B = K_B$

Total energy at C, $E_C = K_C + mgh_C$

By conservation of energy,

$$E_A = E_B = E_C$$

Now, $E_A = E_B \Rightarrow K_B > K_C$ ...(i)

Consider $E_A = E_C$

or $K_A + mgh_A = K_C + mgh_C$

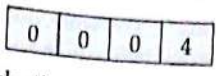
or $h_A - h_C = \dfrac{K_C - K_A}{mg}$

For $h_A > h_C$; $K_C > K_A$

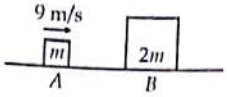
Also, $h_A > h_C$ and from (i), $K_B > K_C$

$\therefore$ Option (a) is correct.

19.



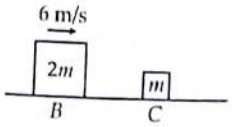
For elastic collision between A and B,



$$v_B = \frac{2m}{m+2m} \times 9 + \frac{2m-m}{2m+m} \times 0$$

$$= 6 \text{ m/s}$$

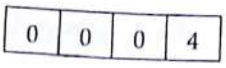
For inelastic collision between B and C,



$$2m \times 6 + m \times 0 = (2m + m)v$$

$$v = 4 \text{ m/s}.$$

20.



Loss in K.E. of the block

= Gain in P.E. of the spring

+ Work done against friction

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 + \mu mgx$$

or $\dfrac{1}{2} \times 0.18v^2 = \dfrac{1}{2} \times 2 \times (0.06)^2 + 0.1 \times 0.18 \times 10 \times 0.06$

or $0.09v^2 = 36 \times 10^{-4} \times 108 \times 10^{-4} = 144 \times 10^{-4}$

or $v = \dfrac{12 \times 10^{-2}}{0.3} = \dfrac{4}{10}$

Given $v = \dfrac{N}{10}$

$\therefore$ $N = 4$.

AIEEE

1. A force $\vec{F} = (5\hat{i} + 3\hat{j} + 2\hat{k})N$ is applied over a particle which displaces it from its origin to the point $\vec{r} = (2\hat{i} - \hat{j})m$. The work done on the particle (in joule) is

(a) –7 (b) +7

(c) +10 (d) +13 [AIEEE 04]

2. An athlete in the olympic games covers a distance of 100 m in 10 s. His kinetic energy can be estimated in the range

(a) 200 J – 500 J (b) 2×10^5 J – 3×10^5 J

(c) 20,000 J – 50,000 J (d) 2000 J – 5000 J [AIEEE 08]

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3. A particle of mass 100 g is thrown vertically upwards with a speed of 5 ms^{-1}. The work done by the force of gravity during the time the particle goes up is

(a) 1.25 J (b) 0.5 J
(c) −0.5 J (d) −1.25 J [AIEEE 06]

4. A ball whose kinetic energy is E, is projected at an angle of 45° to the horizontal. The kinetic energy of the ball at the highest point of its flight will be

(a) E (b) $E/\sqrt{2}$
(c) $E/2$ (d) zero [AIEEE 02]

5. A particle is projected at an angle of 60° to the horizontal with a kinetic energy E. The kinetic energy at the highest point is

(a) E (b) $E/4$
(c) $E/2$ (d) zero [AIEEE 07]

6. A particle moves in a straight line with retardation proportional to its displacement. Its loss of kinetic energy for any displacement x is proportional to

(a) x^2 (b) e^x
(c) x (d) $\log_e x$ [AIEEE 04]

7. The potential energy of a 1 kg particle free to move along the X-axis is given by

$$V(x) = \frac{x^4}{4} - \frac{x^2}{2} \text{ (in joule)}$$

The total mechanical energy of the particle is 2 J. Then the maximum speed (in ms^{-1}) is

(a) $1/\sqrt{2}$ (b) 2
(c) $3/\sqrt{2}$ (d) $\sqrt{2}$ [AIEEE 06]

8. Consider the following *two* statements :

(A) Linear momentum of a system of particles is zero.

(B) Kinetic energy of a system of particles is zero.

Then

(a) A does not imply B and B does not imply A
(b) A implies B but B does not imply A
(c) A does not imply B but B implies A
(d) A implies B and B implies A. [AIEEE 03]

9. A spherical ball of mass 20 kg is stationary at the top of a hill of height 100 m. It rolls down a smooth surface to the ground, then climbs up another hill of height 30 m and finally rolls down to a horizontal base at a height of 20 m above the ground. The velocity attained by the ball is

(a) 10 ms^{-1} (b) $10\sqrt{30}$ ms^{-1}
(c) 40 ms^{-1} (d) 20 ms^{-1} [AIEEE 05]

10. A uniform chain of length 2 m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain on the table?

(a) 7.2 J (b) 3.6 J
(c) 120 J (d) 1,200 J [AIEEE 04]

11. A mass of M kg is suspended by a weightless string. The horizontal force that is required to displace it, until the string makes an angle 45° with the initial vertical direction, is

(a) $Mg/\sqrt{2}$ (b) $(\sqrt{2}-1)Mg$
(c) $(\sqrt{2}+1)Mg$ (d) $\sqrt{2}\,Mg$ [AIEEE 06]

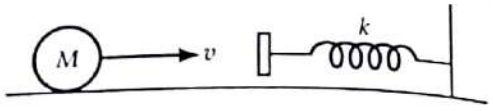
12. A spring of spring constant 5×10^3 Nm^{-1} is stretched initially by 5 cm from the unstretched position. Then the work required to stretch it further by another 5 cm is

(a) 12.50 Nm (b) 18.75 Nm
(c) 25.00 Nm (d) 6.25 Nm [AIEEE 03]

13. A spring of force constant 800 Nm^{-1} has an extension of 5 cm. The work done in extending it from 5 cm to 15 cm is

(a) 16 J (b) 8 J
(c) 32 J (d) 24 J [AIEEE 02]

14. The block of mass M moving on the frictionless horizontal surface collides with the spring of spring



constant k and compresses it by length L. The maximum momentum of the block after collision is

(a) $\frac{ML^2}{k}$ (b) $\frac{kL^2}{2M}$
(c) $\sqrt{Mk}\,L$ (d) zero [AIEEE 05]

15. A 2 kg block slides on a horizontal floor with a speed of 4 ms^{-1}. It strikes an uncompressed spring and compresses it, till the block is motionless. The force of kinetic friction is 15 N and spring constant is 10,000 Nm^{-1}.

The spring compresses by

(a) 5.5 cm (b) 2.5 cm
(c) 11.0 cm (d) 8.5 cm [AIEEE 05]

16. A body of mass m accelerates uniformly from rest to v_1 in time t_1. The instantaneous power delivered to the body as a function of time t is

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(a) $\frac{mv_1 t}{t_1}$ (b) $\frac{mv_1^2 t}{t_1^2}$

(c) $\frac{mv_1 t^2}{t_1}$ (d) $\frac{mv_1^2 t}{t_1}$ [AIEEE 04, 05]

17. A body is moved along a straight line by a machine delivering a constant power. The distance moved by the body in time t is proportional to

(a) $t^{3/4}$ (b) $t^{3/2}$

(c) $t^{1/4}$ (d) $t^{1/2}$ [AIEEE 03]

18. If mass-energy equivalence is taken into account, when water is cooled to form ice, the mass of water should

(a) increase (b) remain unchanged

(c) decrease

(d) first increase then decrease [AIEEE 02]

19. A bomb of mass 16 kg at rest explodes into two pieces of masses 4 kg and 12 kg. The velocity of the 12 kg mass is $4\,ms^{-1}$. The kinetic energy of the other mass is

(a) 192 J (b) 96 J

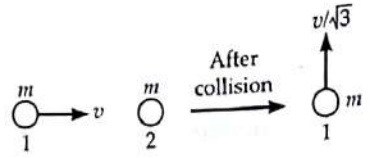
(c) 144 J (d) 288 J [AIEEE 06]

20. A block of mass 0.50 kg is moving with a speed of $2.00\,ms^{-1}$ on a smooth surface. It strikes another mass of 1.00 kg and then move together as a single body. The energy loss during the collision is

(a) 0.16 J (b) 1.00 J

(c) 0.67 J (d) 0.34 J [AIEEE 08]

21. A mss m moves with a velocity v and collides elastically with another identical mass at rest. After collision the first mass moves with velocity $v/\sqrt{3}$ in a



direction perpendicular to the initial direction of motion. Find the speed of the second mass after collision.

(a) v (b) $\sqrt{3}\,v$

(c) $2v/\sqrt{3}$ (d) $v/\sqrt{3}$ [AIEEE 05]

22. **Statement – 1.** Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.

Statement – 2. Principle of conservation of momentum holds true for all kinds of collision.

(a) Statement – 1 is true, Statement – 2 is false

(b) Statement – 1 is true, Statement – 2 is true ; Statement – 2 is the correct explanation of Statement – 1

(c) Statement – 1 is true, Statement – 2 is true ; Statement – 2 is *not* the correct explanation of Statement – 1

(d) Statement – 1 is false, Statement – 2 is true

[AIEEE 2010]

Answers and Explanations

1. (b) $W = \vec{F} \cdot \vec{r} = (5\hat{i} + 3\hat{j} + 2\hat{k}) \cdot (2\hat{i} - \hat{j})$

$= 5 \times 2 + 3(-1) + 2 \times 0 = +7$ J.

2. (d) Average speed of the athlete,

$$v = \frac{s}{t} = \frac{100}{10} = 10 \text{ ms}^{-1}$$

Assuming the mass of the athelete to 60 kg, his average K.E. would be

$$K = \frac{1}{2} \times 60 \times (10)^2 = \mathbf{3000\ J}.$$

3. (d) Work done by the force of gravity

= Loss in K.E. of the body

$$= \frac{1}{2} m(v^2 - u^2) = \frac{1}{2} \times \frac{100}{1000} (0^2 - 5^2)\ \text{J}$$

$$= \mathbf{-1.25\ J}.$$

4. (c) Refer to the solution of Problem 11 on page 6.50.

5. (b) Kinetic energy at the highest point,

$$E' = \frac{1}{2} m(u\cos 60°)^2 = \frac{1}{2} mu^2 \times \frac{1}{4} = \frac{E}{4}.$$

6. (a) Refer to the solution of Problem 12 on page 6.50.

7. (c) Potential energy,

$$V(x) = \frac{x^4}{4} - \frac{x^2}{2} \text{ joule}$$

For maxima or minima,

$$\frac{dV}{dx} = 0$$

$$x^3 - x = 0$$

or $$x = 0, -1, 1$$

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Now $\frac{d^2V}{dx^2} = 3x^2 - 1$

At $x = 0$, $\frac{d^2V}{dx^2} = -1$

At $x = \pm 1$ $\frac{d^2V}{dx^2} = 2 > 0$

Hence P.E. is minimum at $x = \pm 1$.

$V_{min} + K_{max}$ = Total energy

$\left(\frac{1}{4} - \frac{1}{2}\right) + K_{max} = 2$

or $K_{max} = 2 + \frac{1}{4} = \frac{9}{4}$

$\frac{1}{2}mv^2 = \frac{9}{4}$

or $v = \sqrt{\frac{9}{2m}} = \sqrt{\frac{9}{2\times 1}} = \frac{3}{\sqrt{2}}$ **ms^{-1}**.

8. *(c)* When the linear momentum of a system of particles is zero, the velocities of the individual particles may not be zero. Then the kinetic energy of the system of particles may be non-zero. Thus *A* does not imply *B*.

When the kinetic energy of a system of particles is zero, then kinetic energy and hence velocity of each particle is zero. As such, the linear momentum of the system of particles is also zero. Thus *B* implies *A*.

Hence the correct option is *(c)*.

9. *(c)* Total energy at 100 m height

= Total energy at 20 m height

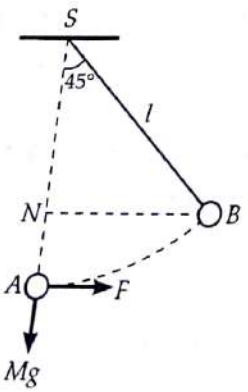
$mgh_1 = mgh_2 + \frac{1}{2}mv^2$

$v = \sqrt{2g(h_1 - h_2)} = \sqrt{2\times 10\times (100-20)}$

$= \mathbf{40\ ms^{-1}}$.

10. *(b)* Refer to the solution of Example 6 on page 6.4.

11. *(b)*



By conservation of energy,

Work done by force *F*

= Change in P.E. of mass *M*

$F \times BN = Mg \times AN$

$F \times l\sin 45° = Mg \times (l - l\cos 45°)$

$F = Mg\left(\frac{1}{\sin 45°} - \cot 45°\right)$

$= Mg(\sqrt{2} - 1)$

12. *(b)* $W = \frac{1}{2}k(x_f^2 - x_i^2)$

$x_i = 5$ cm $= 0.05$ m

$x_f = 5 + 5 = 10$ cm $= 0.10$ m

$\therefore$ $W = \frac{1}{2}\times 5\times 10^3(0.10^2 - 0.05^2) = 18.75$ J

13. *(b)* $W = \frac{1}{2}\times 800(0.15^2 - 0.05^2)$

$= 400 \times 0.20 \times 0.10 = 8$ J.

14. *(c)* When the spring gets compressed by length *L*,

K.E. lost by mass *M*

= P.E. stored in the compressed spring

$\frac{1}{2}Mv_{max}^2 = \frac{1}{2}kL^2$

$\therefore$ $v_{max} = \sqrt{\frac{k}{M}}L$

Maximum momentum of the block,

$p_{max} = Mv_{max} = \sqrt{Mk}\,L$.

15. *(a)* Suppose the spring gets compressed by length *x*. Then

Initial K.E. of the block

= P.E. stored in the spring

+ Work done against friction

$\frac{1}{2}\times 2\times 4^2 = \frac{1}{2}\times 10000\times x^2 + 15x$

or $5000x^2 + 15x - 16 = 0$

or $x = 0.055\,\text{m} = \mathbf{5.5\ cm}$.

16. *(b)* Refer to the solution of Problem 13 on page 6.50.

17. *(b)* Refer to the solution of Problem 14 on page 6.46.

18. *(a)* The heat energy possessed by water gets converted into mass, when ice is formed. This increases mass.

19. *(d)* By conservation of momentum,

$16\times 0 = 4\times v + 12\times 4$

$v = -12$ **ms^{-1}**

K.E. of 4 kg mass

$= \frac{1}{2}mv^2 = \frac{1}{2}\times 4\times 12^2 = \mathbf{288}$ J.

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20. (c) Momentum after collision = Momentum before collision

$$(m_1 + m_2)v = m_1u_1 + m_2u_2$$

$$(0.50 + 1.00)v = 0.50 \times 2.00 + 1.00 \times 0$$

$$v = \frac{1}{1.5} = \frac{2}{3}\ \text{ms}^{-1}$$

Loss of energy,

$$= \frac{1}{2}m_1u_1^2 - \frac{1}{2}(m_1 + m_2)v^2$$

$$= \frac{1}{2} \times 0.5 \times 2^2 - \frac{1}{2}(0.50 + 1.0)\left(\frac{2}{3}\right)^2$$

$$= 1 - \frac{1}{3} = \mathbf{0.67\ J}.$$

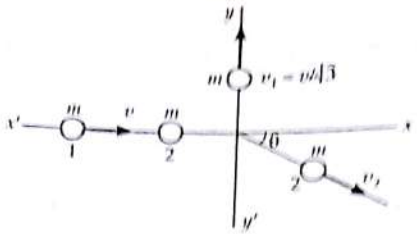
21. (c) The situation is shown in the figure.

Considering conservation of momentum along x-axis,

$$mv + m \times 0 = m \times 0 + m \times v_2 \cos\theta$$

$$v_2 \cos\theta = v \quad \ldots(i)$$

or



Along y-axis, we have

$$m \times 0 + m \times 0 = m \times \frac{v}{\sqrt{3}} + m(-v_2 \sin\theta)$$

or

$$v_2 \sin\theta = \frac{v}{\sqrt{3}} \quad \ldots(ii)$$

Squaring and adding (i) and (ii), we get

$$v_2^2 \cos^2\theta + v_2^2 \sin^2\theta = v^2 + \left(\frac{v}{\sqrt{3}}\right)^2$$

or

$$v_2^2 = \frac{4v^2}{3} \quad \text{or} \quad v_2 = \frac{2v}{\sqrt{3}}.$$

22. (b) If the particles moving in same direction lose all their energy, final momentum will become zero, whereas initial momentum is not zero.

DCE and G.G.S. Indraprastha University Engineering Entrance Exam

(Similar Questions)

1. A body moves a distance of 10 m under the action of force $F = 10$ N. If the work done is 25 J, the angle which the force makes with the direction of motion is

(a) 0° (b) 30°
(c) 60° (d) none of these

[IPUEE 04]

2. What is $\vec{F}.\vec{ds}$?

(a) Torque (b) Impulse
(c) Momentum (d) Work

[DCE 97]

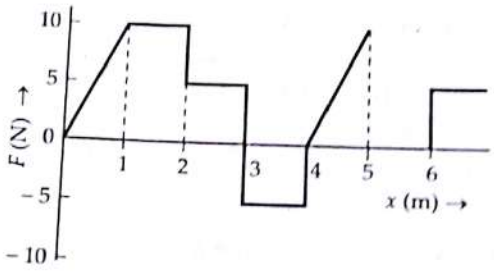
3. A particle moves under a force $F = CX$ from $X = 0$ to $X = X_1$.

The work done is

(a) CX_1^2 (b) $CX_1^2/2$
(c) zero (d) CX_1^3

[DCE 02]

4. The relationship between the force F and position x of a body is as shown in figure. The work done in displacing the body from $x = 1$ m to $x = 5$ m will be



(a) 30 J (b) 15 J
(c) 25 J (d) 20 J

[IPUEE 04]

5. Two bodies of mass m and 4 m have equal kinetic energy. What is the ratio of their momentum ?

(a) 1 : 4 (b) 1 : 2
(c) 1 : 1 (d) 2 : 1

[IPUEE 05]

6. If momentum decreases by 20%, K.E. will decrease by

(a) 40% (b) 36%
(c) 18% (d) 8%

[DCE 98]

7. A hockey player receives a corner shot at a speed of 15 m/s at an angle of 30° with the y-axis and then

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shoots the ball of mass 100 g along the positive x-axis with a speed of 30 m/s. If it remains in contact with the hockey stick for 0.01 s, the force imparted to the ball in the x-direction is

(a) 281.25 N (b) 187.5 N
(c) 562.5 N (d) 375 N [DCE 05]

8. Which is odd out ?

(a) Displacement (b) Momentum
(c) Potential energy (d) Torque [IPUEE 02]

9. The dimension of k in the equation $W = \frac{1}{2}kx^2$ is

(a) $[M^1L^0T^{-2}]$ (b) $[M^0L^1T^{-1}]$
(c) $[M^1L^1T^{-2}]$ (d) $[M^1L^0T^{-1}]$ [IPUEE 05]

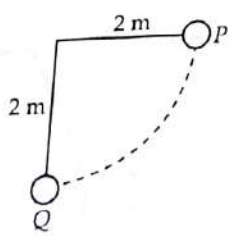
10. The work done in stretching a spring of force constant k from length l_1 to l_2 is

(a) $k(l_2^2 - l_1^2)$ (b) $\frac{1}{2}k(l_2^2 - l_1^2)$
(c) $k(l_2 - l_1)$ (d) $\frac{k}{2}(l_2 + l_1)$ [DCE 07]

11. When a spring is stretched by 2 cm, it stores 100 J of energy. If it is stretched further by 2 cm, the stored energy will be increased by

(a) 100 J (b) 200 J
(c) 300 J (d) 400 J [IPUEE 04]

12. The bob of a pendulum of length 2 m lies at P. When it reaches Q, it loses 10% of its total energy due to air resistance. The velocity at Q is



(a) 6 m/s
(b) 1 m/s
(c) 2 m/s (d) 8 m/s [DCE 98]

13. Unit of power is

(a) kilowatt hour (b) kilowatt/hour
(c) watt (d) erg [DCE 99]

14. Power can be expressed as

(a) $\vec{F}.\vec{v}$ (b) $\frac{1}{2}\vec{F}.v^2$
(c) $\vec{F}.t$ (d) $\vec{F} \times \vec{v}$ [DCE 99]

15. Power applied to a particle varies with time as $= (3t^2 - 2t + 1)$ watt, where t is in second. Find the ange in its kinetic energy between $t = 2$ s and $t = 4$ s.

(a) 32 J (b) 46 J
(c) 61 J (d) 102 J [DCE 05]

16. A block is kept on an inclined plane of inclination 0 and length l. The velocity of particle at the bottom of incline (the coefficient of friction is μ) is

(a) $\sqrt{2gl(\mu\cos\theta - \sin\theta)}$
(b) $\sqrt{2gl(\sin\theta - \mu\cos\theta)}$
(c) $\sqrt{2gl(\sin\theta + \mu\cos\theta)}$
(d) $\sqrt{2gl(\cos\theta + \mu\sin\theta)}$ [DCE 08]

17. In an elastic collision

(a) both momentum and K.E. are conserved
(b) both momentum and K.E. are not conserved
(c) only energy is conserved
(d) only momentum is conserved [DCE 07]

18. In an inelastic collision, what is conserved ?

(a) Kinetic energy (b) Momentum
(c) Both (a) and (b) (d) Neither (a) nor (b) [DCE 04]

19. A body moving with a velocity v, breaks up into two equal parts. One of the parts retraces back with velocity v. Then the velocity of the other part is

(a) v in forward direction
(b) $3v$ in forward direction
(c) v in backward direction
(d) $3v$ in backward direction [DCE 04]

20. A bomb is kept stationary at a point. It suddenly explodes into two fragments of masses 1 g and 3 g. The total K.E. of the fragments is 6.4×10^4 J. What is the K.E. of the smaller fragment ?

(a) 2.5×10^4 J (b) 3.5×10^4 J
(c) 4.8×10^4 J (d) 5.2×10^4 J [DCE 04]

21. A particle of mass m_1 moving with velocity v collides with a mass m_2 at rest, then they get embedded. At the instant of collision, velocity of the system

(a) increases (b) decreases
(c) remains constant (d) becomes zero [IPUEE 02]

22. A body of mass M_1 collides elastically with another mass M_2 at rest. There is maximum transfer of energy when

(a) $M_1 > M_2$
(b) $M_1 < M_2$
(c) $M_1 = M_2$
(d) same of all values of M_1 and M_2 [IPUEE 04]

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23. A spacecraft of mass M and moving with velocity v suddenly breaks in two pieces of same mass m. After the explosion one of the masses m becomes stationary. What is the velocity of the other part of craft ?

(a) $\frac{Mv}{M-m}$ (b) v

(c) $\frac{Mv}{m}$ (d) $\frac{M-m}{m}v$ [DCE 03]

24. If a neutron collides with a stationary α-particle with velocity v, what is resultant velocity of neutron ?

(a) $\frac{1}{5}v$ (b) $\frac{2}{5}v$

(c) $\frac{3}{5}v$ (d) $\frac{4}{5}v$ [DCE 97]

25. A bullet hits and gets embedded in a solid block resting on a frictionless surface. In this process which one of the following is correct ?

(a) only momentum is conserved

(b) only kinetic energy is conserved

(c) neither momentum nor kinetic energy is conserved

(d) both momentum and energy are conserved. [DCE 09]

Answers and Explanations

1. (d) $W = Fs\cos\theta$

$25 = 10 \times 10\cos\theta$

$\cos\theta = \frac{1}{4}$, $\theta = \cos^{-1}\left(\frac{1}{4}\right)$

2. (d) $dW = \vec{F} \cdot d\vec{s}$

3. (b) $W = \int F\,dX = \int_0^{X_1} CX\,dX = C\left[\frac{X^2}{2}\right]_0^{X_1} = \frac{1}{2}CX_1^2$.

4. (b) W = Area under F-x graph from $x = 1$ m to $x = 5$ m.

$= \frac{1}{2}\times 10(1-0) + 10\times(3-2) - 5(4-3) + \frac{1}{2}\times 10(5-4)$

$= 5 + 10 - 5 + 5 = \mathbf{15\ J}$.

5. (b) $p = \sqrt{2mK}$

For same K, $p \propto \sqrt{m}$

$\therefore \quad \frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{m}{4m}} = \frac{1}{2} = \mathbf{1:2}$.

6. (b) Initial K.E., $K = \frac{p^2}{2m}$

Decrease in momentum

$= 20\%$ of $p = \frac{20}{100}\times p = \frac{p}{5}$

Final momentum $= p - \frac{p}{5} = \frac{4p}{5}$

Final K.E., $K' = \frac{(4p/5)^2}{2m} = \frac{16}{25}\frac{p^2}{2m} = \frac{16}{25}K$

Decrease in K.E. $= K - K' = K - \frac{16}{25}K = \frac{9}{25}K$.

% Decrease in K.E.

$= \frac{K-K'}{K}\times 100 = \frac{\frac{9}{25}K}{K}\times 100 = \mathbf{36\%}$.

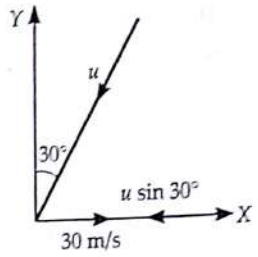
7. (d) Change in momentum of the ball along x-direction

$= m(30 - u\sin 30°)$

$= 0.1\left[30 - (-15)\times\frac{1}{2}\right]$

$= 3.75$ kg ms^{-1}

$F = \frac{\Delta p}{\Delta t} = \frac{3.75}{0.01} = \mathbf{375\ N}$.



8. (c) Only potential energy is scalar, other quantities are vectors.

9. (a) $W = \frac{1}{2}kx^2$

$\therefore [M^1L^2T^{-2}] = [k][L^2]$ or $[k] = [M^1L^0T^{-2}]$

10. (b) $W = U_2 - U_1 = \frac{1}{2}kl_2^2 - \frac{1}{2}l_1^2 = \frac{1}{2}k(l_2^2 - l_1^2)$.

11. (c) $U_1 = \frac{1}{2}kx_1^2 = 100$ J

$U_2 = \frac{1}{2}kx_2^2$

$\frac{U_2}{U_1} = \frac{x_2^2}{x_1^2} = \left(\frac{4}{2}\right)^2 = 4$

$U_2 = 4U_1 = 400$ J

$\therefore \quad U_2 - U_1 = \mathbf{300\ J}$.

12. (a) K.E. at $Q = 90\%$ of P.E. at P

$\frac{1}{2}mv^2 = \frac{90}{100}\times mgh$

$v = \sqrt{1.8gh}$

$= \sqrt{1.8\times 10\times 2} = \mathbf{6\ ms^{-1}}$.

13. (a) The S.I. unit of power is watt (W).

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14. (*a*) $P = \vec{F} \cdot \vec{v}$

15. (*b*) $\Delta K = W = \int P\,dt = \int_2^4 (3t^2 - 2t + 1)\,dt$

$= [t^3 - t^2 + t]_2^4$

$= (64 - 8) - (16 - 4) + (4 - 2)$

$= 56 - 12 + 2 = \mathbf{46\ J}.$

16. (*b*) For an inclination greater than the angle of repose,

$$a = (\sin\theta - \mu\cos\theta)g$$

Also, $u = 0,\ s = l$

As $v^2 - u^2 = 2as$ or $v^2 - 0^2 = 2al$

$$v = \sqrt{2al} = \sqrt{2gl(\sin\theta - \mu\cos\theta)}$$

17. (*a*) In an elastic collision, both momentum and K.E. are conserved.

18. (*b*) In an inelastic collision, momentum is conserved and K.E. is not conserved.

19. (*d*) By conservation of momentum,

$$2m \times v = m(-v) + mv'$$

$$2v = -v + v'$$

or $$v' = 3v.$$

20. (*c*) By conservation of momentum,

$$m_1 v_1 = m_2 v_2$$

$$\frac{m_1}{m_2} = \frac{1}{3} = \frac{v_2}{v_1}$$

$$\frac{K_1}{K_2} = \frac{\frac{1}{2}m_1 v_1^2}{\frac{1}{2}m_2 v_2^2} = \frac{1}{3} \times \left(\frac{3}{1}\right)^2 = \frac{3}{1}.$$

$$\therefore \quad K_1 = \frac{3}{4} \times 6.4 \times 10^4$$

$$= \mathbf{4.8 \times 10^4\ J}.$$

21. (*c*) By conservation of momentum,

$$m_1 v + m_2 \times 0 = (m_1 + m_2)v'$$

$$\therefore \quad v' = \frac{m_1}{m_1 + m_2} \cdot v$$

As $$\frac{m_1}{m_1 + m_2} < 1$$

so, $$v' > v.$$

Hence velocity of the system decreases at the time of collision.

22. (*c*) When both the colliding bodies have equal masses, maximum transfer of energy occurs at the time of collision.

23. (*c*) By conservation of momentum,

$$Mv = m \times 0 + mv' \quad \therefore \quad v' = \frac{Mv}{m}$$

24. (*c*) Mass of an α-particle is four times the mass of a neutron

$$\therefore \quad m_1 = m,\ m_2 = 4m,$$

$$u_1 = v,\ u_2 = 0,\ v_1 = ?$$

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2} \cdot u_1 + \frac{2m_2}{m_1 + m_2} \cdot u_2$$

$$= \frac{m - 4m}{m + 4m} \cdot v + 0 = -\frac{3}{5}v.$$

25. (*a*) This is an example of perfectly inelastic collision in which K.E. is not conserved.

AIIMS Entrance Exam

1. *A boy carrying a box on his head is walking on a level load from one place to another on a straight road is doing no work.*

This statement is

(*a*) correct (*b*) incorrect

(*c*) partly correct (*d*) insufficient data

[AIIMS 99]

2. *Kinetic energy, with any reference, must be*

(*a*) zero (*b*) positive

(*c*) negative (*d*) both (*b*) and (*c*)

[AIIMS 94]

3. If the kinetic energy of a body becomes four times of its initial value, then new momentum will

(*a*) become twice its initial value

(*b*) become thrice its initial value

(*c*) become four times its initial value

(*d*) remain constant

[AIIMS 98]

4. A body of mass 5 kg has momentum of 10 kg m s^{-1}. When a force of 0.2 N is applied on it for 10 s, what is the change in kinetic energy ?

(*a*) 1.1 J (*b*) 2.2 J

(*c*) 3.3 J (*d*) 4.4 J

[AIIMS 2K]

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5. A block of mass 10 kg is moving in x-direction with a constant speed of $10\,m\,s^{-1}$, it is subjected to a retarding force, $F = -0.1x\,Jm^{-1}$ during its travel from $x = 20\,m$ to $x = 30\,m$. Its final kinetic energy will be

(a) 475 J (b) 450 J
(c) 275 J (d) 250 J [AIIMS 05]

6. The decrease in the potential energy of a ball of mass 20 kg, which falls from a height 50 cm is

(a) 968 J (b) 98 J
(c) 1,980 J (d) none of these [AIIMS 97]

7. If the water falls from a dam into a turbine wheel 19.6 m below, then the velocity of water at the turbine (Take $g = 9.8\,ms^{-1}$) is

(a) $9.8\ ms^{-1}$ (b) $19.6\ ms^{-1}$
(c) $39.2\ ms^{-1}$ (d) $98.0\ ms^{-1}$ [AIIMS 98, 07]

8. A gun fires a bullet of mass 50 g with a velocity of $30\,ms^{-1}$. Because of this, the gun is pushed back with a velocity of $1\,ms^{-1}$. The mass of the gun is

(a) 5.5 kg (b) 3.5 kg
(c) 1.5 kg (d) 0.5 kg [AIIMS 01]

9. A body of mass 5 kg is raised vertically to a height of 10 m by a force of 170 N. The velocity of the body at this height will be

(a) $37\,ms^{-1}$ (b) $22\ ms^{-1}$
(c) $15\,ms^{-1}$ (d) $9.8\ ms^{-1}$ [AIIMS 95]

10. A bomb of mass 3.0 kg explodes in air into two pieces of masses 2.0 kg and 1.0 kg. The smaller mass goes at a speed of $80\,ms^{-1}$. The total energy imparted to the two fragments is

(a) 1.07 kJ (b) 2.14 kJ
(c) 2.4 kJ (d) 4.8 kJ [AIIMS 04]

11. If a spring extends by x on loading, then the energy stored by the spring is (if T is tension in the spring and k is spring constant)

(a) $T^2/2x$ (b) $T^2/2k$
(c) $2k/T^2$ (d) $2T^2/k$ [AIIMS 97]

12. A spring 40 mm long is stretched by the application of a force. If 10 N force is required to stretch the spring through 1 mm, then work done in stretching the spring through 40 mm is

(a) 23 J (b) 68 J
(c) 84 J (d) 8 J [AIIMS 98]

13. Which of the following is true ?

(a) Momentum is conserved in all collisions, but kinetic energy is conserved only in inelastic collisions.
(b) Neither momentum nor kinetic energy is conserved in inelastic collisions.
(c) Momentum is conserved in all collisions but not kinetic energy.
(d) Both momentum and kinetic energy are conserved in all collisions. [AIIMS 2K]

14. Two particles are seen to collide and move jointly together after the collision. During such a collision, for the total system,

(a) linear momentum is conserved, but not the mechanical energy
(b) mechanical energy is conserved, but not the linear momentum
(c) both the mechanical energy and the linear momentum are conserved
(d) neither the mechanical energy nor the linear momentum is conserved. [AIIMS 95]

15. A metal ball of mass 2 kg moving with speed of $36\,km\,h^{-1}$ has a head on collision with a stationary ball of mass 3 kg. If after collision, both the balls move together, then the loss in kinetic energy due to collision is

(a) 40 J (b) 60 J
(c) 100 J (d) 140 J [AIIMS 01]

16. A particle of mass m moving with velocity v collides with a stationary particle of mass 2 m. After collision, the speed of the combined particle is

(a) $v/2$ (b) $2v$
(c) $v/3$ (d) $3v$ [AIIMS 99]

17. In elastic collision, 100% energy transfer takes place when

(a) $m_1 = m_2$ (b) $m_1 > m_2$
(c) $m_1 < m_2$ (d) $m_1 = 2m_2$ [AIIMS 01]

18. A ball is bouncing down a flight of stairs. The coefficient of restitution is e. The height of each step is d and the ball descends one step each bounce. After each bounce it rebounds to a height h above the next lower step. The height is large enough compared with the width of step so that the impacts are effectively head-on. Find the relationship between h and d.

(a) $h = \dfrac{d}{1-e^2}$ (b) $h = \dfrac{d}{1+e^2}$
(c) $h = \dfrac{d}{1+e}$ (d) $h = \sqrt{\dfrac{d}{1-e^2}}$ [AIIMS 2009]

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Assertions and Reasons

Directions : In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as

(*a*) If both assertion and reason are true and reason is the correct explanation of the assertion.

(*b*) If both assertion and reason are true but reason is not correct explanation of the assertion.

(*c*) If assertion is true, but reason is false.

(*d*) If both assertion and reason are false.

19. **Assertion.** When a body moves along a circular path, no work is done by the centripetal force.

Reason. The centripetal force is used in moving the body along the circular path and hence no work is done.

20. **Assertion.** Mass and energy are not conserved separately, but are conserved as a single entity called mass-energy.

Reason. Mass and energy are inter-convertible in accordance with Einstein's relation,

$$E = mc^2$$

21. **Assertion.** In an elastic collision of two billiard balls, the total kinetic energy is conserved during the short time of collision of the balls (*i.e.*, when they are in contact).

Reason. Energy spent against friction does not follow the law of conservation of energy. [AIIMS 02]

22. **Assertion.** If momentum of a body increases by 50%, its kinetic energy will increase by 125%

Reason. Kinetic energy is proportional to square of velocity. [AIIMS 2010]

Answers and Explanations

1. (*a*) The body applies force on the load in the upward direction. The displacement of the load is in the horizontal direction.

$$W = F s \cos 90° = 0.$$

2. (*b*) $KE = \frac{1}{2}mv^2$

It is always positive.

3. (*a*) $p = \sqrt{2mK}$

$$p' = \sqrt{2m \times 4K} = 2\sqrt{2mK} = 2p$$

4. (*d*) $u = \frac{p}{m} = \frac{10}{5} = 2 \text{ ms}^{-1}$

$$a = \frac{F}{m} = \frac{02}{5} \text{ ms}^{-2}$$

$$t = 10 \text{ s}$$

$$v = u + at = 2 + \frac{02}{5} \times 10 = 2.4 \text{ ms}^{-1}$$

$$\Delta K = \frac{1}{2}m(v^2 - u^2) = \frac{1}{2} \times 5(2.4^2 - 2^2) = \mathbf{4.4\ J}.$$

5. (*a*) $K_i = \frac{1}{2}mv_i^2 = \frac{1}{2} \times 10 \times (10)^2 = 500 \text{ J}$

$$W = \int F\,dx = -0.1 \int_{20}^{30} x\,dx$$

$$= -0.1\left[\frac{x^2}{2}\right]_{20}^{30} = -\frac{0.1}{2}[30^2 - 20^2] = -25 \text{ J}$$

By work-energy theorem,

$$K_f - K_i = W$$

$$K_f = W + K_i = -25 + 500 = \mathbf{475\ J}.$$

6. (*b*) $\Delta U = mgh = 20 \times 9.8 \times 0.50 = \mathbf{98\ J}.$

7. (*b*) Loss in P.E. = Gain in K.E.

$$mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 19.6}$$

$$= \mathbf{19.6\ ms^{-1}}.$$

8. (*c*) By conservation of momentum,

$$MV = mv$$

$$M = \frac{mv}{V} = \frac{0.05 \times 30}{1} = \mathbf{1.5\ kg}.$$

9. (*b*) W = Gain in P.E. + Gain in K.E.

$$Fh = mgh + \frac{1}{2}mv^2$$

$$170 \times 10 = 5 \times 10 \times 10 + \frac{1}{2} \times 5 \times v^2$$

$$v^2 = \frac{2 \times 1200}{5} = 480$$

$$v = \sqrt{480} = \mathbf{22\ ms^{-1}}.$$

10. (*d*) By conservation of momentum,

$$MV = m_1v_1 + m_2v_2$$

$$3 \times 0 = 2 \times v_1 + 1 \times 80$$

$$v_1 = -40 \text{ ms}^{-1}$$

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Total energy imparted to the two fragements

$$= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$= \frac{1}{2}\times 2\times(40)^2 + \frac{1}{2}\times 1\times(80)^2$$

$$= 4800\ \text{J} = \mathbf{4.8\ kJ}.$$

11. (*b*) Tension,

$$T = kx$$

$$\therefore \quad x = \frac{T}{k}$$

Energy stored in the spring,

$$U = \frac{1}{2}kx^2 = \frac{1}{2}k\left(\frac{T}{k}\right)^2 = \frac{\mathbf{T^2}}{\mathbf{2k}}.$$

12. (*d*) Spring constant,

$$k = \frac{F}{x} = \frac{10\ \text{N}}{1\ \text{mm}} = \frac{10\ \text{N}}{10^{-3}\ \text{m}} = 10^4\ \text{Nm}^{-1}$$

Work done in stretching the spring through 40 m,

$$W = \frac{1}{2}kx^2 = \frac{1}{2}\times 10^4\times(40\times 10^{-3})^2$$

$$= \mathbf{8\ J}.$$

13. (*c*) Momentum is conserved in all collisions but not kinetic energy. Kinetic energy is conserved only in elastic collision.

14. (*a*) The collision is inelastic. Only momentum is conserved and not the mechanical energy. There is loss of K.E. in such a collision.

15. (*b*) Refer to the solution of Problem 33 on page 6.84.

16. (*c*) By conservation of momentum,

$$mv + 2m\times 0 = (m+2m)v'$$

$$\therefore \quad v' = \frac{mv}{3m} = \frac{\mathbf{v}}{\mathbf{3}}.$$

17. (*a*) During elastic collision between two equal masses, the velocities get exchanged. Hence energy transfer is maximum when $m_1 = m_2$.

18. (*a*) The ball falls through distance h from its highest (rest) position and rebounds a distance $(h-d)$

Now $\quad u = \sqrt{2gh}$

$$v = eu = e\sqrt{2gh}$$

$$0^2 - v^2 = 2(-g)(h-d)$$

or $\quad e^2\cdot 2gh = 2g(h-d)$

$$\therefore \quad e^2 = \frac{h-d}{h} \quad \text{or} \quad h = \frac{d}{1-e^2}$$

19. (*c*) The assertion is true but the reason is false. The work done by centripetal force is zero because it acts perpendicular to the circular path.

20. (*a*) Both reason and assertion are true and reason is the correct explanation of the assertion.

21. (*d*) Both assertion and reason are false. The billiard balls in an elastic collision are in a deformed state. Their total energy is partly kinetic and partly potential. So, K.E. is less than the total energy. The energy spent against friction is dissipated as heat which is not available for doing work.

22. (*a*) $\quad p_2 = p_1 + 50\%\ \text{of}\ p_1 = \frac{3}{2}p_1$

$$\therefore \quad v_2 = \frac{3}{2}v_1$$

As $\quad K \propto v^2$, so $K_2 = \frac{9}{4}K_1$

Increase in K.E.

$$= \frac{K_2 - K_1}{K_1}\times 100 = \mathbf{125\ \%}$$

CBSE PMT Prelims and Final Exams

1. A body moves a distance of 10 m along a straight line under the action of a 5 N force. If the work done is 25 J, then angle between the force and direction of motion of the body is

(*a*) 60° (*b*) 75°

(*c*) 30° (*d*) 45° [CBSE PMT 97]

2. When a body moves with a constant speed along a circle

(*a*) no work is done on it

(*b*) no acceleration is produced in it

(*c*) its velocity remains constant

(*d*) no force acts on it.

[CBSE PMT 94]

3. A body, constrained to move in y-direction, is subjected to a force given by $\vec{F} = (-2\hat{i} + 15\hat{j} + 6\hat{k})$ N. The work done by this force in moving the body through a distance of $10\hat{j}$ m along y-axis, is

(*a*) 150 J (*b*) 20 J

(*c*) 190 J (*d*) 160 J [CBSE PMT 94]

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4. If $\vec{F} = (60\hat{i} + 15\hat{j} - 3\hat{k})$ N

and $\vec{v} = (2\hat{i} - 4\hat{j} + 5\hat{k})$ m/s,

then instantaneous power is

(a) 195 watt (b) 45 watt

(c) 75 watt (d) 100 watt [CBSE PMT 2K]

5. A position dependent force, $F = 7 - 2x + 3x^2$ N acts on a small body of mass 2 kg and displaces it from $x = 0$ to $x = 5$ m. The work done in joule is

(a) 135 (b) 270

(c) 35 (d) 70 [CBSE PMT 97]

6. A force acts on a 3 g particle in such a way that the position of the particle as a function of time is given by $x = 3t - 4t^2 + t^3$, where x is in metres and t is in seconds. The work done during the first 4 seconds is

(a) 490 mJ (b) 450 mJ

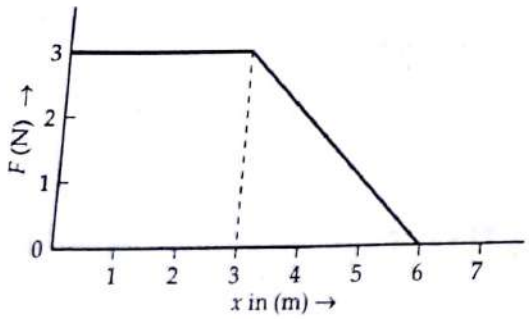
(c) 576 mJ (d) 530 mJ [CBSE PMT 98]

7. A body of mass 3 kg is under a constant force which causes a displacement s in metres in it, given by the relation $s = \frac{1}{3}t^2$, where t is in seconds. Work done by the force in 2 seconds is

(a) $\frac{19}{5}$ J (b) $\frac{5}{19}$ J

(c) $\frac{3}{8}$ J (d) $\frac{8}{3}$ J [CBSE PMT 06]

8. A force F acting on an object varies with distance x as shown here. The force is in N and x in m. The work



done by the force in moving the object from $x = 0$ to $x = 6$ m is

(a) 18.0 J (b) 13.5 J

(c) 9.0 J (d) 4.5 J [CBSE PMT 05]

9. A bomb of mass 30 kg at rest explodes into two pieces of masses 18 kg and 12 kg. The velocity of 18 kg mass is 6 ms^{-1}. The kinetic energy of the other mass is

(a) 324 J (b) 486 J

(c) 256 J (d) 524 J [CBSE PMT 05]

10. A mass of 1 kg is thrown up with a velocity of 100 m/s. After 5 seconds, it explodes into two parts. One part of mass 400 g comes down with a velocity 25 m/s. The velocity of other part is (Take $g = 10$ m/s^2)

(a) 40 m/s ↑ (b) 40 m/s ↓

(c) 100 m/s ↑ (d) 60 m/s ↑ [CBSE PMT 2K]

11. A ball of mass 2 kg and another of mass 4 kg are dropped together from a 60 feet tall building. After a fall of 30 feet each towards earth, their respective kinetic energies will be in the ratio of

(a) $\sqrt{2} : 1$ (b) 1 : 4

(c) 1 : 2 (d) $1 : \sqrt{2}$ [CBSE PMT 1K]

12. A stationary particle explodes into two particles of masses m_1 and m_2 which move in opposite directions with velocities v_1 and v_2. The ratio of their kinetic energies E_1 / E_2 is

(a) m_2 / m_1 (b) m_1 / m_2

(c) 1 (d) $m_1 v_2 / m_2 v_1$ [CBSE PMT 99]

13. A particle of mass m_1 is moving with a velocity v_1 and another particle of mass m_2 is moving with a velocity v_2. Both of them have the same momentum but their different kinetic energies are E_1 and E_2 respectively. If $m_1 > m_2$, then

(a) $E_1 < E_2$ (b) $\frac{E_1}{E_2} = \frac{m_1}{m_2}$

(c) $E_1 > E_2$ (d) $E_1 = E_2$ [CBSE PMT 04]

14. Two bodies of masses m and $4m$ are moving with equal kinetic energies. The ratio of their linear momenta is [CBSE PMT 97, 99; DCE 09]

(a) 1 : 2 (b) 1 : 4

(c) 4 : 1 (d) 1 : 1

15. Two bodies with kinetic energies in the ratio of 4 : 1 are moving with equal linear momentum. The ratio of their masses is

(a) 4 : 1 (b) 1 : 1

(c) 1 : 2 (d) 1 : 4 [CBSE PMT 99]

16. The kinetic energy acquired by a mass m in travelling distance d, starting from rest, under the action of a constant force is directly proportional to

(a) m (b) m^0

(c) $\sqrt{m}$ (d) $1/\sqrt{m}$ [CBSE PMT 94]

17. If kinetic energy of a body is increased by 300%, then percentage change in momentum will be

(a) 100% (b) 150%

(c) 265% (d) 73.2% [CBSE PMT 02]

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18. A particle is projected making an angle of 45° with horizontal having kinetic energy K. The kinetic energy at highest point will be

(a) $K/\sqrt{2}$ (b) $K/2$
(c) $2K$ (d) K [CBSE PMT 92, 01]

19. A particle of mass M is moving in a horizontal circle of radius R with uniform speed v. When it moves from one point to a diametrically opposite point, its

(a) kinetic energy changes by $Mv^2/4$
(b) momentum does not change
(c) momentum changes by $2Mv$
(d) kinetic energy changes by Mv^2 [CBSE PMT 02]

20. A bullet of mass 10 g leaves a rifle at an initial velocity of 1000 m/s and strikes the earth at the same level with a velocity of 500 m/s. The work done in joule overcoming the resistance of air will be

(a) 375 (b) 3750
(c) 5000 (d) 500 [CBSE PMT 89]

21. A child is sitting on a swing. Its minimum and maximum heights from the ground 0.75 m and 2 m respectively, its maximum speed will be

(a) 10 m/s (b) 5 m/s
(c) 8 m/s (d) 15 m/s [CBSE PMT 01]

22. 300 J of work is done in sliding a 2 kg block up an inclined plane of height 10 m. Work done against friction (take $g = 10\,m/s^2$) is

(a) 1000 J (b) 200 J
(c) 100 J (d) zero [CBSE PMT 06]

23. The potential energy of a long spring when stretched by 2 cm is U. If the spring is stretched by 8 cm, the potential energy stored in it is

(a) $U/4$ (b) $4U$
(c) $8U$ (d) $16U$ [CBSE PMT 03, 06]

24. Two springs A and B having spring constants k_A and $k_B = (k_A = 2k_B)$ are stretched by applying force of equal magnitude. If energy stored in spring A is E_A, then energy stored in B will be

(a) $2E_A$ (b) $E_A/4$
(c) $E_A/2$ (d) $4E_A$ [CBSE PMT 01]

25. A mass of 0.5 kg moving with a speed of 1.5 m/s on a horizontal smooth surface, collides with a nearly weightless spring of force constant $k = 50$ N/m. The maximum compression of the spring would be

(a) 0.15 m (b) 0.12 m
(c) 1.5 m (d) 0.5 m [CBSE PMT 04]

26. A vertical spring with force constant k is fixed on a table. A ball of mass m at a height h above the free upper end of the spring falls vertically on the spring so that the spring is compressed by a distance d. The net work done in the process is

(a) $mg(h+d) - \frac{1}{2}kd^2$ (b) $mg(h-d) - \frac{1}{2}kd^2$
(c) $mg(h-d) + \frac{1}{2}kd^2$ (d) $mg(h+d) + \frac{1}{2}kd^2$
[CBSE PMT 07]

27. The potential energy between two atoms in a molecule, is given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are positive constants and x is the distance between the atoms. The atom is in stable equilibrium, when

(a) $x = \left(\frac{2a}{b}\right)^{1/6}$ (b) $x = \left(\frac{11a}{5b}\right)^{1/6}$
(c) $x = 0$ (d) $x = \left(\frac{a}{2b}\right)^{1/6}$ [CBSE PMT 95]

28. Water falls from a height of 60 m at the rate 15 kg/s to operate a turbine. The losses due to frictional forces are 10% of energy. How much power is generated by the turbine ? Take $g = 10\,m/s^2$.

(a) 8.1 kW (b) 10.2 kW
(c) 12.3 kW (d) 7.0 kW
[CBSE PMT 08]

29. How much water a pump of 2 kW can raise in one minute to a height of 10 m ? (take $g = 10\,m/s^2$)

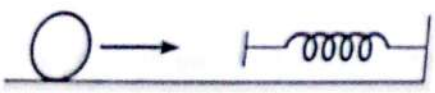
(a) 1000 litres (b) 1200 litres
(c) 10 litres (d) 2000 litres
[CBSE PMT 90]

30. 250 N force is required to raise 75 kg mass from a pulley. If rope is pulled 12 m, then the load is lifted to 3 m, the efficiency of pulley system will be

(a) 25% (b) 33.3%
(c) 75% (d) 90% [CBSE PMT 01]

31. A shell, in flight, explodes into four unequal parts. Which of the following is conserved ?

(a) Potential energy (b) Momentum
(c) Kinetic energy (d) Both (a) and (c)
[CBSE PMT 97]



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32. A moving body of mass m and velocity 3 km/h collides with a rest body of mass 2 m and stricks to it. Now the combined mass starts to move. What will be the combined velocity ?

(*a*) 3 km/hour (*b*) 4 km/hour
(*c*) 1 km/hour (*d*) 2 km/hour [CBSE PMT 96]

33. A metal ball of mass 2 kg moving with speed of 36 km/h has a head on collision with a stationary ball of mass 3 kg. If after collision, both the balls move as a single mass, then the loss in K.E. due to collision is

(*a*) 100 J (*b*) 140 J
(*c*) 40 J (*d*) 60 J [CBSE PMT 97]

34. Two identical balls A and B collide head on elastically. If velocities of A and B, before the collision are +0.5 m/s and −0.3 m/s respectively, then their velocities, after the collision, are respectively

(*a*) −0.5 m/s and +0.3 m/s
(*b*) +0.5 m/s and +0.3 m/s
(*c*) +0.3 m/s and −0.5 m/s
(*d*) −0.3 m/s and +0.5 m/s

[CBSE PMT 91, 94, 98 ; DCE 08]

35. The coefficient of restitution e for a perfectly elastic collision is

(*a*) 1 (*b*) 0
(*c*) ∞ (*d*) −1 [CBSE PMT 88]

36. A body of mass 1 kg is thrown upwards with a velocity of 20 m/s. It momentarily comes to rest after attaining a height of 18 m. How much energy is lost due to air friction ? [Take $g = 10$ m/s^2].

(*a*) 30 J (*b*) 40 J
(*c*) 10 J (*d*) 20 J [CBSE PMT 09]

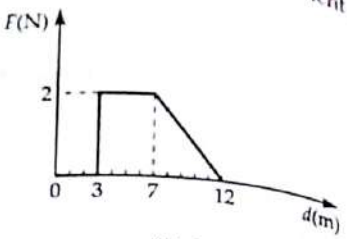
37. An engine pumps water continuously through a hose. Water leaves the hose with a velocity v and mass m is the mass per unit length of the water jet. What is the rate at which kinetic energy is imparted to water ?

(*a*) $\frac{1}{2}m^2v^2$ (*b*) $\frac{1}{2}mv^3$
(*c*) mv^3 (*d*) $\frac{1}{2}mv^2$ [CBSE PMT 09]

38. A block of mass M is attached to the lower end of a vertical spring. The spring is hung from the ceiling and has force constant value k. The mass is released from rest with the spring initially unstretched. The maximum extension produced in the length of the spring will be

(*a*) 2 Mg/k (*b*) 4 Mg/k
(*c*) $Mg/2k$ (*d*) Mg/k

39. Force F on a particle moving in a straight line varies with distance d as shown in the figure. The work done on the particle during its displacement of 12 m is

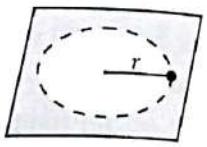


(*a*) 18 J (*b*) 21 J
(*c*) 26 J (*d*) 13 J [CBSE Pre 2011]

40. The potential energy of a system increases if work is done

(*a*) upon the system by a non-conservative force
(*b*) by the system against a conservative force
(*c*) by the system against a non-conservative force
(*d*) upon the system by a conservative force [CBSE Pre 2011]

41. A small mass attached to string rotates on a frictionless table top as shown. If the tension in the



string is increased by pulling the string causing the radius of the circular motion to decrease by a factor of 2, the kinetic energy of the mass will

(*a*) increase by a factor of 4
(*b*) decrease by a factor of 2
(*c*) remain constant
(*d*) increase by a factor of 2 [CBSE Final 2011]

42. A body projected vertically from the earth reaches a height equal to earth's radius before returning to the earth. The power exerted by the gravitational force is greatest

(*a*) at the highest position of the body
(*b*) at the instant just before the body hits the earth
(*c*) it remains constant all through
(*d*) at the instant just after the body is projected [CBSE Pre 2011]

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43. An engine pumps water through a hose pipe. Water passes through the pipe and leaves it with a velocity of 2 m/s. The mass per unit length of water in the pipe is 100 kg/m. What is the power of the engine?

(a) 400 W (b) 200 W
(c) 100 W (d) 800 W [CBSE Pre 2010]

44. A ball moving with velocity 2 m/s collides head on with another stationary ball of double the mass. If the coefficient of restitution is 0.5, then their velocities (in m/s) after collision will be

(a) 0, 1 (b) 1, 1
(c) 1, 0.5 (d) 0, 2 [CBSE Pre 2010]

Answers and Explanations

1. (a) $W = F \cdot s\cos\theta$

$25 = 5 \times 10 \times \cos\theta$

$\cos\theta = 0.5$ or $\theta = 60°$.

2. (a) For a body in uniform circular motion, the centripetal force acts perpendicular to the circular path.

3. (a) $W = \vec{F} \cdot \vec{s}$

$= (-2\hat{i} + 15\hat{j} + 6\hat{k}) \cdot 10\hat{j}$

$= 15 \times 10 = \mathbf{150\ J}$.

4. (b) $P = \vec{F} \cdot \vec{v}$

$= (60\hat{i} + 15\hat{j} - 3\hat{k}) \cdot (2\hat{i} - 4\hat{j} + 5\hat{k})$

$= 60 \times 2 + 15 \times (-4) - 3 \times 5 = \mathbf{45\ W}$.

5. (a) $W = \int F\,dx = \int_0^5 (7 - 2x + 3x^2)dx$

$= [7x - x^2 + x^3]_0^5$

$= 35 - 25 + 125 = \mathbf{135\ J}$.

6. (c) $x = 3t - 4t^2 + t^3$

$v = \frac{dx}{dt} = 3 - 8t + 3t^2$

$a = \frac{dv}{dt} = -8 + 6t$

At $t = 4$ s,

$a = -8 + 6 \times 4 = 16\ \text{ms}^{-2}$

$x = 12 - 64 + 64 = 12$ m

Now $m = 3\text{g} = 3 \times 10^{-3}$ kg

$W = Fx = max = 3 \times 10^{-3} \times 16 \times 12$

$= 576 \times 10^{-3}\ \text{J} = \mathbf{576\ mJ}$.

7. (d) $s = \frac{1}{3}t^2$

$v = \frac{ds}{dt} = \frac{2}{3}t$

$a = \frac{dv}{dt} = \frac{2}{3}\ \text{ms}^{-2}$

At $t = 2$ s, $s = \frac{1}{3} \times 2^2 = \frac{4}{3}$ m

$W = Fs = mas = 3 \times \frac{2}{3} \times \frac{4}{3} = \frac{8}{3}$ J.

8. (b) W = Area of $F - x$ trapezium

$= \frac{1}{2}(6 + 3) \times 3 = \mathbf{13.5\ J}$.

9. (b) By conservation of momentum,

$30 \times 0 = 18 \times 6 + 12 \times v$

$v = -\frac{108}{12} = -9\ \text{ms}^{-1}$.

K.E. of 12 kg mass $= \frac{1}{2} \times 12 \times 81 = 486$ J.

10. (c) Velocity of 1 kg mass after 5 s

$v = u - gt = 100 - 10 \times 5 = 50\ \text{ms}^{-1}$

By conservation of momentum,

$1 \times 50 = 0.4 \times (-25) + 0.6 \times v'$

$v' = \frac{60}{0.6} = \mathbf{100\ ms^{-1}}$, **upwards**.

11. (c) Both balls attain same velocity v after falling through 30 feet height.

$\therefore \quad \frac{E_1}{E_2} = \frac{\frac{1}{2}m_1v_1^2}{\frac{1}{2}m_2v_2^2} = \frac{2v^2}{4v^2} = \frac{1}{2} = \mathbf{1 : 2}$.

12. (a) By conservation of momentum,

$m_1v_1 = m_2v_2$

$\therefore \quad \frac{E_1}{E_2} = \frac{\frac{1}{2}m_1v_1^2}{\frac{1}{2}m_2v_2^2}$

$= \frac{m_1^2v_1^2}{m_2^2v_2^2} \cdot \frac{m_2}{m_1} = \frac{\boldsymbol{m_2}}{\boldsymbol{m_1}}$.

13. (a) Kinetic energy, $= \frac{p^2}{2m}$

For same p, $E \propto \frac{1}{m}$ $\therefore \quad \frac{E_1}{E_2} = \frac{m_2}{m_1}$

As $m_1 > m_2$

So, $E_1 < E_2$.

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14. (*a*) $p = \sqrt{2mE}$

For same K.E. $p \propto m$

$$\frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{m}{4m}} = 1 : 2.$$

15. (*d*) For same p,

$$E \propto \frac{1}{m} \quad \text{or} \quad m \propto \frac{1}{E}$$

$$\therefore \quad \frac{m_1}{m_2} = \frac{E_2}{E_1} = \frac{1}{4} = 1 : 4.$$

16. (*b*) $v^2 - u^2 = 2as$

$$v^2 - 0^2 = 2 \times \frac{F}{m} \times d = \frac{2Fd}{m}$$

$$\text{K.E.} = \frac{1}{2}mv^2 = \frac{1}{2}m \times 2\frac{Fd}{m} = Fd$$

Hence, K.E. does not depend on m or it is directly proportional to m^0.

17. (*a*) Refer to the solution of Example 22 on page 6.11.

18. (*b*) Initial kinetic energy, $K = \frac{1}{2}mu^2$

Velocity at the highest point

= Horizontal component of u

$$= u\cos 45^\circ = \frac{u}{\sqrt{2}}$$

Hence K.E. at the highest point,

$$K' = \frac{1}{2}m\left(\frac{u}{\sqrt{2}}\right)^2 = \frac{1}{2} \cdot \frac{1}{2}mu^2 = \frac{K}{2}.$$

19. (*c*) At any two dimetrically opposite points, velocities of the particle have same magnitude but opposite directions.

$\therefore$ Change in momentum $= Mv - (-Mv) = 2Mv$.

20. (*b*) W = Decrease in K.E.

$$= \frac{1}{2} \times \frac{10}{1000} \times (1000^2 - 500^2)$$

$$= \mathbf{3750\ J}.$$

21. (*b*) Maximum K.E. = Drop in P.E.

$$\frac{1}{2}mv_{max}^2 = mg(h_2 - h_1)$$

$$v_{max} = \sqrt{2g(h_2 - h_1)}$$

$$= \sqrt{2 \times 10(2 - 075)} = \mathbf{5\ ms^{-1}}.$$

22. (*c*) Total work done

= Gain in P.E. + Work done against friction

$$300 = 2 \times 10 \times 10 + W$$

$$W = 300 - 200 = \mathbf{100\ J}.$$

23. (*d*) $U = \frac{1}{2}kx^2$

$$\therefore \quad \frac{U_2}{U_1} = \left(\frac{x_2}{x_1}\right)^2 = \left(\frac{8}{2}\right)^2 = 16$$

$$U_2 = 16U_1 = \mathbf{16\ U}.$$

24. (*a*) $E = \frac{1}{2}kx^2 = \frac{1}{2}\frac{F^2}{k}$

For same E, $F \propto \frac{1}{k}$

$$\therefore \quad \frac{F_B}{F_A} = \frac{k_A}{k_B} = \frac{2k_B}{k_B} = 2 \quad \text{or} \quad F_B = 2F_A.$$

25. (*a*) At the maximum compression,

K.E. of moving mass = P.E. stored in spring

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$$x = \sqrt{\frac{mv^2}{k}} = \sqrt{\frac{0.5 \times 1.5 \times 1.5}{50}} = 0.15\ \text{m}.$$

26. (*a*) Net work done

= P.E. lost by the ball

– Energy stored in compressed spring

$$= mg(h + d) - \frac{1}{2}kd^2$$

27. (*a*) $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$

For stable equilibrium, U must be minimum. Then

$$\frac{dU}{dx} = 0$$

$$-\frac{12a}{x^{13}} + \frac{6b}{x^7} = 0 \quad \text{or} \quad x^6 = \frac{2a}{b}$$

or $$x = \left(\frac{2a}{b}\right)^{1/6}.$$

28. (*a*) $P = \frac{W}{t} = \frac{mgh}{t} = \frac{15 \times 10 \times 60}{1} = 9000\ \text{W} = 9\ \text{kW}$

Loss = 10%

Useful power = 90% of 9 kW $= \frac{90 \times 9}{100} = \mathbf{8.1\ kW}$.

29. (*b*) $P = \frac{W}{t} = \frac{mgh}{t}$

$$\therefore \quad m = \frac{Pt}{gh} = \frac{2000 \times 60}{10 \times 10} = 1200\ \text{kg}$$

As 1 litre of water weighs 1 kg, so volume of water raised = **1200 litres**.

30. (*c*) Load, $W = mg = 75 \times 10 = 750$ N

Effort, $P = 250$ N

$$\text{Mechanical advantage} = \frac{\text{Load}}{\text{Effort}} = \frac{750}{250} = 3$$

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$$\text{Velocity ratio} = \frac{\text{Distance travelled by effort}}{\text{Distance travelled by load}} = \frac{12}{3} = 4.$$

$$\text{Efficiency, } \eta = \frac{\text{Mechanical advantage}}{\text{Velocity ratio}} = \frac{3}{4} \times 100 = 75\%.$$

31. (*b*) Linear momentum is conserved in the absence of external force.

32. (*c*) By conservation of momentum,

$$m \times 3 + 2m \times 0 = (m + 2m)v$$

$$v = \mathbf{1\ km/\ hour}$$

33. (*d*) 36 km/h $= 36 \times \frac{5}{18} = 10$ m/s

By conservation of momentum,

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

$$2 \times 10 + 3 \times 0 = (2 + 3)v$$

$$v = \frac{20}{5} = 4\ \text{ms}^{-1}.$$

Loss in K.E.

$$= \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - \frac{1}{2}(m_1 + m_2)v^2$$

$$= \frac{1}{2} \times 2 \times 10^2 + \frac{1}{2} \times 3 \times (0)^2 - \frac{1}{2}(2 + 3)4^2$$

$$= 100 - 40 = \mathbf{60\ J}.$$

34. (*d*) In an elastic collision between two bodies of equal masses, velocities get exchanged after the collision.

$\therefore$ $v_A = \mathbf{-0.3\ m/s}$, $v_B = \mathbf{+0.5\ m/s}$.

35. (*a*) For a perfectly elastic collision,

$$|v_1 - v_2| = |u_1 - u_2| \quad \therefore \quad e = \frac{|v_1 - v_2|}{|u_1 - u_2|} = \mathbf{1}.$$

36. (*d*) Loss of energy = Initial K.E. – Final P.E.

$$= \frac{1}{2}mv^2 - mgh = \frac{1}{2} \times 1 \times (20)^2 - 1 \times 10 \times 18$$

$$= 200 - 180 = \mathbf{20\ J}.$$

37. (*b*) $K = \frac{1}{2}mv^2$

For constant v,

$$\frac{dK}{dt} = \frac{1}{2}v^2\frac{dm}{dt}$$

$$= \frac{1}{2}v^2\frac{dm}{dx} \cdot \frac{dx}{dt} = \frac{1}{2}v^2 \cdot mv = \frac{1}{2}mv^3$$

$$\left[\frac{dm}{dx} = m = \text{mass per unit length}\right]$$

38. (*a*) Loss of gravitational P.E.

= Gain of spring P.E.

$$Mgx = \frac{1}{2}kx^2 \quad \therefore \quad x = \frac{2Mg}{k}.$$

39. (*d*) Work done = Area under F- d graph

$$= 0 + 4 \times 2 + \frac{1}{2} \times 5 \times 2 = 8 + 5 = \mathbf{13\ J}$$

40. (*b*) The work done by a conservative force is equal to the negative of the potential energy.

$$W_{\text{conservative}} = -\Delta U \quad \Rightarrow \quad \Delta U > 0 \text{ if } W < 0$$

$\therefore$ The potential energy of the system increases if work is done by the system against a conservative force.

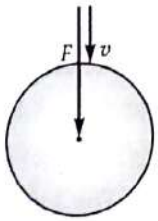
41. (*c*) If T is the tension and R the radius, then

$$K_1 = \frac{1}{2}T_1R_1 \text{ and } K_2 = \frac{1}{2}T_2R_2$$

$$\therefore \quad \frac{K_2}{K_1} = \frac{T_2}{T_1} \cdot \frac{R_2}{R_1}, \quad R_2 = \frac{R_1}{2}, \quad T_2 = 2T_1$$

$$\Rightarrow \quad \frac{K_2}{K_1} = 1$$

42. (*b*) $P = Fv$, both F and v are maximum at the instant just before the body hits the earth.

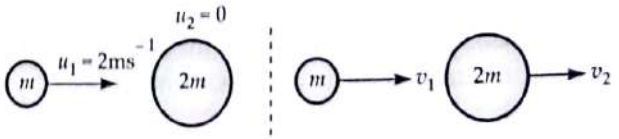


43. (*d*) $F = \frac{dp}{dt} = \frac{d}{dt}(mv)$

$$= v\frac{dm}{dt} = v\frac{dm}{dl} \cdot \frac{dl}{dt} = v\frac{dm}{dl} \cdot v = v^2\frac{dm}{dl}$$

$$\therefore \quad P = Fv = v^3\frac{dm}{dl} = (2)^3 \times 100 = \mathbf{800\ W}$$

44. (*a*)



$$m \times 2 + 2m \times 0 = mv_1 + 2mv_2$$

$$\Rightarrow \quad v_1 + 2v_2 = 2 \qquad \ldots(i)$$

$$e = \frac{v_2 - v_1}{u_1 - u_2} \quad \text{or} \quad 0.5 = \frac{v_2 - v_1}{2 - 0}$$

$$\Rightarrow \quad v_2 - v_1 = 1 \qquad \ldots(ii)$$

Adding (*i*) and (*ii*),

$$3v_2 = 3 \quad \text{or} \quad v_2 = \mathbf{1\ ms^{-1}}$$

$$\therefore \quad v_1 = \mathbf{0}$$

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COMPETITION

Delhi PMT and VMMC Entrance Exam

1. When a body moves with constant speed in a circular path, then
   (*a*) work done will be zero
   (*b*) acceleration will be zero
   (*c*) no force acts on the body
   (*d*) its velocity remains constant [VMMC 06]

2. A force $\vec{F} = 3\hat{i} + c\hat{j} + 2\hat{k}$ acting on a particle causes a displacement $\vec{s} = -4\hat{i} + 2\hat{j} + 3\hat{k}$ in its own direction. If the work done is 6 J, then value of c is
   (*a*) 0 (*b*) 6
   (*c*) 1 (*d*) 12 [DPMT 97]

3. A body of mass 2 kg is placed on rough horizontal plane. The coefficient of friction between body and plane is 0.2 Then
   (*a*) body will move in forward direction if $F = 5$ N
   (*b*) body will move in backward direction with acceleration 0.5 m / s^2, if force $F = 3$ N
   (*c*) If $F = 3$ N, then body will be in rest condition
   (*d*) both (*a*) and (*c*) are correct [VMMC 06]

4. The kinetic energy of body of mass 2 kg and momentum of 2 N is
   (*a*) 1 J (*b*) 3 J
   (*c*) 2 J (*d*) 4 J [DPMT 97]

5. If momentum is increased by 20%, then kinetic energy increases by
   (*a*) 48% (*b*) 40%
   (*c*) 44% (*d*) 35% [DPMT 96]

6. Kinetic energy of particles of mass 10 g and 40 g is same, the ratio of their linear momentum is
   (*a*) 1/4 (*b*) 1/2
   (*c*) $1/\sqrt{2}$ (*d*) $\sqrt{2}/1$ [DPMT 07]

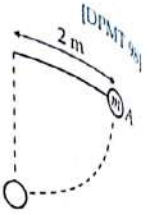
7. If the potential energy of two molecules is given by $U = \frac{A}{r^6} - \frac{B}{r^{12}}$, then at equilibrium position, its potential energy is equal to
   (*a*) $\frac{A^2}{4B}$ (*b*) $\frac{B^2}{4A}$
   (*c*) $\frac{2B}{A}$ (*d*) $-\frac{B^2}{4A}$ [VMMC 02]

(Similar Questions)

8. The potential energy of a particle of mass 5 kg moving in the x-y plane is given by $U = (-7x + 24y)$ J. x and y being in metre. If the particle starts from rest from origin, then speed of particle at $t = 2$ s is
   (*a*) 5 m/s (*b*) 14 m/s
   (*c*) 17.5 m/s (*d*) 10 m/s [VMMC 06]

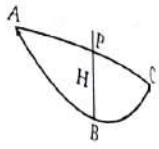
9. A simple pendulum hanging freely and at rest is vertical because in that position
   (*a*) kinetic energy is zero
   (*b*) potential energy is zero
   (*c*) kinetic energy is minimum
   (*d*) potential energy is minimum

10. A body m of mass 1 kg is dropped from position A. If 10% of its energy is lost as heat, what will be its velocity at B ? [DPMT 96]



   (*a*) 6 ms^{-1}
   (*b*) 5.5 ms^{-1}
   (*c*) 6.32 ms^{-1} (*d*) 5.6 ms^{-1}

11. A simple pendulum, with a bob of mass m, oscillates from A to C and back to A such that PB is H. If the acceleration due to gravity is g, then the velocity of the bob as it passes through B is [DPMT 05]



   (*a*) zero (*b*) mgH
   (*c*) $2gH$ (*d*) $\sqrt{2gH}$

12. 300 J of work is done in sliding a 2 kg block up an inclined plane of height 10 m. Taking $g = 10$ m/s^2, the work done against friction is [DPMT 95]
   (*a*) 200 J (*b*) 100 J
   (*c*) zero (*d*) 1000 J

13. Two springs of spring constant 1500 N/m and 3000 N/m respectively are stretched with a same force. Their potential energies will be in the ratio of [VMMC 07]
   (*a*) 4 : 1 (*b*) 2 : 1
   (*c*) 1 : 4 (*d*) 1 : 2

14. A block of mass m falls through a height h on a spring of spring constant k. The maximum displacement in the spring is [DPMT 94]

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(b) $\frac{mg}{k}$

(c) $\sqrt{\frac{2mgh}{k}}$

(d) none of these [DPMT 06]

15. A 15 g ball is shot from a spring gun whose spring has a force constant of 600 N/m. The spring is compressed by 5 cm. The greatest possible horizontal range of the ball for this compression ($g = 10\,m/s^2$) is

(a) 6.0 m (b) 10.0 m

(c) 12.0 m (d) 8.0 m. [DPMT 04]

16. An electric motor exerts a force of 40 N on a cable and pulls it by distance of 30 m in one minute. The power supplied by the motor (in watt) is

(a) 10 (b) 2

(c) 200 (d) 20 [VMMC 03]

17. A particle moves with a velocity $(5\hat{i}-3\hat{j}+6\hat{k})$ m/s under the influence of a constant force $\vec{F}=(10\hat{i}+10\hat{j}+20\hat{k})$ N. The instantaneous power applied to the particle is

(a) $200\,Js^{-1}$ (b) $40\,Js^{-1}$

(c) $140\,Js^{-1}$ (d) $170\,Js^{-1}$ [VMMC 06]

18. A bullet is fired and gets embedded in a block kept on table. If table is frictionless, then

(a) kinetic energy gets conserved

(b) potential energy gets conserved

(c) momentum gets conserved

(d) both (a) and (c) [DPMT 97]

19. A body of mass 5 kg, moving with velocity 10 m/sec collides with another body of the mass 20 kg at rest and comes to rest. The velocity of the second body due to collision is

(a) 2.5 m/sec (b) 7.5 m/sec

(c) 5 m/sec (d) 10 m/sec [DPMT 93]

20. A 10 kg ball moving with velocity 2 m/s collides with a 20 kg mass initially at rest. If both of them coalesce, the final velocity of combined mass is

(a) $\frac{3}{4}$ m/s (b) $\frac{1}{3}$ m/s

(c) $\frac{3}{2}$ m/s (d) $\frac{2}{3}$ m/s [VMMC 05]

21. A body P moving with a velocity of 20 m/sec collides with another body Q of same mass at rest. If after collision P comes to rest, then what is the velocity of body Q ?

(a) 10 m/sec (b) 30 m/sec

(c) 20 m/sec (d) 40 m/sec [DPMT 93]

22. A body of mass $4m$ at rest explodes into three pieces. Two of the pieces each of mass m move with a speed v each in mutually perpendicular directions. The total kinetic energy released is

(a) $\frac{1}{2}mv^2$ (b) mv^2

(c) $\frac{3}{2}mv^2$ (d) $\frac{5}{2}mv^2$ [VMMC 03]

23. A particle of mass m having velocity v moving towards north collides with similar particle moving with same velocity towards east. The two particles stick together and move towards north east with a velocity

(a) $\sqrt{2}\,v$ (b) $v/\sqrt{2}$

(c) $v/2$ (d) $2v$ [DPMT 02]

24. A ball is dropped from height h on the ground where coefficient of restitution is e. After one bounce the maximum height is

(a) e^2h (b) $e\sqrt{h}$

(c) eh (d) $\sqrt{eh}$ [DPMT 07]

25. A body of mass 10 kg initially at rest acquires velocity of 10 ms^{-1}. What is the work done ?

(a) – 500 J (b) 500 J

(c) 50 J (d) – 50 J [DPMT 08]

26. A ball moves in a frictionless inclined table without slipping. The work done by the table surface on the ball is

(a) positive (b) negative

(c) zero (d) none of these. [DPMT 09]

27. The work done by an applied variable force $F = x + x^3$ from $x = 0$ m to $x = 2$ m, where x is displacement, is

(a) 6 J (b) 8 J

(c) 10 J (d) 12 J [DPMT 2011]

28. A particle of mass m_1 moves with velocity v_1 and collides with another particle at rest of equal mass. The velocity of the second particle after the elastic collision is

(a) $2v_1$ (b) v_1

(c) $-v_1$ (d) 0 [DPMT 2011]

29. The coefficient of restitute, e, for a perfectly elastic collision is

(a) 0 (b) –1

(c) 1 (d) ∞ [DPMT 2011]

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Answers and Explanations

1. *(a)* The body is in uniform circular motion. The centripetal force acts perpendicular to the circular path.

2. *(b)* $W = \vec{F} . \vec{s}$

$$6 = (3\hat{i} + c\hat{j} + 2\hat{k}) . (-4\hat{i} + 2\hat{j} + 3\hat{k})$$

$$6 = -12 + 2c + 6$$

$$c = 6.$$

3. *(d)* Limiting friction,

$$f = \mu R = \mu mg$$

$$= 0.2 \times 2 \times 9.8 = 3.92 \text{ N}$$

Option *(a)* is correct because $F > 3.92$ N and body will move in forward direction.

When $F = 3$N, the body will remain at rest. So, option *(c)* is correct.

Hence the correct option is *(d)*.

4. *(a)* $K = \dfrac{p^2}{2m} = \dfrac{2^2}{2 \times 2} = 1$ J.

5. *(c)* Refer to the solution of Example 21 on page 6.11.

6. *(b)* $p = \sqrt{2mK}$, For same K, $p \propto \sqrt{m}$

$$\therefore \quad \frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{10}{40}} = \frac{1}{2}.$$

7. *(a)* $U = \dfrac{A}{r^6} - \dfrac{B}{r^{12}}$

At equilibrium,

$$F = -\frac{dU}{dr} = 0$$

For $\dfrac{dU}{dr} = 0$

$$\frac{-6A}{r^7} + \frac{12B}{r^{13}} = 0$$

or $$-A + \frac{2B}{r^6} = 0$$

or $$r^6 = \frac{2B}{A}$$

At $r^6 = \dfrac{2B}{A}$,

$$U_0 = \frac{A}{2B/A} - \frac{B}{(2B/A)^2}$$

$$= \frac{A^2}{2B} - \frac{A^2}{4B} = \frac{A^2}{4B}.$$

8. *(d)* $U = (-7x + 24y)$ J

$$\vec{F} = -\frac{dU}{dx}\hat{i} - \frac{dU}{dy}\hat{j} = 7\hat{i} - 24\hat{j}$$

$$a_x = \frac{F_x}{m} = \frac{7}{5} = 1.4 \text{ ms}^{-2},$$

along positive x-axis

$$a_y = \frac{F_y}{m} = \frac{-24}{5} = -4.8 \text{ ms}^{-2},$$

along negative y-axis

$$v_x = a_x t = 1.4 \times 2 = 2.8 \text{ ms}^{-1}$$

$$v_y = a_y t = 4.8 \times 2 = 9.6 \text{ ms}^{-1}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{2.8^2 + 9.6^2}$$

$$= 10 \text{ ms}^{-1}.$$

9. *(d)* In the position of equilibrium, the potential energy of the simple pendulum is minimum.

10. *(a)* Refer to the solution of Problem 12 on page 6.75.

11. *(d)* By conservation of energy,

$$\frac{1}{2}mv^2 = mgH$$

$$\therefore \quad v = \sqrt{2gH}$$

12. *(b)* Total work done

= Work done against friction + Increase in P.E.

$$300 \text{ J} = W + 2 \times 10 \times 10$$

$$W = 300 - 200 = \mathbf{100 \text{ J}}.$$

13. *(b)* $U = \dfrac{1}{2}kx^2 = \dfrac{1}{2}k\left(\dfrac{F}{k}\right)^2 = \dfrac{F^2}{2k}$

For constant F,

$$U \propto \frac{1}{k}$$

$$\therefore \quad \frac{U_1}{U_2} = \frac{k_2}{k_1} = \frac{3000}{1500} = \mathbf{2 : 1}.$$

14. *(d)* If the spring is compressed by the block through length x, then

$$mg(h + x) = \frac{1}{2}kx^2$$

On solving, $x = \dfrac{1}{2}\left[\dfrac{2mg}{k} \pm \sqrt{\dfrac{2mg^2}{k} + \dfrac{8mgh}{k}}\right]$

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15. (a) $\mathcal{K}_{max} = \frac{u^2}{g} = \frac{1}{2}mu^2 \times \frac{2}{mg}$

but $\frac{1}{2}mu^2 = \frac{1}{2}kx^2$

$\mathcal{K}_{max} = \frac{1}{2}kx^2 \times \frac{2}{mg} = \frac{kx^2}{mg}$

$\therefore \quad = \frac{600 \times (0.05)^2}{0.015 \times 10} = 10 \text{ m}.$

16. (b) $P = \frac{W}{t} = \frac{Fs}{t} = \frac{40 \times 30}{60} = 2 \text{ W}.$

17. (c) $P = \vec{F} \cdot \vec{v}$

$= (10\hat{i} + 10\hat{j} + 20\hat{k}) \cdot (5\hat{i} - 3\hat{j} + 6\hat{k})$

$= 10 \times 5 + 10 \times (-3) + 20 \times 6$

$= 50 - 30 + 120 = 140 \text{ J}.$

18. (c) Only momentum is conserved. Some kinetic energy is lost when bullet penetrates the block.

19. (a) By conservation of momentum,

$5 \times 10 + 20 \times 0 = 5 \times 0 + 20 \times v$

$\therefore \quad v = \frac{50}{20} = 2.5 \text{ ms}^{-1}.$

20. (d) By conservation of momentum,

$10 \times 2 + 20 \times 0 = (10 + 20) \times v$

$v = \frac{20}{30} = \frac{2}{3} \text{ ms}^{-1}.$

21. (c) By conservation of momentum,

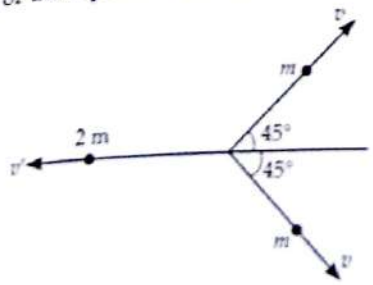
$m \times 20 + m \times 0 = m \times 0 + m \times v$

$\therefore \quad v = 20 \text{ ms}^{-1}.$

22. (c) Total mass $= m$

Mass of each of the two small pieces $= m$

Mass of third piece $= 4m - 2m = 2m$



By conservation of momentum along horizontal direction,

$2mv' = mv\cos 45° + mv\cos 45°$

$= 2mv\frac{1}{\sqrt{2}}$

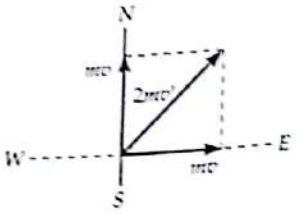
$\therefore \quad v' = \frac{v}{\sqrt{2}}$

Total kinetic energy released

$= \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2} \times 2m \times \left(\frac{v}{\sqrt{2}}\right)^2$

$= \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = \frac{3}{2}mv^2.$

23. (b)



Resultant momentum,

$2mv' = \sqrt{(mv)^2 + (mv)^2} = mv\sqrt{2}$

$v' = \frac{v}{\sqrt{2}}.$

24. (a) The velocity with which the ball strikes the ground,

$u = \sqrt{2gh}$

If the ball rebounces with velocity v, then

$e = \frac{v}{u}$

or $\quad v = eu = e\sqrt{2gh}$

If h' is the maximum height after one bounce, then

$0^2 - v^2 = 2(-g)h'$

or $\quad h' = \frac{v^2}{2g} = \frac{1}{2g} \cdot e^2 \cdot 2gh = e^2h.$

25. (b) W = Gain in K.E. $= \frac{1}{2}mv^2 = \frac{1}{2} \times 10 \times (10)^2$

$= 500 \text{ J}.$

26. (c) Motion without slipping implies pure rolling. During pure rolling work done by friction force is zero.

27. (a) $W = \int F\,dx = \int_0^2 (x + x^3)\,dx = \left[\frac{x^2}{2} + \frac{x^4}{4}\right]_0^2$

$= 6 \text{ J}$

28. (c) When two bodies of equal masses collide, velocities get exchanged after the collision.

29. (c) For a perfectly elastic collision, $e = 1$

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CHAPTER 7

SYSTEMS OF PARTICLES & ROTATIONAL MOTION

7.1 INTRODUCTORY CONCEPTS

1. *What is meant by a particle, a system and internal and external forces ?*

Particle. *A particle is defined as an object whose mass is finite but whose size and internal structure can be neglected.*

System. *A system is a collection of a very large number of particles which mutually interact with one another.* A body of finite size can be regarded as a system because it is composed of a large number of particles interacting with one another.

Internal forces. *The mutual forces exerted by the particles of a system on one another are called internal forces.* These forces are responsible for holding together the particles as a single object.

External forces. *The outside force exerted on an object by any external agency is called an external force. Such a force changes the velocity of an object.*

7.2 CENTRE OF MASS

2. *What do you mean by centre of mass of a system ? How does it differ from centre of gravity ?*

Centre of mass. Newton's laws of motion are applicable to point objects. The introduction of the concept of centre of mass enables us to apply them equally well to the motion of finite or extended objects. *The centre of mass of a body is a point where the whole mass of the body is supposed to be concentrated for describing its translatory motion.*

*The **centre of mass** of a system of particles is that single point which moves in the same way in which a single particle having the total mass of the system and acted upon by the same external force would move.*

If a single force acts on a body and the line of action of the force passes through the centre of mass, the body will have only linear acceleration and no angular acceleration. For example, consider a hammer resting

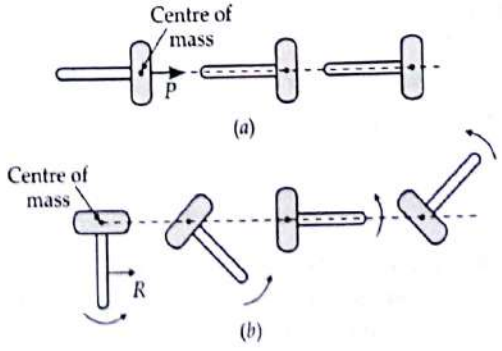

Fig. 7.1 Concept of centre of mass.

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on a plane surface. If a force P is applied on the hammer in such a way that its line of action passes through the centre of mass of the hammer, then the hammer moves along a straight line path, as shown in Fig. 7.1(*a*). But when a force R is applied along a line not passing through its centre of mass, then the hammer rotates about its centre of mass, as shown in Fig. 7.1(*b*).

Centre of mass vs. centre of gravity. The centre of mass of body is point where whole mass of the body may be assumed to be concentrated for describing its translatory motion. On the other hand, the centre of gravity is a point at which the resultant of the gravitational forces on all the particles of the body acts *i.e.*, a point where whole weight may be assumed to act. In a uniform gravitational field such as that of the earth on a small body, the centre of gravity coincides with the centre of mass. But in the case of Mount Everest, the centre of gravity lies a little below its centre of mass because the gravitational force decreases with altitude.

7.3 CENTRE OF MASS OF A TWO-PARTICLE SYSTEM

3. *Write an expression for the location of centre of mass of a two particle system. Discuss the result.*

Centre of mass of a two particle-system. Consider a system of two particles P_1 and P_2 of masses m_1 and m_2. Let $\vec{r}_1$ and $\vec{r}_2$ be their position vectors with respect to the origin O, as shown in Fig. 7.2.

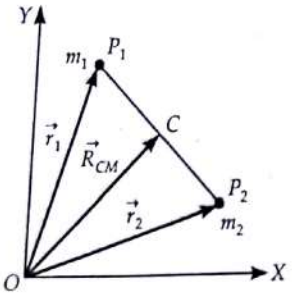

Fig. 7.2 Centre of mass of a two-particle system.

The position vector $\vec{R}_{CM}$ of the centre of mass C of the two-particle system is given by

$$\vec{R}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

Discussion. (*i*) The above equation shows that *the position vector of a system of particles is the weighted average of the position vectors of the particles making the system*, each particle making a contribution proportional to its mass.

(*ii*) We can write the above equation as

$$(m_1 + m_2)\,\vec{R}_{CM} = m_1\vec{r}_1 + m_2\vec{r}_2$$

Thus *the product of the total mass of the system and the position vector of its centre of mass is equal to the sum of the products of individual masses and their respective position vectors.*

(*iii*) If $m_1 = m_2 = m$ (say), then

$$\vec{R}_{CM} = \frac{\vec{r}_1 + \vec{r}_2}{2}$$

Thus *the centre of mass of two equal masses lies exactly at the centre of the line joining the two masses.*

(*iv*) If (x_1, y_1) and (x_2, y_2) are the coordinates of the locations of the two particles, the coordinates of their centre of mass are given by

$$x_{CM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

and $$y_{CM} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2}.$$

7.4 CENTRE OF MASS OF A TWO PARTICLE SYSTEM FROM ab-initio

4. *Derive an expression for the centre of mass of a two particle system from ab-initio.*

Derivation of expression for the centre of mass of a two-particle system. Consider a system of two particles P_1 and P_2 of masses m_1 and m_2. Let $\vec{r}_1$ and $\vec{r}_2$ be their position vectors at any instant t with respect to the origin O, as shown in Fig. 7.3.

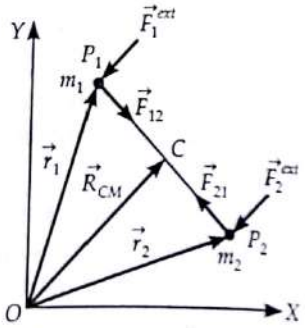

Fig. 7.3 Equations of motion of a two-particle system.

The velocity and acceleration vectors of the two particles are

$$\vec{v}_1 = \frac{d\vec{r}_1}{dt} \quad \text{and} \quad \vec{a}_1 = \frac{d\vec{v}_1}{dt} = \frac{d^2\vec{r}_1}{dt^2}$$

$$\vec{v}_2 = \frac{d\vec{r}_2}{dt} \quad \text{and} \quad \vec{a}_2 = \frac{d\vec{v}_2}{dt} = \frac{d^2\vec{r}_2}{dt^2} \quad \ldots(1)$$

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Total force $\vec{F}_1$ acting on particle P_1 is the sum of the *internal force* $\vec{F}_{12}$ due to P_2 and external force $\vec{F}_1^{ext}$ on it. Thus

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_1^{ext}$$

Similarly, total force acting on particle P_2 is given by

$$\vec{F}_2 = \vec{F}_{21} + \vec{F}_2^{ext}$$

According to Newton's second law of motion, the equations of motion for the two particles can be written as

$$m_1 \vec{a}_1 = \vec{F}_1 = \vec{F}_{12} + \vec{F}_1^{ext} \quad ...(2)$$

$$m_2 \vec{a}_2 = \vec{F}_2 = \vec{F}_{21} + \vec{F}_2^{ext} \quad ...(3)$$

On adding equations (2) and (3), we get

$$m_1 \vec{a}_1 + m_2 \vec{a}_2 = \vec{F}_{12} + \vec{F}_{21} + \vec{F}_1^{ext} + \vec{F}_2^{ext}$$

According to Newton's third law, the internal forces mutually exerted by the two particles are equal and opposite, *i.e.*,

$$\vec{F}_{12} = -\vec{F}_{21} \quad \text{or} \quad \vec{F}_{12} + \vec{F}_{21} = 0$$

$$\therefore \quad m_1 \vec{a}_1 + m_2 \vec{a}_2 = \vec{F} \quad ...(4)$$

where $\vec{F} = \vec{F}_1^{ext} + \vec{F}_2^{ext}$ is the total external force acting on the system.

Suppose the total mass of the two-particle system is M. Then

$$M = m_1 + m_2$$

Let us assume that the total external force $\vec{F}$ acting on the system of mass M produces acceleration $\vec{a}_{CM}$. Then according to Newton's second law,

$$M\vec{a}_{CM} = \vec{F} \quad ...(5)$$

From equations (4) and (5), we get

$$M\vec{a}_{CM} = m_1 \vec{a}_1 + m_2 \vec{a}_2$$

$$= m_1 \frac{d^2 \vec{r}_1}{dt^2} + m_2 \frac{d^2 \vec{r}_2}{dt^2} \quad \text{[Using (1)]}$$

$$= \frac{d^2}{dt^2}(m_1 \vec{r}_1 + m_2 \vec{r}_2)$$

or $$\vec{a}_{CM} = \frac{1}{M}\frac{d^2}{dt^2}(m_1 \vec{r}_1 + m_2 \vec{r}_2)$$

or $$\frac{d^2 \vec{R}_{CM}}{dt^2} = \frac{d^2}{dt^2}\left(\frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}\right) \quad [\because M = m_1 + m_2]$$

The acceleration $\vec{a}_{CM}$ is called the acceleration vector of the centre of mass of the system and $\vec{R}_{CM}$ is called the position vector of the centre of mass.

Clearly, $$\vec{R}_{CM} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}. \quad ...(6)$$

This equation defines the position of the centre of mass of a system of two particles of masses m_1 and m_2 and having position vectors $\vec{r}_1$ and $\vec{r}_2$. Here the entire mass of the two-particle system may be supposed to be concentrated. Clearly, Newton's second law, as applied to the individual particles of the system, also holds for the entire system provided the external force acts at the centre of mass as defined by the above equation. Newton's third law helps us to get rid of the mutual internal forces between the particles. Hence *while applying Newton's second law to the motion of the centre of mass, we need to consider only the external forces acting on the system.*

7.5 CENTRE OF MASS OF *n*-PARTICLE SYSTEM

5. *Write an expression for the position vector of the centre of mass of n-particle system. Also write the equations of motion which govern the motion of the centre of mass.*

Centre of mass of *n*-particle system. Consider a system of n particles having masses $m_1, m_2, m_3, ..., m_n$ and position vectors $\vec{r}_1, \vec{r}_2, \vec{r}_3, ... \vec{r}_n$ relative to the origin O, as shown in Fig. 7.4.

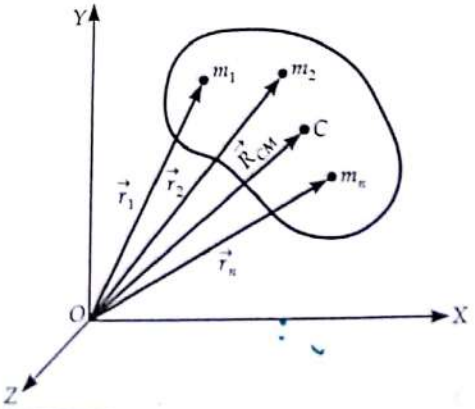

Fig. 7.4 Centre of mass of *n*-particle system.

The total mass of the system is

$$m_1 + m_2 + m_3 + ... + m_n = M \quad \text{(say)}$$

The position vector $\vec{R}_{CM}$ of the centre of mass C can be obtained by adding the products $m_1 \vec{r}_1, m_2 \vec{r}_2, ..., m_n \vec{r}_n$ and dividing it by the total mass of the system.

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Thus

$$\vec{R}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + ... + m_n\vec{r}_n}{m_1 + m_2 + ... + m_n}$$

or $$\vec{R}_{CM} = \frac{\sum_{i=1}^{n} m_i\vec{r}_i}{\sum_{i=1}^{n} m_i} = \frac{\sum_{i=1}^{n} m_i\vec{r}_i}{M} \quad ...(1)$$

Clearly, $\vec{R}_{CM}$ *is the weighted average of the position vectors of all the particles of the system, the contribution of each particle being proportional to its mass.*

Cartesian coordinates of the centre of mass. If x_{CM}, y_{CM} and z_{CM} are the cartesian coordinates of the centre of mass of the n-particle system, then

$$x_{CM} = \frac{m_1x_1 + m_2x_2 + ... + m_nx_n}{m_1 + m_2 + ... + m_n} = \frac{\Sigma mx}{M} \quad ...2(i)$$

$$y_{CM} = \frac{m_1y_1 + m_2y_2 + ... + m_ny_n}{m_1 + m_2 + ... + m_n} = \frac{\Sigma my}{M} \quad ...2(ii)$$

$$z_{CM} = \frac{m_1z_1 + m_2z_2 + ... + m_nz_n}{m_1 + m_2 + ... + m_n} = \frac{\Sigma mz}{M} \quad ...2(iii)$$

Equations of motion for the centre of mass. Let $\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$, ..., $\vec{F}_n$ be the external forces acting on the particles of masses m_1, m_2, m_3, ..., m_n respectively. Let $\vec{F}_{tot}$ be the vector sum of all these external forces acting on the system. If $\vec{a}_{CM}$ is the acceleration of the centre of mass of the system, then the motion of the centre of mass is governed by the equation

$$M\vec{a}_{CM} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ... + \vec{F}_n$$

or $$M\vec{a}_{CM} = \vec{F}_{tot} \quad ...(3)$$

where $$\vec{a}_{CM} = \frac{d^2\vec{R}_{CM}}{dt^2}$$

The equation (3) shows that *the centre of mass of the system moves as if the entire mass of the system is concentrated at this point and the total external force acts on this point.* The internal forces between various particles cancel out in pairs in accordance with Newton's third law of motion. The definition of centre of mass given by equation (1) holds even though there may not be any actual matter present at the centre of mass.

Note In case of a body with a continuous mass distribution, we can replace the summations in equations (2) by the following integrals :

$$\Sigma mx \to \int x\,dm$$

$$\Sigma my \to \int y\,dm$$

$$\Sigma mz \to \int z\,dm$$

Then the coordinates of the centre of mass of a body of mass M will be

$$x_{CM} = \frac{1}{M}\int x\,dm,$$

$$y_{CM} = \frac{1}{M}\int y\,dm,$$

$$z_{CM} = \frac{1}{M}\int z\,dm$$

The equivalent vector representation for the centre of mass will be

$$\vec{R}_{CM} = \frac{1}{M}\int \vec{r}\,dm$$

If we choose, the centre of mass as the origin of our coordinate system, then

$$\vec{R}_{CM}(x, y, z) = 0$$

or $$\int \vec{r}\,dm = 0$$

or $$\int x\,dm = \int y\,dm = \int z\,dm = 0$$

7.6 MOMENTUM CONSERVATION AND CENTRE OF MASS MOTION

6. *Show that in the absence of any external force, the velocity of the centre of mass remains constant.*

Velocity of *CM* is constant in the absence of external force. Suppose an external force $\vec{F}_{tot}$ acts on a system of mass M and produces an acceleration $\vec{a}_{CM}$ in its centre of mass. Then

$$\vec{F}_{tot} = M\vec{a}_{CM}$$

In the absence of any external force, $\vec{F}_{tot} = 0$, so

so $$M\vec{a}_{CM} = 0$$

or $$\vec{a}_{CM} = 0$$

or $$\frac{d\vec{v}_{CM}}{dt} = 0$$

As the derivative of a constant is zero, so

$$\vec{v}_{CM} = \text{constant}$$

where $\vec{v}_{CM}$ is the velocity of the centre of mass. Hence *in the absence of any external force, the centre of mass of system moves with a uniform velocity.* This is Newton's first law of motion.

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The position vector of the centre of mass at any instant t is given by

$$\vec{R}_{CM}(t) = \vec{R}_{CM}(0) + \vec{v}_{CM}\, t$$

1. Show that the total linear momentum of a system of particles is conserved in the absence of any external force. Also show that total linear momentum of the system is equal to the product of the total mass of the system and the velocity of its centre of mass.

Momentum conservation and centre-of-mass motion. Consider a system of n particles of masses $m_1, m_2, m_3, \ldots, m_n$. Suppose the forces $\vec{f}_1, \vec{f}_2, \vec{f}_3, \ldots, \vec{f}_n$ exerted on them produce accelerations $\vec{a}_1, \vec{a}_2, \vec{a}_3, \ldots, \vec{a}_n$ respectively. In the absence of any external force,

$$\vec{F}_{tot} = 0$$

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots + \vec{F}_n = 0$$

or $$m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + \ldots + m_n\vec{a}_n = 0$$

or $$m_1\frac{d\vec{v}_1}{dt} + m_2\frac{d\vec{v}_2}{dt} + m_3\frac{d\vec{v}_3}{dt} + \ldots + m_n\frac{d\vec{v}_n}{dt} = 0$$

or $$\frac{d}{dt}(m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \ldots + m_n\vec{v}_n) = 0$$

or $$m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \ldots + m_n\vec{v}_n = \text{constant}$$

or $$\vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \ldots + \vec{p}_n = \text{constant}$$

where $\vec{P}$ is the total linear momentum of the system.

Hence *if no net external force acts on a system, the total linear momentum of the system is conserved. This is the law of conservation of linear momentum.*

Now the position vector of the centre of mass of n-particle system is given by

$$\vec{R}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \ldots + m_n\vec{r}_n}{m_1 + m_2 + m_3 + \ldots + m_n}$$

or $$\vec{R}_{CM} = \frac{1}{M}(m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \ldots + m_n\vec{r}_n)$$

Differentiating both sides w.r.t. time t, we get

$$\frac{d\vec{R}_{CM}}{dt} = \frac{1}{M}\left(m_1\frac{d\vec{r}_1}{dt} + m_2\frac{d\vec{r}_2}{dt} + m_3\frac{d\vec{r}_3}{dt} + \ldots + m_n\frac{d\vec{r}_n}{dt}\right)$$

$$= \frac{1}{M}(m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \ldots + m_n\vec{v}_n)$$

$$= \frac{1}{M}(\vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \ldots + \vec{p}_n)$$

or $$\vec{v}_{CM} = \frac{1}{M}\vec{P} \quad \text{or} \quad \vec{P} = M\vec{v}_{CM}$$

This equation shows that the *total linear momentum of a system of particles is equal to the product of the total mass of the system and the velocity of its centre of mass.*

Examples based on Centre of Mass

Formulae Used

1. For a system of N particles, the position vector of centre mass is

$$\vec{R}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots + m_N\vec{r}_N}{m_1 + m_2 + \ldots + m_N} = \frac{\sum_{i=1}^{N} m_i\vec{r}_i}{M}$$

2. The position vector of the centre mass of a two particle system is

$$\vec{R}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

3. The Cartesian co-ordinates of the centre of mass are given by

$$x = \frac{m_1x_1 + m_2x_2 + \ldots + m_Nx_N}{m_1 + m_2 + \ldots + m_N} = \frac{\sum_{i=1}^{N} m_ix_i}{M}$$

$$y = \frac{m_1y_1 + m_2y_2 + \ldots + m_Ny_N}{m_1 + m_2 + \ldots + m_N} = \frac{\sum_{i=1}^{N} m_iy_i}{M}$$

$$z = \frac{m_1z_1 + m_2z_2 + \ldots + m_Nz_N}{m_1 + m_2 + \ldots + m_N} = \frac{\sum_{i=1}^{N} m_iz_i}{M}$$

4. For a continuous mass distribution,

$$\vec{R}_{CM} = \frac{1}{M}\int \vec{r}\, dm$$

where dm is the mass of small element located at position $\vec{r}$.

Also $$x_{CM} = \frac{1}{M}\int x\,dm, \quad y_{CM} = \frac{1}{M}\int y\,dm,$$

$$z_{CM} = \frac{1}{M}\int z\,dm$$

5. The algebraic sum of the moments of masses of various particles of a system about its centre of mass is zero.

6. Velocity of CM of a two particle system is

$$\vec{v}_{CM} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$$

Units Used

Masses of various particles of a system are in kg and their distances from the axis of rotation are in metre.

EXAMPLE 1. Find the centre of mass of a triangular lamina. **[NCERT]**

Solution. As shown in Fig. 7.5, divide the lamina (ΔLMN) into narrow strips, each parallel to the base MN.

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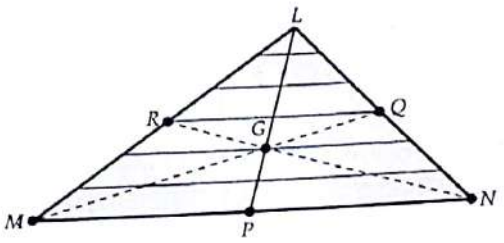

Fig. 7.5

By symmetry, the CM of each strip lies at its mid-point. By joining these midpoints, we get the median *LP*. The CM of triangular lamina must lie on the median *LP*.

By similar arguments as above, we can say that the CM of lamina (ΔLMN) lies on the medians *MQ* and *NR*. This means that CM lies on the point of concurrence of the medians *i.e.*, on the centroid *G* of the triangle.

EXAMPLE 2. Three masses 3, 4 and 5 kg are located at the corners of an equilateral triangle of side 1 m. Locate the centre of mass of the system.

Solution. Suppose the equilateral triangle lies in the *XY*-plane with mass 3 kg at the origin. Let (x, y) be the co-ordinates of CM.

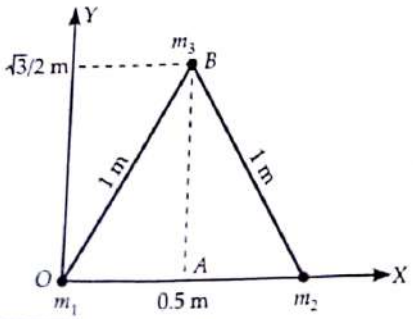

Fig. 7.6

Clearly, $AB = \sqrt{OB^2 - OA^2} = \sqrt{1^2 - \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2}$ m

Now $x_1 = 0,\ x_2 = 1$ m, $x_3 = OA = 0.5$ m

$m_1 = 3$ kg, $m_2 = 4$ kg, $m_3 = 5$ kg

$$\therefore \quad x = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$= \frac{3 \times 0 + 4 \times 1 + 5 \times 0.5}{3 + 4 + 5} = \frac{6.5}{12} = \mathbf{0.54\ m.}$$

Again, $y_1 = 0,\ y_2 = 0,\ y_3 = AB = \frac{\sqrt{3}}{2}$

$$\therefore \quad y = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

$$= \frac{3 \times 0 + 4 \times 0 + 5 \times (\sqrt{3}/2)}{3 + 4 + 5} = \frac{5 \times \sqrt{3}}{2 \times 12} = \mathbf{0.36\ m.}$$

Thus the co-ordinates of CM are (0.54 m, 0.36 m).

EXAMPLE 3. Two particles of masses 100 g and 300 g at a given time have positions $2\hat{i} + 5\hat{j} + 13\hat{k}$ and $-6\hat{i} + 4\hat{j} - 2\hat{k}$ m respectively and velocities $10\hat{i} - 7\hat{j} - 3\hat{k}$ and $7\hat{i} - 9\hat{j} + 6\hat{k}$ ms^{-1} respectively. Determine the instantaneous position and velocity of CM.

Solution. Here $m_1 = 100$ g $= 0.1$ kg,

$m_2 = 300$ g $= 0.3$ kg,

$\vec{r}_1 = 2\hat{i} + 5\hat{j} + 13\hat{k}$ m,

$\vec{r}_2 = -6\hat{i} + 4\hat{j} - 2\hat{k}$ m

The position of CM will be

$$\vec{R}_{CM} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$$

$$= \frac{0.1(2\hat{i} + 5\hat{j} + 13\hat{k}) + 0.3(-6\hat{i} + 4\hat{j} - 2\hat{k})}{0.1 + 0.3}$$

$$= \frac{-16\hat{i} + 17\hat{j} + 7\hat{k}}{4} \text{ m}$$

Again, $\vec{v}_1 = 10\hat{i} - 7\hat{j} - 3\hat{k}$ ms^{-1},

$\vec{v}_2 = 7\hat{i} - 9\hat{j} + 6\hat{k}$ ms^{-1}

The velocity of CM will be

$$\vec{v}_{CM} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2}$$

$$= \frac{0.1(10\hat{i} - 7\hat{j} - 3\hat{k}) + 0.3(7\hat{i} - 9\hat{j} + 6\hat{k})}{0.1 + 0.3}$$

$$= \frac{31\hat{i} - 34\hat{j} + 15\hat{k}}{4} \ ms^{-1}.$$

EXAMPLE 4. If three point masses m_1, m_2 and m_3 are situated at the vertices of an equilateral triangle of side a, then what will be the co-ordinates of the centre of mass of this system?

Solution. Let the point mass m_1 lie at the origin of the co-ordinate system, as shown in Fig. 7.7.

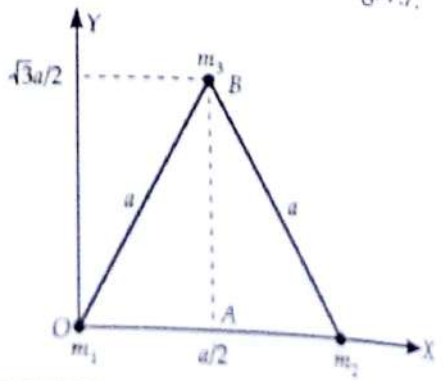

Fig. 7.7

Then
Co-ordinates of m_1 : $x_1 = 0, y_1 = 0$
Co-ordinates of m_2 : $x_2 = a, y_2 = 0$
Co-ordinates of m_3 : $x_3 = \frac{a}{2}, y_3 = \frac{\sqrt{3}}{2}a$

$$x_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$= \frac{m_1 \times 0 + m_2 a + m_3 (a/2)}{m_1 + m_2 + m_3}$$

$$= \frac{m_2 a + m_3 (a/2)}{m_1 + m_2 + m_3}$$

$$y_{CM} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

and

$$= \frac{m_1 \times 0 + m_2 \times 0 + m_3 (\sqrt{3}\, a/2)}{m_1 + m_2 + m_3}$$

$$= \frac{m_3 \sqrt{3}\, a}{2 (m_1 + m_2 + m_3)}.$$

EXAMPLE 5. Find the position of the centre of mass of the T-shaped plate from O in Fig. 7.8.

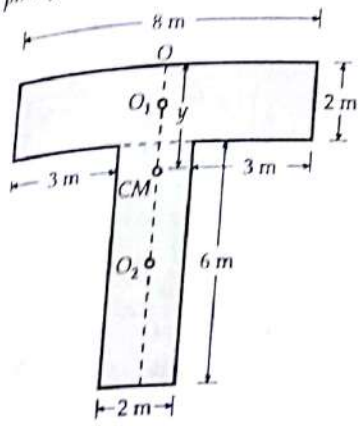

Fig. 7.8

Solution. Let mass per unit area of the plate be σ.

Mass of horizontal portion $= 8 \times 2\ \sigma = 16\ \sigma$

Mass of vertical portion $= 6 \times 2\ \sigma = 12\ \sigma$

The centres O_1 and O_2 of these portions lie at distances 1 m and 2 + 3 = 5 m from the point O. The CM of the T-shaped plate will lie at distance y from the point O which is given by

$$y = \frac{16\ \sigma \times 1 + 12\ \sigma \times 5}{16\ \sigma + 12\ \sigma} = \frac{76\ \sigma}{28\ \sigma} = \mathbf{2.71\ m.}$$

EXAMPLE 6. Find the centre of mass of a uniform L-shaped lamina (a thin flat plate) with dimensions as shown in Fig. 7.9. The mass of lamina is 3 kg. **[NCERT]**

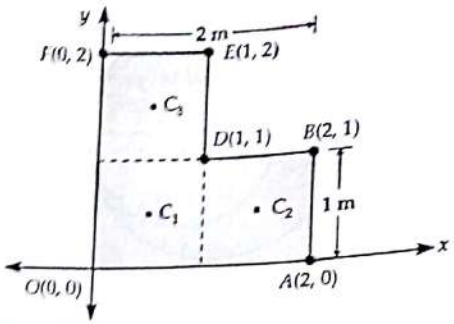

Fig. 7.9

Solution. We divide the L-shaped lamina into three squares each of length 1 m, as shown in Fig. 7.9. As the lamina is uniform, the mass of each square is 1 kg. By symmetry, the centres of mass of the three squares lie at their geometric centres C_1, C_2 and C_3. Their coordinates are as follows :

	C_1	C_2	C_3
x	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{1}{2}$
y	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$

The coordinates of the centre of mass of the L-shaped lamina can be obtained as follows :

$$x_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$= \frac{1(1/2) + 1(3/2) + 1(1/2)}{1 + 1 + 1} = \frac{5}{6}\ \text{m}$$

$$y_{CM} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

$$= \frac{1 \times (1/2) + 1(1/2) + 1(3/2)}{1 + 1 + 1} = \frac{5}{6}\ \text{m}$$

Clearly, the centre of mass lies on the line of symmetry OD.

EXAMPLE 7. A circular plate of uniform thickness has a diameter of 56 cm. A circular portion of diameter 42 cm is removed from one edge of the plate. Find CM of the remaining portion.

Solution. The situation is shown in Fig. 7.10. Let O be CM of the original circular plate, O_1 that of the circular portion removed and O_2 that of the remaining shaded portion. Let m be the mass per unit area of the plate.

Mass of original plate,

$$M = \pi (28)^2\, m$$

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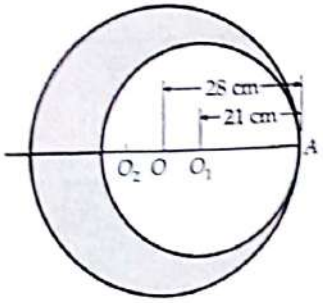

Fig. 7.10

Mass of the circular portion removed,

$$m_1 = \pi (21)^2 \, m$$

Mass of the shaded part,

$$m_2 = M - m_1 = \pi [(28)^2 - (21)^2] \, m = 343 \, \pi \, m$$

Masses m_1 and m_2 may be assumed to be concentrated at O_1 and O_2 respectively and O is their CM.

$\therefore$ Moment of m_1 about O = Moment of m_2 about O

or $$m_1 \times O_1O = m_2 \times O_2 O$$

or $$O_2O = \frac{m_1}{m_2} \times O_1O$$

$$= \frac{441 \, \pi \, m}{343 \, \pi \, m} \times (28 - 21) = 9 \text{ cm.}$$

EXAMPLE 8. A square of side 4 cm and uniform thickness is divided into four equal squares, as shown in Fig. 7.11. If one of the squares is cut off, find the position of the centre of mass of the remaining portion from O.

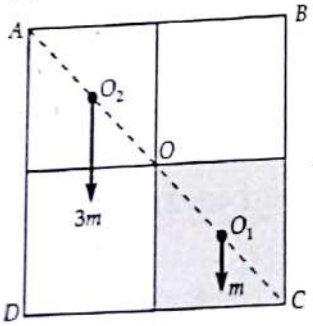

Fig. 7.11

Solution. Let mass of each small square be m. Total mass of the square will be $4m$ which acts at its centre of mass O. Let O_1 be CM of cut off square (shaded square of mass m) and O_2 be CM of the remaining unshaded portion of mass $3m$.

Now $$AC = \sqrt{AB^2 + BC^2} = \sqrt{4^2 + 4^2} = 4\sqrt{2} \text{ cm}$$

$$OC = \frac{1}{2} AC = \frac{4\sqrt{2}}{2} = 2\sqrt{2} \text{ cm}$$

$$OO_1 = \frac{1}{2} OC = \frac{2\sqrt{2}}{2} = \sqrt{2} \text{ cm}$$

Moment of unshaded portion about O = Moment of shaded portion about O

$$3m \times OO_2 = m \times OO_1$$

$$OO_2 = \frac{1}{3} \times OO_1 = \frac{1}{3} \times \sqrt{2} = \frac{\sqrt{2}}{3} \text{ cm.}$$

EXAMPLE 9. Show that the centre of mass of a uniform rod of mass M and length L lies at the middle point of the rod.

Solution. As shown in Fig. 7.12, suppose the rod is placed along X-axis with its left end at the origin O. Consider a small element of thickness dx at distance x from O.

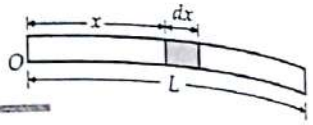

Fig. 7.12

Mass of the small element $= \frac{M}{L} \cdot dx$

Position of the centre of mass is given by

$$x_{CM} = \frac{1}{M} \int_0^L x \, dm$$

$$= \frac{1}{M} \int_0^L x \cdot \frac{M}{L} \cdot dx = \frac{1}{L} \int_0^L x \, dx$$

$$= \frac{1}{L} \left[\frac{x^2}{2} \right]_0^L = \frac{L}{2}.$$

EXAMPLE 10. Determine the position of the centre of mass of a hemisphere of radius R.

Solution. Let ρ be the density of the material of the hemisphere. Take its centre O as the origin. The hemisphere can be assumed to be made of up a large number of co-axial discs. Consider one such elementary disc of radius y and thickness dx at a distance x from the origin.

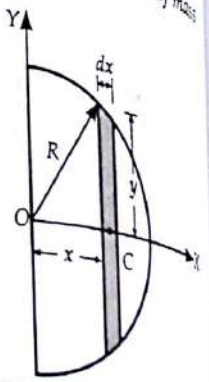

Fig. 7.13

Mass of the elementary disc = Volume × density

$$dm = \pi y^2 \, dx \times \rho = \pi (R^2 - x^2) \, dx \, . \rho$$

The coordinates of the centre of mass of the hemisphere can be determined as follows :

$$x_{CM} = \frac{1}{M} \int x \, dm = \frac{1}{M} \int_0^R x \, \pi (R^2 - x^2) \rho \, dx$$

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$$= \frac{\pi\rho}{M}\int_0^R (R^2 x - x^3)\,dx = \frac{\pi\rho}{M}\left[R^2\frac{x^2}{2} - \frac{x^4}{4}\right]_0^R$$

$$= \frac{\pi\rho}{M}\left[\frac{R^4}{2} - \frac{R^4}{4}\right] = \frac{\pi\rho}{M}\left[\frac{R^4}{4}\right]$$

$$= \frac{\pi\rho}{\frac{2}{3}\pi R^3\rho}\left(\frac{R^4}{4}\right) = \frac{3}{8}R \qquad \left[\because M = \frac{2}{3}\pi R^3 \times \rho\right]$$

Similarly,

$$y_{CM} = \int y\,dm = 0 \text{ and } z_{CM} = \int z\,dm = 0$$

Hence the coordinates of the centre of mass of the hemisphere are $\left(\frac{3}{8}R, 0, 0\right)$

EXAMPLE 11. Determine the coordinates of the centre of mass of a right circular solid cone of base radius R and height h.

Solution. Let ρ be the density of the material of the cone. Take centre of base O as the origin. The cone can be assumed to be made up of a large number of circular discs of different radii and mass. Consider one such elementary disc of radius x and mass dm at a height y from the base.

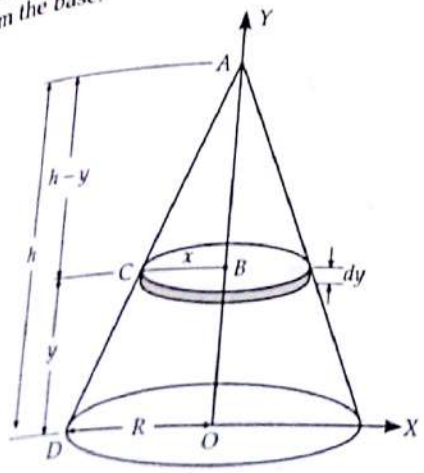

Fig. 7.14

In similar triangles AOD and ABC, we have

$$\frac{h}{h-y} = \frac{R}{x} \text{ or } x = \frac{R(h-y)}{h}$$

Mass of elementary disc is

$$dm = \text{Volume} \times \text{density}$$

$$= \pi x^2\,dy \times \rho = \pi\rho\,\frac{R^2(h-y)^2}{h^2}\,dy$$

The coordinates of the centre of mass can be determined as follows :

$$x_{CM} = \frac{1}{M}\int x\,dm = 0 \qquad [\because x = 0]$$

$$y_{CM} = \frac{1}{M}\int y\,dm = \frac{1}{M}\int_0^h y \cdot \pi\rho\,\frac{R^2(h-y)^2}{h^2}\,dy$$

$$= \frac{\pi\rho R^2}{Mh^2}\int_0^h y(h-y)^2\,dy$$

$$= \frac{\pi\rho R^2}{Mh^2}\int_0^h y(h^2 + y^2 - 2hy)\,dy$$

$$= \frac{\pi\rho R^2}{Mh^2}\int_0^h (h^2 y + y^3 - 2h y^2)\,dy$$

$$= \frac{\pi\rho R^2}{Mh^2}\left[h^2\left|\frac{y^2}{2}\right|_0^h + \left|\frac{y^4}{4}\right|_0^h - 2h\left|\frac{y^3}{3}\right|_0^h\right]$$

$$= \frac{\pi\rho R^2}{Mh^2}\left[\frac{h^4}{2} + \frac{h^4}{4} - \frac{2h^4}{3}\right] = \frac{\pi\rho R^2}{Mh^2}\left[\frac{h^4}{12}\right]$$

$$= \frac{\pi\rho R^2}{\frac{1}{3}\pi R^2 h\rho \times h^2} \times \frac{h^4}{12} = \frac{h}{4} \qquad \left[\because M = \frac{1}{3}\pi r^2 h\rho\right]$$

Similarly,

$$z_{CM} = \frac{1}{m}\int z\,dm = 0 \qquad [\because z = 0]$$

Hence the coordinates of the centre of mass of the right circular solid cone are $\left(0, \frac{h}{4}, 0\right)$.

PROBLEMS FOR PRACTICE

1. Two bodies of masses 1 kg and 2 kg are located at (1, 2) and (− 1, 3) respectively. Calculate the co-ordinates of the centre of mass. **[Central Schools 04, 08]**

   $\left[\text{Ans.}\left(-\frac{1}{3}, \frac{8}{3}\right)\right]$

2. The distance between the centres of carbon and oxygen atoms in the carbon monoxide gas molecule is 1.13 Å. Locate the centre of mass of the gas molecule relative to the carbon atom. **[Central Schools 11]**

   (Ans. 0.6457 Å from C-atom)

3. Three blocks of uniform thickness and masses m, m and 2 m are placed at three corners of a triangle having co-ordinates (2.5, 1.5), (3.5, 1.5) and (3, 3) respectively. Find the centre of mass of the system.

   [Delhi 06] **[Ans.** (3, 2.25)]

4. Find the centre of mass of three particles at the vertices of an equilateral triangle. The masses of the particles are 100 g, 150 g and 200 g respectively. Each side of the equilateral triangle is 0.5 m long. **[NCERT]**

   (Ans. $\frac{5}{18}$ m, $\frac{1}{3\sqrt{3}}$ m)

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5. Three particles each of mass m are placed at three corners of an equilateral triangle of length l. Find the position of centre of mass in terms of coordinates. [Chandigarh 02]

[Ans. $\left(\frac{l}{2}, \frac{l}{2\sqrt{3}}\right)$]

6. Three point masses of 1 kg, 2 kg and 3 kg lie at (1, 2), (0, −1) and (2, −3) respectively. Calculate the coordinates of the centre of mass of the system. [Central Schools 12]

[Ans. $\left(\frac{7}{6}, \frac{-3}{2}\right)$]

7. Two bodies of masses 10 kg and 2 kg are moving with velocities $2\hat{i} - 7\hat{j} + 3\hat{k}$ and $-10\hat{i} + 35\hat{j} - 3\hat{k}$ ms^{-1} respectively. Find the velocity of the centre of mass of the system. (Ans. $2\hat{k}$ ms^{-1})

8. Three particles of masses 0.50 kg, 1.0 kg and 1.5 kg are placed at the corners of a right angle triangle, as shown in Fig. 7.15. Locate the centre of mass of the system. (Ans. 1.33 cm, 1.5 cm)

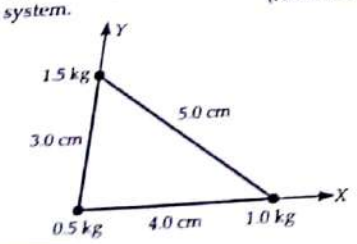


Fig. 7.15

9. Four particles of masses m, m, $2m$ and $2m$ are placed at the four corners of a square of side a. Find the centre of mass of the system.

[Ans. $\left(\frac{a}{2}, \frac{2}{3}a\right)$ with first mass m at the origin]

10. Four particles of masses m, $2m$, $3m$ and $4m$ respectively are placed at the corners of a square of side a, as shown in Fig. 7.16. Locate the centre of mass.

[Ans. $\left(\frac{a}{2}, \frac{7a}{10}\right)$]

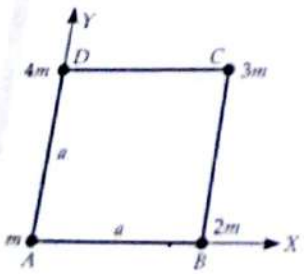


Fig. 7.16

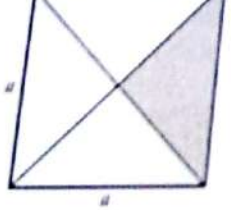


Fig. 7.17

11. From a square sheet of uniform density, a portion is removed shown shaded in Fig. 7.17. Find the centre of mass of the remaining portion if the side of the square is a.

[Ans. $\left(\frac{7}{12}a, \frac{7}{12}a\right)$]

12. The centre of mass of three particles of masses 1 kg, 2 kg and 3 kg lies at the point (3 m, 3 m, 3 m). Where should a fourth particle of mass 4 kg be positioned so that the centre of mass of the four particle system lies at the point (1 m, 1 m, 1 m) ?

[Ans. (−2 m, −2 m, −2 m)]

HINTS

2. $x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{12 \times 0 + 16 \times 1.13}{12 + 16} = 0.646 \text{ m}$

3. $x_{CM} = \frac{m(2.5) + m(3.5) + 2m(3)}{m + m + 2m} = 3$

$y_{CM} = \frac{m(1.5) + m(1.5) + 2m(3)}{m + m + 2m} = 2.25.$

4. The position coordinates for the three particles are shown in Fig. 7.18.

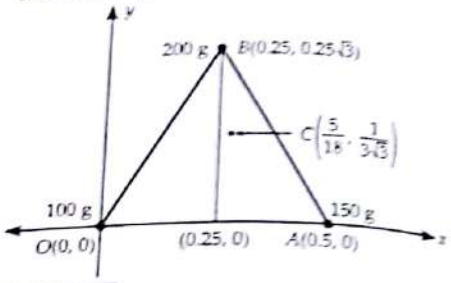


Fig. 7.18

$x_{CM} = \frac{100(0) + 150(0.5) + 200(0.25)}{100 + 150 + 200} = \frac{5}{16} \text{ m}$

$y_{CM} = \frac{100(0) + 150(0) + 200(0.25\sqrt{3})}{100 + 150 + 200} = \frac{1}{3\sqrt{3}} \text{ m}$

5. $x_{CM} = \frac{m(0) + m(l) + m(l/2)}{m + m + m} = \frac{l}{2}$.

$y_{CM} = \frac{m(0) + m(0) + m(\sqrt{3}\, l/2)}{m + m + m} = \frac{l}{2\sqrt{3}}$.

9. $x_{CM} = \frac{m \times 0 + m \times a + 2m \times a + 2m \times 0}{m + m + 2m + 2m} = \frac{3ma}{6m} = \frac{a}{2}$

$y_{CM} = \frac{m \times 0 + m \times 0 + 2m \times a + 2m \times a}{m + m + 2m + 2m} = \frac{4ma}{6m} = \frac{2}{3}a$

10. $x_{CM} = \frac{m \times 0 + 2m \times a + 3m \times a + 4m \times 0}{m + 2m + 3m + 4m} = \frac{a}{2}$

$y_{CM} = \frac{m \times 0 + 2m \times 0 + 3m \times a + 4m \times a}{m + 2m + 3m + 4m} = \frac{7a}{10}$.

12. Suppose the 4 kg m... Then the centre of ... mass and 4 kg mass ... 1 m).

$x_{CM} = \frac{6 \times 3 - }{6}$

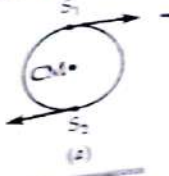
$y_{CM} = \frac{6 \times 3 - }{6}$

$z_{CM} = \frac{6 \times 3}{6}$

7.7 EXAMPLES ... IN NATURE

5. Discuss the mot... following binary syste... (ii) Diatomic molecule...

(i) **Binary stars.** ... gravitational force and ... of mass are called bina... stars S_1 and S_2 of eq... around their common ...



(a)

Fig. 7.19 Orbits ... their (a) CM is at re...

When no exte... centre of mass of ... particle. The orb... complicated, as sh... the combination ... motion of the cen... (ii) the circular or... However, the tw... sides of the CM.

(ii) **Diatomi...** molecule like O... The internal bu... atoms is due ... regarded as a s... (i.e., no collision ... with the walls ... molecule move... line, as shown ... have vibration... the internal for... like a free part...

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12. *Suppose the 4 kg mass be placed at point (x, y, z). Then the centre of mass of 6 kg (1 kg + 2 kg + 3 kg) mass and 4 kg mass must lie at the point (1 m, 1 m, 1 m).*

$$\therefore \quad x_{CM} = \frac{6\times 3 + 4\times x}{6+4} = 1 \text{ m} \quad \text{or} \quad x = -2 \text{ m}$$

$$y_{CM} = \frac{6\times 3 + 4\times y}{6+4} = 1 \text{ m} \quad \text{or} \quad y = -2 \text{ m}$$

$$z_{CM} = \frac{6\times 3 + 4\times z}{6+4} = 1 \text{ m} \quad \text{or} \quad z = -2 \text{ m}.$$

7.7 EXAMPLES OF BINARY SYSTEMS IN NATURE

8. Discuss the motion of the centre of mass of the following binary systems in nature : (i) Binary stars, (ii) Diatomic molecules, and (iii) Earth-moon system.

(*i*) **Binary stars.** *Two stars bound to each other by the gravitational force and orbiting around their common centre of mass are called binary stars.* Fig. 7.19(*a*) shows binary stars S_1 and S_2 of equal mass moving in circular orbits around their common centre of mass, which is at rest.

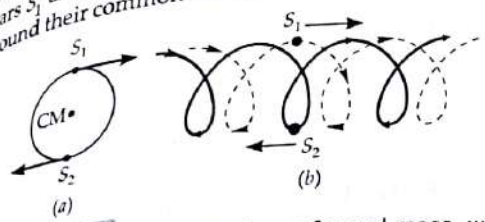

Fig. 7.19 Orbits of binary stars of equal mass, when their (a) CM is at rest, (b) CM is in uniform motion.

When no external force acts on the system, the centre of mass of the double star moves like a free particle. The orbits of the two stars are slightly complicated, as shown in Fig. 7.19(*b*). But these are just the combination of *two* motions : (*i*) the uniform motion of the centre of mass CM in a straight line and (*ii*) the circular orbits of the two stars around their CM. However, the two stars always remain on the opposite sides of the CM.

(*ii*) **Diatomic molecule.** A symmetric diatomic molecule like O_2 is also an example of binary system. The internal binding force between the two oxygen atoms is due to the chemical bond which can be regarded as a spring. When there is no external force (*i.e.*, no collisions between the molecules themselves or with the walls of the vessel), the centre of mass of the molecule moves with uniform velocity in a straight line, as shown in Fig. 7.19(*c*). The molecule can also have vibrational and rotational motions again due to the internal forces. Even then the centre of mass moves like a free particle.

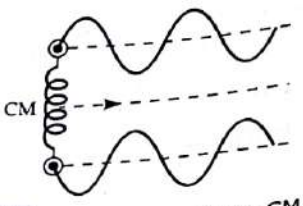

Fig. 7.19. (*c*) Uniform motion of the CM of moving, vibrating symmetric diatomic molecule O_2.

(*iii*) **Earth-moon system.** The moon moves around the earth in a circular orbit and the earth moves around the sun in an elliptical orbit. It will be more correct to say that the centre of mass of the earth-moon system moves around the sun in an elliptical orbit, not the earth and moon themselves. As the mass of the earth is nearly 80 times the mass of the moon, so the centre of mass divides the earth-moon (*E-M*) line in the ratio 1 : 80. In fact, this point lies inside the earth, as shown in Fig. 7.19(*d*).

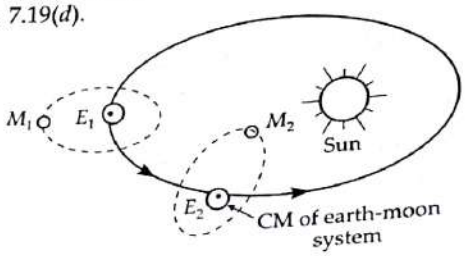

Fig. 7.19. (*d*) Motion of CM of the earth-moon system in an elliptical orbit.

Here the mutual forces of gravitation between the earth and moon are internal forces while the Sun's attraction of both earth and moon are the external forces acting on the centre of mass of the earth-moon system.

7.8 SOME OTHER EXAMPLES OF THE CM MOTION

9. Discuss the trajectory of the motion of the centre of mass of a fire cracker that explodes in air.

Motion of the CM of fire crackers exploding in air. Initially, a fire cracker moves along a parabolic path. It explodes in flight. Each fragment will follow its

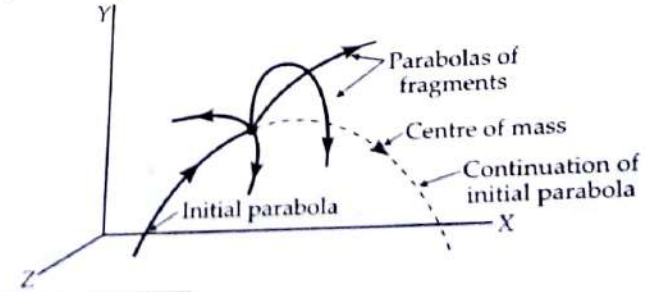

Fig. 7.20 Trajectory of the centre of mass of exploding cracker.

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own parabolic path. Since the explosion is caused by internal forces only, the centre of mass of all the fragments will continue to move along the same parabolic path of the cracker as before explosion.

7.9 RIGID BODIES

10. *What is a rigid body ? Give examples.*

Rigid body. *A body is said to be rigid if it does not undergo any change in its size and shape, however large the external force may be acting on it.* More appropriately, *a rigid body is one whose constituent particles retain their relative positions even when they move under the action of an external force.*

A rigid body cannot be deformed. If the body undergoes some displacement, every particle in it suffers the same displacement. If the body rotates through a certain angle, every particle of it rotates through the same angle about the axis of rotation. No body can be perfectly rigid. In practice, solid bodies of steel, glass etc. ; can be regarded as rigid for moderate forces.

7.10 CENTRE OF MASS OF A RIGID BODY

11. *State the factors on which the position of the centre of mass of a rigid body depends.*

Centre of mass of a rigid body. *The centre of mass of a rigid body is a point at a fixed position with respect to the body as a whole.*

The position of centre of mass of a rigid body depends on *two* factors : (*i*) The geometrical shape of the body. (*ii*) The distribution of mass in the body.

12. *How can we locate the centre of mass of rigid bodies of regular geometrical shape and having uniform mass distribution ? Give examples.*

Centre of mass of regular bodies. *For bodies having regular geometrical shape and uniform mass (or density) distribution, the centre of mass lies at their geometrical centre.* Fig. 7.21 shows the positions of the centre of mass of some regular bodies.

Table 7.1 *Centres of mass of some regular bodies*

S. No.	Shape of body	Position of centre of mass
1.	Long thin rod	Middle point of the rod
2.	Thin circular ring	Geometrical centre of the ring
3.	Circular disc	Geometrical centre of the disc
4.	Rectangular lamina	Point of intersection of diagonals
5.	Rectangular cubical block	Point of intersection of diagonals
6.	Cylinder	Middle point of the axis
7.	Solid or hollow sphere	Geometrical centre of the sphere
8.	Triangular lamina	Point of intersection of the medians
9.	Right circular cone	A point on its axis at a distance of $\frac{h}{4}$ from its base, h = height of the cone

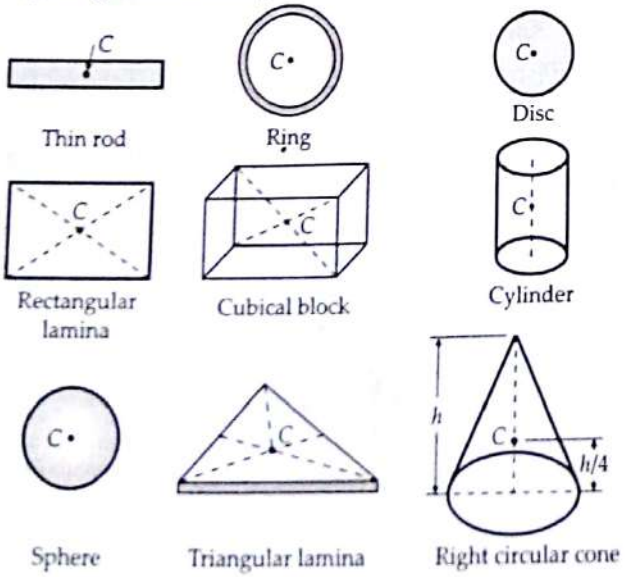

Fig. 7.21 Centres (C) of mass of some regular bodies.

7.11 ROTATIONAL MOTION OF A RIGID BODY

13. *What do you mean by rotational motion of a rigid body ?*

Rotational motion of a rigid body. *A body is said to possess rotational motion if all its particles move along circles in parallel planes. The centres of these circles lie on a fixed line perpendicular to the parallel planes and this line is called the* ***axis of rotation.***

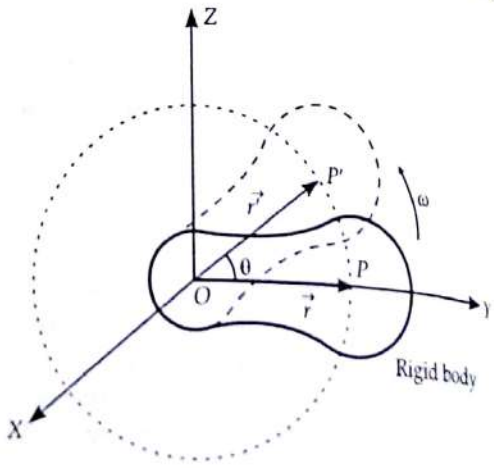

Fig. 7.22 Rotational motion about Z-axis.

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Fig. 7.22 shows a rigid body being rotated anti-clockwise about Z-axis of an inertial frame of reference. Let P be any particle of the body and $\vec{r}$ be its position vector. As the body rotates, the particle P moves along a circle of radius r whose centre lies on the axis of rotation. The radius vector $\vec{r}$ sweeps out an angle θ in certain time t. Similarly, all other particles of body move along circles with their centres on Z-axis and their radius vectors sweep the same angle θ in time t. This implies that *all the particles have the same angular velocity* $\omega (= \theta / t)$ which is also the angular velocity of the body.

7.12 EQUATIONS OF ROTATIONAL MOTION

14. *Derive the three equations of rotational motion under constant angular acceleration from first principles.*

Derivation of first equation of motion. Consider a rigid body rotating about a fixed axis with constant angular acceleration α. By definition,

$$\alpha = \frac{d\omega}{dt}$$

$$d\omega = \alpha\, dt \qquad ...(1)$$

or

At $t = 0$, let $\omega = \omega_0$

At $t = t$, let $\omega = \omega$

Integrating equation (1) within the above limits of time and angular velocity, we get

$$\int_{\omega_0}^{\omega} d\omega = \int_0^t \alpha\, dt = \alpha \int_0^t dt$$

or $$[\omega]_{\omega_0}^{\omega} = \alpha\, [t]_0^t$$

or $$\omega - \omega_0 = \alpha\,(t - 0)$$

or $$\omega = \omega_0 + \alpha t \qquad ...(2)$$

Derivation of second equation of motion. Let ω be the angular velocity of a rigid body at any instant t. By definition,

$$\omega = \frac{d\theta}{dt}$$

or $$d\theta = \omega\, dt \qquad ...(3)$$

At $t = 0$, let $\theta = 0$

At $t = t$, let $\theta = \theta$

Integrating equation (3) within the above limits of time and angular displacement, we get

$$\int_0^{\theta} d\theta = \int_0^t \omega\, dt = \int_0^t (\omega_0 + \alpha t)\, dt \qquad \text{[Using (2)]}$$

$$= \omega_0 \int_0^t dt + \alpha \int_0^t t\, dt$$

or $$[\theta]_0^{\theta} = \omega_0\, [t]_0^t + \alpha \left[\frac{t^2}{2}\right]_0^t$$

or $$\theta - 0 = \omega_0\,(t - 0) + \frac{\alpha}{2}(t^2 - 0)$$

or $$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \qquad ...(4)$$

Derivation of third equation of motion. The angular acceleration α may be expressed as

$$\alpha = \frac{d\omega}{dt} = \frac{d\omega}{d\theta} \cdot \frac{d\theta}{dt} = \frac{d\omega}{d\theta} \cdot \omega$$

or $$\omega\, d\omega = \alpha\, d\theta \qquad ...(5)$$

At $t = 0$, $\theta = 0$ and $\omega = \omega_0$ (initial angular velocity)

At $t = t$, $\theta = \theta$ and $\omega = \omega$ (final angular velocity)

Integrating equation (5) within the above limits of θ and ω, we get

$$\int_{\omega_0}^{\omega} \omega\, d\omega = \int_0^{\theta} \alpha\, d\theta = \alpha \int_0^{\theta} d\theta$$

or $$\left[\frac{\omega^2}{2}\right]_{\omega_0}^{\omega} = \alpha\, [\theta]_0^{\theta}$$

or $$\frac{\omega^2}{2} - \frac{\omega_0^2}{2} = \alpha\,(\theta - 0)$$

or $$\omega^2 - \omega_0^2 = 2\alpha\theta$$

Examples based on Equations of Rotational Motion

FORMULAE USED

For a body in rotational motion under constant angular acceleration, the equations of motion can be written as

1. $\omega = \omega_0 + \alpha t$
2. $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
3. $\omega^2 - \omega_0^2 = 2\alpha t$

UNITS USED

Angular displacement θ is in rad, initial angular velocity ω_0 and final angular velocity ω are in rad s^{-1}, angular acceleration α in rad s^{-2}.

EXAMPLE 12. On the application of a constant torque, a wheel is turned from rest through 400 radians in 10 s. (i) Find angular acceleration. (ii) If same torque continues to act, what will be angular velocity of the wheel after 20 s from start ?

Solution. (i) Here $\theta = 400$ rad, $\omega_0 = 0$, $t = 10$ s, $\alpha = ?$

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As $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\therefore \quad 400 = 0 + \frac{1}{2}\alpha (10)^2$

or $\alpha = \frac{2 \times 400}{100} = 8 \text{ rad s}^{-2}$.

(*ii*) Here $\omega_0 = 0$, $\alpha = 8 \text{ rad s}^{-2}$, $t = 20$ s, $\omega = ?$

$\omega = \omega_0 + \alpha t = 0 + 8 \times 20 = 160 \text{ rad s}^{-1}$.

Example 13. The angular speed of a motor wheel is increased from 1200 rpm to 3120 rpm in 16 seconds. (i) What is its angular acceleration, assuming the acceleration to be uniform ? (ii) How many revolutions does the wheel make during this time ? [NCERT]

Solution. Here

$$\nu_0 = 1200 \text{ rpm} = \frac{1200}{60} = 20 \text{ rps}$$

$$\nu = 3120 \text{ rpm} = \frac{3120}{60} = 52 \text{ rps}$$

$\therefore \quad \omega_0 = 2\pi\nu_0 = 2\pi \times 20 = 40\pi \text{ rads}^{-1}$

$\omega = 2\pi\nu = 2\pi \times 52 = 104\pi \text{ rad s}^{-1}$

(*i*) Angular acceleration,

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{104\pi - 40\pi}{16} = 4\pi \text{ rad s}^{-2}.$$

(*ii*) The angular displacement in time t,

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$= 40\pi \times 16 + \frac{1}{2} \times 4\pi \times (16)^2$$

$$= (640\pi + 512\pi) \text{ rad} = 1152\pi \text{ rad}.$$

Number of revolutions completed in 16 s

$$= \frac{\theta}{2\pi} = \frac{1152\pi}{2\pi} = 576.$$

Example 14. A constant torque is acting on a wheel. If starting from rest, the wheel makes n rotations in t second, show that the angular acceleration is given by

$$\alpha = \frac{4\pi n}{t^2} \text{ rad s}^{-2}.$$

Solution. Initial angular velocity, $\omega_0 = 0$

Number of rotations completed in t second $= n$

$\therefore$ Angular displacement in t second, $\theta = 2\pi n$

As $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\therefore \quad 2\pi n = 0 + \frac{1}{2}\alpha t^2$

or $\alpha = \frac{4\pi n}{t^2} \text{ rad s}^{-2}$.

Example 15. The radius of a wheel of a car is 0.4 m. The car is accelerated from rest by an angular acceleration of 1.5 rad s^{-2} for 20 s. How much distance the wheel covers in this time interval and what will be its linear velocity ?

Solution. Here $r = 0.4$ m, $\omega_0 = 0$,

$\alpha = 1.5 \text{ rad s}^{-2}$, $t = 20$ s

Angular displacement,

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$= 0 + \frac{1}{2} \times 1.5 \times (20)^2 = 300 \text{ rad}$

Distance covered by the wheel,

$s = r\theta = 0.4 \times 300 = 120$ m.

After 20 s, angular velocity of the wheel is

$\omega = \omega_0 + \alpha t = 0 + 1.5 \times 20 = 30 \text{ rad s}^{-1}$

$\therefore$ Linear velocity of the wheel is

$v = r\omega = 0.4 \times 30 = 12 \text{ ms}^{-1}$.

✕ Problems For Practice

1. A gridstone has a constant acceleration of 4 rad s^{-2}. Starting from rest, calculate the angular speed of the grindstone 2.5 s later. (Ans. 10 rad s^{-1})
2. The speed of a motor increases from 600 rpm to 1200 rpm in 20 s. What is its angular acceleration and how many revolutions does it make during this time ? (Ans. π rad s^{-2}, 300)
3. On the application of a constant torque, a wheel is turned from rest through an angle of 200 rad in 8 s. (*i*) What is its angular acceleration ? (*ii*) If the same torque continues to act, what will be the angular velocity of the wheel after 16 s from the start ? [Ans. (*i*) 6.25 rad s^{-2} (*ii*) 100 rad s^{-1}]
4. The motor of an engine is rotating about its axis with an angular velocity of 100 rpm. It comes to rest in 15 s after being switched off. Assuming constant angular deceleration, calculate the number of revolutions made by it before coming to rest. (Ans. 125)
5. A car is moving at a speed of 72 kmh^{-1}. The diameter of its wheels is 0.50 m. If the wheels are stopped in 20 rotations by applying brakes, calculate the angular retardation produced by the brakes. (Ans. – 25.5 rad s^{-2})
6. A flywheel rotating at 420 rpm slows down at a constant rate of 2 rad s^{-2}. What time is required to stop the flywheel ? [Central Schools 12]

✕ Hints

4. Here $\nu_0 = 100 \text{ rpm} = \frac{100}{60} \text{ rps}$

$\therefore \quad \omega_0 = 2\pi\nu_0 = 2\pi \times \frac{100}{60} = \frac{10\pi}{3} \text{ rad s}^{-1}$

$\omega = 0, \quad t = 15$ s

$\therefore \quad \alpha = \frac{\omega - \omega_0}{t} = \frac{0 - 10\pi/3}{15} = -\frac{2\pi}{9} \text{ rad s}^{-2}$

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Now $\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = \frac{10\pi}{3} \times 15 - \frac{1}{2} \times \frac{2\pi}{9} \times (15)^2$

$= 25\pi$ rad

Number of revolutions completed before coming to rest $= \frac{\theta}{2\pi} = \frac{25\pi}{2\pi} = 12.5.$

5. Here $v_0 = 72 \text{ kmh}^{-1} = \frac{72 \times 5}{18} = 20 \text{ ms}^{-1}$, $r = 0.25$ m

Angular speed, $\omega_0 = \frac{v_0}{r} = \frac{20}{0.25} = 80 \text{ rad s}^{-1}$

Angular displacement in 20 rotations,

$\theta = 2\pi n = 2\pi \times 20 = 40\pi$ rad

Also, final angular velocity, $\omega = 0$

As $\omega^2 - \omega_0^2 = 2\alpha\theta$

$\therefore \quad 0 - (80)^2 = 2\alpha \times 40\pi$

or $\alpha = -\frac{(80)^2}{80\pi} = -25.5 \text{ rad s}^{-2}.$

6. $\nu_0 = 420 \text{ rpm} = 7 \text{ rps}$

$\omega_0 = 2\pi\nu_0 = 2 \times \frac{22}{7} \times 7 = 44 \text{ rad s}^{-1}$,

$\omega = 0, \quad \alpha = -2 \text{ rad s}^{-2}$

$t = \frac{\omega - \omega_0}{\alpha} = \frac{0 - 44}{-2} = 22 \text{ s.}$

7.13 MOMENT OF FORCE OR TORQUE

15. *On what factors does the turning effect of a force depend ? What is the turning effect of force called ?*

Turning effect of force. To open a door, we apply a force on its handle. The door turns on its hinges. *The larger the force, the more is its turning effect.* Also, we can easily notice that it is easier to open a door by applying a force near the end than near the hinges. This is because the turning effect of the same force is larger when its distance from the axis of rotation is more.

This turning effect of force is called **moment of force** or **torque**. It depends on *two* factors :

(*i*) The magnitude of the force.

(*ii*) The perpendicular distance of the line of action of the force from the axis of rotation. It is called *lever arm* or *moment arm.*

Thus, greater the magnitude of the force, or greater the perpendicular distance between the line of action of the force and the axis of rotation, the greater is the moment of force, or greater is the turning effect.

16. *Define the term torque or moment of force. Give its units and dimensions.*

Torque or moment of force. *The torque or moment of force is the turning effect of the force about the axis of rotation. It is measured as the product of the magnitude of the force and the perpendicular distance between the line of action of the force and the axis of rotation.*

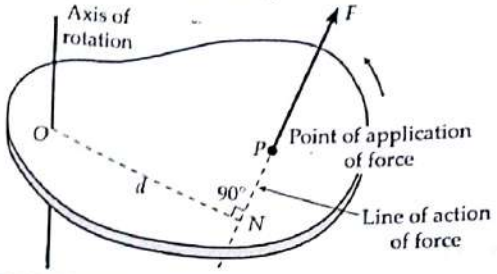

Fig. 7.23 Torque or moment of force.

Fig. 7.23 shows a body free to rotate about a vertical axis through O. A horizontal force F applied on it at point P rotates it about this axis. If d is the perpendicular distance of the line of action of the force from the axis of rotation, then the torque or moment of force F about the axis of rotation is

$$\tau = F \times ON$$

or $$\tau = F \times d$$

or Torque = Force × Lever arm

Dimensions of torque. As

Torque = Force × distance,

so $$[\tau] = [MLT^{-2}][L] = [ML^2T^{-2}].$$

Units of torque. The SI unit of torque is newton metre (Nm) and its CGS unit is dyne cm.

Sign convention. The moment of force is taken *positive* if the turning tendency of the force is *anticlockwise* and *negative* if it is *clockwise.*

7.14 ROTATIONAL EQUILIBRIUM AND THE PRINCIPLE OF MOMENTS

17. *State and explain the principle of moments of rotational equilibrium.*

Principle of moments. *When a body is in rotational equilibrium, the sum of the clockwise moments about any point is equal to the sum of the anticlockwise moments about that point or the algebraic sum of moments about any point is zero.*

As shown in Fig. 7.24, consider a uniform rod free to rotate on a pivot O. Two weights W_1 and W_2 are hung from it at distances d_1 and d_2 from the pivot O.

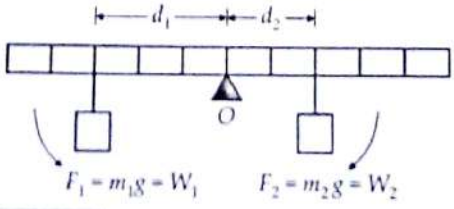

Fig. 7.24 Principle of moments.

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Anticlockwise moment about $O = F_1 \times d_1 = W_1 \times d_1$

Clockwise moment about $O = F_2 \times d_2 = W_2 \times d_2$

According to the principle of moments, the rod will be horizontal or in rotational equilibrium if

Anticlockwise moment = Clockwise moment

or $F_1 \times d_1 = F_2 \times d_2$

or $W_1 \times d_1 = W_2 \times d_2$

i.e., Load × load arm = Effort × effort arm

This is sometimes called the *lever principle*.

18. *What is a couple ? What effect does it have on a body ? Show that the moment of couple is same irrespective of the point of rotation of a body.*

Couple. *A pair of equal and opposite forces acting on a body along two different lines of action constitute a couple.* A couple has a turning effect, but no resultant force acts on a body. So it cannot produce translational motion. When we steer a bicycle round a bend with our both hands on the handle-bars, we apply a couple.

Moment of a couple. The moment of couple can be found by taking the moments of the two forces about any point and then adding them.

In Fig. 7.25, two opposite forces, each of magnitude F act at two points A and B of a rigid body, which can rotate about point O. The turning tendency of the two forces is anticlockwise.

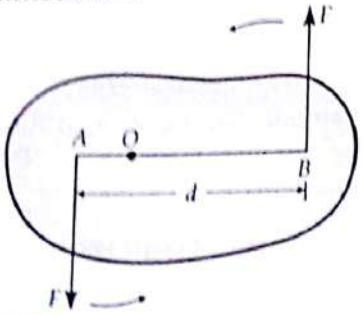

Fig. 7.25 Moment of couple.

Moment or torque of the couple about O is

$$\tau = F \times AO + F \times OB$$

$$= F(AO + OB) = F \times AB$$

or $\tau = Fd$ [$AB = d$, say]

Moment of a couple = Force × perpendicular distance between two forces

Hence *the moment of a couple is equal to the product of either of the forces and the perpendicular distance, called the arm of the couple, between their lines of action.* Note that the torque exerted by couple about O does not depend on the position of O. Hence *torque or moment of a couple is independent of the choice of the fulcrum or the point of rotation.*

Notably, a couple can only be balanced by an equal and opposite couple.

19. *Obtain an expression for the work done by a torque. Hence write the expression for power.*

Work done by a torque. As shown in Fig. 7.26, suppose a body undergoes an angular displacement $\Delta\theta$ under the action of a tangential force F.

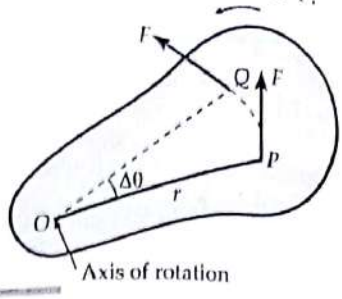

Fig. 7.26 Work done by a torque.

The work done in the rotational motion of the body or the work done by the torque is

$\Delta W = F \times$ distance along the arc PQ

But $\Delta\theta = \dfrac{\text{Arc}}{\text{Radius}} = \dfrac{\text{Arc } PQ}{r}$

$\therefore$ Arc $PQ = r\,\Delta\theta$

Hence $\Delta W = Fr\,\Delta\theta$

or $\Delta W = \tau\,\Delta\theta$

i.e., Work done by a torque = Torque × angular displacement

In case the torque applied is not constant, but variable, the total work done by the torque is given by

$$W = \int_{\theta_1}^{\theta_2} \tau\, d\theta$$

Power delivered by a torque. We know that

$$\Delta W = \tau\,\Delta\theta$$

Dividing both sides by Δt, we get

$$\frac{\Delta W}{\Delta t} = \tau \frac{\Delta\theta}{\Delta t}$$

or $P = \tau\omega$

i.e., Power = Torque × Angular velocity.

20. *Explain how torque can be expressed as a vector product of two vectors. How is the direction of torque determined ?*

Torque acting on a particle. Consider a particle P in the X-Y plane. Suppose its position vector is $\vec{OP} = \vec{r}$ with respect to the origin O of an inertial frame, as shown in Fig. 7.27. Let $\vec{F}$ be the force acting on the particle. *The torque acting on the particle is defined as the*

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vector product of position and force vectors. Thus the *torque of* $\vec{F}$ *about O* is defined as

$$\vec{\tau} = \vec{r} \times \vec{F}$$

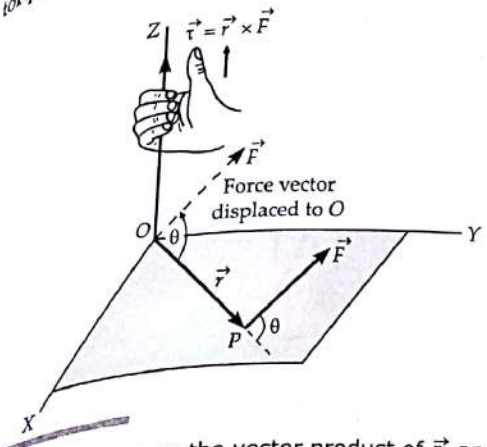

Fig. 7.27 Torque as the vector product of $\vec{r}$ and $\vec{F}$.

Torque $\vec{\tau}$ is a vector quantity. Its magnitude is given by

$$\tau = rF \sin\theta$$

The direction of torque $\vec{\tau}$ is perpendicular to the plane containing vectors $\vec{r}$ and $\vec{F}$ and its sense is determined by *right hand rule.* If we curl the fingers of our right hand in the direction in which vector $\vec{r}$ must be rotated to move into the position of vector $\vec{F}$ through the smaller angle between them, then the extended thumb gives the sense of torque $\vec{\tau}$. In the present case, $\vec{\tau}$ acts along Z-axis.

Special cases. (*i*) **When $\theta = 0°$ or $180°$,** the line of action of the force passes through the origin. In this case $\sin\theta = 0$, so that $\tau = 0$.

(*ii*) **When $\theta = 90°$,** $\tau = Fr \sin 90° = Fr$ and is maximum. This explains why it is easier to open or close a door when we apply force perpendicular to the door at its outer or free edge.

(*iii*) **When *r* is maximum,** torque due to the force is maximum. This explains why we can open or close a door easily by applying force near the outer edge of the door (at maximum distance from the hinges). For this reason, a handle or knob is provided near the free edge of the door.

21. *Show that the magnitude of torque = magnitude of force × moment arm. Also show that only the angular component of the force is responsible for producing torque.*

Dependence of torque on moment arm. Consider a particle capable of rotation in the X-Y plane about the origin O. Suppose the force vector makes an angle θ with the position vector $\vec{r}$ of the particle, as shown in Fig. 7.28.

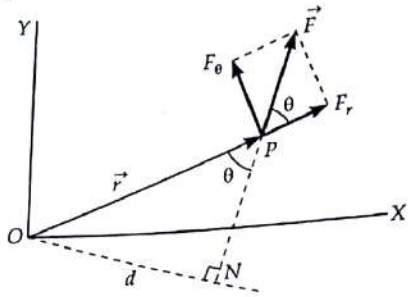

Fig. 7.28 Dependence of torque on moment arm.

Draw *ON* perpendicular to the line of action of the force. Then from the right angled ΔONP, we have

$$\frac{ON}{OP} = \sin\theta \quad \text{or} \quad \frac{d}{r} = \sin\theta \quad \text{or} \quad d = r\sin\theta$$

This is the perpendicular distance of the line of action of force from the axis of rotation through the point O and is called *moment arm* or *lever arm* of the force.

The magnitude of the torque of force F is given by

$$\tau = rF \sin\theta = F(r\sin\theta) = Fd$$

i.e., Torque = Magnitude of force × moment arm

Now the force $\vec{F}$ can be resolved into two rectangular components :

1. *Radial component* F_r along the direction of position vector $\vec{r}$.
2. *Angular or tangential component* F_θ perpendicular to $\vec{r}$.

Clearly, $F_r = F\cos\theta$ and $F_\theta = F\sin\theta$

$\therefore \quad \tau = F\sin\theta \,.\, r = F_\theta\, r$

i.e. Torque = Angular component of the force × its distance from the axis of rotation

Hence *torque due to a force is only due to its angular component.*

The radial component of the force does not contribute to the torque.

22. *Write an expression for torque in three-dimensional motion. Hence write the expressions for the rectangular components of torque.*

Rectangular components of torque. For three dimensional motion, the position vector $\vec{r}$, and torque

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vector $\vec{\tau}$ can be written in terms of their rectangular components as follows :

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

and $$\vec{\tau} = \tau_x\hat{i} + \tau_y\hat{j} + \tau_z\hat{k}$$

Now $$\vec{\tau} = \vec{r} \times \vec{F}$$

$$= (x\hat{i} + y\hat{j} + z\hat{k}) \times (F_x\hat{i} + F_y\hat{j} + F_z\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

or $$\tau_x\hat{i} + \tau_y\hat{j} + \tau_z\hat{k}$$

$$= \hat{i}(yF_z - zF_y) + \hat{j}(zF_x - xF_z) + \hat{k}(xF_y - yF_x)$$

Comparing the coefficients of $\hat{i}$, $\hat{j}$ and $\hat{k}$ on the two sides of the above equation, we get the rectangular components of torque $\vec{\tau}$ as follows :

$$\tau_x = yF_z - zF_y\ ; \quad \tau_y = zF_x - xF_z\ ; \quad \tau_z = xF_y - yF_x.$$

7.15 ANGULAR MOMENTUM

23. *Define the term angular momentum. Give its units and dimensions.*

Angular momentum. In linear motion, the linear momentum of a body gives a measure of its translatory motion. Analogous to it in rotational motion, the angular momentum gives a measure of the turning motion of the body.

The angular momentum of a particle rotating about an axis is defined as the moment of the linear momentum of the particle about that axis. It is measured as the product of linear momentum and the perpendicular distance of its line of action from the axis of rotation.

Angular momentum
= Linear momentum × its perpendicular distance from the axis of rotation.

$$L = pd$$

Dimensions of angular momentum

$$= L \times MLT^{-1} = [ML^2T^{-1}]$$

SI unit of angular momentum is kg m^2s^{-1}.

CGS unit of angular momentum is g cm^2s^{-1}.

24. *Explain how angular momentum can be expressed as the vector product of two vectors. How is its direction determined ?*

Angular momentum of a particle. Consider a particle P of mass m rotating about an axis through O in the X-Y plane. Suppose the particle has linear momentum $\vec{p}$ which makes angle θ with its position vector $\vec{OP} = \vec{r}$. The angular momentum $\vec{L}$ of the particle about the origin O is defined as the vector product of the vectors $\vec{r}$ and $\vec{p}$. Thus

$$\vec{L} = \vec{r} \times \vec{p}$$

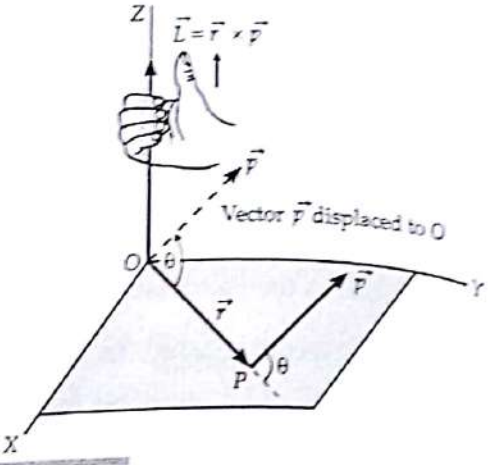

Fig. 7.29 Angular momentum as the vector product of $\vec{r}$ and $\vec{p}$.

Angular momentum is a vector quantity. Its magnitude is given by

$$L = rp \sin\theta$$

The direction of angular momentum $\vec{L}$ is perpendicular to the plane of vectors $\vec{r}$ and $\vec{p}$ in the sense given by *right hand rule*. Thus in the present case, $\vec{L}$ points in Z-direction.

Special cases. (*i*) If $\theta = 0°$ or $180°$, $\sin\theta = 0$

$$\therefore \quad L = rp \times 0 = 0$$

Hence *the angular momentum is zero if the line of action of linear momentum passes through the point of rotation.*

(*ii*) If $\theta = 90°$, $\sin 90° = 1$

$$\therefore \quad L = rp \times 1 = rp = \text{maximum}$$

Hence *the angular momentum is maximum and is equal to rp or mvr, if the line of action of the linear momentum is perpendicular to the position vector.*

25. *Show that the angular momentum of a particle is the product of its linear momentum and the moment arm. Also show that the angular momentum is produced only by the angular component of linear momentum.*

Physical meaning of angular momentum. Consider a particle P of mass m whose position vector relative to

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the origin O is $\vec{r}$. Suppose the momentum vector $\vec{p}$ of the particle makes angle θ with the position vector $\vec{r}$, as shown in Fig. 7.30.

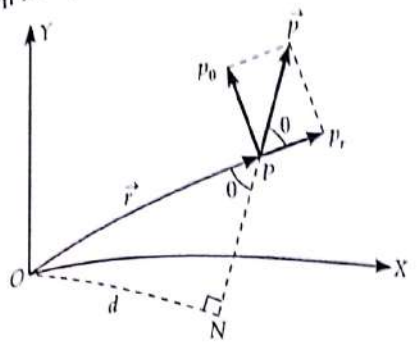

Fig. 7.30 Physical meaning of angular momentum.

Draw ON perpendicular to the line of action of linear momentum $\vec{p}$. From right angled ΔONP, we get

$$\frac{ON}{OP} = \frac{d}{r} = \sin\theta \quad \text{or} \quad d = r\sin\theta.$$

This is the perpendicular distance of the line of action of linear momentum from the point of rotation O and is called *moment arm* of the momentum.

The magnitude of the angular momentum about the point O is

$$L = rp\sin\theta = p(r\sin\theta) = pd$$

i.e., Angular momentum = Linear momentum × moment arm

This is the physical meaning of angular momentum. According to it, *angular momentum is the moment of linear momentum and is a measure of the turning motion of the object.* In contrast to it, we know that

Torque = Force × moment arm

Thus *torque is the moment of force and is a measure of the turning effect of force.*

Moreover, as shown in Fig. 7.30, the momentum vector $\vec{p}$ can be resolved into *two* rectangular components :

1. *Radial component*, p_r along the direction of position vector $\vec{r}$.
2. *Angular or tangential component*, p_θ perpendicular to $\vec{r}$.

Clearly, $p_r = p\cos\theta$ and $p_\theta = p\sin\theta$

$\therefore \quad L = (p\sin\theta)\,r = p_\theta\, r$

or Angular momentum = Angular component of linear momentum × its distance from the axis of rotation.

Hence *only the angular component and not the radial component of the linear momentum contributes towards the angular momentum.*

26. *Express angular momentum in terms of the rectangular components of linear momentum and position vectors.*

Angular momentum in terms of rectangular components. For motion in three dimensions, the position vector $\vec{r}$ and linear momentum vector $\vec{p}$ can be written in terms of their rectangular components as follows :

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

and $$\vec{p} = p_x\hat{i} + p_y\hat{j} + p_z\hat{k}$$

But angular momentum is the cross product of $\vec{r}$ and $\vec{p}$, so

$$\vec{L} = \vec{r}\times\vec{p}$$

$$= (x\hat{i} + y\hat{j} + z\hat{k})\times(p_x\hat{i} + p_y\hat{j} + p_z\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$$

or $$L_x\hat{i} + L_y\hat{j} + L_z\hat{k}$$

$$= \hat{i}(yp_z - zp_y) + \hat{j}(zp_x - xp_z) + \hat{k}(xp_y - yp_x)$$

Comparing the coefficients of $\hat{i}$, $\hat{j}$ and $\hat{k}$ on the two sides of the above equation, we obtain the rectangular components of vector $\vec{L}$ as follows :

$$L_x = yp_z - zp_y \;;\; L_y = zp_x - xp_z \text{ and } L_z = xp_y - yp_x.$$

7.16 RELATION BETWEEN TORQUE AND ANGULAR MOMENTUM

27. *Deduce the relation between torque and angular momentum.*

Relation between torque and angular momentum. We know that

Torque, $\vec{\tau} = \vec{r}\times\vec{F}$

Angular momentum, $\vec{L} = \vec{r}\times\vec{p}$

Differentiating both sides w.r.t. time t, we get

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r}\times\vec{p}) = \frac{d\vec{r}}{dt}\times\vec{p} + \vec{r}\times\frac{d\vec{p}}{dt}$$

$$= \vec{v}\times\vec{p} + \vec{r}\times\vec{F} \qquad \left[\because \frac{d\vec{p}}{dt} = \vec{F}\right]$$

$$= 0 + \vec{\tau} \qquad [\because \vec{v}\times\vec{p} = \vec{v}\times m\vec{v} = 0]$$

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$$\therefore \quad \vec{\tau} = \frac{d\vec{L}}{dt}$$

Thus *the torque acting on a particle is equal to its rate of change of angular momentum.* This equation is the rotational analogue of Newton's second law of linear motion which states that the rate of change of linear momentum of a particle is equal to the force acting on it *i.e.*,

$$\vec{F} = \frac{d\vec{p}}{dt}.$$

7.17 GEOMETRICAL MEANING OF ANGULAR MOMENTUM

28. *Prove that the angular momentum of a particle is equal to twice the product of its mass and areal velocity. How does it lead to Kepler's second law of planetary motion ?*

Geometrical meaning of angular momentum. Consider a particle of mass m rotating in the X-Y plane about the origin O. Let $\vec{r}$ and $(\vec{r} + \Delta\vec{r})$ be the position vectors of the particle at instants t and $(t + \Delta t)$ respectively, as shown in Fig. 7.31. The displacement of the particle in small time Δt is

$$\vec{PQ} = (\vec{r} + \Delta\vec{r}) - \vec{r} = \Delta\vec{r}$$

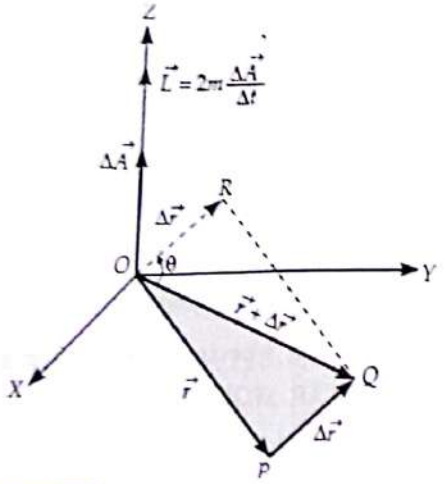

Fig. 7.31 Geometrical meaning of angular momentum.

If $\vec{v}$ is the velocity of the particle at point P, then the small displacement covered in time Δt may be expressed as

$$\Delta\vec{r} = \vec{v}\,\Delta t$$

Complete the parallelogram $OPQR$. Then

$$\vec{OR} = \vec{PQ} = \Delta\vec{r}$$

Area of the parallelogram $OPQR = \vec{r} \times \Delta\vec{r}$

$$\therefore \quad \text{Area of } \Delta OPQ = \frac{1}{2}(\vec{r} \times \Delta\vec{r})$$

The shaded area of ΔOPQ represents the area swept by the position vector in time Δt. By right hand rule, its direction is along Z-axis. If this area is represented by $\Delta\vec{A}$, then

$$\Delta\vec{A} = \frac{1}{2}(\vec{r} \times \Delta\vec{r}) = \frac{1}{2}(\vec{r} \times \vec{v}\,\Delta t)$$

If $\vec{p}$ is the linear momentum of the particle then

$$\vec{p} = m\vec{v} \quad \text{or} \quad \vec{v} = \frac{\vec{p}}{m}$$

$$\therefore \quad \Delta\vec{A} = \frac{1}{2}\vec{r} \times \frac{\vec{p}}{m}\Delta t$$

$$\text{or} \quad \frac{\Delta\vec{A}}{\Delta t} = \frac{1}{2m}(\vec{r} \times \vec{p})$$

But $\vec{r} \times \vec{p} = \vec{L}$, the angular momentum of the particle about Z-axis, so we have

$$\frac{\Delta\vec{A}}{\Delta t} = \frac{\vec{L}}{2m} = \frac{1}{2} \times \text{Angular momentum per unit mass}$$

$$\text{or} \quad \vec{L} = 2m\frac{\Delta\vec{A}}{\Delta t}$$

The quantity $\Delta\vec{A}/\Delta t$ is the *area swept out by the position vector per unit time and is called the areal velocity of the particle.* Thus

$$\text{Angular momentum} = 2 \times \text{Mass} \times \text{areal velocity}$$

This is the geometrical meaning of angular momentum. So geometrically, *the angular momentum of a particle is equal to twice the product of its mass and areal velocity.* Equivalently, we can say that the *areal velocity of a particle is just half its angular momentum per unit mass.*

Kepler's second law of planetary motion. A planet revolves around the sun under the influence of gravitational force which acts towards the sun *i.e.*, the force is purely radial and angular component F_θ of the force is zero.

As torque, $\tau = r F_\theta$,

$$\text{therefore} \quad \tau = 0 \quad \text{or} \quad \frac{dL}{dt} = 0$$

$$\text{or} \quad L = \text{constant}$$

$$\text{or} \quad 2m.\frac{\Delta A}{\Delta t} = \text{constant} \quad [\because L = 2m \times \text{areal velocity}]$$

$$\text{or} \quad \frac{\Delta A}{\Delta t} = \text{constant} \quad [\because m \text{ of planet is constant}]$$

This means that the areal velocity of a planet is constant. This is *Kepler's second law of planetary motion which states that the line joining the planet to the sun sweeps out equal areas in equal intervals of time.*

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7.18 TORQUE AND ANGULAR MOMENTUM FOR A SYSTEM OF PARTICLES

29. *Prove that the rate of change of total angular momentum of a system of particles about a reference point is equal to the total torque acting on the system.*

Torque and angular momentum for a system of particles. Consider a system of n particles. Let $\vec{L}_1$, $\vec{L}_2$, $\vec{L}_3$, ... be the angular momenta of the various particles about the origin O of a reference frame. The total angular momentum of the system about the point O is given by the vector sum of angular momenta of all the individual particles. Thus

$$\vec{L} = \vec{L}_1 + \vec{L}_2 + ... + \vec{L}_n$$
$$= \vec{r}_1 \times \vec{p}_1 + \vec{r}_2 \times \vec{p}_2 + ... + \vec{r}_n \times \vec{p}_n$$
$$\vec{L} = \sum_{i=1}^{n} \vec{L}_i = \sum_{i=1}^{n} \vec{r}_i \times \vec{p}_i$$

or

Similarly, the total torque acting on the system is equal to the vector sum of the torques of all the particles about the origin O. Thus

$$\vec{\tau}^{tot} = \vec{\tau}_1 + \vec{\tau}_2 + ... + \vec{\tau}_n$$
$$= \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + ... + \vec{r}_n \times \vec{F}_n$$
$$\vec{\tau}^{tot} = \sum_{i=1}^{n} \vec{\tau}_i = \sum_{i=1}^{n} \vec{r}_i \times \vec{F}_i$$

or

The total torque acting on the system is due to *two sources* : (*i*) Torques exerted on the particles of the system by mutual *internal forces* between the particles. (*ii*) Torques exerted on the individual particles of the system by the *external forces.*

According to Newton's third law, the internal torque on the system due to internal forces is zero because the forces between any two particles are equal and opposite and directed along the line joining the two particles. Hence the total torque is due to external forces only. So we have

$$\vec{\tau}^{tot} = \vec{\tau}^{ext} = \sum_{i=1}^{n} \vec{\tau}_i^{ext}$$

This is in accordance with the common experience – bodies do not start spinning on their own without external forces acting on them. Hence if the angular momentum of a system changes with time, this change can be due to the torques produced by external forces only. So we can write

$$\vec{\tau}^{ext} = \frac{d\vec{L}}{dt}$$

Thus *the rate of change of total angular momentum of a system of particles about a fixed point is equal to the total external torque acting on the system about that point.*

Examples based on Torque, Power of a Torque, Work done by a Torque and Angular Momentum

FORMULAE USED

1. Torque = Force × its perpendicular distance from the axis of rotation
   or $\tau = Fd$
2. Torque, $\tau = rF\sin\theta$ or $\vec{\tau} = \vec{r} \times \vec{F}$
3. Power of a torque = Torque × Angular velocity
   or $P = \tau\omega$
4. Work done by a torque = Torque × Angular displacement
   or $W = \tau\theta$
5. Angular momentum = Linear momentum × its perpendicular distance from the axis of rotation
   or $L = pd$
6. Angular momentum, $L = rp\sin\theta$ or $\vec{L} = \vec{r} \times \vec{p}$
7. For a particle of mass m moving with uniform speed v along a circle of radius r, $L = mvr$.
8. Torque = Rate of change of angular momentum
   or $\tau = \frac{dL}{dt}$

UNITS USED

Torque is in Nm, work done in joule, power in watt, angular velocity ω in rad s^{-1} and angular momentum L in kg $m^2 s^{-1}$.

EXAMPLE 16. Find the torque of a force $7\hat{i} - 3\hat{j} - 5\hat{k}$ *about the origin which acts on a particle whose position vector is* $\hat{i} + \hat{j} - \hat{k}$. **[Delhi 08]**

Solution. Here $\vec{F} = 7\hat{i} - 3\hat{j} - 5\hat{k}$ and $\vec{r} = \hat{i} + \hat{j} - \hat{k}$

$$\therefore \quad \vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 7 & -3 & -5 \end{vmatrix}$$

or $\vec{\tau} = -8\hat{i} - 2\hat{j} - 10\hat{k}$.

EXAMPLE 17. A metal bar 70 cm long and 4.00 kg in mass is supported on two knife-edges placed 10 cm from each end. A 6.00 kg weight is suspended at 30 cm from one end. Find the reactions at the knife-edges. (Assume the bar to be of uniform cross section and homogeneous.) Take $g = 9.8\ ms^{-2}$ **[NCERT]**

Solution. Fig. 7.32 shows the rod AB and the positions of the knife edges at K_1 and K_2. The centre of gravity G, where the weight W of the rod acts, is at the

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centre of the rod and the weight W_1 is suspended at point P. Let R_1 and R_2 be the reactions of the support at the edges.

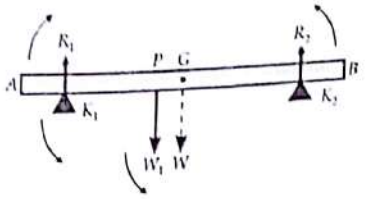

Fig. 7.32

Here $AB = 70$ cm, $AG = 35$ cm, $AP = 30$ cm, $PG = 5$ cm, $AK_1 = BK_2 = 10$ cm, $K_1G = K_2G = 25$ cm

W = Weight of rod = 4.00 kg wt = $4g$ N

W_1 = Weight of suspended mass

= 6.00 kg wt = $6g$ N

Clearly, the reactions R_1 and R_2 act vertically upward. For translational equilibrium of the rod,

$$R_1 + R_2 = W_1 + W = 4g + 6g = 10g$$
$$= 10 \times 9.8 = 98 \text{ N} \quad ...(1)$$

For rotational equilibrium of the rod,

Clockwise moment = Anticlockwise moment

$$R_1 \times K_1G = W_1 \times PG + R_2 \times K_2G$$

or $$R_1 \times 0.25 = W_1 \times 0.05 + R_2 \times 0.25$$

or $$R_1 - R_2 = 0.2\ W_1 = 0.2 \times 6 \times 9.8 \text{ N}$$

or $$R_1 - R_2 = 11.76 \text{ N} \quad ...(2)$$

On solving equations (1) and (2), we get

$$R_1 = 54.88 \text{ N and } R_2 = 43.12 \text{ N}$$

Thus the reactions of the support are about 55 N at K_1 and 43 N at K_2.

EXAMPLE 18. A 3 m long ladder weighing 20 kg leans on a frictionless wall. Its feet rest on the floor 1 m from the wall as shown in Fig. 7.33 Find the reaction forces of the wall and the floor. [NCERT]

Solution. The ladder AB is 3 m long, its foot A is at distance $AC = 1$ m from the wall.

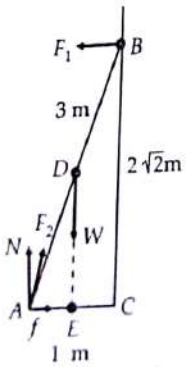

Fig. 7.33

Using pythagoras theorem,

$$BC = \sqrt{AB^2 - AC^2} = \sqrt{3^2 - 1^2} = 2\sqrt{2} \text{ m}$$

The forces acting on the ladder are

1. Its weight W acting at the centre of gravity D.
2. Reaction force F_1 of the wall. It acts perpendicular to the wall because the wall is frictionless.
3. Reaction force F_2 of the floor. This force can be resolved into two components : the normal reaction N and the force of friction f. This friction prevents the ladder sliding away from the wall and hence acts towards the wall.

For translational equilibrium, balancing the forces in the vertical direction,

$$N = W$$

Balancing the forces in the horizontal direction,

$$f = F_1$$

For rotational equilibrium, we consider the moments about the point A.

Clockwise moment = Anticlockwise moment

$$F_1 \times BC = W \times AE$$

or $$F_1 \times 2\sqrt{2} = W \times \frac{1}{2}$$

But $$W = 20g = 20 \times 9.8 = 196.0 \text{ N}$$

$\therefore$ $$N = W = 196.0 \text{ N}$$

$$F_1 = \frac{W}{4\sqrt{2}} = \frac{196.0}{4\sqrt{2}} = 34.6 \text{ N}$$

$$f = F_1 = 34.6 \text{ N}$$

and $$F_2 = \sqrt{f^2 + N^2} = \sqrt{34.6^2 + 196^2} = 199.0 \text{ N}$$

If the force F_2 makes angle α with the horizontal, then

$$\tan \alpha = \frac{N}{f} = 4\sqrt{2} = 5.6568$$

$\therefore$ $$\alpha = 80°$$

EXAMPLE 19. A particle of mass m is released from point p at $x = x_0$ on the X-axis from origin O and falls vertically along the Y-axis, as shown in Fig. 7.34.

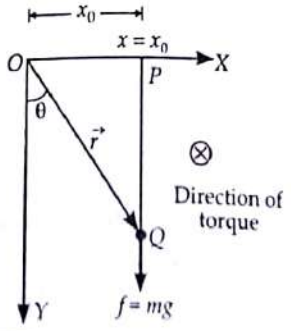

Fig. 7.34

(i) *Find the torque τ acting on the particle at a time t when it is at point Q with respect to O.*

(ii) *Find the angular momentum L of the particle about O at this time t.*

(iii) *Show that $\tau = \frac{dL}{dt}$ in this example.* [NCERT]

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Solution. (*i*) The force of gravity, $F = mg$ produces the torque τ. Let $\vec{r}$ be the position vector of Q. Then the magnitude of the torque is given by

$$\tau = rF \sin\theta$$
$$= r \times mg \times \frac{x_0}{r} = mgx_0. \quad \left[\because \sin\theta = \frac{x_0}{r}\right]$$

The direction of the torque is directed into the plane of paper and perpendicular to it, as shown by ⊗.

(*ii*) The magnitude of the angular momentum is

$$L = rp \sin\theta = rmv \sin\theta$$

But the velocity v at point Q is given by

$$v = u + at = 0 + gt = gt$$

$$\therefore \quad L = rmgt \cdot \frac{x_0}{r} = mgx_0\, t$$

The direction of angular momentum is the same as that of torque.

(*iii*) Now $L = mgx_0 t$

Differentiating both sides with respect to t, we get

$$\frac{dL}{dt} = \frac{d}{dt}(mgx_0 t) = mgx_0 = \tau$$

Hence the relation $\tau = \frac{dL}{dt}$ holds in this example.

EXAMPLE 20. An electron of mass 9×10^{-31} kg revolves in a circle of radius 0.53 Å around the nucleus of hydrogen with a velocity of 2.2×10^6 ms^{-1}. Show that its angular momentum is equal to $h/2\pi$, where h is Planck's constant of value 6.6×10^{-34} Js. [NCERT]

Solution. Here $m = 9 \times 10^{-31}$ kg,

$r = 0.53$ Å $= 0.53 \times 10^{-10}$ m,

$v = 2.2 \times 10^6$ ms^{-1}, $h = 6.6 \times 10^{-34}$ Js

Angular momentum,

$$L = mvr$$
$$= 9 \times 10^{-31} \times 2.2 \times 10^6 \times 0.53 \times 10^{-10}$$
$$= 1.0494 \times 10^{-34} \text{ Js}$$

Also, $\frac{h}{2\pi} = \frac{6.6 \times 10^{-34}}{2 \times 3.142} = 1.0503 \times 10^{-34}$ Js

Hence $L = \frac{h}{2\pi}$.

EXAMPLE 21. Show that the angular momentum of a satellite of mass M_s revolving around the earth having mass M_e in an orbit of radius r is

$$L = \sqrt{GM_e M_s^2 r}.$$

Solution. Let the satellite revolve around the earth with orbital speed v. Then

Centripetal force on the satellite = Gravitational force between the earth and the satellite

or $$\frac{M_s v^2}{r} = G\frac{M_e M_s}{r^2}$$

or $$v^2 = \frac{GM_e}{r} \quad \text{or} \quad v = \sqrt{\frac{GM_e}{r}}$$

As satellite is considered a point mass, its angular momentum is

$$L = M_s vr = M_s \sqrt{\frac{GM_e}{r}} \cdot r = \sqrt{GM_e M_s^2 r}.$$

✠ PROBLEMS FOR PRACTICE

1. Determine the angular momentum of a car of mass 1500 kg moving in a circular track of radius 50 m with a speed of 40 ms^{-1}. (**Ans.** 3×10^6 kgm^2 s^{-1})
2. Mass of an electron is 9.0×10^{-31} kg. It revolves around the nucleus of an atom in a circular orbit of radius 4.0 Å with a speed of 6.0×10^6 ms^{-1}. Calculate the angular momentum of the electron. (**Ans.** 2.16×10^{-33} kgm^2 s^{-1})

7.19 EQUILIBRIUM OF RIGID BODIES

30. *Define a rigid body. Name two kinds of motion which a rigid body can execute. What is meant by the term equilibrium ? For the equilibrium of bodies, two conditions need to be satisfied. State them.*

Equilibrium of rigid bodies. A rigid body is one for which the distances between different particles do not change, even though they move. Under the influence of an external force, a rigid body can execute two kinds of motion : (*i*) *translational motion* in which all particles move with the same velocity and (*ii*) *rotational motion* about an axis. The word 'equilibrium' is related to the Latin word 'Libra' which means equilibrium.

A rigid body is said to be in equilibrium if both the linear momentum and angular momentum of the rigid body remain constant with time. Hence for a body in equilibrium, the linear acceleration of its centre of mass would be zero and also the angular acceleration of the rigid body about any axis would be zero.

A body under the action of several forces will be in equilibrium, if it possesses the following two equilibria simultaneously :

(*i*) **Translational equilibrium.** *The resultant of all the external forces acting on the body must be zero,* otherwise they would produce linear acceleration. Hence for translation equilibrium,

$$\Sigma \vec{F}_{ext} = 0$$

or $$\Sigma F_x = 0, \quad \Sigma F_y = 0 \quad \text{and} \quad \Sigma F_z = 0$$

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Applying Newton's second law,

$$\Sigma \vec{F}_{ext} = M\vec{a}_{CM} = M\frac{d\vec{v}_{CM}}{dt} = 0$$

or $\frac{d\vec{v}_{CM}}{dt} = 0$ or $\vec{v}_{CM}$ = constant.

This implies that a body in translational equilibrium will be either at rest ($v = 0$) or in uniform motion (v = constant). If the body is at rest, it is said to be in ***static equilibrium***. It the body is in uniform motion along a straight path, it is in ***dynamic equilibrium***.

(*ii*) **Rotational equilibrium.** *The resultant of torques due to all the forces acting on the body about any point must be zero*, otherwise they would produce angular acceleration. Hence for rotational equilibrium,

$$\Sigma \vec{\tau}_{ext} = \Sigma \vec{r}_i \times \vec{F}_i^{ext} = 0$$

For rotational equilibrium, the choice of the point of rotation (or fixed point) is unimportant. If the total torque is zero about any point, then it will be zero about any other point when the body is in equilibrium.

31. *Obtain an expression for the work done on a rigid body executing both translational and rotational motions. Hence deduce the condition for the equilibrium of the rigid body.*

Expression for work done in combined rotational and translational motions. A rigid body can have only two kinds of motion : translational and rotational. In translational motion, all the particles of the rigid body move with the same velocity v_0, without rotating.

In small time interval Δt, each particle covers a displacement given by

$$\Delta\vec{s} = \vec{v}_0\,\Delta t$$

In rotational motion, every particle of the rigid body rotates about the axis of rotation with the same angular velocity $\vec{\omega}$. As shown in Fig. 7.35, we choose the origin O on the axis of rotation. Clearly, the particle velocity $\vec{v}$ has magnitude,

$$v = r_\perp\,\omega = \omega r \sin\theta$$

where $r_\perp = r\sin\theta$ is the perpendicular distance of the particle from the axis of rotation and θ is the angle between $\vec{r}$ and the axis of rotation. In vector notation,

$$\vec{v} = \vec{\omega} \times \vec{r}$$

The direction of $\vec{v}$ is perpendicular to both $\vec{\omega}$ and $\vec{r}$. The direction of $\vec{\omega}$ is determined by right hand rule. If we curl the fingers of our right in the direction of rotation of the particle, then the extended thumb gives the direction of the angular velocity vector $\vec{\omega}$.

Displacement covered by the rotating particle in small time interval Δt

$$= \vec{v}\,\Delta t = \vec{\omega} \times \vec{r}\,\Delta t = (\vec{\omega}\,\Delta t) \times \vec{r} = \Delta\vec{\phi} \times \vec{r}$$

where $\Delta\vec{\phi} = \vec{\omega}\,\Delta t$ = the small angular displacement.

So when a rigid body rotates with angular velocity $\vec{\omega}$ and translates with velocity $\vec{v}_0$, the displacement of any point of $\vec{r}$ of the rigid body is given by

$$\Delta\vec{r} = \vec{v}_0\Delta t + \vec{\omega}\Delta t \times \vec{r} = \Delta\vec{s} + \Delta\vec{\phi} \times \vec{r}$$

The work done by an external force $\vec{F}$ acting on point $\vec{r}$ is given by

$$\Delta W = \vec{F}\,.\,\Delta\vec{r} = \vec{F}\,.\,(\Delta\vec{s} + \Delta\vec{\phi} \times \vec{r})$$

$$= \vec{F}\,.\,\Delta\vec{s} + \vec{F}\,.\,(\Delta\vec{\phi} \times \vec{r})$$

As the scalar triple product is cyclic, so

$$\vec{F}\,.\,(\Delta\vec{\phi} \times \vec{r}) = \vec{r}\,.\,(\vec{F} \times \Delta\vec{\phi}) = \Delta\vec{\phi}\,.\,(\vec{r} \times \vec{F})$$

$$= \Delta\vec{\phi}\,.\,\vec{\tau} = \vec{\tau}\,.\,\Delta\vec{\phi}$$

where $\vec{\tau} = \vec{r} \times \vec{F}$, is the torque acting on the particle.

$$\therefore \quad \Delta W = \vec{F}\,.\,\Delta\vec{s} + \vec{\tau}\,.\,\Delta\vec{\phi}$$

When a number of forces $\vec{F}_i$ act on different points $\vec{r}_i$ of the rigid body, the total work done on the rigid body will be

$$W = (\Sigma\vec{F}_i)\,.\,\Delta\vec{s} + (\Sigma\vec{\tau}_i)\,.\,\Delta\vec{\phi}$$

For the rigid body to be in equilibrium, the work done in this displacement plus rotation should be zero

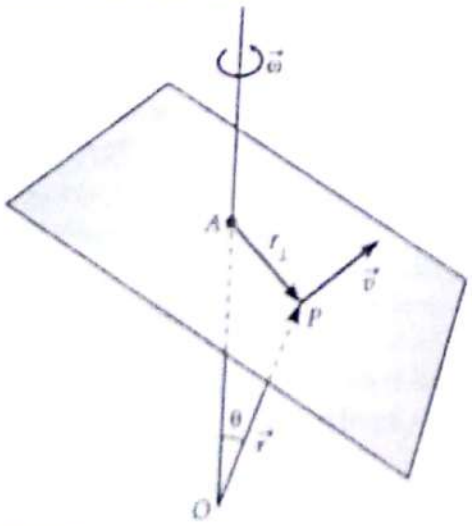

Fig. 7.35 Rotational motion of any particle of a rigid body.

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...all choices of $\Delta\vec{s}$ and $\Delta\vec{\theta}$. We therefore obtain the condition :

$$\Sigma \vec{F}_i = 0 \quad \text{and} \quad \Sigma \vec{\tau}_i = 0$$

Hence for a rigid body in equilibrium, the sum of the forces acting on it must be zero and the sum of the torques acting on it must be zero.

32. *Distinguish between stable, unstable and neutral equilibria of a body.*

(*i*) **Stable equilibrium.** *A body is said to be in stable equilibrium if it tends to regain its equilibrium position after being slightly displaced and released.* In stable equilibrium, a body has *minimum potential energy* and its centre of mass goes higher when it is slightly displaced.

(*ii*) **Unstable equilibrium.** *A body is said to be in unstable equilibrium if it gets further displaced from its equilibrium position after being slightly displaced and released.* In unstable equilibrium, a body possesses *maximum potential energy* and its centre of mass goes lower on being slightly displaced.

(*iii*) **Neutral equilibrium.** *If a body stays in equilibrium position even after being slightly displaced and released, it is said to be in neutral equilibrium.* When a body is slightly displaced from its position of neutral equilibrium, its centre of mass is neither raised nor lowered and its potential energy remains constant.

7.20 MOMENT OF INERTIA AND ITS PHYSICAL SIGNIFICANCE

33. *Define moment of inertia of a body. Give its units and dimensions. Explain the physical significance of moment of inertia.*

Moment of inertia. According to Newton's first law of motion, every body continues in its state of rest or of uniform linear motion, unless an external force acts on it to change that state. This inability of a body to change by itself its state of rest or of linear uniform motion is called *inertia*. Similarly, a body rotating about a given axis tends to maintain its state of uniform rotation, unless an external torque is applied on it to change that state. This property of a body by virtue of which it opposes the torque tending to change its state of rest or of uniform rotation about an axis is called *rotational inertia or moment of inertia.*

The moment of inertia of a rigid body about a fixed axis is defined as the sum of the products of the masses of the particles constituting the body and the squares of their respective distances from the axis of rotation.

Consider a rigid body rotating with uniform angular velocity ω about a vertical axis through O, as shown in Fig. 7.36. Suppose the body consists of n particles of masses $m_1, m_2, m_3, \ldots, m_n$ situated at

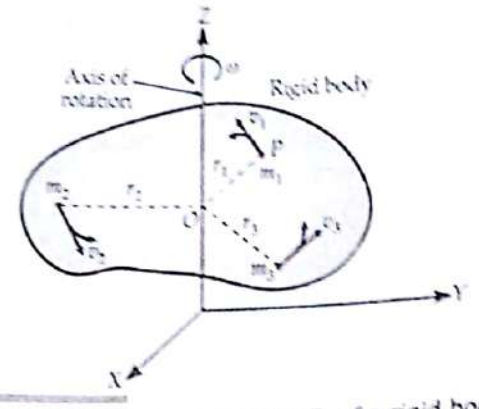

Fig. 7.36 M.I. and rotational K.E. of a rigid body.

distances $r_1, r_2, r_3, \ldots, r_n$ respectively from the axis of rotation. The moment of inertia of the body about the axis OZ is given by

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \ldots + m_n r_n^2$$

or

$$I = \sum_{i=1}^{n} m_i r_i^2$$

The dimensional formula of moment of inertia is $[ML^2T^0]$. The SI unit of moment of inertia is kg m^2 and its CGS unit is g cm^2.

Physical significance of moment of inertia. The mass of a body resists a change in its state of linear motion, it is a measure of its inertia in linear motion. Similarly, the moment of inertia of a body about an axis of rotation resists a change in its rotational motion. The greater the moment of inertia of a body, the greater is the torque required to change its state of rotation. Thus moment of inertia of a body can be regarded as the measure of rotational inertia of the body. The moment of inertia of a body plays the same role in the rotational motion as the mass plays in linear motion. That is why moment of inertia is called the rotational analogue of mass in linear motion.

34. *State the factors on which the moment of inertia of a body depends.*

Factors on which the moment of inertia depends. The moment of inertia of a body is the measure of the manner in which its different parts are distributed at different distances from the axis of rotation. Unlike mass, it is not a fixed quantity as it depends on the position and orientation of the axis of rotation with respect to the body as a whole.

The moment of inertia of a body depends on

(*i*) Mass of the body

(*ii*) Size and shape of the body.

(*iii*) Distribution of mass about the axis of rotation.

(*iv*) Position and orientation of the axis of rotation w.r.t. the body.

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35. *Mention some practical applications which make use of the property of moment of inertia.*

Practical applications of moment of inertia. (*i*) A heavy wheel, called *flywheel*, is attached to the shaft of steam engine, automobile engine, etc. Because of its large moment of inertia, the flywheel opposes the sudden increase or decrease of the speed of the vehicle. It allows a gradual change in the speed and prevents jerky motions and hence ensures smooth ride for the passengers.

(*ii*) In a bicycle, bullock-cart, etc., the moment of inertia is increased by concentrating most of the mass at the rim of the wheel and connecting the rim to the axle through the spokes. Even after we stop paddling, the wheels of a bicycle continue to rotate for some time due to their large moment of inertia.

7.21 RELATION BETWEEN ROTATIONAL K.E. AND MOMENT OF INERTIA

36. *A body is rotating with uniform angular velocity ω about an axis. Establish the formula for its kinetic energy of rotation. Define moment of inertia of the body with respect to the axis of rotation on this basis.*

Relation between rotational kinetic energy and moment of inertia. As shown in Fig. 7.36, consider a rigid body rotating about an axis OZ with uniform angular velocity ω. The body may be assumed to consist of n particles of masses $m_1, m_2, m_3, \ldots, m_n$; situated at distances $r_1, r_2, r_3, \ldots, r_n$ from the axis of rotation. As the angular velocity ω of all the n particles is same, so their linear velocities are

$$v_1 = r_1\omega,\ v_2 = r_2\omega,\ v_3 = r_3\omega, \ldots,\ v_n = r_n\omega$$

Hence the total kinetic energy of rotation of the body about the axis OZ is

Rotational K.E.

$$= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_3v_3^2 + \ldots + \frac{1}{2}m_nv_n^2$$

$$= \frac{1}{2}m_1r_1^2\omega^2 + \frac{1}{2}m_2r_2^2\omega^2 + \frac{1}{2}m_3r_3^2\omega^2 + \ldots + \frac{1}{2}m_nr_n^2\omega^2$$

$$= \frac{1}{2}(m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \ldots + m_nr_n^2)\omega^2$$

$$= \frac{1}{2}(\Sigma mr^2)\omega^2$$

But $\Sigma mr^2 = I$, the moment of inertia of the body about the axis of rotation.

$$\therefore \text{ Rotational K.E. } = \frac{1}{2}I\omega^2$$

When $\omega = 1$, rotational K.E. $= \frac{1}{2}I$

or $I = 2 \times$ Rotational K.E.

Hence the moment of inertia of a rigid body about an axis of rotation is numerically equal to twice the rotational kinetic energy of the body when it is rotating with unit angular velocity about that axis.

7.22 RADIUS OF GYRATION

37. *Define radius of gyration of a body rotating about an axis. Derive an expression for it. On what factors does it depend ?*

Radius of gyration. For any body capable of rotation about a given axis, it is possible to find a radial distance from the axis where, if whole mass of the body is concentrated, its moment of inertia will remain unchanged. This radial distance is called **radius of gyration** and is denoted by k.

The radius of gyration of a body about its axis of rotation may be defined as the distance from the axis of rotation at which, if the whole mass of the body were concentrated, its moment of inertia about the given axis would be the same as with the actual distribution of mass.

The radius of gyration k is a geometrical property of the body and the axis of rotation. It gives a measure of the manner in which the mass of a rotating body is distributed with respect to the axis of rotation.

k has the dimensions of length L and is measured in metre or cm.

Expression for k. Suppose a rigid body consists of n particles of mass m each, situated at distances $r_1, r_2, r_3, \ldots, r_n$ from the axis of rotation AB.

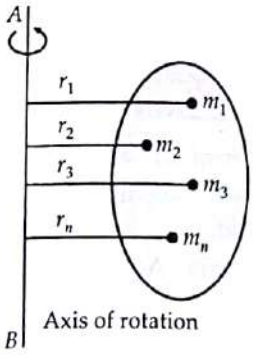

Fig. 7.37 Radius of gyration.

The moment of inertia of the body about the axis AB is

$$I = mr_1^2 + mr_2^2 + mr_3^2 + \ldots + mr_n^2$$

$$= m(r_1^2 + r_2^2 + r_3^2 + \ldots + r_n^2)$$

$$= m \times n\,\frac{(r_1^2 + r_2^2 + r_3^2 + \ldots + r_n^2)}{n}$$

or $$I = M\,\frac{(r_1^2 + r_2^2 + r_3^2 + \ldots + r_n^2)}{n}$$

where $M = m \times n =$ total mass of the body.

If k is the radius of gyration about the axis AB, then

$$I = Mk^2$$

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$$Mk^2 = M\left(\frac{r_1^2 + r_2^2 + r_3^2 + \ldots + r_n^2}{n}\right)$$

$$\therefore \quad k = \sqrt{\frac{r_1^2 + r_2^2 + r_3^2 + \ldots + r_n^2}{n}}$$

or = Root mean square distance

Hence the radius of gyration of a body about an axis of rotation may also be defined as the root mean square distance of its particles from the axis of rotation.

Factors on which radius of gyration of a body depends :

(i) Position and direction of the axis of rotation.

(ii) Distribution of mass about the axis of rotation.

7.23 THEOREMS OF PARALLEL AND PERPENDICULAR AXES

38. *State and prove the theorem of perpendicular axes.*

Theorem of perpendicular axes. *The moment of inertia of a plane lamina about an axis perpendicular to its plane is equal to the sum of the moments of inertia of the lamina about any two mutually perpendicular axes in its own plane and intersecting each other at the point where the perpendicular axis passes through the lamina.*

Proof. Consider a plane lamina lying in the XOY plane. It can be assumed to be made up of large number of particles. Consider one such particle of mass m situated at point $P(x, y)$. Clearly, the distances of the particle from X-, Y- and Z-axes are y, x and r respectively such that

$$r^2 = y^2 + x^2$$

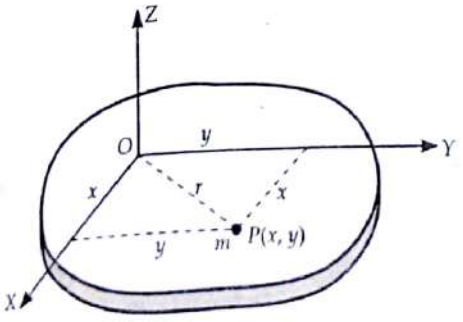

Fig. 7.38 Theorem of perpendicular axes.

Moment inertia of the particle about X-axis

$$= my^2$$

$\therefore$ Moment of inertia of whole lamina about X-axis is

$$I_x = \Sigma my^2$$

Moment of inertia of whole lamina about Y-axis is

$$I_y = \Sigma mx^2$$

Moment of inertia of whole lamina about Z-axis is

$$I_z = \Sigma mr^2 = \Sigma m(y^2 + x^2)$$

$$= \Sigma my^2 + \Sigma mx^2$$

or $$I_z = I_x + I_y$$

This proves the theorem of perpendicular axes.

39. *State and prove the theorem of parallel axes.*

Theorem of parallel axes. *The moment of inertia of a body about any axis is equal to its moment of inertia about a parallel axis through its centre of mass plus the product of the mass of the body and the square of the perpendicular distance between the two parallel axes.*

Proof. Let I be the moment of inertia of a body of mass M about an axis PQ. Let RS be a parallel axis passing through the centre of mass C of the body and at distance d from PQ. Let I_{CM} be the moment of inertia of the body about the axis RS.

Consider a particle P of mass m at distance x from RS and so at distance $(x + d)$ from PQ.

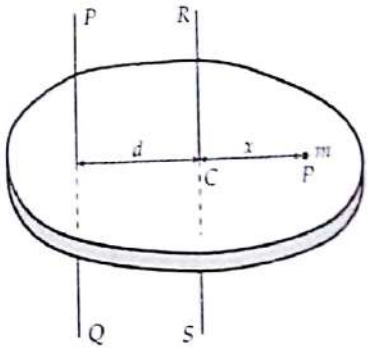

Fig. 7.39 Theorem of parallel axes.

Moment of inertia of the particle about axis PQ

$$= m(x + d)^2$$

$\therefore$ Moment of inertia of the whole body about the axis PQ is

$$I = \Sigma m(x + d)^2 = \Sigma m(x^2 + d^2 + 2xd)$$

$$= \Sigma mx^2 + \Sigma md^2 + \Sigma 2mxd$$

Now $\Sigma mx^2 = I_{CM}$

$$\Sigma md^2 = (\Sigma m)d^2 = Md^2$$

$$\Sigma 2mxd = 2d\,\Sigma mx = 2d \times 0 = 0$$

This is because a body can balance itself about its centre of mass, so the algebraic sum of moments (Σmx) of masses of all its particles about the axis RS is zero.

Hence $$I = I_{CM} + Md^2$$

This proves the theorem of parallel axes.

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7.24 ▼ MOMENT OF INERTIA OF A THIN CIRCULAR RING

40. *Derive an expression for the moment of inertia of a thin uniform circular ring about (a) an axis through its centre and perpendicular to its plane (b) its diameter (c) a tangent in its plane (d) a tangent perpendicular to its plane.*

(*a*) **M.I. of a ring about an axis through its centre and perpendicular to its plane.** Consider a thin uniform circular ring of radius R and mass M. As shown in Fig. 7.40, we wish to determine its moment of inertia I about an axis YY' passing through its centre O and perpendicular to it. The ring can be imagined to be made of a large number of small elements. Consider one such element of length dx.

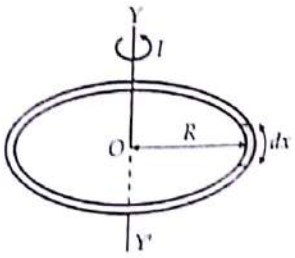

Fig. 7.40 M.I. of a ring about central axis.

Length of the ring = circumference $= 2\pi R$

Mass per unit length of ring $= \dfrac{M}{2\pi R}$

Mass of the small element $= \dfrac{M}{2\pi R}\,dx$

Moment of inertia of the small element about the axis YY',

$$dI = \left(\frac{M}{2\pi R}\,dx\right) R^2 = \frac{MR}{2\pi}\,dx$$

The small elements lie along the entire circumference of the ring *i.e.*, from $x = 0$ to $x = 2\pi R$. Hence the moment of inertia of the whole ring about the axis YY' will be

$$I = \int_0^{2\pi R} \frac{MR}{2\pi}\,dx = \frac{MR}{2\pi}\int_0^{2\pi R} dx$$

$$= \frac{MR}{2\pi}\left[\,x\,\right]_0^{2\pi R} = \frac{MR}{2\pi}(2\pi R - 0)$$

or $$I = MR^2.$$

(*b*) **M.I. of a ring about any diameter.** According to the theorem of perpendicular axes, the moment of inertia about an axis YY' through O and perpendicular to the ring is equal to sum of its moments of inertia about two perpendicular diameters AB and CD, as shown in Fig. 7.41

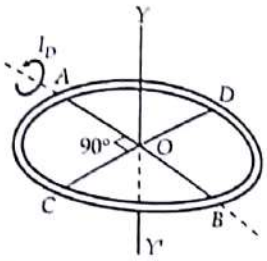

Fig. 7.41 M.I. of a ring about any diameter.

$$I_{AB} + I_{CD} = I_{YY'},$$

$$I_D + I_D = MR^2$$

or $$I_D = \tfrac{1}{2}MR^2.$$

Here I_D is the M.I. of the ring about any diameter.

(*c*) **M.I. of a ring about a tangent in its plane.** Refer to Fig. 7.42. Let I_T be the moment of inertia of the ring about the tangent EBF. Applying the theorem of parallel axes, we get

$$I_T = \text{M.I. about diameter } CD + MR^2$$

$$= \tfrac{1}{2}MR^2 + MR^2$$

or $$I_T = \tfrac{3}{2}MR^2.$$

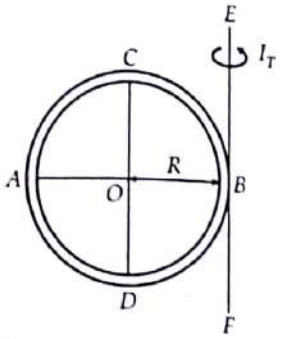

Fig. 7.42 M.I. of a ring about a tangent in its plane.

(*d*) **M.I. of a ring about a tangent perpendicular to its plane.** Let I'_T be the moment of inertia of the ring

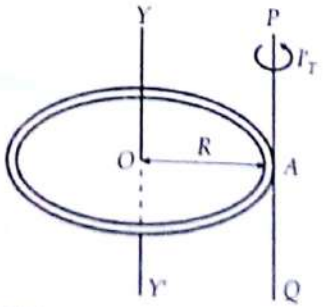

Fig. 7.43 M.I. of a ring about a tangent perpendicular to its plane.

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about the axis PAQ tangent to the plane of the ring. Applying the theorem of parallel axes,

$$I_{PQ} = I_{YY'} + MR^2$$
$$= MR^2 + MR^2$$
$$I_T = 2\,MR^2.$$

Note We can determine the radius of gyration (k) of the ring about any axis by equating its M.I. about that axis to Mk^2. For example, the radius of gyration of a thin ring about any diameter is given by

$$I_D = \frac{1}{2} MR^2 = Mk^2$$
$$k = R/\sqrt{2}.$$

7.25 MOMENT OF INERTIA OF A UNIFORM CIRCULAR DISC

41. *Derive an expression for the moment of inertia of a disc about (a) an axis through the centre and perpendicular to its plane, (b) its diameter, (c) a tangent in its own plane, (d) a tangent perpendicular to its plane.*

(*a*) **M.I. of a circular disc about an axis through its centre and perpendicular to its plane.** As shown in Fig. 7.44, consider a uniform disc of mass M and radius R. Suppose YY' is an axis passing through the centre O of the disc and perpendicular to its plane.

Area of the disc $= \pi R^2$

Mass per unit area of the disc $= \dfrac{M}{\pi R^2}$

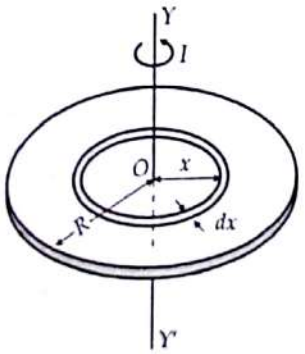

Fig. 7.44 M.I. of a disc about a central axis.

We can imagine the disc to be made up of a large number of concentric rings, whose radii vary from O to R. Let us consider one such concentric ring of radius x and width dx.

Area of the ring

= Circumference × Width $= 2\pi x \times dx$

Mass of the concentric ring,

$$m = \left(\frac{M}{\pi R^2}\right) 2\pi x\, dx = \frac{2\,M\,x\,dx}{R^2}$$

Moment of inertia of the concentric ring about the axis YY'

$$dI = mx^2 = \frac{2\,Mx\,dx}{R^2} \times x^2 = \frac{2\,Mx^3\,dx}{R^2}$$

The moment of inertia of the whole disc about the axis YY' can be obtained by integrating the above expression between the limits 0 to R.

$$I = \int_0^R \frac{2\,Mx^3\,dx}{R^2} = \frac{2\,M}{R^2}\int_0^R x^3\,dx$$

$$= \frac{2\,M}{R^2}\left[\frac{x^4}{4}\right]_0^R = \frac{2\,M}{4\,R^2}[R^4 - 0] = \frac{M}{2\,R^2} \times R^4$$

or $$I = \frac{1}{2} MR^2$$

(*b*) **M.I. of a disc about any diameter.** In Fig. 7.45, AB and CD are two mutually perpendicular diameters in the plane of the disc. Applying the theorem of perpendicular axes, we get

$$I_{AB} + I_{CD} = I_{YY'}$$

or $$I_D + I_D = \frac{1}{2} MR^2$$

or $$I_D = \frac{1}{4} MR^2$$

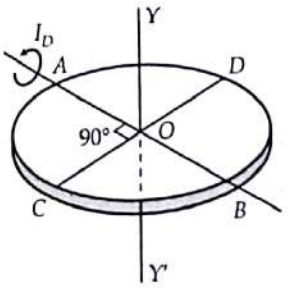

Fig. 7.45 M.I. of disc about any diameter.

(*c*) **M.I. of a disc about a tangent in its plane.** Let I_T the moment of inertia of the disc about a tangent EBF in the plane of the disc. This tangent is parallel to the diameter CD of the disc. Applying the *theorem of parallel axes*, we get

I_T = Moment of inertia of disc about $CD + MR^2$

or $$I_T = \frac{1}{4} MR^2 + MR^2 = \frac{5}{4} MR^2$$

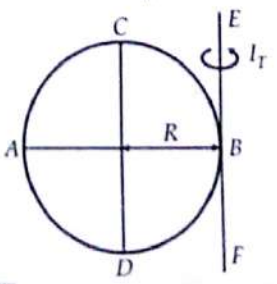

Fig. 7.46 M.I. of a disc about a tangent in its plane.

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(*d*) **M.I. of a disc about a tangent perpendicular to its plane.** In Fig. 7.47, let I_T' be the moment of inertia of disc about the tangent PAQ perpendicular to the plane of the disc. Applying the theorem of parallel axes, we get

$$I_{PQ} = I_{YY'} + MR^2 = \frac{1}{2} MR^2 + MR^2$$

or $$I_T' = \frac{3}{2} MR^2$$

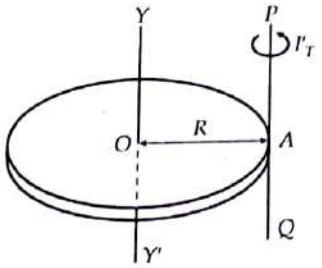

Fig. 7.47 M.I. of a disc about a tangent perpendicular to its plane.

Moreover, the radius of gyration (k) in this case is given by

$$Mk^2 = \frac{3}{2} MR^2 \quad \text{or} \quad k = \sqrt{\frac{3}{2}}\, R.$$

7.26 MOMENT OF INERTIA OF A THIN UNIFORM ROD

42. *Derive an expression for the moment of inertia of a thin uniform rod about an axis through its centre and perpendicular to its length. Also determine the radius of gyration about the same axis.*

M.I. of a thin uniform rod about a perpendicular axis through its centre. Consider a thin uniform rod AB of length L and mass M, free to rotate about an axis YY' through its centre O and perpendicular to its length.

$\therefore$ Mass per unit length of rod $= \dfrac{M}{L}$

Consider a small mass element of length dx at a distance x from O.

Mass of the small element $= \dfrac{M}{L}\, dx$

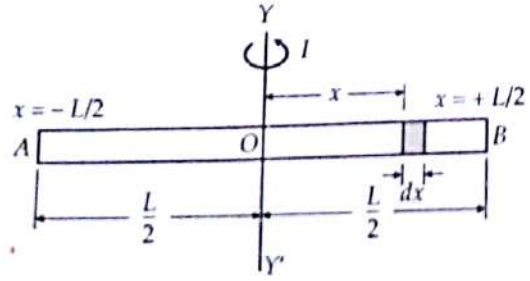

Fig. 7.48 M.I. of a rod about an axis through its centre.

Moment of inertia of the small element about YY',

$$dI = \text{Mass} \times (\text{distance})^2 = \frac{M}{L}\, dx \times x^2$$

The moment of inertia of the whole rod about the axis YY' can be obtained by integrating the above expression between the limits $x = -L/2$ and $x = +L/2$.

$$\therefore \quad I = \int dI = \int_{-L/2}^{+L/2} \frac{M}{L} x^2\, dx$$

$$= \frac{M}{L} \int_{-L/2}^{+L/2} x^2\, dx = \frac{M}{L}\left[\frac{x^3}{3}\right]_{-L/2}^{+L/2}$$

$$= \frac{M}{3L}\left[\left(\frac{L}{2}\right)^3 - \left(-\frac{L}{2}\right)^3\right]$$

$$= \frac{M}{3L}\left[\frac{L^3}{8} + \frac{L^3}{8}\right] = \frac{M}{3L} \times \frac{L^3}{4}$$

or $$I = \frac{ML^2}{12}$$

Radius of gyration. Let k be the radius of gyration of the rod about the axis YY'. Then

$$I = Mk^2$$

$$\therefore \quad Mk^2 = \frac{ML^2}{12} \quad \text{or} \quad k^2 = \frac{L^2}{12}$$

or $$k = \frac{L}{2\sqrt{3}}.$$

Thus, the radius of gyration of a uniform thin rod *rotating about an axis passing through its centre and perpendicular to its length is* $L/2\sqrt{3}$.

43. *Derive an expression for the moment of inertia of a thin uniform rod about an axis passing through its one end and perpendicular to its length. Also determine its radius of gyration about the same axis.*

M.I. of a thin uniform rod about a perpendicular axis through its one end. Consider a thin uniform rod AB of length L and mass M, free to rotate about an axis YY' passing through its one end A and perpendicular to its length, as shown in Fig. 7.49.

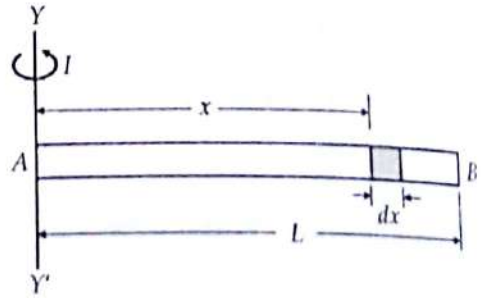

Fig. 7.49 M.I. of a rod about an axis through its one end.

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Mass per unit length of the rod $= \frac{M}{L}$

Consider a small element of length dx of the rod at a distance x from the end A.

Mass of the small element $= \frac{M}{L}\,dx$

Moment of inertia of the small element about the axis YY',

$$dI = \text{Mass} \times (\text{distance})^2 = \frac{M}{L}\,dx \,.\, x^2$$

The moment of inertia of the whole rod about the axis YY' can be obtained by integrating the above expression between the limits $x = 0$ and $x = L$

$$I = \int dI = \int_0^L \frac{M}{L}\,dx \,.\, x^2 = \frac{M}{L}\int_0^L x^2\,dx$$

$$= \frac{M}{L}\left[\frac{x^3}{3}\right]_0^L = \frac{M}{3L}[x^3]_0^L = \frac{M}{3L}[L^3 - 0] = \frac{ML^3}{3L}$$

or $$I = \frac{ML^2}{3}.$$

Radius of gyration. Let k be the radius of gyration of the rod about the axis YY'. Then

$$\frac{ML^2}{3} = Mk^2$$

$$k^2 = \frac{L^2}{3}$$

or $$k = \frac{L}{\sqrt{3}}$$

or

Thus the radius of gyration of the rod about an axis passing through its one end and perpendicular to its length is $L/\sqrt{3}$.

7.27 MOMENT OF INERTIA OF A CYLINDER

44. *Write an expression for the moment of inertia of a hollow cylinder of mass M and radius R about its own axis.*

M.I. of a hollow cylinder about its own axis. Consider a hollow cylinder of mass M and radius R. As shown in Fig. 7.50, every element of the cylinder is at the same perpendicular distance R from its axis. Hence the moment of inertia of the hollow cylinder about its own axis is

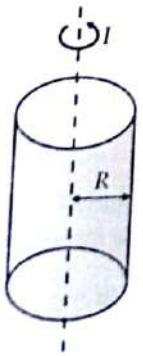

Fig. 7.50 M.I. of a hollow cylinder about its own axis.

$$I = \int r^2\,dm = \int R^2\,dm$$

$$= R^2 \int dm = R^2 \times M$$

or $$I = MR^2.$$

45. *Derive an expression for the moment of inertia of a uniform solid cylinder about its own axis.*

Moment of inertia of uniform solid cylinder about its own axis. Consider a solid cylinder of mass M, radius R and length L. We wish to determine its moment of inertia about its own axis YY'.

Volume of the cylinder

$$= \pi R^2 L$$

Mass per unit volume of the cylinder,

$$\rho = \frac{M}{\pi R^2 L}$$

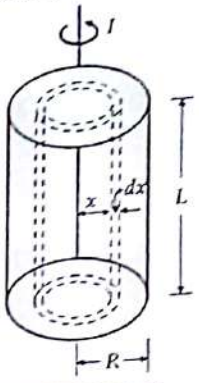

Fig. 7.51 M.I. of a solid cylinder about its own axis.

We can imagine the cylinder to be made of a large number of coaxial cylindrical shells. Consider one such cylindrical shell of inner radius x and outer radius $x + dx$, as shown in Fig. 7.51. The cross-section of the shell is a ring of radius x and thickness dx.

$\therefore$ Cross-sectional area of the cylindrical shell

$$= \text{Circumference} \times \text{thickness} = 2\pi x\,dx$$

Volume of the cylindrical shell

$$= \text{Cross-sectional area} \times \text{length} = 2\pi x\,dx \times L$$

Mass of the cylindrical shell,

$$m = \text{Volume} \times \rho = 2\pi x L\,dx \times \frac{M}{\pi R^2 L} = \frac{2M}{R^2}\,x\,dx$$

As the mass of the shell is distributed at the same distance x from its axis, so its moment of inertia about the axis YY' is

$$dI = mx^2 = \frac{2M}{R^2}\,x\,dx \times x^2 = \frac{2M}{R^2}\,x^3\,dx$$

The moment of inertia of the solid cylinder can be obtained by integrating the above expression between the limits $x = 0$ and $x = R$.

$$\therefore \quad I = \int dI = \int_0^R \frac{2M}{R^2}\,x^3\,dx$$

$$= \frac{2M}{R^2}\int_0^R x^3\,dx = \frac{2M}{R^2}\left[\frac{x^4}{4}\right]_0^R = \frac{2M}{4R^2}[R^4 - 0]$$

or $$I = \frac{1}{2}MR^2.$$

Obviously, the moment of inertia of a cylinder about its own axis does not depend on its length.

46. *Derive an expression for the moment of inertia of a uniform solid cylinder about an axis passing through its centre and perpendicular to its length.*

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M.I. of a solid cylinder about an axis through its centre and perpendicular to its axis. Consider a uniform solid cylinder of mass M, radius R and length L. We wish to determine its moment of inertia about an axis YY' passing through its centre O and perpendicular to its length.

$$\text{Mass per unit length} = \frac{M}{L}$$

We can imagine the cylinder to be made up of a large number of thin circular discs placed perpendicular to the axis of the cylinder. As shown in Fig. 7.52, consider one such thin disc of thickness dx and at distance x from the centre O.

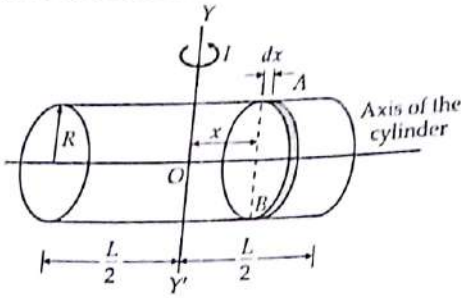

Fig. 7.52 M.I. of a cylinder about an axis through its centre and perpendicular to its axis.

$$\text{Mass of the elementary disc} = \frac{M}{L}\,dx$$

$$\text{Radius of the elementary disc} = R^2$$

Moment of inertia of the elementary disc about the diameter AB

$$= \tfrac{1}{4}\,\text{Mass} \times \text{radius}^2 = \frac{1}{4}\cdot\frac{M}{L}\,dx \times R^2 = \frac{MR^2}{4L}\,dx$$

Applying the theorem of parallel axes, the moment of inertia of the elementary disc about the axis YY',

$$dI = \text{M.I. about the diameter } AB + \text{Mass} \times x^2$$

$$= \frac{MR^2}{4L}\,dx + \frac{M}{L}\,dx \times x^2 = \frac{M}{L}\left(\frac{R^2}{4} + x^2\right)dx$$

The moment of inertia of the cylinder about the axis YY' can be obtained by integrating the above expression between the limits $x = 0$ and $x = L/2$ and multiplying the result by 2 to cover both halves of the cylinder.

Thus
$$I = 2\int dI = 2\int_0^{L/2} \frac{M}{L}\left(\frac{R^2}{4} + x^2\right)dx$$

$$= \frac{2M}{L}\left[\frac{R^2}{4}\int_0^{L/2} dx + \int_0^{L/2} x^2\,dx\right]$$

$$= \frac{2M}{L}\left[\frac{R^2}{4}\,|x|_0^{L/2} + \left|\frac{x^3}{3}\right|_0^{L/2}\right]$$

$$= \frac{2M}{L}\left[\frac{R^2}{4}\left(\frac{L}{2} - 0\right) + \left(\frac{(L/2)^3}{3} - 0\right)\right]$$

$$= \frac{2M}{L}\left[\frac{R^2}{4}\cdot\frac{L}{2} + \frac{L^2}{24}\right]$$

or
$$I = M\left[\frac{R^2}{4} + \frac{L^2}{12}\right].$$

7.28 MOMENT OF INERTIA OF A SOLID SPHERE

47. *Derive an expression for the moment of inertia of a uniform solid sphere about its any diameter. Hence write the expression for its moment of inertia about its tangent.*

Moment of inertia of a solid sphere about its diameter. Consider a uniform solid sphere of mass M and radius R. We wish to determine its moment of inertia about diameter AB.

$$\text{Volume of the sphere} = \frac{4}{3}\pi R^3$$

$$\text{Mass per unit volume, } \rho = \frac{3M}{4\pi R^3}$$

We can imagine the sphere to be made up of a large number of thin slices placed perpendicular to the diameter AB. Consider one such slice of thickness dx placed at distance x from the centre O.

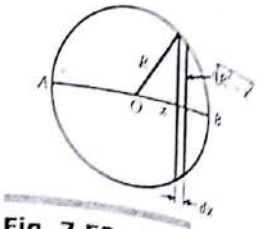

Fig. 7.53 M.I. of a sphere about a diameter.

$$\text{Radius of the elementary slice} = \sqrt{R^2 - x^2}$$

Volume of the elementary slice

$$= \text{Area} \times \text{thickness}$$

$$= \pi\left(\sqrt{R^2 - x^2}\right)^2 \times dx = \pi(R^2 - x^2)\,dx$$

Mass of the elementary slice

$$= \text{Volume} \times \rho = \pi(R^2 - x^2)\,dx \times \frac{3M}{4\pi R^3}$$

$$= \frac{3M(R^2 - x^2)\,dx}{4R^3}$$

Moment of inertia of the thin slice about the axis AB passing through its centre and perpendicular to its plane,

$$dI = \frac{1}{2}\,\text{Mass} \times (\text{radius})^2$$

$$= \frac{1}{2}\cdot\frac{3M(R^2 - x^2)\,dx}{4R^3}\cdot(R^2 - x^2)$$

$$= \frac{3M(R^2 - x^2)^2\,dx}{8R^3}$$

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The moment of inertia of the whole sphere about the diameter AB can be obtained by integrating the above expression between the limits $x = 0$ and $x = R$ and multiplying the result by 2 to include both halves of the sphere.

$$I = 2\int dI = 2\int_0^R \frac{3M(R^2 - x^2)^2\,dx}{8R^3}$$

$$= \frac{2 \times 3M}{8R^3}\int_0^R (R^2 - x^2)^2\,dx$$

$$= \frac{3M}{4R^3}\int_0^R (R^4 - 2R^2x^2 - x^4)\,dx$$

$$= \frac{3M}{4R^3}\left[R^4\int_0^R dx - 2R^2\int_0^R x^2\,dx + \int_0^R x^4\,dx\right]$$

$$= \frac{3M}{4R^3}\left[R^4|x|_0^R - 2R^2\left|\frac{x^3}{3}\right|_0^R + \left|\frac{x^5}{5}\right|_0^R\right]$$

$$= \frac{3M}{4R^3}\left[R^4(R-0) - 2R^2\left(\frac{R^3}{3} - 0\right) + \left(\frac{R^5}{5} - 0\right)\right]$$

$$= \frac{3M}{4R^3}\left[R^5 - \frac{2}{3}R^5 + \frac{R^5}{5}\right] = \frac{3M}{4R^3} \times \frac{8}{15}R^5$$

or $$I = \frac{2}{5}MR^2.$$

Moment of inertia of the solid sphere about a tangent. Applying the theorem of parallel axes, the moment of inertia of a solid sphere about a tangent is given by

$$I_T = \text{M.I. about a diameter} + \text{Mass} \times (\text{radius})^2$$

$$= \frac{2}{5}MR^2 + MR^2$$

or $$I_T = \frac{7}{5}MR^2.$$

Examples based on Moment of Inertia, Radius of Gyration and Rotational K.E.

Formulae Used

1. Moment of inertia of a body about the given axis of rotation,

$$I = m_1 r_1^2 + m_2 r_2^2 + \ldots + m_n r_n^2 = \sum_{i=1}^{n} m_i r_i^2.$$

2. Radius of gyration K is given by

$$I = MK^2 \text{ or } K = \sqrt{\frac{I}{M}}.$$

When all the particles are of same mass,

$$K = \sqrt{\frac{r_1^2 + r_2^2 + \ldots + r_n^2}{n}}$$

3. Theorem of parallel axes : $I_z = I_x + I_y$
4. Theorem of perpendicular axes, $I = I_{CM} + Md^2$
5. M.I. of a circular ring about an axis through its centre and perpendicular to its plane, $I = MR^2$
6. M.I. of a thin ring about any diameter, $I = \frac{1}{2}MR^2$
7. M.I. of a thin ring about any tangent in its plane, $I = \frac{3}{2}MR^2$
8. M.I. of a circular disc about an axis through its centre and perpendicular to its plane, $I = \frac{1}{2}MR^2$
9. M.I. of a circular disc about any diameter, $I = \frac{1}{4}MR^2$
10. M.I. of a circular disc about a tangent in its plane, $I = \frac{5}{4}MR^2$
11. M.I. of a thin rod about an axis through its middle point and perpendicular to rod, $I = \frac{1}{12}ML^2$
12. M.I. of a thin rod about an axis through its one end and perpendicular to rod, $I = \frac{1}{3}ML^2$
13. M.I. of a rectangular lamina of sides l and b about an axis through its centre and perpendicular to its plane, $I = M\left(\frac{l^2 + b^2}{12}\right)$
14. M.I. of a right circular solid cylinder about its symmetry axis, $I = \frac{1}{2}MR^2$.
15. M.I. of a right circular hollow cylinder about its axis $I = MR^2$.
16. M.I. of a solid sphere about an axis through its centre, $I = \frac{2}{5}MR^2$.
17. M.I. of a solid sphere about any tangent, $I = \frac{7}{5}MR^2$
18. M.I. of a hollow sphere about an axis through its centre, $I = \frac{2}{3}MR^2$
19. M.I. of a hollow sphere about any tangent, $I = \frac{5}{3}MR^2$
20. Rotational K.E. $= \frac{1}{2}I\omega^2$
21. Total K.E. = Rotational K.E. + Translational K.E. $= \frac{1}{2}I\omega^2 + \frac{1}{2}Mv^2$

Units Used

Mass M is in kg, radius R in m, moment of inertia I in kg m^2 and radius of gyration K in metre, rotational K.E. in joule and angular velocity ω in rad s^{-1}.

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EXAMPLE 22. A wheel of mass 8 kg and radius of gyration 25 cm is rotating at 300 rpm. What is its moment of inertia ?

Solution. Here $M = 8$ kg, $K = 25$ cm $= 0.25$ m

$\therefore \quad I = MK^2 = 8 \times (0.25)^2 = \mathbf{0.5\ kgm^2}.$

EXAMPLE 23. Three mass points m_1, m_2 and m_3 are located at the vertices of an equilateral triangle of length a. What is the moment of inertia of the system about an axis along the altitude of the triangle passing through m_1 ?

Solution. As shown in Fig. 7.54, the axis of rotation passes through m_1. The distances of m_1, m_2 and m_3 from the axis of rotation are 0, $a/2$ and $a/2$ respectively.

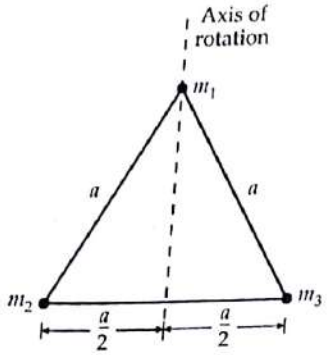


Fig. 7.54

$\therefore$ M.I. of the system about the altitude through m_1 is

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$$

$$= m_1 (0)^2 + m_2 \left(\frac{a}{2}\right)^2 + m_3 \left(\frac{a}{2}\right)^2$$

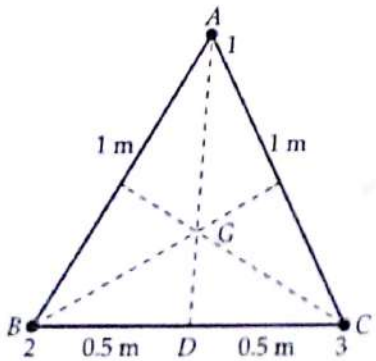
or $$I = \frac{a^2}{4}(m_2 + m_3).$$

EXAMPLE 24. Three balls of masses 1, 2 and 3 kg respectively are arranged at the corners of an equilateral triangle of side 1 m. What will be the moment of inertia of the system about an axis through the centroid and perpendicular to the plane of the triangle ?

Solution.

Median $AD = \sqrt{AB^2 - BD^2} = \sqrt{1^2 - (0.5)^2} = \sqrt{0.75}$

$\therefore \quad AG = BG = CG = \frac{2}{3} AD = \frac{2}{3}\sqrt{0.75}$



g. 7.55

M.I. of the system about an axis through centroid G, and perpendicular to the plane of the triangle is

$$I = 1 \times AG^2 + 2 \times BG^2 + 3 \times CG^2$$

$$= (1 + 2 + 3) \times \left(\frac{2}{3}\sqrt{0.75}\right)^2$$

$$= \frac{6 \times 4 \times 0.75}{9} = \mathbf{2\ kg\ m^2}.$$

EXAMPLE 25. Four particles of masses 4 kg, 2 kg, 3 kg and 5 kg are respectively located at the four corners A, B, C and D of a square of side 1 m as shown in Fig. 7.56. Calculate the moment of inertia of the system about

(i) an axis passing through the point of intersection of the diagonals and perpendicular to the plane of the square,

(ii) the side AB, and (iii) the diagonal BD.

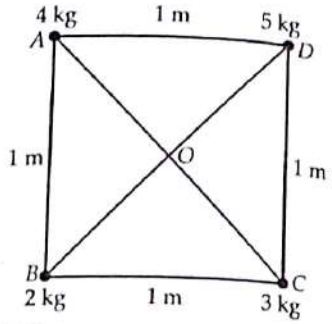


Fig. 7.56

Solution. Here $AB = BC = CD = DA = 1$ m

$$OA = OB = OC = OD = \frac{1}{2}\sqrt{1^2 + 1^2} = \frac{1}{\sqrt{2}}\ \text{m}$$

(*i*) M.I. of the system about an axis through O and perpendicular to the plane of the square,

$$I = 4\,(OA)^2 + 2\,(OB^2) + 3\,(OC)^2 + 5\,(OD)^2$$

$$= (4 + 2 + 3 + 5) \times \left(\frac{1}{\sqrt{2}}\right)^2 = 14 \times \frac{1}{2} = 7\ \text{kg m}^2.$$

(*ii*) M.I. of the system about the side *AB*,

$$I = 3\,(BC)^2 + 5\,(AD)^2 = 3 \times 1 + 5 \times 1 = 8\ \text{kg m}^2.$$

(*iii*) M.I. of the system about the diagonal *BD*,

$$I = 4\,(OA)^2 + 3\,(OC)^2$$

$$= 4 \times \frac{1}{2} + 3 \times \frac{1}{2} = \mathbf{3.5\ kg\ m^2}.$$

EXAMPLE 26. The moment of inertia of a uniform circular disc about its diameter is 100 g cm^2. What is its moment of inertia (i) about its tangent (ii) about an axis perpendicular to its plane ?

Solution. M.I. of disc about its diameter,

$$I_d = \frac{1}{4} MR^2 = 100\ \text{g cm}^2$$

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(*i*) By theorem of parallel axes, M.I. about a tangent parallel to the diameter,

$$I = I_d + MR^2 = \frac{1}{4}MR^2 + MR^2 = \frac{5}{4}MR^2$$

$= 5 \times 100 = \mathbf{500\ g\ cm^2}$.

(*ii*) By theorem of perpendicular axes, M.I. of the disc about an axis perpendicular to its plane,

I = Sum of the moments of inertia about two perpendicular diameters

$= I_d + I_d = 2 \times \frac{1}{4}MR^2 = 2 \times 100 = \mathbf{200\ g\ cm^2}$.

Example 27. Calculate the moment of inertia of a cylinder of length 1.5 m, radius 0.05 m and density 8×10^3 kg m^{-3} about the axis of the cylinder.

Solution. Here $R = 0.05$ m, $l = 1.5$ m,

$$\rho = 8 \times 10^3 \text{ kg m}^{-3}$$

Mass of cylinder,

$$M = \text{Volume} \times \text{density} = \pi R^2 l \cdot \rho$$

$$= 3.14 \times (0.05)^2 \times 1.5 \times 8 \times 10^3 = 94.2 \text{ kg}$$

M.I. of the cylinder about its own axis,

$$I = \frac{1}{2}MR^2 = \frac{1}{2} \times 94.2 \times (0.05)^2$$

$$= \mathbf{0.1175\ kg\ m^2}.$$

Example 28. Calculate the moment of inertia of the earth about its diameter, taking it to be a sphere of 10^{25} kg and diameter 12800 km.

Solution. Here $M = 10^{25}$ kg,

$$R = 6400 \text{ km} = 6.4 \times 10^6 \text{ m}$$

M.I. of the earth about its diameter

$$I = \frac{2}{5}MR^2 = \frac{2}{5} \times 10^{25} \times (6.4 \times 10^6)^2$$

$$= \mathbf{1.64 \times 10^{38}\ kg\ m^2}.$$

Example 29. Four spheres of diameter 2a and mass M each are placed with their centres on the four corners of a square of side b. Calculate the moment of inertia of the system about one side of the square taken as its axis.

Solution. The situation is shown in Fig. 7.57. Let us calculate the moment of inertia of the system about the side CD.

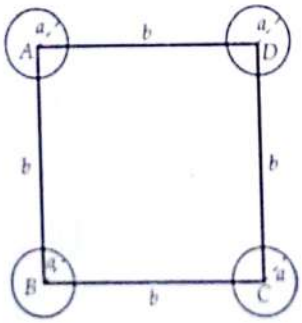

Fig. 7.57

I = M.I. of A about CD + M.I. of B about CD + M.I. of C about CD + M.I. of D about CD

$$= \left(\frac{2}{5}Ma^2 + Mb^2\right) + \left(\frac{2}{5}Ma^2 + Mb^2\right) + \frac{2}{5}Ma^2 + \frac{2}{5}Ma^2$$

$$= \frac{8}{5}Ma^2 + 2\,Mb^2 = \frac{2}{5}M(4a^2 + 5b^2).$$

Example 30. Two point masses of 2 kg and 10 kg are connected by a weightless rod of length 1.2 m. Calculate the M.I. of the system about an axis passing through the centre of mass and perpendicular to the system.

Solution. Here $m_1 = 2$ kg, $m_2 = 10$ kg,

length of rod $= 1.2$ m

Suppose the centre of mass lies at distance x from mass m_1. Then

$$m_1 x = m_2 (1.2 - x)$$

or $2x = 10 \times (1.2 - x)$ or $x = 1$ m

As the rod is weightless, its moment of inertia about any axis is zero.

M.I. of m_1 about CM

$$= m_1 x^2 = 2 \times (1)^2 = 2 \text{ kg m}^2$$

M.I. of m_2 about CM

$$= m_2(1.2 - x)^2 = 10 \times (1.2 - 1)^2 = 0.4 \text{ kgm}^2$$

$\therefore$ Total M.I. $= 2 + 0.4 = \mathbf{2.4\ kg\ m^2}$.

Example 31. Find the moment of inertia of a rectangular bar magnet about an axis passing through its centre and parallel to its thickness. Mass of the magnet is 100 g, length is 12 cm, breadth is 3 cm and thickness is 2 cm.

Solution. Here $M = 100$ g, $l = 12$ cm, $b = 3$ cm, $t = 2$ cm

M.I. of the bar magnet about the axis through its centre and parallel to its thickness is

$$I = M\left(\frac{l^2 + b^2}{12}\right) = 100\left(\frac{12^2 + 3^2}{12}\right) = \frac{100 \times 153}{12}$$

$$= \mathbf{1275\ g\ cm^2}.$$

Example 32. Calculate the ratio of radii of gyration of a circular ring and a disc of the same radius about the axis passing through their centres and perpendicular to their planes.

Solution. Let K_1 and K_2 be the radii of gyration of the ring and the disc about the axis passing through their centres and perpendicular to their planes. Then

M.I. of the ring $= MR^2 = MK_1^2$ or $K_1 = R$

M.I. of the disc $= \frac{1}{2}MR^2 = MK_2^2$ or $K_2 = \frac{1}{\sqrt{2}}R$

$$\therefore \quad \frac{K_1}{K_2} = \frac{R}{R/\sqrt{2}} = \mathbf{\sqrt{2} : 1}.$$

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EXAMPLE 33. Find (i) the radius of gyration and (ii) the moment of inertia of a rod of mass 100 g and length 100 cm about an axis passing through its centre and perpendicular to its length. [Delhi 12]

Solution. (*i*) Here $M = 100\ g = 0.1\ kg$,

$$l = 100\ cm = 1\ m,\quad K = ?,\quad I = ?$$

M.I. of the rod about an axis through its centre and perpendicular to its length is

$$I = \frac{Ml^2}{12} = MK^2 \quad \text{or} \quad K^2 = \frac{l^2}{12}$$

$$\therefore \quad K = \frac{l}{\sqrt{12}} = \frac{1\ m}{3.464} = \mathbf{0.289\ m}.$$

$$(ii)\ I = \frac{Ml^2}{12} = \frac{0.1 \times (1)^2}{12} = \mathbf{0.0083\ kg\ m^2}.$$

EXAMPLE 34. A wheel is rotating at a rate of 1000 rpm and its kinetic energy is 10^6 J. Determine the moment of inertia of the wheel about its axis of rotation.

Solution. Here $\nu = 1000\ rpm = \frac{1000}{60}\ rps = \frac{50}{3}\ rps$,

$$\omega = 2\pi\nu = \frac{100\,\pi}{3}\ rad\ s^{-1}$$

As rotational K.E. $= \frac{1}{2} I\omega^2$

$$\therefore \quad 10^6 = \frac{1}{2} \times I \times \left(\frac{100\,\pi}{3}\right)^2$$

or $$I = \frac{2 \times 10^6 \times 9}{(100)^2\,\pi^2} = \frac{200 \times 9}{9.87} = \mathbf{182.4\ kg\ m^2}.$$

EXAMPLE 35. Calculate the kinetic energy of rotation of a circular disc of mass 1 kg and radius 0.2 m rotating about an axis passing through its centre and perpendicular to its plane. The disc is making $30/\pi$ rotations per minute.

Solution. Here $M = 1\ kg,\ R = 0.2\ m$

$$\nu = \frac{30}{\pi}\ rpm = \frac{30}{\pi \times 60}\ rps = \frac{1}{2\,\pi}\ rps$$

$$\omega = 2\pi\nu = 2\pi \times \frac{1}{2\pi} = 1\ rad\ s^{-1}$$

M.I. of the disc about an axis through its centre and perpendicular to its plane,

$$I = \frac{1}{2} MR^2 = \frac{1}{2} \times 1 \times (0.2)^2 = 0.02\ kgm^2$$

$\therefore$ Rotational K.E.

$$= \frac{1}{2} I\omega^2 = \frac{1}{2} \times 0.02 \times (1)^2 = \mathbf{0.01\ J}.$$

EXAMPLE 36. Energy of 484 J is spent in increasing the speed of a flywheel from 60 rpm to 360 rpm. Find the moment of inertia of the wheel.

Solution. Energy spent = 484 J

$$\nu_1 = 60\ rpm = 1\ rps,\ \nu_2 = 360\ rpm = 6\ rps$$

$$\omega_1 = 2\pi\nu_1 = 2\pi\ rad\ s^{-1},\ \omega_2 = 2\pi\nu_2 = 12\,\pi\ rad\ s^{-1}$$

Let I be the moment of inertia of the wheel.

Initial K.E. of rotation

$$= \frac{1}{2} I\omega_1^2 = \frac{1}{2} I \times (2\pi)^2 = 2\,\pi^2 I$$

Final K.E. of rotation

$$= \frac{1}{2} I\omega_2^2 = \frac{1}{2} I \times (12\,\pi)^2 = 72\,\pi^2 I$$

Increase in K.E. of rotation of wheel

= Energy spent on the wheel

$$\therefore \quad 72\,\pi^2 I - 2\,\pi^2 I = 484$$

or $$I = \frac{484}{70\,\pi^2} = \frac{484 \times 7 \times 7}{70 \times 22 \times 22} = \mathbf{0.7\ kg\ m^2}.$$

EXAMPLE 37. Calculate the rotational K.E. of the earth about its own axis. Mass of the earth $= 6 \times 10^{24}$ kg and radius of the earth = 6400 km.

Solution. Here $R = 6400\ km = 6.4 \times 10^6\ m$,

$$M = 6 \times 10^{24}\ kg,$$

M.I. of the earth about its own axis,

$$I = \frac{2}{5} MR^2 = \frac{2}{5} \times 6 \times 10^{24} \times (6.4 \times 10^6)^2$$

$$= 9.83 \times 10^{37}\ kg\ m^2$$

Angular velocity of the earth,

$$\omega = \frac{2\pi}{T} = \frac{2\pi\ rad}{1\ day} = \frac{2\,\pi\ rad}{24 \times 60 \times 60\ s}$$

Rotational K.E. of the earth

$$= \frac{1}{2} I\omega^2 = \frac{1}{2} \times 9.83 \times 10^{37} \times \left(\frac{2\,\pi}{24 \times 60 \times 60}\right)^2$$

$$= \mathbf{2.6 \times 10^{29}\ J}.$$

EXAMPLE 38. A metre scale AB is held vertically with its one end A on the floor and is then allowed to fall. Find the speed of the other end B when it strikes the floor, assuming that the end on the floor does not slip.

Solution. Let M be the mass and L be the length of the metre scale. When the upper end of the rod strikes the floor, its centre of gravity falls through height $L/2$.

$$\therefore \text{ Loss in P.E.} = Mg \cdot \frac{L}{2}$$

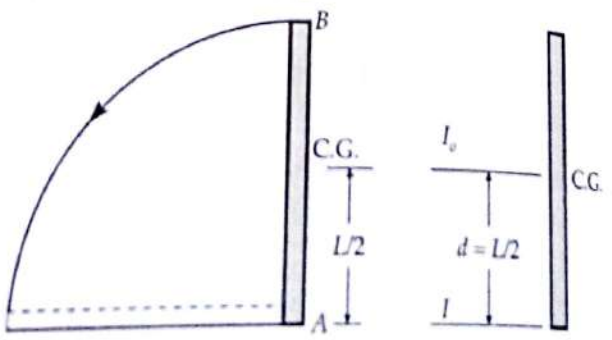

Fig. 7.58

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M.I. of the scale about the lower end A,

I = M.I. of the scale about the parallel axis through C.G. + Md^2

$$= I_0 + Md^2 = \frac{ML^2}{12} + \frac{ML^2}{4} = \frac{ML^2}{3} \quad \left[\because d = \frac{L}{2}\right]$$

Also, $\omega = \frac{v}{r} = \frac{v}{L}$

Gain in rotational K.E.

$$= \frac{1}{2} I\omega^2 = \frac{1}{2} \cdot \frac{ML^2}{3} \cdot \frac{v^2}{L^2} = \frac{Mv^2}{6}$$

Now, Gain in rotational K.E. = Loss in P.E.

$$\frac{Mv^2}{6} = Mg \cdot \frac{L}{2} \quad \text{or} \quad v^2 = 3gl$$

or $$v = \sqrt{3gL} = \sqrt{3 \times 9.8 \times 1} = \mathbf{5.4\ ms^{-1}}.$$

EXAMPLE 39. A uniform circular disc of mass m is set rolling on a smooth horizontal table with a uniform linear velocity v. Find the total K.E. of the disc.

Solution. M.I. of the disc about its own axis,

$$I = \frac{1}{2} mr^2$$

As $v = r\omega \quad \therefore \quad \omega^2 = \frac{v^2}{r^2}$

Rotational K.E. $= \frac{1}{2} I\omega^2 = \frac{1}{2} \times \frac{1}{2} mr^2 \times \frac{v^2}{r^2} = \frac{1}{4} mv^2$

Translational K.E. $= \frac{1}{2} mv^2$

Total K.E. = Rotational K.E. + Translational K.E.

$$= \frac{1}{4} mv^2 + \frac{1}{2} mv^2 = \mathbf{\frac{3}{4} mv^2}.$$

EXAMPLE 40. A solid sphere is rolling on a frictionless plane surface about its axis of symmetry. Find the rotational energy and the ratio of its rotational energy to its total energy.

Solution. Suppose the sphere has mass M and rolls with a uniform speed v.

M.I. of the sphere, $I = \frac{2}{5} MR^2$

Angular velocity, $\omega = \frac{v}{R}$

Rotational K.E.

$$= \frac{1}{2} I\omega^2 = \frac{1}{2} \times \frac{2}{5} MR^2 \times \frac{v^2}{R^2} = \frac{1}{5} \mathbf{Mv^2}.$$

Total energy

= Translational K.E. + Rotational K.E.

$$= \frac{1}{2} Mv^2 + \frac{1}{5} Mv^2 = \frac{7}{10} Mv^2$$

$$\frac{\text{Rotational K.E.}}{\text{Total energy}} = \frac{\frac{1}{5} Mv^2}{\frac{7}{10} Mv^2} = \frac{2}{7} = \mathbf{2:7}.$$

EXAMPLE 41. A wheel of mass 5 kg and radius 0.40 m is rolling on a road without sliding with angular velocity 10 rad s^{-1}. The moment of inertia of the wheel about the axis of rotation is 0.65 kgm^2. What is the percentage of kinetic energy of rotation in the total kinetic energy of the wheel ?

Solution. Here $M = 5$ kg, $R = 0.40$ m,

$$\omega = 10 \text{ rad s}^{-1}, \ I = 0.65 \text{ kg m}^2$$

Linear velocity,

$$v = R\omega = 0.40 \times 10 = 4.0 \text{ ms}^{-1}$$

Translational K.E.

$$= \frac{1}{2} Mv^2 = \frac{1}{2} \times 5 \times 16 = 40 \text{ J}$$

Rotational K.E.

$$= \frac{1}{2} I\omega^2 = \frac{1}{2} \times 0.65 \times 100 = 32.5 \text{ J}$$

Total K.E. = Translational K.E. + Rotational K.E.

$$= 40 + 32.5 = 72.5 \text{ J}$$

$$\frac{\text{Rotational K.E.}}{\text{Total K.E.}} = \frac{32.5}{72.5} = 0.448 = \mathbf{44.8\%}.$$

EXAMPLE 42. A solid cylinder rolls down an inclined plane. Its mass is 2 kg and radius 0.1 m. If the height of the inclined plane is 4 m, what is its rotational K.E. when it reaches the foot of the plane ? **[NCERT]**

Solution. Here $M = 2$ kg, $R = 0.1$ m

Height of inclined plane, $h = 4$ m

At the top of the inclined plane, the cylinder has P.E. $= mgh$

At the bottom of the inclined plane, the cylinder has translational K.E. $\left(= \frac{1}{2} Mv^2\right)$ and rotational K.E. $\left(= \frac{1}{2} I\omega^2\right)$

By conservation of energy,

$$\frac{1}{2} Mv^2 + \frac{1}{2} I\omega^2 = Mgh$$

But $v = R\omega$ and $I = \frac{1}{2} MR^2$

$$\therefore \quad \frac{1}{2} M(R\omega)^2 + \frac{1}{2} \times \frac{1}{2} MR^2 \omega^2 = Mgh$$

or $$\frac{3}{4} Mr^2 \omega^2 = Mgh \quad \text{or} \quad \omega^2 = \frac{4gh}{3R^2}$$

$$\therefore \quad \text{Rotational K.E.} = \frac{1}{2} I\omega^2 = \frac{1}{2} \times \frac{1}{2} MR^2 \times \frac{4gh}{3R^2}$$

$$= \frac{Mgh}{3} = \frac{2 \times 9.8 \times 4}{3} = \mathbf{26.13\ J}.$$

EXAMPLE 43. A bucket of mass 8 kg is supported by a light rope wound around a solid wooden cylinder of mass 12 kg and radius 20 cm free to rotate about its axis. A man holding

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the free end of the rope, with the bucket and the cylinder at rest initially, lets go the bucket freely downwards in a well 50 m deep. Neglecting friction, obtain the speed of the bucket and the angular speed of the cylinder just before the bucket enters water. Take $g = 10\ ms^{-2}$.

Solution. Mass of bucket, $m_1 = 8$ kg.

Mass of cylinder, $m_2 = 12$ kg.

Radius of the cylinder, $R = 20\ cm = 0.20$ m.

When the bucket just enters water,

P.E. lost by bucket = Linear K.E. of the bucket + Rotational K.E. of the cylinder

or $$m_1 gh = \frac{1}{2} m_1 v^2 + \frac{1}{2} I\omega^2$$

$$= \frac{1}{2} m_1 v^2 + \frac{1}{2}\left(\frac{1}{2} m_2 R^2\right)\frac{v^2}{R^2}$$

$$\left[\because I = \frac{1}{2} m_2 R^2,\ \omega = \frac{v}{R}\right]$$

$$= \frac{1}{2} v^2 \left(m_1 + \frac{1}{2} m_2\right) = \frac{1}{2} v^2 (8 + 6) = 7v^2.$$

or $$v = \sqrt{\frac{m_1 gh}{7}} = \sqrt{\frac{8 \times 10 \times 50}{7}}$$

$$= \sqrt{\frac{4000}{7}} = \mathbf{23.9\ ms^{-1}}.$$

The angular speed of cylinder before the bucket touches water,

$$\omega = \frac{v}{R} = \frac{23.9}{0.20} = \mathbf{119.5\ rads^{-1}}.$$

PROBLEMS FOR PRACTICE

1. A body of mass 50 g is revolving about an axis in a circular path. The distance of the centre of mass of the body from the axis of rotation is 50 cm. Find the moment of inertia of the body.
(**Ans.** 0.0125 kg m^2)

2. Find the moment of inertia of the hydrogen molecule about an axis passing through its centre of mass and perpendicular to the internuclear axis. Given mass of H-atom = 1.7×10^{-27} kg, interatomic distance = 4×10^{-10} m. (**Ans.** 13.6×10^{-47} kg m^2)

3. Three particles, each of 10 g are located at the corners of an equilateral triangle of side 5 cm. Determine the moment of inertia of this system about an axis passing through one-corner of the triangle and perpendicular to the plane of the triangle. (**Ans.** 5×10^{-5} kgm^2)

4. Four point masses of 20 g each are placed at the corners of a square *ABCD* of side 5 cm, as shown in Fig. 7.59. Find the moment of inertia of the system (*i*) about an axis coinciding with the side BC and (*ii*) about an axis through *A* and perpendicular to the plane of the square.
[**Ans.** (*i*) 1000 g cm^2 (*ii*) 2000 g cm^2]

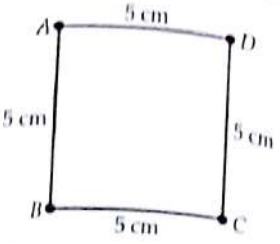

Fig. 7.59

5. The point masses of 0.3 kg, 0.2 kg and 0.1 kg are placed at the corners of a right angled ΔABC, as shown in Fig. 7.60. Find the moment of inertia of the system (*i*) about an axis through *A* and perpendicular to the plane of the diagram and (*ii*) about an axis along *BC*.
[**Ans.** (*i*) 0.043 kg m^2
(*ii*) 0.027 kg m^2]

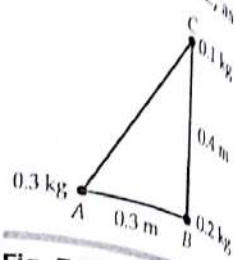

Fig. 7.60

6. Three particles, each of mass *m*, are situated at the vertices of an equilateral ΔABC of side *L*, as shown in Fig. 7.61. Find the moment of inertia of the system about the line *AX* perpendicular to *AB* and in the plane of ΔABC.
$\left(\text{Ans. } \frac{5ml^2}{4}\right)$

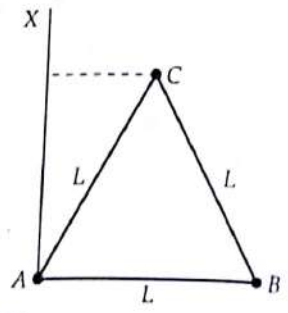

Fig. 7.61

7. Four particles each of mass *m* are kept at the four corners of a square of edge *a*. Find the moment of inertia of the system about an axis perpendicular to the plane of the system and passing through the centre of the square. (**Ans.** $2ma^2$)

8. What is the moment of inertia of a ring of mass 2 kg and radius 50 cm about an axis passing through its centre and perpendicular to its plane? Also find the moment of inertia about a parallel axis through its edge. (**Ans.** 0.5 kgm^2, 1.0 kgm^2)

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9. Calculate the moment of inertia of uniform circular disc of mass 500 g, radius 10 cm about (*i*) diameter of the disc (*ii*) the axis tangent to the disc and parallel to its diameter and (*iii*) the axis through the centre of the disc and perpendicular to its plane.
[**Ans.** (*i*) 12500 g cm^2 (*ii*) 62500 g cm^2 (*iii*) 25000 g cm^2]

10. Calculate moment of inertia of a circular disc of radius 10 cm, thickness 5 mm and uniform density 8 g cm^{-3}, about a transverse axis through the centre of the disc. (**Ans.** 6.28×10^4 gcm^2)

11. The radius of a sphere is 5 cm. Calculate the radius of gyration about (*i*) its diameter and (*ii*) about any tangent. [**Ans.** (*i*) 3.16 cm (*ii*) 5.915 cm]

12. Calculate the radius of gyration of a cylindrical rod of mass M and length L about an axis of rotation perpendicular to its length and passing through its centre. [MNREC 96]
(**Ans.** $K = L/\sqrt{12}$)

13. Two masses of 3 kg and 5 kg are placed at 30 cm and 70 cm marks respectively on a light wooden metre scale, as shown in Fig. 7.62. What will the moment of inertia of the system about an axis through (*i*) 0 cm mark and (*ii*) 100 cm mark, and perpendicular to the metre scale ?
[**Ans.** (*i*) 2.72 kg m^2, (*ii*) 1.92 kg m^2]

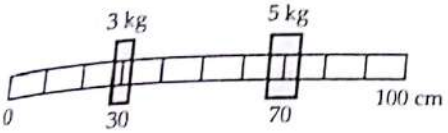

Fig. 7.62

14. Calculate the moment of inertia of a rod of mass 2 kg and length 0.5 m in each of the following cases, as shown in Fig. 7.63.
[**Ans.** (*i*) 0.042 kg m^2 (*ii*) 0.166 kg m^2]

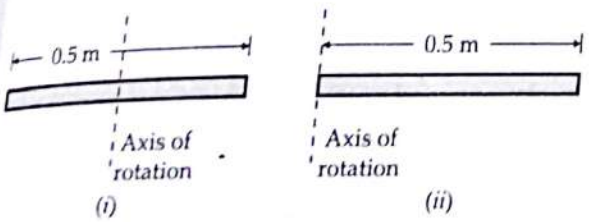

Fig. 7.63

15. A body of mass 2 kg is revolving in a horizontal circle of radius 2 m at the rate of 2 revolutions per second. Determine (*i*) moment of inertia of the body and (*ii*) the rotational kinetic energy of the body.
(**Ans.** 8 kgm^2, 63165 J)

16. A flywheel of mass 500 kg and diameter 1 m makes 500 rpm. Assuming the mass to be concentrated along the rim, calculate (*i*) angular velocity (*ii*) moment of inertia and (*iii*) rotational K.E. of the flywheel. [**Ans.** (*i*) 52.38 rad s^{-1} (*ii*) 125 kg m^2 (*iii*) 1.715×10^5 JJ]

17. A rod revolving 60 times in a minute about an axis passing through an end at right angles to the length, has kinetic energy of 400 J. Find the moment of inertia of the rod. (**Ans.** 20.26 kg m^2)

18. A thin metal hoop of radius 0.25 m and mass 2 kg starts from rest and rolls down an inclined plane. If its linear velocity on reaching the foot of the plane is 2 ms^{-1}, what is its rotation K.E. at that instant ?
(**Ans.** 4 J)

19. The earth has a mass of 6×10^{24} kg and a radius of 6.4×10^6 m. Calculate the amount of work in joules that must be done if its rotation were to be slowed down so that the duration of the day becomes 30 hours instead of 24 hours. Moment of inertia of earth $= \frac{2}{5} MR^2$. (**Ans.** 9.36×10^{28} J)

HINTS

1. Here $m = 50\,g = 0.05$ kg, $r = 50$ cm $= 0.50$ m
$$I = mr^2 = 0.05 \times (0.50)^2 = \mathbf{0.0125\ kgm^2}.$$

2. Mass of each H-atom, $m = 1.7 \times 10^{-27}$ kg
Distance of each H-atom from the axis of rotation
$= 2 \times 10^{-10}$ m
$$\therefore \quad I = mr^2 + mr^2 = 2mr^2$$
$$= 2 \times 1.7 \times 10^{-27} \times (2 \times 10^{-10})^2$$
$$= \mathbf{13.6 \times 10^{-47}\ kg.}$$

4. (*i*) M.I. about BC is
$$I = 20(AB)^2 + 20(BB)^2 + 20(CC)^2 + 20(DC)^2$$
$$= 20(5)^2 + 20(0)^2 + 20(0)^2 + 20(5)^2$$
$$= \mathbf{1000\ g\ cm^2}.$$

(*ii*) $CA = \sqrt{5^2 + 5^2} = 5\sqrt{2}$ cm

M.I. about an axis through A and perpendicular to the plane of the square is
$$I = 20(AA)^2 + 20(BA)^2 + 20(CA)^2 + 20(DA)^2$$
$$= 20(0)^2 + 20(5)^2 + 20(5\sqrt{2})^2 + 20(5)^2$$
$$= \mathbf{2000\ g\ cm^2}.$$

5. (*i*) $I_A = 0.2(AB)^2 + 0.1(AC)^2$
$$= 0.2 \times (0.3)^2 + 0.1 \times (0.5)^2$$
$$= 0.018 + 0.025 = \mathbf{0.043\ kg\ m^2}.$$

(*ii*) $I_{BC} = 0.3 \times (AB)^2 = 0.3 \times (0.3)^2$
$$= \mathbf{0.027\ kg\ m^2}.$$

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7. Distance of each particle from centre $= \frac{a}{\sqrt{2}}$

M.I. of each particle $= m\left(\frac{a}{\sqrt{2}}\right)^2 = \frac{1}{2}ma^2$

Total M.I. of the system $= 4 \times \frac{1}{2}ma^2 = 2ma^2$.

8. Here $M = 2$ kg, $R = 50$ cm $= 0.50$ m

(i) M.I. of the ring about an axis through its centre and perpendicular to its plane,

$I = MR^2 = 2 \times (0.50)^2 = \mathbf{0.5\ kg\ m^2}$.

(ii) M.I. about a parallel axis through its edge,

$I' = I + MR^2 = MR^2 + MR^2 = 2MR^2$

$= 2 \times 0.5 = \mathbf{1.0\ kgm^2}$.

9. (i) M.I. of the disc about any diameter,

$I_d = \frac{1}{4}MR^2 = \frac{1}{4} \times 500 \times (10)^2 = \mathbf{12500\ g\ cm^2}$

(ii) By theorem of parallel axes, M.I. of the disc about a tangent parallel to the diameter of the disc,

$I = I_d + MR^2 = \frac{5}{4}MR^2 = \frac{5}{4} \times 500 \times (10)^2$

$= \mathbf{62500\ g\ cm^2}$.

(iii) M.I. of the disc about an axis through its centre and perpendicular to its plane,

$I = \frac{1}{2}MR^2 = \frac{1}{2} \times 500 \times (10)^2$

$= \mathbf{25000\ g\ cm^2}$.

10. Radius, $R = 10$ cm,

Thickness, $t = 5$ mm $= 0.5$ cm

Density, $\rho = 8\ g\ cm^{-3}$

Mass of disc, M = Area × thickness × density

$= \pi R^2 t \rho = \frac{22}{7} \times (10)^2 \times 0.5 \times 8 = \frac{8800}{7}$ g

M.I. of the disc about a transverse axis through its centre,

$I = \frac{1}{2}MR^2 = \frac{1}{2} \times \frac{8800}{7} \times (10)^2$

$= \mathbf{6.28 \times 10^4\ g\ cm^2}$.

11. (i) $I_{diameter} = \frac{2MR^2}{5} = MK^2$

$\therefore \quad K = \sqrt{\frac{2}{5}}.\ R = \sqrt{0.4} \times 5$

$= 0.632 \times 5 = \mathbf{3.16\ cm}$.

(ii) $I_{tangent} = \frac{7MR^2}{5} = MK^2$

$\therefore \quad K = \sqrt{\frac{7}{5}}.\ R$

$= \sqrt{1.4}\ R = 1.183 \times 5 = \mathbf{5.915\ cm}$.

15. Here $m = 2$ kg, $r = 2$ m, $\nu = 2$ rps

$\omega = 2\pi\nu = 2\pi \times 2 = 4\pi$ rad s^{-1}

(i) Moment of inertia, $I = mr^2 = 2 \times (2)^2 = 8$ kg m^2

(ii) Rotational K.E. $= \frac{1}{2}I\omega^2 = \frac{1}{2} \times 8 \times (4\pi)^2$

$= \frac{1}{2} \times 8 \times 16 \times 9.87 = \mathbf{631.65\ J}$.

16. Here $M = 500$ kg, $R = \frac{1}{2}$ m

$\nu = 500$ rpm $= \frac{500}{60}$ rps $= \frac{25}{3}$ rps

(i) $\omega = 2\pi\nu = 2 \times 3.14 \times \frac{25}{3} = \mathbf{52.38\ rad\ s^{-1}}$.

(ii) As the mass of flywheel is concentrated at its rim, it can be regarded as a ring.

$\therefore \quad I = MR^2 = 500 \times \left(\frac{1}{2}\right)^2 = \mathbf{125\ kg\ m^2}$.

(iii) Rotational K.E. $= \frac{1}{2}I\omega^2 = \frac{1}{2} \times 125 \times (52.38)^2$

$= 1.715 \times 10^5$ J

17. Here $\nu = 60$ rpm $= 1$ rps, $\omega = 2\pi\nu = 2\pi$ rad s^{-1}

Rotational K.E. $= \frac{1}{2}I\omega^2$

$\therefore \quad 400 = \frac{1}{2} \times I \times (2\pi)^2$

or $\quad I = \frac{400}{2\pi^2} = \frac{200}{9.87} = \mathbf{20.26\ kgm^2}$.

18. Here $R = 0.25$ m, $M = 2$ kg, $v = 2\ ms^{-1}$

Rotational K.E. $= \frac{1}{2}I\omega^2 = \frac{1}{2}MR^2 \times \left(\frac{v}{R}\right)^2$

$= \frac{1}{2}Mv^2 = \frac{1}{2} \times 2 \times 4 = \mathbf{4\ J}$.

19. Here $M = 6 \times 10^{24}$ kg, $R = 6.4 \times 10^6$ m,

$T_1 = 24$ h, $T_2 = 30$ h

Work done = Increase in rotational K.E.

$W = K_2 - K_1 = \frac{1}{2}I(\omega_2^2 - \omega_1^2)$

$= \frac{1}{2} \times \frac{2}{5}MR^2 \times \left[\left(\frac{2\pi}{T_2}\right)^2 - \left(\frac{2\pi}{T_1}\right)^2\right]$

$= \frac{4\pi^2}{5} \times MR^2 \left[\frac{1}{T_2^2} - \frac{1}{T_1^2}\right]$

$= \frac{4 \times 9.87 \times 6 \times 10^{24} \times (6.4 \times 10^6)^6}{5}$

$\times \left[\frac{1}{(36 \times 3600)^2} - \frac{1}{(24 \times 3600)^2}\right]$

$= \mathbf{-9.36 \times 10^{28}\ J}$

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7.29 RELATION BETWEEN TORQUE AND MOMENT OF INERTIA

48. *Derive a relation between torque applied and angular acceleration produced in a rigid body and hence define moment of inertia.*

Relation between torque and moment of inertia. When a torque acts on a body capable of rotation about an axis, it produces an angular acceleration in the body. If the angular velocity of each particle is ω, then the angular acceleration, $\alpha = d\omega/dt$ will be same for all particles of the body. The linear acceleration will depend on their distances $r_1, r_2, ..., r_n$ from the axis of rotation.

As shown in Fig. 7.36, consider a particle P of mass m_1 at a distance r_1 from the axis of rotation. Let its linear velocity be v_1.

Linear acceleration of the first particle, $a_1 = r_1 \alpha$

Force acting on the first particle, $F_1 = m_1 r_1 \alpha$

Moment of force F_1 about the axis rotation is

$$\tau_1 = F_1 r_1 = m_1 r_1^2 \alpha$$

Total torque acting on the rigid body is

$$\begin{aligned}\tau &= \tau_1 + \tau_2 + \tau_3 + ... + \tau_n \\ &= m_1 r_1^2 \alpha + m_2 r_2^2 \alpha + m_3 r_3^2 \alpha + ... + m_n r_n^2 \alpha \\ &= (m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + ... + m_n r_n^2)\,\alpha \\ &= (\Sigma mr^2)\,\alpha\end{aligned}$$

But $\Sigma mr^2 = I$, moment of inertia of the body about the given axis

$$\therefore \quad \tau = I\alpha$$

Torque = Moment of inertia × Angular acceleration

When $\alpha = 1$, $\tau = I$

Thus the moment of inertia of a rigid body about an axis of rotation is numerically equal to the external torque required to produce unit angular acceleration in the body about that axis.

7.30 RELATION BETWEEN ANGULAR MOMENTUM AND MOMENT OF INERTIA

49. *Derive a relation between angular momentum, moment of inertia and angular velocity of a rigid body.*

Relation between angular momentum, moment of inertia and angular velocity. As shown in Fig. 7.36, consider a rigid body rotating about a fixed axis with uniform angular velocity ω. The body consists of n particles of masses $m_1, m_2, m_3, ..., m_n$; situated at distance $r_1, r_2, r_3 ..., r_n$ from the axis of rotation. The angular velocity ω of all the n particles will be same but their linear velocities will be different and are given by

$$v_1 = r_1 \omega, \quad v_2 = r_2 \omega, \quad v_3 = r_3 \omega, ..., \quad v_n = r_n \omega$$

Linear momentum of first particle,

$$p_1 = m_1 v_1 = m_1 r_1 \omega$$

Moment of linear momentum of the first particle about the axis YY',

$$L_1 = p_1 r_1 = m_1 r_1^2 \omega$$

The angular momentum of a rigid body about an axis is the sum of moments of linear momenta of all its particles about that axis. Thus

$$\begin{aligned}L &= L_1 + L_2 + L_3 + ... + L_n \\ &= m_1 r_1^2 \omega + m_2 r_2^2 \omega + m_3 r_3^2 \omega + ... + m_n r_n^2 \omega \\ &= (m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + ... + m_n r_n^2)\,\omega \\ &= (\Sigma mr^2)\,\omega\end{aligned}$$

But $\Sigma mr^2 = I$, moment of inertia of the body about the given axis

$$\therefore \quad L = I\omega$$

Angular momentum = M.I. × Angular velocity

When $\omega = 1$, $L = I$

Thus the moment of inertia of a body about an axis is numerically equal to the angular momentum of the rigid body when rotating with unit angular velocity about that axis.

Examples based on Relations between Torque, Angular momentum and Moment of Inertia

FORMULAE USED

1. Torque = M.I. × angular acceleration
   or $\tau = I\alpha$
2. Work done by a torque, $W = \tau\theta$
3. Angular momentum = M.I. × angular velocity
   or $L = I\omega$

UNITS USED

Torque τ is in Nm, moment of inertia I in kgm^2 and angular momentum L in kgm^2s^{-1}.

EXAMPLE 44. A torque of 2.0×10^{-4} Nm is applied to produce an angular acceleration of 4 rad s^{-2} in a rotating body. What is the moment of inertia of the body ?

Solution. Here $\tau = 2.0 \times 10^{-4}$ Nm,

$$\alpha = 4 \text{ rad s}^{-2}, \ I = ?$$

As $\tau = I\alpha$

$$\therefore \quad I = \frac{\tau}{\alpha} = \frac{2.0 \times 10^{-4}}{4} = 0.5 \times 10^{-4} \text{ kg m}^2.$$

EXAMPLE 45. An automobile moves on a road with a speed of 54 kmh^{-1}. The radius of its wheels is 0.35 m. What is the average negative torque transmitted by its brakes to a wheel

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if the vehicle is brought to rest in 15 s ? The moment of inertia of the wheel about the axis of rotation is 3 kgm². [NCERT ; Delhi 14]

Solution. Here $u = 54\ \text{kmh}^{-1} = 15\ \text{ms}^{-1}$,

$R = 0.35$ m, $t = 15$ s, $I = 3\ \text{kg m}^2$

$$\omega_0 = \frac{u}{R} = \frac{15}{0.35}\ \text{rad s}^{-1},\ \omega = 0$$

Average angular acceleration,

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{0 - \frac{15}{0.35}}{15} = -\frac{1}{0.35}\ \text{rad s}^{-2}$$

Average torque transmitted by the brakes,

$$\tau = I.\alpha = -3 \times \frac{1}{0.35} = -8.57\ \text{kgm}^2\ \text{s}^{-2}.$$

EXAMPLE 46. A flywheel of mass 25 kg has a radius of 0.2 m. What force should be applied tangentially to the rim of the flywheel so that it acquires an angular acceleration of 2 rad s⁻² ?

Solution. Here $M = 25$ kg, $R = 0.2$ m, $\alpha = 2\ \text{rad s}^{-2}$

M.I. of the flywheel about its axis,

$$I = \frac{1}{2}MR^2 = \frac{1}{2} \times 25 \times (0.2)^2 = 0.5\ \text{kg m}^2$$

As torque, $\tau = F.R = I\alpha$

$$\therefore \text{Force}, \quad F = \frac{I\alpha}{R} = \frac{0.5 \times 2}{0.2} = 5\ \text{N}.$$

EXAMPLE 47. A torque of 10 Nm is applied to a flywheel of mass 10 kg and radius of gyration 50 cm. What is the resulting angular acceleration ?

Solution. Here

$$\tau = 10\ \text{Nm}, M = 10\ \text{kg}, \quad K = 0.50\ \text{m}, \quad \alpha = ?$$

As $\tau = I\alpha = MK^2\alpha$

$$\therefore \quad \alpha = \frac{\tau}{MK^2} = \frac{10}{10 \times (0.50)^2} = 4\ \text{rad s}^{-2}.$$

EXAMPLE 48. A grindstone has a moment of inertia of 6 kg m². A constant torque is applied and the grindstone is found to have a speed of 150 rpm, 10 seconds after starting from rest. Calculate the torque. [Central Schools 07]

Solution. Here $I = 6\ \text{kg m}^2$, $t = 10$ s, $\omega_0 = 0$

$$\nu = 150\ \text{rpm} = \frac{150}{60}\ \text{rps} = \frac{5}{2}\ \text{rps}$$

$$\omega = 2\pi\nu = 2\pi \times \frac{5}{2} = 5\pi\ \text{rad s}^{-1}$$

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{5\pi - 0}{10} = \frac{\pi}{2}\ \text{rad s}^{-2}$$

Torque, $\tau = I\alpha = 6 \times \frac{\pi}{2} = 3\pi\ \text{Nm}.$

EXAMPLE 49. A flywheel of mass 25 kg has a radius of m. It is making 240 r.p.m. What is the torque necessary to bring it to rest in 20 s ? If the torque is due to a force applied tangentially on the rim of the flywheel, what is the magnitude of the force ? [NCERT]

Solution. $M = 25$ kg, $R = 0.2$ m, $\nu_0 = 240$ rpm $= 4$ rps

$$\omega_0 = 2\pi\nu_0 = 2\pi \times 4 = 8\pi\ \text{rad s}^{-1},\ \omega = 0,\ t = 20\ \text{s}$$

As $\omega = \omega_0 + \alpha t$

$$\therefore \quad 0 = 8\pi + \alpha \times 20$$

or $$\alpha = -\frac{8\pi}{20} = -\frac{2}{5}\pi\ \text{rad s}^{-2}$$

M.I. of the flywheel about its own axis,

$$I = \frac{1}{2}MR^2 = \frac{1}{2} \times 25 \times (0.2)^2 = \frac{1}{2}\ \text{kg m}^2$$

Torque acting on the flywheel,

$$\tau = I\alpha = -\frac{1}{2} \times \frac{2}{5}\pi = -\frac{\pi}{5}\ \text{Nm}$$

The negative sign indicates that the torque is of retarding nature.

Now Torque = Force × perpendicular distance

i.e., $\tau = F \times R$

$$\therefore \quad F = \frac{\tau}{R} = \frac{\pi}{5 \times 0.2} = \pi\ \text{N}.$$

EXAMPLE 50. A cord is wound around the circumference of a wheel of diameter 0.3 m. The axis of the wheel is horizontal. A mass of 0.5 kg is attached at the end of the cord and it is allowed to fall from rest. If the weight falls 1.5 m in 4 s, what is the angular acceleration of the wheel ? Also find out the moment of inertia of the wheel. [NCERT]

Solution. Radius of the wheel, $R = \frac{0.3}{2} = 0.15$ m

For the attached mass :

$$m = 0.5\ \text{kg}, u = 0, s = 1.5\ \text{m}, t = 4\ \text{s}$$

Let *a* be the linear acceleration of the attached mass.

As $s = ut + \frac{1}{2}at^2$

$$\therefore \quad 1.5 = 0 \times 4 + \frac{1}{2}a \times (4)^2$$

or $$a = \frac{1.5}{8} = \frac{3}{16}\ \text{ms}^{-2}$$

If α is angular acceleration of the wheel, then

$$a = R\alpha$$

or $$\alpha = \frac{a}{R} = \frac{3}{16 \times 0.15} = 1.25\ \text{rad s}^{-2}$$

Torque applied by the attached mass,

$$\tau = F \times R = mgR = 0.5 \times 9.8 \times 0.15\ \text{Nm}$$

Now $\tau = I\alpha$

$$\therefore \quad I = \frac{\tau}{\alpha} = \frac{0.5 \times 9.8 \times 0.15}{1.25} = 0.588\ \text{kg m}^2.$$

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Example 51. A cord of negligible mass is wound round the rim of a fly wheel of mass 20 kg and radius 20 cm. A steady pull of 25 N is applied on the cord as shown in Fig. 7.64. The flywheel is mounted on a horizontal axle with frictionless bearings.

(a) Compute the angular acceleration of the wheel.

(b) Find the work done by the pull, when 2 m of the cord is unwound.

(c) Find also the kinetic energy of the wheel at this point. Assume that the wheel starts from rest.

(d) Compare answers to parts (b) and (c). **[NCERT]**

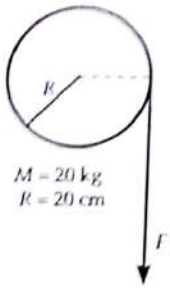

Fig. 7.64

Solution. *(a)* Torque, $\tau = FR = 25\ \text{N} \times 0.20\ \text{m}$

$= 5.0\ \text{Nm}$

Moment of inertia of the wheel about its axis,

$$I = \frac{MR^2}{2} = \frac{20 \times (0.20)^2}{2} = 0.4\ \text{kg m}^2$$

As $\tau = I\alpha$

$\therefore$ Angular acceleration,

$$\alpha = \frac{\tau}{I} = \frac{5.0\ \text{Nm}}{0.4\ \text{kg m}^2} = \mathbf{12.5\ rad\ s^{-2}}.$$

(b) Work done by the pull unwinding 2 m of the cord

$= 25\ \text{N} \times 2\ \text{m} = \mathbf{50\ J}.$

(c) Angular displacement of the wheel,

$$\theta = \frac{\text{Length of unwound string}}{\text{Radius of the wheel}}$$

$$= \frac{2\ \text{m}}{0.20\ \text{m}} = 10\ \text{rad}$$

As the wheel starts from rest, $\omega_0 = 0$

Final angular velocity ω is given by

$$\omega^2 = \omega_0^2 + 2\alpha\theta = 0 + 2 \times 12.5 \times 10$$

$$= 250\ (\text{rad s}^{-1})^2$$

$\therefore$ K.E. gained $= \frac{1}{2} I\omega^2 = \frac{1}{2} \times 0.4 \times 250 = \mathbf{50\ J}.$

(d) The answers are the same, *i.e.*, the kinetic energy gained by the wheel = work done by the force. There is no loss of energy due to friction.

Example 52. A body whose moment of inertia is 3 kgm², is at rest. It is rotated for 20 s with a moment of force 6 Nm. Find the angular displacement of the body. Also calculate the work done.

Solution. Here $I = 3\ \text{kg m}^2$, $t = 20$ s, $\tau = 6$ Nm, $\theta = ?$, $W = ?$

As $\tau = I\alpha$

$$\therefore \quad \alpha = \frac{\tau}{I} = \frac{6}{3} = 2\ \text{rad s}^{-2}$$

Angular displacement in 20 s is

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2} \times 2 \times (20)^2$$

$$= \mathbf{400\ rad}$$

Work done,

$$W = \tau\theta = 6 \times 400 = \mathbf{2400\ J}.$$

Example 53. How much tangential force would be needed to stop the earth in one year, if it were rotating with angular velocity of 7.3×10^{-5} rad s^{-1}? Given the moment of inertia of the earth $= 9.3 \times 10^{37}$ kg m² and radius of the earth $= 6.4 \times 10^6$ m.

Solution. Here $I = 9.3 \times 10^{37}$ kg m², $R = 6.4 \times 10^6$ m,

$\omega_0 = 7.5 \times 10^{-5}$ rad s^{-1},

$t = 1$ year $= 365 \times 24 \times 3600$ s

As $\omega = \omega_0 + \alpha t$

$$\therefore \quad \alpha = \frac{\omega - \omega_0}{t} = \frac{0 - 7.3 \times 10^{-5}}{365 \times 24 \times 3600}$$

$$= -\frac{7.3 \times 10^{-5}}{365 \times 24 \times 3600}\ \text{rad s}^{-2}$$

Torque, $\tau = I\alpha = 9.3 \times 10^{37} \times \dfrac{7.3 \times 10^{-5}}{365 \times 24 \times 3600}\ \text{Nm}$

[Omitting –ve sign]

Let F be the tangential force needed to stop the earth. Then

$$\tau = FR$$

or $$F = \frac{\tau}{R} = \frac{9.3 \times 10^{37} \times 7.3 \times 10^{-5}}{365 \times 24 \times 3600 \times 6.4 \times 10^6}$$

$$= \mathbf{3.363 \times 10^{19}\ N}.$$

Example 54. The angular momentum of a body is 31.4 Js and its rate of revolution is 10 cycles per second. Calculate the moment of inertia of the body about the axis of rotation.

Solution. Here $L = 31.4$ Js, $\nu = 10$ rps,

$$\omega = 2\pi\nu = 2 \times 3.14 \times 10\ \text{rad}^{-1}$$

As $L = I\omega$

$$\therefore \quad I = \frac{L}{\omega} = \frac{31.4}{2 \times 3.14 \times 10} = \mathbf{0.5\ kgm^2}.$$

Example 55. A 40 kg flywheel in the form of a uniform circular disc of 1 m radius is making 120 rpm. Calculate the angular momentum.

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Solution. Here $M = 40$ kg, $R = 1$ m,

$$\nu = 120 \text{ rpm} = \frac{120}{60} \text{ rps} = 2 \text{ rps}$$

$$\therefore \quad I = \frac{1}{2} MR^2 = \frac{1}{2} \times 40 \times (1)^2 = 20 \text{ kg m}^2$$

and $\omega = 2\pi\nu = 2\pi \times 2 = 4\pi$ rad s^{-1}

Angular momentum,

$$L = I\omega = 20 \times 4\pi = 80 \times 3.14$$

$$= \mathbf{251.2 \text{ kg m}^2\text{s}^{-1}}.$$

EXAMPLE 56. A ring of diameter 0.4 m and of mass 10 kg is rotating about its axis at the rate of 2100 rpm. Find (i) moment of inertia (ii) angular momentum and (iii) rotational K.E. of the ring. **[Delhi 03]**

Solution. Here $R = \frac{0.4}{2} = 0.2$ m, $M = 10$ kg

$$\nu = 2100 \text{ rpm} = \frac{2100}{60} \text{ rps} = 35 \text{ s}^{-1}$$

$$\therefore \quad \omega = 2\pi\nu = 2 \times \frac{22}{7} \times 35 = 220 \text{ rad s}^{-1}$$

(*i*) M.I. of the ring about its axis,

$$I = MR^2 = 10 \times (0.2)^2 = \mathbf{0.4 \text{ kg m}^2}.$$

(*ii*) Angular momentum,

$$L = I\omega = 0.4 \times 220 = \mathbf{88 \text{ kg m}^2\text{s}^{-1}}.$$

(*iii*) Rotational K.E.

$$= \frac{1}{2} I\omega^2 = \frac{1}{2} \times 0.4 \times (220)^2 = \mathbf{9680 \text{ J}}.$$

EXAMPLE 57. Calculate the angular momentum of the earth rotating about its own axis. Mass of the earth $= 5.98 \times 10^{27}$ kg, mean radius of the earth $= 6.37 \times 10^6$ m, M.I. of the earth $= \frac{2}{5} MR^2$.

Solution. Here $M = 5.98 \times 10^{24}$ kg,

$R = 6.37 \times 10^6$ m

$$I = \frac{2}{5} MR^2 = \frac{2}{5} \times 5.98 \times 10^{24} \times (6.37 \times 10^6)^2$$

$$= 2.1 \times 10^{38} \text{ kgm}^2$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{1 \text{ day}} = \frac{2\pi}{24 \times 60 \times 60} \text{ rad s}^{-1}$$

Angular momentum,

$$L = I\omega = 2.1 \times 10^{38} \times \frac{2\pi}{24 \times 60 \times 60}$$

$$= \mathbf{1.53 \times 10^{34} \text{ kg m}^2 \text{ s}^{-1}}.$$

EXAMPLE 58. A cylinder of mass 5 kg and radius 30 cm, and free to rotate about its axis, receives an angular impulse of 3 kg m^2 s^{-1} initially followed by a similar impulse after every 4 s. What is the angular speed of the cylinder 30 s after the initial impulse ? The cylinder is at rest initially.

Solution. Here $M = 5$ kg, $R = 30$ cm $= 0.30$ m, $\omega = 0$, $\omega = ?$

Angular impulse = Change in angular momentum

$$\therefore \quad 3 = I(\omega_2 - \omega_1) = \frac{1}{2} MR^2 (\omega - \omega_0)$$

or $$3 = \frac{1}{2} \times 5 \times (0.30)^2 (\omega - 0)$$

or $$\omega = \frac{3 \times 2}{5 \times 0.09} = \frac{40}{3} \text{ rad s}^{-1}$$

Now $$\omega = \omega_0 + \alpha t$$

$$\therefore \quad \frac{40}{3} = 0 + \alpha \times 4 \quad \text{or} \quad \alpha = \frac{10}{3} \text{ rad s}^{-2}$$

The angular impulse is imparted after every 4 seconds. So the pulses are imparted at $t = 0$, 4, 8, 12, 16, 20, 24 and 28 s. But last impulse continues to act upto 32 s, before the next impulse is imparted. So

$$\omega = \omega_0 + \alpha t = 0 + \frac{10}{3} \times 32$$

$$= \mathbf{106.67 \text{ rad s}^{-1}}.$$

PROBLEMS FOR PRACTICE

1. The moment of inertia of a flywheel is 4 kg m^2. What angular acceleration will be produced in it by applying a torque of 10 Nm on it ? **(Ans. 2.5 rad s^{-2})**
2. The moment of inertia of a body is 2.5 kg m^2. Calculate the torque required to produce an angular acceleration of 18 rad s^{-2} in the body. **(Ans. 45 Nm)**
3. A cylinder of length 20 cm and radius 10 cm is rotating about its central axis at an angular speed of 100 rad/s. What tangential force will stop the cylinder at a uniform rate in 10 seconds ? The moment of inertia of the cylinder about its axis of rotation is 0.8 kg m^2. **[Delhi 04] (Ans. 80 N)**
4. A flywheel of moment of inertia 10^7 g cm^2 is rotating at a speed of 120 rotations per minute. Find the constant breaking torque required to stop the wheel in 5 rotations. **(Ans. 2.513×10^7 dyne cm)**
5. If a constant torque of 500 Nm turns a wheel of moment of inertia 100 kg m^2 about an axis through its centre, find the gain in angular velocity in 2 s. **(Ans. 10 rad s^{-1})**
6. A sphere of mass 2 kg and radius 5 cm is rotating at the rate of 300 rpm. Calculate the torque required to stop it in 6.28 revolutions. Moment of inertia of the sphere about any diameter $= \frac{2}{5} MR^2$. **(Ans. 2.542×10^{-2} Nm)**
7. A body of mass 1.0 kg is rotating on a circular path of diameter 2.0 m at the rate of 10 rotations in 31.4 s. Calculate (*i*) angular momentum of the body and (*ii*) rotational kinetic energy. **[Ans. (*i*) 2.0 kgm^2 s^{-1} (*ii*) 20 J]**

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* A wheel, initially at rest, rotates for 2s under the angular acceleration of 3 rad s^{-1}. Find the (*i*) angular velocity acquired by the wheel in 2s (*ii*) angular displacement produced in 2s and (*iii*) torque acting on the wheel if its moment of inertia is 12 kg m^2. **[Central Schools 13]**

[Ans. (*i*) 6 rad s^{-1} (*ii*) 6 rad (*iii*) 36 Nm]

* HINTS

7. $\alpha = \dfrac{\omega - \omega_0}{\tau} = \dfrac{0 - 100}{10} = -10 \text{ rad s}^{-2}$.

$\tau = F.R = I.\alpha$

$\therefore \quad F = \dfrac{I\alpha}{R} = \dfrac{0.8 \text{ kg m}^2 \times (-10 \text{ rad s}^{-2})}{0.1 \text{ m}} = -80 \text{ N}.$

8. Here $m = 1.0$ kg, $r = 1.0$ m, $\nu = \dfrac{10}{31.4}$ s

$\omega = 2\pi\nu = 2 \times 3.14 \times \dfrac{10}{31.4} = 2 \text{ rad s}^{-1}$

$I = mr^2 = 1.0 \times (1.0)^2 = 1.0 \text{ kg m}^2$

$L = I\omega = 1.0 \times 2 = \mathbf{2.0 \text{ kgm}^2 \text{ s}^{-1}}$.

Rotational K.E. $= \dfrac{1}{2} I\omega^2 = \dfrac{1}{2} \times 1.0 \times (2)^2 = \mathbf{2.0 \text{ J}}$.

8. (*i*) $\omega = \omega_0 + \alpha t = 0 + 3 \times 2 = \mathbf{6 \text{ rad s}^{-1}}$.

(*ii*) $\theta = \omega_0 t + \dfrac{1}{2}\alpha t^2 = 0 + \dfrac{1}{2} \times 3 \times (2)^2 = \mathbf{6 \text{ rad}}$.

(*iii*) $\tau = I\alpha = 12 \times 3 = \mathbf{36 \text{ Nm}}$.

7.31 CONSERVATION OF ANGULAR MOMENTUM

50. *State the law of conservation of angular momentum. Give some illustrations of this law.*

Law of conservation of angular momentum. Suppose the external torque acting on a rigid body due to external forces is zero. Then

$$\tau = \frac{dL}{dt} = 0$$

Hence, L = constant.

So when *the total external torque acting on a rigid body is zero, the total angular momentum of the body is conserved. This is the law of conservation of angular momentum.*

Clearly, when $\tau = 0$, $L = I\omega$ = constant

or $$I_1 \omega_1 = I_2 \omega_2$$

This means that when no external torque is acting, the angular velocity ω of the body can be increased or decreased by decreasing or increasing the moment of inertia of the body.

Illustrations of the law of conservation of angular momentum :

(*i*) ***Planetary motion.*** The angular velocity of a planet revolving in an elliptical orbit around the sun increases, when it comes closer to the sun because its moment of inertia about the axis through the sun decreases. When it goes far away from the sun, its moment of inertia increases and hence angular velocity decreases so as to conserve angular momentum.

(*ii*) ***A man carrying heavy weights in his hands and standing on a rotating turn-table can change the angular speed of the turn-table.*** As shown in Fig. 7.65, if a person stands on a turn-table with some heavy weights in his hands stretched out and the table is rotated slowly, his angular speed at once increases, as he draws his hands to his chest. The moment of inertia of man and weights taken together decreases, as he draws his arms inward. As moment of inertia decreases, the angular speed increases so as to conserve total angular momentum.

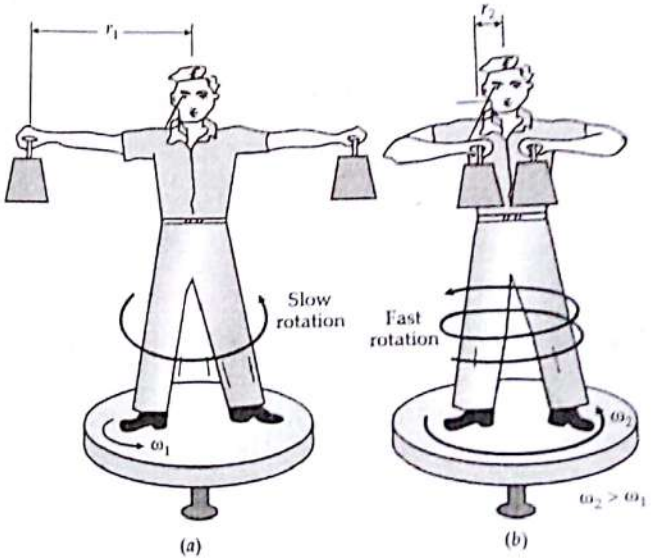

Fig. 7.65 The man begins to rotate faster as he draws his arms inwards.

(*iii*) ***A diver jumping from a spring board exhibits somersaults in air before touching the water surface.*** After leaving the spring board, a diver curls his body by pulling his arms and legs towards the centre of his body. This decreases his moment of inertia and he spins fast in midair. Just before hitting the water surface, he stretches out his arms. This decreases his moment of inertia and the diver enters water at a gentle speed.

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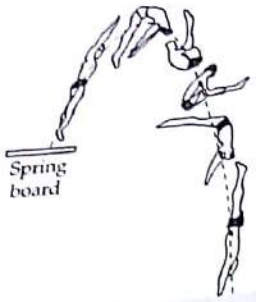

Fig. 7.66 A diver performing somersaults.

(iv) An ice-skater or a ballet dancer can increase her angular velocity by folding her arms and bringing the stretched leg close to the other leg. When she stretches her hands and a leg outward [Fig. 7.67(*a*)], her moment of inertia increases and hence angular speed decreases to conserve angular momentum. When she folds her arms and brings the stretched leg close to the other leg [Fig. 7.67(*b*)], her moment of inertia decreases and hence angular speed increases.

Fig. 7.67 A ballet dancer increases ω by changing to a posture of smaller M.I.

(v) The speed of the inner layers of the whirlwind in a tornado is alarmingly high. The angular velocity of air in a tornado increases as it goes towards the centre. This is because as the air moves towards the centre, its moment of inertia (I) decreases and to conserve angular momentum ($L = I\omega$), the angular velocity ω increases.

Examples based on Law of Conservation of Angular Momentum

Formulae Used

In the absence of any external torque,

$$L = I\omega = \text{a constant}$$

or $\quad I_1\omega_1 = I_2\omega_2$ or $I_1 \cdot \dfrac{2\pi}{T_1} = I_2 \cdot \dfrac{2\pi}{T_2}$

Units Used

Moment of inertia I is in kg m^2 and angular velocity ω in rad s^{-1}.

EXAMPLE 59. A small block is rotating in a horizontal circle at the end of a thread which passes down through a hole at the centre of table top. If the system is rotating at 2.5 rps in a circle of 30 cm radius, what will be the speed of rotation when the thread is pulled inwards to decrease the radius to 10 cm ? Neglect friction.

Solution. Here $\nu_1 = 2.5$ rps, $r_1 = 30$ cm,

$r_2 = 10$ cm, $\nu_2 = ?$

By law of conservation of angular momentum,

$$L_1 = L_2 \quad \text{or} \quad I_1\omega_1 = I_2\omega_2$$

or $\quad mr_1^2 \cdot 2\pi\nu_1 = mr_2^2 \cdot 2\pi\nu_2$

$$\therefore \quad \nu_2 = \frac{r_1^2\nu_1}{r_2^2} = \frac{30 \times 30 \times 2.5}{10 \times 10} = \mathbf{22.5\ rps.}$$

EXAMPLE 60. A star of mass twice the solar mass and radius 10^6 km rotates about its axis with an angular speed of 10^{-6} rad s^{-1}. What is the angular speed of the star when it collapses (due to inward gravitational force) to a radius of 10^4 km ? Solar mass 1.99×10^{30} kg.

Solution. During collapse, the total angular momentum of an isolated star is conserved, hence

$$I_1\omega_1 = I_2\omega_2$$

or $\quad \dfrac{2}{5}MR_1^2\omega_1 = \dfrac{2}{5}MR_2^2\omega_2 \qquad \left[\because I = \dfrac{2}{5}MR^2\right]$

or $\quad R_1^2\omega_1 = R_2^2\omega_2 \quad \therefore \quad \omega_2 = \dfrac{R_1^2}{R_2^2}\omega_1$

But $\quad R_1 = 10^6$ km, $R_2 = 10^4$ km, $\omega_1 = 10^{-6}$ s^{-1}

$$\therefore \quad \omega_2 = \frac{(10^6)^2}{(10^4)^2} \times 10^{-6} = \mathbf{0.01\ rad\ s^{-1}}.$$

EXAMPLE 61. If the earth were to suddenly contract to half of its present radius (without any external torque on it), by what duration would the day be decreased ? Assume earth to be a perfect solid sphere of moment of inertia $\frac{2}{5}MR^2$.

Solution. Present radius of the earth, $R_1 = R$

New radius of the earth after contraction,

$R_2 = R/2$

$T_1 = 24$ h, $T_2 = ?$

By conservation of angular momentum,

$$I_1\omega_1 = I_2\omega_2$$

or $\quad \dfrac{2}{5}MR_1^2 \cdot \dfrac{2\pi}{T_1} = \dfrac{2}{5}MR_2^2 \cdot \dfrac{2\pi}{T_2}$

or $\quad T_2 = \left(\dfrac{R_2}{R_1}\right)^2 \cdot T_1 = \left(\dfrac{R/2}{R}\right)^2 \times 24 = \dfrac{1}{4} \times 24 = 6$ h

$\therefore$ Decrease in the duration of the day

$= 24 - 6 = \mathbf{18\ h.}$

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EXAMPLE 62. What will be the duration of the day, if earth suddenly shrinks to 1/64 of its original volume, mass remaining the same ?

Solution. Original volume of the earth,

$$V = \frac{4}{3}\pi R^3$$

Volume of the earth after shrinking,

$$V' = \frac{V}{64}$$

$$\frac{4}{3}\pi R'^3 = \frac{1}{64} \times \frac{4}{3}\pi R^3$$

or $$R' = \frac{R}{4}$$

or By conservation of angular momentum,

$$I' \omega' = I\omega$$

$$\frac{2}{5} MR'^2 \times \frac{2\pi}{T'} = \frac{2}{5} MR^2 \times \frac{2\pi}{T}$$

or $$T' = \left(\frac{R'}{R}\right)^2 . T = \left(\frac{R/4}{R}\right)^2 \times 24$$

or $$= \frac{1}{16} \times 24 = \mathbf{1.5\ h.}$$

EXAMPLE 63. The maximum and minimum distances of a comet from the sun are 1.4×10^{12} m and 7×10^{10} m. If its velocity nearest to the sun is 6×10^4 ms^{-1}, what is the velocity in the farthest position ? Assume that path of the comet in both the instantaneous positions is circular.

Solution. At minimum distance, $r_1 = 7 \times 10^{10}$ m; velocity, $v_1 = 6 \times 10^4$ ms^{-1}

At maximum distance, $r_2 = 1.4 \times 10^{12}$ m;

velocity, $v_2 = ?$

By conservation of angular momentum,

$$I_1 \omega_1 = I_2 \omega_2$$

or $$mr_1^2 \times \frac{v_1}{r_1} = mr_2^2 . \frac{v_2}{r_2}$$

or $$v_1 r_1 = v_2 r_2$$

or $$v_2 = \frac{v_1 r_1}{r_2} = \frac{6 \times 10^4 \times 7 \times 10^{10}}{1.4 \times 10^{12}} = \mathbf{3000\ ms^{-1}}.$$

EXAMPLE 64. A horizontal disc rotating about a vertical axis passing through its centre makes 180 rpm. A small piece of wax of mass 10 g falls vertically on the disc and adheres to it at a distance of 8 cm from its axis. If the frequency is thus reduced to 150 rpm, calculate the moment of inertia of the disc.

Solution. Here $\nu_1 = 180$ rpm $= 3$ rps,

$$\nu_2 = 150 \text{ rpm} = \frac{150}{60} \text{ rps} = \frac{5}{2} \text{ rps}$$

$$\therefore \quad \omega_1 = 2\pi\nu_1 = 2\pi \times 3 = 6\pi \text{ rad s}^{-1},$$

$$\omega_2 = 2\pi \times \frac{5}{2} = 5\pi \text{ rad s}^{-1}$$

Let I be the M.I. of the disc about the given axis and I_2 be the M.I. when mass m sticks to it at distance r. Then

$$I_2 = I + mr^2$$

By conservation of angular momentum,

$$I_1 \omega_1 = I_2 \omega_2$$

$$I \times 6\pi = (I + mr^2) . 5\pi$$

or $$6I = 5I + 5mr^2$$

or $$I = 5mr^2 = 5 \times 10 \times 10^{-3} \times (8 \times 10^{-2})^2$$

$$= \mathbf{3.2 \times 10^{-8}\ kgm^2}.$$

PROBLEMS FOR PRACTICE

1. An ice skater spins with arms outstretched at 1.9 rps. Her moment of inertia at this instant is 1.33 kg m^2. She pulls in her arms to increase her rate of spin. If the moment of inertia is 0.48 kg m^2 after she pulls in her arms, what is her new rate of rotation ?
(**Ans.** 5.26 rps)

2. A mass of 2 kg is rotating on a circular path of radius 0.8 m with angular velocity of 44 rad s^{-1}. If the radius of the path becomes 1.0 m, what will be the value of angular velocity ?
(**Ans.** 28.16 rad s^{-1})

3. A ball tied to a string takes 4 s to complete revolution along a horizontal circle. If, by pulling the cord, the radius of the circle is reduced to half of the previous value, then how much time the ball will take in one revolution ? (**Ans.** 1 s)

4. The sun rotates round itself once in 27 days. What will be period of revolution if the sun were to expand to twice its present radius ? Assume the sun to be a sphere of uniform density. (**Ans.** 108 days)

5. If the earth suddenly contracts by one-fourth of its present radius, by how much would the day be shortened ? (**Ans.** 10.5 h)

6. Prove that for an earth satellite, the ratio of its velocity at apogee (when farthest from the earth) to its velocity at perigee (when nearest to the earth) is the inverse ratio of its distances from apogee and perigee.

7. A uniform disc rotating freely about a vertical axis makes 90 revolutions per minute. A small piece of wax of mass m gram falls vertically on the disc and sticks to it at a distance of r cm from the axis. If the number of rotations per minute reduces to 60, find the moment of inertia of the disc. (**Ans.** mr^2 g cm^2)

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3. By conservation of angular momentum,

$$I_1 \omega_1 = I_2 \omega_2$$

or $$mr^2 \cdot \frac{2\pi}{T_1} = m\left(\frac{r}{2}\right)^2 \cdot \frac{2\pi}{T_2}$$

$$\therefore \quad T_2 = \frac{1}{4} T_1$$

$$= \frac{1}{4} \times 4 = 1 \text{ s.}$$

4. $I\omega = I' \omega'$

or $$\frac{2}{5} MR^2 \times \frac{2\pi}{T} = \frac{2}{5} M (2R)^2 \times \frac{2\pi}{T'}$$

or $$T' = 4T = 4 \times 27$$

$$= 108 \text{ days.}$$

6. By conservation of angular momentum,
Angular momentum of satellite at apogee
= Angular momentum of satellite at perigee

$$m v_a r_a = m v_p r_p$$

or $$\frac{v_a}{v_p} = \frac{r_p}{r_a}.$$ [$\because L = mvr$]

7.32 ANALOGY BETWEEN TRANSLATIONAL AND ROTATIONAL MOTIONS

51. *Give the analogy between various quantities that describe linear and rotational motions.*

Analogy between translational and rotational motions. Table 7.1 summarises the analogy between the quantities that describe linear motion and the corresponding quantities that describe rotational motion.

Table 7.1 *Analogy between Linear and Rotational Motions.*

Linear motion		Rotational motion	
Quantities :			
displacement	s	angular displacement	θ
velocity	v	angular velocity	ω
acceleration	a	angular acceleration	α or a_θ
force	F	torque	τ
mass	m	moment of inertia	I
Expressions :			
velocity	$v = \frac{ds}{dt}$	angular velocity	$\omega = \frac{d\theta}{dt}$
acceleration	$a = \frac{dv}{dt}$	angular acceleration	$\alpha = \frac{d\omega}{dt}$
force	$F = ma = \frac{d}{dt}(mv)$	torque	$\tau = I\alpha = \frac{d}{dt}(I\omega)$
work done	$W = Fs$	work done	$W = \tau\theta$
linear K.E.	$E = \frac{1}{2}mv^2$	rotational K.E.	$E = \frac{1}{2}I\omega^2$
power	$P = Fv$	power	$P = \tau\omega$
linear momentum	$p = mv$	angular momentum	$L = I\omega$
impulse	$F\Delta t = mv - mu$	angular impulse	$\tau\Delta t = I\omega_f - I\omega_i$
Equations of motion :			
(i) $v = u + at$ (ii) $s = ut + \frac{1}{2}at^2$ (iii) $v^2 - u^2 = 2as$		(i) $\omega = \omega_0 + \alpha t$ (ii) $\theta = \theta_0 t + \frac{1}{2}\alpha t^2$ (iii) $\omega^2 - \omega_0^2 = 2\alpha\theta$	
Dimensions :			
velocity	$[LT^{-1}]$	angular velocity	$[T^{-1}]$
acceleration	$[LT^{-2}]$	angular acceleration	$[T^{-2}]$
mass	$[M]$	moment of inertia $I = \Sigma\ mr^2$	$[ML^2]$
force	$[MLT^{-2}]$	torque $\tau = Fr$	$[ML^2T^{-2}]$
linear K.E.	$[ML^2T^{-2}]$	rotational K.E.	$[ML^2T^{-2}]$
momentum	$[MLT^{-1}]$	angular momentum	$[ML^2T^{-1}]$
power	$[ML^2T^{-3}]$	power	$[ML^2T^{-3}]$

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7.33 ROLLING MOTION

52. *What is rolling motion ? Discuss the motion of a disc rolling without slipping on a level surface. Hence find the condition for rolling without slipping.*

Ans. Rolling motion. *Rolling motion can be regarded as the combination of pure rotation and pure translation.* The wheels of all vehicles running on a road have rolling motion. Consider a disc of radius R, rolling on a level surface without slipping. This means that at any instant of time the bottom of the disc which is in contact with the surface is at rest.

The rolling motion of the disc has *two* simultaneous motions :

(*i*) ***Translational motion.*** The translational velocity of the disc is the velocity $\vec{v}_{CM}$ of its centre of mass. As the centre of mass of the rolling disc lies at its geometric centre C, so $\vec{v}_{CM}$ is the velocity of C. It is parallel to the level surface as shown in Fig. 7.68.

$\therefore \quad v_{trans} = v_{CM}$

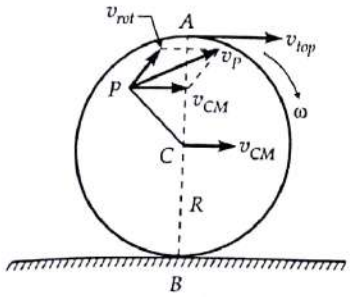

Fig. 7.68 Rolling motion (without slipping) of a disc on a level surface.

(*ii*) ***Rotational motion.*** The disc rotates with angular velocity ω about its symmetry axis through C. The linear velocity of a particle P at distance r from the axis due to the rotational motion is $v_{rot} = r\omega$

The velocity $\vec{v}_{rot}$ is directed perpendicular to the radius vector CP as shown in Fig. 7.68.

The effective linear velocity $\vec{v}$ of particle P is the resultant of the velocities $\vec{v}_{rot}$ and $\vec{v}_{CM}$. It can be shown that $\vec{v}$ is perpendicular to the line AP. Therefore, the line passing through the bottom point B and parallel to the axis through C is called the *instantaneous axis of rotation.*

At the bottom point B, the linear velocity $\vec{v}_{rot}$, due to rotation, is directed exactly opposite to the translational velocity $\vec{v}_{CM}$. Also, $v_{rot} = R\omega$ at the point B. The point B will be instantaneously at rest if $v_{CM} = R\omega$. Hence for the disc the *condition for rolling without slipping is* $v_{CM} = R\omega$

At the top point A of the disc, the linear velocity $R\omega$ due to rotational motion and the translational velocity v_{CM} are in the same direction, parallel to level surface. Therefore,

$$v_{top} = R\omega + v_{CM} = v_{CM} + v_{CM} = 2\,v_{CM}$$

53. *Obtain the expression for the linear acceleration of a cylinder rolling down an inclined plane and hence find the condition for the cylinder to roll down without slipping.*

Solid cylinder rolling down an inclined plane without slipping. Consider a solid cylinder of mass M and radius R rolling down a plane inclined at an angle θ to the horizontal, as shown in Fig. 7.69. Suppose the *cylinder rolls down without slipping.* The condition for rolling without slipping is that at each instant *the line of contact* of the cylinder with the surface at P *is momentarily at rest* and the cylinder rotates about this line as axis. The centre of mass of the cylinder moves in a straight line parallel to the inclined plane. Notably, it is the friction which prevents slipping.

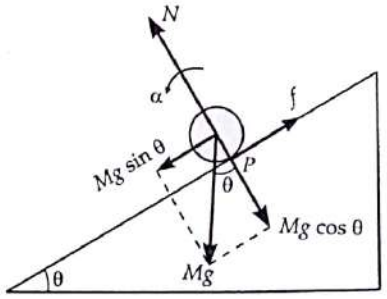

Fig. 7.69 Cylinder rolling without slipping.

The external forces acting on the cylinder are

(*i*) The weight Mg of the cylinder acting vertically downwards through the centre of mass of the cylinder.

(*ii*) The normal reaction N of the inclined plane acting perpendicular to the plane at P.

(*iii*) The frictional force f acting upwards and parallel to the inclined plane.

The weight Mg can be resolved into two rectangular components :

(*i*) $Mg \cos\theta$ perpendicular to the inclined plane.

(*ii*) $Mg \sin\theta$ acting down the inclined plane.

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As there is no motion in a direction normal to the inclined plane, so

$$N = Mg\cos\theta$$

Applying Newton's second law to the linear motion of the centre of mass, the net force on the cylinder rolling down the inclined plane is

$$F = Ma = Mg\sin\theta - f \qquad ...(1)$$

It is only the force of friction f which exerts torque τ on the cylinder and makes it rotate with angular acceleration α. It acts tangentially at the point of contact P and has lever arm equal to R.

$\therefore$ τ = Force × force arm = $f \cdot R$

Also, τ = M.I. × angular acceleration = $I\alpha$

$\therefore$ $f R = I\alpha$

or $$f = \frac{I\alpha}{R} = \frac{Ia}{R^2} \qquad \left[\because \alpha = \frac{a}{R}\right]$$

Putting the value of f in equation (1), we get

$$Ma = Mg\sin\theta - \frac{Ia}{R^2}$$

$$a = g\sin\theta - \frac{Ia}{MR^2}$$

or $$a + \frac{Ia}{MR^2} = g\sin\theta$$

or $$a\left[1 + \frac{I}{MR^2}\right] = g\sin\theta$$

$$a = \frac{g\sin\theta}{1 + \frac{I}{MR^2}}$$

Moment of inertia of the solid cylinder about its axis $= \frac{1}{2}MR^2$

$\therefore$ $$a = \frac{g\sin\theta}{1 + \frac{\frac{1}{2}MR^2}{MR^2}}$$

or $$a = \frac{2}{3}g\sin\theta$$

Clearly, the linear acceleration a of solid cylinder rolling down an inclined plane is less than the acceleration due to gravity g ($a < g$). The linear acceleration of the cylinder is constant for a given inclined plane (or given θ) and is independent of its mass M and radius R. However, for a hollow cylinder, $I = MR^2$, the value of a would decrease to $\frac{1}{2}g\sin\theta$.

From equation (1), the value of force of friction is

$$f = Mg\sin\theta - Ma$$

$$= Mg\sin\theta - M.\frac{2}{3}g\sin\theta = \frac{1}{3}Mg\sin\theta$$

If μ_s is the coefficient of friction between the cylinder and the inclined plane, then

$$\mu_s = \frac{f}{N} = \frac{\frac{1}{3}Mg\sin\theta}{Mg\cos\theta} = \frac{1}{3}\tan\theta$$

To prevent slipping, the coefficient of static friction must be equal to or greater than the above value. That is

$$\mu_s \geq \frac{1}{3}\tan\theta \quad \text{or} \quad \tan\theta \leq 3\mu_s.$$

54. *Write an expression for the kinetic energy of a body rolling without slipping.*

Kinetic energy of rolling motion. The kinetic energy of a body rolling without slipping is the sum of kinetic energies of translation and rotation.

K = K.E. of the translational motion of CM + K.E. of rotational motion of CM

$$= \frac{1}{2}mv_{CM}^2 + \frac{1}{2}I\omega^2$$

where v_{CM} is the velocity of CM and I is the moment of inertia about the symmetry axis of the rolling body. If R is the radius and k the radius of gyration of the rolling body, then

$$v_{CM} = R\omega \quad \text{and} \quad I = mk^2$$

$\therefore$ $$K = \frac{1}{2}mv_{CM}^2 + \frac{1}{2}mk^2\left(\frac{v_{CM}}{R}\right)^2$$

or $$K = \frac{1}{2}mv_{CM}^2\left(1 + \frac{k^2}{R^2}\right)$$

Examples based on Motion of a Cylinder Rolling without Slipping on an Inclined Plane

Formulae Used

For a cylinder of mass M and radius R rolling without slipping down plane inclined at angle θ with the horizontal,

1. Force of friction between the plane and cylinder, $$f = \frac{1}{3}Mg\sin\theta$$
2. Linear acceleration, $a = \frac{2}{3}g\sin\theta$
3. Condition for rolling without slipping is $$\mu_s > \frac{1}{3}\tan\theta$$

Units Used

Accelerations a and g are in ms^{-2} and coefficient of friction μ_s has no units.

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Example 65. A cylinder of mass 5 kg and radius 30 cm is rolling down an inclined plane at an angle of 45° with the horizontal. Calculate (i) force of friction, (ii) acceleration with which the cylinder rolls down and (iii) the minimum value of static friction so that cylinder does not slip while rolling down the plane.

Solution. Here $M = 5$ kg, $R = 30$ cm $= 0.30$ m, $\theta = 45°$

(i) Force of friction,

$$f = \frac{1}{3} Mg \sin\theta = \frac{1}{3} \times 5 \times 9.8 \sin 45° = \mathbf{11.55\ N.}$$

(ii) Acceleration,

$$a = \frac{2}{3} g \sin\theta = \frac{2}{3} \times 9.8 \sin 45° = \mathbf{4.62\ ms^{-2}.}$$

(iii) Minimum value of coefficient of static friction,

$$\mu_s = \frac{1}{3} \tan\theta = \frac{1}{3} \tan 45° = \frac{1}{3}.$$

Example 66. Three bodies, a ring, a solid cylinder and a solid sphere roll down the same inclined plane without slipping. They start from rest. The radii of the bodies are identical. Which of the bodies reaches the ground with maximum velocity ? **[NCERT]**

Solution. Suppose a body of mass m starting from rest rolls down an inclined plane. We assume there is no loss of energy due to friction.

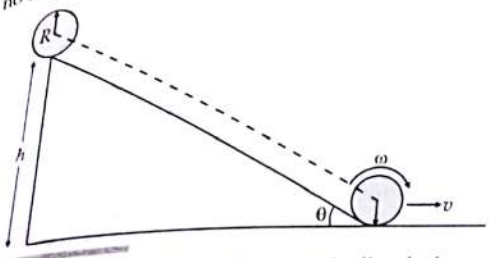

Fig. 7.70 A body rolling down an inclined plane.

By conservation of energy,

P.E. lost by the body in rolling down the inclined plane

= K.E. gained by the body

= Translational K.E. + Rotational K.E.

$$= \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2 = \frac{1}{2} mv^2 + \frac{1}{2} mk^2 \cdot \left(\frac{v}{R}\right)^2$$

or $$mgh = \frac{1}{2} mv^2 \left(1 + \frac{k^2}{R^2}\right)$$

or $$v = \sqrt{\frac{2gh}{1 + k^2/R^2}}$$

Clearly, the velocity v attained by the rolling body at the bottom of the inclined plane is independent of its mass.

For a ring, $k^2 = R^2$

$$\therefore \quad v_{ring} = \sqrt{\frac{2gh}{1+1}} = \sqrt{gh}$$

For a solid cylinder,

$$k^2 = R^2/2$$

$$\therefore \quad v_{cylinder} = \sqrt{\frac{2gh}{1+1/2}} = \sqrt{\frac{4gh}{3}}$$

For a solid sphere,

$$k^2 = 2R^2/5$$

$$\therefore \quad v_{sphere} = \sqrt{\frac{2gh}{1+2/5}} = \sqrt{\frac{10gh}{7}}$$

Clearly, among the three bodies the sphere has the greatest and the ring has the least velocity of the centre of mass at the bottom of the inclined plane.

Example 67. A solid cylinder of radius 4 cm and mass 250 g rolls down an inclined plane (1 in 10). Calculate the acceleration and the total energy of the cylinder after 5 s.

Solution. Here $M = 250$ g $= 0.25$ kg,

$R = 4$ cm $= 0.04$ m, $\sin\theta = \frac{1}{10}$, $t = 5$ s

Acceleration with which the cylinder rolls down,

$$a = \frac{g \sin\theta}{1 + \frac{I}{MR^2}} = \frac{g \sin\theta}{1 + \frac{\frac{1}{2} MR^2}{MR^2}}$$

$$= \frac{2}{3} g \sin\theta = \frac{2}{3} \times 9.8 \times \frac{1}{10} = \mathbf{0.653\ ms^{-2}.}$$

Using first equation of motion,

$$v = u + at = 0 + 0.653 \times 5 = 3.26\ \text{ms}^{-1}$$

Total K.E. of the cylinder

= Translational K.E. + Rotational K.E.

$$= \frac{1}{2} Mv^2 + \frac{1}{2} I\omega^2$$

$$= \frac{1}{2} Mv^2 + \frac{1}{2} \times \frac{1}{2} MR^2 \times \frac{v^2}{R^2}$$

$$= \frac{3}{4} Mv^2 = \frac{3}{4} \times 0.25 \times (3.26)^2 = \mathbf{2.0\ J.}$$

PROBLEMS FOR PRACTICE

1. A solid cylinder of mass 10 kg is rolling perfectly on a plane of inclination 30°. Find the force of friction between the cylinder and the surface of the inclined plane. **(Ans. 16.33 N)**
2. A solid cylinder of mass 8 kg and radius 50 cm is rolling down a plane inclined at an angle of 30° with the horizontal. Calculate (*i*) force of friction,

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(*ii*) acceleration with which the cylinder rolls down and (*iii*) the minimum value of coefficient of friction so that cylinder does not slip while rolling down the plane.

[**Ans.** (*i*) 13.06 N (*ii*) 3.267 ms^{-2} (*iii*) 0.192]

3. If a sphere rolls (starting from rest) in 5.3 s along a plane 1 m in length of which the upper end is raised 0.01 m above the lower, find the acceleration due to gravity. (**Ans.** 9.97 ms^{-2})

HINTS

3. Here $u = 0,\ t = 5.3\,s,\ s = 1\,m,\ \sin\theta = \frac{1}{100} = 0.01$

As $s = ut + \frac{1}{2}at^2 \quad \therefore \quad 1 = 0 + \frac{1}{2}a \times (5.3)^2$

or $a = \frac{2 \times 1}{5.3 \times 5.3} = 0.0712\ ms^{-2}$

But $a = \frac{g\sin\theta}{1 + \frac{I}{MR^2}} = \frac{g\sin\theta}{1 + \frac{\frac{2}{5}MR^2}{MR^2}} = \frac{5}{7}g\sin\theta$

$\therefore \quad g = \frac{7a}{5\sin\theta} = \frac{7 \times 0.0712}{5 \times 0.01} = 9.97\ ms^{-2}.$

7.34 MASS POINT ON STRING WOUND ON A CYLINDER

55. *A light string is wound round a cylinder and carries a mass tied to it at the free end. When the mass is released, calculate (a) the linear acceleration of the descending mass and (b) the angular acceleration of the cylinder and (c) the tension in the string. Show that the acceleration of mass is less than `g'.*

Motion of a mass point attached to a string wound on a cylinder. As shown in Fig. 7.71 consider a solid cylinder of mass m and radius R. It is mounted on a frictionless horizontal axle so that it can freely rotate about its axis. A light string is wound round the cylinder and mass m is suspended from it. When the mass m is released from rest, it moves down with acceleration a. Let T be the tension in the string.

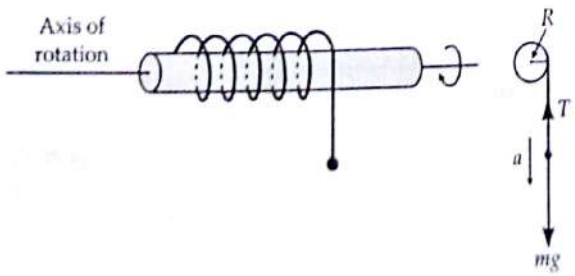

Fig. 7.71 String wound on a cylinder and carrying a point mass.

(*a*) ***Linear acceleration of the point mass.*** The forces acting on the point mass are

(*i*) Its weight mg acting vertically downwards.

(*ii*) Tension T in the string acting upwards.

According to Newton's second law, the net downward force on the point mass is

$$ma = mg - T$$

The tension T in the string acts tangentially on the cylinder and produces a torque τ given by ...(1)

$$\tau = \text{Force} \times \text{lever arm} = T \cdot R$$

If I is the moment of inertia of the cylinder and α the angular acceleration produced in it, then ...(2)

$$\tau = I\alpha$$

From equations (2) and (3), ...(3)

$$TR = I\alpha$$

or $$T = \frac{I}{R}\alpha = \frac{Ia}{R^2} \qquad \left[\because \alpha = \frac{a}{R}\right]$$ ...(4)

From equation (1), we have

$$ma = mg - \frac{Ia}{R^2}$$

or $$ma + \frac{Ia}{R^2} = mg$$

or $$ma\left(1 + \frac{I}{mR^2}\right) = mg$$

or $$a = \frac{g}{1 + \frac{I}{mR^2}}$$ ...(5)

This gives the linear downward acceleration of the point mass.

(*b*) ***Angular acceleration of the point mass.*** As I, m and R are positive quantities, so a is always less than 'g'.

Angular acceleration,

$$\alpha = \frac{a}{R} = \frac{g/R}{1 + \frac{I}{mR^2}}$$ ...(6)

(*c*) ***Tension in the string.*** From equations (4) and (5), we have

$$T = \frac{Ia}{R^2} = \frac{Ig}{R^2\left(1 + \frac{I}{mR^2}\right)}$$

$$= \frac{Ig}{R^2 \cdot \frac{I}{mR^2}\left(\frac{mR^2}{I} + 1\right)}$$

or $$T = \frac{mg}{1 + \frac{mR^2}{I}}$$ ...(7)

Clearly, T is less than the weight mg of the point mass.

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EXAMPLE 68. *A body of mass 5 kg is attached to a weightless string wound round a cylinder of mass 8 kg and radius 0.3 m. The body is allowed to fall. Calculate (i) tension in the string (ii) acceleration with which the body falls and (iii) the angular acceleration of the cylinder.*

Solution. Here $m = 5$ kg, $M = 8$ kg, $R = 0.3$ m

(*i*) M.I. of the cylinder,

$$I = \frac{1}{2} MR^2$$

$\therefore$ Tension in the string,

$$T = \frac{mg}{1 + \frac{mR^2}{I}} = \frac{mg}{1 + \frac{2\,mR^2}{MR^2}} = \frac{mg}{1 + \frac{2m}{M}}$$

$$= \frac{5 \times 9.8}{1 + \frac{2 \times 5}{8}} = \frac{49.0}{2.25} = 21.78 \text{ N.}$$

(*ii*) Linear acceleration,

$$a = \frac{g}{1 + \frac{I}{mR^2}} = \frac{g}{1 + \frac{MR^2}{2\,mR^2}}$$

$$= \frac{g}{1 + \frac{M}{2m}} = \frac{9.8}{1 + \frac{8}{2 \times 5}}$$

$$= \frac{9.8}{1.8} = 5.44 \text{ ms}^{-2}.$$

(*iii*) Angular acceleration,

$$\alpha = \frac{a}{R} = \frac{5.44}{0.3}$$

$$= 18.13 \text{ rad s}^{-1}$$

Very Short Answer Conceptual Problems

Problem 1. What is the advantage of the concept of centre of mass ?

Solution. Centre of mass helps us to describe the behaviour of a macroscopic body in terms of the laws developed for the microscopic bodies. If we are not concerned with the internal motion and structure of a system, the gross motion of the system can be analysed by applying Newton's laws of motion to the CM of the system which is a point where the entire mass of the system may be assumed to be concentrated and where all the external forces are assumed to apply.

Problem 2. Should the centre of mass of a body necessarily lie inside the body ? *[Himachal, 01, 03, 04]*

Solution. Not necessarily. For example, the CM of a ring lies in its hollow portion.

Problem 3. Does the centre of mass of a solid necessarily lie within the body ? If not, give an example.

Solution. No. For example, the CM of *L*-shaped rod lies in the region outside the rod.

Problem 4. Is centre of mass a reality ?

Solution. No. The centre of mass of a system is a hypothetical point which acts as a single mass particle of the system for an external force.

Problem 5. Is it correct to say that the centre of mass of a system of n particles is always given by the average position vectors of the constituent particles ? If not, when is this statement true ?

Solution. No. This is true when all the particles of the system are of same mass.

Problem 6. If two particles of masses m_1 and m_2 move with velocities v_1 and v_2 towards each other on a smooth horizontal table, what is the velocity of their centre of mass ?

Solution. $v_{CM} = \frac{m_1 v_1 - m_2 v_2}{m_1 + m_2}$.

Problem 7. Name the physical quantity that corresponds to the moment of force. On what factors does it depend ?

Solution. The moment of force is called torque. It depends on

(*i*) the magnitude of force.

(*ii*) the perpendicular distance of the line of action of force from the axis of rotation.

Problem 8. A body is in rotational motion. Is it necessary that a torque be acting on it ? *[Delhi 12]*

Solution. No, torque is required only for angular acceleration.

Problem 9. Why are the doors provided with handles near the outer edges far away from the hinges ? *[Central Schools 14]*

Solution. This makes the perpendicular distance of the line of action of force from the hinges maximum. Hence the maximum torque is exerted on the door and the door can be opened or closed easily.

Problem 10. Why do we prefer to use a wrench of longer arm ? *[Central Schools 05]*

Solution. The torque applied on the nut by the wrench is equal to the force multiplied by the perpendicular distance from the axis of rotation. Hence to increase torque a wrench of longer arm is preferred.

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Problem 11. Why in hand driven grinding machine, handle is put near the circumference of the stone or wheel ?

Solution. For a given force, torque can be increased if the perpendicular distance of the point of application of the force from the axis of rotation is increased. Hence the handle put near the circumference produces maximum torque.

Problem 12. It is difficult to open the door by pushing it or pulling it at the hinges. Why ?

Solution. When the force is applied at the hinges, the line of action of the force passes through the axis of rotation i.e., $r = 0$, so $\tau = rF \sin \theta = 0$. So we cannot open the door by pushing or pulling it at the hinges.

Problem 13. Why a force is applied at right angles to the heavy door at the outer edge while closing or opening it ?

Solution. Torque, $\tau = rF \sin \theta$. For a force applied at right angle to the outer edge of the door both $\sin \theta (= \sin 90^\circ = 1)$ and r are maximum. Hence the torque produced is maximum.

Problem 14. A man climbs a tall, old step ladder that has a tendency to sway. He feels much more unstable when standing near the top than when near the bottom. Why ?

Solution. When the man stands at the top of step ladder it increases its distance from the point of contact, the point about which it can slip (rotate). The torque exerted is equal of rF. Hence larger the r, more are the chances to sway away.

Problem 15. Why it is easier to open a tap with two fingers than with one finger ?

Solution. With two fingers, we apply a couple whose moment is equal to the product of the force and perpendicular distance between the two fingers (which is equal to the length of the handle of the tap). When we apply force with one finger, an equal and opposite force of reaction acts at the axis of rotation. This results in a couple of smaller arm and hence lesser moment. So it is easier to open a tap with two fingers than with one finger.

Problem 16. A faulty balance with unequal arms has its beam horizontal. Are the weights of the two pans equal ?

Solution. They are of unequal mass. Their masses are in the inverse ratio of the arms of the balance.

Problem 17. Can the couple acting on a rigid body produce translatory motion ?

Solution. No, the couple acting on a rigid body cannot produce translatory motion ; it can cause only rotatory motion as the resultant force is zero.

Problem 18. A labourer standing near the top of an old wooden step ladder feels unstable. Why ?

Solution. The ladder can rotate about the [illegible] contact of the ladder with the ground. When the [illegible] is at the top of the ladder, the lever arm of the [illegible] large. Hence the turning effect on the ladder will be [illegible]

Problem 19. Which physical quantities are expressed by the following :

(i) the rate of change of angular momentum, and

(ii) moment of linear momentum ?

[Himachal 06 ; Central Schools 0[illegible]

Solution. (i) Torque. (ii) Angular momentum.

Problem 20. If no external torque acts on a body, will its angular velocity remain conserved ?

Solution. No. Angular velocity is not conserved but angular momentum is conserved.

Problem 21. When a labourer cuts down a tree, he makes a cut on the side facing the direction in which he wants it to fall. Why ?

Solution. The weight of tree exerts a torque about the point where the cut is made. This causes rotation of the tree about the cut.

Problem 22. Define the term angular impulse.

Solution. The angular impulse acting on a body is the change of angular momentum of the body about a given axis.

Problem 23. Which component of linear momentum does not contribute to angular momentum ?

Solution. The radial component of linear momentum does not contribute to angular momentum.

Problem 24. A particle revolves uniformly along a circular path, on a smooth horizontal table, by means of a string connected to it. Does its angular momentum change, if the string is suddenly cut ?

Solution. No. The angular momentum remains unaltered when the string is cut.

Problem 25. A heavenly body (such as a comet) revolves around a massive star in a highly elliptical orbit. Is its angular momentum constant over the entire orbit ? (Ignore any mass loss of the comet when it comes too close to the star).

Solution. The heavenly body revolves around the massive star under the effect of gravitational force, which is purely radial. The torque exerted by such a force is zero. Hence the angular momentum of the heavenly body remains constant over its entire orbit.

Problem 26. A projectile acquires angular momentum about the point of projection during its flight. Does it violate the conservation of angular momentum ?

Solution. A projectile will not acquire angular momentum, if no external force acts on it. However during its flight, the projectile is acted upon by the force of gravity and acquires angular momentum.

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Problem 27. Is a body in circular motion in equilibrium ?

Solution. No. A body in circular motion has a centripetal acceleration $\vec{a}$ directed towards the centre of the circle. Since $\vec{a} \neq 0$ the body is not in equilibrium.

Problem 28. When is a body lying in a gravitation field in stable equilibrium ?

Solution. A body in a gravitation field will be in stable equilibrium if the vertical line through its centre of gravity passes through the base of the body.

Problem 29. Can a body in equilibrium while in motion ? If yes, give an example.

Solution. Yes. A body in motion will be in equilibrium if it has no linear and angular accelerations. Hence a body moving with uniform velocity along a straight line will be in equilibrium.

Problem 30. The bottom of a ship is made heavy. Why ?

Solution. The bottom of a ship is made heavy so that the centre of gravity remains low. This ensures the stability of its equilibrium.

Problem 31. Why does a girl lean towards right while carrying a bag in her left hand ?

Solution. When the girl carries a bag in her left hand, her CG shifts towards left. In order to bring it in the middle (for stability of equilibrium), the girl has to lean towards her right.

Problem 32. Some heavy boxes are to be loaded along with some empty boxes on a cart. Which boxes should be put on the cart first and why ?

Solution. The heavy boxes should be loaded first so that the CG of the loaded cart remains in the lowest position. This ensures stability of equilibrium.

Problem 33. Standing is not allowed in a double decker bus. Why ?

Solution. When the passengers stand in the upper deck, the CG of the loaded bus is raised which makes it less stable.

Problem 34. Why we cannot rise from a chair without bending a little forward ?

Solution. Our weight exerts a torque about our feet. This makes difficult for us to rise from the chair. When we bend forward, the CG of our body comes above our feet. The torque due to our weight becomes zero and we can easily rise from the chair.

Problem 35. A system is in stable equilibrium. What can we say about its potential energy ?

Solution. The potential energy of the system is minimum.

Problem 36. Why is moment of inertia also called rotational inertia ?

Solution. The moment of inertia gives a measure of inertia in rotational motion. So it is also called rotational inertia.

Problem 37. Give the physical significance of moment of inertia.

Solution. The moment of inertia plays the same role in rotatory motion as the mass does in translatory motion. It gives a measure of inertia in rotational motion.

Problem 38. Does moment of inertia of a body change with the change of the axis of rotation ?

Solution. Yes. The moment of inertia of a body changes with the change in position and orientation of the axis of rotation.

Problem 39. Does the moment of inertia of a rigid body change with the speed of rotation ? *[Himachal 08]*

Solution. No, because the moment of inertia depends upon the axis of rotation and the distribution of mass.

Problem 40. About which axis, the moment of inertia of a body is minimum ?

Solution. The moment of inertia of a body is minimum about an axis passing through its centre of mass.

Problem 41. Can the mass of body be taken to be concentrated at its centre of mass for the purpose of calculating its rotational inertia ?

Solution. No. The moment of inertia greatly depends on the distribution of mass about the axis of rotation.

Problem 42. About which axis would a uniform cube have a minimum rotational inertia ?

Solution. About a diagonal, because the mass is more concentrated about a diagonal.

Problem 43. Is radius of gyration a constant quantity ? *[Himachal 01, 04]*

Solution. No. It changes with the change in position of the axis of rotation.

Problem 44. Does the radius of gyration depend upon the speed of rotation of the body ?

Solution. No, it depends only on the distribution of mass of the body.

Problem 45. Two lenses of same mass and same radius are given. One is convex and other is concave. Which one will have greater moment of inertia, when rotating about an axis perpendicular to the plane and passing through the centre ?

Solution. The concave lens will have greater moment of inertia because the concave lens is thin at its centre and its mass is more concentrated at the outer edge.

Problem 46. A disc is recast into a thin walled cylinder of same radius. Which will have large moment of inertia ?

Solution. Hollow cylinder will have larger moment of inertia because most of its mass is located at comparatively larger distance from the axis of rotation.

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Problem 47. Two solid spheres of the same mass are made of metals of different densities. Which of them has a larger moment of inertia about the diameter ? Why ? *[Delhi 10]*

Solution. The sphere with small density will have larger radius and hence large moment of inertia.

Problem 48. What is the advantage of the flywheel ?

Solution. In case of a flywheel, the whole mass is practically near the rim which is situated far away from the axis of rotation. This is done to increase the moment of inertia of the wheel, thereby making the motion smooth and less jerky.

Problem 49. Why spokes are provided in a bicycle wheel ?

Solution. By connecting to the rim of wheel to the axle through the spokes, the mass of the wheel gets concentrated at its rim. This increases its moment of inertia. This ensures its uniform speed.

Problem 50. Will two spheres of equal masses, one solid and the other hollow have equal moments of inertia ? Give reason.

Solution. The hollow sphere will have a greater moment of inertia because its entire mass is concentrated at the boundary of the sphere *i.e.*, at maximum distance from the axis of rotation.

Problem 51. Why is it more difficult to revolve a stone tied to a large string than a stone tied to a smaller string ? *[Chandigarh 04]*

Solution. The length of the string increases the distance of rotating mass from the axis of rotation and hence moment of inertia of the system is increased. Now, $\tau = I\,\alpha$ thus a system with large moment of inertia requires large torque for its rotation.

Problem 52. Two satellites of equal masses, which can be considered as particles are orbiting the earth at different heights ? Will their moments of inertia be same or different ? *[Himachal 06]*

Solution. M.I. of a satellite, $I = Mr^2$ *i.e.*, $I \propto r^2$

Hence the two satellites orbiting the earth at different heights will have different moments of inertia. The satellite orbiting at a larger height will have a larger moment of inertia.

Problem 53. What is the use of flywheel in railway engine ?

Solution. In a flywheel, most of the mass is concentrated at its rim. So it has a large moment of inertia. Any change of angular momentum imparted to the wheel by the piston results in a lesser change of angular velocity ($L = I\omega$ or $\omega = L/I$). Moreover, a flywheel stores a large amount of rotational energy ($\frac{1}{2} I\omega^2$). This helps the wheel fly off the dead point.

Problem 54. There is a stick half of which is wooden and half is of steel. It is pivoted at the wooden end and a force is applied at the steel end at right angles to the length. Next, it is pivoted at the steel end and the same force is applied at the wooden end. In which case is the angular acceleration more and why ?

Solution. The distribution of mass is farther from the axis of rotation in the first case than in the second case. So the moment of inertia is more in first case than in second case, but the applied torque τ is same in both cases. As $\tau = I\,\alpha$ or $\alpha = \tau / I$, so the angular acceleration α is less in first case than in second case.

Problem 55. Is the angular momentum of a system always conserved ? If not, under what condition is it conserved ?

Solution. No, angular momentum of a system is not always conserved. It is conserved only when no external torque acts on the system.

Problem 56. A flywheel is revolving with a constant angular velocity. A chip of its rim breaks and flies away. What will be the effect on its angular velocity ?

Solution. Due to the decrease in its mass, the moment of inertia of the flywheel will decrease. To conserve angular momentum, the angular velocity of the flywheel will increase.

Problem 57. A cat is able to land on its feet after a fall. Why ? *[Himachal 05, 07C, 08C]*

Solution. When a cat falls to ground from a height, it stretches its body along with the tail so that its moment of inertia becomes high. Since $I\,\omega$ is to remain constant, the value of angular speed ω decreases and therefore the cat is able to land on the ground gently.

Problem 58. Why there are two propellers in a helicopter ?

Solution. If there were only one propeller in the helicopter then, due to conservation of angular momentum, the helicopter itself would have turned in the opposite direction.

Problem 59. The speed of a whirl wind in a tornado is alarmingly high. Why ?

[Himachal 07C, 08 ; Central Schools 04]

Solution. In a whirl wind, the air from nearby region gets concentrated in a small space thereby decreasing the value of moment of inertia considerably. Since, $I\,\omega =$ constant, due to decrease in moment of inertia, the angular speed becomes quite high.

Problem 60. If earth contracts to half its radius, what would be the length of the day ? *[Himachal 08]*

Solution. The moment of inertia ($I = \frac{2}{5} MR^2$) of the earth about its own axis will become one fourth and so its angular velocity will become four times ($L = I\,\omega =$ constant). Hence the time period will reduce to one fourth ($T = 2\pi / \omega$) *i.e.*, 6 hours.

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Problem 61. Two boys of the same weight sit at the opposite ends of a diameter of a rotating circular table. What happens to the speed of rotation if they move nearer to the axis of rotation ?

Solution. The moment of inertia of the system (circular table + two boys) decreases. To conserve angular momentum ($L = I\omega$ = constant), the speed of rotation of the circular table increases.

Problem 62. A thin wheel can stay up right on its rim for a considerable length of time when rolled with a considerable velocity, while it falls from its upright position at the slightest disturbance when stationary. Give reason.

Solution. When the wheel is rolling upright, it has angular momentum in the horizontal direction *i.e.*, along the axis of the wheel. Because the angular momentum is to remain conserved, the wheel does not fall from its upright position because that would change the direction of angular momentum. The wheel falls only when it loses its angular velocity due to friction.

Problem 63. A person is standing on a rotating table with metal spheres in his hands. If he withdraws his hands to his chest, what will be the effect on his angular velocity ? *[Himachal 08]*

Solution. When the person withdraws his hands to his chest, his moment of inertia decreases. No external torque is acting on the system. So to conserve angular momentum, the angular velocity increases.

Problem 64. A circular turn table rotates at constant angular velocity about a vertical axis. There is no friction and no driving torque. An ice pan containing ice also rotates with it. The ice melts but none of the water escapes from the pan. Is the velocity now greater, the same or less than the original velocity ? Give reason.

Solution. Due to accumulation of water near the edge, the moment of inertia of the system increases. To conserve angular momentum, the angular velocity of the system decreases.

Problem 65. Many rivers flow towards the equator. What effect does the sediment they carry to the seas have on the rotation of the earth ?

Solution. The rivers carry sediments away from the axis of rotation of the earth. This increases the rotational inertia of the earth. To conserve angular momentum, the rotational speed of the earth decreases.

Problem 66. The moments of inertia of two rotating bodies A and B are I_A and I_B ($I_A > I_B$) and their angular momenta are equal. Which one has a greater kinetic energy ? *[Central Schools 2010]*

Solution. Angular momentum,

$$L = I\omega$$

K.E. of rotation, $$K = \frac{1}{2} I\omega^2 = \frac{1}{2}\frac{I^2\omega^2}{I} = \frac{1}{2}\frac{L^2}{I}$$

For constant L, $$K \propto \frac{1}{I}$$

As $$I_A > I_B$$

$$\therefore \quad K_A < K_B \quad \text{or} \quad K_B > K_A.$$

So body B has a greater rotational K.E.

Problem 67. If angular momentum is conserved in a system whose moment of inertia is decreased, will its rotational kinetic energy be conserved ?

[Himachal 06 ; Chandigarh 08]

Solution. Here $L = I\omega$ = constant

Rotational K.E. is given by

$$K = \frac{1}{2} I\omega^2 = \frac{1}{2}\frac{I^2\omega^2}{I} = \frac{1}{2}\cdot\frac{L^2}{I}$$

For constant L, $$K \propto \frac{1}{I}$$

So when the moment of inertia decreases, the rotational K.E. increases. Hence rotational K.E. is not conserved.

Problem 68. How does an ice-skater, a ballet dancer or an acrobat take advantage of the principle of conservation of angular momentum ? *[Himachal 08]*

Or

How does a ballet dancer vary her angular speed by oustretcing her arms and legs ? *[Central Schools 08, 14]*

Solution. An ice-skater, a ballet dancer or an acrobat is able to change his angular speed during the course of the performance. When the performer stretches out his hands and legs, his moment of inertia increases and the angular speed decreases. On the other hand, when he folds his hands and the legs near his body, the moment of inertia decreases and he is able to increase the angular speed.

Problem 69. If earth were to shrink suddenly, what would happen to the length of the day ?

[Chandigarh 04 ; Central Schools 05]

Solution. When the earth shrinks, the moment of inertia $\left(I = \frac{2}{5}MR^2\right)$ decreases about its own axis due to the decrease in radius R. To conserve angular momentum. $(L = I\omega = I\cdot\frac{2\pi}{T})$, the time period T decreases. That is, the length of the day decreases.

Problem 70. A body A of mass M while falling vertically downwards under gravity breaks into two parts ; a body B of mass $M/3$ and a body C of mass $2M/3$. How does the centre of mass of bodies B and C taken together shift compared to that of A ? *[AIEEE 05]*

Solution. The centre of mass of bodies B and C does not shift compared to body A. It continues to move vertically downwards under the effect of gravity. This is because there is no new external force acting on the system.

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Problem 71. Two identical particles move towards each other with velocity $2v$ and v respectively. What is the velocity of the centre of mass ? *[AIEEE 02]*

Solution. Here $m_1 = m_2 = m$; $v_1 = 2v$ and $v_2 = -v$

$$\therefore \quad v_{CM} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{m \times 2v + m(-v)}{m + m} = \frac{v}{2}.$$

Problem 72. A particle moves in a circular path with decreasing speed. What happens to its angular momentum ? *[IIT 05 ; Chandigarh 08]*

Solution. The angular momentum of a particle of mass m moving with velocity $\vec{v}$ along a circular path of radius r is given by

$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v}$$

When the speed v decreases, the magnitude of angular momentum decreases. But the direction of angular momentum remains unchanged.

Problem 73. A particle performing uniform circular motion has angular momentum L. What will be the new angular momentum, if its angular frequency is doubled and its kinetic energy halved ? *[AIEEE 03]*

Solution. Rotational K.E.,

$$K = \frac{1}{2} I\omega^2 \quad \therefore \quad I = \frac{2K}{\omega^2}$$

Angular momentum,

$$L = I\omega = \frac{2K}{\omega^2} \cdot \omega = \frac{2K}{\omega}$$

When angular frequency is doubled and kinetic energy is halved, the angular momentum becomes,

$$L' = \frac{2(K/2)}{2\omega} = \frac{1}{4} \cdot \frac{2K}{\omega} = \frac{L}{4}.$$

Problem 74. The angular velocity of the earth around the sun increases, when it comes closer to the sun. Why ? *[Himachal 09 ; Delhi 01]*

Solution. When the earth comes closer to the sun, its moment of inertia about the axis through the sun decreases. To conserve angular momentum ($L = I\omega$), the angular velocity of the earth increases.

Problem 75. Why are we not able to rotate a wheel by pulling or pushing along its radius ? *[Central Schools 08]*

Solution. This is because radial component of the applied force cannot produce torque.

$$\tau = rF\sin 0 = 0 \times F\sin 0 = 0$$

Problem 76. Two solid spheres of the same mass are made of metals of different densities. Which of them has larger moment of inertia about its diameter ? *[Central Schools 09]*

Solution. The sphere of metal with smaller density, will have larger size and hence it will have larger moment of inertia than the other sphere.

Problem 77. A planet revolves around a massive star in a highly elliptical orbit. Is the angular momentum constant over the entire orbit ? *[Himachal 06]*

Solution. A planet revolves around a star under the effect of the gravitational force, which is purely radial in nature. As radial component of a force does not contribute to torque, so the angular momentum of the planet remains unaffected.

As $\tau = \frac{dL}{dt} = 0$, so L = constant.

Problem 78. If no external torque acts on a body, will its angular velocity remain constant ? Give reason. *[Himachal 05C, 06]*

Solution. When no external torque acts on a body, its angular momentum remains constant. But

$$L = I\omega$$

Clearly, the angular velocity ω will remain constant only so long as the moment of inertia I of the body remains constant.

Short Answer Conceptual Problems

Problem 1. "Newton's laws of motion are applicable to individual particles". How would you explain the motion of a large body ?

Solution. A large body can be considered as made of a number of mass particles and all mass particles interact with each other. The vector sum of all these internal forces is zero. Therefore for a large body, it can be replaced by a single mass particle whose mass is supposed to be situated at its centre of mass and the Newton's laws can be applied.

Problem 2. If an external force can change the state of motion of CM of a body, how does the internal force of the brakes bring a car to rest ?

Solution. Actually, it is not the external force which brings the car to rest. The internal force of the brakes on the wheel locks the wheel. Now a large frictional force comes into play between the wheels and the ground. This force is external to the system and brings the car to rest.

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Problem 3. Two men stand facing each other on two rafts floating on still water at a distance apart. A rope is tied at its ends by both. The two boats are found to meet always at the same point, whether each man pulls forcibly or both pull together, why ? Will the time taken be different in the two cases ? Neglect friction.

Solution. The men on the two boats floating on water constitute a single system. So the forces applied by the men are internal forces. Whether each man pulls forcibly or both pull together, the centre of mass of the system of boats remains fixed due to the absence of any external force. Consequently, the two boats meet at a fixed point which is the centre of mass of the system.

Problem 4. Prove that the centre of mass of two particles divides the line joining the particles in the inverse ratio of their masses.

Solution. As shown in Fig. 7.72, consider a system of two particles of masses m_1 and m_2 situated at points A and B respectively. Suppose the origin O of the frame of reference coincides with their centre of mass.

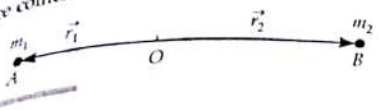

Fig. 7.72

If $\vec{r_1}$ and $\vec{r_2}$ are the position vectors of masses m_1 and m_2 with respect to the centre of mass, then

$$m_1 \vec{r_1} + m_2 \vec{r_2} = (m_1 + m_2)\vec{O} = 0$$

$$\vec{r_1} = -\frac{m_2}{m_1}\vec{r_2}$$

or

$$|\vec{r_1}| = \frac{m_2}{m_1}|\vec{r_2}|$$

or

$$\frac{r_1}{r_2} = \frac{m_2}{m_1}$$

or

Hence the centre of mass of two particles divides the line joining the two particles in the inverse ratio of their masses.

Problem 5. Two balls of mass m each are placed at the two vertices of an equilateral triangle. Another ball of mass $2m$ is placed at the third vertex of the triangle. Locate the centre of mass of the system.

Solution. As shown in Fig. 7.73, the centre of mass of the two balls placed at A and B lies at the midpoint D of the side AB. Now we have the system of two masses, each of mass $2m$ at the points C and D on the perpendicular bisector CD of side AB. Hence the centre of mass of the system lies at the midpoint O of the perpendicular bisector CD.

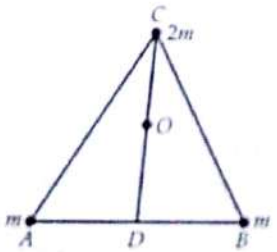

Fig. 7.73

Problem 6. What is the difference between centre of gravity and centre of mass ? *[Himachal 05C]*

Solution. Centre of mass is a point at which whole of the mass of the body may be assumed to be concentrated to describe its motion as a particle. Centre of gravity is a point at which resultant of the gravitational forces on all particles of the body acts. For bodies of normal dimensions, centre of mass and centre of gravity coincide. But centre of mass and centre of gravity relate to two different concepts. Even if the world were devoid of gravitational force, the centre of mass would still have a meaning.

Problem 7. There are 100 passengers in a stationary railway compartment. A physical fight starts between the passengers over some difference of opinion. (*i*) Will the position of CM of the compartment change ? (*ii*) Will the position of CM of system (compartment +100 passengers) change ? Give reason.

Solution. (*i*) The position of the CM of the compartment will change because the passengers are external bodies for the compartment.

(*ii*) The position of CM of the system will not change as no external force is acting on the system.

Problem 8. Show that moment of a couple does not depend on the point about which you take the moments. *[NCERT]*

Solution. Fig. 7.74 shows a couple acting on a rigid body. The forces $-\vec{F}$ and $\vec{F}$ act at points A and B respectively. Let $\vec{r_1}$ and $\vec{r_2}$ be the position vectors of these points with respect to origin O.

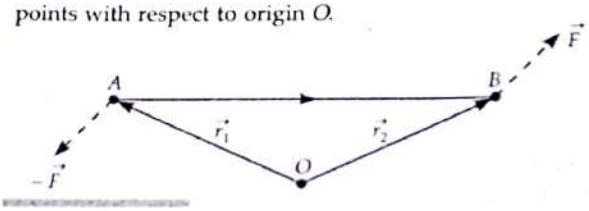

Fig. 7.74

We consider the moments of the forces about the origin O.

Moment of the couple = Sum of the moments of the two forces

$$= \vec{r_1} \times (-\vec{F}) + \vec{r_2} \times \vec{F} = \vec{r_2} \times \vec{F} - \vec{r_1} \times \vec{F}$$

$$= (\vec{r_2} - \vec{r_1}) \times \vec{F} = \vec{AB} \times \vec{F} \qquad [\because \vec{r_1} + \vec{AB} = \vec{r_2}]$$

Clearly, this moment is independent of the point O about which we have considered the moments of the forces.

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Problem 9. Show that the angular momentum about any point of a single particle moving with constant velocity remains constant throughout the motion. Is there any external torque on the particle ? *[NCERT]*

Solution. As shown in Fig. 7.75, suppose a particle has velocity $\vec{v}$ at point P at some instant t. Its angular momentum about any arbitrary point O is

$$\vec{l} = \vec{r} \times m\vec{v}$$

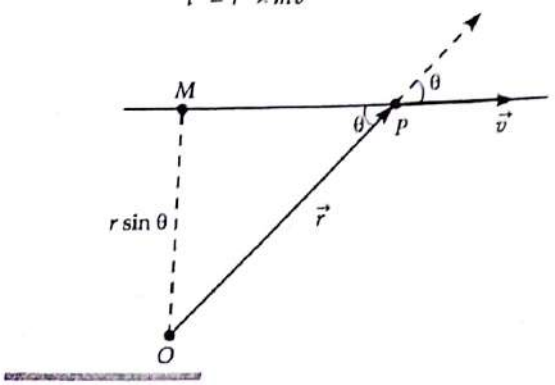

Fig. 7.75

The magnitude of angular momentum is

$$l = mvr \sin\theta$$

where θ is the angle between $\vec{r}$ and $\vec{v}$. Now the particle has a constant velocity. Although the position of the particle changes with time, the line of direction of velocity $\vec{v}$ remains same and hence $OM = r \sin\theta$ is a constant. Consequently, the magnitude l remains constant.

Further, the direction of $\vec{l}$ is perpendicular to the plane of $\vec{r}$ and $\vec{v}$. It is into the plane of the paper. This direction does not change with time.

Thus, $\vec{l}$ remains the same in magnitude and direction and is therefore conserved.

As the velocity of the particle remains constant, so no external force and hence no external torque is acting on the particle.

Problem 10. A rod of weight W is supported by two parallel edges A and B and is in equilibrium in horizontal position. The knives are at a distance d from each other. The centre of mass of the rod is at distance x from A. Find the normal reactions at the knife edges A and B.

Solution. As shown in Fig. 7.76, let R_A and R_B be the normal reactions at the edges.

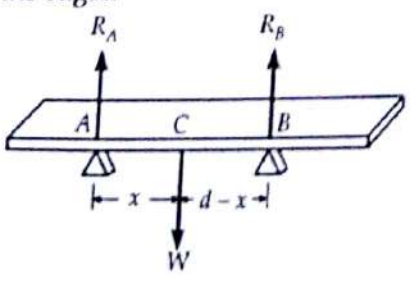

Fig. 7.76

Since the rod is in equilibrium, the sum of moments of the forces about either knife edge must be zero.

Taking moments of forces about point A, we get

$$R_A \times 0 + W \times x - R_B \times d = 0$$

$$\therefore \quad R_B = \frac{x}{d} \cdot W$$

Taking moments of forces about point B, we get

$$R_A \times d - W \times (d - x) + R_B \times 0 = 0$$

$$\therefore \quad R_A = \frac{d - x}{d} \cdot W.$$

Problem 11. Torque and work are both equal to force times distance. Then how do they differ ?

Solution. (*i*) Work is a scalar quantity while torque is a vector quantity.

(*ii*) Work is measured as the product of the applied force and the distance moved by the body in the direction of the force. Torque is measured as the product of the force and its perpendicular distance from the axis of rotation.

Problem 12. When is a rigid body said to be in equilibrium ? State the necessary conditions for a body to be in equilibrium.

Solution. A rigid body is said to be in equilibrium when its linear acceleration and angular acceleration are zero.

(*i*) For translation equilibrium of a rigid body, the sum of all the external forces acting on it must be zero.

$$\Sigma \vec{F}_i = 0$$

(*ii*) For rotational equilibrium of a rigid body, the sum of torques due to all forces acting on it must be zero.

$$\Sigma \vec{\tau}_i = 0$$

Problem 13. Define moment of inertia. On what factors does it depend ?

Solution. The moment of inertia of a rigid body about an axis is the sum of the products of the masses of its various particles and squares of their perpendicular distances from the axis of rotation.

$$I = \Sigma m_i r_i^2$$

The moment of inertia of a body depends on

(*i*) Mass of the body.

(*ii*) Size and shape of the body.

(*iii*) Distribution of mass about the axis of rotation.

(*iv*) Position and orientation of the axis of rotation with respect to the body.

Problem 14. How will you distinguish between a hard boiled egg and a raw egg by spinning it on a table top ?

Solution. Hard boiled egg acts just like a rigid body while rotating while it is not the case in a raw egg because

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of liquid matter present in it. In case of a raw egg, the liquid matter tries to go away from the centre, thereby increasing its moment of inertia. As moment of inertia is more, the raw egg will spin with less angular acceleration provided same torque is applied in both the cases. Thus, hard boiled egg will spin faster.

Problem 15. If two circular discs of the same mass and thickness are made from metals of different densities, which disc will have the larger moment of inertia about its central axis ? Explain.

Solution. $I_1 = \frac{1}{2} m r_1^2$ and $I_2 = \frac{1}{2} m r_2^2$

$$\frac{I_1}{I_2} = \frac{r_1^2}{r_2^2}$$

$$\therefore \quad m = \pi r_1^2 t \, \rho_1 = \pi r_2^2 t \, \rho_2$$

Also, $$\frac{r_1^2}{r_2^2} = \frac{\rho_2}{\rho_1}$$

Hence $$\frac{I_1}{I_2} = \frac{\rho_2}{\rho_1} \quad i.e., \quad I \propto \frac{1}{\rho}$$

Thus the disc with greater density will have less moment of inertia.

Problem 16. Which one is easier to turn—a log or a bench of equal weight and length ? The two have the same coefficient of friction with the ground. Explain.

Solution. In the case of a log, the force of friction is distributed along its entire length and so it is effective midway between the end where the force is applied and the centre of the log, but in the case of a bench, the force of friction is at the end. So to turn the log, the torque required is equal to $\mu \, mg \, l/2$, while to turn the bench, the torque required is equal to $\mu \, mg \, l$. Hence it is easier to turn a log than a bench of equal weight and length.

Problem 17. The Moment of inertia of a disc about an axis passing through its centre and perpendicular to its plane is $1/2MR^2$. Derive the values of moment of inertia of the disc about its diameter and about an axis tangential to the disc lying on its plane ?

[Central Schools 12]

Solution. Refer answer to Q. 41(*b*) and (*c*) on page 7.29.

Problem 18. What is the moment of inertia of a rod of mass M, length l about an axis perpendicular to it through one end ? Given the moment of inertia about the centre of mass is $\frac{1}{12} Ml^2$ *[NCERT ; Central Schools 03]*

Solution. By using *theorem of parallel axes,*

$$I_{end} = I_{CM} + M\left(\frac{l}{2}\right)^2 = \frac{1}{12} Ml^2 + \frac{1}{4} Ml^2 = \frac{1}{3} Ml^2.$$

Problem 19. What is the moment of inertia of a ring about a tangent to the circle of the ring ? *[NCERT]*

Solution. Refer answer to Q. 40(*c*) on page 7.28.

Problem 20. What is the moment of inertia of a uniform circular disc of radius R and mass M about an axis passing through its centre and normal to the disc ? The moment of inertia of the disc about any of its diameters is given to be $(1/4)\,MR^2$. *[Himachal 06]*

Solution. M.I. of the disc about any diameter,

$$I_D = \frac{1}{4} MR^2.$$

By the theorem of perpendicular axes,

M.I. of the disc about an axis through the centre and normal to disc = M.I. about any diameter + M.I. about perpendicular diameter

$$I = I_D + I_D = 2 \times \frac{1}{4} MR^2$$

or $$I = \frac{1}{2} MR^2.$$

Problem 21. The moment of inertia of a solid sphere about a tangent is $\frac{5}{3} MR^2$, where M is mass and R is radius of the sphere. Find the M.I. of the sphere about its diameter.

Solution. Here $I_T = \frac{5}{3} MR^2$.

Now diameter of sphere is an axis passing through its centre of mass. By using theorem of parallel axes,

$$I_T = I_{CM} + MR^2$$

$$\therefore \quad I_{CM} = I_T - MR^2 = \frac{5}{3} MR^2 - MR^2 = \frac{2}{3} MR^2.$$

Problem 22. The moment of inertia of a uniform circular disc about a tangent in its own plane is $\frac{5}{4} MR^2$, where M is mass and R is the radius of the disc. Find its moment of inertia about an axis through its centre and perpendicular to its plane.

Solution. If I_D is the M.I. of the disc about its diameter, then from the theorem of parallel axes,

$$I_T = I_D + MR^2$$

or $$I_D = I_T - MR^2 = \frac{5}{4} MR^2 - MR^2 = \frac{1}{4} MR^2$$

By the theorem of perpendicular axes, the M.I. about an axis through the centre and perpendicular to the plane of the disc,

I = Sum of moments of inertia about two perpendicular diameters

$$= I_D + I_D = 2 \times \frac{1}{4} MR^2$$

or $$I = \frac{1}{2} MR^2.$$

Problem 23. Using expressions for power and kinetic energy of rotational motion, derive the relation. $\tau = I\alpha$, where letters have their usual meanings.

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Solution. Power in rotational motion, $P = \tau\omega$

Rotational K.E., $K = \frac{1}{2} I\omega^2$

Work done in rotational motion,

W = Energy stored as rotational K.E. $= \frac{1}{2} I\omega^2$

$$\therefore \quad P = \frac{dW}{dt} = \frac{d}{dt}\left(\frac{1}{2} I\omega^2\right) = \frac{1}{2} \cdot I \times 2\omega \frac{d\omega}{dt}$$

or $$\tau\omega = I\omega\alpha \qquad \left[\because \frac{d\omega}{dt} = \alpha\right]$$

$$\therefore \quad \tau = I\alpha.$$

Problem 24. Explain if the ice on the polar caps of the earth melts, how will it affect the duration of the day ?

[Himachal 05, 07C, 08 ; Central Schools 09]

Solution. If the ice on the polar caps of the earth melts, the water so formed will spread on the surface of earth. This increases the moment of inertia (I) of the earth about its own axis (due to change in the distribution of mass of the particles of water going away from the axis of rotation). To conserve angular momentum ($= I\omega$), ω (angular velocity of earth about its own axis) will decrease. As $T = 2\pi/\omega$, hence due to decrease in the value of ω, T *i.e.*, the duration of the day will increase.

Problem 25. Two identical cylinders 'run a race' starting from rest at the top of an inclined plane, one slides without rolling and other rolls without slipping. Assuming that no mechanical energy is dissipated as heat, which one will win ?

Solution. When the cylinder slides without rolling,

$$E = \frac{1}{2} mv'^2 \qquad \therefore \quad v' = \sqrt{\frac{2E}{m}}$$

When the cylinder rolls without slipping,

$$E = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2 = \frac{1}{2} mv^2 + \frac{1}{2}\left(\frac{1}{2} mr^2\right)\omega^2$$

$$= \frac{1}{2} mv^2 + \frac{1}{4} mv^2 = \frac{3}{4} mv^2 \qquad \therefore \quad v = \sqrt{\frac{4E}{3m}}$$

As $v' > v$, therefore sliding cylinder will win the race.

Problem 26. A uniform circular disc of radius R is rolling on a horizontal surface. Determine the tangential velocity (*i*) at the upper most point, (*ii*) at the centre of mass and (*iii*) at the point of contact.

Solution. ***Rolling can be regarded as the combination of pure rotation and pure translation.*** As shown in Fig. 7.77 (*a*), in case of pure rotation the, velocity of CM is zero and the tangential velocity at points A and B is $v_A = v_B = R\omega$. As shown in Fig. 7.77(*b*), in case of pure translation, $v_A = v_B = v_{CM} = R\omega = v$ (say).

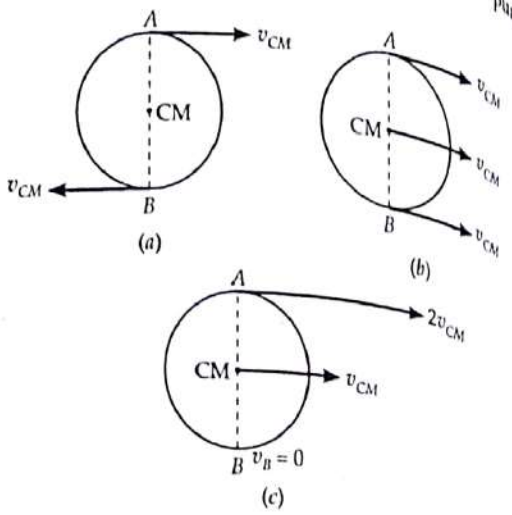

Fig. 7.77 (*a*) Pure rotation. (*b*) Pure translation, (*c*) Rolling.

As shown in Fig. 7.77(*c*), in case of rolling, the tangential velocity at any given point is the vector sum of the velocities in (*a*) and (*b*) at that point.

Hence $v_A = 2v_{CM}$, $v_{CM} = v = R\omega$ and $v_B = 0$.

It may be noted that in case of rolling the instantaneous velocity at the point of contact B with surface is zero, because the body is not slipping.

HOTS

Problems on Higher Order Thinking Skills

Problem 1. *An isolated particle of mass m is moving in a horizontal plane (x-y), along the x-axis at a certain height above the ground. It suddenly explodes into two fragments of masses m/4 and 3m/4. At instant later, the smaller fragment is at y = + 15 cm. What is the position of larger fragment at this instant ?* **[IIT 97]**

Solution. As the isolated particle is initially moving along x-axis, so there is no motion along y-axis. The centre of mass should remain stationary along y-axis even after explosion.

Now $$y_{CM} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$$

But $$y_{CM} = 0$$

$$\therefore \quad m_1 y_1 + m_2 y_2 = 0$$

or $$y_2 = -\frac{m_1}{m_2} \cdot y_1$$

$$= -\frac{m/4}{3m/4} \cdot 15 = -5 \text{ cm}.$$

Thus the larger fragment will be at $y = -5$ cm.

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Problem 2. *A small sphere of radius R is held against the inner surface of a larger sphere of radius 6R [Fig. 7.78]. The masses of large and small spheres are 4M and M, respectively. This arrangement is placed on a horizontal table. There is no friction between any surfaces of contact. The small sphere is now released. Find the co-ordinates of the centre of the larger sphere when the smaller sphere reaches the other extreme position.* **[IIT 96]**

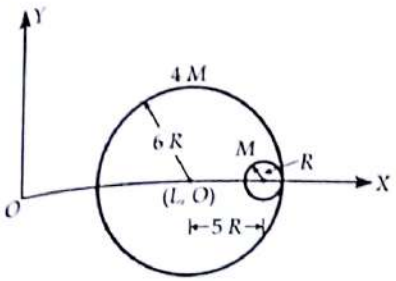

Fig. 7.78

Solution. Initial position of the centre of mass of the system is given by

$$x_{CM} = \frac{4M(L) + M(L + 5R)}{5M} = L + R$$

When the smaller sphere reaches the other extreme [Fig. 7.79], the larger sphere moves to the right so that

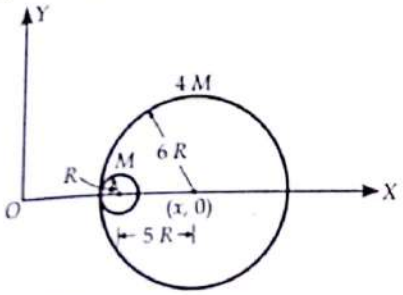

Fig. 7.79

position of CM of the system remains unchanged (as no external force is acting in the system). New position of the CM of the system is

$$x'_{CM} = \frac{4M(x) + M(x - 5R)}{5M} = x - R$$

But $x_{CM} = x'_{CM}$

or $L + R = x - R$

or $x = L + 2R$

∴ Centre of larger sphere will lie at $(L + 2R, 0)$.

Problem 3. *A boat of 90 kg is floating in still water. A boy of mass 30 kg walks from the stern to the bow. The length of the boat is 3 m. Calculate the distance through which the boat will move.* **[REC 92]**

Solution. As shown in Fig. 7.80, let C_1, C_2 and C be the centres of mass of the boy, boat and the system (boy and boat) respectively. Let x_1 and x_2 be the distances of C_1 and C_2 from the shore. Then the centre of mass will be at a distance,

$$x_{CM} = \frac{30x_1 + 90x_2}{30 + 90}$$

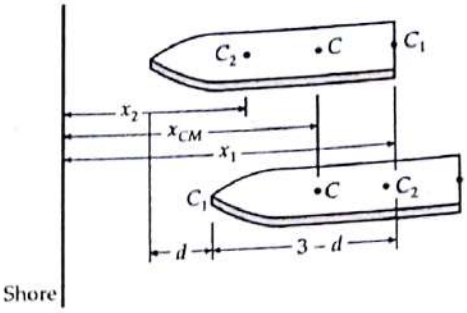

Fig. 7.80

As the boy C_1 moves from the stern to the bow, the boat moves backward through a distance d so that position of the centre of mass of the system remains unchanged.

$$x'_{CM} = \frac{30[x_1 - (3 - d)] + 90(x_2 + d)}{30 + 90}$$

As $x'_{CM} = x_{CM}$

$$\frac{30(x_1 - 3 + d) + 90(x_2 + d)}{120} = \frac{30x_1 + 90x_2}{120}$$

or $-90 + 30d + 90d = 0$

or $d = 0.75$ m.

Problem 4. *From a uniform circular disc of diameter D, a circular disc or hole of diameter D/6 and having its centre at a distance of D/4 from the centre of the disc is scooped out. Determine the centre of mass of the remaining portion.*

Solution. Let mass per unit area of the disc = m

∴ Total mass of the disc, $M = \pi\left(\frac{D}{2}\right)^2 m = \frac{\pi mD^2}{4}$

Mass of the scooped out portion of the disc,

$$M' = \pi\left(\frac{D}{12}\right)^2 m = \frac{\pi mD^2}{144}$$

We take the centre O of the disc as the origin. The masses M and M' can be supposed to be concentrated at the centres of the disc and scooped out portion respectively. The mass M' of the removed portion is taken negative.

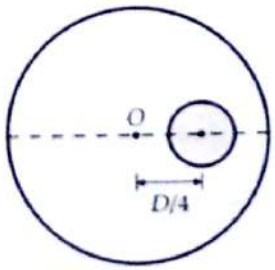

Fig. 7.81

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Then x-coordinate of the CM of the remaining portion of the disc will be

$$x_{CM} = \frac{Mx_1 - Mx_2}{M - M} = \frac{M \times 0 - M\,(D/4)}{\frac{\pi mD^2}{4} - \frac{\pi mD^2}{144}}$$

$$= \frac{MD}{4} \times \frac{144}{35\pi mD^2}$$

$$= -\frac{\pi mD^2}{144} \times \frac{D}{4} \times \frac{144}{35\pi mD^2} = \frac{D}{140}.$$

Thus, the centre of mass of the remaining portion lies at a distance of $D/140$ towards the left of the centre O of the disc.

Problem 5. *A tube of length L is filled completely with an incompressible liquid of mass M and closed at both the ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity ω. Determine the force exerted by the liquid at the other end.* [IIT 92]

Solution. Consider a small element of the liquid of length dx at a distance x from one end.

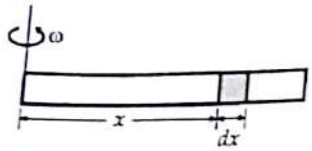

Fig. 7.82

Mass of the small element $= \frac{M}{L}\,dx$

Centripetal force associated with the element

$$dF = \left(\frac{M}{L}\,dx\right) x\,\omega^2 \qquad [\because\ F = mr\omega^2]$$

Force exerted by the liquid = Total centripetal force at the other end

$$F = \int dF = \int_0^L \frac{M}{L}\,\omega^2 x\,dx = \frac{M}{L}\omega^2\left[\frac{x^2}{2}\right]_O^L$$

$$= \frac{M}{L}\,\omega^2\,\frac{L^2}{2} = \frac{1}{2}\,M\omega^2 L$$

Problem 6. *A particle describes a horizontal circle on the smooth surface of an inverted cone. The height of the plane of the circle above the vertex is 9.8 cm. Find the speed of the particle. Take $g = 9.8\ ms^{-2}$.* [MNREC 92]

Solution. As shown in Fig. 7.83, suppose the particle describes a circle of radius r with speed v. Various forces acting on the particle are

(*i*) its weight mg acting vertically downwards,

(*ii*) normal reaction R of the smooth surface of the cone.

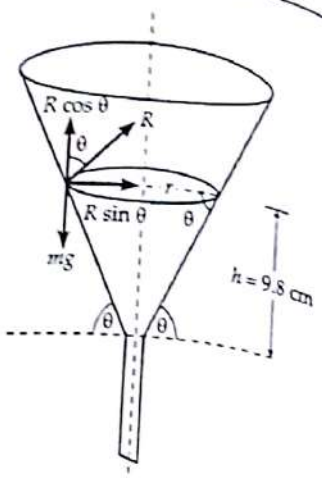

Fig. 7.83

Let the reaction R make angle θ with the vertical. Resolving R into vertical and horizontal components, we get

$$R\cos\theta = mg$$

$$R\sin\theta = \frac{mv^2}{r} \qquad \text{...(ii)}$$

Dividing (*ii*) by (*i*), we get ...(iii)

$$\tan\theta = \frac{v^2}{rg}$$

From Fig. 7.83,

$$\tan\theta = \frac{h}{r}$$

$$\therefore \quad \frac{v^2}{rg} = \frac{h}{r}$$

or $$v = \sqrt{gh} = \sqrt{9.8 \times 9.8 \times 10^{-2}} = 0.98\ \text{ms}^{-1}.$$

Problem 7. *Point masses m_1 and m_2 are placed at the ends of a rigid rod of length L and negligible mass. The rod is to be set rotating about an axis perpendicular to its length. Locate a point on the rod through which the axis of rotation should pass in order that the work required to set the rod rotating with angular velocity ω_0 is minimum.* [REC 96]

Solution. As shown in Fig. 7.84, let the axis of rotation be at a distance x from mass m_1. Then moment of inertia of the system about this axis of rotation is

$$I = m_1 x^2 + m_2 (L - x)^2$$

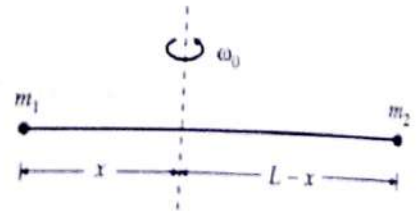

Fig. 7.84

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Work done to set the rod rotating with angular velocity ω_0 = Increase in rotational K.E.

$$W = \frac{1}{2} I\omega_0^2 = \frac{1}{2}[m_1 x^2 + m_2 (L-x)^2]\,\omega_0^2$$

For W to be minimum, $\frac{dW}{dx} = 0$

$$\frac{1}{2}[2m_1 x + 2m_2 (L-x)(-1)]\,\omega_0^2 = 0$$

or $$m_1 x - m_2 (L-x) = 0 \qquad [\because \omega_0 \neq 0]$$

or $$(m_1 + m_2)\, x = m_2 L$$

or $$x = \frac{m_2 L}{m_1 + m_2}.$$

or

Problem 8. *A uniform bar of length 6a and mass 8m lies on a smooth horizontal table. Two point-masses m and 2m moving in the same horizontal plane with speeds 2v and v respectively strike the bar as shown in Fig. 7.85, and stick to the bar after collision.*

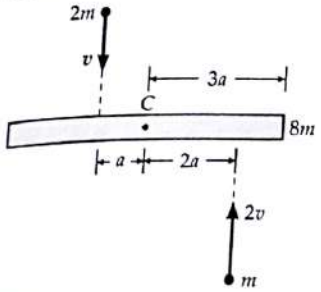

Fig. 7.85

Determine (i) velocity of the centre of mass (ii) angular velocity about centre of mass and (ii) total kinetic energy just after collision. **[IIT 91]**

Solution. (*i*) Let v_c be the velocity of the centre of mass.

By conservation of linear momentum,

$$(8m + m + 2m)\, v_c = 2m \times (-v) + m \times 2v + 8m \times 0$$

or $$v_c = 0.$$

(*ii*) M.I. of the bar about the centre of mass C,

$$I_b = \frac{Ml^2}{12} = \frac{8m \times (6a)^2}{12} = 24\, ma^2$$

By conservation of angular momentum,

$$m_1 v_1 r_1 + m_2 v_2 r_2 = (I_1 + I_2 + I_b)\,\omega$$

or $$2m \times v \times a + m \times 2v \times 2a = [2m \times a^2 + m \times (2a)^2 + 24\, ma^2]\,\omega$$

or $$6\, mva = 30\, ma^2\, \omega$$

or $$\omega = \frac{6\, mva}{30\, ma^2} = \frac{v}{5a}.$$

(*iii*) As the system has no translatory motion, it has only rotational K.E.

$\therefore$ Total K.E.

= Rotational K.E.

$$= \frac{1}{2} I\omega^2 = \frac{1}{2} \times 30\, ma^2 \times \left(\frac{v}{5a}\right)^2 = \frac{3}{5} mv^2.$$

Problem 9. *A carpet of mass M made of inextensible material is rolled along its length in the form of a cylinder of radius R and is kept on a rough floor. The carpet starts unrolling without sliding on the floor when a negligibly small push is given to it. Calculate the horizontal velocity of the axis of the cylindrical part of the carpet when its radius reduces to R/2.* **[IIT 90]**

Solution. Let l be the length of the cylinder and ρ its density.

Mass of carpet, $M = \pi R^2 l \times \rho$

When the carpet unrolls to half the radius, its mass becomes,

$$M' = \pi\left(\frac{R}{2}\right)^2 l \times \rho = \frac{M}{4}$$

Initially, mass is M and CG lies at R.

Finally, mass is $M/4$ and CG lies at $R/2$.

Let v be horizontal velocity of the axis of the cylindrical part of the carpet. Then

Decrease in P.E. = Gain in translational K.E. + Gain in rotational K.E.

$$MgR - \frac{M}{4} \cdot g \cdot \frac{R}{2} = \frac{1}{2} M'v^2 + \frac{1}{2} I\omega^2$$

or $$\frac{7}{8} MgR = \frac{1}{2}\frac{M}{4} \cdot v^2 + \frac{1}{2}\left[\frac{1}{2} M'\left(\frac{R}{2}\right)^2\right]\left[\frac{v}{R/2}\right]^2$$

or $$\frac{7}{8} MgR = \frac{1}{8} Mv^2 + \frac{1}{4} \cdot \frac{M}{4} \cdot \frac{R^2}{4} \cdot \frac{4v^2}{R^2}$$

or $$\frac{7}{8} gR = \frac{v^2}{8} + \frac{v^2}{16} = \frac{3v^2}{16}$$

or $$v = \sqrt{\frac{14}{3} gR}.$$

Problem 10. *Consider a body, shown in Fig. 7.86, consisting of two identical balls, each of mass M connected by a light rigid rod. If an impulse $J = MV$ is imparted to the body at one of its ends, what would be its angular velocity ?* **[IIT Screening 03]**

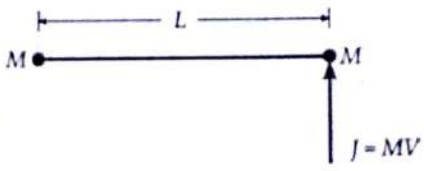

Fig. 7.86

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Solution. Applying conservation of angular momentum about the centre of the rod,

$$I_{CM}\,\omega = J \cdot \frac{L}{2}$$

or $$2 \times M\left(\frac{L}{2}\right)^2 \omega = MV \cdot \frac{L}{2}$$

or $$\omega = V / L.$$

Problem 11. *A cubical block of side L rests on a rough horizontal surface with coefficient of friction μ. A horizontal force F is applied on the block as shown in Fig. 7.87. If the coefficient of friction is sufficiently high so that the block does not slide before toppling, what is the minimum force required to topple the block ?* **[IIT Screening 2000]**

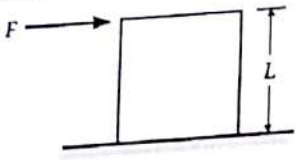

Fig. 7.87

Solution. When F is applied, the normal reaction (N) of the floor moves to the right. The cube topples when N reaches its edge.

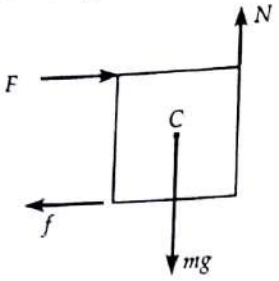

Fig. 7.88

Here, $N = mg$ and the force of friction $= f = F$. Taking torque about the centre C,

$$F \times \frac{L}{2} + f \times \frac{L}{2} = N \times \frac{L}{2}$$

or $$F + f = N$$

or $$F + F = mg$$

or $$F = mg / 2.$$

Problem 12. *One quarter sector is cut from a uniform circular disc of radius R. This sector has mass M. It is made to rotate about a line perpendicular to its plane and passing through the centre of the original disc. What is its moment of inertia about the axis of rotation ?*

[IIT Screening 2001]

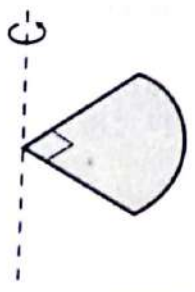

Fig. 7.89

Solution. Mass of the entire disc from which the section has been cut $= 4M$.

Its moment of inertia of the given axis $= \frac{1}{2}(4M)\,R^2$

$\therefore$ Moment of inertia of the quarter section about the same axis

$$= \frac{1}{4} \times \frac{1}{2}(4M)\,R^2 = \frac{1}{2}\,MR^2.$$

Problem 13. *A thin wire of length L and uniform linear mass density ρ is bent into a circular loop with centre at O as shown in Fig. 7.90. What is the moment of inertia of the loop about the axis XX′ ?*

[IIT Screening 2000]

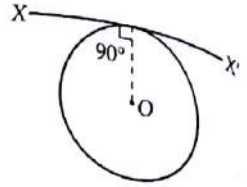

Fig. 7.90

Solution. Let m and r be the mass and radius of the loop.

Then, $m = \rho L$ and $L = 2\pi r$ or $r = \frac{L}{2\pi}$.

Moment of inertia of the loop about an axis through O and perpendicular to its plane $= mr^2$.

By the theorem of perpendicular axes, its moment of inertia about a diameter parallel to $XX' = \frac{1}{2}mr^2$.

By the theorem of parallel axes, its moment of inertia about XX'

$$= \frac{1}{2}mr^2 + mr^2 = \frac{3}{2}mr^2$$

$$= \frac{3}{2}(\rho L)\left(\frac{L}{2\pi}\right)^2 = \frac{3\rho L^3}{8\pi^2}.$$

Problem 14. *A long horizontal rod has a bead which can slide along its length and is initially placed at a distance L from one end A of the rod. The rod is set in angular motion about A with a constant angular acceleration, α. If the coefficient of friction between the rod and the bead is μ, and gravity is neglected then, find the time after which the bead starts slipping.* **[IIT Screening 2000]**

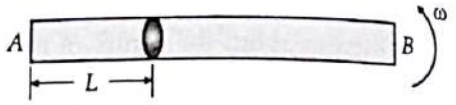

Fig. 7.91

Solution. Linear acceleration of the bead $= a = L\alpha$.

$\therefore$ Reaction force on the bead due to the rod

$$= N = ma = mL\alpha.$$

After time t, the angular velocity of the bead

$$= \omega = \alpha t.$$

$\therefore$ Centripetal acceleration of the bead

$$= \omega^2 L = \alpha^2 t^2 L$$

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Centripetal force on the bead
$= m\alpha^2 t^2 L$

$\therefore$ Force of friction at limiting position
$= \mu N = \mu m L\alpha.$

$\therefore$ For slipping,
$\mu m L\alpha = m\alpha^2 t^2 L$ or $t = \sqrt{\mu/\alpha}.$

Problem 15. *Initial angular velocity of a circular disc of mass M is ω_1. Then two small spheres of mass m are attached gently to two diametrically opposite points on the edge of the disc. What is the final angular velocity of the disc ?* **[AIEEE 02]**

Solution. Let R be the radius of the circular disc. Then initial angular momentum of the disc is

$$L_1 = I_1\omega_1 = \frac{1}{2} MR^2\omega_1$$

When two small spheres are attached on the edge of the disc, the moment of inertia becomes

$$I_2 = \frac{1}{2} MR^2 + mR^2 + mR^2 = \frac{1}{2} MR^2\left(1 + \frac{4m}{M}\right)$$

If ω_2 is the final angular velocity, then the final angular momentum will be

$$L_2 = I_2\omega_2 = \frac{1}{2} MR^2\left(1 + \frac{4m}{M}\right)\omega_2$$

By conservation of angular momentum in the absence of any external torque,

$$L_2 = L_1$$

or $$\frac{1}{2} MR^2\left(\frac{M+4m}{M}\right)\omega_2 = \frac{1}{2} MR^2\,\omega_1$$

or $$\omega_2 = \left(\frac{M}{M+4m}\right)\omega_1.$$

Problem 16. *A T-shaped object with dimensions shown in Fig. 7.92, is lying on a smooth floor. A force $\vec{F}$ is applied at the point P parallel to AB, such that the object has only the translational motion without rotation. Find the location of P with respect to C.* **[AIEEE 05]**

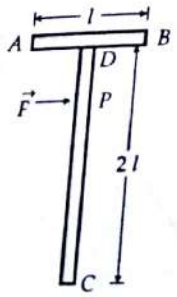

Fig. 7.92

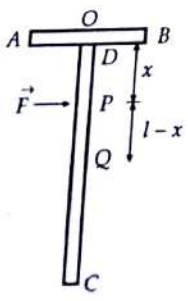

Fig. 7.93

Solution. As shown in Fig. 7.93, let O be the CM of part AB and Q that of CD. Let M be the mass per unit length of the parts AB and CD.

As no rotation is set up about point P, so

Moment of part AB about P = Moment of part CD about P

or $$(ml)\,x = (2\,ml)(l - x)$$

or $$x = 2l - 2x$$

or $$x = 2l/3.$$

Distance of P from the end $C = 2l - 2l/3 = 4l/3.$

Problem 17. *From a circular disc of radius R and mass 9 M, a small disc of radius R/3 is removed from the disc, as shown in Fig. 7.94. Find the moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through the point O.* **[IIT 05]**

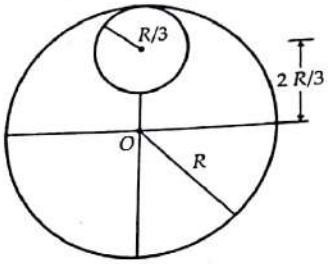

Fig. 7.94

Solution. Total mass of the disc $= 9\,M$

Radius of the disc $= R$

M.I. of the disc about a perpendicular axis through O is

$$I_1 = \frac{1}{2} \times \text{Mass} \times \text{radius}^2$$

$$= \frac{1}{2} \times 9\,M \times R^2 = \frac{9}{2} MR^2$$

Mass of small disc removed

$$= \frac{9\,M}{\pi R^2} \times \pi\left(\frac{R}{3}\right)^2 = M$$

By the theorem of parallel axes, the M.I. of the small disc about the axis through O,

$$I_2 = \frac{1}{2} M\left(\frac{R}{3}\right)^2 + M\left(\frac{2R}{3}\right)^2 = \frac{1}{2} MR^2$$

M.I. of the remaining disc about a perpendicular axis through O,

$$I = I_1 - I_2 = \frac{9}{2} MR^2 - \frac{1}{2} MR^2$$

$$= 4\,MR^2.$$

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Problem 18. *A rod of length L and mass M is hinged at point O. A small bullet of mass m hits the rod with velocity v, as shown in Fig. 7.95. The bullet gets embedded in the rod. Find the angular velocity of the system just after the impact.* [IIT 05]

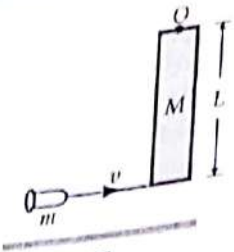

Fig. 7.95

Solution. Before impact, the bullet is moving. The initial angular momentum of the system about O is

$$L_i = mv \times L = mvL$$

After the bullet gets embedded in the rod, suppose the system attains angular velocity ω.

The moment of inertia of the bullet + rod system about the axis through O is

I = (M.I. of bullet + M.I. of rod) about the axis through O

$$= mL^2 + \frac{1}{3} ML^2 = \frac{M+3m}{3} L^2$$

Final angular momentum of the system is

$$L_f = I\omega = \frac{M+3m}{3} L^2 \omega$$

By conservation of angular momentum,

$$\frac{M+3m}{3} L^2\omega = mvL$$

or

$$\omega = \frac{3mv}{(M+3m)\,L}.$$

Guidelines to NCERT Exercises

7.1. *Give the location of the centre of mass of mass of a (i) sphere, (ii) cylinder, (iii) ring, and (iv) cube, each of uniform mass density. Does the centre of mass of a body necessarily lie inside the body ?*

Ans. (*i*) Geometrical centre.

(*ii*) Centre of its axis of symmetry.

(*iii*) Centre of the ring.

(*iv*) Point of intersection of the diagonals.

No, it is not necessary that centre of mass of a body lies inside the body. For example, in a ring, hollow sphere, in a tumbler etc., the centre of mass lies inside their hollow portion.

7.2. *In the HCl molecule, the separation between the nuclei of the two atoms is about 1.27 Å (1 Å = 10^{-10} m). Find the approximate location of the CM of the molecule, given that the chlorine atom is about 35.5 times as massive as a hydrogen atom and nearly all the mass of an atom is concentrated in all its nucleus.*

Ans. As shown in Fig. 7.96, suppose the H nucleus is located at the origin. Then

$$x_1 = 0,\ x_2 = 1.27\ \text{Å},$$

$$m_1 = 1,\ m_2 = 35.5$$

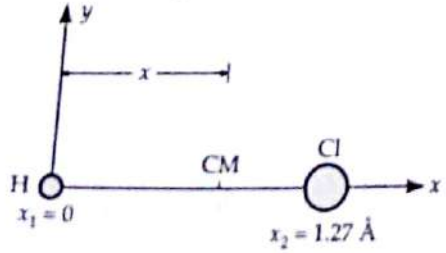

Fig. 7.96

The position of the CM of HCl molecule is

$$x = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$= \frac{1 \times 0 + 35.5 \times 1.27}{1 + 35.5} = 1.235\ \text{Å}.$$

Thus the CM of HCl is located on the line joining H and Cl nuclei at a distance of 1.235 Å from the H nucleus.

7.3. *A child sits stationary at one end of a long trolley moving uniformly with speed v on a smooth horizontal floor. If the child gets up and runs about on the trolley in any manner, then what is the effect of the speed of the centre of mass of the (trolley + child) system ?*

Ans. The forces involved in the given problem are the internal forces of the system. No external force acts on the system when the child runs. So, there will be no change in the speed of the centre of mass of the (trolley + child) system.

7.4. *Show that the area of the triangle contained between the vectors $\vec{a}$ and $\vec{b}$ is one half of the magnitude of $\vec{a} \times \vec{b}$.*

Ans. Area of a triangle. Suppose two vectors $\vec{a}$ and $\vec{b}$ are represented by the sides OP and OQ of ΔPOQ as shown in Fig. 7.97.

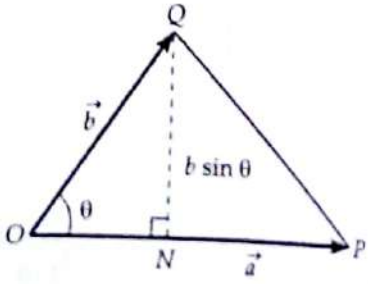

Fig. 7.97

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Let $\angle POQ = \theta$. Draw $QN \perp OP$. The magnitude of the vector product $\vec{a} \times \vec{b}$ is

$$|\vec{a} \times \vec{b}| = ab \sin\theta$$
$$= (OP)(OQ)\sin\theta$$
$$= (OP)(QN) \qquad [\because QN = OQ \sin\theta]$$
$$= 2 \times \frac{1}{2}(OP)(QN)$$
$$= 2 \times \text{Area of } \Delta POQ$$

or Area of $\Delta POQ = \frac{1}{2}|\vec{a} \times \vec{b}|$

Hence the area of the triangle contained between the vectors $\vec{a}$ and $\vec{b}$ is one half of the magnitude of $\vec{a} \times \vec{b}$.

7.5. Show that $\vec{a} \cdot (\vec{b} \times \vec{c})$ is equal in magnitude to the volume of the parallelopiped formed on the three vectors, $\vec{a}$, $\vec{b}$ and $\vec{c}$.

Ans. Volume of a parallelopiped. As shown in Fig. 7.98, consider a parallelopiped having the three non-coplanar vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ as edges meeting at a point O. Let $\vec{OA} = \vec{a}$, $\vec{OB} = \vec{b}$ and $\vec{OC} = \vec{c}$.

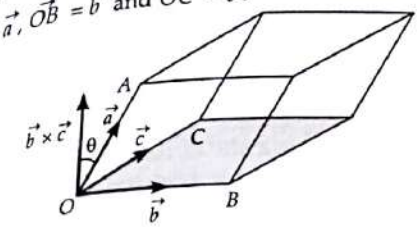

Fig. 7.98 Volume of a parallelopiped.

Then, $\vec{b} \times \vec{c}$ is a vector perpendicular to the plane of $\vec{b}$ and $\vec{c}$. Let θ be the angle between $\vec{a}$ and $\vec{b} \times \vec{c}$. Clearly, $|\vec{a}|\cos\theta$ is the height of the parallelopiped orthogonal to it its base.

Now,

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = |\vec{a}||\vec{b} \times \vec{c}|\cos\theta$$
$$= |\vec{b} \times \vec{c}||\vec{a}|\cos\theta$$
$$= \text{Base area of the parallelopiped} \times \text{height of the parallelopiped on this base}$$
$$= \text{Volume of the parallelopiped having the vectors } \vec{a}, \vec{b}, \vec{c} \text{ along edges meeting at a point.}$$

7.6. Find the components along the x, y, z axes of the angular momentum $\vec{l}$ of a particle, whose position vector is $\vec{r}$ with components x, y, z and momentum is $\vec{p}$ with components p_x, p_y and p_z. Show that if the particle moves only in the x-y plane, the angular momentum has only a z-component.

Ans. We can write

$$\vec{l} = l_x \hat{i} + l_y \hat{j} + l_z \hat{k}$$
$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

and $$\vec{p} = p_x \hat{i} + p_y \hat{j} + p_z \hat{k}$$

But $$\vec{l} = \vec{r} \times \vec{p}$$
$$= (x\hat{i} + y\hat{j} + z\hat{k}) \times (p_x \hat{i} + p_y \hat{j} + p_z \hat{k})$$
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$$

or $$l_x \hat{i} + l_y \hat{j} + l_z \hat{k}$$
$$= \hat{i}(yp_z - zp_y) + \hat{j}(zp_x - xp_z) + \hat{k}(xp_y - yp_x)$$

Comparing the coefficients of $\hat{i}$, $\hat{j}$ and $\hat{k}$ on the two sides, we get the components of $\vec{l}$ as follows :

$$l_x = yp_z - zp_y \; ; \; l_y = zp_x - xp_z \text{ and } l_z = xp_y - yp_x$$

If the particle is constrained to move only in the x-y plane, then $z = 0$ and $p_z = 0$. Hence

$$\vec{l} = \hat{k}(xp_y - yp_x)$$

As only the unit vector $\hat{k}$ corresponding to z-direction survives, the angular momentum $\vec{l}$ has only a z-component.

7.7. Two particles, each of mass m and speed v, travel in opposite directions along parallel lines separated by a distance d. Show that the vector angular momentum of the two particle system is the same whatever be the point about which the angular momentum is taken.

Ans. As shown in Fig. 7.99, suppose the two particles move parallel to the y-axis.

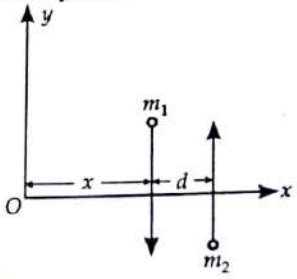

Fig. 7.99

Total angular momentum of the two particle system about O is

$$\vec{l} = \vec{l}_1 + \vec{l}_2 = \vec{r}_1 \times \vec{p}_1 + \vec{r}_2 \times \vec{p}_2$$
$$= x\hat{i} \times (-mv)\hat{j} + (x + d)\hat{i} \times (mv)\hat{j}$$
$$= (-mvx)\hat{i} \times \hat{j} + (mvx + mvd)\hat{i} \times \hat{j}$$
$$= (-mvx + mvx + mvd)\hat{k}$$
$$= mvd\,\hat{k} \qquad [\because \hat{i} \times \hat{j} = \hat{k}]$$

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Clearly, $\vec{l}$ does not depend on x and hence on the origin O. Thus the angular momentum of the two particle system is same whatever be point about which the angular momentum is taken.

7.8. *A non-uniform bar of weight W is suspended at rest by two strings of negligible weights as shown in Fig. 7.100. The angles made by the strings with the vertical are 36.9° and 53.1° respectively. The bar is 2 m long. Calculate the distance d of the centre of gravity of the bar from its left end.*

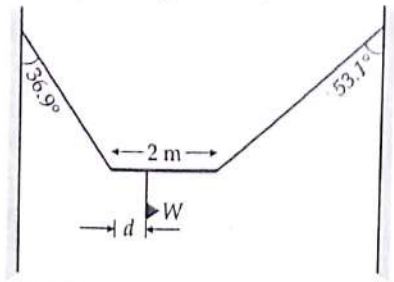

Fig. 7.100

Ans. Let T_1 and T_2 be the tensions in the two strings, as shown in Fig. 7.101.

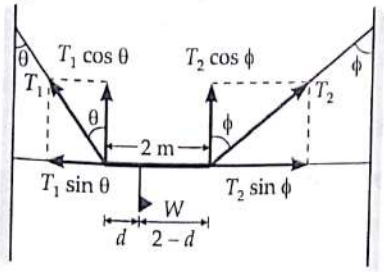

Fig. 7.101

For translational equilibrium, on balancing the vertical components of forces, we get

$$T_1 \cos\theta + T_2 \cos\phi = W \quad \ldots(1)$$

On balancing the horizontal components, we get

$$T_1 \sin\theta = T_2 \sin\phi \quad \ldots(2)$$

For rotational equilibrium, we balance the torques about the CG of the bar.

Clockwise torque = Anticlockwise torque

$$T_1 \cos\theta \times d = T_2 \cos\phi \times (2 - d) \quad \ldots(3)$$

Dividing (3) by (2), we get

$$d \cot\theta = (2 - d)\cot\phi$$

or $$d \cot 36.9° = (2 - d)\cot 53.1°$$

or $$d \cot 36.9° = (2 - d)\tan 36.9°$$

or $$d \times \frac{4}{3} = (2 - d) \times \frac{3}{4}$$

or $$16d = 18 - 9d$$

or $$d = 18/25 = 0.72 \text{ m} = \mathbf{72 \text{ cm}}.$$

7.9. *A car weighs 1800 kg. The distance between its front and back axles is 1.8 m. Its centre of gravity is 1.05 m behind the front axle. Determine the force exerted by the level ground on each front wheel and each back wheel.*

Ans. Let N_B and N_F be the total reaction forces exerted by the level ground on front and back wheels respectively. The situation is shown in Fig. 7.102.

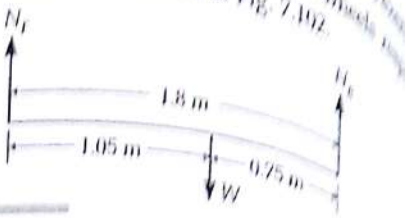

Fig. 7.102

For translational equilibrium of the car,

$$N_F + N_B = W = 1800 \times 9.8 \text{ N}$$

or $$N_F + N_B = 17640 \text{ N}$$

For rotational equilibrium of the car,

$$1.05\, N_F = 0.75\, N_B$$

or $$1.05\, N_F = 0.75\,(17640 - N_F)$$

or $$1.8\, N_F = 13230$$

or $$N_F = 13230/1.8 = 7350 \text{ N}$$

and $$N_B = 17640 - 7350 = 10290 \text{ N}$$

Force on each front wheel = 7350 / 2 = 3675 N.

Force on each back wheel = 10290 / 2 = 5145 N.

7.10. *(a) Find the moment of inertia of a sphere about a tangent to the sphere, given the moment of inertia of the sphere about any of its diameters to be $2MR^2/5$, where M is the mass of the sphere and R is the radius of the sphere.* **[Central Schools II, 16]**

(b) Given the moment of inertia of a disc of mass M and radius R about any of its diameters to be $MR^2/4$, find its moment of inertia about an axis normal to the disc and passing through a point on its edge.

Ans. *(a)* Here $I_D = \frac{2}{5} MR^2$

The tangent EF is parallel to the diameter CD. By the theorem of *parallel axes*,

$$I_{EF} = I_{CD} + MR^2$$

$$I_T = \frac{2}{5} MR^2 + MR^2$$

$$= \frac{7}{5} MR^2.$$

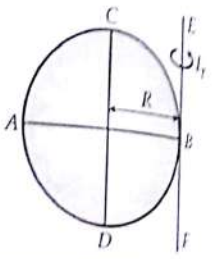

Fig. 7.103

(b) By the *theorem of parallel axes*, M.I. about an axis passing through an edge point and normal to disc

= M.I. about central normal axis + MR^2

$$I' = \frac{1}{2} MR^2 + MR^2 = \frac{3}{2} MR^2.$$

7.11. *Torques of equal magnitude are applied to a hollow cylinder and a solid sphere, both having the same mass and radius. The cylinder is free to rotate about its standard axis of*

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...ngular speed after a given time ?

Ans. Let M and R be mass and radius of the hollow cylinder and the solid sphere. Then

M.I. of hollow cylinder about its axis of symmetry,

$$I_1 = MR^2$$

M.I. of solid sphere about an axis through its centre,

$$I_2 = \frac{2}{5} MR^2$$

Let α_1 and α_2 be angular accelerations produced in the rotational motion of the cylinder and the sphere on applying a torque τ in each case. Then

$$\alpha_1 = \frac{\tau}{I_1} = \frac{\tau}{MR^2}$$

$$\alpha_2 = \frac{\tau}{I_2} = \frac{\tau}{\frac{2}{5} MR^2} = 2.5 \frac{\tau}{MR^2} = 2.5\, \alpha_1$$

and

As $\alpha_2 > \alpha_1$ and $\omega = \omega_0 + \alpha t$, so the solid sphere will acquire a greater angular speed after a given time.

7.12 *A solid cylinder of mass 20 kg rotates about its axis with angular speed 100 rad s^{-1}. The radius of the cylinder is 0.25 m. What is the kinetic energy associated with the rotation of the cylinder ? What is the magnitude of angular momentum of the cylinder about its axis ?* **[Central Schools 13]**

Ans. Here $M = 20$ kg, $\omega = 100$ rad s^{-1}, $R = 0.25$ m

M.I. of the cylinder about its own axis,

$$I = \frac{1}{2} MR^2 = \frac{1}{2}\, 20 \times (0.25)^2 = 0.625 \text{ kgm}^2$$

Rotational K.E.

$$= \frac{1}{2} I\omega^2 = \frac{1}{2} \times 0.625 \times (100)^2 = \mathbf{3125\ J.}$$

Angular momentum,

$$L = I\omega = 0.625 \times 100 = \mathbf{62.5\ kg\ m^2\ s^{-1}.}$$

7.13 *(i) A child stands at the centre of turntable with his two arms out stretched. The turntable is set rotating with an angular speed of 40 rpm. How much is the angular speed of the child if he folds his hands back and thereby reduces his moment of inertia to 2/3 times the initial value ? Assume that the turntable rotates without friction.*

(ii) Show that the child's new kinetic energy of rotation is more than the initial kinetic energy of rotation. How do you account for this increase in kinetic energy ?

Ans. Here $\omega_1 = 40$ rpm, $I_2 = \frac{2}{5} I_1$

By the principle of conservation of angular momentum,

$$I_1 \omega_1 = I_2 \omega_2 \quad \text{or} \quad I_1 \times 40 = \frac{2}{5} I_1 \omega_2$$

or $\omega_2 = \mathbf{100\ rpm.}$

(ii) Initial kinetic energy of rotation

$$= \frac{1}{2} I_1 \omega_1^2 = \frac{1}{2} I_1 (40)^2 = 800\, I_1$$

New kinetic energy of rotation

$$= \frac{1}{2} I_2 \omega_2^2 = \frac{1}{2} \times \frac{2}{3} I_1 \times (100)^2 = 2000\, I_1$$

$$\therefore \quad \frac{\text{New K.E.}}{\text{Initial K.E.}} = \frac{2000\, I_1}{800\, I_1} = \mathbf{2.5.}$$

Thus the child's new kinetic energy of rotation is 2.5 times its initial kinetic energy of rotation. This increase in kinetic energy is due to the internal energy of the child which he uses in folding his hands back from the out stretched position.

7.14. *A rope of negligible mass is wound round a hollow cylinder of mass 3 kg and radius 40 cm. What is angular acceleration of the cylinder if the rope is pulled with a force of 30 N ? What is the linear acceleration of the rope ? Assume that there is no slipping.*

Ans. Here $M = 3$ kg, $R = 40$ cm $= 0.40$ m, $F = 30$ N

Torque, $\tau = F \times R = 30 \times 0.40 = 12$ Nm

M.I. of the hollow cylinder about its own axis,

$$I = MR^2 = 3 \times (0.40)^2 = 0.48 \text{ kg m}^2$$

Angular acceleration,

$$\alpha = \frac{\tau}{I} = \frac{12}{0.48} = \mathbf{25\ rad\ s^{-2}.}$$

Linear acceleration,

$$a = R\alpha = 0.40 \times 25 = \mathbf{10\ ms^{-2}.}$$

7.15. *To maintain a rotor at a uniform angular speed of 200 rad s^{-1}, an engine needs to transmit a torque of 180 Nm. What is the power required by the engine ? Assume that the engine is 100% efficient.*

Ans. Here $\omega = 200$ rad s^{-1}, $\tau = 180$ Nm

$\therefore$ Power, $P = \tau\omega = 180 \times 200 = 36{,}000$ W $= \mathbf{36\ kW.}$

7.16. *From a uniform disc of radius R, a circular hole of radius R / 2 is cut out. The centre of the hole is at R / 2 from the centre of the original disc. Locate the centre of gravity of the resulting flat body.*

Ans. The situation is shown in Fig. 7.104. Let O be the CM of the original circular portion, O_1 that of the circular hole cut out and O_2 that of the remaining shaded portion. Let m be the mass per unit area of the disc.

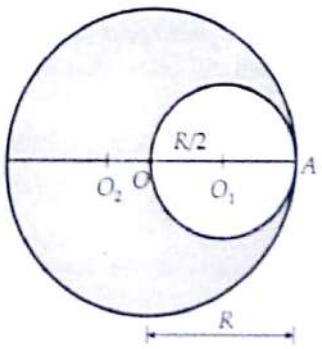

Fig. 7.104

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Mass of the original disc,

$$M = \pi R^2 m$$

Mass of the circular hole cut,

$$m_1 = \pi\left(\frac{R}{2}\right)^2 m = \frac{\pi}{4} R^2 m$$

Mass of the remaining portion,

$$m_2 = \pi R^2 m - \frac{\pi}{4} R^2 m = \frac{3}{4} \pi R^2 m$$

Masses m_1 and m_2 may be assumed to be concentrated at O_1 and O_2 respectively and O is their CM.

$\therefore$ Moment of m_1 about O

= Moment of m_2 about O

or $\quad m_1 \times O_1O = m_2 \times O_2O$

or $\quad \frac{\pi}{4} R^2 m \times \frac{R}{2} = \frac{3}{4} \pi R^2 m \times O_2O$

or $\quad O_2O = \frac{R}{6}$

Thus the CM of the resulting portion lies at $R/6$ from the centre of the original disc in a direction opposite to the centre of the cut out portion.

7.17. *A metre stick is balanced on a knife edge at its centre. When two coins, each of mass 5 g are put one on top of the other at the 12.0 cm mark, the stick is found to be balanced at 45.0 cm. What is the mass of the metre stick ?*

Ans. As the metre stick balances at 50.0 cm mark, its CG must lie at this mark.

The 45.0 cm mark is the CG of the metre stick + 2 coins system.

Let m be the mass of the metre stick.

Distance between 50.0 cm mark and new CG

= 50.0 – 45.0 = 5.0 cm

Distance between 12.0 cm mark and new CG

= 45.0 – 12.0 = 33.0 cm

From the principle of moments (for equilibrium),

$$mg \times 5.0 = (2 \times 5) \times g \times 33.0$$

or $\quad m = \frac{2 \times 5 \times 33.0}{5.0} = \mathbf{66.0\ g.}$

7.18. *A solid sphere rolls down two different inclined planes of the same heights but different angles of inclination. (a) Will it reach the bottom with the same speed in each case ? (b) Will it take longer to roll down one plane than the other ? (c) If so, which one and why ?*

Ans. Acceleration of the rolling sphere,

$$a = \frac{g \sin\theta}{(1 + k^2/R^2)}$$

Velocity of the sphere at the bottom of the inclined plane,

$$v = \sqrt{\frac{2gh}{(1 + k^2/R^2)}}$$

(a) Yes, the sphere will reach the bottom with the same speed v because h is same in both cases.

(b) Yes, the sphere will take longer time to roll down one plane than the other.

(c) The sphere will take larger time in case of the plane with smaller inclination because the acceleration, $a \propto \sin\theta$.

7.19. *A hoop of radius 2 m weighs 100 kg. It rolls along a horizontal floor so that its centre of mass has a speed of 20 cm/s. How much work has to be done to stop it ?*

Ans. Here $R = 2$ m, $M = 100$ kg,

$$v_{cm} = 20\ \text{cms}^{-1} = 0.20\ \text{ms}^{-1}$$

Work required to stop the hoop

= Total K.E. of the hoop

= Rotational K.E. + Translational K.E.

$$= \frac{1}{2} I\omega^2 + \frac{1}{2} M v_{cm}^2$$

$$= \frac{1}{2} \times MR^2 \times \left(\frac{v_{cm}}{R}\right)^2 + \frac{1}{2} M (v_{cm})^2$$

$$= M v_{cm}^2 = 100 \times (0.20)^2 = \mathbf{4\ J.}$$

7.20. *The oxygen molecule has a mass of 5.30×10^{-26} kg and a moment of inertia of 1.94×10^{-46} kg m^2 about an axis through its centre perpendicular to the line joining the two atoms. Suppose the mean speed of such a molecule in a gas is 500 m/s and that its kinetic energy of rotation is two thirds of its kinetic energy of translation. Find the average angular velocity of the molecule.*

Ans. Rotational K.E. $= \frac{2}{3}$ Translational K.E.

or $\quad \frac{1}{2} I\omega^2 = \frac{2}{3} \cdot \frac{1}{2} mv^2$

or $\quad \omega = v \times \sqrt{\frac{2m}{3I}} = 500 \times \sqrt{\frac{2 \times 5.30 \times 10^{-26}}{3 \times 1.94 \times 10^{-46}}}$

$$= 500 \times \sqrt{1.82 \times 10^{20}}$$

$$= 500 \times 1.35 \times 10^{10}\ \text{rad s}^{-1} = \mathbf{6.75 \times 16^{12}\ rad\ s^{-1}}.$$

7.21. *A solid cylinder rolls up an inclined plane of angle of inclination 30°. At the bottom of the inclined plane the centre of mass of the cylinder has a speed of 5 m/s.*

(a) How far will the cylinder go up the plane ?

(b) How long will it take to return to the bottom ?

Ans. (a) Total initial kinetic energy of the cylinder,

$$K_i = \frac{1}{2} M v_{CM}^2 + \frac{1}{2} I_{CM}\, \omega^2$$

$$= \frac{1}{2} M v_{CM}^2 + \frac{1}{2} \times \frac{1}{2} MR^2 \times \frac{v_{CM}^2}{R^2}$$

$$= \frac{1}{2} M v_{CM}^2 + \frac{1}{4} M v_{CM}^2 = \frac{3}{4} M v_{CM}^2$$

Initial potential energy, $U_i = 0$

Final kinetic energy, $\quad K_f = 0$

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Final potential energy,
$U_f = Mgh = Mgs \sin 30° = \frac{1}{2} Mgs$

where s is the distance travelled up the incline and h is the vertical height covered above the bottom.

Gain in P.E. = Loss in K.E.

$$\frac{1}{2} Mgs = \frac{3}{4} Mv_{CM}^2$$

$$s = \frac{3v_{CM}^2}{2g} = \frac{3 \times (5)^2}{2 \times 9.8} = \mathbf{3.8\ m.}$$

(b) Using equation of motion for the motion up the incline, we get

$$0 = v_{CM} + at \quad \text{or} \quad a = -\frac{v_{CM}}{t}$$

Also $0^2 - v_{CM}^2 = 2as$

$$a = -\frac{v_{CM}^2}{2s}$$

or $$\frac{v_{CM}}{t} = \frac{v_{CM}^2}{2s}$$

$$\therefore \quad t = \frac{2s}{v_{CM}} = \frac{2 \times 3.8}{5} = 1.5\ \text{s}$$

or

Total time taken in returning to the bottom

$$= 2 \times 1.5 = \mathbf{3.0\ s.}$$

7.22. *As shown in Fig. 7.105, the two sides of a step ladder BA and CA are 1.6 m long and hinged at A. A rope DE, 0.5 m is tied half way up. A weight 40 kg is suspended from a point F, 1.2 m from B along the ladder BA. Assuming the floor to be frictionless and neglecting the weight of the ladder, find the tension in the rope and forces exerted by the floor on the ladder. (Take $g = 9.8\ m/s^2$).*

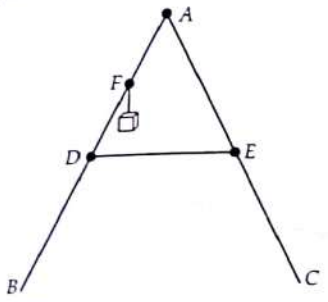

Fig. 7.105

Ans. As shown in Fig. 7.106, let N_B and N_C be the normal reactions of the floor at B and C respectively and T be the tension in the rope DE. Then

$$N_B + N_C = W = 40 \times 9.8\ \text{N}$$

or $$N_B + N_C = 392\ \text{N} \qquad ...(1)$$

Consider the portion AC of the ladder. Balancing torques about A, we get

$$N_C \times LC = T \times AG$$

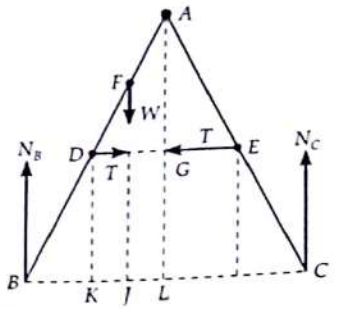

Fig. 7.106

From the geometry of the figure, we get

$$LC = 2GE = DE = 0.5\ \text{m}$$

or $$GE = 0.25\ \text{m}$$

$$\therefore \quad AG = \sqrt{AE^2 - GE^2} = \sqrt{(0.8)^2 - (0.25)^2}$$

$$= \sqrt{0.5775} = 0.76\ \text{m}$$

Hence $N_C \times 0.5 = T \times 0.76$ or $T = 0.66\ N_C$ ...(2)

Now consider the portion AB of the ladder. Balancing the torques about A, we get

$$N_B \times BL - W \times JL = T \times AG$$

But $$JL = \frac{1}{4} DE = \frac{1}{4} \times 0.5 = 0.125\ \text{m},$$

$$BL = DE = 0.5\ \text{m},\ W = 392\ \text{N}$$

$$\therefore \quad N_B \times 0.5 - 392 \times 0.125 = T \times 0.76$$

or $$N_B \times 0.5 - 392 \times 0.125 = 0.66\ N_C \times 0.76$$ [using (2)]

or $$N_B - 98 = N_C$$

or $$N_B - N_C = 98 \qquad ...(3)$$

On solving (1) and (3), we get

$$N_B = \mathbf{245\ N},\ N_C = 245 - 98 = \mathbf{147\ N},$$

and $$T = 0.66\ N_C = 0.66 \times 147 = \mathbf{97\ N.}$$

7.23. *A man stands on a rotating platform, with his arms stretched horizontally holding a 5 kg weight in each hand. The angular speed of the platform is 30 revolutions per minute. The man then brings his arms close to his body with the distance of each weight from the axis changing from 90 cm to 20 cm. The moment of inertia of the man together with the platform may be taken to be constant and equal to 7.6 kg m^2.*

(a) What is his new angular speed ? (Neglect friction)

(b) Is kinetic energy conserved in the process ? If not, from where does the change come about ?

Ans. (a) Total initial moment of inertia,

I_1 = M.I. of man and platform + M.I. of two 5 kg weights

$$= 7.6 + 2 \times 5 \times (0.90)^2 = 7.6 + 8.1$$

$$= 15.7\ \text{kg m}^2$$

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Initial angular speed,

$$\omega_1 = 30 \text{ rpm}$$

Total final moment of inertia,

$$I_2 = 7.6 + 2 \times 5 \times (0.20)^2 = 7.6 + 0.4$$
$$= 8.0 \text{ kg m}^2$$

By the principle of conservation of angular momentum,

$$I_1\omega_1 = I_2\omega_2$$

or $$15.7 \times 30 = 8.0 \times \omega_2$$

or $$\omega_2 = \frac{15.7 \times 30}{8.0} = 58.875 \approx 59 \text{ rpm.}$$

(*b*) $$\frac{\text{Final K.E.}}{\text{Initial K.E.}} = \frac{\frac{1}{2} I_1 \omega_1^2}{\frac{1}{2} I_2 \omega_2^2} = \frac{8.0 \times (59)^2}{15.7 \times (30)^2} = 1.97.$$

Thus the final K.E. is about twice the initial K.E. *i.e.*, K.E. is not conserved in the process. The increase in K.E. is due to the internal energy the man uses in bringing his arms closer to his body.

7.24. *A bullet of mass 10 g and speed 500 m/s is fired into a door and gets embedded exactly at the centre of the door. The door is 1.0 m wide and weighs 12 kg. It is hinged at one end and rotates about a vertical axis practically without friction. Find the angular speed of the door just after the bullet embeds into it.*

(**Hint.** *The moment of inertia of the door about the vertical axis at one end is* $ML^2/3$)

Ans. By the principle of conservation of angular momentum,

Initial angular momentum of the bullet = Final angular momentum of the door

or $$pr = I\omega$$

or $$mvr = \frac{ML^2}{3} \times \omega \quad \text{or} \quad \omega = \frac{3mvr}{ML^2}$$

Here $m = 10 \text{ g} = 10^{-2} \text{ kg}$, $v = 500 \text{ ms}^{-1}$,

$$r = \frac{1.0}{2} = 0.5 \text{ m}, \quad L = 1.0 \text{ m}, \quad M = 12 \text{ kg}$$

$$\therefore \quad \omega = \frac{3 \times 10^{-2} \times 500 \times 0.5}{12 \times (1.0)^2} = 0.625 \text{ rad s}^{-1}.$$

7.25. *Two discs of moments of inertia I_1 and I_2 about their respective axes (normal to the disc and passing through the centre), and rotating with angular speed ω_1 and ω_2 are brought into contact face to face with their axes of rotation coincident. (i) What is the angular speed of the two-disc system ? (ii) Show that the kinetic energy of the combined system is less than the sum of the initial kinetic energies of the two discs. How do you account for this loss in energy ? Take $\omega_1 \neq \omega_2$.*

Ans. (*i*) Let ω be the angular speed of the two-disc system. Then by conservation of angular momentum,

$$(I_1 + I_2)\omega = I_1\omega_1 + I_2\omega_2$$

or $$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}.$$

(*ii*) Initial K.E. of the two discs,

$$K_1 = \frac{1}{2} I_1 \omega_1^2 + \frac{1}{2} I_2 \omega_2$$

Final K.E. of the two-disc system,

$$K_2 = \frac{1}{2}(I_1 + I_2)\omega^2$$

$$= \frac{1}{2}(I_1 + I_2)\left(\frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}\right)^2$$

Loss in K.E. $= K_1 - K_2$

$$= \frac{1}{2}(I_1\omega_1^2 + I_2\omega_2^2) - \frac{1}{2(I_1 + I_2)}(I_1\omega_1 + I_2\omega_2)^2$$

$$= \frac{1}{2(I_1 + I_2)}[I_1^2\omega_1^2 + I_1I_2\omega_2^2 + I_1I_2\omega_1^2 + I_2^2\omega_2^2 - (I_1^2\omega_1^2 + I_2^2\omega_2^2 + 2I_1I_2\omega_1\omega_2)]$$

$$= \frac{1}{2(I_1 + I_2)}[I_1I_2\omega_2^2 + I_1I_2\omega_1^2 - 2I_1I_2\omega_1\omega_2]$$

$$= \frac{I_1I_2}{2(I_1 + I_2)}(\omega_1^2 + \omega_2^2 - 2\omega_1\omega_2)$$

$$= \frac{I_1I_2}{2(I_1 + I_2)}(\omega_1 - \omega_2)^2 = \text{a positive quantity} \quad [\because \omega_1 \neq \omega_2]$$

Hence there is a loss of rotational K.E. which appears as heat. When the two discs are brought together, work is done against friction between the two discs.

7.26. (*a*) *Prove the theorem of perpendicular axes.*

(*b*) *Prove the theorem of parallel axes.*

Ans. (*a*) Refer answer to Q.38 on page 7.27.

(*b*) Refer answer to Q.39 on page 7.27.

7.27. *Prove the result that the velocity v of translation of a rolling body (like a ring, disc, cylinder or sphere) at the bottom of an inclined plane of a height h is given by*

$$v^2 = \frac{2gh}{(1 + k^2/R^2)}$$

using dynamical consideration (i.e., by consideration of forces and torques). Note k is the radius of gyration of the body about its symmetry axis, and R is the radius of the body. The body starts from rest at the top of the plane. [Delhi 13]

Ans. Velocity attained by a body rolling down an inclined plane. Consider a body of mass M and radius R rolling down a plane inclined at an angle θ with the horizontal, as shown in Fig. 7.107. It is only due to friction at the line of contact that body can roll without slipping. The centre of mass of the body moves in a straight line parallel to the inclined plane.

The external forces on the body are

(*i*) The weight Mg acting vertically downwards.

(*ii*) The normal reaction N of the inclined plane.

(*iii*) The force of friction acting up the inclined plane.

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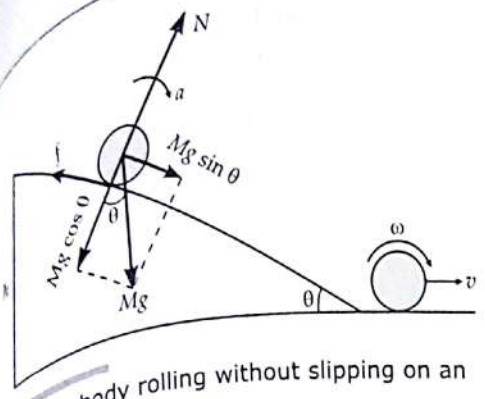

Fig. 7.107 A body rolling without slipping on an inclined plane.

Let a be the downward acceleration of the body. The equations of motion for the body can be written as

$$N - Mg\cos\theta = 0$$

$$F = Ma = Mg\sin\theta - f$$

As the force of friction f provides the necessary torque for rolling, so

$$\tau = f \times R = I\alpha = Mk^2\left(\frac{a}{R}\right)$$

or
$$f = M\frac{k^2}{R^2}\cdot a$$

where k is the radius of gyration of the body about its axis of rotation. Clearly

$$Ma = Mg\sin\theta - M\frac{k^2}{R^2}\cdot a$$

or
$$a = \frac{g\sin\theta}{(1 + k^2/R^2)}$$

Let h be height of the inclined plane and s the distance travelled by the body down the plane. The velocity v attained by the body at the bottom of the inclined plane can be obtained as follows :

$$v^2 - u^2 = 2as$$

or
$$v^2 - 0^2 = 2\cdot\frac{g\sin\theta}{(1 + k^2/R^2)}\cdot s$$

or
$$v^2 = \frac{2gh}{1 + k^2/R^2} \qquad \left[\because \frac{h}{s} = \sin\theta\right]$$

or
$$v = \sqrt{\frac{2gh}{(1 + k^2/R^2)}}$$

7.28. *A disc rotating about its axis with angular speed ω_0 is placed lightly (without any translational push) on a perfectly frictionless table. The radius of the disc is R. What are the linear velocities of the points A, B and C on the disc shown in Fig. 7.108 ? Will the disc roll in the direction indicated ?*

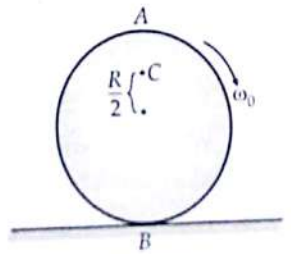

Fig. 7.108

Ans. We know that, $v = R\omega$

For point A : $v_A = R\omega_0$
(in the direction of the arrow)

For point B : $v_B = R\omega_0$
(in the direction opposite to arrow)

For point C : $v_C = \left(\frac{R}{2}\right)\omega_0$
(in the direction of the arrow)

The disc *will not roll* in the direction indicated. It is because the disc is placed on a perfectly frictionless table and without friction, a body cannot roll.

7.29. *Explain why friction is necessary to make the disc in Fig. 7.108 given in previous question roll in the direction indicated.*

(i) Give the direction of frictional force at B, and the sense of frictional torque, before perfect rolling begins.

(ii) What is the force of friction after perfect rolling begins ?

Ans. To roll a disc, a torque is required which in turn requires a tangential force to act on it. As the force of friction is the only tangential force acting on the disc, so it is necessarily required for the rolling of a disc.

(*i*) Frictional force at B opposes the velocity of B. Therefore, frictional force is in the same direction as the arrow. The sense of frictional torque is such as to oppose the angular motion. By right hand rule, both $\vec{\omega_0}$ and $\vec{\tau}$ act normal to the plane of paper, $\vec{\omega_0}$ into the plane of paper and $\vec{\tau}$ out of the paper.

(*ii*) Frictional force decreases the velocity of the point of contact B. Perfect rolling begins when this velocity is zero. Once this is, the force of friction is zero.

7.30. *A solid disc and a ring, both of radius 10 cm are placed on a horizontal table simultaneously, with initial angular speed equal to 10π rad s^{-1}. Which of the two will start to roll earlier ? The co-efficient of kinetic friction is $\mu_k = 0.2$.* **[Delhi 14]**

Ans. In case of pure rotation without translation, the velocity of centre of mass is zero. The friction reduces the speed at the point of contact and as such accelerates the centre of mass till the velocity of centre of mass becomes equal to $v = R\omega$ and the instantaneous velocity at the contact point becomes zero. Thus the force of friction $\mu_k mg$ produces an acceleration a in the centre of mass. So the equation of motion for CM is

$$\mu_k mg = ma \qquad ...(1)$$

The torque due to force of friction is $\mu_k mg \times R$. It produces angular retardation given by

$$\mu_k mgR = -I\alpha \qquad ...(2)$$

Rolling begins when

$$v = R\omega$$

But
$$v = 0 + at = \mu_k gt \qquad ...(3)$$

[From (1), $a = \mu_k g$]

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and $$\omega = \omega_0 + \alpha t = \omega_0 - \frac{\mu_k mgR}{I} t \quad \text{[using (2)]}$$

or $$\frac{v}{R} = \omega_0 - \frac{\mu_k mgR}{I} t$$

or $$\frac{\mu_k gt}{R} = \omega_0 - \frac{\mu_0 mgR}{I} t$$

or $$\frac{\mu_k gt}{R}\left[1 + \frac{mR^2}{I}\right] = \omega_0$$

or $$t = \frac{R\omega_0}{\mu_k g\left(1 + \frac{mR^2}{I}\right)}$$

For a disc : $I = mR^2/2$

$$\therefore \quad t = \frac{R\omega_0}{3\mu_k g} = \frac{0.10 \times 10\pi}{3 \times 0.2 \times 9.8} = \mathbf{0.53\ s.}$$

For a ring : $I = mR^2$

$$\therefore \quad t = \frac{R\omega_0}{2\mu_k g} = \frac{0.10 \times 10\,\pi}{2 \times 0.2 \times 9.8} = \mathbf{0.80\ s.}$$

Thus the disc begins to roll earlier than the ring.

7.31. *A solid cylinder of mass 10 kg and radius 15 cm is rolling perfectly on a plane of inclination 30°. The coefficient of static friction, $\mu_s = 0.25$. (i) Find the force of friction acting on the cylinder. (ii) What is the work done against friction during rolling ? (iii) If the inclination θ of the plane is increased, at what value of θ does the cylinder begin to skid, and not roll perfectly ?* **[Delhi 13]**

Ans. Here $M = 10$ kg, $R = 0.15$ m, $\mu_s = 0.25$, $\theta = 30°$

(*i*) Force of friction,

$$f = \frac{1}{3} Mg \sin\theta = \frac{1}{3} \times 10 \times 9.8 \times \sin 30°$$

$$= \frac{1}{3} \times 10 \times 9.8 \times 0.5 = \mathbf{16.33\ N.}$$

(*ii*) Work done against friction during rolling

$$= \mathbf{0\ J.}$$

(*iii*) Condition for skidding (or no rolling) is

$$\frac{f}{N} \le \mu_s$$

or $$\frac{\frac{1}{3} Mg \sin\theta}{Mg \cos\theta} \le \mu_s$$

or $$\frac{1}{3} \tan\theta \le \mu_s$$

Thus the cylinder will start skidding at an angle of inclination θ given by

$$\tan\theta = 3\mu_s = 3 \times 0.25 = 0.75$$

or $$\theta = \mathbf{36°52'}.$$

7.32. *Read each statement below carefully, and state, with reasons, if it is true or false :*

(*a*) *During rolling, the force of friction acts in the same direction as the direction of motion of the CM of the body.*

(*b*) *The instantaneous speed of the point of contact during rolling is zero.*

(*c*) *The instantaneous acceleration of the point of contact during rolling is zero.*

(*d*) *For perfect rolling motion, work done against friction is zero.*

(*e*) *A wheel moving down a perfectly frictionless inclined plane will undergo slipping (not rolling) motion.*

Ans. (*a*) ***True.*** When a body rolls, the force of friction acts in the same direction as the direction of motion of the CM of the body.

(*b*) ***True.*** A rolling body can be imagined to be rotating about an axis passing through the point of contact of the body and the ground. Hence the instantaneous speed of the point of contact is zero.

(*c*) ***False.*** As the body is rotating, its instantaneous acceleration cannot be zero.

(*d*) ***True.*** Perfect rolling begins when force of friction is zero. So work done against friction is zero.

(*e*) ***True.*** On a perfectly frictionless inclined plane, there is no tangential force of friction. So the wheel cannot roll. It will simply slip under the effect of its own weight.

Text Based Exercises

Type A : Very Short Answer Questions

1 Mark Each

1. Define centre of mass of a system. **[Himachal 08, 09]**
2. On what factors, does the position of centre of mass of a rigid body depend ?
3. Where does the centre of mass of uniform triangular lamina lie ?
4. Where does the centre of mass of a cone lie ?
5. What is an isolated system ? What will be the nature of motion of centre of mass of isolated system ?
6. Where does the centre of mass of a two particle system lie, if one particle is more massive than the other ? **[Delhi 97]**

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7. Give an example each for a body, where the centre of mass lies inside the body and outside the body. [Delhi 99]
8. Where does the centre of mass of two particles of equal mass lie ? What is its position vector ?
9. Write an expression for the centre of mass of a two particle system.
10. Write an expression for the velocity of the centre of mass of a system of particles. [Central Schools 13]
11. What is a rigid body ? [Himachal 08 ; Delhi 09]
12. Name the rotational analogue of force. Give its SI unit.
13. Is torque a scalar or vector ? If vector, which rule is used to determine its direction ?
14. Write the dimensional formula of torque.
15. What is the ratio of SI unit to the CGS unit of torque ?
16. Under what conditions, is the torque zero ?
17. A body is rotating at a steady rate. Is any torque acting on the body ?
18. Which physical quantity corresponds to moment of linear momentum ?
19. Write the dimensional formula of angular momentum. Is it a scalar ?
20. Write the SI unit of angular momentum. [Delhi 08]
21. What is the ratio of the SI unit to the CGS unit of angular momentum ?
22. Name the constant whose dimensions are same as that of angular momentum ?
23. Does the magnitude and direction of angular momentum depend on the choice of the origin ?
24. Does the total momentum of a system of particles depend upon the velocity of the centre of mass ?
25. What is the angular momentum of a body of mass m moving in a circular path of radius r with constant speed v ?
26. State right hand rule to find the direction of angular momentum.
27. Name the physical quantity which is equal to the time rate of change of angular momentum.
28. Which physical quantity is conserved when a planet revolves around the sun ?
29. A planet revolves around the sun under the effect of gravitational force exerted by the sun. Why is the torque on the planet due to the gravitational force zero ?
30. A boat is likely to capsize if the persons in the boat stand up. Why ?
31. A body is moving in a circle of radius r centimetre at a constant speed of v cm s^{-1}. What is the angular velocity ?
32. What is the angular velocity of the earth spinning about its own axis ?
33. Is the angular velocity of rotation of hour hand of a watch greater or smaller than the angular velocity of earth's rotation about its own axis ?
34. Two bodies move in two concentric circular paths of radii r_1 and r_2 in the same time. What is the ratio of their angular velocities ?
35. Name the rotational analogue of linear acceleration.
36. What is the analogous of mass in rotational motion ? Write unit of that physical quantity also. [Delhi 95, 08]
37. Is moment of inertia a scalar or a vector quantity ? [Himachal 01]
38. Write an expression for the moment of inertia of a rod of mass 'M' and length 'L' about an axis perpendicular to it through one end. [Delhi 2003]
39. What is the moment of inertia of a solid sphere about its diameter ? [Delhi 09]
40. What is the moment of inertia of a hollow sphere about an axis passing through its centre ?
41. Find the radius of gyration of a uniform solid sphere about its diameter. [Central Schools 14]
42. What is the dimensional formula of radius of gyration ?
43. State the factors on which radius of gyration of a body depends.
44. A circular ring and a circular disc of the same radius have the same moment of inertia about axis passing through their centres and perpendicular to their planes. What is the ratio of their masses ?
45. Where does the centre of mass of a rectangular lamina lie ? [Himachal 03]
46. What is torque ? Give its SI unit. [Himachal 04, 05C, 09]
47. What are the factors on which the moment of inertia of a body depends ? [Himachal 04]
48. Give the physical significance of moment of inertia. [Himachal 08, 09]
49. Which physical quantity is represented by the product of moment of inertia and angular velocity ? [Delhi 04 ; Central Schools 12]
50. Define angular momentum. [Himachal 05C, 07C, 08, 09]
51. Define the term moment of momentum. [Himachal 07C]
52. Write two factors on which centre of mass of a body does not depend. [Chandigarh 09]

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Answers

1. The centre of mass of a system may be defined as the point at which the entire mass of the system may be supposed to be concentrated. The nature of the motion of the system will remain unaffected if all the forces acting on the system were applied directly on the centre of mass of the system.
2. The centre of mass of a rigid body depends on
   (*i*) The geometrical shape of the body.
   (*ii*) The distribution of mass in the body.
3. Centre of mass of a triangular lamina lies at its centroid *i.e.*, at the point of intersection of the three medians.
4. The centre of mass of a cone lies on the line joining the apex to the centre of the base at a distance equal to $\frac{1}{4}$ of the length of this line from the base.
5. An isolated system is one on which no external force acts. The centre of mass of such a system remains either at rest or moves with uniform velocity.
6. The CM will lie on the line joining the two particles, closer to the more massive particle.
7. (*i*) The CM of a solid sphere lies inside the body.
   (*ii*) The CM of a hollow sphere lies outside the body.
8. The centre of mass of two particles of equal masses lies midway between them. Its position vector is the average of the position vectors of the two particles.
9. $\vec{R}_{CM} = \dfrac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$.
10. $\vec{v}_{CM} = \dfrac{\Sigma m_i \vec{v}_i}{M}$.
11. A rigid body is one whose constituent particles retain their relative positions even when they move under the action of an external force.
12. Torque. Its SI unit is Nm.
13. Torque is a vector, $\vec{\tau} = \vec{r} \times \vec{F}$. Its direction is perpendicular to the plane of $\vec{r}$ and $\vec{F}$ and is given by right handed screw rule.
14. $[\text{Torque}] = [ML^2T^{-2}]$.
15. $\dfrac{\text{SI unit of torque}}{\text{CGS unit of torque}} = 10^7$.

As $\tau = rF\sin\theta$, so torque is zero if r is zero or F is zero. It is also zero if $\theta = 0°$ or $180°$.

17. No. A torque produces angular acceleration.
18. Angular momentum.
19. $[\text{Angular momentum}] = [ML^2T^{-1}]$.
    Angular momentum is not a scalar but a vector.
20. SI unit of angular momentum = kg m^2 s^{-1}.
21. $\dfrac{\text{SI unit of angular momentum}}{\text{CGS unit of angular momentum}} = 10^7$.
22. Planck's constant (h) has the same dimensions as angular momentum.
23. Yes.
24. Yes.
    Total momentum = Mv_{CM}.
25. Angular momentum
    = Linear momentum × moment arm
    or $L = p \times r = mvr$.
26. Curl the fingers of the right hand in the direction of rotation, then the thumb points in the direction of angular momentum.
27. Torque.
28. Angular momentum of the planet.
29. The gravitational force acts along the line joining the planet to the sun. Vectors $\vec{r}$ and $\vec{F}$ are always parallel.
    $\therefore \quad \tau = rF\sin 0° = 0$.
30. If the passengers stand, the CG of the system (boat + passengers) is raised and so it is likely to capsize.
31. $\omega = \dfrac{v}{r}$ rad s^{-1}.
32. Angular velocity of spinning of earth
    $= \dfrac{2\pi}{86400}$ rad s^{-1}.
33. The hour hand completes one rotation in 12 hours while the earth completes one rotation in 24 hours. So, angular velocity of hour hand is double the angular velocity of earth, because $\omega = 2\pi / T$.
34. Angular velocity, $\omega = 2\pi / T$ is same for both bodies. So the ratio is 1 : 1.
35. Angular acceleration is the rotational analogue of linear acceleration.
36. Moment of inertia is the rotational analogue of mass. Its SI unit is kg m^2.
37. M.I. is a scalar quantity.
38. $I = \dfrac{ML^2}{3}$.

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39. $I = \frac{2}{5} MR^2$, where M is the mass and R is the radius of the solid sphere.

40. $I = \frac{2}{3} MR^2$, where M is the mass and R is the radius of the hollow sphere.

41. $Mk^2 = \frac{2}{5} MR^2 \quad \therefore \quad k = \sqrt{\frac{2}{5}} R$

42. The dimensional formula of radius of gyration is $[M^0L^1T^0]$.

43. The radius of gyration of a body depends on
(*i*) the position and orientation of the axis of rotation.
(*ii*) the distribution of mass of the body about the axis of rotation.

44. $I = M_{ring} R^2 = \frac{1}{2} M_{disc} R^2$

$\therefore \quad M_{ring} : M_{disc} = 1 : 2$

45. The CM of a rectangle lies at the point of intersection of the diagonals of the rectangle.

46. Refer to point 8 of Glimpses.

47. Moment of inertia of a body depends on :
(*i*) Mass of the body.
(*ii*) Distribution of mass about the axis of rotation.

48. Moment of inertia of a body is the rotational inertia of the body. It is the rotational analogue of mass in linear motion.

49. Angular momentum ($L = I\omega$).

50. Angular momentum of a particle is the moment of its linear momentum about the axis of rotation.

51. It is equal to the product of linear momentum and the perpendicular distance of its line of action from the axis of rotation.

52. The centre of mass of a system does not depend on (*i*) choice of the coordinate system, and (*ii*) force acting on the particles.

Type B : Short Answer Questions

2 or 3 Marks Each

1. What is meant by a particle, a system and internal and external forces ?
2. Define centre of mass of a system. How does it differ from the centre of gravity ?
3. Write an expression for the centre of mass of a two particle system. What will be the location of centre of mass if the two particles have equal masses ?
4. Show that the centre of mass of two particles is on the line joining them at a point whose distance from each particle is inversely proportional to the mass of that particle.
5. Write an expression for the centre of mass of n-particle system. Also write the equations of motion which govern the motion of the centre of mass.
6. Show that in the absence of any external force, the velocity of the centre of mass remains constant.
7. Show that the total linear momentum of a system of particles is conserved in the absence of any external force.
8. Show that the total linear momentum of a system of particles is equal to the product of the total mass of the system and the velocity of its centre of mass.
9. What are binary stars ? Discuss their motion in respect of their centre of mass.
10. Briefly describe the motion of CM of a diatomic molecule like O_2.
11. Briefly explain the motion of the centre of mass of the earth-moon system.
12. Discuss the motion of the centre of mass of a fire cracker that explodes in air.
13. Define a rigid body. State the factors on which the centre of mass a rigid body depends.
14. Define the term torque or moment of force. Give its units and dimensions. **[Himachal 01C, 04, 08]**
15. Name the physical quantity corresponding to force in rotational motion. How is it related to force ? Give its units. **[Central Schools 08]**
16. State and explain the principle of moments.
17. Define a couple. Show that the moment of a couple is same irrespective of the point of rotation of a body.
18. Derive an expression for the work done by a torque. Hence write the expression for the power delivered by a torque.
19. How torque is expressed as the vector product of two vectors ? State the rule for determining the direction of torque.
20. What is torque ? Give its unit. Show that it is equal to the product of force and the perpendicular distance of its line of action from the axis of rotation. **[Chandigarh 07 ; Himachal 07C]**

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21. What is torque ? Explain that torque is only due to the tangential component of force, radial component of force has nothing to do with the torque. [Central Schools 14]

22. Define angular momentum of a particle. State its SI unit.

23. How angular momentum can be expressed as the vector product of two vectors ? How is its direction determined ?

24. Show that the angular momentum of a particle is the product of its linear momentum and moment arm. Also show that the angular momentum is produced only by the angular component of linear momentum. What is the physical meaning of angular momentum ?

25. Define angular momentum. Prove that angular momentum of a particle is equal to twice the product of its mass and areal velocity. [Himachal 05C, 06]

26. Define angular momentum. Derive the relation between angular momentum and torque. [Central Schools 14 ; Delhi 10]

27. Prove that the time rate of change of the angular momentum of a particle is equal to the torque acting on it. [Delhi 97, 05]

28. Define a rigid body. Name *two* kinds of motion which a rigid body can execute. What is meant by the term equilibrium ? For the equilibrium of a body, *two* conditions need to be satisfied. State them. [Delhi 2003]

29. Distinguish between the stable, unstable and neutral equilibria of a body.

30. Define moment of inertia of a body. Give its units and dimensions. What is the physical significance of moment of inertia ? [Himachal 05C, 06]

31. State the factors on which the moment of inertia of a body depends. [Himachal 06 ; Delhi 03C]

32. Establish the relation between kinetic energy and moment of inertia for a rigid body. [Himachal 07]

33. Show that the moment of inertia of a body about the given axis of rotation is equal to twice the kinetic energy of rotation of the body rotating with unit angular velocity. [Himachal 07C]

34. Derive an expression for the rotational kinetic energy of a body and hence define moment of inertia. [Himachal 07, 09C ; Delhi 08]

35. Define radius of gyration. Derive an expression for the radius of gyration of a body consisting of n particles about an axis of rotation. [Central Schools 14]

36. State and prove the theorem of perpendicular axes on moment of inertia. [Himachal 02, 03, 04]

37. State the theorem of parallel axes on moment of inertia. [Delhi 12]

38. Derive an expression for moment of inertia of a circular disc about an axis passing through its centre and perpendicular to its plane. [Delhi 95, 96 ; Himachal 05, 08]

39. What is the moment of inertia of a rod of mass M and length L about an axis perpendicular to it through one end ? Given the moment of inertia about the centre of mass is $\frac{1}{12} ML^2$. [Central Schools 03, 08, 09]

40. Deduce an expression for the moment of inertia of a hollow cylinder of mass M and radius R about its own axis.

41. Establish the relation between torque and angular acceleration. Hence define moment of inertia. [Delhi 99 ; Himachal 03]

42. Establish the relation between moment of inertia and torque on a rigid body. [Himachal 07, 08, 09C]

43. Establish the relation between angular momentum and moment of inertia for a rigid body. [Himachal 07]

44. Show that angular momentum = moment of inertia × angular acceleration and hence define moment of inertia. [Chandigarh 03]

45. State the law of conservation of angular momentum and illustrate it with the example of planetary motion. [Delhi, 96, 04 ; Himachal 05]

46. State and prove the principle of conservation of angular momentum. [Himachal 03, 05, 08]

47. Define moment of inertia. Write any two factors on which it depends. When the diver leaves the diving board, why he brings his hand and feet closer together in order to make a somersault. [Delhi 05]

48. Derive an expression for moment of inertia of a thin circular ring about an axis passing through the centre and perpendicular to the plane of the ring. [Himachal 07 ; Central Schools 05]

49. (*a*) State theorem of parallel axes for the moment of inertia of a body.

(*b*) Determine the moment of inertia of a thin ring about a tangent to the circle in the plane of the ring. [Delhi 09]

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50. State perpendicular axis theorem for calculation of moment of inertia using appropriate diagram. Also calculate moment of inertia about a diameter if that of an axis perpendicular to the plane of a disc and passing through its centre is given by $\frac{1}{2}MR^2$. [Central Schools 08, 09]

51. Derive an expression for angular momentum of a rigid body rotating at an angular speed of ω. Give its geometrical meaning and hence derive Kepler's second law of planetary motion. [Delhi 06]

52. Prove that the acceleration of a solid cylinder rolling without slipping down an inclined plane is $2/3\, g \sin\theta$. [Central Schools 03]

53. How moment of inertia of a circular disc around its axis will be affected by
   (*a*) increasing its angular speed of rotation ?
   (*b*) increasing its mass without changing its radius ? Justify your answers. [Central Schools 13]

Answers

1. Refer answer to Q. 1 on page 7.1.
2. Refer answer to Q. 2 on page 7.1.
3. Refer answer to Q. 3 on page 7.2.
4. Refer to solution of Problem 4 on page 7.59.
5. Refer answer to Q. 5 on page 7.3.
6. Refer answer to Q. 6 on page 7.4.
7. Refer answer to Q. 7 on page 7.5.
8. Refer answer to Q. 7 on page 7.5.
9. Refer answer to Q. 8(*i*) on page 7.11.
10. Refer answer to Q. 8(*ii*) on page 7.11.
11. Refer answer to Q. 8(*iii*) on page 7.11.
12. Refer answer to Q. 9 on page 7.11.
13. Refer answer to Q. 10 and Q. 11 on page 7.12.
14. Refer answer to Q. 16 on page 7.15.
15. The analogue of force in rotational motion is torque.

    Torque, $\vec{\tau} = \vec{r} \times \vec{F}$

    S.I. unit of torque = Nm.
16. Refer answer to Q. 17 on page 7.15.
17. Refer answer to Q. 18 on page 7.16.
18. Refer answer to Q. 19 on page 7.16.
19. Refer answer to Q. 20 on page 7.16.
20. Refer answer to Q.21 on page 7.17.
21. Refer answer to Q. 21 on page 7.17.
22. Refer answer to Q. 23 on page 7.18.
23. Refer answer to Q. 24 on page 7.18.
24. Refer answer to Q. 25 on page 7.18.
25. Refer answer to Q.28 on page 7.20.
26. Refer answer to Q.27 on page 7.19.
27. Refer answer to Q. 27 on page 7.19.
28. Refer answer to Q. 30 on page 7.23.
29. Refer answer to Q. 32 on page 7.25.
30. Refer answer to Q. 33 on page 7.25.
31. Refer answer to Q. 34 on page 7.25.
32. Refer answer to Q.36 on page 7.26.
33. Refer answer to Q.36 on page 7.26.
34. Refer answer to Q. 36 on page 7.26.
35. Refer answer to Q. 37 on page 7.26.
36. Refer answer to Q. 38 on page 7.27.
37. Refer answer to Q. 39 on page 7.27.
38. Refer answer to Q. 41(*a*) on page 7.29.
39. By using theorem of parallel axes,

$$I_{end} = I_{CM} + M\left(\frac{L}{2}\right)^2$$

$$= \frac{1}{12}ML^2 + \frac{1}{4}ML^2 = \frac{1}{3}ML^2.$$

40. Refer answer to Q. 44 on page 7.31.
41. Refer answer to Q. 48 on page 7.41.
42. Refer answer to Q.48 on page 7.41.
43. Refer answer to Q.49 on page 7.41.
44. Refer answer to Q. 49 on page 7.41.
45. Refer answer to Q. 50 on page 7.45.
46. Refer answer to Q.50 on page 7.45.
47. Refer answer to Q. 50(*iii*) on page 7.45.
48. Refer answer to Q. 41 on page 7.29.
49. (*a*) Refer answer to Q.39 on page 7.27.
    (*b*) Refer answer to Q.40(*c*) on page 7.28.
50. Refer answer to Q.39 on page 7.27 and Q.41(*b*) on page 7.29.
51. Refer answer to Q. 28 on page 7.20.
52. Refer answer to Q. 52 on page 7.49.
53. M.I. of a disc about its own axis = $\frac{1}{2}MR^2$
    (*a*) M.I. is not affected by increasing ω
    (*b*) M.I. increases as M increases.

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Type C : Long Answer Questions

5 Marks Each

1. Define centre of mass of a system. Derive an expression for the centre of mass of a two particle system from *ab*-initio. **[Chandigarh 03]**
2. Define rotational motion of a body. Derive the following equations of rotational motion under constant angular acceleration.

   (*i*) $\omega = \omega_0 + \alpha t$ (*ii*) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

   (*iii*) $\omega^2 - \omega_0^2 = 2\alpha\theta$.
3. Prove that the angular momentum of a particle is twice the product of its mass and areal velocity. How does it lead to the Kepler's second law of planetary motion ?
4. Prove that the rate of change of total angular momentum of a system of particles about a reference point is equal to the total torque acting on the system.
5. Derive an expression for the total work done on a rigid body executing both translational and rotational motions. Hence deduce the condition for the equilibrium of the rigid body.
6. Derive an expression for the moment of inertia of a thin uniform circular ring about an axis through its centre and perpendicular to its plane. Hence write the expressions for its moment of inertia

   (*i*) about its diameter and

   (*ii*) about a tangent in its plane. **[Himachal 05]**
7. Define the term moment of inertia. Derive an expression for moment of inertia of a circular disc about an axis passing through its centre and perpendicular to its plane.

   [Himachal 07 ; Delhi 03C ; Chandigarh 04]
8. Derive an expression for the moment of inertia of a thin uniform rod about an axis through its centre and perpendicular to its length. Also determine the radius of gyration about the same axis.
9. Derive an expression for the moment of inertia of a thin uniform rod about an axis passing through its one end and perpendicular to its length. Also determine the radius of gyration about the same axis.
10. Derive an expression for the moment of inertia of a uniform solid cylinder about its own axis.
11. Derive an expression for the moment of inertia of a uniform cylinder about an axis passing through its centre and perpendicular to its length.
12. Derive an expression for the moment of inertia of a uniform solid sphere about its any diameter. Hence write the expression for its moment of inertia about its tangent.
13. Obtain an expression for the linear acceleration of a cylinder rolling down an inclined plane and hence find the condition for the cylinder to roll down without slipping.
14. A light string is wound round a cylinder and carries a mass tied to it at the free end. When the mass is released, calculate

    (*i*) the linear acceleration of the descending mass,

    (*ii*) the angular acceleration of the cylinder and

    (*iii*) the tension in the string.

    Show that the acceleration of the mass is less than g.

Answers

1. Refer answer to Q. 4 on page 7.2.
2. Refer answer to Q. 13 on page 7.12 and Q. 14 on page 7.13.
3. Refer answer to Q. 28 on page 7.20.
4. Refer answer to Q. 28 on page 7.20.
5. Refer answer to Q. 31 on page 7.24.
6. Refer answer to Q. 41 on page 7.29.
7. Refer answer to Q. 41 on page 7.29.
8. Refer answer to Q. 42 on page 7.30.
9. Refer answer to Q. 43 on page 7.30.
10. Refer answer to Q. 45 on page 7.31.
11. Refer answer to Q. 46 on page 7.31.
12. Refer answer to Q. 47 on page 7.32.
13. Refer answer to Q. 52 on page 7.49.
14. Refer answer to Q. 55 on page 7.52.

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Competition Section

Systems of Particles & Rotational Motion

GLIMPSES

1. **Particle.** A particle is an object whose mass is finite but whose size and internal structure can be neglected.
2. **System.** A system is a collection of a very large number of particles which mutually interact with one another.
3. **Centre of mass.** It is the point at which entire mass of a system may be supposed to be concentrated. The nature of motion of the system remains unaffected when all the forces acting on the system are applied directly on the centre of mass of the system.

If $\vec{r_1}$ and $\vec{r_2}$ are the position vectors of two particles of masses m_1 and m_2, then the position vector of their centre of mass is given by

$$\vec{R}_{CM} = \frac{m_1 \vec{r_1} + m_2 \vec{r_2}}{m_1 + m_2}$$

For a system of N-particles, the centre of mass is given by

$$\vec{R}_{CM} = \frac{m_1 \vec{r_1} + m_2 \vec{r_2} + \ldots + m_N \vec{r_N}}{m_1 + m_2 + \ldots + m_N}$$

$$= \frac{\sum_{i=1}^{N} m_i \vec{r_i}}{M}$$

For a continuous mass distribution, the centre of mass is given by

$$\vec{R}_{CM} = \frac{1}{M} \int \vec{r}\, dm$$

where dm is the mass of small element located at position $\vec{r}$

The velocity of the centre of mass of a two-particle system is given by

$$\vec{v}_{CM} = \frac{m_1 \vec{v_1} + m_2 \vec{v_2}}{m_1 + m_2}$$

4. **Properties of centre of mass :**
   (*i*) The location of the centre of mass is the weighted average of the locations of the particles of the system.
   (*ii*) Centre of mass moves as if the external force acts on the entire mass concentrated at this point.
   (*iii*) In the absence of any external force, the centre of mass moves with a constant velocity.
   (*iv*) For bodies of normal dimensions, centre of mass and centre of gravity coincide.
5. **Rigid body.** A rigid body is one whose constituent particles retain their relative positions even when they move under the action of an external force.
6. **Rotational motion of a rigid body.** A body is said to possess rotational motion if all its particles move along circles in parallel planes. The centres of these circles lie on a fixed line perpendicular to the parallel planes and is called the axis of rotation.
7. **Equations of rotational motion.** For a body moving in circle, let ω_0 be the initial angular velocity, ω the final angular velocity, α the angular acceleration and θ be the angular displacement after the time t. Then equations of rotational motion can be written as

   (*i*) $\omega = \omega_0 + \alpha t$ (*ii*) $\theta = \omega_0 t + \frac{1}{2} \alpha t^2$

   (*iii*) $\omega^2 - \omega_0^2 = 2\alpha\theta$.

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8. **Torque ($\vec{\tau}$).** The turning effect of a force about the axis of rotation is called moment of force or torque due to the force.

Torque = Force × its perpendicular distance from the axis of rotation

or τ = Force × moment arm = Fd

If a force $\vec{F}$ acts at a point whose position vector is $\vec{r}$ with respect to the axis of rotation, then torque is

$$\vec{\tau} = \vec{r} \times \vec{F}$$

(*i*) In Cartesian coordinates : $\tau_z = x F_y - y F_x$

(*ii*) In polar coordinates : $\tau = rF \sin\theta$

where θ is the angle between force vector $\vec{F}$ and position vector $\vec{r}$.

SI Unit of torque is Nm.

9. **Principle of moments of rotational equilibrium.** When a body is in rotational equilibrium, the sum of the clockwise moments about any point is equal to the sum of the anticlockwise moments about that point. Or, the algebraic sum of moments about any point is zero. In rotational equilibrium,

Clockwise moment = Anticlockwise moment

or $F_1 \times d_1 = F_2 \times d_2$

or Load × load arm = Effort × effort arm

This is sometimes called the lever principle.

10. **Couple.** A pair of equal and opposite forces acting on a body along two different lines of action constitute a couple. In a couple, total external force is zero but torque is non-zero. So a couple has a turning effect but cannot produce translational motion.

11. **Moment of a couple.** The moment of a couple is equal to the product of either of the forces and the perpendicular distance, called the arm of the couple, between their lines of action. It is independent of the choice of the fulcrum or the point of rotation.

Moment of couple = Force × arm of the couple

or $\tau = Fd$

A couple can only be balanced by an equal and opposite couple.

12. **Work done by a torque and power of a torque.** If a torque τ applied on a body rotates it through an angle $\Delta\theta$, the work done by the torque is

$$\Delta W = \tau \Delta\theta$$

or Work done = Torque × Angular displacement

Power of a torque is given by

$$P = \frac{\Delta W}{\Delta t} = \frac{\tau \Delta\theta}{\Delta t} = \tau\,\omega$$

i.e., Power of a torque = Torque × Angular velocity

13. **Angular momentum ($\vec{L}$).** It is the moment of linear momentum of a particle about the axis of rotation.

Angular momentum = Linear momentum × its perpendicular distance from the axis of rotation

or $L = p\,d$

If $\vec{p}$ is the linear momentum of a particle and $\vec{r}$ its position vector, then its angular momentum is

$$\vec{L} = \vec{r} \times \vec{p}$$

(*i*) In Cartesian coordinates : $L_z = xp_y - yp_x$

(*ii*) In polar coordinates : $L = rp \sin\theta$

where θ is the angle between the linear momentum vector $\vec{p}$ and the position vector $\vec{r}$.

If a particle of mass m moves with uniform speed v along a circle of radius r, then its angular momentum is $L = mvr$

SI unit of angular momentum is $kgm^2 s^{-1}$.

The linear momentum $\vec{p}$ and velocity vector $\vec{v}$ are always parallel to each other. But the angular momentum $\vec{L}$ and the angular velocity $\vec{\omega}$ are not necessarily parallel vectors.

14. **Geometrical meaning of angular momentum.** Geometrically, the angular momentum of a particle is equal to twice the product of its mass and areal velocity. Equivalently, the areal velocity of a particle is just half its angular momentum per unit mass.

$$\vec{L} = 2m\frac{\Delta\vec{A}}{\Delta t} \quad \text{or} \quad \frac{\Delta\vec{A}}{\Delta t} = \frac{\vec{L}}{2m}$$

15. **Relation between torque and angular momentum.** The rate of change of angular momentum of a system of particles about a fixed point is equal to the total external torque acting on the system about that point.

$$\vec{\tau}^{ext} = \frac{d\vec{L}}{dt}$$

16. **Equilibrium of rigid bodies.** A rigid body is said to be in equilibrium if both the linear momentum and angular momentum of the rigid body remain constant with time. It must possess the following two equilibria simultaneously :

(*i*) **Translational equilibrium.** The resultant of all the external forces acting on the body must be zero.

$$\Sigma \vec{F}_i^{ext} = 0$$

(*ii*) **Rotational equilibrium.** The resultant of all the torques due to all the forces acting on the body about any point must be zero.

$$\Sigma \vec{\tau}_i^{ext} = \Sigma \vec{r}_i \times \vec{F}_i^{ext} = 0$$

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17. **Moment of inertia.** The moment of inertia of a rigid body about an axis of rotation is the sum of the products of masses of the various particles and squares of their perpendicular distances from the axis of rotation. Mathematically

$$I = m_1 r_1^2 + m_2 r_2^2 + ... + m_n r_n^2 = \sum_{i=1}^{n} m_i r_i^2$$

SI unit of moment of inertia = kg m^2.

The moment of inertia of a body can be regarded as the rotational inertia of a body. It is the rotational analogue of mass in linear motion.

18. **Factors on which the moment of inertia depends.** The moment of inertia of a body depends on the following factors :

(*i*) Mass of the body.
(*ii*) Size and shape of the body.
(*iii*) Distribution of mass about the axis of rotation.
(*iv*) Position and orientation of the axis of rotation with respect to the body.

19. **Radius of gyration.** It may be defined as the distance from the axis of rotation at which if whole mass of the body were supposed to be concentrated, the moment of inertia, would be same as with the actual distribution of mass. The relation between moment of inertia I and radius of gyration K is

$$I = MK^2 \quad \text{or} \quad K = \sqrt{\frac{I}{M}}$$

For a body composed of particles of equal masses,

$$K = \sqrt{\frac{r_1^2 + r_2^2 + ... + r_n^2}{n}}$$

i.e. radius of gyration is equal to the root mean square distance of the particles from the axis of rotation.

SI unit of radius of gyration = m.

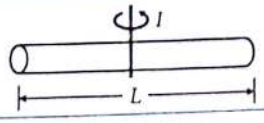
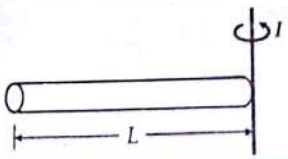
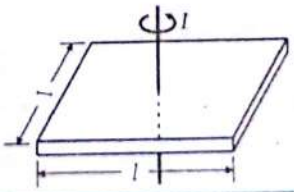
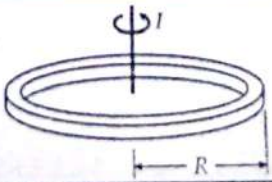
20. **Theorem of perpendicular axes.** It states that the moment of inertia of a plane lamina about an axis perpendicular to its plane is equal to the sum of the moments of inertia of the lamina about any two mutually perpendicular axes in its plane and intersecting each other at the point, where the perpendicular axes pass through the lamina. Mathematically,

$$I_z = I_x + I_y$$

where X- and Y-axes lie in the plane of the lamina and Z-axis is perpendicular to its plane and passes through the point of intersection of X- and Y-axes.

21. **Theorem of parallel axes.** It states that the moment of inertia of a rigid body about any axis is equal to the moment of inertia of the body about a parallel axis through its centre of mass plus the product of mass of the body and the square of the perpendicular distance between the parallel axes. Mathematically, $I = I_{CM} + Md^2$

Table Moments of Inertia of some Bodies of Regular Shape

Body	Axis	Figure	Moment of Inertia
Thin rod of length L	Passing through centre and perpendicular to the rod		$I = \frac{1}{12} ML^2$
Thin rod of length L	Through its one end and perpendicular to its length		$I = \frac{1}{3} ML^2$
Rectangular lamina of length l and breadth b	Through its centre and perpendicular to its plane		$I = M\left(\frac{l^2 + b^2}{12}\right)$
Circular ring of radius R	Passing through its centre and perpendicular to its plane		$I = MR^2$

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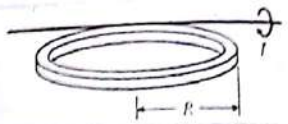
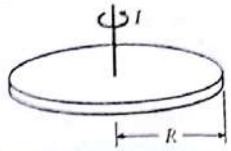
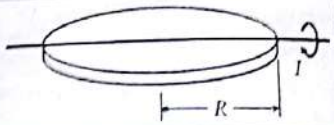
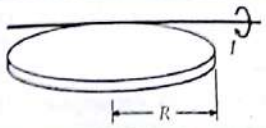
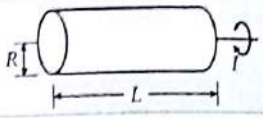
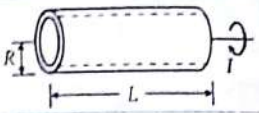
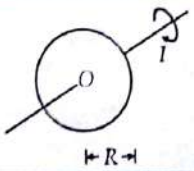
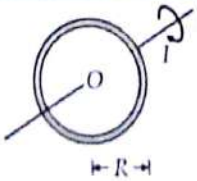
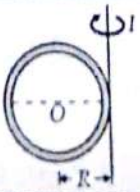
Body	Axis	Figure	Moment of Inertia
Circular ring of radius R	Diameter		$I = \frac{1}{2} MR^2$
Circular ring of radius R	Tangent in its plane		$I = \frac{3}{2} MR^2$
Circular disc of radius R	Passing through its centre and perpendicular to its plane		$I = \frac{1}{2} MR^2$
Circular disc of radius R	Diameter		$I = \frac{1}{4} MR^2$
Circular disc of radius R	Tangent in its plane		$I = \frac{5}{4} MR^2$
Right circular solid cylinder of radius R and length L	Symmetry axis		$I = \frac{1}{2} MR^2$
Hollow cylinder of radius R and length L	Symmetry axis		$I = MR^2$
Solid sphere of radius R	Diameter		$I = \frac{2}{5} MR^2$
Solid sphere of radius R	Any tangent		$I = \frac{7}{5} MR^2$
Hollow sphere of radius R	Symmetry axis		$I = \frac{2}{3} MR^2$
Hollow sphere of radius R	Any tangent		$I = \frac{5}{3} MR^2$

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23. **Rotational K.E.** If a body of mass M and moment of inertia I rotates about an axis of rotation with an angular velocity ω, then

$$\text{Rotational K.E.} = \frac{1}{2} I\omega^2$$

24. **Total K.E. of a rolling body.** The centre of mass of a rolling body moves along a straight line, so it possesses translational K.E. in addition to rotational K.E.

$\therefore$ Total K.E. of a rolling body

= Translational K.E. + Rotational K.E.

$$= \frac{1}{2} Mv^2 + \frac{1}{2} I\omega^2$$

25. **Relation between M.I. and angular momentum.**

Angular momentum = M.I. × Angular velocity

$$L = I\omega$$

i.e.,

26. **Relation between M.I. and torque.**

Torque = M.I. × Angular acceleration

$$\tau = I\alpha.$$

i.e.,

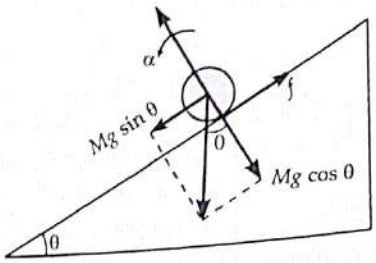
27. **Law of conservation of angular momentum.** If no external torque acts on a system, total angular momentum of the system remains unchanged.

In the absence of any external torque,

$$L = I\omega = \text{constant}$$

or $I_1 \omega_1 = I_2 \omega_2$ or $I_1 \cdot \frac{2\pi}{T_1} = I_2 \cdot \frac{2\pi}{T_2}$.

28. **Motion of a cylinder rolling down an inclined plane without slipping.** As shown in figure, consider a solid cylinder of mass M and radius R rolling down an inclined plane of inclination θ without slipping.



(*i*) Linear acceleration of the cylinder down the inclined plane,

$$a = \frac{g \sin\theta}{1 + \frac{I}{MR^2}} = \frac{2}{3} g \sin\theta$$

(*ii*) Force of friction between inclined plane and cylinder,

$$f = \frac{1}{3} Mg \sin\theta$$

(*iii*) Condition for rolling of cylinder without slipping is

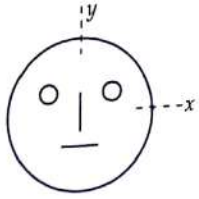
$$\frac{1}{3} \tan\theta \le \mu_s$$

where μ_s is the coefficient of static friction.

IIT Entrance Exam

MULTIPLE CHOICE QUESTIONS WITH ONE CORRECT ANSWER

1. Look at the drawing given in the figure which has been drawn with ink of uniform line-thickness. The mass of ink used to draw each of the two inner circles, and each of the two line segments is m. The mass of the ink used to draw the outer circle is $6m$. The coordinates of the centres of the different parts are : *outer circle* $(0, 0)$, *left inner circle* $(-a, a)$, *right inner circle* (a, a), *vertical line* $(0, 0)$ and *horizontal line* $(0, -a)$. The y-coordinate of the centre of mass of the ink in this drawing is



(*a*) $\frac{a}{10}$ (*b*) $\frac{a}{8}$

(*c*) $\frac{a}{12}$ (*d*) $\frac{a}{3}$ [IIT 09]

2. Two particles A and B, initially at rest, move towards each other under mutual force of attraction. At the instant when the speed of A is v and the speed of B is $2v$, the speed of the centre of mass of the system is

(*a*) $3v$ (*b*) v

(*c*) $1.5v$ (*d*) zero. [IIT 82]

3. Two blocks of masses 10 kg and 4 kg are connected by a spring of negligible mass and placed on a frictionless horizontal surface. An impulse gives a velocity of 14 m/s to the heavier block in the direction of the lighter block.

The velocity of the centre of mass is

(*a*) 30 m/s (*b*) 20 m/s

(*c*) 10 m/s (*d*) 5 m/s [IIT 02]

4. An isolated particle of mass m is moving in horizontal plane (x-y) along the x-axis at a certain height above the ground. It suddenly explodes into two fragments of masses $m/4$ and $3m/4$. An instant later, the smaller fragment is at $y = +15$ cm. The larger fragment at this instant is at

(*a*) $y = -5$ cm (*b*) $y = +20$ cm

(*c*) $y = +5$ cm (*d*) $y = -20$ cm. [IIT 97]

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5. A smooth sphere A is moving on a frictionless horizontal plane with angular speed ω and centre of mass velocity u. It collides elastically and head on with an identical sphere B at rest. Neglect friction everywhere. After the collision, their angular speeds are ω_A and ω_B respectively. Then

(a) $\omega_A < \omega_B$ (b) $\omega_A = \omega_B$

(c) $\omega_A = \omega$ (d) $\omega_B = \omega$ [IIT 97]

6. A mass m is moving with a constant velocity along a line parallel to the z-axis, away from the origin. Its angular momentum with respect to the origin

(a) is zero (b) remains constant

(c) goes on increasing (d) goes on decreasing [IIT 97]

7. A particle of mass m is projected with a velocity v making an angle of 45° with the horizontal. The magnitude of the angular momentum of the projectile about the point of projection, when the particle is at its maximum height h is

(a) zero (b) $\frac{mv^3}{4\sqrt{2}g}$

(c) $\frac{mv^2}{\sqrt{2}g}$ (d) $m\sqrt{2gh^3}$ [IIT 90]

8. A particle undergoes uniform circular motion. About which point on the plane of the circle, will the angular momentum of the particle remain conserved ?

(a) centre of the circle

(b) on the circumference of the circle

(c) inside the circle

(d) outside the circle [IIT 03]

9. A particle is confined to rotate in a circular path with decreasing linear speed. Then which of the following is correct ?

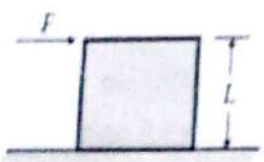
(a) $\vec{L}$ (angular momentum) is conserved about the centre

(b) only direction of angular momentum $\vec{L}$ is conserved

(c) it spirals towards the centre

(d) its acceleration is towards the centre. [IIT 05]

10. A cubical block of side L rests on a rough horizontal surface with coefficient of friction μ. A horizontal face F is applied on the block as shown. If the coefficient of friction is sufficiently high so that the block does not slide before toppling, the minimum force required to topple the block is



(a) infinitesimal (b) $mg/4$

(c) $mg/2$ (d) $mg(1-\mu)$

11. A disc is rolling without slipping with angular velocity ω. P and Q are two points equidistant from the centre C. The order of magnitude of velocity is

(a) $v_Q > v_C > v_P$ (b) $v_P > v_C > v_Q$

(c) $v_P = v_C, v_Q = v_C/2$ (d) $v_P < v_C < v_Q$

12. A thin wire of length L and uniform linear mass density ρ is bent into a circular loop with centre at O as shown. The moment of inertia of the loop about the axis XX' is

(a) $\frac{\rho L^3}{8\pi^2}$ (b) $\frac{\rho L^3}{16\pi^2}$

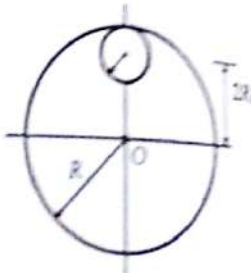
(c) $\frac{5\rho L^3}{16\pi^2}$ (d) $\frac{3\rho L^3}{8\pi^2}$

13. One quarter sector is cut from a uniform circular disc of radius R. This sector has mass M. It is made to rotate about a line perpendicular to its plane and passing through the centre of the original disc. Its moment of inertia about the axis of rotation is

(a) $\frac{1}{2}MR^2$ (b) $\frac{1}{4}MR^2$

(c) $\frac{1}{8}MR^2$ (d) $\sqrt{2}MR^2$

14. From a circular disc of radius R and mass $9M$, a small disc of radius $R/3$ is removed from the disc. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through O is



(a) $4MR^2$

(b) $\frac{40}{9}MR^2$

(c) $10MR^2$

(d) $\frac{37}{9}MR^2$

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16. A solid sphere of mass M and radius R having moment of inertia I about its diameter is recast into a solid disc of radius r and thickness t. The moment of inertia of the disc about an axis passing the edge and perpendicular to the plane remains I. Then R and r are related as

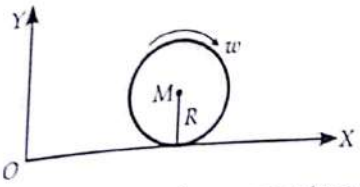
(a) $r = \sqrt{\frac{2}{15}}R$ (b) $r = \frac{2}{\sqrt{15}}R$

(c) $r = \frac{2}{15}R$ (d) $r = \frac{\sqrt{2}}{15}R$ [IIT 06]

18. Two point masses of 0.3 kg and 0.7 kg are fixed at the ends of a rod of length 1.4 m and of negligible mass. The rod is set rotating about an axis perpendicular to its length with a uniform angular speed. The point on the rod through which the axis should pass in order that the work required for rotation of the rod is minimum, is located at a distance of

(a) 0.42 m from mass of 0.3 kg
(b) 0.70 m from mass of 0.7 kg
(c) 0.98 m from mass of 0.3 kg
(d) 0.98 m from mass of 0.7 kg [IIT 95]

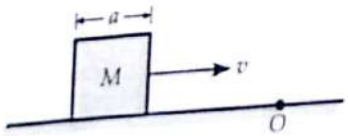
17. A disc of mass M and radius R is rolling with angular speed ω on a horizontal plane as shown in figure.



The magnitude of angular momentum of the disc about the origin O is

(a) $(1/2)MR^2\omega$ (b) $MR^2\omega$
(c) $(3/2)MR^2\omega$ (d) $2MR^2\omega$ [IIT 99]

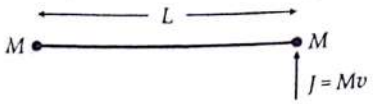
18. A cubical block of side a is moving with velocity v on a horizontal smooth plane as shown in figure. It



hits a ridge at point O. The angular speed of the block after it hits O is

(a) $3v/4a$ (b) $3v/2a$
(c) $3v/\sqrt{2}a$ (d) zero. [IIT 99]

19. Consider a body, shown in figure, consisting of two identical balls, each of mass M connected by a light rigid rod. If an impulse $J = Mv$ is imparted to the body at one of its ends, what would be its angular velocity ?

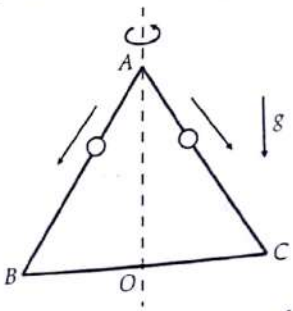


(a) v/L (b) $2v/L$
(c) $v/3L$ (d) $v/4L$ [IIT 03]

20. A cylinder rolls up an inclined plane, reaches some height, and then rolls down (without slipping throughout these motions). The directions of the frictional force acting on the cylinder are

(a) up the incline, while ascending and down the incline, while descending.
(b) up the incline, while ascending as well as descending.
(c) down the incline, while ascending and up the incline, while descending.
(d) down the incline, while ascending as well as descending. [IIT 02]

21. An equilateral triangle ABC formed from a uniform wire has two small identical beads initially located at A. The triangle is set rotating about the vertical axis AO. Then the beads are released from rest simultaneously and allowed to slide down, one along AB and the other along AC as shown. Neglecting frictional effects, the quantities that are conserved as the beads slide down are



(a) angular velocity and total energy (kinetic and potential)
(b) total angular momentum and total energy.
(c) angular velocity and moment of inertia about the axis of rotation.
(d) total angular momentum and moment of inertia about the axis of rotation. [IIT 2K]

22. A horizontal circular plate is rotating about a vertical axis passing through its centre with an angular velocity ω_0. A man sitting at the centre having two blocks in his hands stretches out his hands so that the moment of inertia of the system doubles. If the kinetic energy of the system is K initially, its final kinetic energy will be

(a) $2K$ (b) $K/2$
(c) K (d) $K/4$ [IIT 04]

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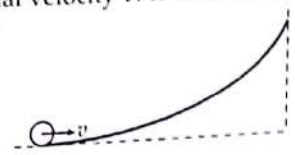
23. A child is standing with folded hands at the centre of a platform rotating about its central axis. The kinetic energy of the system is K. The child now stretches his arms so that the moment of inertia of the system doubles. The kinetic energy of the system now is

(*a*) $2K$ (*b*) $K/2$

(*c*) $K/4$ (*d*) $4K$ [IIT 04]

24. A small object of uniform density rolls up a curved surface with an initial velocity v. It reaches up to a maximum height of $\frac{3v^2}{4g}$ with respect to the initial position. The object is

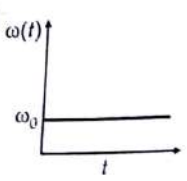


(*a*) ring (*b*) solid sphere

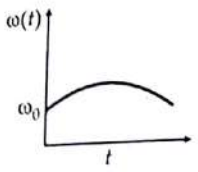
(*c*) hollow sphere (*d*) disc. [IIT 07]

25. A circular platform is free to rotate in horizontal plane about a vertical axis passing through its centre. A tortoise is sitting at the edge of the platform. Now, the platform is given an angular velocity ω_0. When the tortoise moves along a chord of the platform with a constant velocity (with respect to the platform), the angular velocity of the platform $\omega(t)$ will vary with time t as [IIT 02]

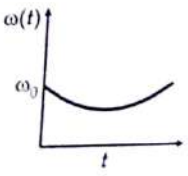
(*a*)



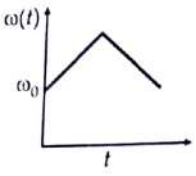
(*b*)



(*c*)



(*d*)



26. A piece of wire is bent in the shape of a parabola $y = kx^2$ (y-axis vertical) with a bead of mass m on it. The bead can slide on the wire without friction. It stays at the lowest point of the parabola when the wire is at rest. The wire is now accelerated parallel to the x-axis with a constant acceleration a. The distance of the new equilibrium position of the bead, where the bead can stay at rest w.r.t. the wire, from the y-axis is

(*a*) $\frac{a}{gk}$ (*b*) $\frac{a}{2gk}$

(*c*) $\frac{2a}{gk}$ (*d*) $\frac{a}{4gk}$ [IIT 09]

MULTIPLE CHOICE QUESTIONS WITH ONE OR MORE THAN ONE CORRECT ANSWER

27. A ball hits the floor and rebounds after an inelastic collision. In this case

(*a*) the momentum of the ball just after the collision is the same as that just before the collision

(*b*) the mechanical energy of the ball remains the same in the collision

(*c*) the total momentum of the ball and the earth is conserved

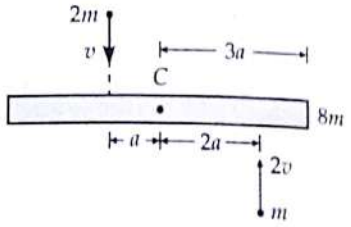
(*d*) the total energy of the ball and the earth is conserved. [IIT 86]

28. A shell is fired from a cannon with a velocity v (m/sec) at an angle θ with the horizontal direction. At the highest point in its path it explodes into two pieces of equal mass. One of the pieces retraces its path to the cannon and the speed (in m/sec) of the other piece immediately after the explosion is

(*a*) $3v\cos\theta$ (*b*) $2v\cos\theta$

(*c*) $\frac{3}{2}v\cos\theta$ (*d*) $\sqrt{\frac{3}{2}}v\cos\theta$ [IIT 86]

29. A uniform bar of length $6a$ and mass $8m$ lies on a smooth horizontal table. Two point masses m and $2m$ moving in the same horizontal plane with speed $2v$ and v respectively, strike the bar [as shown in figure]



and stick to the bar after collision. Denoting angular velocity (about the centre of mass), total energy and centre of mass velocity by ω, E and v_C respectively, we have after collision

(*a*) $v_C = 0$ (*b*) $\omega = \frac{3v}{5a}$

(*c*) $\omega = \frac{v}{5a}$ (*d*) $E = \frac{3mv^2}{5}$. [IIT 91]

30. Two blocks A and B, each of mass m, are connected by a massless spring of natural length L and spring constant k. The blocks are initially resting on a smooth horizontal floor with the spring at its natural length, as shown in the figure. A third identical block

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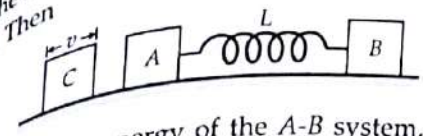
C also of mass m, moves on the floor with a speed v along the line joining A and B, and collides elastically with A. Then



(a) the kinetic energy of the A-B system, at maximum compression of the spring, is zero.

(b) the kinetic energy of the A-B system, at maximum compression of the spring, is $mv^2/4$

(c) the maximum compression of the spring is $v\sqrt{(m/k)}$

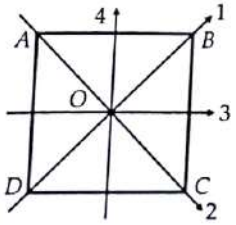
(d) the maximum compression of the spring is $v\sqrt{(m/2k)}$. [IIT 93]

31. A tube of length L is filled completely with an incompressible liquid of mass M and closed at both the ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity ω. The force exerted by the liquid at the other end is

(a) $\frac{M\omega^2 L}{2}$ (b) $M\omega^2 L$

(c) $\frac{M\omega^2 L}{4}$ (d) $\frac{M\omega^2 L^2}{2}$ [IIT 92]

32. The moment of inertia of a thin square plate $ABCD$, as shown in the figure, of uniform thickness about an axis passing through the centre O and perpendicular to the plane of the plate is



(a) $I_1 + I_2$

(b) $I_3 + I_4$

(c) $I_1 + I_3$

(d) $I_1 + I_2 + I_3 + I_4$

where I_1, I_2, I_3 and I_4 are respectively the moments of inertia about axis 1, 2, 3 and 4 which are in the plane of the plate. [IIT 92]

33. Let I be the moment of inertia of a uniform square plate about an axis AB that passes through its centre and is parallel to two of its sides. CD is a line in the plane of the plate that passes through the centre of the plate and makes an angle θ with AB. The moment of inertia of the plate about the axis CD is then equal to

(a) I (b) $I\sin^2\theta$

(c) $I\cos^2\theta$ (d) $I\cos^2\theta/2$ [IIT 98]

34. A solid cylinder is rolling down a rough inclined plane of inclination θ. Then

(a) the friction force is dissipative

(b) the friction force is necessarily changing

(c) the friction force will aid rotation but hinder translation.

(d) the friction force is reduced if θ is reduced. [IIT]

35. If the resultant of all the external forces acting on a system of particles is zero, then from an inertial frame, one can surely say that

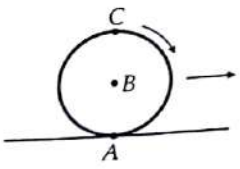
(a) linear momentum of the system does not change in time

(b) kinetic energy of the system does not change in time

(c) angular momentum of the system does not change in time

(d) potential energy of the system does not change in time. [IIT 09]

36. A sphere is rolling without slipping on a fixed horizontal plane surface. In the figure, A is the point of contact, B is the centre of the sphere and C is its topmost point. Then



(a) $\vec{V}_C - \vec{V}_A = 2(\vec{V}_B - \vec{V}_C)$

(b) $\vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$

(c) $|\vec{V}_C - \vec{V}_A| = 2|\vec{V}_B - \vec{V}_C|$

(d) $|\vec{V}_C - \vec{V}_A| = 4|\vec{V}_B|$ [IIT 09]

37. A point mass of 1 kg collides elastically with a stationary point mass of 5 kg. After their collision, the 1 kg mass reverses its direction and moves with a speed of 2 ms^{-1}. Which of the following statement(s) is (are) correct for the system of these two masses ?

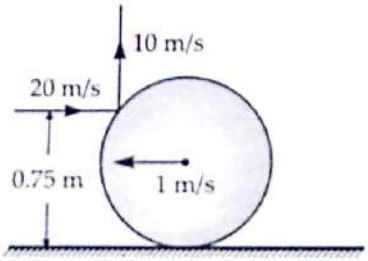
(a) Total momentum of the system is 3 kg ms^{-1}

(b) Momentum of 5 kg mass after collision is 4 kg ms^{-1}

(c) Kinetic energy of the centre of mass is 0.75 J

(d) Total kinetic energy of the system is 4 J [IIT 2010]

38. A thin ring of mass 2 kg and radius 0.5 m is rolling without slipping on a horizontal plane with



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velocity 1 m/s. A small ball of mass 0.1 kg, moving with velocity 20 m/s in the opposite direction, hits the ring at a height of 0.75 m and goes vertically up with velocity 10 m/s. Immediately after the collision

(a) the ring has pure rotation about its stationary CM

(b) the ring comes to a complete stop

(c) friction between the ring and the ground is to the left

(d) there is no friction between the ring and the ground. [IIT 2011]

REASONING TYPE

Instructions. This question contains statement – 1 (assertion) and statement – 2 (reason). Of these statements, mark correct choice if

(a) statements – 1 and 2 are true and statement – 2 is a correct explanation for statement – 1.

(b) statements – 1 and 2 are true and statement – 2 is not a correct explanation for statement – 1.

(c) statement – 1 is true, statement – 2 is false.

(d) statement – 1 is false, statement – 2 is true.

39. **Statement – 1 :** If there is no external torque on a body about its centre of mass, then the velocity of the centre of mass remains constant.

Statement – 2. The linear momentum of an isolated system remains constant. [IIT 07]

40. **Statement – 1 :** Two cylinders, one hollow (metal) and the other solid (wood) with the same mass and identical dimensions are simultaneously allowed to roll without slipping down an inclined plane from the same height. The hollow cylinder will reach the bottom of the inclined plane first.

Statement – 2 : By the principle of conservation of energy, the total kinetic energies of both the cylinders are identical when they reach the bottom of the incline.

COMPREHENSION BASED QUESTIONS

PARAGRAPH FOR QUESTION NOS. 41 TO 43

Two discs A and B are mounted coaxially on a vertical axle. The discs have moments of inertia I and $2I$ respectively about the common axis. Disc A is imparted an initial angular velocity 2ω using the entire potential energy of a spring compressed by a distance x_1. Disc B is imparted an angular velocity ω by a spring having the same spring constant and compressed by a distance x_2. Both the discs rotate in the clockwise direction.

Read the passage given above and answer the following questions

41. The ratio x_1 / x_2 is

(a) 2 (b) 1/2

(c) $\sqrt{2}$ (d) $1/\sqrt{2}$ [IIT 07]

42. When disc B is brought in contact with disc A, they acquire a common angular velocity in time t. The average frictional torque on the disc by the other during this period is

(a) $\frac{2I\omega}{3t}$ (b) $\frac{9I\omega}{2t}$

(c) $\frac{9I\omega}{4t}$ (d) $\frac{3I\omega}{2t}$ [IIT 07]

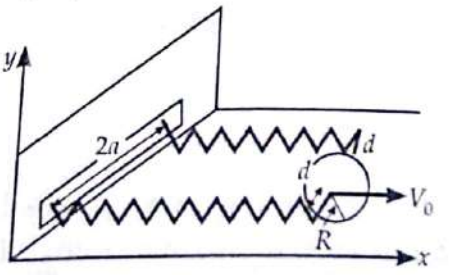
43. The loss of kinetic energy during the above process is

(a) $\frac{I\omega^2}{2}$ (b) $\frac{I\omega^2}{3}$

(c) $\frac{I\omega^2}{4}$ (d) $\frac{I\omega^2}{6}$ [IIT 07]

PARAGRAPH FOR QUESTION NOS. 44 TO 46

A uniform thin cylindrical disk of mass M and radius R is attached to two identical massless springs of spring constant k which are fixed to the wall as shown in the figure. The springs are attached to the axle of the disk symmetrically on either side at a distance d from its centre. The axle is massless and both the springs and the axle are in a horizontal plane. The unstretched length of each spring is L. The disk is initially at its equilibrium position with its centre of mass (CM) at a distance L from the wall. The disk rolls without slipping with velocity $\vec{V}_0 = V_0 \hat{i}$. The coefficient of friction is μ.



Read the passage given above and answer the following questions

44. The net external force acting on the disk when its centre of mass is at displacement x with respect to its equilibrium position is

(a) $-kx$ (b) $-2kx$

(c) $-\frac{2kx}{3}$ (d) $-\frac{4kx}{3}$ [IIT 09]

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45. The centre of mass of the disk undergoes simple harmonic motion with angular frequency ω equal to

(a) $\sqrt{\frac{k}{M}}$ (b) $\sqrt{\frac{2k}{M}}$

(c) $\sqrt{\frac{2k}{3M}}$ (d) $\sqrt{\frac{4k}{3M}}$ [IIT 09]

46. The maximum value of V_0 for which the disk will roll without slipping is

(a) $\mu g\sqrt{\frac{M}{k}}$ (b) $\mu g\sqrt{\frac{M}{2k}}$

(c) $\mu g\sqrt{\frac{3M}{k}}$ (d) $\mu g\sqrt{\frac{5M}{2k}}$ [IIT 09]

INTEGER ANSWER TYPE

47. Four solid spheres each of diameter $\sqrt{5}$ cm and mass 0.5 kg are placed with their centres at the corners of a square of side 4 cm. The moment of inertia of the system about the diagonal of the square is $N \times 10^{-4}$ kgm^2. Find N. [IIT 2011]

48. A binary star consists of two stars A(Mass 2.2 M_s) and B(Mass 11 M_s), where M_s is the mass of the sun. They are separated by distance d and are rotating about their centre of mass, which is stationary. Find the ratio of the total angular momentum of the binary star to the angular momentum of star B about the centre of mass. [IIT 2010]

Answers and Explanations

1. (a) The positions of different masses are $m(-a,a)$, $m(a,a)$, $m(0,0)$, $m(0,-a)$ and $6m(0,0)$.

$$\therefore \quad y_{CM} = \frac{m(a)+m(a)+m(0)+m(-a)+6m(0)}{m+m+m+m+6m} = \frac{a}{10}$$

2. (d) No external force is acting on the centre of mass. It remains at rest. The speed of CM is zero.

3. (c) $v_{CM} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = \frac{10\times 14 + 4\times 0}{4+10} = \mathbf{10\ ms^{-1}}$

4. (a) As the isolated particle is initially moving along x-axis, so there is no motion along y-axis. CM should remain stationary along y-axis even after explosion.

Now $$y_{CM} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2}$$

But $$y_{CM} = 0$$

$$\therefore \quad m_1y_1 + m_2y_2 = 0$$

or $$y_2 = -\frac{m_1}{m_2}\cdot y_1 = -\frac{m/4}{3m/4}\times 15 = -5 \text{ cm}.$$

Thus the larger fragment will be at $y = \mathbf{-5\ cm}$.

5. (c) Only the linear velocities are exchanged. The two spheres cannot exert torques on each other, as their surfaces are frictionless, and so that angular velocities of the spheres do not change.

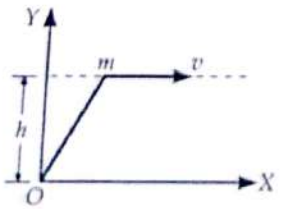
$$\therefore \quad \omega_A = \omega \text{ and } \omega_B = 0.$$

6. (b)

Angular momentum

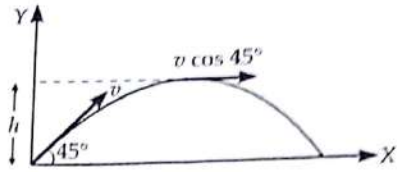
= Mement of momentum

$L = mv \times h$ = constant



As the particle moves, m, v and h all remain unchanged.

7. (b)



When the particle is at its maximum height,

L = Horizontal component of momentum × maximum height

$$= mv\cos 45^\circ \times \frac{v^2\sin^2 45^\circ}{2g}$$

$$= mv \times \frac{1}{\sqrt{2}} \times \frac{v^2}{2g} \times \frac{1}{2} = \frac{mv^3}{4\sqrt{2}g}.$$

8. (a) In uniform circular motion, centripetal force acts towards the centre. Torque due to such a force about the centre is zero. Hence angular momentum is conserved about the centre of the circle.

9. (b) $\vec{L} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$

As the particle has decreasing linear speed ($\vec{v}$), so $|\vec{L}|$ also decreases *i.e.*, $\vec{L}$ is not conserved. Option (a) is not correct.

Direction of $(\vec{r} \times \vec{v})$, hence the direction of $\vec{L}$ remains the same. Option (b) is correct.

As the particle is confined to rotate in a circular path, it cannot spiral towards the centre. Option (c) is incorrect.

As the particle has a decreasing speed, it must have a transverse acceleration in addition to the centripetal acceleration. Option (d) is in correct.

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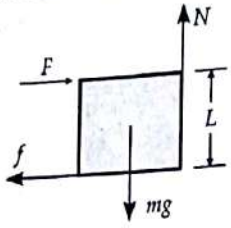
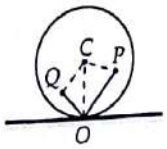
10. (*c*) When force F is applied, the normal reaction N of the floor shifts to the right. The cube topples when N reaches the edge. Consider torque about O.



$$F \times L = mg \times \frac{L}{2}$$

or $$F = \frac{mg}{2}.$$

11. (*b*) In pure rolling, instan- taneous velocity at the lowermost point O is zero. In the figure shown, $CP = CQ$. Velocity of any point on the disc, $v = r\omega$, where r is the distance of the point from O.



Now

$$OP > OC > OQ$$

or $$r_P > r_C > r_Q$$

$$\therefore \quad v_P > v_C > v_Q$$

12. (*d*) Mass of the ring,

$$M = \rho L$$

If R is the radius of the ring, then

$$L = 2\pi R$$

or $$R = \frac{L}{2\pi}$$

M.I. of the ring about a diameter through O and parallel to XX',

$$I_O = \frac{1}{2}MR^2$$

By parallel axes theorem, moment of inertia about XX' will be

$$I_{XX'} = I_O + MR^2 = \frac{1}{2}MR^2 + MR^2$$

$$= \frac{3}{2}MR^2 = \frac{3}{2} \times \rho L \times \left(\frac{L}{2\pi}\right)^2 = \frac{3\rho L^3}{8\pi^2}.$$

13. (*a*) Mass of complete disc = $4M$

M.I. of whole disc about the given axis,

$$I = \frac{1}{2}(4M)R^2 = 2MR^2$$

M.I. of quarter section of the disc,

$$I' = \frac{I}{4} = \frac{1}{2}MR^2$$

14. (*a*) Refer to the solution of Problem 17 on page 7.67.

15. (*b*) M.I. of the solid sphere about a diameter,

$$I = \frac{2}{5}MR^2$$

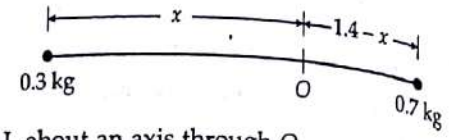
M.I. of the disc about an axis through its edge and perpendicular to its plane (from parallel axes theorem) is

$$I = \frac{Mr^2}{2} + Mr^2$$

$$\therefore \quad \frac{2}{5}MR^2 = \frac{Mr^2}{2} + Mr^2$$

or $$r = \frac{2}{\sqrt{15}}R.$$

16. (*c*) Let the required point be located at a distance x from 0.3 kg mass.



M.I. about an axis through O,

$$I = 0.3x^2 + 0.7(1.4 - x)^2$$

Work done,

$$W = \frac{1}{2}I\omega^2$$

W will be minimum when I is minimum.

$$\therefore \quad \frac{dI}{dx} = 0$$

$$2 \times 0.3x - 2 \times 0.7(1.4 - x) = 0$$

$$0.3x + 0.7x = 0.7 \times 1.4$$

$$x = \mathbf{0.98\ m}.$$

17. (*c*) $L_{tot} = L_{tran} + L_{rot}$

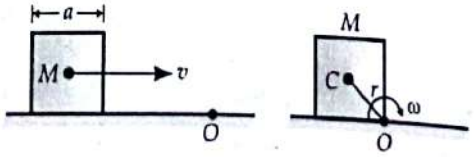
$$= MvR + I\omega$$

$$= M(R\omega)R + \frac{1}{2}MR^2\omega$$

$$= \frac{3}{2}MR^2\omega.$$

18. (*a*) Clearly, $(2r)^2 = a^2 + a^2$

or $$r^2 = \frac{a^2}{2}$$



As net torque about O is zero, angular momentum is conserved,

$$L_i = L_f$$

or $$Mv\left(\frac{a}{2}\right) = I_0\omega = (I_C + Mr^2)\omega$$

$$= \left[\frac{Ma^2}{6} + M\left(\frac{a^2}{2}\right)\right]\omega = \frac{2}{3}Ma^2\omega$$

$$\therefore \quad \omega = \mathbf{\frac{3v}{4a}}.$$

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19. (*a*) Applying conservation of angular momentum about the centre of the rod,

$$I_{CM}\omega = J \cdot \frac{L}{2}$$

$$2 \times M\left(\frac{L}{2}\right)^2 \omega = Mv \cdot \frac{L}{2}$$

or $$\omega = \frac{v}{L}.$$

20. (*b*) Whether the cylinder rolls up or down, the CM and hence the point of contact of the cylinder has an acceleration $g\sin\theta$ in the downward direction. Hence in both cases, the force of friction acts up the inclined plane.

21. (*b*) No external torque acts on system. Therefore, angular momentum is conserved. Again, forces acting on the system are conservative. Therefore, total mechanical energy of the system is conserved.

22. (*b*) Initial K.E., $K = \frac{1}{2} I_0 \omega_0^2$

By conservation of angular momentum ($L_0 = I_0\omega_0$), when M.I. is doubled, angular velocity is halved. So, the final K.E. is

$$K' = \frac{1}{2}(2I_0)\left(\frac{\omega_0}{2}\right)^2 = \frac{1}{2} \cdot \frac{1}{2} I_0 \omega_0^2 = \frac{K}{2}.$$

23. (*b*) Same reasoning as in the above problem.

24. (*d*) Loss in K.E. = Gain in P.E.

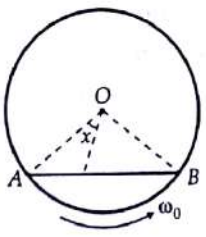
$$\frac{1}{2}Mv_{CM}^2 + \frac{1}{2}I_{CM}\omega^2 = Mgh$$

$$\frac{1}{2}Mv_{CM}^2 + \frac{1}{2}I_{CM}\left(\frac{v}{r}\right)^2 = Mg\frac{3v^2}{gh}$$

$$\therefore \quad I_{CM} = \frac{1}{2}MR^2$$

Clearly, the small object is a disc.

25. (*b*) Suppose initially the tortoise is at *A*. Its distance from the centre = *OA* = radius of the platform. As the tortoise moves along the chord *AB*, its distance from the axis of rotation varies. Let it be *x* at any instant *t*.



By conservation of angular momentum,

$$m(OA)^2\omega_0 = m \cdot x^2\omega$$

$$\omega = \frac{(OA)^2}{x^2} \cdot \omega_0$$

$$\therefore \quad \omega \propto \frac{1}{x^2}$$

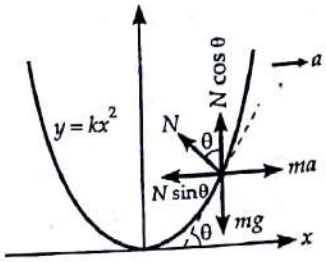
Thus the variation of ω with x is non-linear. Moreover, then value of x first decreases, takes a minimum value and then increases to maximum value (equals to radius). Consequently, ω first increases non-linearly, takes a maximum value (at *C*) and again falls to original value (at *B*). Hence option (*b*) is correct.

26. (*b*) In equilibrium,

$$N\cos\theta = mg$$

and $$N\sin\theta = ma$$

$$\therefore \quad \tan\theta = \frac{a}{g}$$



Given :

$$y = kx^2$$

$$\therefore \quad \frac{dy}{dx} = 2kx$$

But $\frac{dy}{dx}$ = Slope of parabolic curve = $\tan\theta$

$$\therefore \quad 2kx = \frac{a}{g} \quad \text{or} \quad x = \frac{a}{2gk}.$$

27. (*c*), (*d*) Option (*a*) is incorrect because after the collision, momentum of the ball changes both in magnitude and direction. Option (*b*) is also incorrect because during collision some mechanical energy gets converted into heat and sound.

Options (*c*) and (*d*) are correct.

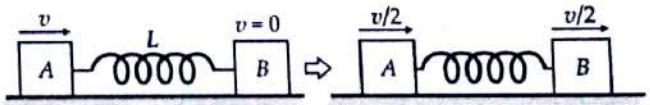
28. (*a*) The shell follows a parabolic path. At the highest point, it has horizontal velocity $v\cos\theta$. After explosion, one piece retraces its path with velocity $-v\cos\theta$. Let velocity of other piece be v'. Then conservation of momentum,

$$2m \times v\cos\theta = m(-v\cos\theta) + m \cdot v'$$

or $$v' = \mathbf{3}v\cos\theta.$$

29. (*a*), (*c*), (*d*) Refer to the solution of Problem 8 on page 7.65.

30. (*b*), (*d*) As a result of head-on collision between *C* and *A*, *C* stops and *A* begins to move with speed *v*. It compresses the spring *L* which pushes the block *B* towards right. At maximum compression, *A* and *B* both have same speed *v*/2 (from conservation of momentum).



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Let x be the maximum compression of the spring. Then the K.E. of A-B system at maximum compression,

$$K = \frac{1}{2}m\left(\frac{v}{2}\right)^2 + \frac{1}{2}m\left(\frac{v}{2}\right)^2 = \frac{mv^2}{4}$$

Again by conservation of mechanical energy,

$$\frac{1}{2}mv^2 = \frac{1}{4}mv^4 + \frac{1}{2}kx^2$$

or $$\frac{1}{2}kx^2 = \frac{1}{4}mv^2$$

or $$x = v\sqrt{\frac{m}{2k}}.$$

31. (*a*) Refer to the solution of Problem 5 on page 7.64.

32. (*a*), (*b*), (*c*) By symmetry,

$$I_1 = I_2 \text{ and } I_3 = I_4$$

By perpendicular axes theorem,

$$I_O = I_1 + I_2$$

and $$I_O = I_3 + I_4$$

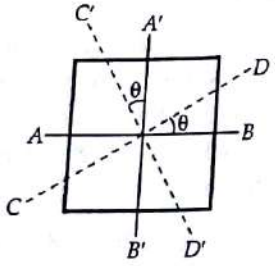
On adding,

$$2I_O = (I_1 + I_2) + (I_3 + I_4)$$
$$= (I_1 + I_1) + (I_3 + I_3)$$
$$= 2(I_1 + I_3)$$

or $$I_O = I_1 + I_3$$

Hence options (*a*), (*b*) and (*c*) are correct.

33. (*a*) Let $A'B' \perp AB$ and $C'D' \perp CD$.



By symmetry,

$$I_{AB} = I_{A'B'} \text{ and } I_{CD} = I_{C'D'}$$

By perpendicular axes theorem, the M.I. about O, perpendicular to the plane of the plate,

$$I_O = I_{AB} + I_{A'B'} = I_{CD} + I_{C'D'}$$

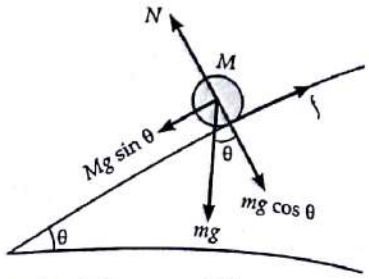
or $$2I_{AB} = 2I_{CD}$$

But $$I_{AB} = I$$

$$\therefore \quad I_{CD} = I$$

This is independent of θ.

34. (*c*), (*d*) The component $Mg\sin\theta$ tends to slide the point of contact down the inclined plane. The sliding friction acts in its opposite direction. The cylinder rolls because of friction. Thus frictional force aids rotation but hinders translation.



Hence option (*c*) is correct. If a_c is the acceleration of CM of the cylinder, then

$$Ma_c = Mg\sin\theta - f$$

or $$f = Mg\sin\theta - Ma_c$$

Also, $$a_c = \frac{g\sin\theta}{1 + \frac{I_C}{MR^2}} = \frac{g\sin\theta}{1 + \frac{\frac{1}{2}MR^2}{MR^2}} \quad \ldots(i)$$

$$= \frac{2}{3}g\sin\theta$$

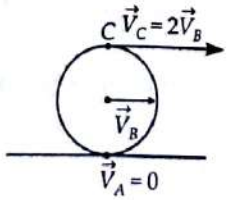
$$\therefore \quad f = Mg\sin\theta - M \times \frac{2}{3}g\sin\theta$$

or $$f = \frac{1}{3}Mg\sin\theta$$

Clearly, if θ is reduced, f also reduces. Hence option (*d*) is correct.

35. (*a*) According to law of conservation of linear momentum if $\vec{F}_{ext} = 0$, then the linear momentum of the system does not change in time. There may be external forces acting due to which K.E. or P.E. or both may change. Also, net force is zero does not mean net torque is zero. So, angular momentum may change. Hence only option (*a*) is correct.

36. (*b*), (*c*) The velocities at points A, B and C are as shown



$$\vec{V}_C - \vec{V}_A = 2V_B$$

$$\vec{V}_B - \vec{V}_C = -\vec{V}_B$$

$$\vec{V}_B - \vec{V}_A = \vec{V}_B$$

Clearly, only options (*b*) and (*c*) are correct.

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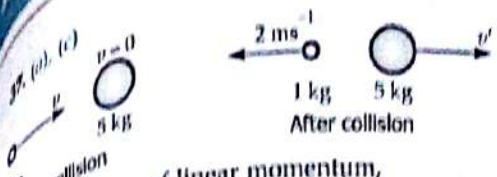


Before collision

By conservation of linear momentum,

$$1\times v + 5\times 0 = 1\times(-2) + 5v'$$

$$v = -2 + 5v' \qquad ...(i)$$

As the collision is elastic,

$$-\frac{v_1 - v_2}{u_1 - u_2} = 1$$

$$-\frac{2 - v'}{v - 0} = 1$$

$$2 + v' = v \qquad ...(ii)$$

On solving (*i*) and (*ii*), $v = 3\,\text{ms}^{-1}$, $v' = 1\,\text{ms}^{-1}$

Total momentum of the system

$$= 1\times 3 + 5\times 0 = 3\,\text{kgms}^{-1}$$

$$v_{CM} = \frac{1\times 3 + 5\times 0}{1+5} = \frac{1}{2}\,\text{ms}^{-1}$$

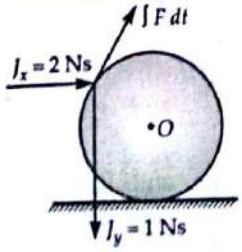
∴ K.E. of the centre of mass

$$= \frac{1}{2}\times(1+5)^2\times\left(\frac{1}{2}\right)^2 = 0.75\ \text{J}$$

Total K.E. $= \frac{1}{2}\times 1\times 3^2 = 4.5\ \text{J}$

Hence the correct options are (*a*) and (*c*).

38. (*c*) During collision, friction is impulsive. The angular impulse created by the impulsive forces tends to decrease the angular speed of the ring about *O* but the ring continues to rotate anticlockwise. Hence the friction between the ring and the ground at the point of contact will act towards left.



39. (*d*) The absence of external torque does not ensure the absence of external force. If external force is present, then the velocity of CM will not remain constant. Thus statement – 1 is false.

The linear momentum of an isolated system remains constant because no external force acts on such a system. Thus statement – 2 is true.

Hence option (*d*) is correct.

40. (*d*) From energy conservation

$$1/2\,mv_c^2 + 1/2\,I_c\omega^2 = mgh$$

$$\omega = v_c / R$$

$$(I_c)_{\text{solid}} < (I_c)_{\text{hollow}}$$

Hence $(v_c)_{\text{solid}} > (v_c)_{\text{hollow}}$

Hence solid cylinder will reach the bottom first.

41. (*c*) Clearly, $\frac{1}{2}kx_1^2 = \frac{1}{2}I(2\omega)^2$

and $\frac{1}{2}kx_2^2 = \frac{1}{2}(2I)\omega^2$

$$\therefore \quad \frac{x_1}{x_2} = \sqrt{2}\,.$$

42. (*a*) By conservation of angular momentum,

$$(I+2I)\omega' = I.2\omega + 2I.\omega$$

or $$\omega' = \frac{4I\omega}{3I} = \frac{4\omega}{3} \qquad ...(i)$$

Angular acceleration of disc *B*,

$$\alpha = \frac{\text{Torque}}{\text{M.I.}} = \frac{\tau}{2I}$$

$$\therefore \quad \omega' = \omega + \alpha t = \omega + \frac{\tau}{2I}t \qquad ...(ii)$$

From (*i*) and (*ii*),

$$\omega + \frac{\tau}{2I}t = \frac{4\omega}{3}$$

or $$\tau = \frac{2I\omega}{3t}\,.$$

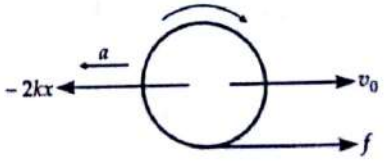
43. (*b*) Loss in K.E.

$$= K_i - K_f$$

$$= \left[\frac{1}{2}I(2\omega)^2 + \frac{1}{2}(2I)\omega^2\right] - \frac{1}{2}(I+2I)\left(\frac{4\omega}{3}\right)^2$$

$$= \frac{I\omega^3}{3}\,.$$

44. (*d*)



$$F_{net} = 2kx - f = ma$$

Torque, $fR = I_C\alpha$

$$a = \alpha R$$

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$$\therefore \quad f = I_C \frac{\alpha}{R}$$

$$= \frac{1}{2}mR^2 \cdot \frac{a}{R^2} = \frac{1}{2}ma$$

$$F_{net} = 2kx - \frac{1}{2}ma = ma$$

$$2kx = \frac{3}{2}ma$$

$$ma = \frac{4kx}{3}$$

$$\vec{F}_{net} = -\frac{4}{2}k\vec{x}.$$

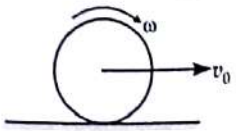
45. (*d*) $-(2kx)R = I_p\alpha$

$$2k.R\theta.R = \frac{3}{2}MR^2\alpha$$

$$\alpha = -\frac{4k}{3M}\theta = -\omega^2\theta$$

$$\therefore \quad \omega = \sqrt{\frac{4k}{3M}}$$

46. (*a*)



$$f \le \mu N$$

$$\frac{I_c.\alpha}{R} \le \mu N$$

$$\alpha \le \frac{\mu NR}{I_c}$$

$$\alpha \le \frac{\mu mgR}{mR^2/2}$$

$$\frac{a_c}{R} = \alpha \le \frac{2\mu g}{R}$$

$$\frac{4}{3}\frac{kA}{M} = a_c \le 2\mu g$$

$$A \le \frac{2\mu gM \times 3}{4k} = \frac{3}{2}\frac{\mu gM}{k}$$

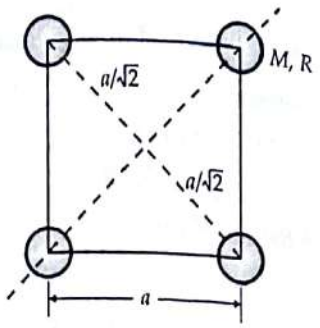
$$v \le \omega A = \sqrt{\frac{4k}{3M}} \times \frac{3}{2}\frac{\mu gM}{k}$$

$$v \le \mu g\sqrt{\frac{M}{k}}$$

$$v_0 = \mu g\sqrt{\frac{M}{k}}.$$

47.

0	0	0	9



$$I = 2\left(\frac{2}{5}MR^2\right) + 2\left[\frac{2}{5}MR^2 + M\left(\frac{a}{\sqrt{2}}\right)^2\right]$$

$$= \frac{8}{5}MR^2 + Ma^2$$

$$= \frac{8}{5} \times 0.5 \times \left(\frac{\sqrt{5}}{2} \times 10^{-2}\right)^2 + 0.5 \times (4 \times 10^{-2})^2$$

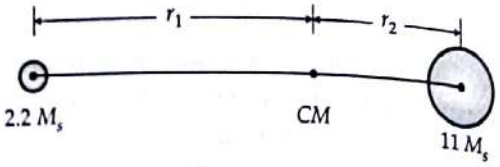
$$= 1 \times 10^{-4} + 8 \times 10^{-4} = 9 \times 10^{-4}\ \text{kgm}^2$$

But $I = N \times 10^{-4}\ \text{kgm}^2$

Hence, $N = 9$

48.

0	0	0	6



Clearly $2.2\,r_1 = 11\,r_2$

$\Rightarrow \quad r_1 = 5r_2$

But $r_1 + r_2 = d$

$\therefore \quad r_1 = \frac{5d}{6}$ and $r_2 = \frac{d}{6}$

Now $L_1 = I_1\omega$ and $L_2 = I_2\omega$

$\therefore \quad \frac{L_1}{L_2} = \frac{I_1}{I_2}$

$$= \frac{2.2\,M_s(5d/6)^2 + 11M_s(d/6)^2}{11M_s(d/6)^2}$$

$$= \frac{\frac{55}{36} + \frac{11}{36}}{\frac{11}{36}} = 6$$

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AIEEE

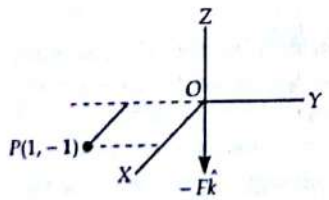
1. A body A of mass M while falling vertically downwards under gravity breaks into two parts ; a body of mass $\frac{1}{3}M$ and body C of mass $\frac{2}{3}M$. The centre of mass of bodies B and C taken together shifts compared to that of body A towards
(a) body C (b) body B
(c) depends on height of breaking (d) does not shift [AIEEE 05]

2. Two identical particles move towards each other with velocities $2v$ and v respectively. The velocity of centre of mass is
(a) v (b) $v/3$
(c) $v/2$ (d) zero. [AIEEE 02]

3. Consider a two particle system with particles having masses m_1 and m_2. If the first particle is pushed towards the centre of mass through a distance d, by what distance should the second particle be moved, so as to keep the centre of mass at the same position ?
(a) d (b) $\frac{m_2}{m_1}d$
(c) $\frac{m_1}{m_1+m_2}d$ (d) $\frac{m_1}{m_2}d$ [AIEEE 06 ; DCE 08]

4. A circular disc of radius R is removed from a bigger circular disc of radius $2R$, such that the circumferences of the discs coincide. The centre of mass of the new disc is αR from the centre of the bigger disc. The value of α is
(a) 1/3 (b) 1/2
(c) 1/6 (d) 1/4 [AIEEE 07]

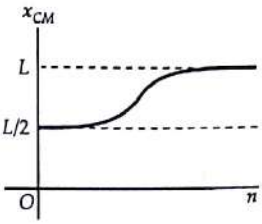
5. A force of $-F\hat{k}$ acts on O, the origin of the coordinate system.



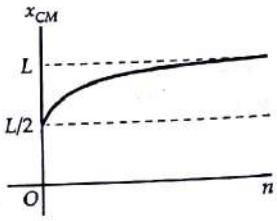
The torque about the point (1, –1) is
(a) $F(\hat{i}+\hat{j})$ (b) $-F(\hat{i}-\hat{j})$
(c) $F(\hat{i}-\hat{j})$ (d) $-F(\hat{i}+\hat{j})$ [AIEEE 06]

6. A thin rod of length L is lying along the x-axis with its ends at $x=0$ and $x=L$. Its linear density (mass/length) varies with x as $k(x/L)^n$, where n can be zero or any positive number. If the position x_{CM} of the centre of mass of the rod is plotted against n, which of the following graphs best approximates the dependence of x_{CM} on n ? [AIEEE 08]

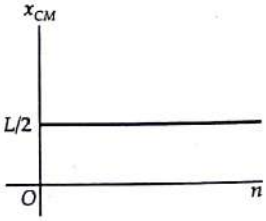
(a)



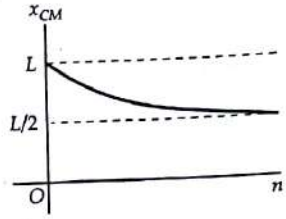
(b)



(c)



(d)



7. Let $\vec{F}$ be the force acting on a particle having position vector $\vec{r}$ and $\vec{\tau}$ be the torque of this force about the origin. Then
(a) $\vec{r}.\vec{F}=0$ and $\vec{F}.\vec{\tau}\neq 0$
(b) $\vec{r}.\vec{\tau}\neq 0$ and $\vec{F}.\vec{\tau}=0$
(c) $\vec{r}.\vec{\tau}\neq 0$ and $\vec{F}.\vec{\tau}\neq 0$
(d) $\vec{r}.\vec{\tau}=0$ and $\vec{F}.\vec{\tau}=0$ [AIEEE 03 ; CBSE PMT 09]

8. Angular momentum of the particle rotating with a central force is constant due to
(a) constant force
(b) constant linear momentum
(c) constant torque
(d) zero torque. [AIEEE 07]

9. A particle performing uniform circular motion has angular momentum L. If its angular frequency is doubled and kinetic energy is halved, then the angular momentum becomes
(a) $L/4$ (b) $2L$
(c) $4L$ (d) $L/2$ [AIEEE 03]

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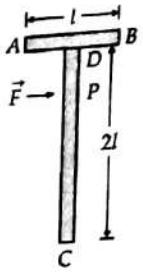
10. A T-shaped object with dimensions shown in the figure is lying on a smooth floor. A force $\vec{F}$ is applied at the point P parallel to AB, such that the object has only the translational motion without rotation. Find the location of P with respect to C

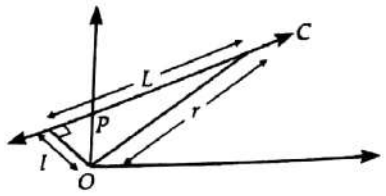


(a) $-2l/3$ (b) $3l/2$

(c) $4l/3$ (d) l

[AIEEE 05]

11. A particle of mass m moves along line PC with velocity v as shown. What is the angular momentum of the particle about P ?



(a) mvL (b) mvl

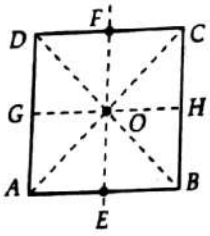
(c) mvr (d) zero. [AIEEE 02]

12. Four point masses, each of the value m, are placed at the corners of a square $ABCD$ of side l. The moment of inertia of this system about an axis passing through A and parallel to BD is

(a) $3ml^2$ (b) ml^2

(c) $2ml^2$ (d) $\sqrt{3}ml^2$ [AIEEE 06]

13. For the given uniform square lamina $ABCD$, whose centre is O,



(a) $I_{AC} = \sqrt{2}\, I_{EF}$

(b) $\sqrt{2}\, I_{AC} = I_{EF}$

(c) $I_{AD} = 3 I_{EF}$

(d) $I_{AC} = I_{EF}$ [AIEEE 07]

14. Moment of inertia of a circular wire of mass M and radius R about is its diameter is

(a) $\frac{1}{2}MR^2$ (b) $\frac{1}{4}MR^2$

(c) $2MR^2$ (d) MR^2 [AIEEE 02]

15. A circular disc X of radius R is made from an iron plate of thickness t and another disc Y of radius $4R$ is made from an iron plate of thickness $t/4$. Then the relation between the moment of inertia I_X and I_Y is

(a) $I_Y = I_X$ (b) $I_Y = 16 I_X$

(c) $I_Y = 32 I_X$ (d) $I_Y = 64 I_X$ [AIEEE 03]

16. Consider a uniform square plate of side a and mass m. The moment of inertia of this plate about an axis perpendicular to its plane and passing through one of its corners is

(a) $\frac{2}{3}ma^2$ (b) $\frac{5}{6}ma^2$

(c) $\frac{1}{12}ma^2$ (d) $\frac{7}{12}ma^2$

[AIEEE 05]

17. The moment of inertia of a uniform semi-circular disc of mass M and radius R about a line perpendicular to the plane of the disc through the centre is

(a) $\frac{1}{4}Mr^2$ (b) $\frac{2}{5}Mr^2$

(c) Mr^2 (d) $\frac{1}{2}Mr^2$

[AIEEE 05]

18. One solid sphere A and another hollow sphere B are of same mass and same outer radii. Their moments of inertia about their diameters are respectively I_A and I_B, such that

(a) $I_A = I_B$ (b) $I_A > I_B$

(c) $I_A < I_B$ (d) $I_A / I_B = \rho_A / \rho_B$

Here ρ_A and ρ_B represent their densities.

[AIEEE 04]

19. A solid sphere is rotating in free space. If the radius of the sphere is increased keeping mass same, which one of the following will not be affected ?

(a) Moment of inertia (b) Angular momentum

(c) Angular velocity (d) Rotational kinetic energy

[AIEEE 04]

20. Initial angular velocity of a circular disc of mass M is ω_1. Then, two small spheres of mass m are attached gently to two diametrically opposite points on the edge of the disc. What is the final angular velocity of the disc?

(a) $\frac{M+m}{M}\omega_1$ (b) $\frac{M}{M+m}\omega_1$

(c) $\frac{M}{M+4m}\omega_1$ (d) $\frac{M}{M+2m}\omega_1$

[AIEEE 02]

21. A thin circular ring of mass m and radius R is rotating about its axis with a constant angular velocity ω. Two objects each of mass M are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with an angular velocity ω', which is equal to

(a) $\frac{m\omega}{m+M}$ (b) $\frac{m\omega}{m+2M}$

(c) $\frac{(m+2M)\omega}{m}$ (d) $\frac{(m-2M)\omega}{(m+2M)}$

[IIT 83 ; AIEEE 06 ; CBSE PMT 09]

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22. A solid sphere, a hollow sphere and a ring are released from top of an inclined plane (frictionless) so that they slide down the plane. Then, maximum acceleration down the plane is for (no rolling)

(a) solid sphere (b) hollow sphere
(c) ring (d) all same [AIEEE 02]

23. A uniform round body of radius R, mass M and moment of inertia I rolls down (without slipping) an inclined plane making an angle θ with the horizontal. Then the acceleration is

(a) $\frac{g\sin\theta}{2+I/MR^2}$ (b) $\frac{g\sin\theta}{1+MR^2/I}$

(c) $\frac{g\sin\theta}{1-I/MR^2}$ (d) $\frac{g\sin\theta}{1-MR^2/I}$ [AIEEE 07]

24. A thin uniform rod of length l and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω. Its centre of mass rises to a maximum height of

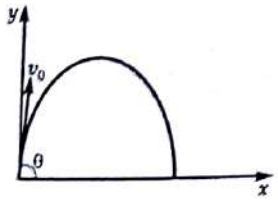
(a) $\frac{1}{3}\frac{l^2\omega^2}{g}$ (b) $\frac{1}{6}\frac{l\omega}{g}$

(c) $\frac{1}{2}\frac{l^2\omega^2}{g}$ (d) $\frac{1}{6}\frac{l^2\omega^2}{g}$. [AIEEE 09]

25. A pulley of radius 2 m is rotated about its axis by a force $F=(20t-5t^2)$ newton (where t is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10 kgm^2, the number of rotations made by the pulley before its direction of motion is reversed, is

(a) less than 3
(b) more than 3 but less than 6
(c) more than 6 but less than 9
(d) more than 9 [AIEEE 2011]

26. A small particle of mass m is projected at an angle θ with the x-axis with an initial velocity v_0 in the x-y plane as shown in the figure. At a time $t<\frac{v_0\sin\theta}{g}$, the angular momentum of the particle is



(a) $\frac{1}{2}mgv_0t^2\cos\theta\,\hat{i}$ (b) $-mgv_0t^2\cos\theta\,\hat{j}$

(c) $mgv_0t\cos\theta\hat{k}$ (d) $-\frac{1}{2}mgv_0t^2\cos\theta\hat{k}$

where $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axis respectively [AIEEE 2010]

27. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect, the angular speed of the disc

(a) remains unchanged
(b) continuously decreases
(c) continuously increases
(d) first increases and then decreases [AIEEE 2011]

28. A mass m hangs with the help of a string wrapped around a pulley on a frictionless bearing. The pulley has mass m and radius R. Assuming pulley to be a perfect uniform circular disc, the acceleration of the mass m, if the string does not slip on the pulley, is

(a) $\frac{3}{2}g$ (b) g

(c) $\frac{2}{3}g$ (d) $\frac{g}{3}$ [AIEEE 2011]

Answers and Explanations

1. (d) Refer to the solution of Problem 70 on page 7.57.

2. (c) Refer to the solution of Problem 71 on page 7.58.

3. (d) Refer to the solution of Problem 4 on page 7.112.

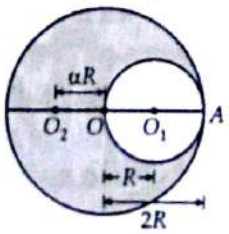
4. (a) Mass of original disc = M

Mass of disc removed,

$$m_1=\frac{M}{\pi(2R^2)}\times\pi R^2=\frac{M}{4}$$

Mass of the remaining disc,

$$m_2=M-\frac{M}{4}=\frac{3M}{4}$$



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Masses m_1 and m_2 are concentrated at O_1 and O_2 respectively and O is their CM.

$\therefore$ Moment of m_1 about O
= Moment of m_2 about O

$$\frac{M}{4} \times R = \frac{3M}{4} \times \alpha R$$

$$\alpha = \frac{1}{3}.$$

5. (*a*) Here $\vec{F} = -F\hat{k}$

Position vector of point (1, –1),

$$\vec{r} = \hat{i} - \hat{j}$$

$$\vec{\tau} = \vec{r} \times \vec{F} = (\hat{i} - \hat{j}) \times (-F\hat{k})$$

$$= F(-\hat{i} \times \hat{k} + \hat{j} \times \hat{k}) = F(\hat{k} \times \hat{i} + \hat{j} \times \hat{k})$$

$$= F(\hat{j} + \hat{i}) = F(\hat{i} + \hat{j}).$$

6. (*b*) Linear density (mass/length),

$$m = k\left(\frac{x}{L}\right)^n$$

For $n = 0$, $m =$ constant. For this uniform distribution, CM will lie at the centre of the rod *i.e.*, at $x = \frac{L}{2}$.

As the value of n increases to 1, 2, 3, ..., the linear density increases. The CM shifts beyond $x = \frac{L}{2}$, towards $x = L$. For very large values of n, CM approaches the other end $x = L$ of the rod. Hence graph (*b*) is the best approximation of dependence of x_{CM} on n.

7. (*d*) $\vec{\tau} = \vec{r} \times \vec{F}$

Vector $\vec{\tau}$ is perpendicular to both $\vec{r}$ and $\vec{F}$.

$\therefore$ $\vec{r} \cdot \vec{\tau} = 0$ and $\vec{F} \cdot \vec{\tau} = 0$

8. (*d*) Torque due to the central force is zero.

$$\tau = \frac{dL}{dt} = 0$$

$\therefore$ $L =$ constant.

9. (*a*) Refer to the solution of Problem 73 on page 7.58.

10. (*c*) Refer to the solution of Problem 16 on page 7.67.

11. (*d*) As the particle moves with velocity v along line PC, the line of motion passes through P. Hence angular momentum about P is zero.

12. (*a*) $AC = BD = \sqrt{l^2 + l^2} = \sqrt{2} \cdot l$

$$I_{BD} = m(AO)^2 + m(CO)^2$$

$$= m\left(\frac{\sqrt{2}l}{2}\right)^2 + m\left(\frac{\sqrt{2}l}{2}\right)^2$$

$$= \frac{ml^2}{2} + \frac{ml^2}{2} = ml^2$$

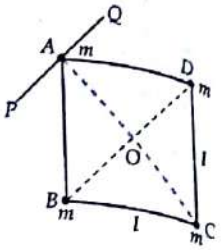
By theorem of parallel axes

$$I_{PAQ} = I_{BD} + 4m(AO)^2$$

$$= ml^2 + 4m\left(\frac{\sqrt{2}l}{2}\right)^2$$

$$= ml^2 + 2ml^2$$

$$= 3ml^2.$$



13. (*d*) Let I be the M.I. of the square lamina about an axis through O and perpendicular to its plane. Then by perpendicular axes theorem,

$$I_{AC} + I_{BD} = I$$

or $$I_{AC} = \frac{I}{2} \qquad [\because I_{AC} = I_{BD}]$$

Again by perpendicular axes theorem,

$$I_{EF} + I_{GH} = I$$

or $$I_{EF} = \frac{I}{2} \qquad [\because I_{EF} = I_{GH}]$$

Hence $$I_{AC} = I_{EF}$$

14. (*b*) A circular wire behaves like a ring. By perpendicular axes theorem,

$$I_D + I_D = \frac{1}{2}MR^2 \quad \therefore \quad I_D = \frac{1}{4}MR^2.$$

15. (*d*) Let ρ be the density of iron. M.I. of a circular disc about its axis,

$$I = \frac{1}{2}MR^2$$

$\therefore$ $$I_X = \frac{1}{2}M_X R_X^2 = \frac{1}{2}(\pi R^2 t\rho)R^2$$

$$I_Y = \frac{1}{2}M_Y R_Y^2 = \left[\pi(4R^2)\left(\frac{t}{4}\right)\rho\right](4R)^2$$

$$= \frac{1}{2}(\pi R^2 t\rho)R^2 \times 64$$

Hence $I_Y = 64\, I_X$.

16. (*a*) M.I. of a square plate about an axis through its centre and perpendicular to its plane is

$$I = \frac{m(a^2 + a^2)}{12} = \frac{ma^2}{6}$$

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By parallel axes theorem, the M.I. of the plate about an axis through one of its corners,

$$I' = I + m\left(\frac{\sqrt{2}a}{2}\right)^2 = \frac{ma^2}{6} + \frac{ma^2}{2} = \frac{2}{3}ma^2.$$

17. (d) M.I. of the circular disc will be

$$2I = \frac{(2M)r^2}{2}$$

M.I. of the semicircular disc, $I = \frac{1}{2}Mr^2$.

18. (c) In a hollow sphere, the mass is distributed away from the axis of rotation. So, its moment of inertia is greater than that of a solid sphere.

19. (b) As the radius increases, the M.I. of the sphere increases. As no external torque acts in free space, the speed of rotation decreases but the angular momentum remains constant.

20. (c) Refer to the solution of Problem 15 on page 7.67.

21. (b) By conservation of angular momentum,

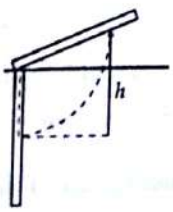
$$(m+2M)R^2\omega' = mR^2\omega$$

$$\omega' = \frac{m\omega}{m+2M}$$

22. (d) As the bodies slide and do not roll, they all have same acceleration, $a = g\sin\theta$

23. (a) The acceleration is given by, $a = \frac{g\sin\theta}{1+\frac{I}{MR^2}}$

24. (d) Loss in kinetic energy
= Gain in potential energy



$$\frac{1}{2}I\omega^2 = mgh$$

$$\frac{1}{2}\left(\frac{ml^3}{3}\right)\omega^2 = mgh$$

or $$h = \frac{l^2\omega^2}{6g}.$$

25. (b) $\tau = rF = I\alpha$

$$2(20t - 5t^2) = 10\alpha$$

$$\therefore \quad \alpha = \frac{d\omega}{dt} = 4t - t^2$$

$$\int d\omega = \int (4t - t^2)\,dt$$

$$\omega = 2t^2 - \frac{t^3}{3}$$

$$\omega = 0 \Rightarrow t = 6\ s$$

$$\omega = \frac{d\theta}{dt} = 2t^2 - \frac{t^3}{3}$$

$$\therefore \quad \int d\theta = \int\left(2t^2 - \frac{t^3}{3}\right)dt$$

or $$\theta = \frac{2t^3}{3} - \frac{t^3}{12}$$

For $t = 6$ s, $\theta = 36$ rad

or $$2\pi n = 36$$

$$\therefore \quad n = \frac{36}{2\pi} < 6.$$

26. (d) Angular momentum,

$$L = \int \tau\,dt$$

$$L = -\int mgx\,dt\,\hat{k}$$

$$= -\int mgv_0\cos\theta\, t\,dt\,\hat{k}$$

$$= -\frac{mgv_0\cos\theta\, t^2}{2}\hat{k}$$

27. (d) $\tau = 0$

$\therefore$ Angular momentum L is conserved

$$L = I\omega = mr^2\omega = \text{constant}$$

As r first decreases and then increases, angular speed ω first increases and then decreases.

28. (c) Equations of motion are

$$mg - T = ma$$

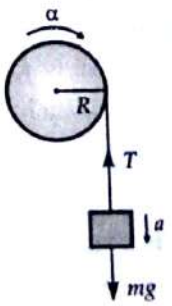
and $$T.R = \frac{1}{2}mR^2\alpha$$

and $$a = R\alpha$$

$$\therefore \quad T = \frac{1}{2}mR\,\alpha = \frac{1}{2}ma$$

and $$mg - \frac{1}{2}ma = ma$$

$$\Rightarrow \quad a = \frac{2}{3}g$$



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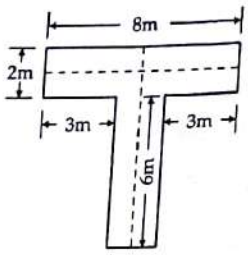
DCE and G.G.S. Indraprastha University Engineering Entrance Exam

(Similar Questions)

1. The separation between C and O atoms in CO is 1.2 Å. The distance of carbon atom from the centre of mass is

(a) 0.3 Å (b) 0.7 Å
(c) 0.5 Å (d) 0.9 Å [IPUEE 04]

2. The distance of the centre of mass of the T-shaped plate from O is



(a) 7 m (b) 2.7 m
(c) 4 m (d) 1 m [DCE 05]

3. In a bicycle the radius of rear wheel is twice the radius of front wheel. If r_f and r_r are the radii v_f and v_r are the speeds of top most points of wheels, then

(a) $v_r = 2v_f$ (b) $v_f = 2v_r$
(c) $v_f = v_r$ (d) $v_f > v_r$ [IPUEE 05]

4. If force acts on a body, whose line of action does not pass through its centre of gravity, then the body will experience

(a) angular acceleration
(b) linear acceleration
(c) both (a) and (b)
(d) none of the above. [DCE 97]

5. A couple is acting on a two particle system. The resultant motion will be

(a) purely rotational motion
(b) purely linear motion
(c) both (a) and (b)
(d) neither (a) nor (b) [DCE 04]

6. Angular momentum is

(a) moment of momentum
(b) product of mass and angular velocity
(c) product of M.I. and velocity
(d) moment of angular motion. [DCE 2K]

7. Which is a vector quantity ?

(a) angular momentum
(b) work
(c) potential energy
(d) electric current [DCE 01]

8. The dimensions of angular momentum are :

(a) $[MLT^{-2}]$ (b) $[ML^2T^{-1}]$
(c) $[ML^2T^{-2}]$ (d) $[ML^2T]$ [DCE 97 ; DPMT 2K]

9. The angular momentum of a system of particles is conserved

(a) when no external force acts upon the system
(b) when no external torque acts upon the system
(c) when no external impulse acts upon the system
(d) when axis of rotation remains same. [DCE 2K]

10. Planetary motion in the solar system describes

(a) conservation of kinetic energy
(b) conservation of linear momentum
(c) conservation of angular momentum
(d) none of the above. [DCE 98, 03]

11. If a person standing on a rotating disc stretches out his hands, the angular speed will

(a) increase (b) decrease
(c) remain same (d) none of these. [DCE 99]

12. The torque $\vec{\tau}$ on a body about a given point is found to be equal to $\vec{A} \times \vec{L}$, where $\vec{A}$ is a constant vector and $\vec{L}$ is the angular momentum of the body about that point. From this it follows that

(a) $\frac{d\vec{L}}{dt}$ is perpendicular to $\vec{L}$ at all instants of time
(b) the component of $\vec{L}$ in the direction of $\vec{A}$ does not change with time
(c) the magnitude of $\vec{L}$ does not change with time.
(d) all the above. [IIT 98 ; DCE 01]

13. Analogue of mass in rotational motion is

(a) moment of inertia (b) angular momentum
(c) gyration (d) none of the above. [DCE 2K]

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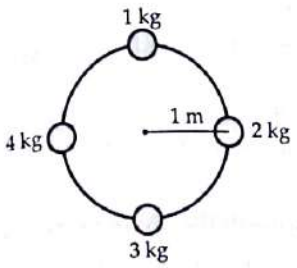
14. Moment of inertia depends upon
(a) axis of rotation (b) torque applied
(c) angular speed (d) angular momentum [DCE 97]

15. Moment of inertia of an object does not depend upon
(a) mass of object (b) mass distribution
(c) angular velocity (d) axis of rotation. [IPUEE 05]

16. Four balls each of radius 10 cm and mass 1 kg, 2 kg, 3 kg and 4 kg are attached to the periphery of massless plate of radius 1 m.



What is moment of inertia of the system about the centre of the plate ?
(a) 12.04 kg m^2 (b) 10.04 kg m^2
(c) 11.50 kg m^2 (d) 5.04 kg m^2 [DCE 06]

17. One circular ring and one circular disc both are having the same mass and radius. The ratio of their moments of inertia about the axis passing through their centres and perpendicular to their planes will be
(a) 1 : 1 (b) 2 : 1
(c) 1 : 2 (d) 4 : 1 [DCE 2K, 03]

18. What is the ratio of the moments of inertia of two rings of radii r and nr about an axis perpendicular to their plane and passing through their centres ?
(a) $1 : n^2$ (b) $1 : n$
(c) $1 : 2n$ (d) $n^2 : 1$ [DCE 98]

19. Two rings of radii R and nR made from the same wire have the ratio of moments of inertia about an axis passing through their centre equal to 1 : 8. The value of n is
(a) 2 (b) $2\sqrt{2}$
(c) 4 (d) 1/2 [DCE 05]

20. A round disc of moment of inertia I_2 about its axis perpendicular to its plane and passing through its centre is placed over another disc of moment of inertia I_1 rotating with an angular velocity ω about the same axis. The final angular velocity of the combination of discs is
(a) $\frac{I_2\omega}{I_1 + I_2}$ (b) ω
(c) $\frac{I_1\omega}{I_1 + I_2}$ (d) $\frac{(I_1 + I_2)\omega}{I_1}$ [DCE 06]

21. A sphere of radius r is rolling without sliding. What is ratio of rotational kinetic energy and total kinetic energy associated with the sphere ?
(a) $\frac{2}{7}$ (b) $\frac{2}{5}$
(c) 1 (d) $\frac{1}{2}$ [DCE 06]

22. A solid sphere of radius r is rolling with velocity v on a smooth plane. The total kinetic energy of sphere is
(a) $\frac{7}{10}mv^2$ (b) $\frac{3}{4}mv^2$
(c) $\frac{1}{2}mv^2$ (d) $\frac{1}{4}mv^2$ [DCE 05]

23. A disc is rolling on the inclined plane. What is the ratio of its rotationl KE to the total KE ?
(a) 1 : 3 (b) 3 : 1
(c) 1 : 2 (d) 2 : 1 [DCE 07, 09]

24. Two bodies have their moments of inertia I and $2I$ respectively about their axis of rotation. If their kinetic energies of rotation are equal, their angular momenta will be in the ratio
(a) 1 : 2 (b) $\sqrt{2}$: 1
(c) 2 : 1 (d) $1 : \sqrt{2}$. [DCE 01]

25. A coin of mass 4.8 kg and radius one metre is rolling on a horizontal surface without sliding with angular velocity 600 rotations/min. What is total kinetic energy of the coin ?
(a) 360 J (b) 1440 π^2J
(c) 4000 π^2J (d) 600 π^2J [DCE 06]

26. A sphere rolls down an inclined plane of inclination θ. What is the acceleration as the sphere reaches bottom ?
(a) $\frac{5}{7}g\sin\theta$ (b) $\frac{3}{5}g\sin\theta$
(c) $\frac{2}{7}g\sin\theta$ (d) $\frac{2}{5}g\sin\theta$ [IPUEE 05]

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27. An inclined plane makes an angle 30° with horizontal. A solid sphere rolling down this inclined plane has a linear acceleration of

(a) $\frac{5g}{14}$ (b) $\frac{2g}{3}$

(c) $\frac{g}{3}$ (d) $\frac{5g}{7}$ [DCE 05 ; IPUEE 04]

28. Moment of inertia of a ring of mass M and radius R about an axis passing through the centre and perpendicular to the plane is I. What is the moment of inertia about its diameter ?

(a) I (b) $I/2$

(c) $I/\sqrt{2}$ (d) $I + MR^2$ [DCE 09]

Answers and Explanations

1. (b) $x_{CM} = \frac{12\times 0 + 16\times 1.2}{12+16} = 0.7$ Å.

2. (b) Refer to the solution of Example 5 on page 7.7.

3. (c) Speeds of the top most points of both wheels will be equal and equal to that of CM of the car.

4. (c) The body will experience both linear and angular accelerations.

5. (a) The resultant of the two forces forming a couple is zero. So a couple produces a rotational motion not a linear motion.

6. (a) Angular momentum is moment of momentum.

7. (a) Of the given quantities, only angular momentum is a vector quantity.

8. (b) [Angular momentum] $= [ML^2T^{-1}]$

9. (b) Angular momentum of a system is conserved only when no external torque acts on it.

10. (c) Planetary motion obeys the conservation of angular momentum.

11. (b) As the person stretches out his hands, his moment of inertia increases. To conserve angular momentum, his angular speed decreases.

12. (d) Torque, $\vec{\tau} = \frac{d\vec{L}}{dt} = \vec{A}\times\vec{L}$

Clearly, $\frac{d\vec{L}}{dt}$ is perpendicular to $\vec{L}$. After time Δt, the new angular momentum vector $\vec{L} + \Delta\vec{L}$ has the same magnitude as the initial angular momentum vector $\vec{L}$, but has a different direction. Thus the magnitude of $\vec{L}$ remains constant while its direction changes. $d\vec{L}$ has no component in the direction of $\vec{A}$. So the component of $\vec{L}$ in the direction of $\vec{A}$ does not change with time.

Hence all given options are true.

13. (a) Analogue of mass in rotational motion is the angular momentum.

14. (a) Moment of inertia depends upon axis of rotation.

15. (c) M.I. does not depend on the angular velocity of the object but depends on all factors given in other options.

16. (b) M.I. about the centre of the plate,

$$I = \left[\frac{2}{5}\times 1\times(0.1)^2 + 1\times 1^2\right] + \left[\frac{2}{5}\times 2\times(0.1)^2 + 2\times 1^2\right] + \left[\frac{2}{5}\times 3\times(0.1)^2 + 3\times 1^2\right] + \left[\frac{2}{5}\times 4\times(0.1)^2 + 4\times 1^2\right]$$

$$= 1.004 + 2.008 + 3.012 + 4.016 = \mathbf{10.04\ kg\ m^2}.$$

17. (b) $\frac{I_{ring}}{I_{disc}} = \frac{MR^2}{\frac{1}{2}MR^2} = \mathbf{2:1}.$

18. (a) $\frac{I_1}{I_2} = \frac{Mr^2}{M(nr)^2} = \frac{1}{n^2} = \mathbf{1:n^2}.$

19. (a) As radius of second ring is n times, length and hence mass of wire used is also n times.

$$\therefore \frac{I_1}{I_2} = \frac{MR^2}{nM(nR)^2} = \frac{1}{n^3} = \frac{1}{8} \quad \therefore \quad n = 2.$$

20. (c) By conservation of angular momentum,

$$I_1\omega = (I_1 + I_2)\omega \quad \therefore \quad \omega' = \frac{I_1\omega}{I_1 + I_2}$$

21. (a) $E_{tot} = E_{tran} + E_{rot} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$$= \frac{1}{2}mv^2 + \frac{1}{2}\times\frac{2}{5}mr^2.\frac{v^2}{r^2}$$

$$= \frac{1}{2}mv^2 + \frac{2}{10}mv^2 = \frac{7}{10}mv^2$$

$$\frac{E_{rot}}{E_{tot}} = \frac{\frac{1}{5}mv^2}{\frac{7}{10}mv^2} = \frac{2}{7}.$$

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22. (*a*) Refer to the solution of the above problem.

23. (*a*) $E_{tot} = E_{tran} + E_{rot} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$$= \frac{1}{2}mv^2 + \frac{1}{2}\times\frac{1}{2}mr^2\times\frac{v^2}{r^2}$$

$$= \frac{1}{2}mv^2 + \frac{1}{4}mv^2 = \frac{3}{4}mv^2$$

$$\frac{E_{rot}}{E_{tot}} = \frac{\frac{1}{4}mv^2}{\frac{3}{4}mv^2} = \frac{1}{3} = \mathbf{1:3}.$$

24. (*d*) Rotational K.E. $= \frac{1}{2}I\omega^2$

$$\therefore \quad \frac{1}{2}I\omega^2 = \frac{1}{2}\cdot 2I\cdot\omega'^2$$

$$\omega' = \frac{\omega}{\sqrt{2}}$$

$$L = I\omega$$

$$L' = 2I\cdot\omega' = 2I\cdot\frac{\omega}{\sqrt{2}} = \sqrt{2}\,L$$

$$\therefore \quad \frac{L}{L'} = \frac{1}{\sqrt{2}} = 1:\sqrt{2}.$$

25. (*b*) $\omega = 600$ rotations/min $= \frac{600\times 2\pi}{60}$ rad s^{-1}

$$= 20\,\pi \text{ rad s}^{-1}$$

Total K.E. $= E_{tran} + E_{rot} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$$= \frac{1}{2}m\omega^2r^2 + \frac{1}{2}\times\frac{1}{2}mr^2\times\omega^2$$

$$= \frac{3}{4}mr^2\omega^2 = \frac{3}{4}\times 4.8\times(1)^2\times(20\pi)^2$$

$$= \mathbf{1440\ \pi^2\ J}.$$

26. (*a*) $a = \dfrac{g\sin\theta}{1+\dfrac{I}{MR^2}}$

For a sphere,

$$I = \frac{2}{5}MR^2 \quad \therefore \quad a = \frac{5}{7}g\sin\theta$$

27. (*a*) $a = \frac{5}{7}g\sin 30° = \frac{5g}{14}$.

28. (*b*) Using perpendicular axes theorem,

$$I_x + I_y = I_z \quad \text{or} \quad I_D + I_D = I$$

$$\therefore \quad I_D = I/2.$$

AIIMS Entrance Exam

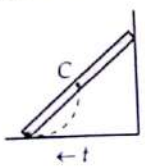
1. Two spheres of masses M and $2M$ are initially at rest at a distance R apart. Due to mutual force of attraction, they approach each other. When they are at separation $R/2$, the acceleration of their centre of mass would be

(*a*) 0 (*b*) g

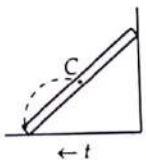
(*c*) $3g$ (*d*) $12g$ [AIIMS 2K]

2. A ladder is leaned against a smooth wall and it is allowed to slip on a frictionless floor. Which figure represents trace of its centre of mass ? [AIIMS 05]

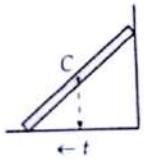
(*a*)



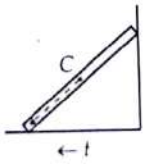
(*b*)



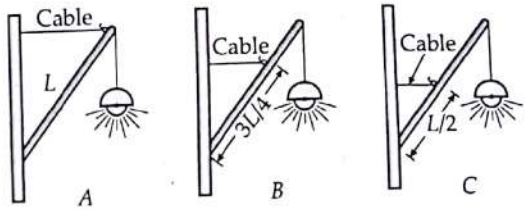
(*c*)



(*d*)



3. If a street light of mass M is suspended from the end of a uniform rod of length L in different possible patterns as shown in figure, then



(*a*) pattern A is more sturdy

(*b*) pattern B is more sturdy

(*c*) pattern C is more sturdy

(*d*) all will have same sturdiness. [AIIMS 06]

4. A solid sphere is rolling on a frictionless surface with a translational velocity v (in ms^{-1}) as shown in the

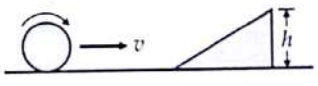


figure. If it is to climb the inclined surface, then v should be

(*a*) $\sqrt{10gh/7}$ (*b*) $\geq\sqrt{10gh/7}$

(*c*) $\sqrt{2gh}$ (*d*) $\geq\sqrt{2gh}$ [AIIMS 05]

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5. In an orbital motion, the angular momentum vector is

(*a*) along the radius vector

(*b*) parallel to the linear momentum

(*c*) in the orbital plane

(*d*) perpendicular to the orbital plane. [AIIMS 04]

6. The direction of angular velocity vector is along

(*a*) the tangent to the circular path

(*b*) the inward radius

(*c*) the outward radius

(*d*) the axis of rotation. [AIIMS 07]

7. The motion of planets in the solar system is an example of the conservation of

(*a*) mass

(*b*) linear momentum

(*c*) angular momentum

(*d*) energy [AIIMS 03]

8. The angular momentum of a moving body remains constant, if

(*a*) net external force is applied

(*b*) net pressure is applied

(*c*) net external torque is applied

(*d*) net external torque is not applied. [AIIMS 02]

9. If there is change of angular momentum from J to $4J$ in 4 s, then the torque is

(*a*) $(3/4)J$ (*b*) $1J$

(*c*) $(5/4)J$ (*d*) $(4/3)J$ [AIIMS 97]

10. A body is projected from the ground with some angle to the horizontal. What happens to the angular momentum about the initial position in this motion ?

(*a*) decreases

(*b*) increases

(*c*) remains same

(*d*) first increases and then decreases. [AIIMS 2K]

11. Radius of gyration of a body depends upon

(*a*) axis of rotation (*b*) translational motion

(*c*) shape of the body (*d*) area of the body [AIIMS 95]

12. A rod of length 1.4 m and negligible mass has two masses of 0.3 kg and 0.7 kg tied to its two ends. Find the location of the point on this rod, where the rotational energy is minimum, when the rod is rotated about the point.

(*a*) 0.98 m from 0.3 kg

(*b*) 0.98 m from 0.7 kg

(*c*) 0.7 m from 0.3 kg

(*d*) 0.7 m from 0.7 kg [AIIMS 2K]

13. The moment of inertia of a rod about an axis through its centre and perpendicular to it is $\frac{1}{12}ML^2$ (where M is the mass and L, the length of the rod). The rod is bent in the middle, so that the two halves make an angle of 60°. The moment of inertia of the bent rod about the same axis would be

(*a*) $\frac{1}{12}ML^2$ (*b*) $\frac{1}{8\sqrt{3}}ML^2$

(*c*) $\frac{1}{24}ML^2$ (*d*) $\frac{1}{48}ML^2$ [AIIMS 06]

14. If the earth is treated as a sphere of radius R and mass M having period of rotation T, then its angular momentum about its axis of rotation is

(*a*) $\frac{4\pi MR^2}{5T}$ (*b*) $\frac{2\pi MR^2}{5T}$

(*c*) $\frac{MR^2T}{2\pi}$ (*d*) $\frac{MR^2T}{4\pi}$ [AIIMS 94]

15. A horizontal platform is rotating with uniform angular velocity ω around the vertical axis passing through its centre. At some instant of time, a viscous liquid of mass m is dropped at the centre and is allowed to spread out and finally fall. The angular velocity during this period

(*a*) decreases continuously

(*b*) decreases initially and increases again

(*c*) remains unaltered

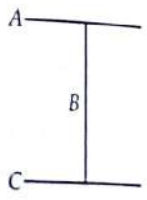
(*d*) increases continuously [AIIMS 05]

16. If a solid sphere of mass 1 kg and radius 0.1 m rolls without slipping at a uniform velocity of 1 m/s along a straight line on a horizontal floor, the kinetic energy is

(*a*) $\frac{7}{5}$ J (*b*) $\frac{2}{5}$ J

(*c*) $\frac{7}{10}$ J (*d*) 1 J [AIIMS 07]

17. In the diagram shown all three rods are of equal length and equal mass M. The system is rotated such that the rod B is the axis. What is the moment of inertia of the system ?



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(a) $\frac{ML^2}{6}$ (b) $\frac{4}{3}ML^2$

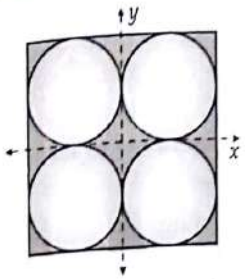
(c) $\frac{ML^2}{3}$ (d) $\frac{2}{3}ML^2$ [AIIMS 07]

18. What is moment of inertia of a cylinder of radius along its height ?

(a) mr^2 (b) $\frac{mr^2}{2}$

(c) $\frac{2mr^2}{5}$ (d) $\frac{mr^2}{5}$ [AIIMS 2010]

19. Four holes of radius R are cut from a thin square plate of side $4R$ and mass M. The moment of inertia of the remaining portion about z-axis is



(a) $\frac{\pi}{12}MR^2$ (b) $\left(\frac{4}{3}-\frac{\pi}{4}\right)MR^2$

(c) $\left(\frac{4}{3}-\frac{\pi}{6}\right)MR^2$ (d) $\left(\frac{8}{3}-\frac{10\pi}{16}\right)MR^2$ [AIIMS 2010]

ASSERTIONS AND REASONS

Directions : In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as

(a) If both assertion and reason are true and reason is the correct explanation of the assertion.

(b) If both assertion and reason are true but reason is not correct explanation of the assertion.

(c) If assertion is true, but reason is false.

(d) If both assertion and reason are false.

20. **Assertion.** A judo fighter in order to throw his opponent on to the mat tries to initially bend his opponent and then rotate him around his hip.

Reason. As the mass of the opponent is brought closer to the fighter's hip, the force required to throw the opponent is reduced. [AIIMS 06]

21. **Assertion.** For a system of particles under central force field, the total angular momentum is conserved.

Reason. The torque acting on such a system is zero. [AIIMS 05]

22. **Assertion.** There are very small sporadic changes in the period of rotation of the earth.

Reason. Shifting of large air masses in the earth's atmosphere produces a change in the moment of inertia of the earth causing its period of rotation to change. [AIIMS 04]

23. **Assertion.** The earth is slowing down and as a result the moon is coming nearer to it

Reason. The angular momentum of the earth moon system is not conserved. [AIIMS 03]

24. **Assertion.** If polar ice melts, days will be longer.

Reason. Moment of inertia decreases and thus angular velocity increases. [AIIMS 2K]

25. **Assertion.** If ice caps of the pole melts, the day length will shorten.

Reason. Ice flows towards the equator and decreases the moment of inertia of the earth and hence increases the frequency of rotation of the earth. [AIIMS 97]

26. **Assertion.** The size and shape of the rigid body remains unaffected under the effect of external forces.

Reason. The distance between two particles remains constant in a rigid body. [AIIMS 2009]

27. **Assertion.** Angular speed of a planet around the sun increases, when it is closer to the sun.

Reason. Total angular momentum of the system remains constant. [AIIMS 2009]

28. **Assertion.** A wheel moving down a perfectly frictionless inclined plane will undergo slipping (not rolling motion).

Reason. For perfect rolling motion, work done against friction is zero.

Answers and Explanations

1. (a) As there is no external force acting on the system, the centre of mass continues to remain at rest.

2. (c) The centre of mass remains at the centre of the ladder. Hence Fig. (a) represents the correct trace of CM.

3. (a) Pattern A is more sturdy. Here the moment of tension about the fulcurm required to counterbalance the moment of mg acting at CM of the rod is maximum.

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4. (*b*) Initial K.E. ≥ Final P.E.

$$E_{tran} + E_{rot} \geq mgh$$

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \geq mgh$$

$$\frac{1}{2}mv^2 + \frac{1}{2}\cdot\frac{2}{5}mr^2\frac{v^2}{r^2} \geq mgh$$

$$\frac{7}{10}mv^2 \geq mgh$$

$$v \geq \sqrt{\frac{10}{7}gh}\,.$$

5. (*d*) In an orbital motion, the direction of angular momentum vector is perpendicular to the orbital plane.

6. (*d*) If we wrap the right hand around the axis of rotation with the fingers pointing in the direction of rotation, then the thumb points in the direction of angular velocity.

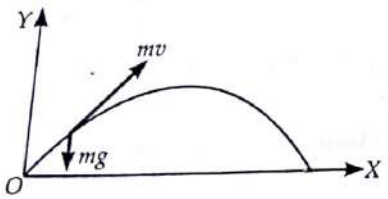
7. (*a*) The motion of planets around the sun is an example of the conservation of angular momentum.

8. (*d*) Angular momentum is conserved only in the absence of external torque.

9. (*a*) $\tau = \frac{dL}{dt}$

$$= \frac{4J - J}{4} = \frac{3}{4}J.$$

10. (*b*) About the initial position, the angular momentum of the particle is in clockwise sense. Also the torque due to its weight *mg* is in clockwise sense. Hence the angular momentum increases.



11. (*a*) Radius of gyration of a body depends on the axis rotation.

12. (*a*) Rotational energy is minimum when M.I. is minimum about the axis of rotation. If this point is at distance *x* from 0.3 kg mass, then

$$I = 0.3x^2 + 0.7(1.4 - x)^2$$

Now $\frac{dI}{dx} = 0$

or $0.6x + 1.4(1.4 - x)(-1) = 0$

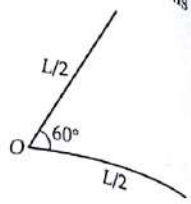
or $x = \mathbf{0.98\ m}$.

13. (*a*) M.I. of a rod about a perpendicular axis through its one end

$$= \frac{ML^2}{3}$$

Total M.I. of two halves about the axis through *O* will be

$$I = 2\left(\frac{M}{2}\right)\frac{(L/2)^2}{3} = \frac{ML^2}{12}$$



14. (*a*) $L = I\omega = \frac{2}{5}MR^2\cdot\frac{2\pi}{T}$

$$= \frac{4\pi MR^2}{2T}.$$

15. (*b*) By conservation of angular momentum,

$$L = I\omega = \text{constant}$$

As the liquid is dropped, it starts spreading out. The moment of inertia increases and angular velocity decreases. As the liquid starts falling, the moment of inertia again decreases and angular velocity increases.

16. (*c*) K.E. of rolling

$$= E_{tran} + E_{rot}$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}\cdot\frac{2}{5}mr^2\cdot\frac{v^2}{r^2}$$

$$= \frac{7}{10}mv^2 = \frac{7}{10}\times 1\times(1)^2 = \frac{7}{10}\text{ J}.$$

17. (*a*) The M.I. of *B* about its own length is zero. We need to find M.I. of *A* and *C* about a perpendicular axis through their centre.

$$\therefore \quad I = I_A + I_C = \frac{ML^2}{12} + \frac{ML^2}{12} = \frac{ML^2}{6}$$

18. (*b*) M.I. of a cylinder of radius *r* along its axis

$$= \frac{1}{2}mr^2$$

19. (*d*) Mass of each hole cut of radius *R*,

$$m = \frac{M}{16R^2}\times\pi R^2 = \frac{M\pi}{16}$$

M.I. of the remaining portion about *z*-axis

$$I = I_{\text{square}} - 4I_{\text{hole}}$$

$$= \frac{M}{12}[(4R)^2 + (4R)^2] - 4\left[\frac{mR^2}{2} + m(\sqrt{2}R)^2\right]$$

$$= \frac{M}{12}\times 32\,R^2 - 10mR^2$$

$$= \frac{8}{3}MR^2 - \frac{10}{16}MR^2\pi = \left(\frac{8}{3} - \frac{10\pi}{16}\right)MR^2.$$

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20. (*d*) Both assertion and reason are correct. rotation of the opponent makes it difficult for the person rotated to resist. By bringing him closer to the axis, the moment inertia of the opponent is reduced.

21. (*a*) Both the assertion and reason are true.

For central forces,

$$\tau = \frac{dL}{dt} = 0$$

L = Constant

22. (*a*) Both the assertion and reason are true.

23. (*d*) Both the assertion and reason are false. The earth is not slowing down. The angular momentum of the earth-moon system is conserved.

24. (*a*) As the polar ice melts, water so formed flows towards the equator. The moment of inertia of the earth increases. To conserve angular momentum, angular velocity decreases. This increases the length ($T = 2\pi/\omega$) of the day.

25. (*d*) Both the assertion and reason are false. As the polar ice melts, water so formed flows towards the equator. This increases the moment of inertia. To conserve angular momentum, frequency of rotation decreases and the day length becomes longer.

26. (*a*) Assertion is true and the reason is its correct explanation.

27. (*a*) When a planet moves closer to the sun, its moment of inertia decreses and its angular speed increases so as to conserve total angular momentum.

28. (*b*) Both assertion and reason are true but rolling motion is possible only due to friction which is a tangential force and can exert torque.

CBSE PMT Prelims and Final Exams

1. Three identical metal balls, each of the radius r are placed touching each other on a horizontal surface such that an equilateral triangle is formed when centres of three balls are joined. The centre of the mass of the system is located at

(*a*) line joining centres of any two balls
(*b*) centre of one of the balls
(*c*) horizontal surface
(*d*) point of intersection of the medians. [CBSE PMT 99]

2. The centre of mass of a system of particles does not depend on

(*a*) position of the particles
(*b*) relative distance between the particles
(*c*) masses of the particles
(*d*) forces acting on the particles [CBSE PMT 97]

3. A solid sphere of radius R is placed on smooth horizontal surface. A horizontal force F is applied at height h from the lowest point. For the maximum acceleration of centre of mass, which is correct ?

(*a*) $h = R$ (*b*) $h = 2R$
(*c*) $h = 0$
(*d*) no relation between h and R [CBSE PMT 02]

4. Consider a system of two particles having masses m_1 and m_2. If the particle of mass m_1 is pushed towards the mass centre of particles through a distance d, by what distance would the particle of mass m_2 move so as to keep the mass centre of particles at the original position ?

(*a*) $\frac{m_1}{m_1 + m_2} d$ (*b*) $\frac{m_1}{m_2} d$
(*c*) d (*d*) $\frac{m_2}{m_1} d$ [CBSE PMT 04]

5. A rod has length 3 m and its mass acting per unit length is directly proportional to distance x from one of its end, then its centre of gravity from that end will be at

(*a*) 1.5 m (*b*) 2 m
(*c*) 2.5 m (*d*) 3.0 m [CBSE PMT 02]

6. A disc is rolling, the velocity of its centre of mass is v_{cm}.

Which one will be correct ?

(*a*) the velocity of highest point is $2v_{cm}$ and point of contact is zero.
(*b*) the velocity of highest point is v_{cm} and point of contact is v_{cm}.
(*c*) the velocity of highest point is $2v_{cm}$ and point of contact is v_{cm}.
(*d*) the velocity of highest point is $2v_{cm}$ and point of contact is $2v_{cm}$. [CBSE PMT 01]

7. Identify the vector quantity among the following :

(*a*) distance (*b*) angular momentum
(*c*) heat (*d*) energy. [CBSE PMT 97]

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8. Angular momentum is
(a) axial vector (b) polar vector
(c) scalar (d) none of the above. [CBSE PMT 93]

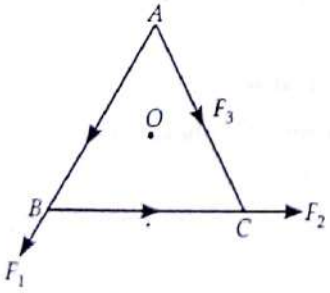
9. A couple produces
(a) linear and rotational motion
(b) no motion
(c) purely linear motion
(d) purely rotational motion. [CBSE PMT 97]

10. A wheel has angular acceleration of 3.0 rad/sec^2 and an initial angular speed of 2.00 rad/sec. In a time of 2 sec, it has rotated through an angle (in radian) of
(a) 10 (b) 12
(c) 4 (d) 6 [CBSE PMT 07]

11. Find the torque of a force $\vec{F} = -3\hat{i} + \hat{j} + 5\hat{k}$ acting at the point $\vec{r} = 7\hat{i} + 3\hat{j} + \hat{k}$
(a) $-21\hat{i} + 4\hat{j} + 4\hat{k}$ (b) $-14\hat{i} + 34\hat{j} - 16\hat{k}$
(c) $14\hat{i} - 38\hat{j} + 16\hat{k}$ (d) $4\hat{i} + 4\hat{j} + 6\hat{k}$
[CBSE PMT 97]

12. What is the torque of the force $\vec{F} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ N acting at the point $\vec{r} = 3\hat{i} + 2\hat{j} + 3\hat{k}$ m about origin ?
(a) $-6\hat{i} + 6\hat{j} - 12\hat{k}$ (b) $-17\hat{i} + 6\hat{j} + 13\hat{k}$
(c) $6\hat{i} - 6\hat{j} + 12\hat{k}$ (d) $17\hat{i} - 6\hat{j} - 13\hat{k}$
[CBSE PMT 95]

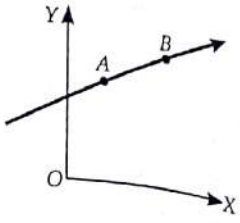
13. O is the centre of an equilateral triangle ABC. F_1, F_2 and F_3 are three forces acting along the sides AB, BC



and AC as shown in the figure. What should be the magnitude of F_3, so that the total torque about O is zero ?
(a) $(F_1 + F_2)$ (b) $2(F_1 + F_2)$
(c) $(F_1 + F_2)/2$ (d) $(F_1 - F_2)$ [CBSE PMT 98]

14. A particle of mass $m = 5$ is moving with a uniform speed $v = 3\sqrt{2}$ in the XOY plane along the line $y = x + 4$. The magnitude of the angular momentum of the particle about the origin is
(a) 60 units (b) $40\sqrt{2}$ units
(c) zero (d) 7.5 units [CBSE PMT 91]

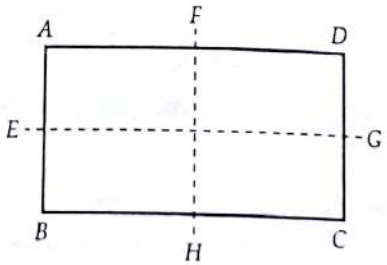
15. A particle of mass m moves in the XY plane with a velocity v along the straight line AB. If the angular momentum of the particle with respect to origin O is L_A when it is at A and L_B when it is at B, then



(a) $L_A = L_B$
(b) the relationship between L_A and L_B depends upon the slope of the line AB.
(c) $L_A < L_B$ (d) $L_A > L_B$ [CBSE PMT 07]

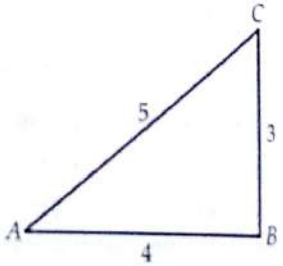
16. A circular disc is to be made by using iron and aluminimum so that it acquired maximum moment of inertia about geometrical axis. It is possible with
(a) aluminium at interior and iron surround to it
(b) iron at interior and aluminium surround to it
(c) using iron and aluminium layers in alternate order.
(d) sheet of iron is used at both external surfaces and aluminium sheet as internal layers.
[CBSE PMT 02]

17. In a rectangle $ABCD(BC = 2AB)$. The moment of inertia is minimum along axis through



(a) BC (b) BD
(c) HF (d) EG [CBSE PMT 93]

18. ABC is a triangular plate of uniform thickness. The sides are in the ratio shown in the figure. I_{AB}, I_{BC} and I_{CA} are the moments of inertia of the plate about AB, BC, and



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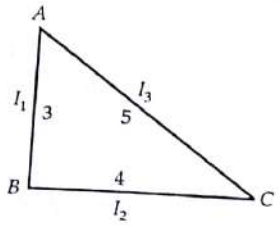
CA respectively. Which one of the following relations is correct ?

(a) $I_{AB} + I_{BC} = I_{CA}$ (b) I_{CA} is maximum

(c) $I_{AB} > I_{BC}$ (d) $I_{BC} > I_{AB}$. [CBSE PMT 95]

19. For the adjoining diagram, the correct relation between I_1, I_2 and I_3 is (I = moment of inertia)



(a) $I_1 > I_2$

(b) $I_2 > I_1$

(c) $I_3 > I_1$

(d) $I_3 > I_2$ [CBSE PMT 2K]

20. The moment of inertia of a uniform circular disc of radius R and mass M about an axis touching the disc at its diameter and normal to the disc is

(a) $\frac{1}{2}MR^2$ (b) MR^2

(c) $\frac{2}{5}MR^2$ (d) $\frac{3}{2}MR^2$ [CBSE PMT 05, 06]

21. Moment of inertia of a uniform circular disc about a diameter is I. Its moment of inertia about an axis perpendicular to its plane and passing through a point on its rim will be

(a) $5I$ (b) $3I$

(c) $6I$ (d) $4I$ [CBSE PMT 90]

22. The moment of inertia of a disc of mass M and radius R about an axis, which is tangential to the circumference of the disc and parallel to its diameter is

(a) $\frac{5}{4}MR^2$ (b) $\frac{2}{3}MR^2$

(c) $\frac{3}{2}MR^2$ (d) $\frac{4}{5}MR^2$ [CBSE PMT 99]

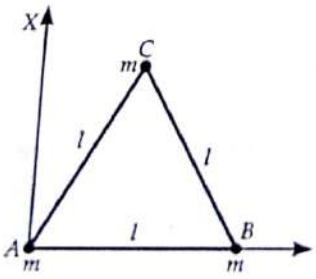
23. Three particles, each of mass m gram, are situated at the vertices of an equilateral triangle ABC of side l cm (as shown in the figure). The moment of inertia of the system about a line AX perpen- dicular to AB and in the plane of ABC, in gram-cm^2 units will be

(a) $\frac{3}{4}ml^2$

(b) $2ml^2$

(c) $\frac{5}{4}ml^2$

(d) $\frac{3}{2}ml^2$ [CBSE PMT 04]



24. A thin rod of length L and mass M is bent at its midpoint into two halves so that the angle between them is 90°. The moment of inertia of the bent rod about an axis passing through the bending point and perpendicular to the plane defined by the two halves of the rod is

(a) $\frac{ML^2}{24}$ (b) $\frac{ML^2}{12}$

(c) $\frac{ML^2}{6}$ (d) $\frac{\sqrt{2}ML^2}{24}$ [CBSE PMT 08]

25. A ring of mass m and radius r rotates about an axis passing through its centre and perpendicular to its plane with angular velocity ω. Its kinetic energy is

(a) $\frac{1}{2}mr^2\omega^2$ (b) $mr\omega^2$

(c) $mr^2\omega^2$ (d) $\frac{1}{2}mr\omega^2$ [CBSE PMT 88]

26. A flywheel rotating about fixed axis has a kinetic energy of 360 joule when its angular speed is 30 radian/sec. The moment of inertia of the wheel about the axis of rotation is

(a) 0.6 kgm^2 (b) 0.15 kgm^2

(c) 0.8 kgm^2 (d) 0.75 kgm^2 [CBSE PMT 90]

27. Two bodies have their moments of inertia I and $2I$ respectively about their axis of rotation. If their kinetic energies of rotation are equal, their angular velocity will be in the ratio

(a) 2 : 1 (b) 1 : 2

(c) $\sqrt{2}$: 1 (d) 1 : $\sqrt{2}$ [CBSE PMT 05]

28. The ratio of the radii of gyration of a circular disc about a tangential axis in the plane of the disc and of a circular ring of the same radius about a tangential axis in the plane of the ring is

(a) 2 : 3 (b) 2 : 1

(c) $\sqrt{5}$: $\sqrt{6}$ (d) 1 : $\sqrt{2}$ [CBSE PMT 04]

29. The ratio of the radii of gyration of a circular disc to that of circular ring, each of same mass and same radius about their axes is

(a) $\sqrt{3}$: $\sqrt{2}$ (b) 1 : $\sqrt{2}$

(c) $\sqrt{2}$: 1 (d) $\sqrt{2}$: $\sqrt{3}$ [CBSE PMT 08]

30. A wheel having moment of inertia 2 kg m^2 about its vertical axis, rotates at the rate of 60 rpm about this axis. The torque which can stop the wheel's rotation in one minute would be

(a) $\frac{2\pi}{15}$ N m (b) $\frac{\pi}{12}$ N m

(c) $\frac{\pi}{15}$ N m (d) $\frac{\pi}{18}$ N m [CBSE PMT 04]

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31. The moment of inertia of a body about a given axis is 1.2 kg m^2. Initially, the body is at rest. In order to produce a rotational kinetic energy of 1500 joule, an angular acceleration of 25 radian/sec^2 must be applied about that axis for a duration of

(a) 4 s (b) 2 s
(c) 8 s (d) 10 s [CBSE PMT 90]

32. A disc is rotating with angular speed ω. If a child sits on it, what is conserved

(a) linear momentum (b) angular momentum
(c) kinetic energy (d) potential energy
[CBSE PMT 02]

33. A round disc of moment of inertia I_2 about its axis perpendicular to its plane and passing through its centre is placed over another disc of moment of inertia I_1 rotating with an angular velocity ω about the same axis. The final angular velocity of the combination of discs is

(a) $\frac{I_2\omega}{I_1+I_2}$ (b) ω
(c) $\frac{I_1\omega}{I_1+I_2}$ (d) $\frac{(I_1+I_2)\omega}{I_1}$ [CBSE PMT 04]

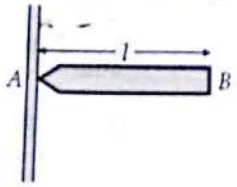
34. A thin circular ring of mass M and radius r is rotating about its axis with a constant angular velocity ω. Two objects each of mass m are attached gently to the opposite ends of a diameter of the ring. The ring will now rotate with an angular velocity

(a) $\frac{\omega(M+2m)}{M}$ (b) $\frac{\omega M}{M+2m}$
(c) $\frac{\omega(M-2m)}{M+2m}$ (d) $\frac{\omega M}{M+m}$ [CBSE PMT 98, 09]

35. A thin circular ring of mass M and radius r is rotating about its axis with a constant angular velocity ω. Four objects each of mass m, are kept gently to the opposite ends of two perpendicular diameters of the ring. The angular velocity of the ring will be

(a) $\frac{M\omega}{4m}$ (b) $\frac{M\omega}{M+4m}$
(c) $\frac{(M+4m)\omega}{M}$ (d) $\frac{(M-4m)\omega}{M+4m}$ [CBSE PMT 03]

36. A uniform rod AB of length l and mass m is free to rotate in a vertical plane about point A. The rod is released from rest in the horizontal position. Given that the moment of inertia of the rod about A is $ml^2/3$, the initial angular acceleration of the rod will be



(a) $\frac{mgl}{2}$ (b) $\frac{3}{2}gl$
(c) $\frac{3g}{2l}$ (d) $\frac{2g}{3l}$ [CBSE PMT 06, 07]

37. If a sphere is rolling, the ratio of translational energy to total kinetic energy is given by

(a) 7 : 10 (b) 2 : 5
(c) 10 : 7 (d) 5 : 7 [CBSE PMT 94]

38. A solid spherical ball rolls on a table. Ratio of its rotational kinetic energy to total kinetic energy is

(a) $\frac{1}{2}$ (b) $\frac{1}{6}$
(c) $\frac{7}{10}$ (d) $\frac{2}{7}$ [CBSE PMT 92]

39. The speed of a homogenous solid sphere after rolling down an inclined plane of vertical height h from rest without sliding is

(a) $\sqrt{\frac{10}{7}gh}$ (b) $\sqrt{gh}$
(c) $\sqrt{\frac{6}{5}gh}$ (d) $\sqrt{\frac{4}{3}gh}$ [CBSE PMT 92]

40. For a hollow cylinder and a solid cylinder rolling without slipping on an inclined plane, then which of these reaches earlier

(a) solid cylinder (b) hollow cylinder
(c) both simultaneously
(d) can't say anything. [CBSE PMT 2K]

41. A solid sphere, disc and solid cylinder all of the same mass and made of the same material are allowed to roll down (from rest) on the inclined plane, then

(a) solid sphere reaches the bottom first
(b) solid sphere reaches the bottom last
(c) disc will reach the bottom first
(d) all reach the bottom at the same time.
[CBSE PMT 93]

42. A solid homogenous sphere of mass M and radius R is moving on a rough horizontal surface, partly rolling and partly sliding. During this kind of motion of this sphere

(a) total kinetic energy is conserved
(b) the angular momentum of the sphere about the point of contact with the plane is conserved

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(c) only the rotational kinetic energy about the centre of mass is conserved
(d) angular momentum about the centre of mass is conserved. [CBSE PMT 88]

43. A drum of radius R and mass M, rolls down without slipping along an inclined plane of angle θ. The frictional force
(a) dissipates energy as heat
(b) decreases the rotational motion
(c) decreases the rotational and translational motion
(d) converts translational energy to rotational energy. [CBSE PMT 05 ; Delhi 06]

44. A solid cylinder of mass M and radius R rolls without slipping down an inclined plane of length L and height h. What is the speed of its centre of mass when the cylinder reaches its bottom ?

(a) $\sqrt{2gh}$ (b) $\sqrt{\frac{3}{4}gh}$
(c) $\sqrt{\frac{4}{3}gh}$ (d) $\sqrt{4gh}$ [CBSE PMT 89, 03]

45. A ball rolls without slipping. The radius of gyration of the ball about an axis passing through its centre of mass is k. If radius of the ball be R, then the fraction of total energy associated with its rotational energy will be

(a) $\frac{k^2+R^2}{R^2}$ (b) $\frac{k^2}{R^2}$
(c) $\frac{k^2}{k^2+R^2}$ (d) $\frac{R^2}{k^2+R^2}$ [CBSE PMT 03]

46. If point P is the point of contact of a wheel on ground which rolls on ground without slipping, then value of displacement of point P when wheel completes half of rotation (if radius of wheel is 1 m) is

(a) 2 m (b) $\sqrt{\pi^2+4}$ m
(c) π m (d) $\sqrt{\pi^2+2}$ m. [CBSE PMT 02]

47. Two bodies of masses 1 kg and 3 kg have position vectors $\hat{i}+2\hat{j}+\hat{k}$ and $-3\hat{i}-2\hat{j}+\hat{k}$ respectively. The centre of mass of this system has a position vector

(a) $-2\hat{i}-\hat{j}+\hat{k}$ (b) $2\hat{i}-\hat{j}-2\hat{k}$
(c) $-\hat{i}+\hat{j}+\hat{k}$ (d) $-2\hat{i}+2\hat{k}$.

[CBSE PMT 09]

48. Four thin rods of mass M and length l, form a square frame. Moment of inertia of this frame about an axis through the centre of the square and perpendicular to its plane is

(a) $\frac{2}{3}Ml^2$ (b) $\frac{13}{3}Ml^2$
(c) $\frac{1}{3}Ml^2$ (d) $\frac{4}{3}Ml^2$ [CBSE PMT 09]

49. Two particles which are initially at rest, move towards each other under the action of their internal attraction. If their speeds are v and $2v$ at any instant, then the speed of centre of mass of the system will be

(a) $2v$ (b) zero
(c) $1.5v$ (d) v [CBSE Pre 2010]

50. The instantaneous angular position of a point on a rotating wheel is given by the equation $\theta(t)=2t^3-6t^2$

The torque on the wheel becomes zero at

(a) $t=1$ s (b) $t=0.5$ s
(c) $t=0.25$ s (d) $t=2$ s [CBSE Pre 2011]

51. The moment of inertia of a thin uniform rod of mass M and length L about an axis passing through its midpoint and perpendicular to its length is I_o. Its moment of inertia about an axis passing through one of its ends and perpendicular to its length is

(a) $I_o+ML^2/2$ (b) $I_o+ML^2/4$
(c) I_o+2ML^2 (d) I_o+ML^2 [CBSE Pre 2011]

52. A circular disk of moment of inertia I_t is rotating in a horizontal plane, about its symmetry axis, with a constant angular speed ω_i. Another disk of moment of inertia I_b is dropped coaxially onto the rotating disk. Initially the second disk has zero angular speed. Eventually both the disks rotate with a constant angular speed ω_f. The energy lost by the initially rotating disk due to friction is

(a) $\frac{1}{2}\frac{I_b^2}{(I_t+I_b)}\omega_i^2$
(b) $\frac{1}{2}\frac{I_t^2}{(I_t+I_b)}\omega_i^2$
(c) $\frac{I_b-I_t}{(I_t+I_b)}\omega_i^2$
(d) $\frac{1}{2}\frac{I_bI_t}{(I_t+I_b)}\omega_i^2$ [CBSE Pre 2010]

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53. A planet moving along an elliptical orbit is closest to the sun at a distance r_1 and farthest away at a distance of r_2. If v_1 and v_2 are the linear velocities at these points respectively, then the ratio $\frac{v_1}{v_2}$ is

(a) $\left(\frac{r_1}{r_2}\right)^2$ (b) $\frac{r_2}{r_1}$

(c) $\left(\frac{r_2}{r_1}\right)^2$ (d) $\frac{r_1}{r_2}$

[CBSE Pre 2011]

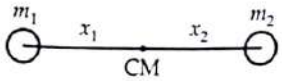
Answers and Explanations

1. (*d*) The centre of mass of the system lies at the point of intersection of the medians *i.e.*, at the centroid of the equilateral triangle.

2. (*d*) The centre of mass of a system of particles does not depend on the forces acting on the particles.

3. (*d*) The smooth horizontal surface does not provide any friction. The sphere does not roll. Hence the acceleration of CM of the sphere is independent of the position of the applied force. There is no relation between h and R.

4. (*b*)



$$x_{CM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} \quad ...(i)$$

After moving m_1 through distance d towards CM and to keep the position of CM unchanged,

$$x_{CM} = \frac{m_1(x_1 - d) + m_2(x_2 + d')}{m_1 + m_2} \quad ...(ii)$$

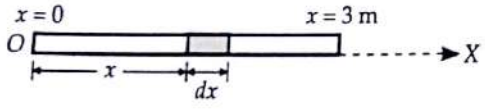
From (*i*) and (*ii*), we get

$$\frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{m_1(x_1 - d) + m_2(x_2 + d')}{m_1 + m_2}$$

or $$-m_1 + d + m_2d' = 0$$

or $$d' = \frac{m_1}{m_2} \cdot d$$

5. (*b*) Suppose the rod is placed along x-axis. Consider a small element of thickness dx at distance x from its left end.



As the mass acting per unit length is directly proportional to distance x from one end, mass of the small element is

$$dm = kx\,dx$$

The position of CM of the rod will be

$$x_{CM} = \frac{\int_0^3 x\,dm}{\int_0^3 dm} = \frac{\int_0^3 kx^2\,dx}{\int_0^3 kx\,dx}$$

$$= \frac{\left[\frac{x^3}{3}\right]_0^3}{\left[\frac{x^2}{2}\right]_0^3} = \frac{27}{3} \times \frac{2}{9} = 2 \text{ m}.$$

6. (*a*) Velocity at the highest point is $2v_{CM}$ and it is zero at the point of contact. For details, refer to solution of Problem 26 on page 7.62.

7. (*b*) Only angular momentum is a vector quantity. Other given quantities are scalars.

8. (*a*) Angular momentum is an axial vector.

9. (*d*) A couple produces purely rotational motion.

10. (*a*) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$$= 2 \times 2 + \frac{1}{2} \times 3.0 \times 2^2$$

$$= 4 + 6 = \mathbf{10\ rad}.$$

11. (*c*) $\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 3 & 1 \\ -3 & 1 & 5 \end{vmatrix}$

$$= 14\hat{i} - 38\hat{j} + 16\hat{k}.$$

12. (*d*) $\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 3 \\ 2 & -3 & 4 \end{vmatrix}$

$$= 17\hat{i} - 6\hat{j} - 13\hat{k}$$

13. (*a*) Let x be the distance of each side of the ΔABC from the centre O. Then total torque above O will be

$$F_1 \times x + F_2 \times x - F_3 \times x = 0$$

or $$F_1 + F_2 = F_3$$

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14. (*a*) Perpendicular distance of the line $y = x + 4$

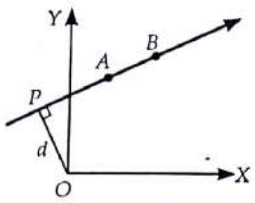
$x - y + 4 = 0$ from the origin (0, 0) is

$$d = \frac{|0 - 0 + 4|}{\sqrt{1^2 + (-1)^2}} = \frac{4}{\sqrt{2}}.$$

Angular momentum of the particle about the origin will be

$$L = pd = mv \times d$$
$$= 5 \times 3\sqrt{2} \times \frac{4}{\sqrt{2}} = \mathbf{60\ units.}$$

15. (*a*) Whether the particle is at A or B, the perpendicular distance, $OP = d$ is same from the origin O in both cases.



Hence $L_A = L_B$.

16. (*d*) Density of iron is more than that of aluminium. For moment of inertia to be maximum, heavier mass should lie farther from the axis of rotation. Hence aluminium should be at the interior and iron surrounding it.

17. (*d*) The moment of inertia will be minimum about the axis through EG because the mass distribution of the rectangle is closest to EG.

18. (*d*) The CM of a triangular lamina lies at centroid. But the distance of a side of a triangle from its centroid is inversely proportional to the length of that side, so

$$r_{BC} > r_{AB} > r_{CA}$$
$$\therefore \quad I_{BC} > I_{AB} > I_{CA}$$

Hence option (*d*) is correct.

19. (*b*) As explained in the above problem,

$$r_{BC} > r_{AB} > r_{CA}$$
$$\therefore \quad I_2 > I_1 > I_3$$

Only option (*b*) is correct.

20. (*d*) By using theorem of parallel axes, the moment of inertia of a disc about a tangent perpendicular to its plane,

$$I'_T = I_C + MR^2$$
$$= \frac{1}{2}MR^2 + MR^2$$
$$= \frac{3}{2}MR^2.$$

21. (*c*) M.I. of a disc about its diameter = I

By theorem of perpendicular axes,

M.I. of the disc about its central axis

$$= I + I = 2I = \frac{1}{2}MR^2$$

Now, by theorem of parallel axes, M.I. of the disc about an axis perpendicular to its plane and tangential its rim

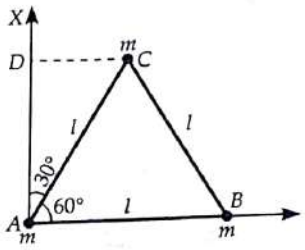
$$= I_C + MR^2$$
$$= 2I + 4I = 6I.$$

22. (*a*) M.I. of a disc about its diameter $= \frac{1}{4}MR^2$.

By theorem of parallel axes, M.I. of the disc and an axis tangent to disc and parallel to its diameter

$$= \frac{1}{4}MR^2 + MR^2 = \frac{5}{4}MR^2.$$

23. (*c*)

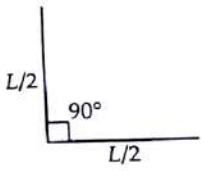


M.I. about the axis AX

$$= m(0)^2 + m(AB)^2 + m(CD)^2$$
$$= 0 + ml^2 + m(l\sin 30°)^2$$
$$= ml^2 + ml^2 \cdot \frac{1}{4} = \frac{5}{4}ml^2.$$

24. (*b*) $I = I_1 + I_2$

$$= \frac{M}{2}\frac{(L/2)^2}{3} + \frac{M}{2}\frac{(L/2)^2}{3}$$
$$= \frac{ML^2}{12}.$$



25. (*a*) For a ring, $I = mr^2$

$$\text{Rotational K.E.} = \frac{1}{2}I\omega^2$$
$$= \frac{1}{2}mr^2\omega^2.$$

26. (*c*) $\text{Rotational K.E.} = \frac{1}{2}I\omega^2$

$$360 = \frac{1}{2} I \times (30)^2$$
$$\therefore \quad I = \frac{720}{900} = \mathbf{0.8\ kg\ m^2}.$$

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27. (*c*) $\omega = \sqrt{\dfrac{2 \times \text{Rotational K.E.}}{I}}$

For same rotational K.E.,

$$\omega \propto \frac{1}{\sqrt{I}}$$

$$\frac{\omega_1}{\omega_2} = \sqrt{\frac{I_2}{I_1}} = \sqrt{\frac{2I}{I}} = \sqrt{2} : 1.$$

28. (*c*) M.I. of a circular disc about a tangential axis in the plane of the disc,

$$I_1 = Mk_1^2 = \frac{5}{4}MR^2$$

$$\therefore \quad k_1 = \sqrt{\frac{5}{4}}R$$

M.I. of a circular ring of same radius about a tangential axis in the plane of the ring,

$$I_2 = Mk_2^2 = \frac{3}{2}MR^2$$

$$\therefore \quad k_2 = \sqrt{\frac{3}{2}}R$$

$$\frac{k_1}{k_2} = \frac{\sqrt{\frac{5}{4}}R}{\sqrt{\frac{3}{2}}R} = \sqrt{\frac{5}{4} \times \frac{2}{3}} = \sqrt{5} : \sqrt{6}.$$

29. (*b*) $\dfrac{I_1}{I_2} = \dfrac{Mk_1^2}{Mk_2^2} = \dfrac{\frac{1}{2}MR^2}{MR^2} = \dfrac{1}{2}.$

$$\therefore \quad \frac{k_1}{k_2} = \frac{1}{\sqrt{2}} = \mathbf{1:\sqrt{2}}$$

30. (*c*) $I = 2 \text{ kg m}^2$, $\nu_0 = 60 \text{ rpm} = 1 \text{ rps}$

$$\omega_0 = 2\pi\nu_0 = 2\pi \text{ rad s}^{-1}, \omega = 0$$

$$t = 1 \text{ min} = 60 \text{ s}$$

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{0 - 2\pi}{60} = -\frac{\pi}{30} \text{ rad s}^{-2}$$

$$\tau = I\alpha = 2 \times \frac{\pi}{30} = \frac{\pi}{15} \textbf{ Nm}.$$

31. (*b*) Rotational K.E. $= \dfrac{1}{2}I\omega^2$

$$1500 = \frac{1}{2} \times 1.2 \times \omega^2$$

$$\omega = \sqrt{\frac{3000}{1.2}} = 50 \text{ rad s}^{-1}$$

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{50 - 0}{25} = \mathbf{2 \text{ s}}.$$

32. (*b*) When child sits on the rotating disc, his weight acts downwards and does not produce any torque. Hence the *angular momentum* of the system is conserved.

33. (*c*) By conservation of angular momentum,

$$I_1\omega = (I_1 + I_2)\omega_1$$

$$\omega_1 = \frac{I_1\omega}{I_1 + I_2}$$

34. (*b*) By conservation of angular momentum,

$$I'\omega' = I\omega$$

$$(M + 2m)r^2\omega' = Mr^2\omega$$

or $$\omega' = \frac{Mr^2\omega}{(M+2m)r^2} = \frac{\omega M}{M+2m}.$$

35. (*b*) By conservation of angular momentum,

$$(M + 4m)r^2\omega' = Mr^2\omega$$

or $$\omega' = \frac{M\omega}{M+4m}$$

36. (*c*) M.I. of the rod about point A,

$$I = \frac{Ml^2}{3}$$

Torque about point A due to the weight mg acting on the C.G. of the rod,

$$\tau = mg \times \frac{l}{2}$$

or $$I\alpha = mg \times \frac{l}{2}$$

or $$\frac{ml^2}{3} \cdot \alpha = mg \times \frac{l}{2}$$

or $$\alpha = \frac{3g}{2l}.$$

37. (*d*) $E_{tran} = \dfrac{1}{2}mv^2$

$$E_{rot} = \frac{1}{2}I\omega^2$$

$$= \frac{1}{2} \times \frac{2}{5}mr^2 \times \omega^2 = \frac{1}{5}mv^2$$

$$E_{tot} = E_{tran} + E_{rot}$$

$$= \left(\frac{1}{2} + \frac{1}{5}\right)mv^2 = \frac{7}{10}mv^2$$

$$\frac{E_{tran}}{E_{total}} = \frac{\frac{1}{2}mv^2}{\frac{7}{10}mv^2} = \frac{5}{7} = 5 : 7.$$

38. (*d*) From the above problem, we have

$$\frac{E_{rot}}{E_{tot}} = \frac{\frac{1}{5}mv^2}{\frac{7}{10}mv^2}$$

$$= \frac{2}{7}.$$

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39. (*a*) P.E. lost by the sphere

$$= E_{rot} + E_{tran} = E_{tot}$$

$$mgh = \frac{7}{10}mv^2$$

$$\therefore \quad v = \sqrt{\frac{10gh}{7}}$$

or

40. (*a*) Acceleration of a cylinder rolling down an inclined plane without slipping,

$$a = \frac{g\sin\theta}{1+\frac{I}{MR^2}}$$

For a solid cylinder,

$$I = \frac{1}{2}MR^2 \quad \therefore \quad a = \frac{2}{3}g\sin\theta$$

For a hollow cylinder,

$$I = MR^2 \quad \therefore \quad a' = \frac{1}{2}g\sin\theta$$

As solid cylinder has a greater acceleration than hollow cylinder, so solid cylinder reaches the bottom earlier.

41. (*a*) $$a = \frac{g\sin\theta}{1+\frac{I}{MR^2}}$$

For a solid sphere,

$$I = \frac{2}{5}MR^2 \quad \therefore \quad a = \frac{5}{7}g\sin\theta$$

For a disc,

$$I = \frac{1}{2}MR^2 \quad \therefore \quad a = \frac{2}{3}g\sin\theta$$

For a solid cylinder,

$$I = \frac{1}{2}MR^2 \quad \therefore \quad a = \frac{2}{3}g\sin\theta$$

As solid sphere has maximum acceleration, it reaches the bottom first. Both disc and cylinder reach together later.

42. (*b*) Angular momentum about the point of contact also includes the angular momentum about the centre. Because of friction, linear momentum is not conserved.

43. (*d*) Frictional force converts translational energy into kinetic energy.

44. (*c*) P.E. lost $= E_{tran} + E_{rot}$

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}Mv^2 + \frac{1}{2}\cdot\frac{1}{2}MR^2\cdot\left(\frac{v}{R}\right)^2$$

$$gh = \frac{1}{2}v^2 + \frac{1}{4}v^2 = \frac{3}{4}v^2$$

$$v = \sqrt{\frac{4}{3}gh}\,.$$

45. (*c*) $$E_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2}Mk^2\left(\frac{v}{R}\right)^2$$

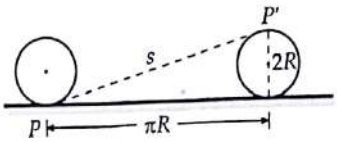
$$E_{tot} = E_{tran} + E_{rot}$$

$$= \frac{1}{2}Mv^2 + \frac{1}{2}Mk^2\left(\frac{v}{R}\right)^2$$

$$= \frac{1}{2}Mv^2\left(1+\frac{k^2}{R^2}\right)$$

$$\frac{E_{rot}}{E_{tot}} = \frac{\frac{1}{2}Mv^2\left(\frac{k^2}{R^2}\right)}{\frac{1}{2}Mv^2\left(1+\frac{k^2}{R^2}\right)} = \frac{k^2}{k^2+R^2}\,.$$

46. (*b*) When the wheel completes half of rotation without slipping, the point of contact P takes the new position P' as shown in the figure.



Horizontal displacement covered, $x = \pi R$

Vertical displacement covered, $y = 2R$

Net displacement of point P',

$$s = \sqrt{x^2+y^2} = \sqrt{\pi^2R^2+4R^2}$$

But $R = 1$ m,

$$s = \sqrt{\pi^2+4}\ \text{m}\,.$$

47. (*a*) $$\vec{R}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2}$$

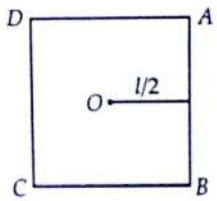
$$= \frac{1(\hat{i}+2\hat{j}+\hat{k})+3(-3\hat{i}-2\hat{j}+\hat{k})}{1+3}$$

$$= \frac{-8\hat{i}-4\hat{j}+4\hat{k}}{4} = -2\hat{i}-\hat{j}+\hat{k}\,.$$

48. (*d*) We use parallel axes theorem,

$$I_z = 4I_{AB} = 4\times\left[\frac{Ml^2}{12} + M\left(\frac{l}{2}\right)^2\right]$$

$$= \frac{4Ml^2}{3}\,.$$



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49. (*b*) No external force is acting on the centre of mass of system. It remains at rest. The speed of the *CM* is zero.

50. (*a*) $\theta(t) = 2t^3 - 6t^2$

$$\omega = \frac{d\theta}{dt} = 6t^2 - 12t$$

$$\alpha = \frac{d\omega}{dt} = 12t - 12 = 0$$

$\Rightarrow \quad t = 1\ \text{s}$

When angular acceleration is zero, torque on the wheel is also zero.

51. (*b*) By parallel axes theorem,

$$I_{end} = I_{CM} + Md^2 = I_o + M(L/2)^2$$
$$= I_o + ML^2/4$$

52. (*d*) By conservation of angular momentum,

$$I_t\omega_i = (I_t + I_b)\omega_f \quad \Rightarrow \quad \omega_f = \frac{I_t}{I_t + I_b}\omega_i$$

$\therefore$ Loss of rotational K.E.,

$$\Delta K = \frac{1}{2}I_t\omega_i^2 - \frac{1}{2}(I_t + I_b)\frac{I_t^2}{(I_t + I_b)^2}\omega_i^2$$
$$= \frac{1}{2}\omega_i^2\frac{I_t}{(I_t + I_b)}(I_t + I_b - I_t)$$
$$= \frac{1}{2}\omega_i^2\frac{I_t I_b}{I_t + I_b}$$

53. (*b*) By conservation of angular momentum,

$$L_1 = L_2$$

or $$mv_1r_1 = mv_2r_2$$

$\therefore$ $$\frac{v_1}{v_2} = \frac{r_2}{r_1}$$

Delhi PMT and VMMC Entrance Exam

(Similar Questions)

1. Two bodies of masses 2 kg and 4 kg are moving with velocities 2 m/s and 10 m/s respectively. What is velocity of their centre of mass ?

(*a*) 5.3 m/s (*b*) 7.3 m/s

(*c*) 6.4 m/s (*d*) 8.1 m/s [DPMT 93]

2. Two racing cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2 respectively. Their speeds are such that each makes a complete circle in the same time *t*. The ratio of the angular speeds of the first to the second car will be

(*a*) 1 : 1 (*b*) $r_1 : r_2$

(*c*) $m_1 : m_2$ (*d*) $m_1m_2 : r_1r_2$ [DPMT 94]

3. Two wheels having radii in the ratio 1 : 3 are connected by a common belt. If the smaller wheel is accelerated from rest at a rate 1.5 rads^{-2} for 10 s, find the velocity of bigger wheel.

(*a*) 5 rads^{-1} (*b*) 15 rads^{-1}

(*c*) 45 rads^{-1} (*d*) none of these.

[DPMT 06]

4. Angular acceleration α of a body is given by the relation $\alpha = 4at^3 - 3bt^2$. If initial angular velocity of the body is ω_0, then its velocity at time *t* will be

(*a*) $\omega_0 + at^4 - bt^3$ (*b*) $\omega_0 + 4at^4 - 4bt^3$

(*c*) $\omega_0 + 12at^2 - 6bt$ (*d*) $\omega_0 - at^4 + bt^3$

[DPMT 05]

5. If $\vec{F}$ is force and $\vec{r}$ is the radius vector, then torque is given by

(*a*) $\vec{r} \times \vec{F}$ (*b*) $\vec{r} \cdot \vec{F}$

(*c*) $|\vec{r}|\,|\vec{F}|$ (*d*) none of these. [DPMT 93]

6. Find the torque of a force $\vec{F} = -3\hat{i} + \hat{j} + 5\hat{k}$ acting at the point $\vec{r} = 7\hat{i} + 3\hat{j} + \hat{k}$.

(*a*) $14\hat{i} - 38\hat{j} + 16\hat{k}$ (*b*) $4\hat{i} + 4\hat{j} + 6\hat{k}$

(*c*) $-14\hat{i} + 38\hat{j} - 16\hat{k}$ (*d*) $-21\hat{i} + 3\hat{j} + 5\hat{k}$ [DPMT 04]

7. Which one is a vector quantity ?

(*a*) energy (*b*) torque

(*c*) both of these (*d*) none of these. [DPMT 2K]

8. A car is moving at a speed of 72 km/h. The diameter of its wheels is 0.5 m. If the wheels are stopped in 20 rotations by applying brakes, then angular retardation produced by the brakes is

(*a*) –25.5 rad/s^2 (*b*) –33.5 rad/s^2

(*c*) – 29.5 rad/s^2 (*d*) – 45.5 rad/s^2 [DPMT 92]

9. An automobile engine develops 100 kW when rotating at a speed of 1800 rev/min. What torque does it deliver ?

(*a*) 350 Nm (*b*) 531 Nm

(*c*) 440 Nm (*d*) 628 Nm [DPMT 98]

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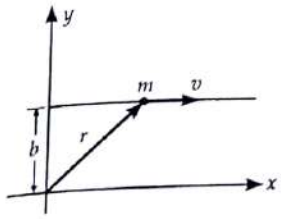
10. Joule second is the unit of
(a) momentum (b) angular momentum
(c) work (d) pressure [DPMT 05]

11. Under a constant torque, the angular momentum of a body changes from A to $4A$ in 4 sec. The torque on the body will be
(a) $\frac{3}{4}A$ (b) $\frac{1}{4}A$
(c) $\frac{4}{3}A$ (d) $4A$ [VMMC 02 ; DPMT 93]

12. A particle of mass m is moving with a constant velocity v parallel to x-axis in an x-y plane as shown in figure. Calculate angular momentum with respect to origin at any instant.



(a) $-mvb\hat{k}$ (b) zero
(c) $\frac{mvb}{2}\hat{k}$ (d) $mvb\cos\theta\hat{k}$ [DPMT 06]

13. Moment of inertia of a body depends upon
(a) axis of rotation
(b) earth's gravitational constant
(c) relativistic effect of motion of earth around sun
(d) none of these. [DPMT 2K]

14. Moment of inertia of a thin circular disc of mass M and radius R about any diameter is
(a) $\frac{MR^2}{4}$ (b) MR^2
(c) $\frac{MR^2}{2}$ (d) $2MR^2$ [DPMT 94]

15. Moment of inertia of ring about its diameter is I. The moment of inertia of the same ring about the axis perpendicular to its plane and passing through centre is
(a) $I/2$ (b) $2I$
(c) $I/4$ (d) $4I$ [DPMT 94]

16. The ratio of radii of gyration of a circular disc and a circular ring of the same radii and same mass about a tangential axis in the plane is
(a) 1 : 2 (b) $\sqrt{5} : \sqrt{6}$
(c) 2 : 3 (d) 2 : 1 [VMMC 06]

17. The radius of gyration of a disc of mass 100 g and radius 5 cm about an axis passing through its centre of gravity and perpendicular to the plane is
(a) 1.52 cm (b) 3.54 cm
(c) 2.51 cm (d) 6.54 cm [DPMT 96]

18. A flywheel is attached to an engine to
(a) increase its speed (b) decrease its speed
(c) help in overcoming the dead point
(d) decrease its energy [DPMT 92]

19. The angular momentum of a system of particles is not conserved
(a) when a net external force acts upon the system
(b) when a net external torque is acting upon the system.
(c) when a net external impulse is acting upon the system
(d) none of these. [DPMT 04]

20. A diver in a swimming pool bends his head before diving. It
(a) increases his linear velocity
(b) decreases his angular velocity
(c) increases his moment of inertia
(d) decreases his moment of inertia [DPMT 04]

21. A disc having mass M and radius R is rotating with angular velocity ω, another disc of mass $2M$ and radius $R/2$ is placed coaxially on the first disc gently. The angular velocity of system will now be
(a) $\frac{4\omega}{5}$ (b) $\frac{2\omega}{5}$
(c) $\frac{3\omega}{2}$ (d) $\frac{2\omega}{3}$ [VMMC 02]

22. A thin circular ring of mass M and radius r is rotating about its axis with a constant angular velocity ω. If two objects of mass m are attached gently to opposite ends of a diameter of ring, ring will now rotate with an angular velocity given by
(a) $\frac{2\omega M}{(M-2m)}$ (b) $\frac{(M-2m)}{M}$
(c) $\frac{\omega M}{(M+2m)}$ (d) $\frac{2\omega M}{(M-2m)}$ [VMMC 04]

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23. A circular disc of mass M and radius R is rotating with an angular velocity ω about an axis passing through its centre and perpendicular to the plane of the disc. A small point like part of mass m datches from the rim of the disc and continues to move with same angular speed. The angular velocity of remaining disc just after detaching will become

(a) $\left(\frac{M-2m}{M+m}\right)\omega$ (b) $\left(\frac{M+2m}{M+m}\right)\omega$

(c) $\left(\frac{M-2m}{M-m}\right)\omega$ (d) $\left(\frac{M+2m}{M-m}\right)\omega$

[VMMC 07]

24. Moment of inertia of a body is 1 kg m^2. If the body makes 2 revolutions per second, when its angular momentum is

(a) 2π Js (b) 4π Js

(c) $\pi/2$ Js (d) π Js [DPMT 05]

25. A constant torque of 31.4 N m is exerted on a pivoted wheel. If the angular acceleration of wheel is 4π rad/s^2, then the moment of inertia of the wheel is

(a) 2.5 kg m^2 (b) 4.5 kg m^2

(c) 3.5 kg m^2 (d) 5.5 kg m^2 [DPMT 96]

26. A particle performs uniform circular motion with an angular momentum L. If the frequency of particle motion is doubled and its KE is halved, the angular momentum becomes

(a) $2L$ (b) $4L$

(c) $\frac{L}{2}$ (d) $\frac{L}{4}$ [VMMC 02, 06]

27. Angular momentum L of body with moment of inertia I and angular velocity ω rad/sec is equal to

(a) $\frac{I}{\omega}$ (b) $I\omega^2$

(c) $I\omega$ (d) none of these.

[DPMT 98, 03]

28. A circular disc of mass m and radius r is rolling forward on horizontal table with a velocity v. Its total kinetic energy is

(a) mv^2 (b) $\frac{3}{4}mv^2$

(c) $\frac{1}{4}mv^2$ (d) $\frac{1}{2}mv^2$ [VMMC 03]

29. When a sphere of moment of inertia I about an axis through centre of gravity and mass m rolls from rest down an inclined plane without slipping, its kinetic energy is

(a) $\frac{1}{2}I\omega^2$ (b) $\frac{1}{2}mv^2$

(c) $I\omega + mv$ (d) $\frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$

[DPMT 04]

30. The acceleration of a solid cylinder rolling down an inclined plane of inclination 30° is

(a) $g/2$ (b) $2g/2$

(c) $g/3$ (d) $g/4$ [DPMT 02]

31. A cylinder is rolling down an inclined plane of inclination 60°. What is its acceleration ?

(a) $g/\sqrt{3}$ (b) $g\sqrt{3}$

(c) $\sqrt{\frac{2}{3}}g$ (d) none of these.

[DPMT 08]

32. Two identical co-centric rings each of mass m and radius R are placed perpendicularly. What is the moment of inertia about axis of one of the rings ?

(a) $\frac{1}{2}MR^2$ (b) MR^2

(c) $\frac{3}{2}MR^2$ (d) $2MR^2$

[DPMT 08]

33. A uniform rod of Length L and mass 1.8 kg is made to rest on two measuring scales at its two ends. A uniform block of mass 2.7 kg is placed on the rod at a distance of $L/4$ from the left end. The force experienced by the measuring scale on the right end is

(a) 16 N (b) 27 N

(c) 29 N (d) 45 N [DPMT 09]

34. The centre of mass of a solid cone along the line from the centre of the base to the vertex is at

(a) one-fourth of the height

(b) one-third of the height

(c) one-fifth of the height

(d) none of these [DPMT 2011]

35. The reduced mass of two particles having masses m and $2m$ is

(a) $2m$ (b) $3m$

(c) $2m/3$ (d) $m/2$ [DPMT 2011]

36. A solid cylinder is rolling without slipping on a plane having inclination θ and the coefficient of static friction μ_s. The relation between θ and μ_s is

(a) $\tan\theta > 3\mu_s$ (b) $\tan\theta \le 3\mu_s$

(c) $\tan\theta < 3\mu_s^2$ (d) none of these

[DPMT 2011]

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ASSERTIONS AND REASONS

Directions : In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as

(a) If both assertion and reason are true and reason is the correct explanation of the assertion.

(b) If both assertion and reason are true but reason is not correct explanation of the assertion.

(c) If assertion is true, but reason is false.

(d) If both assertion and reason are false.

37. **Assertion.** There are very small sporadic changes in the speed of rotation of the earth.

Reason. Shifting of large air masses in the earth's atmosphere produces a change in the moment of inertia of the earth causing its speed of rotation to change. [VMMC 04]

38. **Assertion.** If the ice on the polar caps of the earth melts, then length of day will increase.

Reason. Moment of inertia of earth increases, as ice on polar caps melts. [VMMC 06]

Answers and Explanations

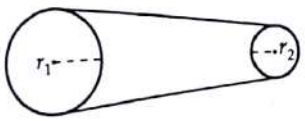
1. (b) $v_{CM} = \dfrac{m_1v_1 + m_2v_2}{m_1 + m_2} = \dfrac{2\times 2 + 4\times 10}{2+4} = 7.3$ **m/s.**

2. (a) $\omega = \dfrac{v}{r} = \dfrac{2\pi}{T}$

As both cars take same time for one revolution, so ω is same for both the cars.

3. (a) For smaller wheel,

$$\omega_1 = \omega_0 + \alpha t = 0 + 1.5\times 10 = 15 \text{ rad s}^{-1}.$$



As both the wheels are connected by a belt, they have common linear velocity,

$$v_1 = v_2$$
$$r_1\omega_1 = r_2\omega_2$$
$$\omega_2 = \frac{r_1}{r_2}.\omega_1 = \frac{1}{3}\times 15$$
$$= \mathbf{5 \text{ rad s}^{-1}}.$$

4. (a) $\omega = \int \alpha\, dt = \int (4at^3 - 3bt^2)\, dt$

$$= at^4 - bt^3 + c$$

At $t = 0$, $\omega_0 = c$

$\therefore$ $\omega = \omega_0 + at^4 - bt^3$.

5. (a) $\vec{\tau} = \vec{r}\times\vec{F}$

6. (a) $\vec{\tau} = \vec{r}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 3 & 1 \\ -3 & 1 & 5 \end{vmatrix}$

$$= 14\hat{i} - 38\hat{j} + 16\hat{k}.$$

7. (b) Torque is a vector while energy is a scalar.

8. (a) $v_0 = 72 \text{ kmh}^{-1}$

$$= 72\times\frac{5}{18} \text{ ms}^{-1} = 20 \text{ ms}^{-1}$$

$$\omega_0 = \frac{v_0}{r} = \frac{20}{0.25} = 80 \text{ rad s}^{-1}$$

$$\theta = 2\pi\times 20 \text{ rad}$$

As $\omega^2 - \omega_0^2 = 2\alpha\theta$

$\therefore$ $0^2 - (80)^2 = 2\alpha\times 2\pi\times 20$

$$\alpha = \mathbf{-25.5 \text{ rad/s}^2}.$$

9. (b) $P = 100 \text{ kW} = 10^5 \text{ W}$

$$\omega = 1800\frac{\text{rev}}{\text{min}} = 30 \text{ rps} = 30\times 2\pi \text{ rad s}^{-1}$$

$$\tau = \frac{P}{\omega} = \frac{10^5}{60\pi} = \mathbf{531 \text{ Nm}}.$$

10. (b) [Angular momentum] = [Work] [Time]

$\therefore$ Js is the unit of angular momentum.

11. (a) $\tau = \dfrac{\Delta L}{\Delta t} = \dfrac{4A - A}{4} = \dfrac{3}{4}A$

12. (a) As the particle is moving in x-y plane,

$z = 0$ and $p_z = 0$

$$\vec{\tau} = \vec{r}\times\vec{p} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & 0 \\ p_x & p_y & 0 \end{vmatrix}$$

$$= \hat{k}\,(xp_y - yp_x)$$

But $x = vt$, $y = b$, $p_x = mv$, $p_y = 0$.

$\therefore$ $\vec{L} = \hat{k}\,(vt\times 0 - bmv) = -mvb\hat{k}$.

13. (a) Of the given options, moment of inertia depends only on the axis of rotation.

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14. (*a*) By perpendicular axes theorem,

$$I_C = I_D + I_D$$

$$\frac{1}{2}MR^2 = 2I_D$$

$$I_D = \frac{1}{4}MR^2.$$

15. (*b*) By perpendicular axes theorem,

$$I_{centre} = I_D + I_D$$

$$= I + I = 2I.$$

16. (*b*) By perpendicular axes theorem,

$$I_{ring} = I_D + MR^2$$

$$= \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

$$I_{disc} = I_D + MR^2$$

$$= \frac{1}{4}MR^2 + MR^2 = \frac{5}{4}MR^2$$

$$\frac{I_{disc}}{I_{ring}} = \frac{5/4}{3/2} = \frac{5}{6}$$

or $$\frac{Mk^2_{disc}}{Mk^2_{ring}} = \frac{5}{6}$$

$$\therefore \quad \frac{k_{disc}}{k_{ring}} = \sqrt{5} : \sqrt{6}.$$

17. (*b*) M.I. of a disc about the central perpendicular axis,

$$Mk^2 = \frac{1}{2}MR^2$$

$$\therefore \quad k = \frac{R}{\sqrt{2}} = 0.707\ R = 0.707 \times 5 \text{ cm}$$

$$\simeq \mathbf{3.54\ cm.}$$

18. (*c*) Due to its large inertia of motion, a flywheel helps the engine to overcome the dead points.

19. (*b*) Angular momentum is not conserved in the presence of an external torque.

20. (*d*) It decreases his moment of inertia and increases his angular velocity.

21. (*d*) By conservation of angular momentum,

$$\frac{1}{2}MR^2\omega = \left[\frac{1}{2}MR^2 + \frac{1}{2} \times 2M \times \left(\frac{R}{2}\right)^2\right]\omega'$$

$$\omega = \left(1 + \frac{1}{2}\right)\omega'$$

$$\omega' = \frac{2\omega}{3}.$$

22. (*c*) By conservation of angular momentum,

$$Mr^2\omega = (M + 2m)r^2\omega'$$

$$\omega' = \frac{\omega M}{M + 2m}$$

23. (*c*) By conservation of angular momentum,

$$\frac{1}{2}MR^2\omega = \frac{1}{2}(M - m)R^2\omega' + mR^2\omega$$

or $$(M - m)\omega' = (M - 2m)\omega$$

or $$\omega' = \left(\frac{M - 2m}{M - m}\right)\omega.$$

24. (*b*) $\omega = 2$ rps $= 2 \times 2\pi$ rad s^{-1}

$$L = I\omega = 1 \times 4\pi = 4\pi \text{ Js}.$$

25. (*a*) $\tau = I\alpha$

$$I = \frac{\tau}{\alpha}$$

$$= \frac{31.4}{4\pi} = \frac{31.4}{4 \times 3.14} = \mathbf{2.5\ kg\ m^2}.$$

26. (*d*) Refer to the solution of Problem 73 on page 7.58.

27. (*c*) $L = I\omega$

28. (*b*) Total K.E. of rolling circular disc

$$= E_{tran} + E_{rot}$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2} \times \frac{1}{2}mr^2 \times \frac{v^2}{r^2} = \frac{3}{4}mv^2.$$

29. (*d*) Total K.E.

$$= E_{rot} + E_{tran}$$

$$= \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

30. (*c*) $$a = \frac{g\sin\theta}{1 + \frac{I}{MR^2}} = \frac{g\sin 30^\circ}{1 + \frac{\frac{1}{2}MR^2}{MR^2}} = \frac{g}{3}.$$

31. (*a*) $$a = \frac{g\sin\theta}{1 + \frac{I}{MR^2}} = \frac{g\sin 60^\circ}{1 + \frac{\frac{1}{2}MR^2}{MR^2}} = \frac{g}{\sqrt{3}}.$$

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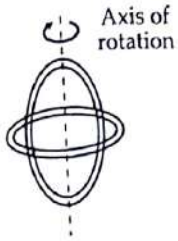
32. (*c*) For horizontal ring,

$$I_1 = MR^2$$

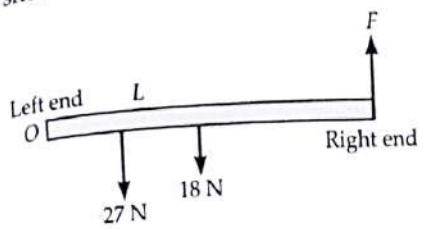


For vertical ring, axis of rotation is along a diameter

$$I_2 = \frac{1}{2}MR^2$$

$$\therefore \quad I = I_1 + I_2 = \frac{3}{2}MR^2.$$

33. (*a*) As the rod is at rest, the net torque about the left end should be zero.



Net torque about *O*,

$$\tau = 27 \times \frac{L}{4} + 18 \times \frac{L}{2} - F \times L = 0$$

or $$F = \frac{27}{4} + \frac{18}{2} = \frac{63}{4}$$

$$\simeq 16 \text{ N}.$$

34. (*a*) The *CM* of a solid cone lies on its axis at a distance of $h/4$ from its base.

35. (*c*) Reduced mass

$$= \frac{m \times 2m}{m + 2m} = \frac{2m}{3}$$

36. (*b*) The condition for rolling of cylinder without slipping is $\tan \leq 3\mu_s$

37. (*a*) Both assertion and reason are true.

38. (*a*) Both assertion and reason are true. Refer to the solution of Problem 24 on page 7.62.

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CHAPTER

8

GRAVITATION

8.1 GRAVITATION AND GRAVITY

1. *Distinguish between the terms gravitation and gravity. Give examples.*

Gravitation. Every body in the universe attracts every other body with a force called force of gravitation. Thus

Gravitation is the force of attraction between any two bodies in the universe.

The attraction between the sun and earth, the attraction between a table and a chair lying in a room, etc., are examples of gravitation. Gravitation is the weakest of the four basic forces in nature. However, it is the most important force as it has played an important role in initiating the birth of stars and in controlling the structure and evolution of the entire universe.

Gravity. Gravity is a special case of gravitation. If one of the attracting bodies is the earth, then gravitation is called gravity.

Gravity is the force of attraction between the earth and any object lying on or near its surface.

A body thrown up falls back on the surface of the earth due to earth's force of gravity.

8.2 ACCELERATION DUE TO GRAVITY AND WEIGHT OF A BODY

2. *What is meant by the term free fall ?*

Free fall. *The motion of a body under the influence of gravity alone is called a free fall.* If we neglect the resistance or friction offered by air, the fall of a body in air is a free fall. In fact, a body falls freely only in vacuum. The motion of a small heavy body in air may be taken as a free fall because air resistance on it is very small.

3. *What is meant by acceleration due to gravity ? Is it a scalar or a vector ?*

Acceleration due to gravity. When a body falls freely towards the surface of the earth, its velocity continuously increases. The acceleration developed in its motion is called acceleration due to gravity.

The acceleration produced in a freely falling body under the gravitational pull of the earth is called acceleration due to gravity.

It is denoted by g. It is a vector having direction towards the centre of the earth. It does not depend on the mass, size and shape of the body. The value of g is constant at a given place. However, it varies from place to place on the surface of the earth. It depends on altitude, depth, rotation of the earth and shape of the earth.

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Using lasers, it is possible to measure distances up to 10^{-9} m and by electronic devices time can be measured upto 10^{-9} s. Using these techniques, g can be determined to an accuracy of 1 part in 10^8 by observing the free fall of a body in vacuum.

Near the surface of the earth, $g = 9.8\ ms^{-2}$ or $32\ ft\ s^{-2}$.

4. *What do you mean by weight of a body ? Is it a scalar or vector ? What are its units ?*

Weight of a body. The weight of a body is the measure of the gravitational pull exerted by the earth upon it.

Weight of a body is defined as the gravitational force with which a body is attracted towards the centre of the earth.

If the g is the acceleration due to gravity at a place, then a body of mass of m is attracted towards the centre of the earth with a force equal to mg at that place. Hence weight of a body is given by

$$W = mg$$

In vector notation, $\vec{W} = m\vec{g}$

Weight is a vector quantity. It is measured in the units of force such as newton, kgwt, etc. As the value of g varies from place to place, the weight of a body also varies from place to place.

8.3 NEWTON'S UNIVERSAL LAW OF GRAVITATION

5. *How did Newton discover the universal law of gravitation ?*

Discovery of Newton's law of gravitation. One day in the year 1665, seeing an apple falling from a tree, Newton was inspired to think about the law of gravitation. He thought that the force which attracts the apple towards the earth might be the same as the force attracting the moon towards the earth. By comparing the acceleration due to gravity on the earth with the acceleration required to keep the moon in orbit around the earth, Newton was able to deduce the law of gravitation as discussed below.

Newton assumed that the moon revolved around the earth in a circular orbit of radius $R\ (= 3.84 \times 10^8\ m)$, as shown in Fig. 8.1.

Period of moon around the earth,

$$T = 27.3 \text{ days} = 27.3 \times 86{,}400 \text{ s}$$

Speed of the moon

$$v = \frac{\text{Circumference of orbit}}{\text{Orbital period}}$$

$$= \frac{2\pi \times (3.84 \times 10^8\ m)}{27.3 \times 86400\ s} = 1.02 \times 10^3\ ms^{-1}$$

Centripetal acceleration of the moon,

$$a_c = \frac{v^2}{R} = \frac{(1.02 \times 10^3)^2}{3.84 \times 10^8}$$

$$= 2.72 \times 10^{-3}\ ms^{-2} \quad \ldots(i)$$

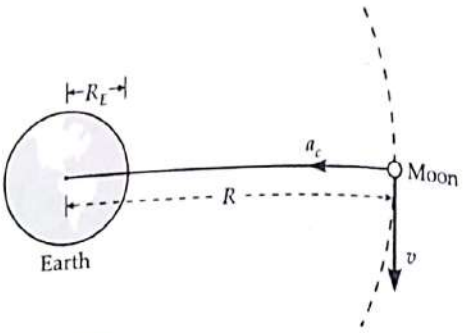

Fig. 8.1 Centripetal acceleration of the moon.

Acceleration due to gravity at the earth's surface, $g = 9.8\ ms^{-2}$.

Clearly $a_c << g$. Newton assumed that both acceleration of the moon and acceleration of the falling object are provided by earth's gravitational attraction. Newton argued that force and hence the acceleration produced must decrease with distance from the centre of the earth. From the relative values of a_c and g, he proposed that the gravitational force should be inversely proportional to the square of the distance. If R_E is the radius of the earth, then

$$\frac{a_c}{g} = \frac{1/R^2}{1/R_E^2} = \left(\frac{R_E}{R}\right)^2$$

Newton knew that $\frac{R_E}{R} = \frac{1}{60}$

$$\therefore\ a_c = \left(\frac{R_E}{R}\right)^2 \times g = \left(\frac{1}{60}\right)^2 \times 9.8 = 2.72 \times 10^{-3}\ ms^{-2}.$$

This value is in close argument with the value obtained in equation (*i*), thus verifying the inverse square law. It is called *Newton's moon test.*

Newton further analysed that the force of gravitation exerted by an object should be proportional to its mass. By the third law of motion, the second object should exert an equal and opposite force on the first one. This force should be proportional to the mass of the second object. Taking into account all these facts, he arrived at his famous universal law of gravitation.

6. *State Newton's law of gravitation. Hence define G. What are the units and dimensions of G ? Why is G called a universal gravitational constant ?*

Statement of Newton's law of gravitation. In 1687, Newton published the universal law of gravitation in his book *Principia.* This law can be stated as follows :

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Every particle in the universe attracts every other particle with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. This force acts along the line joining the two particles.

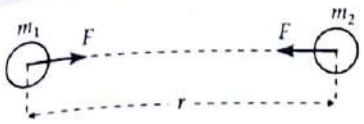

Fig. 8.2 Law of gravitation.

Consider two bodies of masses m_1 and m_2 and separated by distance r. According to the law of gravitation, the force of attraction F between them is such that

$$F \propto m_1 m_2 \quad \text{and} \quad F \propto \frac{1}{r^2}$$

$$\therefore \quad F \propto \frac{m_1 m_2}{r^2}$$

or
$$F = G\frac{m_1 m_2}{r^2}$$

where G is a constant called *universal gravitational constant*.

Definition of G. If $m_1 = m_2 = 1$ and $r = 1$, then

$$F = G$$

The universal gravitational constant may be defined as the force of attraction between two bodies of unit mass each and placed unit distance apart.

In SI, the gravitational constant is equal to the force of attraction between two bodies of 1 kg each and placed 1 m apart.

In cgs system, the gravitational constant is equal to the force of attraction between two bodies of 1 g each and placed 1 cm apart.

Dimensions of G. As $F = G\frac{m_1 m_2}{r^2}$

$$\therefore \quad G = \frac{Fr^2}{m_1 m_2}$$

$$\text{Dimensions of } G = \frac{MLT^{-2} \times L^2}{M \times M} = [M^{-1}L^3T^{-2}]$$

Units of G. As $G = \frac{Fr^2}{m_1 m_2}$

$$\therefore \quad \text{SI unit of } G = \frac{Nm^2}{kg \times kg} = \mathbf{Nm^2kg^{-2}}.$$

Similarly, cgs unit of $G = \mathbf{dyn\ cm^2g^{-2}}$.

Value of G. In SI, $G = 6.67 \times 10^{-11}\ Nm^2\ kg^{-2}$

In cgs system, $G = 6.67 \times 10^{-8}\ dyn\ cm^2g^{-2}$.

The value of G does not depend on the nature and size of the bodies. It also does not depend on the nature of the medium between the two bodies. That is why G is called universal gravitational constant.

7. *Briefly explain the Cavendish's experiment for the determination of the universal constant G.*

Cavendish's experiment for the determination of G. The value of the gravitational constant G was first determined experimentally by English scientist *Henry Cavendish* in 1798. The apparatus used is shown in Fig. 8.3.

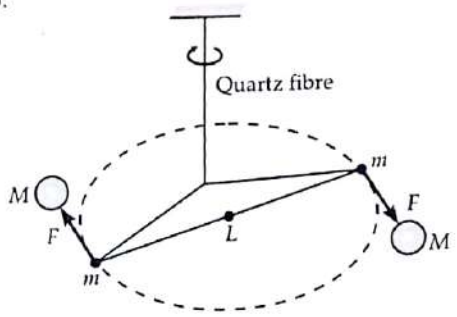

Fig. 8.3 Cavendish method.

Here two small identical spheres of lead, each of mass m are connected to the two ends of a light rod to form a dumb-bell. The rod is supported by a vertical length of quartz fibre. Two large lead spheres of mass M each are placed near the ends of the dumb-bell on the opposite sides in such a way that all the four spheres lie on a horizontal circle. The small spheres move towards the larger ones under the gravitational attraction,

$$F = \frac{GMm}{r^2}$$

where r is the distance between the centre of the large and its neighbouring small sphere.

The forces on the two small spheres form a couple which exerts a torque. This torque deflects the rod and twists the suspension till such time as the restoring torque of the fibre equals the deflecting gravitational torque. The angle of deflection θ is noted by measuring the deflection of a light beam by a lamp and scale arrangement. Let L be the length of the light rod.

Then,

$$\text{Deflecting torque} = F \times L = \frac{GMm}{r^2} L$$

$$\text{Restoring torque} = k\theta$$

where k is the restoring torque per unit angle of twist and is called *torsion constant* of the suspension fibre.

In rotational equilibrium, both the torques are equal and opposite.

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$$\frac{GMm}{r^2} L = k\theta$$

or
$$G = \frac{k\theta r^2}{MmL}$$

Knowing all the quantities on the right hand side from the experiment, the value of G can be determined.

Since Cavendish's experiment, the measurement of G has been improved upon. The currently accepted value is

$$G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}.$$

8.4 EVIDENCES IN SUPPORT OF LAW OF GRAVITATION

8. *Mention some experimental evidences in support of Newton's law of gravitation.*

Experimental evidences in support of the law of gravitation. Many of the results predicted theoretically on the basis of the law of gravitation are found to be in close agreement with the experimental observations. Some of such evidences are as follows :

(*i*) The rotation of the earth around the sun or that of the moon around the earth is explained on the basis of this law.

(*ii*) The tides are formed in oceans due to the gravitational force of attraction between the moon and sea-water.

(*iii*) The times of solar and lunar eclipses calculated on the basis of the law of gravitation are found to be reasonably accurate.

(*iv*) The orbits and periods of revolutions of artificial satellites can be predicted very accurately on the basis of this law.

(*v*) The value of g varies from place to place on the surface of the earth in accordance with the law of gravitation.

8.5 VECTOR FORM OF THE LAW OF GRAVITATION

9. *Express law of gravitation in vector form. What are its implications ?*

Vector form of Newton's Law of Gravitation. As shown in Fig. 8.4, consider two particles A and B of masses m_1 and m_2 and separated by distance r.

Fig. 8.4 Law of gravitation in vector form.

Let

$\hat{r}_{12}$ = a unit vector from A to B

$\hat{r}_{21}$ = a unit vector from B to A

$\vec{F}_{12}$ = gravitational force exerted on A by B

$\vec{F}_{21}$ = gravitational force exerted on B by A

In vector form, Newton's law of gravitation can be expressed as

$$\vec{F}_{12} = -G\frac{m_1 m_2}{r^2}\hat{r}_{21}$$

The negative sign shows that the direction of force $\vec{F}_{12}$ is opposite to that of $\hat{r}_{21}$ *i.e.*, the gravitational force is attractive in nature so that m_1 is attracted towards m_2.

Similarly, $$\vec{F}_{21} = -\frac{G m_1 m_2}{r^2}\hat{r}_{12}$$

But $$\hat{r}_{21} = -\hat{r}_{12}$$

Hence $$\vec{F}_{21} = -\vec{F}_{12}$$

Thus the vector form of the law of gravitation implies that the gravitational forces acting between two particles form *action and reaction pair*. As $\vec{F}_{12}$ and $\vec{F}_{21}$ are directed towards the centres of the two particles, so gravitational force is a *central force*.

8.6 IMPORTANT FEATURES OF GRAVITATIONAL FORCE

10. *Mention the characteristic features of gravitational force.*

Important features of gravitational force :

(*i*) The gravitational force between two masses is *independent of intervening medium.*

(*ii*) The mutual gravitational forces between two bodies are equal and opposite *i.e.*, they form action and reaction pair. Hence *gravitational forces obey Newton's third law of motion.*

(*iii*) The law of gravitation strictly holds for point masses.

(*iv*) The gravitational force between two point masses is a central force. Its magnitude depends only on r and has no angular dependence. Thus the gravitational force possesses *spherical symmetry.*

(*v*) The gravitational force is a conservati e iorce.

(*vi*) The gravitational force between two bodies is independent of the presence of other bodies.

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8.7 PRINCIPLE OF SUPERPOSITION OF GRAVITATIONAL FORCES

11. *State and illustrate the principle of superposition of gravitational forces.*

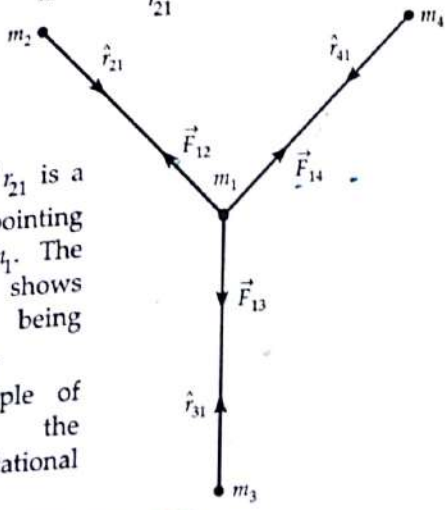
Principle of superposition of gravitational forces. According to the principle of superposition, the gravitational force between two masses acts independently and is unaffected by the presence of other bodies. Hence the resultant gravitational force acting on a particle due to a number of masses is the vector sum of the gravitational forces exerted by the individual masses on the given particle. Mathematically,

$$\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots\ldots + \vec{F}_n = \sum_{i=1}^{n} \vec{F}_i$$

where $\vec{F}_1, \vec{F}_2, \vec{F}_3, \ldots\ldots, \vec{F}_n$ are the gravitational forces exerted individually by n masses $m_1, m_2, m_3, \ldots\ldots, m_n$ on the particle of mass m. Each force is determined by the law of gravitation.

Illustration. In Fig. 8.5, the total gravitational force on point mass m_1 is the vector sum of the gravitational forces exerted by the point masses m_2, m_3 and m_4. According to the law of gravitation, the force on m_1 exerted by m_2 is

$$\vec{F}_{12} = -G\frac{m_2 m_1}{r_{21}^2}\hat{r}_{21}$$



where $\hat{r}_{21} = \vec{r}_{21}/r_{21}$ is a unit vector pointing from m_2 to m_1. The negative sign shows that m_1 is being attracted by m_2.

By the principle of superposition, the total gravitational force on m_1 is

Fig. 8.5 Principle of superposition of gravitational forces.

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14}$$

$$= -G\left[\frac{m_2 m_1}{r_{21}^2}\hat{r}_{21} + \frac{m_3 m_1}{r_{31}^2}\hat{r}_{31} + \frac{m_4 m_1}{r_{41}^2}\hat{r}_{41}\right]$$

8.8 SHELL THEOREM

12. *State Newton's shell theorem for the gravitational force.*

Newton's shell theorem. This theorem gives gravitational force on a point mass due to a spherical shell or a solid sphere. It can be stated as follows :

(i) If a point mass lies outside a uniform spherical shell / sphere with a spherically symmetric internal mass distribution, the shell/sphere attracts the point mass as if the entire mass of the shell/sphere were concentrated at its centre.

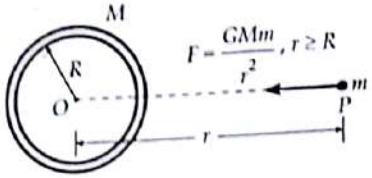


Fig. 8.6 Force on an outside particle due to a shell.

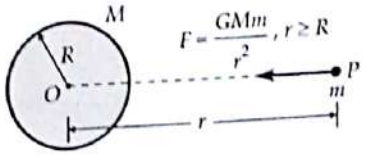
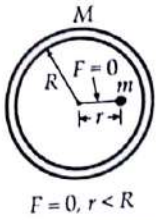


Fig. 8.7 Force on an outside particle due to a solid sphere.

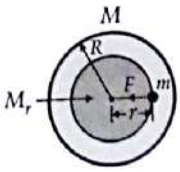
Explanation. The gravitational forces exerted by the various regions of the shell/sphere have components along the line joining the point mass to the centre as well as along a direction perpendicular to this line. When summed over all the regions of the shell/sphere, the perpendicular components cancel out. Thus a resultant force acts along the joining the point mass to the centre.

(ii) If a point mass lies inside a uniform spherical shell, the gravitational force on the point mass is zero. But if a point mass lies inside a homogeneous solid sphere, the force on the point mass acts towards the centre of the sphere. This force is exerted by the spherical mass situated interior to the point mass.



$F = 0, r < R$

Fig. 8.8 No force acts on a particle inside a shell.



$$F = \frac{GM_r m}{r^2} = \frac{GMm}{R^3}r \quad \left[\because \frac{M_r}{M} = \frac{r^3}{R^3}\right]$$

Fig. 8.9 A net force acts towards the centre on a particle inside ($r < R$) a solid sphere.

Explanation. Various regions of the spherical shell attract the point mass inside it in various directions. These forces cancel each other completely.

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13. *Is gravitational shielding possible ?*

Gravitational shielding is not possible. Although the gravitational force on a particle inside a spherical shell is zero, yet the shell does not shield the other bodies outside it from exerting gravitational forces on the particle lying inside it. Thus gravitational shielding is not possible.

For Your Knowledge

- The *law of gravitation is strictly true for point masses.* However, it is valid in the following circumstances also :
  - For two bodies of any size provided they both have spherical symmetry.
  - When one body has spherical symmetry and the other is small compared with the separation between their centres.
  - When neither body has spherical symmetry but both are small compared with the separation between their centres.
- The value of G does not depend on the nature and size of the masses.
- The law of gravitation is universally valid. It applies to small objects on the earth, planets in the solar system and to galaxies.
- Though the gravitational force between two particles is central, the force between two finite rigid bodies is not necessarily along the line joining their centres of mass. For a particle lying outside a spherical symmetric body, the force acts as if the mass is concentrated at the centre of the body. Therefore, this force is central.
- The gravitational force on a particle lying inside a hollow body of any arbitrary shape is zero.

Examples based on Newton's Law of Gravitation and Principle of Superposition

FORMULAE USED

1. Newton's law of gravitation, $F = \dfrac{G m_1 m_2}{r^2}$
2. Mass of planet or satellite, $M = \dfrac{4\pi^2 r^3}{GT^2}$
3. Principle of superposition of gravitational forces, $\vec{F}_R = \vec{F}_1 + \vec{F}_2 + ... + \vec{F}_n$

UNITS USED

Force of gravitation F is in newton, masses m_1, m_2 and M are in kg, distance r in metre and period of revolution in second.

CONSTANT USED

Universal gravitational constant,

$G = 6.67 \times 10^{-11}\ \text{Nm}^2\text{kg}^{-2}$.

EXAMPLE 1. *Calculate the force of attraction between two balls each of mass 1 kg each, when their centres are 10 cm apart. Given* $G = 6.67 \times 10^{-11}\ Nm^2kg^{-2}$. [Delhi 1997]

Solution. Here $m_1 = m_2 = 1$ kg, $r = 10$ cm $= 0.10$ m

$$\therefore \quad F = \frac{G m_1 m_2}{r^2} = \frac{6.67 \times 10^{-11} \times 1 \times 1}{(0.10)^2}$$

$$= \mathbf{6.67 \times 10^{-9}\ N.}$$

EXAMPLE 2. *The mass of planet Jupiter is* 1.9×10^{27} *kg and that of the sun is* 1.99×10^{30} *kg. The mean distance of the Jupiter from the Sun is* 7.8×10^{11} *m. Calculate the gravitational force which the sun exerts on Jupiter. Assuming that Jupiter moves in a circular orbit around the sun, calculate the speed of the Jupiter.*

Solution. Here $M = 1.99 \times 10^{30}$ kg,

$m = 1.9 \times 10^{27}$ kg, $r = 7.8 \times 10^{11}$ m,

$G = 6.67 \times 10^{-11}\ \text{Nm}^2\ \text{kg}^{-2}$

$$\therefore \quad F = \frac{GMm}{r^2} = \frac{6.67 \times 10^{-11} \times 1.99 \times 10^{30} \times 1.9 \times 10^{27}}{(7.8 \times 10^{11})^2}$$

$$= \mathbf{4.145 \times 10^{23}\ N.}$$

The gravitational force of attraction due to the sun provides the necessary centripetal force to the Jupiter to move in the circular orbit. If v is the orbital speed of Jupiter, then

$$\frac{mv^2}{r} = F$$

or
$$v = \sqrt{\frac{rF}{m}} = \sqrt{\frac{7.8 \times 10^{11} \times 4.145 \times 10^{23}}{1.9 \times 10^{27}}}$$

$$= \mathbf{1.304 \times 10^4\ ms^{-1}.}$$

EXAMPLE 3. *Two particles, each of mass m, go round a circle of radius R under the action of their mutual gravitational attraction. Find the speed of each particle.*

Solution. The force on each particle is directed along the radius of the circle. The two particles will always lie at the ends of a diameter so that distance between them is $2R$.

$$\therefore \quad F = G\frac{m \times m}{(2R)^2} = \frac{Gm^2}{4R^2}$$

As this force provides the centripetal force, so

$$\frac{Gm^2}{4R^2} = \frac{mv^2}{R}$$

or
$$v = \sqrt{\frac{Gm}{4R}}.$$

EXAMPLE 4. *The mean orbital radius of the earth around the sun is* 1.5×10^8 *km. Calculate the mass of the sun if* $G = 6.67 \times 10^{-11}\ Nm^2kg^{-2}$.

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Solution. Here $r = 1.5 \times 10^8$ km $= 1.5 \times 10^{11}$ m

$T = 365$ days $= 365 \times 24 \times 3600$ s

Centripetal force required = Force of gravitation between the earth and the sun

$$\frac{mv^2}{r} = \frac{GMm}{r^2} \quad \text{or} \quad \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2 = \frac{GMm}{r^2}$$

$$M = \frac{4\pi^2 r^3}{GT^2}$$

$$= \frac{4 \times 9.87 \times (1.5 \times 10^{11})^3}{6.67 \times 10^{-11} \times (365 \times 24 \times 3600)^2}$$

$$= 2.01 \times 10^{30} \text{ kg.}$$

EXAMPLE 5. A mass M is broken into two parts of masses m_1 and m_2. How are m_1 and m_2 related so that force of gravitational attraction between the two parts is maximum ?

Solution. Let $m_1 = m$, then $m_2 = M - m$

Force of gravitation between the two parts when they are placed distance r apart is

$$F = G\frac{m(M-m)}{r^2} = \frac{G}{r^2}(Mm - m^2)$$

Differentiating w.r.t m, we get

$$\frac{dF}{dm} = \frac{G}{r^2}(M - 2m)$$

For F to be maximum, $\dfrac{dF}{dm} = 0$

$$\frac{G}{r^2}(M - 2m) = 0$$

or

or $\quad M = 2m \quad$ or $\quad m = M/2$

$\therefore \; m_1 = m_2 = M/2.$

EXAMPLE 6. Three equal masses of m kg each are fixed at the vertices of an equilateral triangle ABC, as shown in Fig. 8.10.

(a) What is the force acting on a mass 2m placed at the centroid G of the triangle ?

(b) What is the force if the mass at the vertex A is doubled ?

Take AG = BG = CG = 1 m **[NCERT, Delhi 06]**

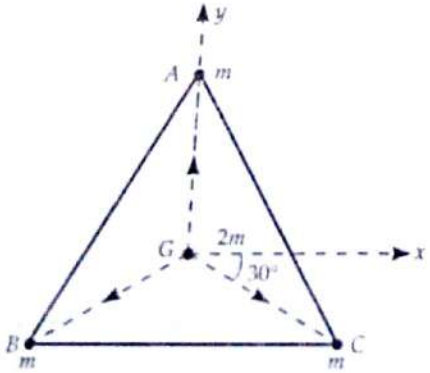

Fig. 8.10

Solution. We choose the co-ordinate axes as shown in Fig. 8.10.

Angle between GC and positive x-axis $= 30°$.

Angle between GB and negative x-axis $= 30°$.

Force on mass at G due to mass at A,

$$\vec{F}_{GA} = \frac{Gm(2m)}{1^2}\hat{j}$$

Force on mass at G due to mass at B,

$$\vec{F}_{GB} = \frac{Gm(2m)}{1^2}(-\hat{i}\cos 30° - \hat{j}\sin 30°)$$

Force on mass at G due to mass at C,

$$\vec{F}_{GC} = \frac{Gm(2m)}{1^2}(+\hat{i}\cos 30° - \hat{j}\sin 30°)$$

By the principle of superposition, the resultant force on mass $2m$ is

$$\vec{F}_R = \vec{F}_{GA} + \vec{F}_{GB} + \vec{F}_{GC}$$

$$= 2Gm^2[\hat{j} + (-\hat{i}\cos 30° - \hat{j}\sin 30°) + (\hat{i}\cos 30° - \hat{j}\sin 30°)]$$

$$= 2\,Gm^2[\hat{j} - 2\hat{j}\sin 30°]$$

$$= 2\,Gm^2(\hat{j} - \hat{j}) = \mathbf{0}.$$

From symmetry consideration also, the resultant force on mass $2m = 0$.

(*b*) Now the mass at the vertex A is $2m$. Then

$$\vec{F}_{GA} = \frac{G(2m)(2m)}{1^2}\hat{j} = 4\,Gm^2\hat{j}$$

$\vec{F}_{GB}$ and $\vec{F}_{GC}$ remain same as above.

$$\therefore \quad \vec{F}_R = \vec{F}_{GA} + \vec{F}_{GB} + \vec{F}_{GC}$$

$$= 2Gm^2[2\hat{j} + (-\hat{i}\cos 30° - \hat{j}\sin 30°) + (\hat{i}\cos 30° - \hat{j}\sin 30°)]$$

$$= 2\,Gm^2[2\hat{j} - 2\hat{j}\sin 30°]$$

$$= 2\,Gm^2(2\hat{j} - \hat{j}) = \mathbf{2\,Gm^2\,\hat{j}}.$$

PROBLEMS FOR PRACTICE

1. A sphere of mass 40 kg is being attracted by another sphere of mass 80 kg with a force equal to 1/4 of a milligram weight when their centres are 30 cm apart. Calculate the value of G.

(**Ans.** 6.88×10^{-11} Nm^2kg^{-2})

2. The centres of two identical spheres are 1.0 m apart. If the gravitational force between the spheres be 1.0 N, then what is the mass of each sphere ? ($G = 6.67 \times 10^{-11}$ $m^3kg^{-1}s^{-2}$).

(**Ans.** 1.225×10^5 kg)

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3. Find the gravitational attraction between two H-atoms of a hydrogen molecule. Given $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}, mass of the atom $= 1.67 \times 10^{-27}$ kg and distance between two H-atoms = 1Å. (**Ans.** 1.86×10^{-44} N)

4. Calculate the force of gravitation between two bodies, each of mass 100 kg and 1 m apart on the surface of the earth. Will the force of attraction be different if the same bodies are taken on the moon, their separation remaining constant ?
(**Ans.** 6.67×10^{-7} N, No)

5. An apple of mass 0.25 kg falls from a tree. What is the acceleration of the apple towards the earth ? Also calculate the acceleration of the earth towards the apple. Mass of the earth $= 5.983 \times 10^{24}$ kg, Radius of the earth $= 6.378 \times 10^6$ m and $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}.
(**Ans.** 9.810 ms^{-2}, 4.099×10^{-25} ms^{-2})

6. If the mass of the sun is 2×10^{30} kg, the distance of the earth from the sun is 1.5×10^{11} m and period of revolution of the earth around the sun is one year (= 365.3 days), calculate the value of gravitational constant. (**Ans.** 6.69×10^{-11} Nm^2 kg^{-2})

7. How far from earth must a body be along a line towards the sun so that the sun's gravitational pull on it balances that of the earth. Distance between sun and earth's centre is 1.5×10^{10} km. Mass of sun is 3.24×10^5 times mass of earth. (**Ans.** 2.63×10^7 km)

✕ HINTS

1. Here $m_1 = 80$ kg, $m_2 = 40$ kg, $r = 30$ cm $= 0.30$ m

$$F = \frac{1}{4} \text{ mg wt} = \frac{1}{4} \times 10^{-6} \text{ kg wt}$$

$$= \frac{10^{-6} \times 9.8}{4} \text{ N}$$

$$G = \frac{Fr^2}{m_1 m_2} = \frac{10^{-6} \times 9.8 \times 0.30 \times 0.30}{4 \times 80 \times 40}$$

$$= \mathbf{6.88 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}}.$$

3. $$F = G\frac{m_1 m_2}{r^2}$$

$$= \frac{6.67 \times 10^{-11} \times 1.67 \times 10^{-27} \times 1.67 \times 10^{-27}}{(10^{-10})^2}$$

$$= \mathbf{1.86 \times 10^{-44} \text{ N}}.$$

4. $$F = \frac{Gm_1 m_2}{r^2} = \frac{6.67 \times 10^{-11} \times 100 \times 100}{(1)^2}$$

$$= 6.67 \times 10^{-7} \text{ N}.$$

The force of attraction between the two bodies will remain same on the moon as the masses of two bodies also remain the same.

5. Here $m = 0.25$ kg, $M = 5.983 \times 10^{24}$ kg, $R = 6.378 \times 10^6$ m

Force of gravitation between earth and apple,

$$F = \frac{GMm}{R^2}$$

Acceleration of the apple towards the earth,

$$a = \frac{F}{m} = \frac{GM}{R^2}$$

$$= \frac{6.67 \times 10^{-11} \times 5.983 \times 10^{24}}{(6.378 \times 10^6)^2} = \mathbf{9.810 \text{ ms}^{-2}}$$

Acceleration of the earth towards the apple,

$$a' = \frac{F}{m} = \frac{Gm}{R^2} = \frac{6.67 \times 10^{-11} \times 0.25}{(6.378 \times 10^6)^2}$$

$$= \mathbf{4.099 \times 10^{-25} \text{ ms}^{-2}}.$$

7. Let P be a point at distance x where the gravitational pull due to the earth and the sun balance. Then

$$\frac{GM_E m}{x^2} = \frac{GM_S m}{(r-x)^2}$$

or $$\frac{(r-x)^2}{x^2} = \frac{M_S}{M_E} = 3.24 \times 10^5$$

or $$\frac{r-x}{x} = \sqrt{3.24 \times 10^5} \simeq 570$$

or $$x = \frac{r}{571} = \frac{1.5 \times 10^{10}}{571} = \mathbf{2.63 \times 10^7 \text{ km}}.$$

8.9 ACCELERATION DUE TO GRAVITY OF THE EARTH

14. *Write expressions for the gravitational force exerted by the earth on a point mass m located above, below and on the surface of the earth. Hence deduce expression for g on the earth's surface.*

Acceleration due to gravity of the earth. Consider the earth to be a sphere of radius R and uniform density ρ. Then its mass will be

$$M = \frac{4}{3}\pi R^3 \rho$$

(*i*) ***At points above the earth's surface.*** Suppose a point mass m is situated outside the earth at a distance r from its centre. According to shell theorem, the gravitational force at a point outside the earth is just as if the entire mass of the earth is concentrated at its centre. Hence the gravitational force on the point mass m is

$$F = G\frac{Mm}{r^2} \qquad (\text{For } r > R)$$

(*ii*) ***At points below the earth's surface.*** As shown in Fig. 8.11, suppose a point mass m is located at a point

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f in a mine at a depth d below the earth's surface. Thus the point P lies outside the sphere of radius r and inside the shell of thickness

$$d = R - r.$$

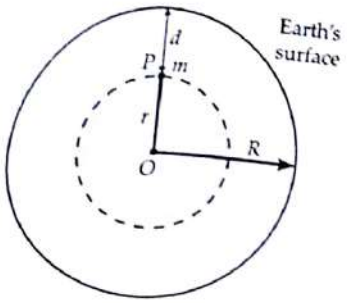

Fig. 8.11 Gravitational force on a mass *m* at depth *d*.

According to shell theorem, the outer shell exerts no force on the mass m kept at point P. The inner sphere of radius r exerts a force on point mass m at point P as if its mass m_r is concentrated at the centre. Thus the gravitational force on mass m is

$$F = \frac{Gmm_r}{r^2}$$

But $$m_r = \frac{4}{3}\pi r^3 \rho = \frac{4}{3}\pi R^3 \rho \frac{r^3}{R^3} = M\frac{r^3}{R^3}$$

$$\therefore \quad F = \frac{Gm}{r^2} \cdot \frac{Mr^3}{R^3}$$

or $$F = \frac{GMm}{R^3} r \qquad \text{(For } r < R\text{)}$$

(*iii*) **At points on the earth's surface.** If the point mass m is situated on the earth's surface, then $r = R$, and the gravitational force on mass m is

$$F = \frac{GMm}{R^3} \cdot R$$

or $$F = \frac{GMm}{R^2} \qquad \text{(For } r = R\text{)}$$

Suppose the mass m experiences acceleration g, called the acceleration due to gravity, then according to Newton's second law of motion,

$$F = mg$$

$$\therefore \quad mg = \frac{GMm}{R^2}$$

or $$g = \frac{GM}{R^2}$$

This gives the acceleration due to gravity on the surface of the earth.

15. *Obtain an expression for the acceleration due to gravity g in terms of mass of the earth M and gravitational constant G.*

Relation between *g* and *G*. Consider the earth to be a sphere of mass M and radius R. Suppose a body of mass m is lying on its surface, as shown in Fig. 8.12. According to the law of gravitation, the force of attraction between the earth and the body is

$$F = \frac{GMm}{R^2}$$

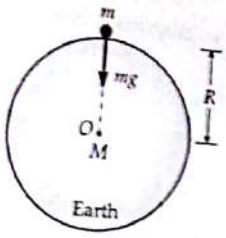

Fig. 8.12 Relation between *g* and *G*.

Here we have used shell theorem according to which the gravitational force due to a sphere on a mass outside it acts as if the entire mass of the sphere is concentrated at its centre. The force of gravity F produces an acceleration g (called acceleration due to gravity) in the body of mass m. From Newton's second law of motion, we get

$$F = mg$$

From the above two equations, we have

$$mg = \frac{GMm}{R^2} \qquad \text{or} \qquad g = \frac{GM}{R^2}$$

This gives acceleration due to gravity on the surface of the earth. *The value of g is independent of the mass, size and shape of the body falling under gravity.*

8.10 MASS AND DENSITY OF THE EARTH

16. *Explain how the mass and average density of the earth can be estimated from the knowledge of G ?*

Mass of the earth. As $g = \frac{GM}{R^2}$, therefore

Mass of the earth, $$M = \frac{gR^2}{G}$$

Knowing the values of g, G and R, the mass of the earth can be determined.

As $g = 9.8\ \text{ms}^{-2}$, $R = 6.37 \times 10^6\ \text{m}$,

$G = 6.67 \times 10^{-11}\ \text{Nm}^2\text{kg}^{-2}$

$$\therefore \quad M = \frac{9.8 \times (6.37 \times 10^6)^2}{6.67 \times 10^{-11}}$$

$$= 5.97 \times 10^{24}\ \text{kg} \simeq 6 \times 10^{24}\ \text{kg}.$$

As the value G was first experimentally determined by *Cavendish*, so he is regarded as the *first person to have weighed the earth.*

Average density of the earth. If the earth is taken as a sphere of radius R and average density ρ, then its mass would be

$$M = \text{Volume} \times \text{density} = \frac{4}{3}\pi R^3 \rho$$

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But mass of the earth,

$$M = \frac{gR^2}{G}$$

$$\therefore \quad \frac{4}{3}\pi R^3 \rho = \frac{gR^2}{G} \quad \text{or} \quad \rho = \frac{3g}{4\pi GR}$$

Knowing the values of g, G and R, the average density ρ of the earth can be estimated. Thus

$$\rho = \frac{3 \times 9.8}{4 \times 3.142 \times 6.67 \times 10^{-11} \times 6.37 \times 10^6}$$

$$= 5497 \text{ kgm}^{-3} \simeq 5500 \text{ kgm}^{-3}.$$

In fact, the density of the upper layers of the earth is around 2700 kgm^{-3} while the density of the inner layer is much larger than average value of 5500 kgm^{-3}.

Examples based on Mass and Density of Earth

FORMULAE USED

1. Acceleration due to gravity on the earth's surface, $g = \frac{GM}{R^2}$
2. Mass of the earth, $M = \frac{gR^2}{G}$
3. Mean density of earth, $\rho = \frac{3g}{4\pi GR}$
4. From Kepler's law of periods, $M = \frac{4\pi^2 r^3}{GT^2}$
5. Weight of a body, $W = mg$

UNITS USED

The masses M and m are in kg, acceleration due to gravity g in ms^{-2}, period of revolution T in second and density ρ in kgm^{-3}.

EXAMPLE 7. ***Weighing the Earth :*** *You are given the following data : $g = 9.81\ ms^{-2}$. $R_E = 6.37 \times 10^6\ m$, the distance to the moon $r = 3.84 \times 10^8\ m$ and the time period of the moon's revolution is 27.3 days. Obtain the mass of the Earth M_E in two different ways.* [NCERT]

Solution. (*i*) $M_E = \frac{gR_E^2}{G} = \frac{9.81 \times (6.37 \times 10^6)^2}{6.67 \times 10^{-11}}$

$$= 5.97 \times 10^{24} \text{ kg}.$$

(*ii*) From Kepler's law of periods,

$$M_E = \frac{4\pi^2 r^3}{GT^2} = \frac{4 \times 3.14 \times 3.14 \times (3.84)^3 \times 10^{24}}{6.67 \times 10^{-11} \times (27.3 \times 24 \times 60 \times 60)^2}$$

$$= 6.02 \times 10^{24} \text{ kg}.$$

Both the methods give almost the same mass, the difference being less than 1%.

EXAMPLE 8. *If the earth were made of lead of relative density 11.3, what then would be the value of acceleration due to gravity on the surface of the earth ? Radius of the earth $= 6.4 \times 10^6\ m$ and $G = 6.67 \times 10^{-11}\ Nm^2\ kg^{-2}$.*

Solution. Density of the earth,

ρ = Relative density × density of water

$= 11.3 \times 10^3\ \text{kgm}^{-3}$

Acceleration due to gravity on the earth's surface,

$$g = \frac{GM}{R^2} = \frac{G}{R^2} \cdot \frac{4}{3}\pi R^3 \times \rho = \frac{4}{3}\pi GR\rho$$

$$= \frac{4}{3} \times \frac{22}{7} \times 6.67 \times 10^{-11} \times 6.4 \times 10^6 \times 11.3 \times 10^3$$

$$= 22.21 \text{ ms}^{-2}.$$

EXAMPLE 9. *The acceleration due to gravity at the moon's surface is $1.67\ ms^{-2}$. If the radius of the moon is $1.74 \times 10^6\ m$, calculate the mass of the moon. Use the known value of G.* [NCERT]

Solution. Here $G = 6.67 \times 10^{-11}\ \text{Nm}^2\ \text{kg}^{-2}$,

$g = 1.67\ \text{ms}^{-2}$, $R = 1.74 \times 10^6$ m

$$M = \frac{gR^2}{G} = \frac{1.67 \times (1.74 \times 10^6)^2}{6.67 \times 10^{-11}} = 7.58 \times 10^{22} \text{ kg}.$$

EXAMPLE 10. *Two lead spheres of 20 cm and 2 cm diameter respectively are placed with centres 100 cm apart. Calculate the attraction between them, given the radius of the earth as 6.37×10^8 cm and its mean density as $5.53 \times 10^3\ kg\ m^{-3}$. Specific gravity of lead = 11.5. If the lead spheres are replaced by brass spheres of same radii, would the force of attraction be same ?*

Solution. Here $r_1 = \frac{20}{2}$ cm $= 0.10$ m,

$$r_2 = \frac{2}{2} \text{ cm} = 0.01 \text{ m}$$

$r = 1.0$ m, $\rho' = 11.5 \times 10^3\ \text{kg m}^{-3}$

$R = 6.37 \times 10^8$ cm $= 6.37 \times 10^6$ m,

$\rho = 5.53 \times 10^3\ \text{kgm}^{-3}$

Masses of the two lead spheres will be

$$m_1 = \frac{4}{3}\pi r_1^3 \rho'$$

$$= \frac{4}{3} \times 3.14 \times (0.10)^3 \times 11.5 \times 10^3 = 48.15 \text{ kg}$$

$$m_2 = \frac{4}{3}\pi r_2^3 \rho'$$

$$= \frac{4}{3} \times 3.14 \times (0.01)^3 \times 11.5 \times 10^3 = 0.04815 \text{ kg}$$

As $g = \frac{GM}{R^2} = \frac{G}{R^2} \times \frac{4}{3} \times \pi R^3 \rho$

$$\therefore \quad G = \frac{3g}{4\pi R\rho}$$

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The force of attraction between the two lead spheres is

$$F = G\frac{m_1 m_2}{r^2} = \frac{3g}{4\pi R\rho} \times \frac{m_1 m_2}{r^2}$$

$$= \frac{3 \times 9.8 \times 48.15 \times 0.04815}{4 \times 3.14 \times (6.37 \times 10^6) \times 5.53 \times 10^3 \times (1)^2}$$

$$= 15.4 \times 10^{-11} \text{ N}$$

As the density of brass is less than that of lead, the masses of brass spheres will be smaller than those of lead spheres, so the force of attraction ($F \propto m_1 m_2$) will decrease when lead spheres are replaced by brass spheres.

EXAMPLE 11. Compare the gravitational acceleration of the earth due to attraction of the sun with that due to attraction of the moon. Given that mass of sun, $M_s = 1.99 \times 10^{30}$ kg, mass of moon, $M_m = 7.35 \times 10^{22}$ kg, distance of sun from earth, $r_{es} = 1.49 \times 10^{11}$ m and distance of moon from earth $r_{em} = 3.84 \times 10^8$ m.

Solution. Here $M_s = 1.99 \times 10^{30}$ kg

$M_m = 7.35 \times 10^{22}$ kg

$r_{es} = 1.49 \times 10^{11}$ m

and $r_{em} = 3.84 \times 10^8$ m

Let M_e be mass of earth. If g_{es} is acceleration of the earth due to the attraction of the sun, then

$$M_e g_{es} = G\frac{M_e M_s}{r_{es}^2} \quad \text{or} \quad g_{es} = \frac{GM_s}{r_{es}^2} \qquad ...(i)$$

If g_{em} is acceleration of earth due to the attraction of the moon, then

$$M_e g_{em} = G\frac{M_e M_m}{r_{em}^2} \quad \text{or} \quad g_{em} = \frac{GM_m}{r_{em}^2} \qquad ...(ii)$$

Dividing equation (*i*) by (*ii*), we get

$$\frac{g_{es}}{g_{em}} = \frac{GM_s}{r_{es}^2} \times \frac{r_{em}^2}{GM_m} = \frac{M_s}{M_m} \times \frac{r_{em}^2}{r_{es}^2}$$

$$= \frac{1.99 \times 10^{30}}{7.35 \times 10^{22}} \times \frac{(3.84 \times 10^8)^2}{(1.49 \times 10^{11})^2} = 179.8$$

or $g_{es} : g_{em} = 179.8 : 1$

EXAMPLE 12. A body weighs 90 kg f on the surface of the earth. How much will it weigh on the surface of Mars whose mass is 1/9 and the radius is 1/2 of that of the earth ?

Solution. The acceleration due to gravity on the surface of the earth is given by

$$g_e = \frac{GM_e}{R_e^2} \qquad ...(i)$$

The acceleration due to gravity on the surface of Mars is given by

$$g_m = \frac{GM_m}{R_m^2} \qquad ...(ii)$$

Dividing equation (*ii*) by (*i*), we get

$$\frac{g_m}{g_e} = \frac{M_m}{M_e}\left[\frac{R_e}{R_m}\right]^2 = \frac{1}{9} \times \left[\frac{2}{1}\right]^2 = \frac{4}{9}$$

$$\therefore \quad g_m = \frac{4}{9} g_e$$

Weight on the surface of Mars,

$$W_m = mg_m = \frac{4}{9} mg_e = \frac{4}{9} \times 90 \text{ kg f} = \mathbf{40 \text{ kg f.}}$$

EXAMPLE 13. If the radius of the earth shrinks by 2.0%, mass remaining constant, then how would the value of acceleration due to gravity change ? **[Central Schools 09]**

Solution. Acceleration due to gravity on the surface of the earth is given by

$$g = \frac{GM}{R^2}$$

Taking logarithm of both sides, we get

$$\log g = \log G + \log M - 2 \log R$$

As G and M are constant, so differentiation of the above equation gives

$$\frac{dg}{g} = 0 + 0 - 2\frac{dR}{R}$$

As radius of the earth decreases by 2%, so

$$\frac{dR}{R} = -\frac{2}{100}$$

$$\frac{dg}{g} \times 100 = -2\frac{dR}{R} \times 100$$

$$= -2 \times \left(-\frac{2}{100}\right) \times 100 = \mathbf{4\%}.$$

Thus the value of g increases by 4%.

EXAMPLE 14. A man can jump 1.5 m high on the earth. Calculate the approximate height he might be able to jump on a planet whose density is one-quarter that of the earth and whose radius is one-third of the earth's radius.

Solution. Acceleration due to gravity on the earth's surface is given by

$$g = \frac{4}{3}\pi GR\rho$$

On the planet, $g' = \frac{4}{3}\pi GR'\rho'$

But $R' = \frac{R}{3}$, $\rho' = \frac{\rho}{4}$

$$\therefore \quad g' = \frac{4}{3}\pi G \times \frac{R}{3} \times \frac{\rho}{4} = \frac{1}{12} \times \frac{4}{3}\pi GR\rho = \frac{1}{12} g$$

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Assuming that the man puts in the same energy in jumping high on the earth and the planet, then

$$mg'h' = mgh$$

or $$m \times \frac{1}{12} g \times h' = mgh$$

or $$h' = 12\ h = 12 \times 1.5 = 18 \text{ m}.$$

✕ PROBLEMS FOR PRACTICE

1. A spherical mass of 20 kg lying on the surface of the earth is attracted by another spherical mass of 150 kg with a force equal to the weight of 0.25 mg. The centres of the two masses are 30 cm apart. Calculate the mass of the earth. Radius of the earth $= 6 \times 10^6$ m. **(Ans. 4.8×10^{24} kg)**

2. The period of moon around the earth is 27.3 days and radius of the orbit is 3.9×10^5 km. $G = 6.67 \times 10^{-11}$ Nm^{-2} kg^{-2}, find the mass of the earth. **(Ans. 6.31×10^{24} kg)**

3. Assuming the earth to be a uniform sphere of radius 6400 km and density 5.5 g cm^{-3}, find the value of g on its surface.
Given $G = 6.66 \times 10^{-11}$ Nm^2 kg^{-2}.
(Ans. 9.82 ms^{-2})

4. The mass of Jupiter is 314 times that of earth and the diameter of Jupiter is 11.35 times that of earth. If 'g' has a value of 9.8 ms^{-2} on the earth, what is its value on Jupiter ? **(Ans. 23.90 ms^{-2})**

5. The value of 'g' on the surface of the earth is 9.81 ms^{-2}. Find its value on the surface of the moon. Given mass of earth $= 6.4 \times 10^{24}$ kg, radius of earth $= 6.4 \times 10^6$ m, mass of moon $= 7.4 \times 10^{22}$ kg, radius of moon $= 1.76 \times 10^6$ m. **(Ans. 1.63 ms^{-2})**

6. An astronaut on the moon measures the acceleration due to gravity to be 1.7 ms^{-2}. He knows that the radius of the moon is about 0.27 times that of the earth. Find the ratio of the mass of the earth to that of the moon, if the value of g on the earth's surface is 9.8 ms^{-2}. **(Ans. 79)**

7. The acceleration due to gravity on the surface of the earth is 10 ms^{-2}. The mass of the planet Mars as compared to earth is 1/10 and radius is 1/2. Determine the gravitational acceleration of a body on the surface of Mars. **(Ans. 4 ms^{-2})**

8. A body weighs 100 kg on earth. Find its weight on Mars. The mass and radius of Mars are 1/10 and 1/2 of the mass and radius of earth. **(Ans. 40 kg wt)**

9. The weight of a person on the earth is 80 kg. What will be his weight on the moon ? Mass of the moon $= 7.34 \times 10^{22}$ kg, radius $= 1.75 \times 10^6$ m and gravitational constant $= 6.67 \times 10^{-11}$ Nm^2 kg^{-2}. What will be the mass of the person at the moon and acceleration due to gravity there ? If this person can jump 2 m high on the earth, how much high can he jump at the moon ?
(Ans. 128 N, 80 kg, 1.6 ms^{-2}, about 12 m)

10. On a planet whose size is the same and mass 4 times as that of the earth, find the energy needed to lift a 2 kg mass vertically upwards through 2 m distance in joule. The value of g on the surface of earth is 10 ms^{-2}. **(Ans. 160 J)**

✕ HINTS

1. Here $m_1 = 20$ kg, $m_2 = 150$ kg, $r = 30$ cm $= 0.30$ m
$F = 0.25$ mg wt $= 0.25 \times 10^{-6}$ kg wt
$= 25 \times 10^{-8} \times 9.8$ N

$$G = \frac{Fr^2}{m_1 m_2} = \frac{25 \times 9.8 \times 10^{-8} \times (0.3)^2}{20 \times 150}$$

$$= 7.35 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$$

Mass of the earth,

$$M = \frac{g R^2}{G} = \frac{9.8 \times (6 \times 10^6)^2}{7.35 \times 10^{-11}} = 4.8 \times 10^{24} \text{ kg}.$$

4. Given $\frac{M_j}{M_e} = 3\ M$

$$\frac{\text{Diameter of Jupiter}}{\text{Diameter of earth}} = 11.35$$

$$\therefore \quad \frac{R_j}{R_e} = 11.35$$

Now $$g_e = \frac{GM_e}{R_e^2} \text{ and } g_j = \frac{GM_j}{R_j^2}$$

$$\therefore \quad \frac{g_j}{g_e} = \frac{M_j}{M_e}\left[\frac{R_e}{R_j}\right]^2 = 314 \times \left(\frac{1}{11.35}\right)^2$$

or $$g_j = \frac{314}{11.35 \times 11.35} \times g_e = \frac{314 \times 9.8}{11.35 \times 11.35}$$

$$= 23.90 \text{ ms}^{-2}.$$

5. $$g_m = \frac{M_m}{M_e}\left[\frac{R_e}{R_m}\right]^2 \times g_e$$

$$= \frac{7.4 \times 10^{22}}{6 \times 10^{24}} \times \left[\frac{6.4 \times 10^6}{1.74 \times 10^6}\right]^2 \times 9.8$$

$$= \frac{7.4 \times 6.4 \times 6.4 \times 9.8 \times 10^{-2}}{6 \times 1.74 \times 1.74} = 1.63 \text{ ms}^{-2}.$$

6. Here $$g_e = \frac{GM_e}{R_e^2}$$

and $$g_m = \frac{GM_m}{R_m^2}$$

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$$\therefore \quad \frac{g_e}{g_m} = \frac{M_e}{M_m} \cdot \frac{R_m^2}{R_e^2}$$

or $$\frac{M_e}{M_m} = \frac{g_e}{g_m}\left[\frac{R_e}{R_m}\right]^2 = \frac{9.8}{1.7} \times \frac{1}{(0.27)^2} = 79.$$

9. Here $$g_m = \frac{GM_m}{R_m^2} = \frac{6.67 \times 10^{-11} \times 7.34 \times 10^{22}}{(1.75 \times 10^6)^2}$$

$$= 1.6 \text{ ms}^{-2}$$

Mass of the person on the moon = 80 kg

Weight on the moon = mg = 80 × 1.6 = **128 N**

As $mg'h' = mgh$

$$\therefore \quad h' = \frac{gh}{g'} = \frac{9.8 \times 2}{1.6} \simeq 12 \text{ m.}$$

10. On the earth, $$g = \frac{GM}{R^2};$$

On the planet, $$g' = \frac{4GM}{R^2} = 4g$$

8.11 VARIATIONS IN ACCELERATION DUE TO GRAVITY

17. *What are the various factors on which the value of g at any place on the earth depends ?*

Factors on which g depends. The value of g changes from place to place. It depends on various factors such as (*i*) altitude (*ii*) depth (*iii*) shape of the earth and (*iv*) rotation of the earth.

8.12 VARIATION OF g WITH ALTITUDE (HEIGHT)

18. *Discuss the variation of g with altitude.*

Effect of altitude on g. Consider the earth to be a sphere of mass M, radius R and centre O. Then the acceleration due to gravity at a point A on the surface of the earth will be

$$g = \frac{GM}{R^2} \qquad ...(i)$$

If g_h is the acceleration due to gravity at a point B at a height h from the earth's surface, then

$$g_h = \frac{GM}{(R+h)^2} \qquad ...(ii)$$

Dividing equation (*ii*) by (*i*), we get

$$\frac{g_h}{g} = \frac{GM}{(R+h)^2} \times \frac{R^2}{GM}$$

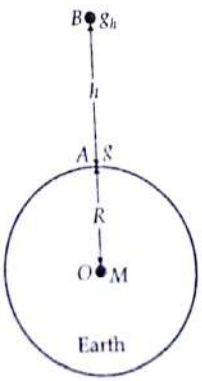

Fig. 8.13 Effect of altitude on g.

or $$\frac{g_h}{g} = \frac{R^2}{(R+h)^2} \qquad ...(iii)$$

or $$\frac{g_h}{g} = \frac{R^2}{R^2\left(1+\frac{h}{R}\right)^2} = \left(1+\frac{h}{R}\right)^{-2}$$

Expanding R.H.S. by using binomial theorem, we get

$$\frac{g_h}{g} = 1 - \frac{2h}{R} + \text{terms containing higher powers of } \frac{h}{R}$$

If $h << R$, then $\frac{h}{R} << 1$, so that higher powers of $\frac{h}{R}$ can be neglected, we get

$$\frac{g_h}{g} = 1 - \frac{2h}{R}$$

or $$g_h = g\left(1 - \frac{2h}{R}\right) \qquad ...(iv)$$

Both equations (*iii*) and (*iv*) show that the value of acceleration due to gravity decreases with the increase in height h, that is why the value of g is less at mountains than at plains. While solving numerical problems, equation (*iii*) should be used when h is comparable to R and equation (*iv*) should be used when $h << R$.

For Your Knowledge

- The decrease in the value of g at height h is
$$g - g_h = \frac{2gh}{R}$$
Clearly $g - g_h \propto h$
- The percentage decrease in the value of g at height h is
$$\frac{g - g_h}{g} \times 100 = \frac{2h}{R} \times 100\%$$
- The loss in weight of a body at a height h
$$= mg - mg_h = \frac{2mgh}{R}.$$
- At an altitude h = 320 km, g_h = 0.9 g, *i.e.*, the value of g decreases by 10%.
- A typical space shuttle altitude is h = 400 km.

Examples based on Variation of 'g' with Altitude

FORMULAE USED

1. $g_h = g \cdot \frac{R^2}{(R+h)^2}$, when h is comparable to R
2. $g_h = g\left(1 - \frac{2h}{R}\right)$, when $h << R$

UNITS USED

Here g and g_h are in ms^{-2}, and h and R are in metre.

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Example 15. At what height from the surface of the earth, will the value of g be reduced by 36% from the value at the surface ? Radius of the earth = 6400 km. **[Delhi 11]**

Solution. Suppose at height h, the value of g reduces by 36% *i.e.*, it becomes 64% of that at the surface. Thus

$$g_h = 64\% \text{ of } g = \frac{64}{100} g$$

But $$g_h = g \frac{R^2}{(R+h)^2}$$

$$\frac{64}{100} g = g \frac{R^2}{(R+h)^2} \quad \text{or} \quad \frac{8}{10} = \frac{R}{R+h}$$

or $$h = \frac{R}{4} = \frac{6400}{4} = \mathbf{1600\ km.}$$

Example 16. At what height above the earth's surface, the value of g is half of its value on earth's surface ? Given its radius is 6400 km.

Solution. Here $g_h = g/2$

But $$g_h = g\left(\frac{R}{R+h}\right)^2$$

$$\therefore \quad \frac{g}{2} = g\left(\frac{R}{R+h}\right)^2 \quad \text{or} \quad \left(\frac{R}{R+h}\right)^2 = \frac{1}{2}$$

or $$\frac{R+h}{R} = \sqrt{2}$$

or $$h = (\sqrt{2} - 1)\, R = 0.414\ R = 0.414 \times 6400$$
$$= \mathbf{2649.6\ km.}$$

Example 17. Find the percentage decrease in the weight of a body when taken to a height of 32 km above the surface of the earth. Radius of the earth is 6400 km. **[Delhi 05]**

Solution. Here $h = 32$ km, $R = 6400$ km

As $h << R$, so

$$g_h = g\left(1 - \frac{2h}{R}\right) = g - \frac{2gh}{R}$$

or $$g - g_h = \frac{2gh}{R}$$

Percent decrease in weight

$$= \frac{mg - mg_h}{mg} \times 100 = \frac{g - g_h}{g} \times 100$$

$$= \frac{2gh}{g \times R} \times 100 = \frac{2h}{R} \times 100$$

$$= \frac{2 \times 32}{6400} \times 100 = \mathbf{1\%.}$$

Example 18. A mass of 0.5 kg is weighed on a balance at the top of a tower 20 m high. The mass is then suspended from the pan of the balance by a fine wire 20 m long and is reweighed. Find the change in weight. Assume that the radius of the earth is 6400 km.

Solution. At a height h ($<< R$), we have

$$g_h = g\left(1 - \frac{2h}{R}\right) = g - \frac{2gh}{R}$$

$$\therefore \quad g - g_h = \frac{2gh}{R}$$

Change in weight = Wt. at the foot of tower − Wt. at the top of the tower

$$= mg - mg_h = m(g - g_h) = \frac{2\ mgh}{R}$$

But $mg = 0.5$ kg f, $h = 20$ m,

$R = 6400$ km $= 6.4 \times 10^6$ m

$\therefore$ Change in weight

$$= \frac{2 \times 0.5 \times 20}{6.4 \times 10^6} = 3.125 \times 10^{-6} \text{ kg f.}$$

Example 19. A body hanging from a spring stretches it by 1 cm at the earth's surface. How much will the same body stretch the spring at a place 1600 km above the earth's surface ? Radius of the earth = 6400 km.

Solution. In equilibrium, weight of the suspended body = Stretching force

$\therefore$ At the earth's surface, $mg = k \times x$

At a height h, $mg' = k \times x'$

$$\frac{g'}{g} = \frac{x'}{x} = \frac{R^2}{(R+h)^2} = \frac{(6400)^2}{(6400+1600)^2}$$

$$= \left(\frac{6400}{8000}\right)^2 = \frac{16}{25}.$$

or $$x' = \frac{16}{25} \times x = \frac{16}{25} \times 1 \text{ cm} = \mathbf{0.64\ cm.}$$

Problems For Practice

1. The radius of the earth is 6000 km. What will be the weight of a 120 kg body if it is taken to a height of 2000 km above the surface of the earth ?
(Ans. 67.5 kg f)

2. A body of mass m is raised to a height h from the surface of the earth where the acceleration due to gravity is g. Prove that the loss in weight due to variation in g is approximately $2\ mgh/R$, where R is the radius of the earth.

3. The Mount Everest is 8848 m above sea level. Estimate the acceleration due to gravity at this height, given that mean g on the surface of the earth is 9.8 ms^{-2} and mean radius of the earth is 6.37×10^6 m.
(Ans. 9.772 ms^{-2})

4. At what height above the surface of the earth will the acceleration due to gravity be 25% of its value on the surface of the earth ? Assume that the radius of the earth is 6400 km.
(Ans. 6400 km)

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5. Find the value of g at a height of 400 km above the surface of the earth. Given radius of the earth, $R = 6400$ km and value of g at the surface of the earth = 9.8 ms^{-2}.
[Central Schools 12] (Ans. 8.575 ms^{-2})

6. How far away from the surface of earth does the acceleration due to gravity become 4% of its value on the surface of earth ? Radius of earth = 6400 km.
[Delhi 98] (Ans. 25,600 km)

HINTS

1. Here $mg = 120$ kg f, $h = 200$ km, $R = 6000$ km

$$g_h = \left(\frac{R}{R+h}\right)^2 \quad g = \left(\frac{6000}{8000}\right)^2 \times g = \frac{9}{16} g$$

$$\therefore \quad mg_h = \frac{9}{16} mg = \frac{9}{16} \times 120 = 67.5 \text{ kg f.}$$

2. For $h < R$,

$$g_h = g\left(1 - \frac{2h}{R}\right) = g - \frac{2gh}{R}$$

or $$g - g_h = \frac{2gh}{R}$$

$\therefore$ Loss in weight due to variation in g

$$= mg - mg_h = m(g - g_h)$$

$$= \frac{m \times 2gh}{R} = \frac{2mgh}{R}.$$

3. Here $h = 8848$ m, $g = 9.8$ ms^{-2}, $R = 6.37 \times 10^6$ m

Acceleration due to gravity at height h (when $h < R$) is given by

$$g_h = g\left(1 - \frac{2h}{R}\right) = 9.8\left(1 - \frac{2 \times 8848}{6.37 \times 10^6}\right)$$

$$= 9.8(1 - 0.002778) = 9.772 \text{ ms}^{-2}.$$

6. $$\frac{4}{100} g = g\left(\frac{R}{R+h}\right)^2$$

$$\frac{2}{10} = \frac{R}{R+h}$$

$\therefore$ $$h = 4R = 4 \times 6400 = 25{,}600 \text{ km.}$$

8.13 VARIATION OF g WITH DEPTH

19. *Discuss the variation of g with depth.*

Effect of depth on g. Consider the earth to be a sphere of mass M, radius R and centre O. The acceleration due to gravity at any point A on the surface of the earth will be

$$g = \frac{GM}{R^2}$$

Assuming the earth to be a homogeneous sphere of average density ρ, then its total mass will be

$$M = \text{Volume} \times \text{density} = \frac{4}{3}\pi R^3 \rho$$

$\therefore$ $$g = \frac{G \times \frac{4}{3}\pi R^3 \rho}{R^2}$$

or $$g = \frac{4}{3}\pi G R \rho$$

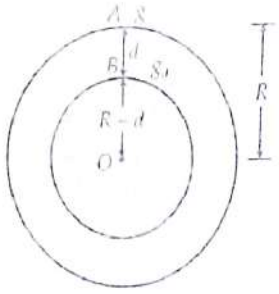

Fig. 8.14 Effect of depth on g.

Let g_d be the acceleration due to gravity at a point B at depth d below the surface of the earth. A body at B is situated at the surface of inner solid sphere and lies inside the spherical shell of thickness d. The gravitational force of attraction on a body inside a spherical shell is always zero. Therefore, a body at B experiences gravitational force due to inner shaded sphere of radius $(R - d)$ and mass M', where

$$M' = \frac{4}{3}\pi(R-d)^3\rho$$

$$\therefore \quad g_d = \frac{GM'}{(R-d)^2} = \frac{G}{(R-d)^2} \times \frac{4}{3}\pi(R-d)^3\rho$$

or $$g_d = \frac{4}{3}\pi G(R-d)\rho$$

$$\frac{g_d}{g} = \frac{\frac{4}{3}\pi G(R-d)\rho}{\frac{4}{3}\pi GR\rho} = \frac{R-d}{R} = 1 - \frac{d}{R}$$

or $$g_d = g\left(1 - \frac{d}{R}\right)$$

Clearly, *the acceleration due to gravity decreases with the increase in depth d.* That is why the acceleration due to gravity is less in mines than that on earth's surface.

20. *How much will be the weight of a body at the centre of the earth ?*

Weight of a body at the centre of the earth. At the centre of the earth, $d = R$,

$$g_d = g\left(1 - \frac{R}{R}\right) = 0$$

Weight of a body of mass m at the centre of the earth,

$$= mg_d = m \times 0 = 0.$$

Hence *the weight of a body at the centre of the earth is zero though its mass is not zero.*

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For Your Knowledge

- The acceleration due to gravity decreases both with the increase in height and increase in depth. So *it is maximum at the surface of the earth and zero at the centre of the earth.*
- Decrease in the value of g at depth d is
$$g - g_d = \frac{d}{R} g$$
- Percentage decrease in the value of g at depth d is
$$\frac{g - g_d}{g} \times 100 = \frac{d}{R} \times 100\%$$

21. *What is the relation between height h and depth d for the same change in g ?*

Relation between height h and depth d for the same change in g. Acceleration due to gravity at a height h above the earth's surface,
$$g_h = g\left(1 - \frac{2h}{R}\right)$$

Acceleration due to gravity at a depth d below the earth's surface,
$$g_d = g\left(1 - \frac{d}{R}\right)$$

For the same change in g, we have
$$g_h = g_d$$
$$\therefore \quad 1 - \frac{2h}{R} = 1 - \frac{d}{R} \quad \text{or} \quad \frac{2h}{R} = \frac{d}{R} \quad \text{or} \quad d = 2h$$

Hence the *acceleration due to gravity at a height h above the earth's surface will be same as that at depth $d = 2h$, below the earth's surface.* But this fact holds only when $h << R$.

Examples based on Variation of 'g' with Depth

FORMULAE USED

1. At a depth d, $g_d = g\left(1 - \frac{d}{R}\right)$
2. When $d = 2h$, $g_h = g_d$

UNITS USED

Here g and g_h and g_d are in ms^{-2}, and d, h and R are in metre.

EXAMPLE 20. Find the percentage decrease in weight of a body, when taken 16 km below the surface of the earth. Take radius of the earth as 6400 km.

Solution. Here $R = 6400$ km, $d = 16$ km
$$g_d = g\left(1 - \frac{d}{R}\right) = g\left(1 - \frac{16}{6400}\right) = \frac{399}{400} g$$
$$\therefore \quad g - g_d = g - \frac{399}{400} g = \frac{1}{400} g$$

The percentage decrease in weight of the body
$$= \frac{mg - mg_d}{mg} \times 100 = \frac{g - g_d}{g} \times 100$$
$$= \frac{(1/400)\, g}{g} \times 100 = \mathbf{0.25\%}.$$

EXAMPLE 21. How much below the surface of the earth does the acceleration due to gravity become 1% of its value at the earth's surface ? Radius of the earth = 6400 km.

Solution. Here $g_d = 1\%$ of $g = \frac{g}{100}$

But $\quad g_d = g\left(1 - \frac{d}{R}\right)$

$$\therefore \quad \frac{g}{100} = g\left(1 - \frac{d}{R}\right)$$

or $\quad \frac{d}{R} = 1 - \frac{1}{100} = \frac{99}{100}$

or $\quad d = \frac{99}{100} \times R = \frac{99}{100} \times 6400 = \mathbf{6336\ km}.$

EXAMPLE 22. At what height above the earth's surface, the value of g is same as in a mine 80 km deep ?

Solution. Let h be the height at which 'g' is same as that at depth d. Now
$$g_h = g_d \quad \text{or} \quad g\left(1 - \frac{2h}{R}\right) = g\left(1 - \frac{d}{R}\right)$$

or $\quad \frac{2h}{R} = \frac{d}{R}$

$$\therefore \quad h = \frac{d}{2} = \frac{80}{2} = \mathbf{40\ km}.$$

EXAMPLE 23. Imagine a tunnel dug along a diameter of the earth. Show that a particle dropped from one end of the tunnel executes simple harmonic motion. What is the time period of this motion ? Assume the earth to be a sphere of uniform mass density (equal to its known average density $= 5520\ kg\ m^{-3}$.) $G = 6.67 \times 10^{-11}\ Nm^2\ kg^{-2}$. Neglect all damping forces.

Solution. The acceleration due to gravity at a depth d below the earth's surface is given by
$$g_d = g\left(1 - \frac{d}{R}\right)$$

or $\quad g_d = g\left(\frac{R - d}{R}\right) = \frac{g}{R} y$

where $y = R - d =$ Distance of the body from the centre of the earth.

Thus $\quad g_d \propto y$

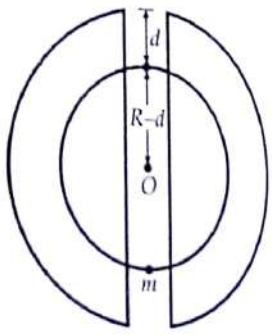

Fig. 8.15

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As the acceleration is proportional to displacement and is directed towards the mean position, so the motion of the body is simple harmonic. Its time period is given by

$$T = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}} = 2\pi\sqrt{\frac{y}{g_d}} = 2\pi\sqrt{\frac{R}{g}}$$

But $$g = \frac{GM}{R^2} = \frac{G}{R^2} \times \frac{4}{3}\pi R^3 \rho = \frac{4}{3}\pi GR\rho$$

$$\therefore \quad T = 2\pi\sqrt{\frac{R}{\frac{4}{3}\pi GR\rho}} = 2\pi\sqrt{\frac{3}{4\pi G\rho}} = \sqrt{\frac{3\pi}{G\rho}}$$

$$\therefore \quad = \sqrt{\frac{3 \times 3.142}{6.67 \times 10^{-11} \times 5520}} = 5059.77 \text{ s.}$$

✕ PROBLEMS FOR PRACTICE

1. Find the value of acceleration due to gravity in a mine at a depth of 80 km from the surface of the earth. Radius of the earth = 6400 km. **(Ans. 9.68 ms^{-2})**
2. Calculate the depth below the surface of the earth where acceleration due to gravity becomes half of its value at the surface of the earth. Radius of the earth = 6400 km. **(Ans. 3200 km)**
3. How much below the surface of the earth does the acceleration due to gravity become 70% of its value at the surface of the earth ? Radius of the earth is 6400 km. **(Ans. 1920 km)**
4. How much below the surface of the earth does the acceleration due to gravity (*i*) reduces to 36% (*ii*) reduces by 36% of its value on the surface of the earth ? Radius of the earth = 6400 km. **[Delhi 13]**

[Ans. (*i*) 4096 km (*ii*) 2304 km]

5. Compare the weights of a body when it is (*i*) 100 km above the surface of the earth and (*ii*) 100 km below the surface of the earth. Radius of the earth is 6300 km. **(Ans. 0.984)**

✕ HINTS

5. At a height h, $g_h = g\left(1 - \frac{2h}{R}\right)$

At a depth d, $g_d = g\left(1 - \frac{d}{R}\right)$

$$\therefore \quad \frac{g_h}{g_d} = \frac{1 - \frac{2h}{R}}{1 - \frac{d}{R}} = \frac{R - 2h}{R - d}$$

or $$\frac{W_h}{W_d} = \frac{mg_h}{mg_d} = \frac{R - 2h}{R - d}$$

But $R = 6300$ km, $h = 100$ km, $d = 100$ km

$$\therefore \quad \frac{W_h}{W_d} = \frac{6300 - 200}{6300 - 100} = \frac{6100}{6200} = 0.984.$$

8.14 VARIATION OF g WITH SHAPE OF THE EARTH

22. *Discuss the variation of g on earth's surface due to shape of the earth. Why does the weight of a body increase when it is taken from the equator to the pole ?*

Effect of shape of earth on g. Earth is not perfect sphere. It is flattened at the poles and bulges out at the equator. So the equatorial radius R_e of the earth is greater than the polar radius R_p by about 21 km.

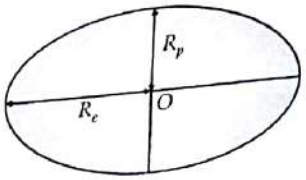

Fig. 8.16 Equatorial and polar radii of the earth.

Acceleration due to gravity on the earth's surface is given by

$$g = \frac{GM}{R^2}$$

i.e., $$g \propto \frac{1}{R^2} \qquad [\because G, M \text{ are constant}]$$

As $R_e > R_p$, so $g_e < g_p$.

Thus the value of g is minimum at the equator and maximum at the poles. That is why the weight of a body increases when it is taken from the equator to the pole. The variation of g between the poles and the equator is about 0.5%.

8.15 VARIATION OF 'g' WITH LATITUDE (OR ROTATION OF THE EARTH)

23. *Define latitude at a place.*

Latitude. The plane passing through the centre of the earth and perpendicular to its axis of rotation is called its equatorial plane. *The latitude of a place is defined as the angle which the line joining the place to the centre of the earth makes with the equatorial plane.* In Fig. 8.17, the latitude of place $P = \angle POE = \lambda$. Clearly, $\lambda = 0°$ at the equator and $\lambda = 90°$ at the poles.

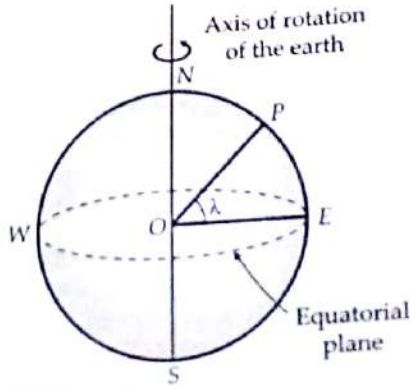

Fig. 8.17 Latitude.

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24. *Explain how is the acceleration due to gravity affected at a latitude due to the rotational motion of the earth.*

Effect of latitude or rotation of earth on g. Refer to Fig. 8.18. As the earth rotates about its polar axis, every particle lying on its surface also revolves along a horizontal circle with the same angular velocity ω.

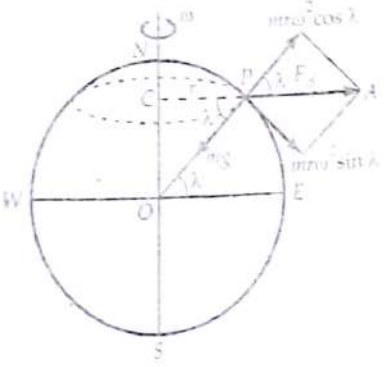

Fig. 8.18 Effect of rotation on g.

Consider a particle of mass m lying at point P, whose latitude is λ. The particle P describes a horizontal circle of radius,

$$r = PC = R\cos\lambda$$

The centrifugal force acting on the particle is $F_{cf} = mr\omega^2$, acting along PA.

This force has two rectangular components : $mr\omega^2\cos\lambda$ and $mr\omega^2\sin\lambda$. The component $mr\omega^2\sin\lambda$ acts perpendicular to mg and has no effect on it. The component $mr\omega^2\cos\lambda$ acts opposite to mg. So the apparent weight of the particle P is

$$mg_\lambda = mg - mr\omega^2\cos\lambda$$

or $$g_\lambda = g - r\omega^2\cos\lambda$$

or $$g_\lambda = g - R\omega^2\cos^2\lambda \qquad [\because r = R\cos\lambda]$$

As λ increases, $\cos\lambda$ decreases and g_λ increases. So *as we move from equator to pole, the acceleration due to gravity increases.*

Special cases :

(i) At the equator, $\lambda = 0°$, $\cos\lambda = 1$, hence

$$g_e = g - R\omega^2$$

(ii) At the poles, $\lambda = 90°$, $\cos\lambda = 0$, hence

$$g_p = g - R\omega^2 \times 0 = g$$

Thus acceleration due to gravity is minimum at the equator and maximum at the poles. The difference in the two values is

$$g_p - g_e = g - (g - R\omega^2) = R\omega^2$$

25. *Draw a graph showing the variation of acceleration due to gravity g with distance r from the centre of the earth.*

For *points lying outside* the earth ($r > R$)

$$\frac{g_h}{g} = \frac{R^2}{(R+h)^2} = \frac{R^2}{r^2} \quad \text{or} \quad g_h = \frac{gR^2}{r^2}$$

$$\Rightarrow \qquad g_h \propto \frac{1}{r^2}$$

For *points lying inside* the earth ($r < R$)

$$g_d = \frac{g(R-d)}{R} = \frac{gr}{R} \quad \text{or} \quad g_d = \frac{gr}{R}$$

$$\Rightarrow \qquad g_d \propto r$$

Hence the graph showing the variation of acceleration due to gravity g with distance r from the centre of the earth is of the type as shown in Fig. 8.19.

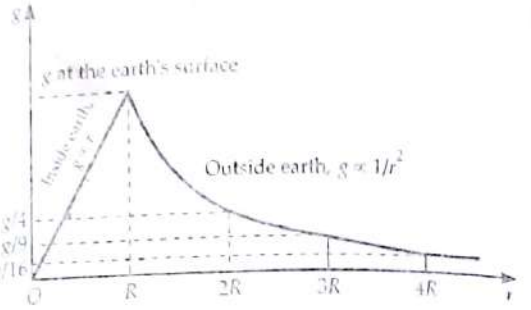

Figure 8.19 Variation of g with distance r from the centre of the earth.

8.16 VARIATION OF 'g' DUE TO OTHER FACTORS

26. *State some factors (other than altitude, depth and latitude) on which g depends. What is the utility of measuring g accurately on earth surface ?*

Variation of g due to other factors. The earth's surface is uneven. It has mountains, plateaus and valleys. This causes a variation in the value of g. Moreover, the density of earth is not uniform. Its inner core is heavier than the mantle. Also, the density of earth's crust varies from place to place. Hence the value of g is different at different places.

When g is measured accurately, its variations indicate the presence of oil and heavy minerals. Hence such studies are useful in oil and mineral explorations.

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- Acceleration due to gravity decreases due to rotation of the earth ($g' < g$).
- Acceleration due to gravity increases with the increase in latitude of the place.
- The effect of rotation of the earth is maximum at the equator and minimum at the poles. In fact, rotational motion of the earth has no effect on the value of g at the poles.
- If the earth stops rotating, the weight of a body would increase due to the absence of the centrifugal force.
- Both the rotation of the earth and its equatorial bulge contribute additively to lower the value of g at the equator than at the poles.
- Even for the rotating earth, the direction of acceleration due to gravity is towards the centre of the earth both at the equator ($\lambda = 0°$) and at the poles ($\lambda = \pm 90°$). At intermediate latitudes, this direction slightly deviates from the centre of the earth. The maximum deviation is about 0.1°.

Examples based on Variation of 'g' with Rotation of the Earth

Formulae Used

1. At latitude λ, $g_\lambda = g - R\omega^2 \cos^2 \lambda$
2. At equator, $\lambda = 0°$, so $g_e = g - R\omega^2$
3. At poles, $\lambda = 90°$, so $g_p = g$
4. $g_p - g_e = R\omega^2$

Units Used

Here g, g_e, g_p, g_λ are in ms^{-2}, latitude λ in degrees, angular velocity ω in $rad\ s^{-1}$, and R in metre.

Example 24. Calculate that imaginary angular velocity of the earth for which effective acceleration due to gravity at the equator becomes zero. In this condition what will be the length (in hours) of the day ? Given radius of the earth = 6400 km and $g = 10\ ms^{-2}$.

Solution. At latitude λ, $g_\lambda = g - R\omega^2 \cos^2 \lambda$

At equator, $\lambda = 0°$, so $g_e = g - R\omega^2$

When $g_e = 0$, we have,

$$\omega = \sqrt{\frac{g}{R}} = \sqrt{\frac{10}{6400 \times 10^3}} = 1.25 \times 10^{-3}\ rad\ s^{-1}.$$

In this condition, the new period of rotation or the length of the day will be

$$T = \frac{2\pi}{\omega} = \frac{2 \times 3.14}{1.25 \times 10^{-3}} = 5024\ s = 1.4\ h.$$

Example 25. Determine the speed with which the earth would have to rotate on its axis so that a person on the equator would weigh 3/5th as much as at present. Take the equatorial radius as 6400 km. **[Roorkee 88]**

Solution. Acceleration due to gravity at the equator is

$$g_e = g - R\omega^2$$

$$\therefore \quad mg_e = mg - mR\omega^2$$

or $$\frac{3}{5}mg = mg - mR\omega^2 \qquad \left[\because\ mg_e = \frac{3}{5}mg\right]$$

$$\therefore\ \omega = \sqrt{\frac{2}{5}\frac{g}{R}} = \sqrt{\frac{2 \times 9.8}{5 \times 6400 \times 10^3}} = 7.8 \times 10^{-4}\ rad\ s^{-1}.$$

Example 26. If the earth were a perfect sphere of radius 6.37×10^6 m, rotating about its axis with a period of 1 day ($= 8.64 \times 10^4$ s), how much would the acceleration due to gravity (g) differ from the poles to the equator ?

Solution. Acceleration due to gravity at the latitude λ is given by $g_\lambda = g - R\omega^2 \cos^2 \lambda$

At equator, $\lambda = 0°$,

so $$g_e = g - R\omega^2$$

At poles, $\lambda = 90°$, so $g_p = g$

$$\therefore \quad g_p - g_e = g - (g - R\omega^2)$$

$$= R\omega^2 = \frac{4\pi^2 R}{T^2} \qquad \left[\because\ \omega = \frac{2\pi}{T}\right]$$

But $T = 1$ day $= 8.64 \times 10^4$ s, $R = 6.37 \times 10^6$ m

$$\therefore \quad g_p - g_e = \frac{4 \times 9.87 \times 6.37 \times 10^6}{(8.64 \times 10^4)^2}$$

$$= 3.37 \times 10^{-2}\ ms^{-2} = 3.4\ cms^{-2}.$$

Problems for Practice

1. Calculate the value of acceleration due to gravity at a place of latitude 45°. Radius of the earth $= 6.38 \times 10^3$ km. (Ans. 9.783 ms^{-2})
2. If the earth stops rotating about its axis, then what will be the change in the value of g at a place in the equatorial plane ? Radius of the earth = 6400 km. (Ans. 3.4 cms^{-2})
3. Assuming that the whole variation of the weight of a body with its position on the surface of the earth is due to its rotation, find the difference in the weight of 5 kg as measured at the equator and at the poles. Radius of the earth = 6.4×10^6 m. (Ans. 17.2 gf)
4. How many times faster than its present speed the earth should rotate so that the apparent weight of an object at equator becomes zero ? Given radius of the earth = 6.37×10^6 m. What would be the duration of the day in that case ? (Ans. 17 times faster, 1.412 h)

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✕ HINTS

1. Here $\lambda = 45°$, $R = 6.38 \times 10^6$ m, $g = 9.8$ ms^{-2}

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{24 \times 60 \times 60} = 7.27 \times 10^{-5} \text{ rad s}^{-1}$$

$$\therefore \quad g_\lambda = g - R\omega^2 \cos^2 \lambda$$
$$= 9.8 - 6.38 \times 10^6 \times (7.27 \times 10^{-5})^2 \times (\cos 45°)^2$$
$$= 9.8 - 0.01687 = 9.783 \text{ ms}^{-2}.$$

2. Change in the value of g

$$= g - g_e = R\omega^2 = R\left(\frac{2\pi}{T}\right)^2$$
$$= 6.4 \times 10^6 \times \left(\frac{2\pi}{24 \times 60 \times 60}\right)^2$$
$$= 3.37 \times 10^{-2} \text{ ms}^{-2} = \mathbf{3.4 \text{ cms}^{-2}}.$$

3. Change in the weight of the body when taken from pole to equator

$$= m(g_p - g_e) = mR\omega^2 = mR\left(\frac{2\pi}{T}\right)^2$$
$$= 5 \times 3.37 \times 10^{-2} \text{ newton}$$
$$= \frac{5 \times 3.37 \times 10^{-2}}{9.8} \text{ kg f} = \mathbf{17.2 \text{ g f}}.$$

4. At the equator, we have

$$g_\lambda = g - R\omega_1^2 \quad \text{or} \quad mg_\lambda = mg - mR\omega_1^2$$

As the apparent weight at equator is zero,

so $mg_\lambda = 0$

$\therefore \quad mg - mR\omega_1^2 = 0$

or $g = R\omega_1^2$

$$\therefore \quad \omega_1 = \sqrt{\frac{g}{R}} = \sqrt{\frac{9.8}{6.37 \times 10^6}} = 1.241 \times 10^{-3} \text{ rad s}^{-1}.$$

But the present speed of the earth is

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{24 \times 60 \times 60} = 7.27 \times 10^{-5} \text{ rad s}^{-1}$$

$$\therefore \quad \frac{\omega_1}{\omega} = \frac{1.241 \times 10^{-3}}{7.27 \times 10^{-5}} = 17.06 = \mathbf{17}.$$

or $\omega_1 = 17\,\omega$

i.e. the earth should rotate 17 times faster than its present speed.

New duration of the day $= \frac{24}{17} = \mathbf{1.412 \text{ h}}$.

8.17 GRAVITATIONAL FIELD

27. *Give the concept of gravitational field.*

Gravitational field. Two bodies attract each other by the gravitational force even if they are not in direct contact. This interaction is called *action at a distance*. It can be best explained in terms of concept of field. According to the field concept,

(*i*) Every mass modifies the space around it. This modified space is called gravitational field.

(*ii*) When any other mass is placed in this field, it feels a gravitational force of attraction due to its interaction with the gravitational field.

The space surrounding a material body within which its gravitational force of attraction can be experienced is called its gravitational field.

The earth is surrounded by a gravitational field. Any body brought in this field experiences a force of attraction towards the centre of the earth.

8.18 INTENSITY OF GRAVITATIONAL FIELD

28. *Define intensity of gravitational field at any point. Is it a scalar or vector ?*

Intensity of gravitational field or gravitational field strength. *The gravitational field intensity at any point in the gravitational field due to a given mass is defined as the force experienced by a unit mass placed at that point provided the presence of unit mass does not disturb the original gravitational field.*

The gravitational field intensity is a vector quantity, denoted by $\vec{E}$. It always acts towards the mass producing the gravitational field.

29. *Show that gravitational field intensity at any point is equal to the free acceleration of a test mass placed at that point.*

Ans. Intensity of gravitational field due to a body. Consider a body of mass M. To determine its gravitational field intensity at a point P at distance r from its centre O, place a test mass m ($m << M$) at the point P.

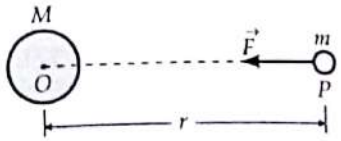

Fig. 8.20 Gravitational field intensity.

Let $\vec{F}$ be the force of gravitation experienced by test mass m. The gravitational field intensity at point P will be

$$\vec{E} = \frac{\vec{F}}{m} \qquad \text{...(i)}$$

The direction of $\vec{E}$ is same as that of $\vec{F}$.

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According to Newton's law of gravitation,

$$F = \frac{GMm}{r^2}$$

$$\therefore \quad E = \frac{F}{m} = \frac{GM}{r^2} \qquad ...(ii)$$

At $r = \infty$, $E = 0$. Thus gravitational field intensity decreases as distance r increases and becomes zero at infinity.

If the test mass m is free to move, it will move towards the mass M with acceleration a under the force F, so

$$a = \frac{F}{m} \qquad ...(iii)$$

From equations (*ii*) and (*iii*), we get $a = E$

Thus the intensity of gravitational field at any point is equal to the free acceleration produced in the test mass when placed at that point.

30. *Show that the gravitational field intensity of the earth at any point is equal to the acceleration produced in the freely falling body at that point.*

Intensity of gravitational field due to earth. As shown in Fig. 8.21, let earth be a sphere of radius R and mass M. Suppose a test mass m be placed at a point P at distance r from its centre O. According to Newton's law of gravitation, the force of attraction on test mass m is

$$F = \frac{GMm}{r^2}$$

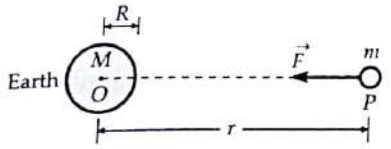

Fig. 8.21. Gravitational field intensity due to earth.

The gravitational field intensity at point P will be

$$E = \frac{F}{m} = \frac{GM}{r^2}$$

But GM/r^2 is equal to the acceleration due to gravity at the point P. *Hence the gravitational field intensity of the earth at any point is equal to acceleration produced in the freely falling body at that point.*

For any point on the surface of the earth, $r = R$, so

$$E_{surface} = \frac{GM}{R^2} = g$$

This is the acceleration due to gravity at the surface of the earth.

31. *Give the units and dimensions of gravitational field intensity.*

Units of E. As gravitational field intensity is force per unit mass, so its SI unit is Nkg^{-1} and cgs unit is dyn g^{-1}.

Dimensions of E. As $E = \frac{F}{m}$

$$\therefore \text{ Dimensions of } E = \frac{MLT^{-2}}{M} = [LT^{-2}].$$

8.19 GRAVITATIONAL POTENTIAL ENERGY

32. *What is meant by gravitational potential energy of a body ? What is the zero level of potential energy ?*

Gravitational potential energy. When two bodies are placed close to one another, they interact through the gravitational force. Due to this, they possess mutual gravitational potential energy. When the distance between the two bodies is changed, work is done either by the gravitational force between the two bodies or against this force. In either case, the gravitational potential energy of the bodies changes.

The gravitational potential energy of a body is the energy associated with it due to its position in the gravitational field of another body and is measured by the amount of work done in bringing a body from infinity to a given point in the gravitational field of the other.

When one body lies at infinity from another body, the gravitational force on it is zero. Consequently its potential energy is zero. This is called *zero level* of potential energy.

33. *Derive an expression for the gravitational potential energy of a body of m located at distance r from the centre of the earth.*

Expression for gravitational potential energy. As shown in Fig. 8.22, suppose the earth is a uniform sphere of mass M and radius R. We wish to calculate the potential energy of a body of mass m located at point P such that $OP = r$ and $r > R$.

Suppose at any instant the body is at point A such that

$$OA = x$$

The gravitational force of attraction on the body at A is

$$F = \frac{GMm}{x^2}$$

The small work done in moving the body through small distance $AB (= dx)$ is given by

$$dW = Fdx = \frac{GMm}{x^2}\,dx$$

The total work done in bringing the body from infinity ($x = \infty$) to the point P ($x = r$) will be

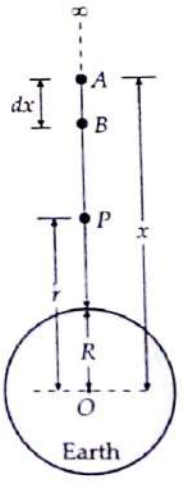

Fig. 8.22 Gravitational P.E. due to earth.

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$$W = \int dW = \int_{\infty}^{r} \frac{GMm}{x^2} dx = GMm \int_{\infty}^{r} x^{-2}\, dx$$

$$= GMm\left[-\frac{1}{x}\right]_{\infty}^{r} = -GMm\left[\frac{1}{r} - \frac{1}{\infty}\right] = -\frac{GMm}{r}$$

By definition, this work done is the gravitational potential energy U of the body of mass m located at distance r from the centre of the earth.

$$\therefore \qquad U = -\frac{GMm}{r} \qquad ...(i)$$

Some important points :

1. The negative sign in equation (*i*) indicates that the *potential energy is due to the gravitational attraction between the earth and the body.* When the body is brought from infinity to a distance r, work is done by the gravitational force of attraction. As the mutual energy of the two bodies is expended, so their energy reduces by this amount.

2. As the distance r increases, the gravitational P.E. increases because it becomes zero *i.e.*, maximum.

3. If a body of mass m is moved from a point at distance r_1 to a point at distance r_2, then the change in potential energy of the body will be

$$\Delta U = \int_{r_1}^{r_2} \frac{GMm}{x^2} dx = GMm\left[-\frac{1}{x}\right]_{r_1}^{r_2}$$

$$= GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

If $r_1 > r_2$, then ΔU be negative. *So when a body is brought closer to the earth, its gravitation P.E. decreases.*

4. If a body is moved from the surface of the earth ($r_1 = R$) to a point at height h above the surface of the earth ($r_2 = R + h$), then the change in its gravitation P.E. will be

$$\Delta U = GMm\left[\frac{1}{R} - \frac{1}{R+h}\right] = \frac{GMm}{R}\left[1 - \frac{R}{R+h}\right]$$

$$= \frac{GMm}{R}\left[1 - \frac{1}{\left(1 + \frac{h}{R}\right)}\right]$$

$$= \frac{GMm}{R}\left[1 - \left(1 + \frac{h}{R}\right)^{-1}\right]$$

Applying binomial theorem, we get

$$\Delta U = \frac{GMm}{R}\left[1 - \left(1 - \frac{h}{R} + \text{terms containing higher powers of } \frac{h}{R}\right)\right]$$

If $h << R$, then higher powers of h / R can be neglected.

Hence $\Delta U = \dfrac{GMm}{R}\left[1 - \left(1 - \dfrac{h}{R}\right)\right] = \dfrac{GMmh}{R^2}$

But $\dfrac{GM}{R^2} = g$ = acceleration due to gravity on the earth's surface.

$\therefore \quad \Delta U = mgh$

8.20 GRAVITATIONAL POTENTIAL

34. *Define the term gravitational potential. Is it a scalar or vector ? Give the units and dimensions of gravitational potential.*

Gravitational potential. *The gravitational potential at a point is the potential energy associated with a unit mass due to its position in the gravitational field of another body.*

The gravitational potential at a point in the gravitational field of a body is defined as the amount of work done in bringing a body of unit mass from infinity to that point.

Gravitational potential,

$$V = \frac{\text{Work done}}{\text{Mass}} = \frac{W}{m}$$

The gravitational potential is a scalar quantity. Its SI unit is Jkg^{-1} and cgs units is erg g^{-1}.

The dimensional formula of gravitational potential is $[M^0L^2T^{-2}]$.

35. *Derive expression for the gravitational potential at a point in the gravitational field of the earth. How is gravitational P.E. related to gravitational potential ?*

Gravitational potential at a point due to the earth. The work done in bringing a body of mass m from infinity to a point at distance r from the centre of the earth is

$$W = -\frac{GMm}{r}$$

Hence the gravitational potential due to the earth at distance r from its centre is

$$V = \frac{W}{m} = -\frac{GM}{r}$$

At the surface of the earth, $r = R$, therefore

$$V_{\text{surface}} = -\frac{GM}{R}$$

Relation between gravitational potential energy and gravitational potential. From the above equations, we find that

$$U = -\frac{GMm}{r} = \left(-\frac{GM}{r}\right) \times m$$

$\therefore$ Gravitational potential energy
= Gravitational potential × mass

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Examples based on Gravitational Intensity, Potential and Potential Energy

FORMULAE USED

1. Intensity of gravitational field, $E = \frac{F}{m} = \frac{GM}{r^2}$
2. Gravitational potential, $V = \frac{\text{Work done}}{\text{Mass}} = -\frac{GM}{r}$
3. Gravitational potential energy,
   U = Gravitational potential × mass $= -\frac{GMm}{r}$
4. $E = -\frac{dV}{dr}$
5. Total energy of a body in a gravitational field,
   $= \text{K.E.} + \text{P.E.} = \frac{1}{2}mv^2 + \left(-\frac{GMm}{r}\right)$

UNITS USED

Gravitational intensity E is in Nkg^{-1} or ms^{-2}, gravitational potential V in $J\,kg^{-1}$ and gravitational P.E. in joule.

EXAMPLE 27. Find the intensity of gravitational field when a force of 100 N acts on a body of mass 10 kg in the gravitational field.

Solution. Here $F = 100$ N, $m = 10$ kg

Intensity of gravitational field,

$$E = \frac{F}{m} = \frac{100\text{ N}}{10\text{ kg}} = \mathbf{10\ Nkg^{-1}}.$$

EXAMPLE 28. Two bodies of masses 10 kg and 1000 kg are at a distance 1 m apart. At which point on the line joining them will the gravitational field intensity be zero ?

Solution. Let the resultant gravitational intensity be zero at distance x from the mass of 10 kg on the line joining the centres of the two bodies. At this point, the gravitational intensities due to the two bodies must be equal and opposite.

$$\therefore \quad \frac{G\times 10}{x^2} = \frac{G\times 1000}{(1-x)^2}$$

or $\quad 100\,x^2 = (1-x)^2 \quad$ or $\quad 10\,x = 1 - x$

or $\quad 11\,x = 1 \quad$ or $\quad x = \mathbf{1/11\ m}.$

EXAMPLE 29. Two masses, 800 kg and 600 kg, are at a distance 0.25 m apart. Compute the magnitude of the intensity of the gravitational field at a point distant 0.20 m from the 800 kg mass and 0.15 m from the 600 kg mass.

Solution. Let A and B be the positions of the two masses and P the point at which the intensity of the gravitational field is to be computed.

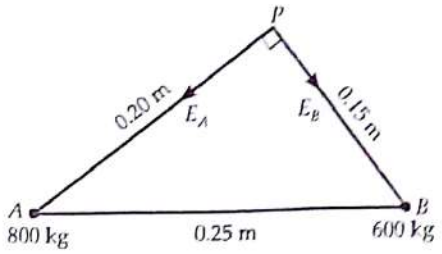

Fig. 8.23

Gravitational intensity at P due to mass at A,

$$E_A = \frac{GM}{r^2} = G\frac{800}{(0.20)^2} = 20{,}000\ G, \text{ along } PA$$

Gravitational intensity at P due to mass at B,

$$E_B = G\frac{600}{(0.15)^2} = \frac{80{,}000}{3}\ G, \text{ along } PB$$

In $\Delta\ APB$,

$$PA^2 + PB^2 = AB^2$$

$$\therefore \quad \angle\ APB = 90°.$$

Hence the magnitude of resultant gravitational intensity at P is

$$E = \sqrt{E_A^2 + E_B^2} = G\sqrt{(20{,}000)^2 + \left(\frac{80{,}000}{3}\right)^2}$$

$$= 6.66\times 10^{-11} \times \frac{10{,}000}{3} = \mathbf{2.22\times 10^{-6}\ N}.$$

EXAMPLE 30. At a point above the surface of the earth, the gravitational potential is $-5.12\times 10^7\ J\,kg^{-1}$ and the acceleration due to gravity is 6.4 ms^{-2}. Assuming the mean radius of the earth to be 6400 km, calculate the height of this point above the earth's surface.

Solution. Let r be the distance of the given point from the centre of the earth. Then

Gravitational potential,

$$V = -\frac{GM}{r} = -5.12\times 10^7\ \text{J kg}^{-1} \qquad ...(i)$$

Acceleration due to gravity,

$$g = \frac{GM}{r^2} = 6.4\ \text{ms}^{-2} \qquad ...(ii)$$

Dividing (i) by (ii),

$$r = \frac{5.12\times 10^7}{6.4} = 8\times 10^6\ \text{m} = 8000\ \text{km}$$

Height of the point from the earth's surface

$$= 8000 - 6400 = \mathbf{1600\ km}.$$

EXAMPLE 31. The radius of the earth is 6.37×10^6 m, its mean density is $5.5\times 10^3\ kg\,m^{-3}$ and $G = 6.66\times 10^{-11}\ Nm^2\,kg^{-2}$. Determine the gravitational potential on the surface of the earth.

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Solution. Here $R = 6.37 \times 10^6$ m,

$\rho = 5.5 \times 10^3$ kgm^{-3}, $G = 6.66 \times 10^{-11}$ Nm^2 kg^{-2}

Mass of the earth,

$$M = \text{Volume} \times \text{density} = \frac{4}{3}\pi R^3 \rho$$

Gravitational potential on the earth's surface

$$V = -\frac{GM}{R} = -\frac{G}{R} \times \frac{4}{3}\pi R^3 \rho = -\frac{4}{3}\pi G R^2 \rho$$

$$= -\frac{4}{3} \times 3.14 \times 6.66 \times 10^{-11} \times (6.37 \times 10^6)^2 \times 5.5 \times 10^3$$

$$= -6.22 \times 10^7 \text{ J kg}^{-1}.$$

EXAMPLE 32. Three mass points each of mass m are placed at the vertices of an equilateral triangle of side l. What is the gravitational field and potential due to three masses at the centroid of the triangle ?

Solution. In Fig. 8.24, three mass points, each of mass m, are placed at three vertices of equilateral ΔABC of side l. If O is the centroid of the triangle, then $OA = OB = OC$.

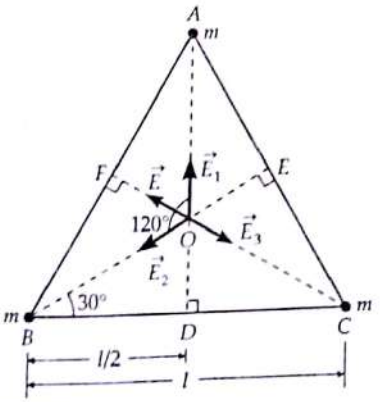

Fig. 8.24

From right ΔODB,

$$\cos 30° = \frac{BD}{OB} = \frac{l/2}{OB}$$

or $$OB = \frac{l/2}{\cos 30°} = \frac{l/2}{\sqrt{3}/2} = \frac{l}{\sqrt{3}}$$

Gravitational fields at O due to mass points at A, B and C are as follows :

$$E_1 = \frac{Gm}{(OA)^2} = \frac{Gm}{(l/\sqrt{3})^2} = \frac{3Gm}{l^2}, \text{ along } \vec{OA}$$

$$E_2 = \frac{Gm}{(OB)^2} = \frac{3Gm}{l^2}, \text{ along } \vec{OB}$$

$$E_3 = \frac{Gm}{(OC)^2} = \frac{3Gm}{l^2}, \text{ along } \vec{OC}$$

Angle between $\vec{E}_1$ and $\vec{E}_2$ is 120°. Their resultant is

$$E = \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos 120°}$$

$$= \frac{3Gm}{l^2}\sqrt{1 + 1 + 2 \times \left(-\frac{1}{2}\right)} = \frac{3Gm}{l^2}, \text{ along } \vec{OF}$$

Clearly, $\vec{E}$ is equal and opposite to $\vec{E}_3$, hence the resultant gravitational field at O is zero.

As gravitational potential is a scalar quantity, so the total gravitational potential at O is

$$V = V_1 + V_2 + V_3 = -\frac{Gm}{OA} - \frac{Gm}{OB} - \frac{Gm}{OC}$$

$$= -\frac{3Gm}{OA} = -\frac{3Gm}{l/\sqrt{3}} \quad \left[\because OA = OB = OC = \frac{l}{\sqrt{3}}\right]$$

or $$V = -3\sqrt{3}\,\frac{Gm}{l}.$$

EXAMPLE 33. Find the potential energy of a system of four particles, each of mass m, placed at the vertices of a square of side l. Also obtain the potential at the centre of the square. [NCERT]

Solution. In Fig. 8.25,

$$AB = BC = CD = DA = l$$

$$\therefore \quad AC = BD = \sqrt{l^2 + l^2} = \sqrt{2}\,l$$

$$OA = OB = OC = OD = \sqrt{2}\,l/2 = l/\sqrt{2}$$

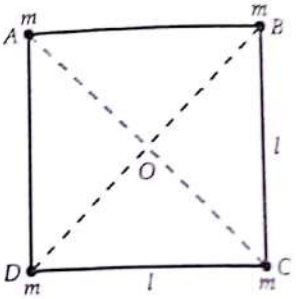

Fig. 8.25

By the principle of superposition, total potential energy of the system of particles is

$$U = U_{BA} + (U_{CB} + U_{CA}) + (U_{DA} + U_{DB} + U_{DC})$$

$$= 4\,U_{BA} + 2\,U_{DB}$$

$$[\because U_{BA} = U_{DA} = U_{DC} = U_{CB},\ U_{CA} = U_{DB}]$$

$$= 4\left(-\frac{Gmm}{l}\right) + 2\left(-\frac{Gmm}{l/\sqrt{2}}\right)$$

$$= -\frac{2Gm^2}{l}\left[2 + \frac{1}{\sqrt{2}}\right] = -\frac{2Gm^2}{l}[2 + 0.707]$$

$$= -\frac{5.41\,Gm^2}{l}$$

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Total gravitational potential at the centre O,

$$V = V_A + V_B + V_C + V_D = 4\,V_A = 4\left(-\frac{Gm}{OA}\right)$$

$$= 4\left(-\frac{Gm}{l/\sqrt{2}}\right) = -\frac{4\sqrt{2}\,Gm}{l}.$$

EXAMPLE 34. *Two bodies of masses m_1 and m_2 are placed at a distance r apart. Show that at the position where the gravitational field due to them is zero, the potential is given by*

$$V = -\frac{G}{r}\left[m_1 + m_2 + 2\sqrt{m_1 m_2}\right]$$

[IIT]

Solution. Let the gravitational field be zero at a point P at distance x from m_1 and $(r-x)$ from m_2. Then

$$\frac{Gm_1}{x^2} = \frac{Gm_2}{(r-x)^2} \quad \text{or} \quad \frac{\sqrt{m_1}}{x} = \frac{\sqrt{m_2}}{(r-x)}$$

or $(r-x)\sqrt{m_1} = x\sqrt{m_2}$ or $x(\sqrt{m_1} + \sqrt{m_2}) = r\sqrt{m_1}$

$$\therefore \quad x = \frac{r\sqrt{m_1}}{\sqrt{m_1} + \sqrt{m_2}} \quad \text{or} \quad \frac{1}{x} = \frac{\sqrt{m_1} + \sqrt{m_2}}{r\sqrt{m_1}}$$

and $$(r-x) = r - \frac{r\sqrt{m_1}}{\sqrt{m_1} + \sqrt{m_2}} = \frac{r\sqrt{m_2}}{\sqrt{m_1} + \sqrt{m_2}}$$

or $$\frac{1}{r-x} = \frac{\sqrt{m_1} + \sqrt{m_2}}{r\sqrt{m_2}}$$

Gravitational potential at point P due to masses m_1 and m_2 will be

$$V = V_1 + V_2 = -\frac{Gm_1}{x} - \frac{Gm_2}{r-x} = -G\left[\frac{m_1}{x} + \frac{m_2}{r-x}\right]$$

$$= -G\left[\frac{m_1(\sqrt{m_1} + \sqrt{m_2})}{r\sqrt{m_1}} + \frac{m_2(\sqrt{m_1} + \sqrt{m_2})}{r\sqrt{m_2}}\right]$$

$$= -\frac{G}{r}\left[m_1 + m_2 + 2\sqrt{m_1 m_2}\right].$$

EXAMPLE 35. *A non-homogeneous sphere of radius R has the following density variation :*

$\rho = \rho_0$ *for* $r \le R/3$

$\rho = \rho_0/2$ *for* $R/3 < r \le 3R/4$

$\rho = \rho_0/8$ *for* $3R/4 < r \le R$.

What is the gravitational field due to the sphere at $r = R/4$, $R/2$, $5R/6$ and $2R$?

Solution. The gravitational field due to a spherical body of mass M at a distance r is given by

$$E = \frac{GM}{r^2}$$

(*i*) **For $r = R/4$,** density $= \rho_0$

Mass of the spherical portion of radius $R/4$,

$$M = \frac{4}{3}\pi\left(\frac{R}{4}\right)^3 \rho_0$$

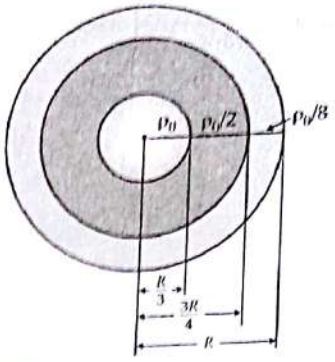

Fig. 8.26

Gravitational field at distance $r = R/4$ from the centre,

$$E_1 = G \times \frac{4}{3}\pi\left(\frac{R}{4}\right)^3 \rho_0 \times \frac{1}{(R/4)^2}$$

$$= 0.33\,\pi\,GR\rho_0\,.$$

(*ii*) **For $r = R/2$,** density of the portion of sphere of radius $R/3$ is ρ_0 and of the portion between $R/3$ and $R/2$, the density is $\rho_0/2$.

$\therefore$ Mass of the spherical portion of radius $R/2$,

$$\therefore \quad M = \frac{4}{3}\pi\left(\frac{R}{3}\right)^3 \rho_0 + \left[\frac{4}{3}\pi\left(\frac{R}{2}\right)^3 - \frac{4}{3}\pi\left(\frac{R}{3}\right)^3\right]\frac{\rho_0}{2}$$

$$= \frac{4}{3}\pi R^3 \rho_0\left[\frac{1}{27} + \frac{1}{16} - \frac{1}{54}\right] = 0.108\,\pi R^3 \rho_0$$

The gravitational field at the distance $r = R/2$,

$$E_2 = \frac{G \times 0.108\,\pi R^3 \rho_0}{(R/2)^2} = 0.43\,\pi GR\,\rho_0.$$

(*iii*) **For $r = 5R/6$,** the density of portion of radius $R/3$ is ρ_0, of the portion between $r = R/3$ to $r = 3R/4$ is $\rho_0/2$ and of the portion between $r = 3R/4$ to $r = 5R/6$ is $\rho_0/8$.

$$\therefore \quad M = \frac{4}{3}\pi\left(\frac{R}{3}\right)^3 \rho_0 + \left[\frac{4}{3}\pi\left(\frac{3R}{4}\right)^3 - \frac{4}{3}\pi\left(\frac{R}{3}\right)^3\right]\frac{\rho_0}{2}$$

$$+ \left[\frac{4}{3}\pi\left(\frac{5R}{6}\right)^3 - \frac{4}{3}\pi\left(\frac{3R}{4}\right)^3\right]\frac{\rho_0}{8}$$

$$= \frac{4}{3}\pi R^3 \rho_0\left[\frac{1}{27} + \frac{27}{128} - \frac{1}{54} + \frac{125}{1728} - \frac{27}{512}\right]$$

$$= 0.332\,\pi R^3 \rho_0.$$

The gravitational field at the distance $r = 5R/6$,

$$E_3 = \frac{G \times 0.332\,\pi R^3 \rho_0}{(5R/6)^2} = 0.48\,G\,\pi\,R\,\rho_0\,.$$

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(*iv*) For $r = 2R$, the gravitational field is due to the whole of the mass of the sphere.

$$\therefore \quad M = \frac{4}{3}\pi\left(\frac{R}{3}\right)^3 \rho_0 + \left[\frac{4}{3}\pi\left(\frac{3R}{4}\right)^3 - \frac{4}{3}\pi\left(\frac{R}{3}\right)^3\right]\frac{\rho_0}{2} + \left[\frac{4}{3}\pi R^3 - \frac{4}{3}\pi\left(\frac{3R}{4}\right)^3\right]\frac{\rho_0}{8}$$

$$= \frac{4}{3}\pi R^3 \rho_0\left[\frac{1}{27} + \frac{27}{128} - \frac{1}{54} + \frac{1}{8} - \frac{27}{512}\right]$$

$$= 0.402\ \pi R^3 \rho_0$$

The gravitational field at the distance $r = 2R$,

$$E_4 = \frac{G \times 0.402\ \pi R^3 \rho_0}{(2R)^2} = 0.1\ \pi GR\rho_0.$$

PROBLEMS FOR PRACTICE

1. The gravitational field intensity at a point 10,000 km from the centre of the earth is 4.8 N kg^{-1}. Calculate the gravitational potential at that point.

   (**Ans.** -4.8×10^7 J kg^{-1})

2. The distance between the earth and the moon is 3.85×10^8 metre. At what point in between the two will the gravitational field intensity be zero ? Mass of the earth $= 6.0 \times 10^{24}$ kg, mass of the moon $= 7.26 \times 10^{22}$ kg.

   (**Ans.** 3.47×10^8 m from the centre of the earth)

3. Two bodies of masses 100 kg and 1000 kg are at a distance 1.00 metre apart. Calculate the gravitational field intensity and the potential at the middle-point of the line joining them.
   Take $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}.

   (**Ans.** 2.40×10^{-7} Nkg^{-1}, -1.47×10^{-7} J kg^{-1})

4. The mass of the earth is 6.0×10^{24} kg. Calculate (*i*) the potential energy of a body of mass 33.5 kg and (*ii*) the gravitational potential, at a distance of 3.35×10^{10} m from the centre of the earth. Take $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}.

   [**Ans.** (*i*) -4.02×10^5 J (*ii*) -12×10^3 J kg^{-1}]

5. The radius of the earth is R and the acceleration due to gravity at its surface is g. Calculate the work required in raising a body of mass m to a height h from the surface of the earth. $\left(\text{Ans. } \frac{mgh}{1 + \frac{h}{R}}\right)$

6. Find the work done to bring 4 particles each of mass 100 gram from large distances to the vertices of a square of side 20 cm. (**Ans.** -1.80×10^{-11} J)

HINTS

1. Gravitational field intensity, $E = \frac{GM}{R^2}$

   Gravitational potential, $V = -\frac{GM}{R}$

   $\therefore \quad \frac{V}{E} = R$

   or $\quad V = -E \times R = -4.8 \times 10{,}000 \times 10^3$
   $= -4.8 \times 10^7$ J kg^{-1}.

5. Let M be the mass of the earth. Then
   Work done = Change in P.E.

$$= -\frac{GMm}{R+h} - \left[-\frac{GMm}{R}\right]$$

$$= GMm\left[\frac{1}{R} - \frac{1}{R+h}\right]$$

$$= \frac{GMmh}{R(R+h)} = \frac{gR^2\, mh}{R(R+h)}$$

$$= \frac{mgh}{1 + h/R} \qquad [\because GM = gR^2]$$

6. As shown in Fig. 8.27, suppose the four particles are placed at the vertices of A, B, C and D of square $ABCD$. Here $AB = BC = CD = DA = 20$ cm $= 0.2$ m

$$AC = BC = \sqrt{(0.2)^2 + (0.2)^2} = 0.2\sqrt{2}\ \text{m}.$$

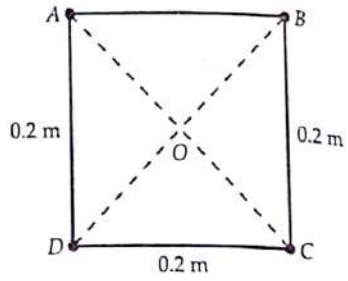

Fig. 8.27

Total work done = Gain in P.E. of the system

or $\quad W = U_{BA} + (U_{CB} + U_{CA}) + (U_{DA} + U_{DC} + U_{DB})$

$$= 4U_{BA} + 2U_{DB}$$

$[\because U_{BA} = U_{DA} = U_{DC} = U_{CB}$ and $U_{CA} = U_{DB}]$

$$= 4 \times \left[-\frac{Gm_1 m_2}{AB}\right] + 2 \times \left[-\frac{Gm_1 m_2}{BD}\right]$$

$$= 4 \times \left[-\frac{6.67 \times 10^{-11} \times 0.1 \times 0.1}{0.2}\right] + 2 \times \left[-\frac{6.67 \times 10^{-11} \times 0.1 \times 0.1}{0.2\sqrt{2}}\right]$$

$$= -1.33 \times 10^{-11} - 0.47 \times 10^{-11} = -1.80 \times 10^{-11}\ \text{J}.$$

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8.21 ESCAPE VELOCITY

36. *Define escape velocity. Obtain an expression for the escape velocity of a body from the surface of the earth.*

Escape velocity. If we throw a ball into air, it rises to a certain height and falls back. If we throw it with a greater velocity, it rises to a greater height. If we throw it with a sufficient velocity, it will never come back. It will escape from the gravitational pull of the earth. This minimum velocity is called escape velocity.

Escape velocity is the minimum velocity with which a body must be projected vertically upwards in order that it may just escape the gravitational field of the earth.

Expression for escape velocity. Consider the earth to be a sphere of mass M and radius R with centre O. Suppose a body of mass m lies at point P at distance x from its centre, as shown in Fig. 8.28. The gravitational force of attraction on the body at P is

$$F = \frac{GMm}{x^2}$$

The small work done in moving the body through small distance $PQ = dx$ against the gravitational force is given by

$$dW = Fdx = \frac{GMm}{x^2}\,dx$$

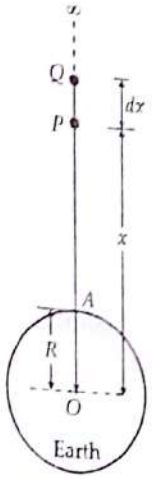

Fig. 8.28

The total work done in moving the body from the surface of the earth ($x = R$) to a region beyond the gravitational field of the earth ($x = \infty$) will be

$$W = \int dW = \int_R^\infty \frac{GMm}{x^2}\,dx$$

$$= GMm\int_R^\infty x^{-2}\,dx = GMm\left[-\frac{1}{x}\right]_R^\infty$$

$$= GMm\left[-\frac{1}{\infty} + \frac{1}{R}\right] = \frac{GMm}{R}$$

If v_e is the escape velocity of the body, then the kinetic energy $\frac{1}{2}mv_e^2$ imparted to the body at the surface of the earth will be just sufficient to perform work W.

$$\therefore \quad \frac{1}{2}mv_e^2 = \frac{GMm}{R} \quad \text{or} \quad v_e^2 = \frac{2GM}{R}$$

Escape velocity $$v_e = \sqrt{\frac{2GM}{R}} \qquad ...(i)$$

As $$g = \frac{GM}{R^2} \quad \text{or} \quad GM = gR^2$$

$$\therefore \quad v_e = \sqrt{\frac{2gR^2}{R}} \quad \text{or} \quad v_e = \sqrt{2gR} \qquad ...(ii)$$

If ρ is the mean density of the earth, then

$$M = \frac{4}{3}\pi R^3\rho$$

$$\therefore \quad v_e = \sqrt{\frac{2G}{R} \times \frac{4}{3}\pi R^3\rho} = \sqrt{\frac{8\pi\rho\, G R^2}{3}} \qquad ...(iii)$$

Equations (*i*), (*ii*) and (*iii*) give different expressions for the escape velocity of a body. Clearly, *the escape velocity does not depend on the mass of the body projected.*

For Your Knowledge

- *For the earth,* $g = 9.8\ ms^{-2}$ and $R = 6.4 \times 10^6$ m, so
$$v_e = \sqrt{2gR} = \sqrt{2 \times 9.8 \times 6.4 \times 10^6}$$
$$= 11.2 \times 10^3\ ms^{-1} = 11.2\ kms^{-1}.$$
- In deriving the expression for escape velocity, we have neglected the air resistance on the body. In actual practice, the value of escape velocity is slightly greater than the above calculated value.
- The escape velocity does not depend on angle of projection from the earth's surface. But as the earth rotates about its axis, so it becomes easier to attain escape velocity if the body is projected in the direction in which the launch site is moving.
- As the escape velocity depends on the mass and radius of the planet from the surface of which the body is projected, so value of escape velocity is different for different planets.
- *Some important escape velocities*

Heavenly body	*Escape velocity*
Moon	$2.3\ kms^{-1}$
Mercury	$4.28\ kms^{-1}$
Earth	$11.2\ kms^{-1}$
Jupiter	$60\ kms^{-1}$
Sun	$618\ kms^{-1}$
Neutron star	$2 \times 10^5\ kms^{-1}$

- A planet will have atmosphere if the root mean square velocity of its atmospheric molecules is less than the escape velocity for the given planet. That is why moon has no atmosphere ($v_e = 2.3\ kms^{-1}$) while Jupiter has a thick atmosphere ($v_e = 60\ kms^{-1}$). Even the lightest hydrogen cannot escape from its surface.

Examples based on Escape Velocity of a Satellite

FORMULAE USED

1. $v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR} = \sqrt{\frac{8}{3}\pi G\rho\, R^2}$

UNITS USED

Velocity v_e is in ms^{-1}, radius R in metre.

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Example 36. Find the velocity of escape at the earth given that its radius is 6.4×10^6 m and the value of g at its surface is 9.8 ms^{-2}.

Solution. Here $R = 6.4 \times 10^6$ m, $g = 9.8$ ms^{-2}

$$v_e = \sqrt{2gR} = \sqrt{2 \times 9.8 \times 6.4 \times 10^6}$$

$$= 11.2 \times 10^3 \text{ ms}^{-1} = \mathbf{11.2 \text{ kms}^{-1}}.$$

Example 37. Determine the escape velocity of a body from the moon. Take the moon to be a uniform sphere of radius 1.76×10^6 m, and mass 7.36×10^{22} kg. Given $G = 6.67 \times 10^{-11}$ $Nm^2 kg^{-2}$.

Solution. Here $R = 1.76 \times 10^6$ m, $M = 7.36 \times 10^{22}$ kg

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 7.36 \times 10^{22}}{1.76 \times 10^6}}$$

$$= 2375 \text{ ms}^{-1} = \mathbf{2.375 \text{ km s}^{-1}}.$$

Example 38. A black hole is a body from whose surface nothing may even escape. What is the condition for a uniform spherical body of mass M to be a black hole ? What should be the radius of such a black hole if its mass is nine times the mass of the earth ? **[Delhi 03C]**

Solution. From Einstein's special theory of relativity, we know that speed of any object cannot exceed the speed of light, $c = 3 \times 10^8$ ms^{-1}. Thus c is the upper limit to the projectile's escape velocity. Hence for a body to be a black hole,

$$v_e = \sqrt{\frac{2GM}{R}} \le c$$

If $M = 9\ M_E = 9 \times 6 \times 10^{24}$ kg, then

$$R = \frac{2GM}{c^2} = \frac{2 \times 6.67 \times 10^{-11} \times 9 \times 6 \times 10^{24}}{(3 \times 10^8)^2}$$

$$\simeq 8 \times 10^{-2} \text{ m } \textit{or } \mathbf{\text{nearly 8 cm}}.$$

Example 39. Jupiter has a mass 318 times that of the earth, and its radius is 11.2 times the earth's radius. Estimate the escape velocity of a body from Jupiter's surface, given that the escape velocity from the earth's surface is 11.2 $km\,s^{-1}$.

Solution. Escape velocity from the earth's surface is

$$v_e = \sqrt{\frac{2GM}{R}} = 11.2 \text{ kms}^{-1}$$

Escape velocity from Jupiter's surface will be

$$v_e' = \sqrt{\frac{2GM'}{R'}}$$

But $M' = 318\ M$, $R' = 11.2\ R$

$$\therefore \quad v_e' = \sqrt{\frac{2G(318\ M)}{11.2}} = \sqrt{\frac{2GM}{R} \times \frac{318}{11.2}}$$

$$= v_e \times \sqrt{\frac{318}{11.2}} = 11.2 \times \sqrt{\frac{318}{11.2}} = \mathbf{59.7 \text{ kms}^{-1}}.$$

Example 40. Show that the moon would depart for ever if its speed were increased by 42% .

Solution. The centripetal force required by the moon to revolve around the earth is provided by gravitational attraction.

$$\therefore \quad \frac{mv_0^2}{R} = \frac{GMm}{R^2}$$

or $$v_0 = \sqrt{\frac{GM}{R}} = \sqrt{\frac{gR^2}{R}} = \sqrt{gR}$$

Velocity required to escape, $v_e = \sqrt{2gR}$

% increase in the velocity of moon

$$= \frac{v_e - v_0}{v_0} \times 100 = \frac{\sqrt{2gR} - \sqrt{gR}}{\sqrt{gR}} \times 100$$

$$= \frac{\sqrt{2} - 1}{1} \times 100 = (1.414 - 1) \times 100 = 41.4\% \simeq \mathbf{42\%}.$$

Example 41. Calculate the escape velocity for an atmospheric particle 1600 km above the earth's surface, given that the radius of the earth is 6400 km and acceleration due to gravity on the surface of earth is 9.8 ms^{-2}.

Solution. At a height h above the earth's surface, we have

$$v_e = \sqrt{2g_h (R + h)}, \quad g_h = \frac{gR^2}{(R + h)^2}$$

$$\therefore \quad v_e = \sqrt{\frac{2 \times gR^2}{(R + h)^2} \times (R + h)} = \sqrt{\frac{2gR^2}{R + h}}$$

But $g = 9.8$ ms^{-2}, $R = 6.4 \times 10^6$ m,

$h = 1600$ km $= 1.6 \times 10^6$ m,

$$R + h = (6.4 + 1.6) \times 10^6 = 8 \times 10^6 \text{ m}$$

$$\therefore \quad v_e = \sqrt{\frac{2 \times 9.8 \times (6.4 \times 10^6)^2}{8 \times 10^6}}$$

$$= 10.02 \times 10^3 \text{ ms}^{-1} = \mathbf{10.02 \text{ kms}^{-1}}.$$

Example 42. The radius of a planet is double that of the earth but their average densities are the same. If the escape velocities at the planet and at the earth are v_p and v_E respectively, then prove that $v_p = 2\ v_E$.

Solution. If ρ is the average density of the earth, then mass of the earth,

$$M_E = \frac{4}{3} \pi R_E^3 \rho$$

Escape velocity on the earth,

$$v_E = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{\frac{2G}{R_E} \times \frac{4}{3} \pi R_E^3 \rho}$$

$$= R_E \sqrt{\frac{8}{3} G \pi \rho}$$

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Similarly, escape velocity on the planet,

$$v_p = R_p \sqrt{\frac{8}{3} G \pi \rho}$$

$$\frac{v_p}{v_E} = \frac{R_p}{R_E}$$

$$\therefore \quad R_p = 2\, R_E \quad \therefore \quad v_p = 2\, v_E.$$

But

Example 33. Two uniform solid spheres of equal radii R, but mass M and 4M have a centre to centre separation 6 R, as shown in Fig. 8.29. The two spheres are held fixed. A projectile of mass m is projected from the surface of the sphere of mass M directly towards the centre of the second sphere. Obtain an expression for the minimum speed v of the projectile so that it reaches the surface of the second sphere. [NCERT]

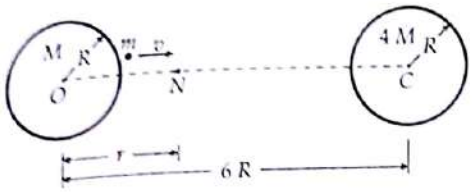

Fig. 8.29

Solution. The two spheres exert gravitational forces on the projectile in mutually opposite directions. At the neutral point N, these two forces cancel each other. If $ON = r$, then

$$\frac{GMm}{r^2} = \frac{G(4M)\,m}{(6R - r)^2}$$

or $\quad (6R - r)^2 = 4r^2 \quad$ or $\quad 6R - r = \pm 2r$

or $\quad r = 2R \quad$ or $\quad -6R$

The neutral point $r = -6R$ is inadmissible.

$$\therefore \quad ON = r = 2R$$

It will be sufficient to project the particle m with a minimum speed v which enables it to reach the point N. Thereafter, the particle m gets attracted by the gravitational pull of $4M$.

The total mechanical energy of m at surface of left sphere is

E_i = K.E. of m + P.E. due to left sphere + P.E. due to right sphere

$$= \frac{1}{2} mv^2 - \frac{GMm}{R} - \frac{4\,GMm}{5R}.$$

At the neutral point, speed of the particle becomes zero. The energy is purely potential.

$\therefore \quad E_N$ = P.E. due to left sphere + P.E. due to right sphere

$$= -\frac{GMm}{2R} - \frac{4GMm}{4R}$$

By conservation of mechanical energy,

$$E_i = E_N$$

or $$\frac{1}{2} mv^2 - \frac{GMm}{R} - \frac{4GMm}{5R} = -\frac{GMm}{2R} - \frac{4GMm}{4R}$$

or $$v^2 = \frac{2GM}{R}\left(\frac{4}{5} - \frac{1}{2}\right) = \frac{3GM}{5R}$$

$$\therefore \quad v = \sqrt{\frac{3GM}{5R}}.$$

PROBLEMS FOR PRACTICE

1. Find the velocity of escape at the moon. Given that its radius is 1.7×10^6 m and the value of 'g' is 1.63 ms^{-2}. **(Ans. 2.354 kms^{-1})**

2. The mass of Jupiter is 1.91×10^{36} kg and its diameter is 13.1×10^7 m. Calculate the escape velocity on the surface of Jupiter. **(Ans. 1.972×10^9 ms^{-1})**

3. If earth has a mass 9 times and radius twice that of a planet Mars, calculate the minimum velocity required by a rocket to pull out of gravitational force of Mars. Take the escape velocity on the surface of earth to be 11.2 kms^{-1}. **[Chandigarh 04 ; Himachal 09]** **(Ans. 5.28 kms^{-1})**

4. The escape velocity of a projectile on the surface of the earth is 11.2 kms^{-1}. A body is projected out with twice this speed. What is the speed of the body far away from the earth *i.e.* at infinity ? Ignore the presence of the sun and other planets, etc. **(Ans. 19.4 kms^{-1})**

5. Find the velocity of escape from the sun, if its mass is 1.89×10^{30} kg and its distance from the earth is 1.59×10^8 km. Take $G = 6.67 \times 10^{-11}$ $Nm^2\,kg^{-2}$. **(Ans. 3.98×10^4 ms^{-1})**

6. A body is at a height equal to the radius of the earth from the surface of the earth. With what velocity be it thrown so that it goes out of the gravitational field of the earth ? Given $M_e = 6.0 \times 10^{24}$ kg, $R_e = 6.4 \times 10^6$ m and $G = 6.67 \times 10^{-11}$ $Nm^2\,kg^{-2}$. **(Ans. 7.9 kms^{-1})**

7. A body of mass 100 kg falls on the earth from infinity. What will be its velocity on reaching the earth ? What will be its K.E. ? Radius of the earth is 6400 km and $g = 9.8$ ms^{-2}. Air friction is negligible. **(Ans. 11.2 kms^{-1}, 6.27×10^9 J)**

HINTS

3. For earth, $v_e = \sqrt{\frac{2GM}{R}}$

For Mars, $v_e' = \sqrt{\frac{2GM'}{R'}}$

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$$\therefore \quad \frac{v_e'}{v_e} = \sqrt{\frac{M'}{M} \times \frac{R}{R'}} = \sqrt{\frac{1}{9} \times \frac{2}{1}} = \frac{\sqrt{2}}{3}$$

$$\text{or} \quad v_e' = \frac{\sqrt{2}}{3} \times v_e = \frac{1.414 \times 11.2}{3} = 5.28 \text{ kms}^{-1}.$$

5. $$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 1.89 \times 10^{30}}{1.59 \times 10^{11}}}$$

$$= 3.98 \times 10^4 \text{ ms}^{-1}.$$

6. Escape velocity from earth's surface,

$$v_e = \sqrt{\frac{2GM_e}{R_e}}$$

If the body is at a height R_e from the earth's surface, then the distance of the body from the centre of the earth will be $2R_e$. Hence in this case, the escape velocity of the body will be

$$v_e' = \sqrt{\frac{2GM_e}{2R_e}} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{2 \times 6.4 \times 10^6}}$$

$$= 7.9 \times 10^3 \text{ ms}^{-1} = 7.9 \text{ kms}^{-1}.$$

7. A body thrown up with escape velocity v_e reaches infinity. Hence a body falling on the earth from infinity should come back with velocity v_e given by

$$v_e = \sqrt{2gR_e} = \sqrt{2 \times 9.8 \times 6400 \times 10^3}$$

$$= 11.2 \times 10^3 \text{ ms}^{-1} = 11.2 \text{ kms}^{-1}.$$

$$\text{K.E.} = \frac{1}{2} mv_e^2 = \frac{1}{2} \times 100 \times (11.2 \times 10^3)^2$$

$$= 6.27 \times 10^9 \text{ J}.$$

8.22 NATURAL AND ARTIFICIAL SATELLITES

37. *What is a satellite ?*

Satellite. A *satellite is a body which continuously revolves on its own around a much larger body in a stable orbit.*

38. *What are natural and artificial satellites ? Give examples.*

Natural satellite. *A satellite created by nature is called a natural satellite.* Moon is a natural satellite of the earth which, in turn, is a satellite of the sun. In fact, each planet is a satellite of the sun. The planets Jupiter and Saturn have fourteen and twelve satellites respectively.

Artificial satellite. *A man made satellite is called an artificial satellite.* Russians were the first to put an artificial satellite, SPUTNIK-I, in an orbit around the earth on October 4, 1957. Since then, many artificial satellites have been put into orbits around the earth to study various phenomena in the outer regions of the earth's atmosphere. India entered space age on April 19, 1975 by putting in orbit its first satellite Aryabhatta from Russian soil. India's list includes important satellites like INSAT-IA, INSAT-IB, INSAT-2B, IRS-IC, INSAT-2D etc.

8.23 LAUNCHING OF A SATELLITE

39. *What is the principle of launching an artificial satellite ?*

Ans. Principle of a launching a satellite. Consider a high tower with its top projecting outside the earth's atmosphere. Let us throw a body horizontally from the top of the tower with different velocities. When the velocity is low, the body describes a parabolic path under the effect of gravity and hits the earth's surface at *A*. With somewhat larger velocity, its path is still parabolic but hits the surface at *B* covering a larger horizontal range. As we go on increasing the velocity of horizontal projection, the body will hit the ground at a point farther and farther from the foot of the tower. At a certain horizontal velocity, the body will not hit the earth, but will always be in a state of free fall under gravity and attempt to fall to the earth but missing it all the time. Then the body will follow a stable circular path around the earth and will become a satellite of the earth. This horizontal velocity is called orbital velocity.

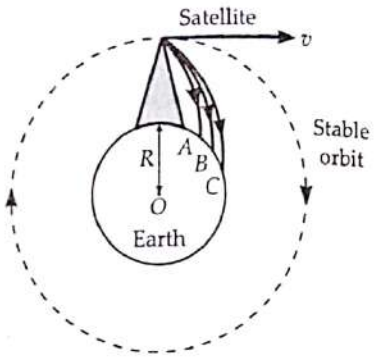

Fig. 8.30 Launching of a satellite.

Hence to put a satellite into an orbit around the earth, we need to give it *two* velocities :

1. A minimum vertical velocity (escape velocity) to take the satellite to a suitable height.
2. After the satellite has ascended the required height, it is given a suitable horizontal velocity to make it move in a circular orbit around the earth.

40. *Explain the use of multistage rockets in launching a satellite.*

Use of multistage rockets. Escape velocity on the earth's surface is 11.2 kms^{-1}. At this high velocity, the air resistance is very large. Still much higher escape velocities are required to take the satellite to a suitable height. Such high velocities can be imparted to the satellites by using multistage rockets. Generally 3-stage rockets are used. The satellite is placed on the third

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stage. At lift off, the exhaust gases build up a very large upthrust so that the rocket accelerates upwards. The rocket rises vertically through the denser atmosphere with a minimum time. When the fuel of the first stage gets exhausted, its casing is detached. Now the rocket is tilted gradually, the second stage comes into operation and its velocity increases further. The second stage gets detached. The final stage of the rocket turns the satellite in a horizontal direction and gives it a proper speed. With this speed, the satellite moves around the earth in a stable orbit.

8.24 ORBITAL VELOCITY

41. *Define orbital velocity of a satellite. Derive expressions for the orbital velocity of a satellite. Show that the escape velocity of a body from the earth's surface is $\sqrt{2}$ times its velocity in a circular orbit just above the earth's surface.*

Orbital velocity. *Orbital velocity is the velocity required to put the satellite into its orbit around the earth*

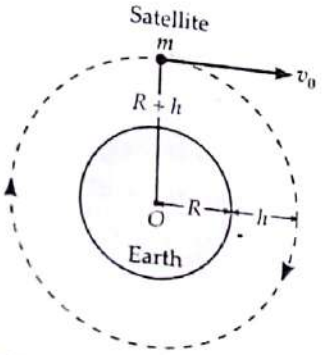

Fig. 8.31 Orbital velocity.

Expression for orbital velocity. In Fig. 8.31, let

M = mass of the earth,

R = radius of the earth,

m = mass of the satellite

v_0 = orbital velocity of the satellite

h = height of the satellite above the earth's surface

$R + h$ = orbital radius of the satellite

According to the law of gravitation, the force of gravity on the satellite is

$$F = \frac{GMm}{(R+h)^2}$$

The centripetal force required by the satellite to keep it in its orbit is

$$F = \frac{mv_0^2}{R+h}$$

In equilibrium, the centripetal force is just provided by the gravitational pull of the earth, so

$$\frac{mv_0^2}{R+h} = \frac{GMm}{(R+h)^2}$$

or $$v_0^2 = \frac{GM}{R+h}$$

$\therefore$ Orbital velocity, $$v_0 = \sqrt{\frac{GM}{R+h}} \quad ...(i)$$

If g is the acceleration due to gravity on the earth's surface, then

$$g = \frac{GM}{R^2}$$

or $$GM = gR^2$$

Hence $$v_0 = \sqrt{\frac{gR^2}{R+h}} = R\sqrt{\frac{g}{R+h}}$$

When the satellite revolves close to the surface of the earth, $h = 0$ and the orbital velocity will become

$$v_0 = \sqrt{gR}$$

As $g = 9.8\ \text{ms}^{-2}$ and $R = 6.4 \times 10^6$ m, so

$$v_0 = \sqrt{9.8 \times 6.4 \times 10^6} = 7.92 \times 10^3\ \text{ms}^{-1}$$

$$= 7.92\ \text{kms}^{-1}.$$

Some important points

From equation (*i*), it is clear that the orbital velocity of a satellite

(*i*) is independent of the mass of the satellite.

(*ii*) decreases with the increase in the radius of the orbit and with increase in the height of the satellite.

(*iii*) depends on the mass and radius of the planet about which the satellite revolves.

Relation between orbital velocity and escape velocity. The escape velocity of a body from the earth's surface is

$$v_e = \sqrt{2gR}$$

The orbital velocity of a satellite revolving close to the earth's surface is

$$v_0 = \sqrt{gR}$$

$\therefore$ $$\frac{v_e}{v_0} = \frac{\sqrt{2gR}}{\sqrt{gR}} = \sqrt{2}$$

or $$v_e = \sqrt{2}\,v_0.$$

Hence the escape velocity of a body from the earth's surface is $\sqrt{2}$ times its velocity in a circular orbit just above the earth's surface.

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42. *Derive expressions for time period, height and angular momentum of a satellite.*

Time period of a satellite. *It is the time taken by a satellite to complete one revolution around the earth.* It is given by

$$T = \frac{\text{Circumference of the orbit}}{\text{Orbital velocity}} = \frac{2\pi(R+h)}{v_0}$$

As orbital velocity,

$$v_0 = \sqrt{\frac{GM}{R+h}}$$

$$\therefore \quad T = \frac{2\pi(R+h)}{\sqrt{\frac{GM}{R+h}}} = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$$

But $g = GM/R^2$ or $GM = gR^2$, therefore

$$T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}} \qquad ...(i)$$

If the earth is a sphere of mean density ρ, then its mass would be

$$M = \text{Volume} \times \text{density} = \frac{4}{3}\pi R^3\rho$$

$$\therefore \quad T = 2\pi\sqrt{\frac{(R+h)^3}{G.\frac{4}{3}\pi R^3\rho}} = \sqrt{\frac{3\pi(R+h)^3}{G\rho R^3}}$$

When the satellite revolves close to the earth, $h = 0$ and the time period will be

$$T = 2\pi\sqrt{\frac{R^3}{GM}} = 2\pi\sqrt{\frac{R}{g}} = \sqrt{\frac{3\pi}{G\rho}}$$

Putting $g = 9.8\ \text{ms}^{-2}$ and $R = 6.4 \times 10^6$ m, we get

$$T = 2\pi\sqrt{\frac{6.4\times10^6}{9.8}} = 5078\ \text{s} = \mathbf{84.6\ min.}$$

Height of a satellite above the earth's surface. Squaring both sides of equation (*i*), we get

$$T^2 = \frac{4\pi^2(R+h)^3}{gR^2}$$

$$\text{or} \quad (R+h)^3 = \frac{T^2R^2g}{4\pi^2} \quad \text{or} \quad R+h = \left[\frac{T^2R^2g}{4\pi^2}\right]^{1/3}$$

$$\therefore \text{ Height of satellite, } \quad h = \left[\frac{T^2R^2g}{4\pi^2}\right]^{1/3} - R$$

Angular momentum. The angular momentum of a satellite of mass m moving with velocity v_0 in an orbit of radius $r\ (= R + h)$ is given by

$$L = mv_0r = m\sqrt{\frac{GM}{r}}\,r = \sqrt{GMm^2r}$$

Examples based on Orbital Velocity of Satellites

Formulae Used

1. Orbital velocity at a height h,

$$v_0 = \sqrt{\frac{GM}{R+h}} = \sqrt{\frac{gR^2}{R+h}} = R\sqrt{\frac{g}{R+h}}$$

2. When a satellite revolves close to earth's surface

$$v_0 = \sqrt{gR}$$

$$v_e = \sqrt{2}\,v_0$$

3. Time period of a satellite

$$T = \frac{2\pi(R+h)}{v_0} = 2\pi\sqrt{\frac{(R+h)^3}{GM}} = \frac{2\pi}{R}\sqrt{\frac{(R+h)^3}{g}}$$

$$= \sqrt{\frac{3\pi(R+h)^3}{G\rho R^3}}$$

4. Height of a satellite, $h = \left[\frac{T^2R^2g}{4\pi^2}\right]^{1/3} - R$

Units Used

Velocities v_0 and v_e are in ms^{-1}, R and h in metre.

Example 44. An artificial satellite revolves around the earth at a height of 1000 km. The radius of the earth is 6.38×10^3 *km. Mass of the earth is* 6×10^{24} kg *and* $G = 6.67 \times 10^{-11}\ Nm^2\ kg^{-2}$. *Find its orbital velocity and period of revolution.*

Solution. Here : $h = 1000\ \text{km} = 10^6$ m,

$$R = 6.38 \times 10^3\ \text{km} = 6.38 \times 10^6\ \text{m}$$

$$\therefore \quad R + h = 7.38 \times 10^6\ \text{m}, \quad M = 6 \times 10^{24}\ \text{kg}$$

Orbital velocity,

$$v_0 = \sqrt{\frac{GM}{R+h}} = \sqrt{\frac{6.67\times10^{-11}\times6\times10^{24}}{7.38\times10^6}}$$

$$= \mathbf{7364\ ms^{-1}}.$$

Period of revolution,

$$T = \frac{2\pi(R+h)}{v_0} = \frac{2\pi\times7.38\times10^6}{7364} = 6297\ \text{s}.$$

Example 45. A remote sensing satellite of the earth revolves in a circular orbit at a height of 250 *km above the earth's surface. What is the (i) orbital speed and (ii) period of revolution of the satellite ? Radius of the earth,* $R = 6.38 \times 10^6$ *m, and acceleration due to gravity on the surface of the earth,* $g = 9.8\ ms^{-2}$.

Solution. (*i*) Here $g = 9.8\ \text{ms}^{-2}$,

$$R = 6.38 \times 10^6\ \text{m}, \quad h = 250{,}000\ \text{m}$$

$$R + h = 6.38 \times 10^6 + 250{,}000 = 6.63 \times 10^6\ \text{m}$$

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The orbital speed is given by

$$v_0 = \sqrt{\frac{gR^2}{R+h}} = \sqrt{\frac{9.8 \times (6.38 \times 10^6)^2}{6.63 \times 10^6}}$$

$$= 7.76 \times 10^3 \text{ ms}^{-1} = \mathbf{7.76 \text{ kms}^{-1}}.$$

(ii) The period of revolution of the satellite will be

$$T = \frac{2\pi(R+h)}{v_0} = \frac{2 \times 22 \times 6.63 \times 10^6}{7 \times 7.76 \times 10^3}$$

$$= \mathbf{5370 \text{ s}}.$$

EXAMPLE 46. An artificial satellite is going round the earth, close to its surface. What is the time taken by it to complete one round ? Given radius of the earth = 6400 km.

Solution. Here $R = 6400 \text{ km} = 6.4 \times 10^6 \text{ m}$,

$$g = 9.8 \text{ ms}^{-2}$$

Orbital velocity near the earth's surface is

$$v_0 = \sqrt{gR} = \sqrt{9.8 \times 6.4 \times 10^6} = 7290 \text{ ms}^{-1}$$

Time period,

$$T = \frac{2\pi R}{v_0} = \frac{2 \times 22 \times 6.4 \times 10^6}{7 \times 7290} = 5079 \text{ s}$$

$$= \mathbf{1.411 \text{ hour}}.$$

EXAMPLE 47. A satellite revolves in an orbit close to the surface of a planet of mean density $5.51 \times 10^3 \text{ kgm}^{-3}$. Calculate the time period of the satellite.

Given $G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$.

Solution. Here $\rho = 5.51 \times 10^3 \text{ kgm}^{-3}$,

$$G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}.$$

Time period of the satellite near the surface of the planet ($h = 0$) is

$$T = \sqrt{\frac{3\pi}{G\rho}}$$

$$= \sqrt{\frac{3 \times 3.14}{6.67 \times 10^{-11} \times 5.51 \times 10^3}} = \mathbf{5062.7 \text{ s}}.$$

EXAMPLE 48. An earth's satellite makes a circle around the earth in 90 minutes. Calculate the height of the satellite above the earth's surface. Given radius of the earth is 6400 km and $g = 980 \text{ cms}^{-2}$.

Solution. Here $T = 90 \text{ minutes} = 5400 \text{ s}$,

$$R = 6400 \text{ km} = 6.4 \times 10^6 \text{ m}$$

$$g = 980 \text{ cms}^{-2} = 9.8 \text{ ms}^{-2}$$

As $T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}}$

$$\therefore \quad R + h = \left[\frac{gR^2T^2}{4\pi^2}\right]^{1/3}$$

$$= \left[\frac{9.8 \times (6.4 \times 10^6)^2 \times (5400)^2}{4 \times 9.87}\right]^{1/3}$$

$$= 6.668 \times 10^6 \text{ m} = 6668 \text{ km}$$

Hence $h = 6668 - R = 6668 - 6400 = \mathbf{268 \text{ km}}$.

EXAMPLE 49. If the period of revolution of an artificial satellite just above the earth's surface be T and the density of earth be ρ, then prove that ρT^2 is a universal constant. Also calculate the value of this constant.

Given : $G = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$.

Solution. As $T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$

$$\therefore \quad T^2 = \frac{4\pi^2(R+h)^3}{GM}$$

or $$M = \frac{4\pi^2(R+h)^3}{GT^2}$$

For the satellite revolving just above the earth's surface, $h = 0$. So

$$M = \frac{4\pi^2 R^3}{GT^2}$$

Also, $$M = \frac{4}{3}\pi R^3 \rho$$

$$\therefore \quad \frac{4}{3}\pi R^3 \rho = \frac{4\pi^2 R^3}{GT^2}$$

$$\therefore \quad \rho T^2 = \frac{3\pi}{G}, \text{ which is a universal constant.}$$

and $$\rho T^2 = \frac{3 \times 3.14}{6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}}$$

$$= \mathbf{1.41 \times 10^{11} \text{ kgs}^2 \text{ m}^{-3}}.$$

EXAMPLE 50. In a two-stage launch of a satellite, the first stage brings the satellite to a height of 150 km and the second stage gives it the necessary critical speed to put it in a circular orbit around the Earth. Which stage requires more expenditure of fuel ? (Neglect damping due to air resistance, especially in the first stage).

Mass of the Earth $= 6.0 \times 10^{24} \text{ kg}$, radius $= 6400 \text{ km}$, $G = 6.67 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$.

Solution. Work done on the satellite in first stage,

W_1 = P.E. at height of 150 km

− P.E. at the surface of the earth

$$= -\frac{GMm}{R+h} - \left(-\frac{GMm}{R}\right)$$

$$= GMm\left(\frac{1}{R} - \frac{1}{R+h}\right) = \frac{GMmh}{R(R+h)}$$

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Work done on the satellite in second stage,

W_2 = Energy required to give it orbital velocity v_0

$$= \frac{1}{2}mv_0^2 = \frac{1}{2}m\left(\frac{GM}{R+h}\right) = \frac{1}{2}\left(\frac{GMm}{(R+h)}\right)$$

$$\therefore \quad \frac{W_1}{W_2} = \frac{2h}{R} = \frac{2\times 150}{6400} = \frac{3}{64} < 1.$$

As $W_2 > W_1$, so the second stage requires more expenditure of fuel.

✕ Problems For Practice

1. An artificial satellite circled around the earth at a distance of 3400 km. Calculate its orbital velocity and period of revolution. Radius of earth = 6400 km and $g = 9.8\ ms^{-2}$. **(Ans. 6400 ms^{-1}, 9621 s)**

2. The orbit of a geostationary satellite is concentric and coplanar with the equator of earth and rotates along the direction of rotation of earth. Calculate the height and speed. Take mass of earth = 5.98×10^{27} g and its radius = 6400 km. Given, $\pi^2 = 9.87$.
(Ans. 35850 km, 3.071 kms^{-1})

3. A satellite revolves round a planet in an orbit just above the surface of planet. Taking $G = 6.67 \times 10^{-11}\ Nm^2\ kg^{-2}$ and the mean density of the planet = $8.0 \times 10^3\ kg\ m^{-3}$, find the period of satellite. **[Ans. 4206.7 s]**

4. An artificial satellite of mass 100 kg is in a circular orbit of 500 km above the earth's surface. Take radius of earth as 6.5×10^6 m. *(i)* Find the acceleration due to gravity at any point along the satellite path. *(ii)* What is the centripetal acceleration of the satellite ? Take $g = 9.8\ ms^{-2}$.
[Ans. *(i)* 8.45 ms^{-2} *(ii)* 8.45 ms^{-2}]

5. A space-ship is launched into a circular orbit close to the earth's surface. What additional velocity has now to be imparted to the space-ship in the orbit to overcome the gravitational pull ? (Radius of the earth = 6400 km, $g = 9.8\ ms^{-2}$). **[Roorkee 88]**
(Ans. 3.278 kms^{-1})

✕ Hints

1. $v_0 = R\sqrt{\frac{g}{R+h}} = 64\times 10^5\sqrt{\frac{9.8}{(64+34)\times 10^5}}$

$= 6400\ ms^{-1}$

$$T = \frac{2\pi(R+h)}{v_0} = \frac{2\pi\times 98\times 10^5}{6400} = 9621\ s.$$

2. Here $T = 24\times 3600$ s, $G = 6.67\times 10^{-11}\ Nm^2\ kg^{-2}$,

$M = 5.98\times 10^{27}\ g = 5.98\times 10^{24}$ kg

$R = 6400$ km

$$R+h = \left[\frac{T^2\,GM}{4\pi^2}\right]^{1/3}$$

$$= \left[\frac{(24\times 3600)^2\times 6.67\times 10^{-11}\times 5.98\times 10^{24}}{4\times 9.87}\right]^{1/3}$$

$= 4.225\times 10^7\ m = 42250$ km

$\therefore \quad h = 42250 - 6400 = 35850$ km

Orbital speed,

$$v_0 = \frac{2\pi(R+h)}{T} = \frac{2\times 3.14\times 42250}{24\times 3600} = 3.071\ kms^{-1}.$$

4. Here $h = 500$ km $= 0.5\times 10^6$ m, $R = 6.5\times 10^6$ m;

$\therefore \quad R+h = 7.0\times 10^6$ m

(i) $$g' = g\left(\frac{R}{R+h}\right)^2 = 9.8\left(\frac{6.5\times 10^6}{7.0\times 10^6}\right)^2 = 8.45\ ms^{-2}.$$

(ii) $$a_c = \frac{v_0^2}{R+h} = \frac{1}{R+h}\times\frac{gR^2}{R+h} = 8.45\ ms^{-2}.$$

5. Orbital velocity near earth's surface, $v_0 = \sqrt{gR_e}$

Escape velocity, $v_e = \sqrt{2gR_e} = 1.414\sqrt{gR_e}$

Additional velocity required

$= v_e - v_0 = (1.414 - 1)\sqrt{gR_e}$

$= 0.414\times\sqrt{9.8\times 6400\times 10^3} = 3.278\times 10^3\ ms^{-1}$

$= 3.278\ kms^{-1}$

8.25 GEOSTATIONARY SATELLITES

43. *What are geostationary satellites ? Calculate the height of the orbit above the surface of the earth in which a satellite, if placed, will appear stationary.*

Geostationary satellite. If a satellite is made to revolve from west to east with a period of revolution equal to 24 hours in a circular orbit concentric and coplanar with the equatorial plane of the earth, its relative velocity with respect to the earth will be zero. Such a satellite is called *geostationary satellite* because it appears stationary to an observer on the earth. It is called **synchronous satellite** because its angular speed is same as that of the earth about its own axis. When such a satellite is used for communication purposes, it is known as *communication satellite*. *Telstar* was the first communication satellite sent by U.S.A. into space in 1962.

A satellite which revolves around the earth in its equatorial plane with the same angular speed and in the same direction as the earth rotates about its own axis is called a geostationary or synchronous satellite.

Height of a geostationary satellite. The height of a satellite above the earth's surface is given by

$$h = \left[\frac{T^2R^2g}{4\pi^2}\right]^{1/3} - R$$

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But $T = 24\text{ h} = 86400\text{ s}$,

R = radius of the earth = 6400 km,

$g = 9.8\text{ ms}^{-2} = 0.0098\text{ kms}^{-2}$

$$\therefore \quad h = \left[\frac{(86400)^2 \times (6400)^2 \times 0.0098}{4 \times 9.87}\right]^{1/3} - 6400$$

$$= 42330 - 6400 = 35930\text{ km}.$$

44. *State the necessary conditions for a satellite to be geostationary.*

Necessary conditions for a geostationary satellite. These are as follows :

1. It should revolve in an orbit concentric and coplanar with the equatorial plane of the earth.
2. Its sense of rotation should be same as that of the earth *i.e.*, from west to east.
3. Its period of revolution around the earth should be exactly same as that of the earth about its own axis *i.e.*, 24 hours.
4. It should revolve at a height of nearly 36,000 km above the earth's surface.

45. *Discuss the use of geostationary satellites in global communication.*

Use of geostationary satellites in global transmission. A satellite cannot establish communication link over the entire earth. This is because, the curvature of the earth keeps a large part of the earth out of sight. However, three uniformly spaced satellites (120° apart from each other) placed in a geostationary orbit and equipped with radio transponders can be used to provide line of sight communication between any two points on the earth, as shown in Fig. 8.32.

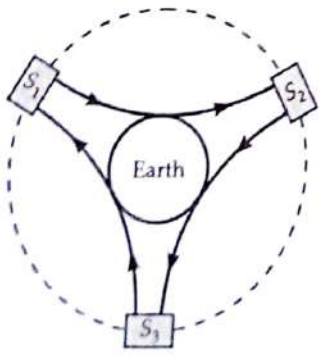

Fig. 8.32 Three uniformly spaced satellites placed in a geostationary orbit.

Such satellites are called synchronous communication satellites (SYNCOMS). The geostationary orbit is also called *Clarke geosynchronous orbit* or *Clarke arc* after the name of famous science writer *Arthur C. Clarke* who first proposed the idea of a communication satellite in 1945.

46. *Give some uses of geostationary satellites.*

Uses of geostationary satellites :

1. In communicating radio, T.V. and telephone signals across the world. Geostationary satellites act as reflectors of such signals.
2. In studying upper regions of the atmosphere.
3. In forecasting weather.
4. In determining the exact shape and dimensions of the earth.
5. In studying meteorites.
6. In studying solar radiations and cosmic rays.

EXAMPLE 51. *To what latitude does the SYNCOMS coverage extend ? What is the orbital speed of a SYNCOMS ?* **[NCERT]**

Solution. Clearly, the latitude of the coverage extends upto the tangent SP, as shown in Fig. 8.33. From right ΔOPS,

$$\cos\lambda = \frac{OP}{OS} = \frac{R_E}{OS} = \frac{6.37 \times 10^3\text{ km}}{4.22 \times 10^4\text{ km}} = 0.151.$$

$$\therefore \quad \lambda = 81.3°.$$

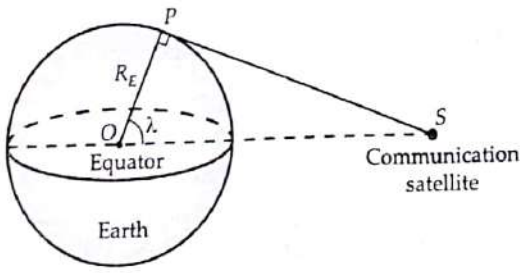

Fig. 8.33

Thus a circular arc of about 90° is left uncovered around the pole. That is why we need three satellites to cover the entire earth.

Orbital speed of the SYNCOMS

$$v = \frac{2\pi r}{T} = \frac{2 \times 3.14 \times 4.22 \times 10^7\text{ m}}{86400\text{ s}} = \mathbf{3067\ ms^{-1}}.$$

8.26 ▼ POLAR SATELLITES

47. *What are polar satellites ? Give some of their uses.*

Polar satellite. *A satellite that revolves in a polar orbit is called a polar satellite.* A polar orbit is one whose plane is perpendicular to the equatorial plane of the earth. A polar orbit passes over north and south poles of the earth and has a smaller radius of 500 – 800 km. As the earth rotates about its axis, the polar satellite successively passes across the different parts of earth's surface. Thus the polar satellite eventually scans the

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entire surface of the earth. Examples of polar satellites are European SPOT and the Indian Earth resources satellites (IERS).

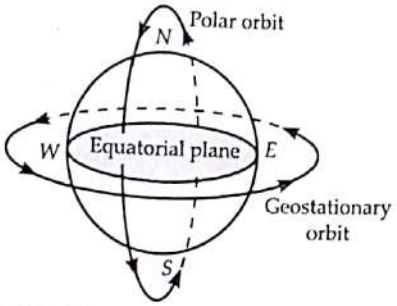

Fig. 8.34 Polar and geostationary orbits.

Uses of polar satellites :

(*i*) Polar satellites are used in weather and environment monitoring. They provide more reliable information than geostationary satellites because their orbits are closed to the earth.

(*ii*) They are used in spying work for military purposes.

(*iii*) British polar satellite first detected hole in the ozone layer.

(*iv*) They are used to study topography of Moon, Venus and Mars.

8.27 TOTAL ENERGY AND BINDING ENERGY OF A SATELLITE

48. *Derive an expression for the total energy of a satellite orbiting the earth. What is the significance of negative total energy ?*

Total energy of a satellite. Consider a satellite of mass m moving around the earth with velocity v_0 in an orbit of radius r. Because of gravitational pull of the earth, the satellite has *potential energy* which is given by

$$U = -\frac{GMm}{r}$$

The *kinetic energy* of a satellite due to its orbital motion is

$$K = \frac{1}{2}mv_0^2 = \frac{1}{2}m\left(\frac{GM}{r}\right)\left[\because v_0 = \sqrt{\frac{GM}{r}}\right]$$

Total energy of the satellite is

$$E = U + K = -\frac{GMm}{r} + \frac{1}{2}\frac{GMm}{r}$$

or
$$E = -\frac{GMm}{2r}$$

The total energy of the satellite is *negative*. It indicates that the *satellite is bound to the earth*. At infinity ($r = \infty$), the potential energy is zero and also the kinetic energy is zero. Hence total energy at infinity is zero. Thus a negative total energy means that in order to send the satellite to infinity, it needs to be given extra energy (to make total energy zero) ; otherwise it will continue revolving in a closed orbit. This fact is expressed by saying that the satellite is bound to the earth.

49. *What do you mean by binding energy of a satellite ? Write an expression for it.*

Binding energy of a satellite. *The energy required by a satellite to leave its orbit around the earth and escape to infinity is called its binding energy.*

The total energy of a satellite is $-\frac{GMm}{2r}$. In order to escape to infinity, it must be supplied an extra energy equal to $+\frac{GMm}{2r}$ so that its total energy E becomes equal to zero. Hence

Binding energy of a satellite $= \frac{GMm}{2r}$.

For Your Knowledge

▲ The total mechanical energy of an object (say satellite in orbit), is negative if it is bound, *e.g.*, its orbit is an ellipse or circle. But it is *not always negative*. It can be positive in which case its trajectory is a hyperbola and the object is not bound to the central star or its equivalent. These statements are evidently true when the zero of potential energy is chosen at infinity.

Examples based on Total Energy and Binding Energy of Satellite

FORMULAE USED

1. Potential energy, $U = -\frac{GMm}{r}$
2. Kinetic energy, $K = \frac{1}{2}mv_0^2 = \frac{1}{2}m\left(\frac{GM}{r}\right) = \frac{1}{2}\frac{GMm}{r}$
3. Total energy, $E = K + U = \frac{1}{2}mv_0^2 - \frac{GMm}{r} = -\frac{GMm}{2r}$
4. As $E = \frac{U}{2} = -K \quad \therefore \Delta K = -\Delta E$ and $\Delta U = 2\Delta E$
5. Binding energy $= \frac{GMm}{2r}$

UNITS USED

Masses M and m are in kg, distance r in metre, energies K, U and E in joule.

EXAMPLE 52. A 400 kg satellite is in a circular orbit of radius $2R_E$ about the earth. How much energy is required to transfer it to a circular orbit of radius $4R_E$? What are the changes in the kinetic and potential energies ? [NCERT]

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Solution. Total energy in orbit of radius $2R_E$,

$$E_i = -\frac{GM_E m}{4R_E} \qquad \left[\because E = \frac{GMm}{2r}\right]$$

Total energy in orbit of radius $4R_E$,

$$E_f = -\frac{GM_E m}{8R_E}$$

Energy required to transfer the satellite to orbit of radius $4R_E$,

$$\Delta E = E_f - E_i = -\frac{GM_E m}{8R_E} + \frac{GM_E m}{4R_E}$$

$$= \frac{GM_E m}{8R_E} = \left(\frac{GM_E}{R_E^2}\right)\frac{mR_E}{8} = \frac{gmR_E}{8}$$

But $g = 9.8\ \text{ms}^{-2}$, $m = 400$ kg, $R_E = 6.37 \times 10^6$ m

$$\therefore \quad \Delta E = \frac{9.8 \times 400 \times 6.37 \times 10^6}{8} = 3.13 \times 10^9 \text{ J.}$$

Change in K.E., $\Delta K = -\Delta E = -3.13 \times 10^9$ J

Change in P.E., $\Delta U = 2\Delta E = 6.26 \times 10^9$ J.

EXAMPLE 53. A satellite orbits the earth at a height of 500 km from its surface. Compute its (i) kinetic energy, (ii) potential energy, and (iii) total energy. Mass of the satellite = 300 kg, Mass of the earth = 6.0×10^{24} kg, radius of the earth = 6.4×10^6 m, $G = 6.67 \times 10^{-11}$ $Nm^2\ kg^{-2}$. Will your answer alter if the earth were to shrink suddenly to half its size ?

Solution. Here

$h = 500\ \text{km} = 500 \times 10^3$ m,

$m = 300$ kg, $M = 6.0 \times 10^{24}$ kg

$R = 6.4 \times 10^6$ m, $G = 6.67 \times 10^{-11}\ \text{Nm}^2\ \text{kg}^{-2}$

(*i*) Kinetic energy

$$= \frac{1}{2}mv^2 = \frac{1}{2}m.\frac{GM}{R+h} \qquad \left[\because v = \sqrt{\frac{GM}{R+h}}\right]$$

$$= \frac{1}{2} \times \frac{300 \times 6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{6.4 \times 10^6 + 500 \times 10^3}$$

$$= \mathbf{8.7 \times 10^9\ J.}$$

(*ii*) Potential energy

$$= -\frac{GMm}{(R+h)} = -\frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 300}{6.4 \times 10^6 + 500 \times 10^3}$$

$$= \mathbf{-17.4 \times 10^9\ J.}$$

(*iii*) Total energy

$$= \text{K.E.} + \text{P.E.} = 8.7 \times 10^9 - 17.4 \times 10^9$$

$$= \mathbf{-8.7 \times 10^9\ J.}$$

PROBLEMS FOR PRACTICE

1. A rocket is launched vertically from the surface of the earth with an initial velocity of 10 kms^{-1}. How far above the surface of the earth would it go ? Radius of the earth = 6400 km and $g = 9.8\ \text{ms}^{-2}$. **(Ans. 2.5×10^4 km)**

2. A satellite of mass 250 kg is orbiting the earth at a height of 500 km above the surface of earth. How much energy must be expended to rocket the satellite out of the gravitational influence of the earth ? Given mass of the earth = 6.0×10^{24} kg, radius of the earth = 6400 km and $G = 6.67 \times 10^{-11}\ \text{Nm}^2\ \text{kg}^{-2}$. **(Ans. 7.25×10^9 J)**

3. A body is to be projected vertically upwards from earth's surface to reach a height of $9R$, where R is the radius of earth. What is the velocity required to do so ? Given $g = 10\ \text{ms}^{-2}$ and radius of earth $= 6.4 \times 10^6$ m. **(Ans. $1.073 \times 10^4\ \text{ms}^{-1}$)**

4. Show that the velocity of a body released at a distance r from the centre of the earth, when it strikes the surface of the earth is given by

$$v = \sqrt{2GM\left(\frac{1}{R} - \frac{1}{r}\right)}$$

where R and M are the radius and mass of the earth respectively. Also show that the velocity with which the meteorites strike the surface of the earth is equal to the escape velocity.

5. Calculate the energy required to move an earth satellite of mass 10^3 kg from a circular orbit of radius $2R$ to that of radius $3R$. Given mass of the earth, $M = 5.98 \times 10^{24}$ kg and radius of the earth, $R = 6.37 \times 10^6$ m. **[Delhi 14]** **(Ans. 5.02×10^9 J)**

HINTS

1. Initial K.E. of the rocket = Gain in gravitational P.E.

$$\frac{1}{2}mv^2 = -\frac{GMm}{(R+h)} - \left[-\frac{GMm}{R}\right] = \frac{GMmh}{R(R+h)}$$

or $\quad \frac{1}{2}v^2 = \frac{gR^2h}{R(R+h)} = \frac{gRh}{R+h}$

or $\quad \frac{R+h}{h} = \frac{2gR}{v^2}$ or $\frac{R}{h} + 1 = \frac{2gR}{v^2}$

or $\quad h = R\left[\frac{2gR}{v^2} - 1\right]^{-1}$

$$= 6.4 \times 10^6\left[\frac{2 \times 9.8 \times 6.4 \times 10^6}{(10^4)^2} - 1\right]^{-1}$$

$$= 6.4 \times 10^6 \times (0.2544)^{-1} = 2.5 \times 10^7\ \text{m}$$

$$= \mathbf{2.5 \times 10^4\ km.}$$

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2. Total energy of the satellite at height h,

$$E = \text{P.E.} + \text{K.E.}$$

$$= -\frac{GMm}{R+h} + \frac{1}{2}mv^2 = -\frac{GMm}{R+h} + \frac{GMm}{2(R+h)}$$

$$= -\frac{GMm}{2(R+h)} \qquad \left[\because v = \sqrt{\frac{GM}{R+h}}\right]$$

Energy needed to rocket out the satellite

$$= -E = +\frac{GMm}{2(R+h)}$$

$$= \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 2.5}{2 \times (6.4 \times 10^6 + 5 \times 10^5)}$$

$$= \mathbf{7.25 \times 10^9\ J.}$$

3. By the conservation of energy,

$$-\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{10R} + 0$$

or $$\frac{v^2}{2} = -\frac{GM}{10R} + \frac{GM}{R} = \frac{9GM}{10R} = \frac{9gR^2}{10R} = \frac{9gR}{10}$$

or $$v = \sqrt{\frac{18gR}{10}} = \sqrt{\frac{18 \times 10 \times 6.4 \times 10^6}{10}}$$

$$= \mathbf{1.073 \times 10^4\ ms^{-1}.}$$

4. As the body is initially at rest, its total initial energy

$$E_i = \text{K.E.} + \text{P.E.} = 0 - \frac{GMm}{r}$$

If v is the velocity of the body on reaching the earth's surface, then its total final energy

$$E_f = \text{K.E.} + \text{P.E.} = \frac{1}{2}mv^2 - \frac{GMm}{R}$$

By conservation of energy,

$$-\frac{GMm}{r} = \frac{1}{2}mv^2 - \frac{GMm}{R}$$

or $$v = \sqrt{2GM\left(\frac{1}{R} - \frac{1}{r}\right)}$$

As meteorites come from a large distance, $r = \infty$.

$$\therefore \quad v = \sqrt{\frac{2GM}{R}} = \mathbf{11.2\ kms^{-1}.}$$

5. Total energy of a satellite,

$$E = \text{P.E.} + \text{K.E.} = -\frac{GMm}{r} + \frac{1}{2}mv^2$$

$$= -\frac{GMm}{r} + \frac{1}{2}m \cdot \frac{GM}{r} = -\frac{GMm}{2r}$$

$$W = E_f - E_i = -\frac{GMm}{2 \times 3R} + \frac{GMm}{2 \times 2R} = \frac{GMm}{12R}$$

$$= \frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24} \times 10^3}{12 \times 6.37 \times 10^6}$$

$$= \mathbf{5.02 \times 10^9\ J.}$$

8.28 THEORIES ABOUT PLANETARY MOTION

50. *Discuss the various theories about the planetary motion.*

Different theories about planetary motion. Since ancient times, scientists have been studying the motion of celestial objects like sun, planets, moon, etc. Some of these noteworthy theories are as follows :

(*i*) **Geocentric model.** Around 100 A.D., Greek astronomer *Ptolmey* wrote a book, *The Almagest*, in which he proposed the **geocentric model** of the planetary motion. According to this model, *the earth remained stationary at the centre and all planets, moon, sun and other stars revolved around it.* The planets revolved in small circles called *epicycles* and the centres of these epicycles moved in larger circles called deferents, around the earth.

(*ii*) **Aryabhata's contribution.** In 498 AD, the great Indian mathematician and astronomer, *Aryabhata*, proposed that *the earth revolves around the sun along with other planets and also rotates about its own axis.* He was able to explain phenomena like solar eclipse, lunar eclipse, formation of days and nights, etc. However, his ideas could not be communicated to the western world.

(*iii*) **Heliocentric model.** In 1543, the Polish astronomer *Nicolaus Copernicus* suggested that the *sun is at the centre of the solar system and the earth and other planets revolve around it.* This theory was called **heliocentric theory.**

(*iv*) **Contributions of Brahe and Kepler.** To test the validity of Copernicus model, the great Danish astronomer *Tycho Brahe* (1546–1601) made extra-ordinary observations by studying the motions of planets and stars without the aid of a telescope. His data were critically analysed over a period of twenty years by *Johannes Kepler* (1571 – 1630), who was Brahe's assistant. From these complicated data, Kepler deduced simple relations that governed planetary motion. These are three famous laws of Kepler which strongly supported the Copernicus model of solar system and played major role in the discovery of Newton's law of gravitation.

8.29 KEPLER'S LAWS OF PLANETARY MOTION

51. *State and explain the Kepler's laws of planetary motion.*

Kepler's laws of planetary motion. To explain the motion of the planets, Kepler formulated the following three laws :

1. Law of orbits (first law). *Each planet revolves around the sun in an elliptical orbit with the sun situated at one of the two foci.*

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As shown in Fig. 8.35, the planets move around the sun in an elliptical orbit. An ellipse has two foci S and S' the sun remains at one focus S.

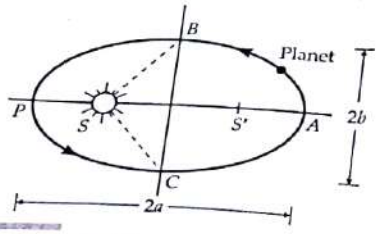

Fig. 8.35 Elliptical orbit of a planet, $PA = 2a$ = major axis, $BC = 2b$ = minor axis.

The points P and A on the orbit are called the **perihelion** *and the* **aphelion** *and represent the closest and farthest distances from the sun respectively.* The orbits of Pluto and Mercury are highly elliptical. The orbits of Neptune and Venus are circular. The orbits of other planets have slight ellipticity and may be taken as nearly circular.

2. Law of areas (second law). *The radius vector drawn from the sun to a planet sweeps out equal areas in equal intervals of time i.e., the areal velocity (area covered per unit time) of a planet around the sun is constant.*

Suppose a planet takes same time to go from position A to B as in going from C to D [Fig. 8.36]. From Kepler's second law, the areas ASB and CSD (covered in equal time) must be equal. Clearly, the planet covers a larger distance CD when it is near the sun than AB when it is farther away in the same interval of time. Hence *the linear velocity of a planet is more when it is closer to the sun than its linear velocity when away from the sun.*

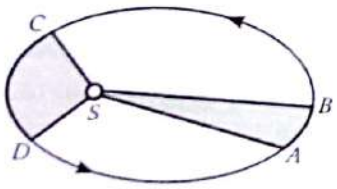

Fig. 8.36 Kepler's second law of areas.

3. Law of periods (Third law). *The square of the period of revolution of a planet around the sun is proportional to the cube of the semimajor axis of its elliptical orbit.*

If T is the period of revolution of a planet and R is the length of semimajor axis of its elliptical orbit, then

$$T^2 \propto R^3$$

or

$$T^2 = KR^3$$

where K is a proportionality constant.

For two different planets, we can write

$$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}$$

Thus *larger the distance of a planet from the sun, the larger will be its period of revolution around the sun.* The period of revolution of the farthest planet Pluto around the sun is 247 years while that of the nearest planet mercury is 81 days.

8.30 DERIVATION OF KEPLER'S LAWS

52. *What is Kepler problem ?*

Kepler problem. *The derivation of the Kepler's three laws of planetary motion from Newton's law of gravitation is called the Kepler problem.*

53. *Prove the Kepler's first law of planetary motion.*

Proof of Kepler's first law of planetary motion. The planetary motion takes place under the action of the gravitational force exerted by the sun,

$$\vec{F} = -\frac{GM_s m_p}{r^3}\vec{r}$$

where M_s is the mass of the sun. This force is radial and central. Negative sign indicates that $\vec{F}$ is oppositely directed to $\vec{r}$.

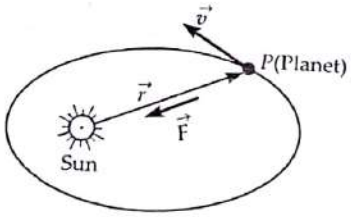

Fig. 8.37 Gravitational force on a planet due to the sun.

The torque exerted on the planet P about the sun is

$$\vec{\tau} = \vec{r} \times \vec{F} = \vec{r} \times \left(-\frac{GM_s m_p}{r^3}\right)\vec{r} = \vec{0}$$

$$[\because \vec{r} \times \vec{r} = \vec{0}]$$

But $\vec{\tau}$ = Rate of change of angular momentum

$$= \frac{d\vec{L}}{dt}$$

$$\therefore \frac{d\vec{L}}{dt} = 0 \quad \text{or} \quad \vec{L} = \text{constant.}$$

This shows that the angular momentum of the planet about the sun remains constant both in magnitude and direction. Since the direction of $\vec{L}(= \vec{r} \times \vec{p})$ is

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fixed, $\vec{r}$ and $\vec{v}$ lie in a plane normal to $\vec{L}$. Moreover, it can be shown that the central force under the action of which the planet moves varies as the square of the distance between the planet and sun and this orbit is an ellipse.

54. *Prove the Kepler's second law of planetary motion.*

Proof of Kepler's second law. As shown in Fig. 8.38, consider a planet moving in an elliptical orbit with the sun at focus S. Let $\vec{r}$ be the position vector of the planet w.r.t. the sun and $\vec{F}$ be the gravitational force on the planet due to the sun. Torque exerted on the planet by this force about the sun is

$$\vec{\tau} = \vec{r} \times \vec{F} = 0$$

[$\because$ $\vec{r}$ and $\vec{F}$ are oppositely directed]

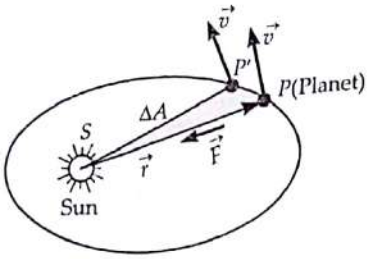

Fig. 8.38 Area swept by the radius vector in time Δt.

But $$\vec{\tau} = \frac{d\vec{L}}{dt}$$

$\therefore$ $$\frac{d\vec{L}}{dt} = 0 \quad \text{or} \quad \vec{L} = \text{constant}$$

Suppose the planet moves from position P to P′ in time Δt. The area swept by the radius vector $\vec{r}$ is

$$\Delta\vec{A} = \text{Area of triangular region } SPP'$$
$$= \frac{1}{2}\vec{r} \times \vec{P}P'$$

But $$\vec{PP'} = \Delta\vec{r} = \vec{v}\,\Delta t = \frac{\vec{p}}{m}\Delta t$$

$\therefore$ $$\Delta\vec{A} = \frac{1}{2}\vec{r} \times \frac{\vec{p}}{m}\Delta t$$

or $$\frac{\Delta\vec{A}}{\Delta t} = \frac{1}{2m}(\vec{r} \times \vec{p}) = \frac{\vec{L}}{2m}$$

or $$\frac{\Delta\vec{A}}{\Delta t} = \text{constant} \quad [\because \vec{L} \text{ and } m \text{ are constant}]$$

Thus the areal velocity of the planet remains constant *i.e., the radius vector joining planet to the sun sweeps out equal areas in equal intervals of time.* This proves Kepler's second law of planetary motion.

55. *Using Newton's law of gravitation, prove Kepler's third law of planetary motion for circular orbits.*

Proof of Kepler's third law. Suppose a planet of mass m moves around the sun in a circular orbit of radius r with orbital speed v. Let M be the mass of the sun. The force of gravitation between the sun and the planet provides the necessary centripetal force.

$\therefore$ $$\frac{mv^2}{r} = \frac{GMm}{r^2} \quad \text{or} \quad v^2 = \frac{GM}{r}$$

But orbital speed,

$$v = \frac{\text{Circumference}}{\text{Period of revolution}} = \frac{2\pi r}{T}$$

$\therefore$ $$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$$

or $$T^2 = \frac{4\pi^2}{GM} r^3 = K_s r^3$$

Thus $$T^2 \propto r^3$$

This proves Kepler's third law. The constant K_s is same for all planets. Its value is $2.97 \times 10^{-19}\ s^2 m^{-3}$. For an elliptical orbit, r gets replaced by semi-major axis a.

8.31 DEDUCTION OF LAW OF GRAVITATION FROM KEPLER'S THIRD LAW

56. *Using Kepler's law of periods, derive Newton's law of gravitation.*

Proof of Newton's law of gravitation. Suppose a planet of mass m moves around the sun in a circular orbit of radius r. Let M be the mass of the sun. If v is the orbital velocity of the planet, then the required centripetal force is

$$F = \frac{mv^2}{r}$$

But orbital velocity,

$$v = \frac{\text{Circumference}}{\text{Period of revolution}} = \frac{2\pi r}{T}$$

$\therefore$ $$F = \frac{m}{r} \times \left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 mr}{T^2}$$

According to Kepler's law of periods,

$$T^2 \propto r^3$$

or $$T^2 = k r^3$$

where k is a constant. Therefore,

$$F = \frac{4\pi^2 mr}{k r^3} = \frac{4\pi^2}{k} \cdot \frac{m}{r^2}$$

As the force between the sun and the planet is mutual, it should also be proportional to the mass M of the sun.

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Hence the factor,

$$\frac{4\pi^2}{k} \propto M \quad \text{or} \quad \frac{4\pi^2}{k} = GM$$

where G is another constant. So we have

$$F = G\frac{Mm}{r^2}$$

This is Newton's law of gravitation and it is applicable to any two bodies in the universe.

On the basis of Kepler's laws, Newton made the following conclusions :

1. A planet is acted upon by a centripetal force directed towards the sun.
2. The force acting on the planet is inversely proportional to the square of the distance between the planet and the sun.
3. The force acting on the planet is directly proportional to the product of the masses of the planet and the sun.

Examples based on Kepler's Law of Planetary Motion

FORMULAE USED

1. Angular momentum, $L = mvr = \text{constant}$
2. Law of areas, $\frac{\Delta A}{\Delta t} = \text{constant}$
3. Law of periods, $T^2 \propto r^3$ or $T^2 = k r^3$. For a satellite of earth, $k = \frac{4\pi^2}{GM_E} = 10^{-13}\ s^2 m^{-3}$
4. $\frac{T_2^2}{T_1^2} = \frac{r_2^3}{r_1^3}$

UNITS USED

Time periods T_1 and T_2 are in second, distances r_1 and r_2 in metre.

EXAMPLE 54. Calculate the period of revolution of Neptune around the sun, given that diameter of its orbit is 30 times the diameter of earth's orbit around the sun, both orbits being assumed to be circular.

Solution. According to Kepler's law of periods,

$$\left[\frac{T_N}{T_E}\right]^2 = \left[\frac{r_N}{r_E}\right]^3$$

But $\frac{r_N}{r_E} = 30$ and $T_E = 1$ year

$$\therefore \quad T_N^2 = T_E^2\left[\frac{r_N}{r_E}\right]^3 = (1)^2 \times (30)^3 = 27000$$

$$T_N = \sqrt{27000} = \mathbf{164.3\ years.}$$

EXAMPLE 55. In Kepler's law of periods : $T^2 = kr^3$, the constant $k = 10^{-13}\ s^2 m^{-3}$. Express the constant k in days and kilometres. The moon is at a distance of 3.84×10^5 km from earth. Obtain its time-period of revolution in days. **[NCERT]**

Solution. Given $k = 10^{-13}\frac{s^2}{m^3}$

As $1\,s = \frac{1}{24 \times 60 \times 60}$ day and $1\,m = \frac{1}{1000}$ km

$$\therefore \quad k = 10^{-13} \times \frac{1}{(24 \times 60 \times 60)^2}\, d^2\, \frac{1}{(1/1000)^3}\, km^{-3}$$

$$= \mathbf{1.33 \times 10^{-14}\ d^2 km^{-3}.}$$

For the moon, $r = 3.84 \times 10^5$ km.

$$\therefore \quad T^2 = kr^3 = 1.33 \times 10^{-14} \times (3.84 \times 10^5)^3$$

$$= \mathbf{27.3\ days.}$$

EXAMPLE 56. In an imaginary planetary system, the central star has the same mass as our sun, but is brighter so that only a planet twice the distance between the earth and the sun can support life. Assuming biological evolution (including aging process etc.) on that planet similar to ours, what would be the average life span of a 'human' on that planet in terms of its natural year ? The average life span of a human on the earth may be taken to be 70 years.

Solution. According to Kepler's law of periods,

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3$$

Here,

T_1 = Average life span of a human on the earth = 70 years.

T_2 = Average life span of a human on the planet = ?

R_1 = Distance between the earth and the planet = $2\ R_2$

R_2 = Distance between the earth and the sun.

$$\therefore \quad \left(\frac{70}{T_2}\right)^2 = \left(\frac{2\ R_2}{R_2}\right)^3$$

$$\text{or} \quad \frac{70 \times 70}{T_2^2} = 8$$

$$\text{or} \quad T_2^2 = \frac{70 \times 70}{8}$$

$$\therefore \quad T_2 = \frac{70}{\sqrt{8}} = \mathbf{25\ planet\ years.}$$

EXAMPLE 57. The planet Mars has two moons, Phobos and Delmos. (i) Phobos has a period 7 hours, 39 minutes and an orbital radius of 9.4×10^3 km. Calculate the mass of Mars.

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(ii) Assume that Earth and Mars move in circular orbits around the Sun, with the Martian orbit being 1.52 times the orbital radius of the Earth. What is the length of the Martian year in days ? **[NCERT ; Delhi 08]**

Solution. (*i*) Here $r = 9.4 \times 10^6$ m

$T = 7$ h 39 min $= 459$ min $= 459 \times 60$ s

Mass of Mars,

$$M_M = \frac{4\pi^2 r^3}{GT^2} = \frac{4 \times 9.87 \times (9.4)^3 \times 10^{18}}{6.67 \times 10^{-11} \times (459 \times 60)^2}$$

$$= 6.48 \times 10^{23} \text{ kg.}$$

(*ii*) Here $r_M = 1.52\ r_E$, $T_E = 365$ days

According to Kepler's law of periods,

$$\frac{T_M^2}{T_E^2} = \frac{R_M^3}{R_E^3}$$

$$\therefore \quad T_M = \left(\frac{R_M}{R_E}\right)^{3/2} T_E$$

$$= (1.52)^{3/2} \times 365 = 684 \text{ days.}$$

EXAMPLE 58. The distances of two planets from the sun are 10^{13} m and 10^{12} m respectively. Find the ratio of time periods and speeds of the two planets.

Solution. Here $r_1 = 10^{13}$ m, $r_2 = 10^{12}$ m

From Kepler's third law,

$$\frac{T_1}{T_2} = \left[\frac{r_1}{r_2}\right]^{3/2} = \left[\frac{10^{13}}{10^{12}}\right]^{3/2} = 10\sqrt{10}.$$

If v_1 and v_2 are the orbital speeds of the planets, then

$$v_1 = \frac{2\pi r_1}{T_1} \quad \text{and} \quad v_2 = \frac{2\pi r_2}{T_2}$$

$$\therefore \quad \frac{v_1}{v_2} = \frac{r_1}{r_2} \cdot \frac{T_2}{T_1} = \frac{r_1}{r_2}\left[\frac{r_2}{r_1}\right]^{3/2} = \left[\frac{r_2}{r_1}\right]^{1/2}$$

$$= \left[\frac{10^{12}}{10^{13}}\right]^{1/2} = 1/\sqrt{10}.$$

EXAMPLE 59. Let the speed of the planet at the perihelion P in Fig. 8.35 be v_P and the sun-planet distance SP be r_P. Relate (r_P, v_P) to the corresponding quantities at the aphelion (r_A, v_A). Will the planet take equal times to traverse BAC and CPB ? **[NCERT]**

Solution. At any point, the radius vector and velocity vector of the planet are mutually perpendicular. Angular momentum of the planet at the perihelion P is

$$L_P = m_P r_P v_P$$

Similarly, at the aphelion,

$$L_A = m_P r_A v_A$$

By conservation of angular momentum,

$$L_P = L_A$$

or $$m_P r_P v_P = m_P r_A v_A$$

or $$\frac{v_P}{v_A} = \frac{r_A}{r_P}$$

As $r_A > r_P$, so $v_P > v_A$.

To traverse BAC, area swept by radius vector = Area $SBAC$.

To traverse CPB, area swept by radius vector = Area $SBPC$.

But Area $SBAC$ > Area $SBPC$

From Kepler's second law, equal areas are swept in equal times. Hence the planet will take a longer time to traverse BAC than CPB.

PROBLEMS FOR PRACTICE

1. The distance of Venus from the sun is 0.72 AU. Find the orbital period of Venus. **[CBSE 93C ; Delhi 06]** **(Ans. 223 days)**
2. If the earth be one half its present distance from the sun, how many days will the present one year on the surface of earth change ? **(Ans.** Year decreases by 236 days**)**
3. The distance of planet Jupiter from the sun is 5.2 times that of the earth. Find the period of revolution of Jupiter around the sun. **(Ans. 11.86 years)**
4. The planet Neptune travels around the sun with a period of 165 years. Show that the radius of its orbit is approximately 30 times that of earth's orbit, both being considered as circular.
5. A geostationary satellite is orbiting the earth at a height $6R$ above the surface of earth, where R is the radius of the earth. Find the time period of another satellite at a height of $2.5R$ from the surface of earth in hours. **[IIT 87]** **(Ans. $6\sqrt{2}$ h)**
6. The radius of earth's orbit is 1.5×10^8 km and that of Mars is 2.5×10^{11} m. In how many years, does the Mars complete its one revolution ? **(Ans. 2.15 years)**
7. A planet of mass m moves around the sun of mass M in an elliptical orbit. The maximum and minimum distances of the planet from the sun are r_1 and r_2 respectively. Find the relation for the time period of the planet in terms of r_1 and r_2. **(Ans. $T \propto (r_1 + r_2)^{3/2}$)**

HINTS

1. For the earth, $T_1 = 1$ year, $r_1 = 1$ AU
   For the planet Venus, $T_2 = ?$ $r_2 = 0.72$ AU

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$$\therefore \quad T_2^2 = T_1^2 \cdot \left[\frac{r_2}{r_1}\right]^3 = 1^2 \times \left[\frac{0.72}{1}\right]^3 = 0.37 \text{ yr}^2$$

or $T_2 = \sqrt{0.37} = 0.61 \text{ yr} = \mathbf{223 \text{ days}}$.

2. Here $T_1 = 365$ days, $R_1 = r$, $R_2 = r/2$

$$T_2 = T_1\left(\frac{r_2}{r_1}\right)^{3/2} = 365\left(\frac{r/2}{r}\right)^{3/2} = 129 \text{ days}$$

Decrease in the number of days in one year
$= 365 - 129 = \mathbf{236 \text{ days}}$.

3. $T_J = T_E\left[\frac{r_J}{r_E}\right]^{3/2} = 1 \times \left[\frac{5.2\, r_E}{r_E}\right]^{3/2} = \mathbf{11.86 \text{ years}}$.

4. Here $T_1 = 1$ year, $T_2 = 165$ years

$$\therefore \quad R_2 = R_1\left[\frac{T_2}{T_1}\right]^{2/3} = R_1 \times \left[\frac{165}{1}\right]^{2/3} \simeq 30\, R_1.$$

5. Here $r_1 = 6R + R = 7R$, $T_1 = 24$ h

$r_2 = 2.5\, R + R = 3.5\, R$, $T_2 = ?$

$$T_2 = T_1\left[\frac{r_2}{r_1}\right]^{3/2} = 24\left[\frac{3.5\, R}{7R}\right]^{3/2} = \frac{24}{2\sqrt{2}} = 6\sqrt{2} \text{ h}.$$

6. Here $r_1 = 1.5 \times 10^8$ km $= 1.5 \times 10^{11}$ m,

$r_2 = 2.5 \times 10^{11}$ m, $T_1 = 1$ year

$$T_2 = \left[\frac{R_2}{R_1}\right]^{3/2} \times T_1 = \left[\frac{2.5 \times 10^{11}}{1.5 \times 10^{11}}\right]^{3/2} \times 1 = \left[\frac{5}{3}\right]^{3/2}$$

$= \mathbf{2.15 \text{ years}}$.

7. Semimajor axis of the elliptical orbit of the plane around the sun, $r = (r_1 + r_2)/2$. According to Kepler's third law,

$$T^2 \propto \left(\frac{r_1 + r_2}{2}\right)^3 \quad \text{or} \quad T \propto \left(\frac{r_1 + r_2}{2}\right)^{3/2}$$

or $T \propto (r_1 + r_2)^{3/2}$.

8.32 WEIGHTLESSNESS

57. *What is weightlessness ? How does weightlessness arise in various situations ? Give some problems of weightlessness.*

Weightlessness. When a body presses against a supporting surface, the supporting surface exerts a force of reaction on him. This force of reaction produces the feeling of weight in the body. If somehow the force of reaction becomes zero, the apparent weight of the body becomes zero.

A body is said to be in a state of weightlessness when the reaction of the supporting surface is zero or its apparent weight is zero.

A body can be in the state of weightlessness under the following circumstances :

(*i*) **In a freely falling lift.** Consider a person of true weight mg standing in a lift which is moving vertically downwards with acceleration a. If R is the reaction of the floor on the man, then

$$mg - R = ma$$

$\therefore$ Apparent weight,

$$R = m(g - a)$$

If the cable of the lift breaks, it begins to fall freely. Then $a = g$, and

$$R = m(g - g) = 0$$

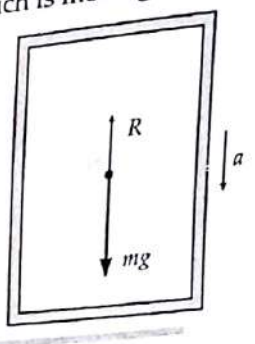

Fig. 8.39 Lift moving downwards.

Thus the apparent weight of the man becomes zero. Both the man and the lift are moving downwards with the same acceleration g. There are no forces of action and reaction between them. Hence *the man falling freely develops a feeling of weightlessness.*

(*ii*) **Inside a spacecraft.** Consider a spacecraft revolving around the earth in an orbit of radius r. The acceleration of the satellite is GM/r^2, towards the centre of the earth. Here M is the mass of the earth. Suppose a body of mass m lies on an inside surface of the satellite. Forces acting on this body will be

(*a*) Gravitational pull of the earth $= \frac{GMm}{r^2}$.

(*b*) Reaction force R of the surface.

By Newton's second law,

$$\frac{GMm}{r^2} - R = ma = m\left(\frac{GM}{r^2}\right)$$

$$\therefore \quad R = 0$$

Thus the surface does not exert any force on the body and hence its apparent weight is zero. That is why, an astronaut sees all the bodies floating weightlessly inside the spacecraft.

(*iii*) **At null points in space.** At certain points in space, called the null points, the gravitational forces due to various masses cancel out. As the value of g is zero at these points, so the effective weight of the body is zero.

(*iv*) **At the centre of the earth.** As the value of g is zero at the centre of the earth, so weight of a body is zero at the centre of the earth.

Problems of weightlessness :

(*i*) Eating and drinking become difficult in the state of weightlessness. An astronaut cannot

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drink water from a glass because on tilting the glass, the water comes out in the form of floating drops. He takes food in the form of paste from tube squeezed into his mouth.

(*ii*) Space flight for a long time adversely affects the human organism.

(*iii*) While walking in a space craft, an astronaut is pushed away from the floor and he may crash against the ceiling of the spacecraft.

(*iv*) It is not possible to perform experiments on simple pendulum in the state of weightlessness. As $g = 0$, so $T = 2\pi\sqrt{l/g} = \infty$.

For Your Knowledge

- When a body is in a free fall, a gravitational pull mg does act on it. It is said to be weightless because it exerts no force on its support.
- An astronaut experiences weightlessness in space. This is not because the gravitational force is small on him. It is because the astronaut and the satellite both are in a state of free fall towards the earth.
- In the state of weightlessness, though the bodies have no weight, they have inertia on account of their mass. So bodies floating in a space craft may collide with each other and crash.

8.33 INERTIAL AND GRAVITATIONAL MASS

58. *What is inertial mass of a body ? Give its important properties.*

Inertial mass. *The mass of a body which measures its inertia is called its inertial mass. It is equal to the ratio of the external force applied on the body to the acceleration produced in it along a smooth horizontal surface.*

$$\text{Inertial mass} = \frac{\text{Applied force}}{\text{Acceleration produced}}$$

or $$m_i = \frac{F}{a}$$

The inertial mass of a body is a measure of its ability to resist the production of acceleration by an external force. If some force is applied on two different bodies, then the inertial mass of that body will be more in which the acceleration produced is less and vice versa.

Properties of inertial mass :

1. Inertial mass of a body is directly proportional to the quantity of matter possessed by it.
2. It is independent of size, shape and state of the body.
3. It is conserved both in physical and chemical processes.
4. It is not affected by the presence of other bodies.
5. When different bodies are put together, their inertial masses get added together irrespective of the nature of their materials.
6. The inertial mass of a body increases with its speed. When a body of rest mass m_0 moves with speed v, its inertial mass will be

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

59. *What is gravitational mass of a body ? Explain its equivalence with inertial mass.*

Gravitational mass. *The mass of a body which determines the gravitational pull due to earth acting upon it is called its gravitational mass.* If M is the mass of the earth and R its radius, then according to Newton's law of gravitation, the gravitational pull acting on a body of mass m_g placed on the earth's surface is given by

$$F = \frac{GMm_g}{R^2}$$

$$\therefore \quad \text{Gravitational mass,} \quad m_g = \frac{FR^2}{GM}$$

Greater the gravitational mass of a body, greater is the gravitational pull of the earth on it. So if two bodies lying at equal heights from the ground experience equal force of gravity, then their gravitational masses must be equal. This forms the principle of a pan balance.

Equivalence of inertial and gravitational masses. According to Newton's law of gravitation, the gravitational force acting on a body of gravitational mass m_g placed on the surface of the earth is given by

$$F = \frac{GMm_g}{R^2}$$

If a body of inertial mass m_i is allowed to fall freely, then from Newton's second law,

F = Inertial mass × acceleration due to gravity = $m_i g$

From the above two equations, we get

$$m_i g = \frac{GMm_g}{R^2}$$

or $$\frac{m_i}{m_g} = \frac{GM}{gR^2} = k \text{ (a constant)}$$

or $$m_i = km_g \quad \text{or} \quad m_i \propto m_g$$

Thus the inertial mass of a body is proportional to its gravitational mass. In fact the proportionality constant, $k = 1$, so that $m_i = m_g$ *i.e.*, inertial and gravitational masses are equivalent. Hence in the force equation $F = ma$ and weight equation $W = mg$, we need to consider only one type of mass m.

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60. *Give a comparison of inertial and gravitational masses.*

Comparison between inertial and gravitational masses :

Similarities :

1. Both represent the quantity of matter in a body.
2. Both are equivalent in magnitude and have same units of measurement.
3. Both do not depend on the shape or state of matter.
4. Both are not affected by the presence of other bodies.
5. Both are scalar quantities.

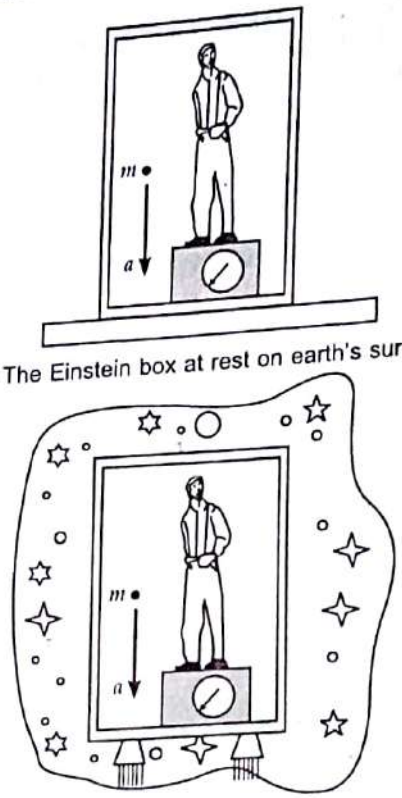
Differences :

1. Inertial mass is the measure of difficulty of accelerating a body while gravitational mass measures the force of attraction between the body and the earth.
2. Inertial mass is determined from Newton's second law of motion while gravitational mass is determined from Newton's law of gravitation.
3. Inertial mass can be measured only under dynamic conditions *i.e.*, when the body is in motion, which is neither convenient nor practical. Gravitational mass can be easily measured by using a common balance.

61. *Give an illustration to distinguish between inertial and gravitational masses.*

How did Einstein experimentally prove the equivalence of inertial and gravitational masses. How did it help in forming the general theory of relativity ?

Einstein's view of gravitation. Einstein pointed out that *gravitation and acceleration are equivalent*. This is called **principle of equivalence**. Einstein used this principle as one of the basic ideas in forming his *general theory of relativity* that explains gravitational effects in terms of curvature of space. He proved the equivalence of inertial and gravitation masses by the following simple experiment.



(*a*) The Einstein box at rest on earth's surface.

(*b*) The Einstein box accelerating through interstellar space at 9.8 ms^{-2}.

Fig. 8.40

Consider a person enclosed in small box, called **Einstein box**. In Fig. 8.40(*a*), the box is at rest on earth's surface and is subject only to earth's gravitational force. In Fig. 8.40(*b*), the box is accelerating through interstellar space at 9.8 ms^{-2} and is subject only to the external force producing this much acceleration. It is not possible, by doing experiments within the box, for the person to tell which situation he is in [*i.e.*, (*a*) or (*b*)]. For example, the platform scale on which he stands reads the same weight in both cases. Moreover, if he observes an object falling past him, the object has the same acceleration relative to him in both cases. This conclusively proves the equivalence of inertial and gravitational masses.

Very Short Answer Conceptual Problems

Problem 1. Why is the law of gravitation called the universal law ?

Solution. This is because of the fact that the law of gravitation holds good for any pair of bodies in the universe whether microscopically small or astronomically large in size.

Problem 2. Why is *G* called the universal gravitational constant ?

Solution. This is because the value of *G* is same for any pair of the bodies in the universe. It does not depend on the nature of the medium between the two bodies or on the nature of the bodies themselves.

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Problem 3. What is the ratio of the force of attraction between two bodies kept in air and the same distance apart in water ?

Solution. 1:1, because the gravitational force does not depend on the nature of the medium.

Problem 4. If spheres of same material and same radius r are touching each other, then show that the gravitational force between them is directly proportional to r^4.

Solution. If ρ is the density of the material, then

$$F = G\frac{m_1 m_2}{(2r)^2} = G\frac{\frac{4}{3}\pi r^3 \rho \times \frac{4}{3}\pi r^3 \rho}{(2r)^2} = \frac{4}{9}\pi^2\rho^2 G r^4$$

Clearly, $F \propto r^4$.

Problem 5. If the density of planet is doubled without any change in its radius, how does g change on the planet ?

Solution. It gets doubled, because $g \propto \rho$.

Problem 6. Is it possible to shield a body from gravitational effects ?

Solution. No, it is not possible to shield a body from gravitational effects because gravitational interaction does not depend upon the nature of the intervening medium.

Problem 7. If the force of gravity acts on all bodies in proportion to their masses, why does a heavy body not fall faster than a light body ? *[Delhi 12]*

Solution. If F be the gravitational force on a body of mass m, then

$$F = G\frac{Mm}{R^2} = mg \quad \text{or} \quad g = \frac{GM}{R^2}$$

Clearly, $F \propto m$ but g does not depend on m. Hence all bodies fall with same rapidness if there is no air resistance.

Problem 8. The mass of the moon is nearly 10% of the mass of the earth. What will be the gravitational force of the earth on the moon, in comparison to the gravitational force of the moon on the earth ?

Solution. Both forces will be equal in magnitude as gravitational force between two bodies is a mutual force.

Problem 9. Earth is continuously pulling the moon towards its centre, still it does not fall to the earth. Why ?

Solution. Gravitational force of attraction due to earth provides the centripetal force, which keeps the moon in orbit around the earth. Moreover, this gravitational force acts perpendicular to the velocity of the moon.

Problem 10. We cannot move finger without disturbing all stars. Why ?

Solution. When we move our finger, the distance between the objects and our finger changes. Hence, the force of attraction changes, disturbing the entire universe, including the stars.

Problem 11. According to Newton's law of gravitation, the apple and the earth experience equal and opposite forces due to gravitation. But it is the apple that falls towards the earth and not vice-versa. Why ? *[Delhi 1999]*

Solution. According to Newton's third law of motion, the force with which the earth is attracted towards the apple is equal to the force with which earth attracts the apple. However, the mass of the earth is extremely large as compared to that of apple. So acceleration of the earth is very small and is not noticeable.

Problem 12. According to Newton's law of gravitation, every particle of matter attracts every other particle. But bodies on the surface of earth never move towards each other on account of this force of attraction. Why ? *[NCERT]*

Solution. Because of their small masses, the gravitaional attraction between two bodies is too small to produce any motion in them. But due to the large mass of the earth, the gravitational attraction between the bodies and earth is very large and so all bodies are attracted towards the centre of the earth.

Problem 13. Does the gravitational force of attraction of the earth become zero at some height above the earth ? Give reason.

Solution. No, the gravitational force of attraction of earth on a body at height h is

$$F = G\frac{Mm}{(R+h)^2}$$

Clearly, F will be zero only when h is infinity.

Problem 14. Which is more fundamental-mass or weight of a body ?

Solution. Mass is more fundamental than weight because the mass of a body remains constant while its weight changes from place to place due to change in the value of g.

Problem 15. If the diameter of the earth becomes twice its present value but its mass remains unchanged, then how would be the weight of an object on the surface of the earth affected ? *[Roorkee 80]*

Solution. Weight of body,

$$W = mg = \frac{GMm}{R^2}$$

When the diameter or radius of the earth becomes double its present value, the weight of the body will be

$$W' = \frac{GMm}{(2R)^2} = \frac{1}{4}W$$

i.e., weight will become one-fourth of the present value.

Problem 16. If the diameter of the earth becomes half its present value but its average density remains unchanged then how would be the weight of an object on the surface of the earth affected ?

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Solution. Acceleration due to gravity on the surface of the earth

$$g = \frac{GM}{R^2} = \frac{G}{R^2} \times \frac{4}{3}\pi R^3 \rho = \frac{4}{3}\pi GR\rho$$

When the diameter or radius becomes half its present value,

$$g' = \frac{4}{3}\pi G\left(\frac{R}{2}\right)\rho = \frac{g}{2}.$$

Hence the weight of the object will be halved.

Problem 17. The mass and diameter of a planet are twice those of the earth. What will be the time-period of that pendulum on this planet, which is a second's pendulum on the earth ? *[IIT]*

Solution. Initially, $g = \frac{GM}{R^2}$ and $T = 2\pi\sqrt{\frac{l}{g}} = 2\,s$

(For a second's pendulum)

When M and R are doubled,

$$g' = \frac{G(2M)}{(2R)^2} = \frac{g}{2}$$

and $$T' = 2\pi\sqrt{\frac{l}{g/2}} = \sqrt{2}\,T = 2\sqrt{2}\ s.$$

Problem 18. If the radii of two planets be R_1 and R_2 and their mean densities be ρ_1 and ρ_2, then prove that the ratio of accelerations due to gravity on the planets will be $R_1\rho_1 : R_2\rho_2$.

Solution. On the surface of any planet, $g = \frac{4}{3}\pi GR\rho$

$$\therefore \quad \frac{g_1}{g_2} = \frac{\frac{4}{3}\pi GR_1\rho_1}{\frac{4}{3}\pi GR_2\rho_2} = R_1\rho_1 : R_2\rho_2.$$

Problem 19. The distance between two bodies A and B is r. Taking the gravitational force according to the law of inverse square of r, the acceleration of the body is a. If the gravitational force follows an inverse fourth power law, then what would be the acceleration of the body A ?

Solution. For inverse square law, $F = \frac{Gm_1m_2}{r^2}$

$\therefore$ Acceleration of body $A = \frac{F}{m_1} = \frac{Gm_2}{r^2} = a$

For inverse fourth power law, $F' = \frac{Gm_1m_2}{r^4}$

$\therefore$ Acceleration of body $A = \frac{F'}{m_1} = \frac{Gm_2}{r^4} = \frac{a}{r^2}$.

Problem 20. Why is centre of mass of a body often called its centre of gravity ?

Solution. The torque due to gravity on a body acts as if its entire mass were concentrated at its centre of mass. That is why centre of mass of a body is often called its centre of gravity.

Problem 21. Why the value of g is more at the poles than at the equator ?

Solution. As $g = \left(\frac{GM}{R^2}\right)$ and the value of R at equator is greater than that at poles, hence g at poles is greater than g at equator.

Problem 22. Why a body weighs more at the poles than at the equator ? *[Himachal 03]*

Solution. At the poles $g_p > g_e$, hence, $mg_p > mg_e$.

Problem 23. Where does a body weigh more ? At the sea level or on the mountains ?

Solution. At the sea level because weight decreases with altitude.

Problem 24. Where will a body weigh more – 1 km above the surface of earth or 1 km below the surface of earth ?

Solution. As $g_h = g\left(1 - \frac{2h}{d}\right)$ and $g_d = g\left(1 - \frac{d}{h}\right)$

For $h = d = 1$ km, clearly $g_d > g_p$.

Hence the body will weigh more at a depth of 1 km below the surface of earth.

Problem 25. Does the concentration of the earth's mass near its centre change the variation of g with height compared with a homogeneous sphere ? How ?

Solution. Any change in the distribution of the earth's mass will not affect the variation of acceleration due to gravity with height. This is because for a point outside the earth, the whole mass of the earth is 'effective' and the earth behaves as a homogeneous sphere.

Problem 26. The weight of a body is less inside the earth than on the surface. Why ?

Solution. As we go inside the earth, the value of the attracting mass M decreases and hence the value of g decreases. Therefore, the weight of the body mg is less inside the earth than on the surface.

Problem 27. Why do you feel giddy while moving on a merry-go-round ?

Solution. When moving in a merry-go-round, our weight appears to decrease when we move down and appears to increase when we move up.

Problem 28. At which place on earth's surface, the value of g is largest and why ?

Solution. The value of g is largest at the poles due to following *two* reasons :

(*a*) Distance of poles from the centre of earth is smaller than the distance of any other point on earth's surface from its centre.

(*b*) At poles, no centrifugal force acts on the body.

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Problem 29. When dropped from the same height a body reaches the ground quicker at poles than at the equator. Why ?

Solution. The acceleration due to gravity is more at the poles than at the equator. When the initial velocities and distances travelled are the same, the time taken by the body is smaller if the acceleration due to gravity is large. Hence, when dropped from the same height, a body reaches the ground quicker at the poles than at the equator.

Problem 30. When a clock controlled by a pendulum is taken from the plains to a mountain, it becomes slow but a wrist-watch controlled by a spring remains unaffected. Explain the reason for the difference in the behaviour of the two watches.

Solution. Due to decrease in the value of g at the mountain, the time period of the pendulum of the clock increases. On the other hand, the spring watch remains unaffected by the variation in g.

Problem 31. A clock fitted with a pendulum and another with a spring indicate correct time on earth. Which shows correct time on the moon ?

Solution. A clock fitted with a spring will show correct time on the moon, because its time period is not affected by the variation in g.

Problem 32. Why does a tennis ball bounce higher on a hill than on plains ? *[Himachal 03]*

Solution. The value of g is less on hills because they are comparatively at a greater distance from the centre of the earth. Therefore, the gravitational pull on the tennis ball is less on hill tops and so it bounces higher on hills than on plains.

Problem 33. A man can jump six times as high on the moon as that on the earth. Justify.

Or

Explain, why one can jump higher on the surface of the moon than that on the earth. *[Himachal 07]*

Solution. The value of 'g' on the surface of moon is about $\frac{1}{6}$th of its value on the surface of earth.

For the same gain of P.E. in both cases, we have

$$mg_m h_m = mg_e h_e$$

or $$h_m = \frac{g_e h_e}{g_m} = \frac{g_e h_e}{g_e/6} = 6h_e.$$

Problem 34. Moon-travellers tie heavy weight at their back before landing on the moon. Why ?

Solution. Due to the small value of g on the moon, the traveller would weigh less. To compensate for this loss in weight, the travellers load their backs with heavy weight.

Problem 35. Why is earth flat at the poles ?

Solution. Due to its rotation about the polar axis.

Problem 36. What is the effect of rotation of the earth on the acceleration due to gravity ?

Solution. The acceleration due to gravity decreases due to rotation of the earth. This effect is zero at the poles and maximum at the equator.

Problem 37. Name two factors which determine whether a planet would have an atmosphere or not. *[Himachal 06 ; Delhi 99]*

Solution. The two such factors are

(*i*) The value of acceleration due to gravity on the planet.

(*ii*) Surface temperature of the planet.

Problem 38. If the earth stops rotating about its axis, what will be the effect on the value of g ? Will this effect be same at all places ?

Solution. The value of g will increase at all places except at the poles. The increase will be different at different places, maximum at the equator.

Problem 38(*a*). If the earth stops rotating about its axis, then by what value will the acceleration due to gravity change at the equator ?

Solution. The value of g increases by $\omega^2 R$.

Problem 39. Explain why tidal waves (high tide and low tide) are formed on seas.

Solution. The gravitational attraction of moon on sea water causes high tides. Tides at one place cause low tides (ebbs) at another. Attraction by sun also causes tides but only of half the magnitude. Hence on new moon and full moon days, when both effects add, tides are very high.

Problem 40. Why are we not thrown off the surface of the earth by the centrifugal force ?

Solution. The force of gravity exerted on our body by the earth towards its centre is greater than centrifugal force acting an our body away from the centre.

Problem 41. A satellite does not need any fuel to circle around the earth. Why ? *[Delhi 13]*

Solution. The gravitational force between satellite and earth provides the centripetal force required by the satellite to move in a circular orbit.

Problem 42. Why a multi-stage rocket is required to launch a satellite ?

Solution. The multistage rocket helps to economise the consumption of fuel.

Problem 43. Why do different planets have different escape velocities ? *[Himachal 06]*

Solution. As $v_e = \sqrt{\frac{2GM}{R}}$, therefore escape velocities have different values on different planets which are of different masses and different sizes.

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Problem 44. An elephant and an ant are to be projected out of earth into space. Do we need different velocities to do so ?

Solution. The escape velocity, $v_e = \sqrt{2gR}$, does not depend upon the mass of the projected body. Thus we need the same velocity to project an elephant and an ant into space.

Problem 45. Does a rocket really need the escape velocity of 11.2 kms^{-1} initially to escape from the earth ?

Solution. No, rocket can have any velocity at the start. The rocket can continue to increase the velocity due to thrust provided by the escaping gases that will carry it to a desired position.

Problem 46. The moon has no atmosphere. Why ?

[Himachal 03, 05 ; Delhi 05C, 10]

Solution. Due to the small value of 'g', the escape velocity on the moon surface is small (2.38 kms^{-1}). The air molecules have thermal velocities greater than the escape velocity. Therefore, the air molecules escape away and cannot form atmosphere on the moon.

Problem 47. The earth has atmosphere. Why ?

Solution. Because the r.m.s. velocity of air molecules is less than escape velocity on the earth, the air cannot escape from the surface of the earth. Hence the earth has atmosphere.

Problem 48. Lighter gases like H_2, He, etc. are rare in the atmosphere of the earth. Why ?

Solution. Usually the average velocity of the molecules of *lighter gases* is greater than the *escape velocity* at the earth's surface. So lighter gases have escaped from the earth's atmosphere.

Problem 49. The gravitational force exerted by the sun on the moon is greater than (about twice as great as) the gravitational force exerted by the earth on the moon. Why then doesn't the moon escape from the earth (during a solar eclipse, for example) ?

Solution. The moon can escape only if the moon has no orbital motion. In fact, while revolving around the earth, the moon has orbital motion around the sun also. The gravitational attraction of the sun on the moon provides the centripetal force required for the orbital motion around the sun.

Problem 50. The escape velocity for a satellite is 11.2 kms^{-1}. If the satellite is launched at an angle of 60° with the vertical, what will be the escape velocity ?

Solution. 11.2 kms^{-1}, because escape velocity ($v_e = \sqrt{2GM/R}$) does not depend on the angle of projection.

Problem 51. An astronaut, while revolving in a circular orbit happens to throw a spoon outside. Will the spoon reach the surface of the earth ?

Solution. The spoon will continue to move in the same circular orbit, and chase the astronaut. It will never reach the surface of the earth.

Problem 52. An artificial satellite revolves around the earth without using any fuel. On the other hand, an aeroplane requires fuel to fly. Why ?

Solution. Air is essential for an aeroplane to fly. To overcome the resistive forces of air, aeroplane requires fuel. The satellite orbits at a much higher height where air resistance is negligible and so it needs no fuel.

Problem 53. Why are space rockets usually launched from west to east in the equatorial plane ?

Solution. Due to rotation of the earth about its polar axis, every particle on the earth has a linear velocity directed from west to east. This velocity ($v = R\omega$) is maximum at the equator. When a rocket is launched from west to east, this maximum velocity gets added to the launching velocity, so the launching becomes easier.

Problem 54. A satellite of small mass burns during its descent and not during ascent. Why ?

Solution. The speed of the satellite during descent is much larger than that during its ascent. As the air resistance is directly proportional to velocity, so heat produced during descent is very large and the satellite burns up.

Problem 55. Is it possible to place an artificial satellite in an orbit such that it is always visible over New Delhi ?

Solution. No. A satellite remains always visible only if it revolves in the equatorial plane with a period of revolution equal to that of the earth. New Delhi does not lie in the region of equatorial plane.

Problem 56. The astronauts in a satellite orbiting the earth feel weightlessness. Does the weightlessness depend upon the distance of the satellite from the earth ? Give reason.

Solution. No. The gravitational acceleration of the astronaut relative to the satellite is zero, whatever be the distance of the satellite from the earth.

Problem 57. Can we determine the gravitational mass of a body inside an artificial satellite ?

Solution. No, because both the body and the artificial satellite are in a state of free fall.

Problem 58. A satellite is orbiting the earth with speed v_0. To make the satellite escape, what should be the minimum percenta8ge increase in its velocity ?

Solution. Required percentage increase in velocity

$$= \frac{v_e - v_0}{v_0} \times 100 = \left(\frac{v_e}{v_0} - 1\right) \times 100$$

$$= (\sqrt{2} - 1) \times 100 = \mathbf{41.4\%}.$$

Problem 59. Why an astronaut in an orbiting space-craft is not in zero gravity although weightless ?

[Delhi 1997]

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Solution. The astronaut in the orbiting spacecraft is still in the gravitational field of the earth and experiences appreciable gravity. However, the gravity is used up in providing the necessary centripetal force.

Problem 60. Why does an astronaut in a spacecraft feel weightlessness ?

Solution. This is because the orbiting spacecraft alongwith the astronaut is in a state of free fall towards the earth.

Problem 61. Two identical geostationary satellites are moving with equal speeds in the same orbit but their sense of rotation brings them on a collision course. What will happen to the debris ?

Solution. The collision between the two satellites is inelastic. They stick together.

$$mv + (-mv) = (m + m)\,V \quad \text{or} \quad V = 0$$

The kinetic energy of the debris in its orbit will be zero. Due to gravity, it falls to the surface of the earth and in the course of its journey, it may get burnt up.

Problem 62. The linear speed of a rocket is not constant in its orbit. Comment.

Solution. The areal velocity of a planet around the sun is constant but its linear speed continuously changes. The linear speed of the planet is large when it passes close to the sun and is small when the planet is away from the sun.

Problem 63. Identify the portion of sun in the following diagram if the linear speed of the planet is greater at *C* compared to that at *D*. *[Central Schools 03]*

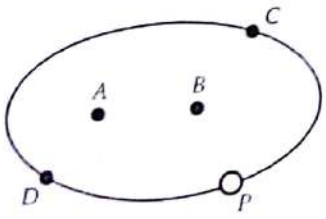

Fig. 8.41

Solution. The sun should be at position *B*, because the speed of a planet is greater when it is closer to the sun.

Problem 64. The largest and shortest distances of the earth from the sun are r_1 and r_2 respectively. What is the distance from the sun when it is perpendicular to the major axis of the orbit from the sun ?

Solution. Let *R* be the distance of the earth from the sun in the position perpendicular to the major axis of its elliptical orbit.

Using the property of an ellipse, we can write

$$\frac{1}{R} + \frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2}$$

or
$$\frac{2}{R} = \frac{r_1 + r_2}{r_1 r_2}$$

$$\therefore \quad R = \frac{2r_1 r_2}{r_1 + r_2}.$$

Problem 65. Why does the weight of a body become zero at the centre of the earth ? *[Himachal 06]*

Solution. As we go inside the earth, the value of attracting mass *M* decreases and its value becomes zero at the centre of the earth. Consequently, a body at the centre of the earth feels no gravitational attraction or its weight becomes zero at the centre of the earth.

Problem 66. Imagine a spacecraft going from the earth to the moon. How does its weight vary as it goes from the earth to the moon ? *[Himachal 07C]*

Solution.

(*i*) As the spacecraft moves from the earth towards the moon, its weight decreases.

(*ii*) The weight of the spacecraft becomes zero at the null point where the gravitational forces of the earth and the moon are equal and opposite.

(*iii*) Again the weight increases, becoming *mg*/6 on the moon's surface.

Problem 67. The artificial satellite does not have any fuel, but even then it remains in its orbit around the earth. Why. *[Himachal 05]*

Solution. The centripetal force necessary for orbital motion of the satellite is provided by the force of gravitation exerted by the earth on the satellite. So the satellite does not need any fuel for its orbital motion.

Problem 68. According to Albert Einstein, no particle can have velocity equal to or greater than that of light. Hence what may be the value of escape velocity from the surface of a 'Black hole' such that it holds everything to itself ? *[Central Schools 08]*

Ans. $v_e = \sqrt{\dfrac{2GM}{R}} \le c.$

Short Answer Conceptual Problems

Problem 1. Answer the following :

(*a*) **Among the known types of forces in nature, the gravitational force is the weakest. Why then does it play a dominant role for motion of bodies on the terrestrial, astronomical and cosmological scale ?**

(*b*) **Do the forces of friction and other contact forces arise due to gravitational attraction ? If not, what is the origin of these forces ?**

Solution. (*a*) The strong nuclear forces operate only over a range of distance of the order of 10^{-14} to 10^{-15} m. The

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forces involved in weak interactions operate only during radiactive decay. Electrical forces are stronger than gravitational forces for a given distance, but they can be attractive as well as repulsive unlike gravitational force, which is always attractive. As a result, the forces between massive neutral bodies are predominantly gravitational.

(*b*) No, the forces of friction and other contact forces have electromagnetic origin.

Problem 2. Choose the correct alternatives :

(*i*) If the gravitational potential energy of two mass points infinite distance away is taken to be zero, the gravitational potential energy of a galaxy is (positive/negative/zero).

(*ii*) The universe on the large scale is shaped by (gravitational/electromagnetic forces), on the atomic scale by (gravitational/electromagnetic) forces, on the nuclear scale by (gravitational / electromagnetic/strong nuclear) forces.

Solution. (*i*) The gravitational potential energy of a galaxy is negative because it is a bound system.

(*ii*) The universe on the large scale is shaped by gravitational forces, on the atomic scale by *electromagnetic forces* and on the nuclear scale by *strong nuclear forces.*

Problem 3. What is the difference between inertial mass and gravitational mass of a body ?

Solution. The inertial mass of a body is a measure of its inertia and is given by the ratio of the external force applied on it to the acceleration produced in it. The gravitational mass of a body, on the other hand, is a measure of the gravitational pull acting on it due to the earth. The gravitational mass is measured by a common balance.

Problem 4. Which of the following observations point to the equivalence of inertial and gravitational mass :

(*a*) Two spheres of different masses dropped from the top of a long evacuated tube reach the bottom of the tube at the same time.

(*b*) The time-period of a simple pendulum is independent of its mass.

(*c*) The gravitational force on a particle inside a hollow isolated sphere is zero.

(*d*) For a man in closed cabin that is falling freely under gravity, gravity 'disappears'.

(*e*) An astronaut inside a spaceship orbiting around the Earth feels weightless.

(*f*) Planets orbiting around the sun obey Kepler's Third Law (approximately).

(*g*) The gravitational force on a body due to the Earth is equal and opposite to the gravitational force on the Earth due to the body. *[NCERT]*

Solution. The observations (*a*), (*b*), (*d*), (*e*) and (*f*) point to the equivalence of gravitational masses, because in these situations the bodies are in motion.

The observations (*c*) and (*g*) do not establish the equivalence of inertial and gravitational masses because the bodies are not in the state of motion.

Problem 5. A body is taken from the centre of the earth to the moon. What will be the changes in the weight of the body ?

Solution. (*i*) At the centre of the earth, the weight of the body is zero ($g = 0$).

(*ii*) As the body is moved from centre to the earth surface, its weight increases (due to increase in g)

(*iii*) As the body is moved from earth's surface towards the moon, the weight decreases (due to the decrease in g), becomes zero at the point where the gravitational forces of the earth and moon are equal and opposite.

(*iv*) Again the weight increases, becoming $mg / 6$ on the moon's surface.

Problem 6. Mention the conditions under which the weight of a person can become zero.

Solution. The weight of a person can become zero under the following conditions :

(*i*) When the person is at the centre of the earth (as $g = 0$ at the centre of the earth).

(*ii*) When the person is at the null points in space (At these points, the gravitational forces due to different masses cancel out).

(*iii*) When the person is standing in a freely falling lift.

(*iv*) When the person is inside a spacecraft which is orbiting around the earth.

Problem 7. How will the value of g be affected if (*i*) the rotation of the earth stops (*ii*) the rotational speed of the earth is doubled (*iii*) the rotational speed of the earth is increased to seventeen times its present value ?

Solution. (*i*) If the rotation of the earth stops, no centrifugal force will act on the bodies lying on it. The value of g increases maximum at the equator. As no centrifugal force acts on body at poles, so the value g is not affected.

(*ii*) If the rotational speed of the earth is doubled, the centrifugal force on the bodies increases. The value of g decreases maximum at the equator and is not affected at poles.

(*iii*) If the rotational speed of the earth is increased to seventeen times its present value, the value of g at the equator will become zero. At the poles, the value of g remains unchanged.

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Problem 8. When a satellite moves to a lower orbit in the atmosphere of the earth, it becomes hot. This indicates that there is some dissipation in its mechanical energy. But the satellite spirals down towards the earth with an increasing speed, why ?

Solution. For a satellite orbiting the earth,

Total energy = – Kinetic energy *i.e.*, $E = -K$

When the satellite enters the atmosphere of the earth, it dissipates its mechanical energy (which is negative) against atmospheric friction. Energy E becomes more negative. As a result, the kinetic energy of the satellite increases and hence its speed increases. But its orbital speed $v_0 = \sqrt{GM/(R+h)}$ can increase only if its height h becomes smaller. Hence the satellite moves to a lower orbit with an increased speed. Thus due to the atmospheric friction, the satellite spirals down towards the earth with increasing speed and ultimately burns out in the lower dense atmosphere.

Problem 9. What are the conditions under which a rocket fired from the earth becomes a satellite of the earth and orbits in a circle ?

Solution. (*i*) First the rocket should be given a sufficient vertical velocity so that it reaches a height at which it is supposed to revolve around the earth.

(*ii*) At this height, the rocket must be given a horizontal orbital velocity given by $v_0 = \sqrt{\dfrac{GM}{R+h}}$

(*iii*) The air resistance should be negligible at the height of its orbit.

Problem 10. What would happen if the force of gravity were to disappear suddenly ?

Solution. If the force of gravity suddenly disappears, then

(*i*) All bodies will lose weight.

(*ii*) We would be thrown away from the earth due to the centrifugal force.

(*iii*) Eating, drinking and in fact all operations would become impossible.

(*iv*) Motion of satellites around the planets and the motion of planets around the sun would cease.

Problem 11. The radii of two planets are R and $2R$ respectively and their densities ρ and $\rho/2$ respectively. What is the ratio of acceleration due to gravity at their surfaces ? *[Central Schools 05]*

Solution. Here,

$$g = \frac{GM}{R^2} = \frac{G}{R^2} \cdot \frac{4}{3}\pi R^3 \rho = \frac{4}{3}\pi GR\rho$$

or $g \propto R\rho$

$$\therefore \quad \frac{g_1}{g_2} = \frac{R\rho}{2R \cdot \dfrac{\rho}{2}} = 1:1.$$

Problem 12. The time period of the satellite of the earth is 5 hours. If the separation between the earth and the satellite is increased to 4 times the previous value, then what will be the new time period of the satellite ? *[AIEEE 03]*

Solution. According to Kepler's law of periods,

$$\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{R_2}{R_1}\right)^3 \quad \text{or} \quad \left(\frac{T_2}{5}\right)^2 = \left(\frac{4R_1}{R_1}\right)^3$$

or $$T_2 = \sqrt{64 \times 25} = 40 \text{ h}.$$

Problem 13. Prove that acceleration due to gravity on the surface of the earth is given by $g = \frac{4}{3}\pi\rho GR$, where G is gravitational constant, ρ is mean density and R is the radius of the earth. *[Delhi 01, 05C]*

Solution. Mass of the earth, $M = \frac{4}{3}\pi R^3 \rho$

Acceleration due to gravity on the earth's surface,

$$g = \frac{GM}{R^2} = \frac{G}{R^2} \times \frac{4}{3}\pi R^3 \rho = \frac{4}{3}\pi \rho GR.$$

Problem 14. Why is the weight of a body at the poles more than the weight at the equator ? Explain.

[Himachal 03 ; Delhi 05C]

Solution. As $g = GM/R^2$ and the value of R at the poles is less than that at the equator, so g at poles is greater that g at the equator. Now, as $g_p > g_e$, hence $mg_p > mg_e$ *i.e.*, the weight of a body at the poles is more than the weight at the equator.

Problem 15. Why does the earth impart the same acceleration to all bodies ? *[Chandigarh 02]*

Solution. The force of gravitation exerted by the earth on a body of mass m is

$$F = G\frac{Mm}{R^2} = mg$$

Acceleration imparted to the body,

$$g = \frac{GM}{R^2}$$

Clearly, g does not depend on m. Hence the earth imparts same acceleration to all bodies.

Problem 16. If suddenly the gravitational force of attraction between the earth and a satellite revolving around it becomes zero, what will happen to the satellite ? *[AIEEE 02]*

Solution. If the gravitational force of the earth suddenly becomes zero, the satellite will stop revolving around the earth and it will move in a direction tangential to its original orbit with a speed with which it was revolving around the earth.

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HOTS

Problem 1. *Draw graphs showing the variation of acceleration due to gravity with (i) height above the earth's surface and (ii) depth below the earth's surface.*

Solution. (i) The value of g varies with height h as

$$g \propto \frac{1}{(R+h)^2} \quad \text{or} \quad g \propto \frac{1}{r^2}$$

Thus the graph of g versus r is the parabolic curve AB, as shown in Fig. 8.42.

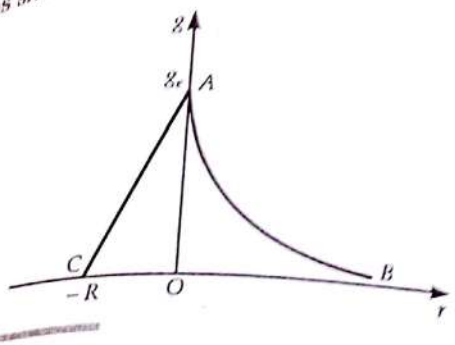

Fig. 8.42

(ii) The value of g varies with depth d as

$$g = g\left(1 - \frac{d}{R}\right) \quad \text{i.e., } g \propto (R - d)$$

Thus the graph of g versus depth d is the straight line AC, as shown in Fig. 8.42.

Problem 2. *Are we living at the bottom of a gravitational well ? Give reason.*

Solution. Yes, we are living at the bottom of a gravitational well. Fig. 8.43 shows the variation of gravitational force F with distance r from the centre of the earth. Clearly, the graph has a force minimum at the surface of the earth ($r = R$).

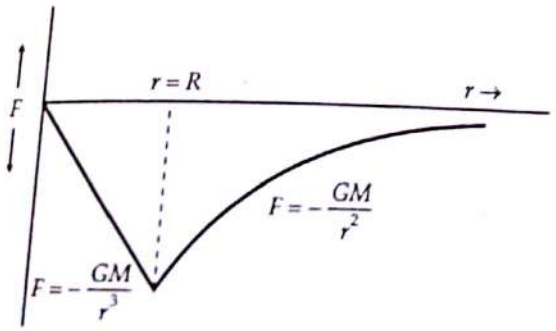

Fig. 8.43

Problem 3. *Generally the path of a projectile from the earth is parabolic but it is elliptical for projectiles going to a very great height. Why ?*

Problems on Higher Order Thinking Skills

Solution. Under ordinary heights, the change in the distance of a projectile from the centre of the earth is negligible compared to the radius of the earth. Hence the projectile moves under a nearly uniform gravitational force and its path is parabolic. But for projectile going to a very great height, the gravitational force decreases in inverse proportion to the square of the distance of the projectile from the centre of the earth. Under such a variable force, the path of projectile is elliptical.

Problem 4. *A person sitting in a satellite feels weightlessness but a person standing on moon has weight though moon is also a satellite of the earth. Give reason.*

Solution. For the person sitting in the artificial satellite, the gravitational attraction on him due to the earth provides the necessary centripetal force. The net force on him is zero, so he feels weightlessness. For the person standing on the moon, the gravitational attraction acting on him due to the moon is left unbalanced, which accounts for his weight on the moon.

Problem 5. *The sun attracts all bodies on the earth. At midnight, when the sun is directly below, it pulls on a body in the same direction as the pull of the earth on that body ; at noon, when the sun is directly above, it pulls on a body in a direction opposite to the pull of the earth. Then will the weight of the body be greater at mid-night than at noon ?*

Solution. No, earth is a satellite of sun. Any body placed on earth is also the satellite of sun. The body and the earth both will have the same acceleration towards the sun. Hence there will be no relative sun's gravitational acceleration between the body and the earth. That is, a body placed on earth will experience no gravitational effect due to the sun. It will experience a gravitational force only due to the earth. This will be weight of the body measured on the earth and will remain same for all the twenty four hours.

Problem 6. *Suppose the gravitational force varies inversely as the nth power of distance. Then, find the expression for the time period of a planet in a circular orbit of radius r around the sun.* **[AIEEE 04]**

Solution. As the gravitational force varies inversely as the nth power of the distance, so the gravitational force on the planet is given by

$$F = \frac{GMm}{r^n}$$

This force provides the centripetal force $mr\omega^2$ to the planet.

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$$\therefore \quad \frac{GMm}{r^n} = mr\omega^2 = mr\left(\frac{2\pi}{T}\right)^2$$

or $$T^2 = \frac{r \times 4\pi^2 \times r^n}{GM} = \frac{4\pi^2 r^{(n+1)}}{GM}$$

or $$T = \frac{2\pi}{\sqrt{GM}} \cdot r^{(n+1)/2}$$

Clearly, $T \propto r^{(n+1)/2}$

Problem 7. *A simple pendulum has a time period T_1 when on the earth's surface, and T_2 when taken to a height R above the earth's surface, where R is the radius of the earth. What is the value of T_2 / T_1 ?* **[IIT Screening 01]**

Solution. Let g and g' be the acceleration due to gravity on the earth's surface and at a height R above the earth's surface respectively. Then

$$g' = \frac{g}{\left(1 + \frac{h}{R}\right)^2} = \frac{g}{\left(1 + \frac{R}{R}\right)^2} = \frac{g}{4}$$

Time period of simple pendulum,

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\therefore \quad \frac{T_2}{T_1} = \sqrt{\frac{g}{g'}} = \sqrt{\frac{g}{g/4}} = 2.$$

Problem 8. *A geo-stationary satellite orbits around the earth in a circular orbit of radius 36,000 km. Then, what will be the time period of a spy satellite orbiting a few hundred km above the earth's surface (R_{earth} = 6,400 km) ?* **[IIT Screening 02]**

Solution. As $T^2 \propto R^3$ and R_e = 6400 km , therefore

$$\frac{T^2}{(24)^2} = \left(\frac{6400}{36000}\right)^3 \quad \text{or} \quad T = \mathbf{1.7\ h}.$$

For the spy satellite, R is slightly greater than R_e .

So $\quad T_s > T \quad$ or $\quad T_s \simeq \mathbf{2\ h}$.

Problem 9. *Find the period of oscillation of a simple pendulum of length L suspended from the roof of a vehicle which moves without friction down an inclined plane of inclination α.* **[IIT Screening 2K]**

Solution. The effective value of acceleration due to gravity,

$$g' = g\cos\alpha$$

$$\therefore \quad T = 2\pi\sqrt{\frac{l}{g'}} = 2\pi\sqrt{\frac{l}{\cos a}}.$$

Problem 10. *Two bodies of masses m_1 and m_2 are initially at rest placed infinite distance apart. They are then allowed to move towards each other under mutual gravitational attraction. Show that their relative velocity of approach at separation r between them is*

$$v = \sqrt{\frac{2G(m_1 + m_2)}{r}}$$ **[IIT 94]**

Solution. By conservation of energy,

K.E. of first body = Its gravitation P.E.

$$\frac{1}{2} m_1 v_1^2 = \frac{Gm_1 m_2}{r}$$

or $$v_1 = \sqrt{\frac{2Gm_2}{r}}$$

Similarly, $$v_2 = \sqrt{\frac{2Gm_1}{r}}$$

∴ Relative velocity of approach

$$= v_1 + v_2 = \sqrt{\frac{2Gm_2}{r}} + \sqrt{\frac{2Gm_1}{r}}$$

$$= \sqrt{\frac{2G(m_1 + m_2)}{r}}.$$

Problem 11. *Imagine a light planet revolving around a massive star in a circular orbit of radius r with a period of revolution T. If the gravitational force of attraction between planet and the star is proportional to $r^{-5/2}$, then find the relation between T and r.* **[IIT 89]**

Solution. Force of gravitation on the planet

= Centripetal force

$$kr^{-5/2} = mr\omega^2 = mr\left(\frac{2\pi}{T}\right)^2$$

or $$T^2 = \frac{4\pi^2 mr}{kr^{-5/2}} = \frac{4\pi^2 m}{k} \cdot r^{7/2}$$

$$\therefore \quad T^2 \propto r^{7/2}.$$

Problem 12. *A spherical cavity is made inside a sphere of density ρ. If its centre lies at a distance l from the centre of the sphere, show that the gravitational field strength of the field inside the cavity is*

$$E = \frac{4\pi}{3} Gl\rho.$$ **[Roorkee 86]**

Solution. In Fig. 8.44, the gravitational field strength at the centre C of the cavity will be due to the mass of the shaded solid sphere of radius OC = l.

Mass of the shaded sphere,

$$M = \frac{4}{3}\pi l^3 \rho$$

Gravitational field strength at the centre C of the cavity is

$$E = \frac{GM}{l^2} = \frac{G}{l^2} \times \frac{4}{3}\pi l^3 \rho$$

$$= \frac{4}{3}\pi G l\rho.$$

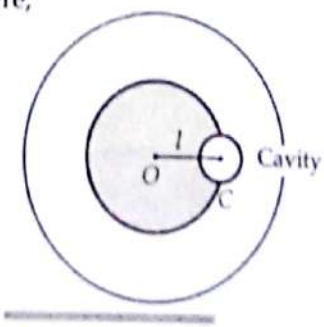

Fig. 8.44

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Problem 13. *The distance between the centres of two stars is 10a. The masses of these stars are M and 16 M and their radii a and 2a respectively. A body of mass m is fired straight from the surface of the larger star towards the smaller star. What should be its minimum initial speed to reach the surface of the smaller star ? Obtain the expression in terms of G, M and a.* [IIT 96]

Solution. Let the gravitational field due to the two stars be zero at some point O lying at a distance x from the centre of smaller star.

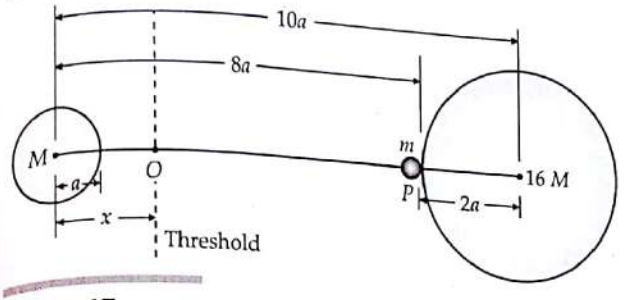

Fig. 8.45

Then

$$G\frac{Mm}{x^2} = G\frac{(16\ M)\ m}{(10\ a - x)^2}$$

or $$\frac{1}{x^2} = \frac{16}{(10a - x)^2}$$

or $$16x^2 = (10a - x)^2$$

or $$4x = \pm(10a - x)$$

The negative sign is inadmissible, so $x = 2a$

The body of mass m when fired from point P lying on the surface of heavier star must cross the threshold (the point O), otherwise it would return back.

The gravitational potential energies when the body of mass m lies at positions P and O are given by

$$U_p = -\frac{GMm}{8a} - \frac{G \times 16\ M \times m}{2a} = -\frac{65\ GMm}{8a}$$

$$U_O = -\frac{GMm}{2a} - \frac{G \times 16M \times m}{8a} = -\frac{5GMm}{2a}$$

$\therefore$ Increase in potential energy,

$$\Delta U = U_p - U_O = -\frac{5\ GMm}{2a} + \frac{65\ GMm}{8a} = \frac{45\ GMm}{8a}$$

If v is the minimum speed with which the body is fired from P so as to reach O, then

$$\frac{1}{2}mv^2 = \Delta U = \frac{45\ GMm}{8\ a}$$

or $$v = \sqrt{\frac{45}{4}\frac{GM}{a}} = \frac{3}{2}\sqrt{\frac{5GM}{a}}.$$

Problem 14. *An artificial satellite is moving in a circular orbit around the earth with a speed equal to half the magnitude of escape velocity from the earth. (i) Determine the height of the satellite above the earth's surface. (ii) If the satellite is stopped suddenly in its orbit and allowed to fall freely on to the earth, find the speed with which it hits the surface of the earth. Take $g = 9.8\ ms^{-2}$, radius of the earth $= 6400\ km$.* [IIT 90]

Solution. (*i*) Orbital velocity of a satellite at a height h above the earth's surface is

$$v_0 = \frac{GM}{R + h}$$

Escape velocity from the earth's surface,

$$v_e = \sqrt{\frac{2GM}{R}}$$

Given $$v_0 = \frac{v_e}{2}$$

$\therefore$ $$\sqrt{\frac{GM}{R+h}} = \frac{1}{2}\sqrt{\frac{2GM}{R}}$$

or $$\frac{GM}{R+h} = \frac{1}{4} \times \frac{2GM}{R} = \frac{GM}{2R}$$

or $$h = R = \mathbf{6.4 \times 10^6\ m}.$$

(*ii*) Let V be the speed with which the satellite hits the earth when suddenly stopped. If m be the mass of the satellite, then by the conservation of energy,

Initial P.E. at height $h\ (= R)$

= Final P.E. on the surface of earth + K.E. of the satellite.

or $$-\frac{GMm}{2R} = -\frac{GMm}{R} + \frac{1}{2}mV^2$$

or $$V = \sqrt{\frac{GM}{R}} = \sqrt{\frac{gR^2}{R}} = \sqrt{gR}$$

$$= \sqrt{9.8 \times 6.4 \times 10^6}$$

$$= 7.92 \times 10^3\ \text{ms}^{-1} = \mathbf{7.92\ kms^{-1}}.$$

Problem 15. *A particle is projected upward from the surface of the earth (radius R) with a K.E. equal to half the minimum value needed for it to escape. To which height does it rise above the surface of the earth ?* [IIT 97]

Solution. For the particle to escape, K.E. = P.E

$$\frac{1}{2}mv_e^2 = \frac{GMm}{R}$$

But supplied K.E. $$= \frac{1}{2} \times \frac{1}{2}mv_e^2 = \frac{GMm}{2R}$$

Suppose the particle rises to a height h, then

$$\frac{1}{2} \times \frac{1}{2}mv_e^2 = \frac{GMm}{R+h}$$

or $$\frac{GMm}{2R} = \frac{GMm}{R+h}$$

$\therefore$ $$h = R$$

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Guidelines to NCERT Exercises

8.1. *Answer the following :*

(*a*) *You can shield a charge from electrical forces by putting it inside a hollow conductor. Can you shield a body from the gravitational influence of nearby matter by putting it inside a hollow sphere or by some other means ?* [Central Schools 08]

(*b*) *An astronaut inside a small space ship orbiting around the earth cannot detect gravity. If the space station orbiting around the Earth has a large size, can he hope to detect gravity ?*

(*c*) *If you compare the gravitational force on the Earth due to the Sun to that due to the moon, you would find that the Sun's pull is greater than the moon's pull. However, the tidal effect of the moon's pull is greater than the tidal effect of Sun. Why ?*

Ans. (*a*) No. The shell does not shield other bodies lying outside it from exerting gravitational forces on a particle lying inside it.

(*b*) Yes. If the size of spaceship orbiting around the earth is large enough, the astronaut inside the spaceship can detect the variation in g.

(*c*) The tidal effect depends inversely upon the cube of the distance whereas the gravitational force varies inversely as the square of the distance. As the moon is closer to earth than the sun, so its tidal effect is greater than that of the sun. The ratio of these two effects is

$$\frac{T_m}{T_s} = \left(\frac{d_s}{d_m}\right)^3 = \left(\frac{1.5 \times 10^{11}}{3.8 \times 10^{8}}\right)^3 = 61.5 \times 10^6.$$

8.2. *Choose the correct alternative :*

(*i*) *Acceleration due to gravity increases/decreases with increasing altitude.*

(*ii*) *Acceleration due to gravity increases/decreases with increasing depth (assume the Earth to be a sphere of uniform density).*

(*iii*) *The effect of rotation on the effective value of acceleration due to gravity is greatest at the equator/poles.*

(*iv*) *Acceleration due to gravity is independent of mass of the Earth/mass of the body.*

(*v*) *The formula* $-GMm(1/r_2 - 1/r_1)$ *is more/less accurate than the formula* $mg(r_2 - r_1)$ *for the difference of potential energy between two points* r_2 *and* r_1 *distance away from the centre of the Earth.*

Ans. (*i*) At altitude h, $g_h = g\left(1 - \frac{2h}{R}\right)$.

Thus acceleration due to gravity *decreases* with increasing altitude.

(*ii*) At depth d, $g_d = g\left(1 - \frac{d}{R}\right)$

Thus acceleration due to gravity *decreases* with increasing depth.

(*iii*) At latitude λ, $g_\lambda = g\left(1 - \frac{R\omega^2 \cos^2\lambda}{g}\right)$

For maximum variation (decrease) of g, $\cos\lambda = 1$ or $\lambda = 0°$.

Thus the effect of rotation on the effective value g is greatest at the *equator* (for which $\lambda = 0°$.)

(*iv*) As $g = \frac{GM}{R^2}$, where M is the mass of the earth.

Thus acceleration due to gravity is independent of mass m of the body.

(*v*) The formula $-GMm\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$ is *more* accurate than the formula $mg(r_2 - r_1)$, because the value of g varies from place to place.

8.3. *Suppose there existed a planet that went around the sun twice as fast as the earth. What would be its orbital size as compared to that of the earth ?*

Ans. Let period of revolution of the earth

$= T_e$

$\therefore$ Period of revolution of planet,

$T_p = \frac{1}{2} T_e$ [$\because$ The planet goes around the sun twice as fast as earth]

Orbital size of the earth, $r_e = 1\,\text{AU}$

Orbital size of the planet, $r_p = ?$

From Kepler's third law of planetary motion

$$\frac{T_p^2}{T_e^2} = \frac{r_p^3}{r_e^3}$$

$$\therefore \quad r_p = \left(\frac{T_p}{T_e}\right)^{2/3} r_e = \left(\frac{\frac{1}{2} T_e}{T_e}\right)^{2/3} \times 1\,\text{AU}$$

$$= (0.5)^{2/3}\,\text{AU} = 0.63\ \text{AU}.$$

8.4. *I_o, one of the satellites of Jupiter, has an orbital period of 1.769 days and the radius of the orbit is 4.22×10^8 m. Show that the mass of Jupiter is about one thousandth that of the sun.*

Ans. Orbital period of Jupiter's satellite,

$T = 1.769$ days $= 1769 \times 24 \times 60 \times 60$ s

Orbital radius of Jupiter's satellite,

$R = 4.22 \times 10^8$ m

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Mass of Jupiter is given by

$$M_J = \frac{4\pi^2}{G}\cdot\frac{R^3}{T^2} = \frac{4\pi^2}{G}\cdot\frac{(4.22\times10^8)^3}{(1.769\times24\times60\times60)^2}$$

Orbital period of the earth around the sun,

$$T = 1 \text{ year} = 365.25\times24\times60\times60 \text{ s}$$

Orbital radius of the earth,

$$R = 1.496\times10^{11} \text{ m}$$

Mass of the sun,

$$M_S = \frac{4\pi^2}{G}\cdot\frac{(1.496\times10^{11})^3}{(365.25\times24\times60\times60)^2}$$

$$\therefore \frac{M_J}{M_S} = \frac{(4.22\times10^8)^3}{(1.769\times24\times60\times60)^2}\times\frac{(365.25\times24\times60\times60)^2}{(1.496\times10^{11})^3}$$

$$= \frac{1}{1046} \approx \frac{1}{1000}.$$

Hence the mass of Jupiter is about one thousandth that of the sun.

8.5. *Let us assume that our galaxy consists of 2.5×10^{11} stars each of one solar mass. How long will a star at a distance of 50,000 ly from the galactic centre take to complete one revolution ? Take the diameter of the milky way to be 10^5 ly.*

Ans. Mass of galaxy,

$$M = 2.5\times10^{11} \text{ solar masses}$$

$$= 2.5\times10^{11}\times2\times10^{30} \text{ kg} = 5\times10^{41} \text{ kg}$$

Orbital radius of the star,

$$R = 50{,}000 \text{ ly} = 50{,}000\times9.46\times10^{15} \text{ m}$$

$$= 4.73\times10^{20} \text{ m}$$

As $$M = \frac{4\pi^2}{G}\cdot\frac{R^3}{T^2}$$

$$\therefore T^2 = \frac{4\pi^2}{G}\cdot\frac{R^3}{M} = \frac{4\times9.87\times(4.73\times10^{20})^3}{6.67\times10^{-11}\times5\times10^{41}}$$

$$= 1.25275\times10^{32}$$

$$\therefore T = 1.12\times10^{16} \text{ s} = \frac{1.12\times10^{16}}{365.25\times24\times60\times60} \text{ years}$$

$$= \mathbf{3.54\times10^8 \text{ years}}.$$

8.6. *Choose the correct alternatives :*

(a) If the zero of potential energy is at infinity, the total energy of an orbiting satellite is negative of its kinetic/potential energy.

(b) The energy required to rocket an orbiting satellite out of Earth's gravitational influence is more/less than the energy required to project a stationary object at the same height (as the satellite) out of the Earth's influence.

Ans. (*a*) The total energy of an orbiting satellite is negative of its kinetic energy.

(*b*) The energy required to rocket an orbiting satellite out of Earth's gravitational influence is less than the energy required to project a stationary object at the same height out of Earth's influence.

8.7. *Does the escape speed of a body from the Earth depend on (a) the mass of the body, (b) the location from where it is projected, (c) the direction of projection, (d) the height of the location from where the body is launched ? Explain your answer.*

Ans. Escape velocity, $v_e = \sqrt{\frac{2GM}{r}}$.

(*a*) Escape velocity is independent of the mass *m* of the body to be projected.

(*b*) As the gravitational potential, $V = GM / R$ depends slightly on the latitude of the point, so escape velocity also depends (slightly) on the latitude of the location from where the body is projected.

(*c*) Escape velocity is independent of the direction of projection.

(*d*) As the gravitational potential depends on the height of location, so escape velocity also depends on height of location.

8.8. *A comet orbits the sun in a highly elliptical obit. Does the comet have a constant (a) linear speed, (b) angular speed, (c) angular momentum, (d) kinetic energy, (e) potential energy, (f) total energy throughout its orbit ? Neglect any mass loss of the comet when it comes very close to the sun.*

Ans. (*a*) According to Kepler's second law of planetary motion, a planet moves faster when it is close to the sun and moves slower, when away from the sun. So its linear speed keeps on changing.

(*b*) As the planet moves under the influence of a purely radial force, its angular momentum remains constant.

(*c*) As the linear speed of the planet continuously changes, so its kinetic energy also keeps on changing.

(*d*) As the distance of the planet from the sun continuously changes, so its potential energy keeps on changing.

(*e*) Total energy of the planet always remains constant.

8.9. *Which of the following symptoms is likely to afflict an astronaut in space (a) swollen feet, (b) swollen face, (c) headache, (d) orientational problem.*

Ans. (*b*), (*c*) and (*d*) are affected in space.

In the following two exercises, choose the correct answer from among the given ones :

8.10. *The gravitation intensity at the centre C of the drumhead defined by a hemispherical shell has the direction indicated by the arrow [see Fig. 8.46]*

(i) a, (ii) b, (iii) c, (iv) zero.

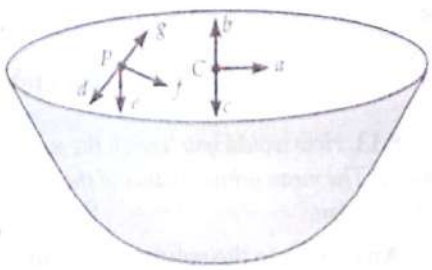

Fig. 8.46

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Ans. Complete the hemisphere to sphere, as shown in Fig. 8.47. Gravitational potential V is a constant. So gravitational intensity $\left(E = -\frac{dV}{dr}\right)$ is zero at C (for the complete sphere). For the hemisphere, the net gravitational intensity will point along (c) at the centre C.

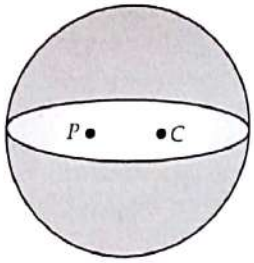

Fig. 8.47

8.11. *For the above problem, the direction of the gravitational intensity at an arbitrary point P is indicated by the arrow (i) d, (ii) e, (iii) f, (iv) g.*

Ans. By the similar reasoning as in the above problem, the direction of gravitational intensity at P will be along (e).

8.12. *A rocket is fired from the earth towards the sun. At what point on its path is the gravitational force on the rocket zero ? Mass of sun $= 2 \times 10^{30}$ kg, mass of the earth $= 6 \times 10^{24}$ kg. Neglect the effect of other planets etc. Orbital radius $= 1.5 \times 10^{11}$ m.*

Ans. Given $M_s = 2 \times 10^{30}$ kg,

$$M_e = 6 \times 10^{24} \text{ kg}, \quad r = 1.5 \times 10^{11} \text{ m}$$

Let m be the mass of the rocket. Let at distance x from the earth, the gravitational force on the rocket be zero. Then at this distance,

Gravitational pull of the earth on the rocket

= Gravitational pull of the sun on the rocket

$$\text{i.e.,} \quad \frac{GM_e m}{x^2} = \frac{GM_s m}{(r-x)^2}$$

$$\text{or} \quad \frac{(r-x)^2}{x^2} = \frac{M_s}{M_e}$$

$$\text{or} \quad \frac{r-x}{x} = \sqrt{\frac{M_s}{M_e}} = \sqrt{\frac{2 \times 10^{30}}{6 \times 10^{24}}} = \frac{10^3}{\sqrt{3}} = 577.35$$

$$\text{or} \quad r - x = 577.35\, x$$

$$\text{or} \quad 578.35\, x = r = 1.5 \times 10^{11}$$

$$\text{or} \quad x = \frac{1.5 \times 10^{11}}{578.35} = \mathbf{2.59 \times 10^8 \text{ m}}.$$

8.13. *How would you 'weigh the sun', that is estimate its mass ? The mean orbital radius of the earth around the sun is 1.5×10^8 km.*

Ans. Refer to the solution of Example 4 on page 8.6.

8.14. *A Saturn year is 29.5 times the earth year. How far is the Saturn from the sun if the earth is 1.50×10^8 km away from the sun ?*

Ans. According to Kepler's law of periods,

$$\left[\frac{T_S}{T_E}\right]^2 = \left[\frac{r_S}{r_E}\right]^3$$

But $\frac{T_S}{T_E} = 29.5$ and $r_E = 1.5 \times 10^8$ km

$$\therefore \quad (29.5)^2 = \left(\frac{r_S}{1.5 \times 10^8}\right)^3$$

or $$r_S = 1.5 \times 10^8 \times (29.5)^{2/3} = \mathbf{14.32 \times 10^8 \text{ km}}.$$

8.15. *A body weighs 63 N on the surface of the earth. What is the gravitational force on it due to the earth at a height equal to half the radius of the earth ?*

Ans. Here $mg = 63$ N, $h = R/2$

As $$\frac{g_h}{g} = \left(\frac{R}{R+h}\right)^2 = \left(\frac{R}{R+\frac{R}{2}}\right) = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

$$g_h = \frac{4}{9} g$$

$$\therefore \quad mg_h = \frac{4}{9} mg = \frac{4}{9} \times 63 = \mathbf{28 \text{ N}}.$$

8.16. *Assuming the earth to be a sphere of uniform mass density, how much would a body weigh half way down to the centre of the earth if it weighed 250 N on the surface ?*

Ans. Here $mg = 250$ N, $d = R/2$

Acceleration due to gravity at depth $d = R/2$, below the earth's surface will be

$$g_d = g\left(1 - \frac{d}{R}\right) = g\left(1 - \frac{R/2}{R}\right) = \frac{g}{2}$$

$$\therefore \text{ New weight } = mg_d = \frac{mg}{2} = \frac{250}{2} = \mathbf{125 \text{ N}}.$$

8.17. *A rocket is fired vertically with a speed of 5 kms^{-1} from the earth's surface. How far from the earth does the rocket go before returning to the earth ? Mass of earth $= 6.0 \times 10^{24}$ kg, mean radius of the earth $= 6.4 \times 10^6$ m, $G = 6.67 \times 10^{-11}$ $Nm^2 kg^{-2}$.*

Ans. Here $v = 5 \text{ kms}^{-1} = 5000 \text{ ms}^{-1}$,

$$M = 6.0 \times 10^{24} \text{ kg}, \quad R = 6.4 \times 10^6 \text{ m},$$
$$G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}.$$

Suppose the rocket goes upto a height h before returning to the earth. Clearly at this height, velocity of rocket will become zero. By the law of conservation of energy,

(K.E. + P.E.) at the earth's surface

= (K.E. + P.E.) at height h

$$\text{or} \quad \frac{1}{2} mv^2 - \frac{GMm}{R} = 0 - \frac{GMm}{(R+h)}$$

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or $$v^2 - \frac{2GM}{R} = -\frac{2GM}{R+h}$$

or $$\frac{2GM}{R+h} = \frac{2GM}{R} - v^2 = \frac{2GM - v^2 R}{R}$$

or $$R + h = \frac{2GMR}{2GM - v^2 R}$$

$$= \frac{2 \times 6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 6.4 \times 10^6}{2 \times 6.67 \times 10^{-11} \times 6.0 \times 10^{24} - (5000)^2 \times 6.4 \times 10^6}$$

$$= \frac{5.12 \times 10^{21}}{6.4 \times 10^{14}} = 8.0 \times 10^6 \text{ m}$$

$$h = 8.0 \times 10^6 - R = 8.0 \times 10^6 - 6.4 \times 10^6$$

$$= \mathbf{1.6 \times 10^6 \text{ m}}.$$

8.18. *The escape velocity of a projectile on the earth's surface is 11.2 km s^{-1}. A body is projected out with thrice this speed. What is the speed of the body far away from the earth ? Ignore the presence of the sun and other planets.*

Ans. Escape velocity, $v_e = 11.2 \text{ kms}^{-1}$

Velocity of projection, $v = 3v_e$

Let m be the mass of projectile and v_0 be the velocity of the projectile after escaping gravitational pull.

By the law of conservation of energy,

$$\frac{1}{2} mv_0^2 = \frac{1}{2} mv^2 - \frac{1}{2} mv_e^2$$

or $$v_0 = \sqrt{v^2 - v_e^2} = \sqrt{(3v_e)^2 - v_e^2} = \sqrt{8v_e^2}$$

$$= \sqrt{8 \times (11.2)^2} = 22.4\sqrt{2} = \mathbf{31.68 \text{ kms}^{-1}}.$$

8.19. *A satellite orbits the earth at a height of 400 km above the surface. How much energy must be expended to rocket the satellite out of the earth's gravitational influence ? Mass of the satellite = 200 kg, mass of the earth = 6.0×10^{24} kg, radius of the earth = 6.4×10^6 m, G = 6.67×10^{-11} Nm^2 kg^{-2}.*

Ans. Total energy of the satellite in the orbit is

$$E = \text{K.E.} + \text{P.E.} = \frac{1}{2} mv^2 - \frac{GMm}{R+h}$$

$$= \frac{1}{2} m \frac{GM}{R+h} - \frac{GMm}{R+h} = -\frac{GMm}{2(R+h)}$$

$$\left[\because v = \sqrt{\frac{GM}{R+h}}\right]$$

Here $G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$, $M = 6.0 \times 10^{24}$ kg

$m = 200$ kg, $R = 6.4 \times 10^6$ m,

$h = 400 \text{ km} = 400 \times 10^3$ m

$$\therefore \quad E = -\frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 200}{2(6.4 \times 10^6 + 400 \times 10^3)}$$

$$= -\frac{6.67 \times 6.0}{6.8} \times 10^9 = -5.89 \times 10^9 \text{ J}$$

Energy required to rocket the satellite

$$= \mathbf{5.89 \times 10^9 \text{ J}}.$$

8.20. *Two stars each of 1 solar mass ($= 2 \times 10^{30}$ kg) are approaching each other for a head-on collision. When they are at a distance 10^9 km, their speeds are negligible. What is the speed with which they collide ? The radius of each star is 10^4 km. Assume the stars to remain undistorted until they collide. Use the known value of G.*

Ans. Mass of each star, $M = 2 \times 10^{30}$ kg

Radius of each star, $R = 10^7$ m

Initial P.E. of the two stars when they are 10^{12} m apart,

$$U_i = -\frac{GM \times M}{r} = -\frac{GM^2}{10^{12}} \qquad [r = 10^9 \text{ km} = 10^{12} \text{ m}]$$

When the stars are just to collide, distance between their centres

= twice the radius of each star = $2R = 2 \times 10^7$ m

Final P.E. of the two stars when they just collide,

$$U_f = -\frac{GM \times M}{2R} = -\frac{GM^2}{2 \times 10^7}$$

Change in P.E. of the stars

$$= U_i - U_f = -\frac{GM^2}{10^{12}} - \left(-\frac{GM^2}{2 \times 10^7}\right)$$

$$= \frac{GM^2}{2 \times 10^7} - \frac{GM^2}{10^{12}} = \frac{GM^2}{2 \times 10^7} \quad \left[\because \frac{GM^2}{10^{12}} << \frac{GM^2}{2 \times 10^7}\right]$$

Let v be the velocity of each star just before collision. Then

Change in K.E. of the stars

= Final K.E. – Initial K.E.

$$= 2 \times \tfrac{1}{2} Mv^2 - 0 = Mv^2$$

By conservation of energy,

$$Mv^2 = \frac{Gm^2}{2 \times 10^7}$$

or $$v = \sqrt{\frac{GM}{2 \times 10^7}} = \sqrt{\frac{6.67 \times 10^{-11} \times 2 \times 10^{30}}{2 \times 10^7}}$$

$$= \mathbf{2.6 \times 10^6 \text{ ms}^{-1}}.$$

8.21. *Two heavy spheres each of mass 100 kg and radius 0.1 m are placed 1.0 m apart on a horizontal table. What is the gravitational field and potential at the mid point of the line joining the centres of the spheres ?*

Take G = 6.67×10^{-11} Nm^2 kg^{-2}.

Ans. Let A and B be the positions of the two spheres and P be the midpoint of AB.

Gravitational field at P due to mass at A,

$$E_1 = \frac{GM}{r^2} = \frac{G \times 100}{(0.5)^2}, \text{ along } PA$$

Gravitational field at P due to mass at B,

$$E_2 = \frac{G \times 100}{(0.5)^2}, \text{ along } PB$$

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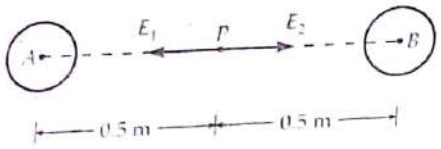

Fig. 8.48

As E_1 and E_2 have equal magnitudes but opposite directions, so the resultant gravitational field at P is zero.

As gravitational potential is a scalar quantity, so total potential at P is

$$V = V_A + V_B = -\frac{GM}{r} - \frac{GM}{r} = -\frac{2GM}{r}$$

$$= -\frac{2 \times 6.67 \times 10^{-11} \times 100}{0.5} = -2.668 \times 10^{-8} \text{ J kg}^{-1}.$$

8.22. *A geostationary satellite orbits the earth at a height of nearly 36,000 km from the surface of the earth. What is the potential due to earth's gravity at the site of the satellite ? Mass of the earth = 6×10^{24} kg and radius = 6400 km.*

Ans. Here $M = 6 \times 10^{24}$ kg, $R = 6400$ km,

$h = 36{,}000$ km

$r = R + h = 6400 + 36{,}000 = 42{,}400$ km $= 4.24 \times 10^7$ m

Gravitational potential,

$$V = -\frac{GM}{r} = -\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{4.24 \times 10^7}$$

$$= -9.43 \times 10^6 \text{ J kg}^{-1}.$$

8.23. *A star 2.5 times the mass of the sun and collapsed to a size of 12 km rotates with a speed of 1.5 rev/s. Will an object placed on its equator remain stuck to its surface due to gravity ? Mass of the sun = 2×10^{30} kg.*

Ans. Here $M = 2.5 \times$ Mass of the sun

$$= 2.5 \times 2 \times 10^{30} = 5 \times 10^{30} \text{ kg}$$

$$R = 12 \text{ km} = 12 \times 10^3 \text{ m}$$

Acceleration due to gravity on the surface of the star is

$$g = \frac{GM}{R^2} = \frac{6.67 \times 10^{-11} \times 5 \times 10^{30}}{(12 \times 10^3)^2} = 2.316 \times 10^{12} \text{ ms}^{-2}$$

Now $\nu = 1.5$ rps, so $\omega = 2\pi\nu = 3\pi$ rad s^{-1}

Centripetal acceleration of the object $= R\omega^2$

$$= 12 \times 10^3 \times (3 \times 3.14)^2 = 1.065 \times 10^6 \text{ ms}^{-2}$$

As the acceleration due to gravity on the surface of the star is greater than the centripetal acceleration of the object, so the object will remain struck to its surface.

8.24. *A spaceship is stationed on Mars. How much energy must be expended on the spaceship to rocket it out of the solar system ? Mass of the spaceship 1000 kg, mass of the sun = 2×10^{30} kg, mass of Mars = 6.4×10^{23} kg, radius of Mars 3395 km, radius of the orbit of Mars = 2.28×10^8 km, $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}.*

Ans. Energy of the spaceship in the orbit is

$$E = \text{K.E.} + \text{P.E.} = \frac{1}{2}mv^2 - \frac{GMm}{(R+h)}$$

$$= \frac{1}{2}m \cdot \frac{GM}{R+h} - \frac{GMm}{R+h} \qquad \left[\because v = \sqrt{\frac{GM}{R+h}}\right]$$

$$= -\frac{GMm}{2(R+h)}$$

But $m = 1000$ kg, $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2},

$M = 2 \times 10^{30}$ kg

$R = 2.28 \times 10^8$ km $= 2.28 \times 10^8 \times 10^3$ m,

$h = 3395$ km $= 3395 \times 10^3$ m

$$\therefore \quad E = -\frac{6.67 \times 10^{-11} \times 2 \times 10^{30} \times 1000}{2(2.28 \times 10^8 \times 10^3 + 3395 \times 10^3)}$$

$$= -2.9 \times 10^{11} \text{ J}.$$

Energy needed to rocket the spaceship out of solar system $= 2.9 \times 10^{11}$ J.

8.25. *A rocket is fired vertically from the surface of Mars with a speed of 2 kms^{-1}. If 20% of its initial energy is lost due to Martian atmospheric resistance, how far will the rocket go from the surface of Mars before returning to it ? Mass of Mars = 6.4×10^{23} kg, radius of Mars = 3395 km, $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}.*

Ans. Total energy of the rocket

$$E = \frac{1}{2}mv^2 - \frac{GMm}{R}$$

Since 20% energy is lost, hence energy remained

$= 80\%$ or $E = \frac{80}{100}E = \frac{4}{5}E = \frac{4}{5}\left[\frac{1}{2}mv^2 - \frac{GMm}{R}\right]$.

At the highest point, distant h from the surface of Mars, its total energy will be potential. Hence

$$\frac{4}{5}\left[\frac{1}{2}mv^2 - \frac{GMm}{R}\right] = -\frac{GMm}{R+h}$$

or $$\frac{2}{5}m\left(v^2 - \frac{2GM}{R}\right) = -\frac{GMm}{R+h}$$

or $$\frac{2}{5} \cdot \frac{v^2 R - 2GM}{R} = -\frac{GM}{R+h}$$

or $$R + h = -\frac{5RGM}{2(v^2 R - 2GM)}$$

But $R = 3395$ km $= 3395 \times 10^3$ m,

$G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}

$M = 6.4 \times 10^{23}$ kg, $v = 2$ kms^{-1} $= 2 \times 10^3$ ms^{-1}

$$\therefore \quad R + h = -\frac{5 \times 3395 \times 10^3 \times 6.67 \times 10^{-11} \times 6.4 \times 10^{23}}{2[(2 \times 10^3)^2 \times 3395 \times 10^3 - 2 \times 6.67 \times 10^{-11} \times 6.4 \times 10^{23}]}$$

$$= -\frac{7.25 \times 10^{20}}{2[1.36 \times 10^{13} - 8.54 \times 10^{13}]}$$

$$= \frac{7.25 \times 10^7}{2 \times 7.18} = 5.05 \times 10^6 \text{ m}$$

or $$h = 5.05 \times 10^6 - 3395 \times 10^3$$

$$= 1655 \times 10^3 \text{ m} = \mathbf{1655 \text{ km}}.$$

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Text Based Exercises

Type A : Very Short Answer Questions

1 Mark Each

1. What is difference between gravitation and gravity ?
2. What do you mean by free fall of a body ?
3. State Newton's law of gravitation. **[Delhi 05]**
4. Does the force of attraction between two bodies depend upon the presence of other bodies and properties of intervening medium ?
5. The value of G on the surface of earth is 6.67×10^{-11} $Nm^2 kg^{-2}$. What is its value on the moon ?
6. What is the dimensional formula of gravitational constant? **[Delhi 13]**
7. Do all the bodies fall with the same constant acceleration in the absence of air resistance ?
8. What are the values of g and G at the centre of the earth ?
9. Give one point of difference between g and G.
10. Which is scalar and which is vector amongst g and G ?
11. Name the scientist who first determined the value of G experimentally.
12. Name the apparatus used for the experimental determination of G.
13. Do the gravitational forces obey Newton's third law of motion ?
14. The gravitational force between two bodies is 1 N. If the distance between them is doubled, what will be the force ? **[Himachal 06]**
15. Does the acceleration with which a body falls towards the centre of the earth depend on mass of the body ?
16. Calculate the force of attraction between two balls each of mass 1 kg when their centres are 10 cm apart. The value of gravitational constant $G = 6.67 \times 10^{-11}$ $Nm^2 kg^{-2}$. **[Delhi 97]**
17. The distance of Pluto from the sun is 40 times the distance of earth. If the masses of earth and Pluto be equal, what will be ratio of gravitational forces of sun on these planets ?
18. Write an expression for the acceleration due to gravity on the earth's surface.
19. Write an equation for the mean density of the earth.
20. The gravitational force acting on a rocket at a height h from the earth's surface is 1/3 rd of the force acting on a body at sea level. What is relation between h and R_e (radius of the earth) ?
21. The mass and diameter of a planet are twice those of the earth. What will be acceleration due to gravity at the planet, if acceleration due to gravity on the earth is g ?
22. What is the mass of a body that weighs 1 N at a place where $g = 9.80\ ms^{-2}$? **[Delhi 97]**
23. How much is the torque due to gravity on a body about its centre of mass ?
24. Which is greater – the attraction of earth for 1 kg of iron or attraction of 1 kg of iron for the earth ? Give reason.
25. Has gravity any effect on inertial mass ?
26. Why do different planets have different escape velocities ?
27. On what factors does the value of g depend ?
28. What is the effect of altitude on acceleration due to gravity ?
29. What is the effect of non-sphericity of the earth on the value of 'g' ? **[Delhi 03]**
30. If accelerations due to gravity at a height h and at a depth d below the surface of the earth are equal, how are h and d related ?
31. How does the weight of a body vary enroute from the earth to the moon ? Would its inertial mass change and gravitational mass change ?
32. What is the time period of a sample pendulum at the centre of the earth ?
33. What will be our weight at the centre of earth, if the earth were a hollow sphere ?
34. A body of mass 5 kg is taken to the centre of the earth. What will be its mass there ?
35. If a man goes from the surface of the earth to a height equal to the radius of the earth, then what will be his weight relative to that on the earth ? What if he goes equally below the surface of the earth ?

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36. The earth is acted upon by the gravitational force of attraction due to the sun. Then why does the earth not fall towards the sun ?

37. Suppose the earth stops rotating about its axis. What will be the effect on the weight of bodies ?

38. If the earth rotates faster, how does g change at poles ?

39. What is the effect on our weight due to revolution of the earth about the sun ?

40. Write the formula for the gravitational potential energy of mass m at a finite distance r in the gravitational field of mass M.

41. What is the work done in bringing a body of mass m from infinity to the surface of the earth ?

42. What is the unit of intensity of the gravitational field ?

43. What is the value of gravitational potential energy at infinity ?

44. What is the value of the gravitational intensity at the earth's surface and at the earth's centre ?

45. What is the relation between gravitational intensity and gravitational potential at a point ?

46. Why is gravitational potential energy negative ?

47. The gravitational potential energy of a body at a distance r from the centre of the earth is U. What is the weight of the body at that point ?

48. From where does a satellite get centripetal force for moving around its planet ?

49. What is escape velocity ? Write down its minimum value on the surface of the earth ?

50. Define orbital velocity of a satellite.

51. How does the orbital velocity of a satellite depend on the mass of the satellite ? **[Delhi 04]**

52. A satellite revolves close to the surface of a planet. How is its orbital velocity related with velocity of escape from that planet ?

[Chandigarh 07]

53. What are the values of the escape velocities for the moon and the sun respectively ?

54. Which has greater value of escape velocity-Mercury or Jupiter ?

55. Does the escape velocity of a body depend upon the density of a planet ?

56. Why does hydrogen escape from the earth's atmosphere more readily than oxygen ?

57. Does the speed of a satellite remain constant in an orbit ?

58. The speed of revolution of an artificial satellite revolving close to the surface of the earth, is 8 kms^{-1}. What will be the escape velocity for a body on the earth ? **[Manipur 99]**

59. A satellite revolving around the earth loses height. What happens to its time period ?

60. If the kinetic energy of a satellite revolving in an orbit close to the earth's surface happens to be doubled, will the satellite escape ?

61. The escape velocity on earth is 11.2 kms^{-1}. What will be its value on a planet having double the radius and eight times the mass of the earth ?

62. What is Geosynchronous satellite ? **[Delhi 95]**

63. What is a parking orbit ?

64. What is the use of stationary orbit ?

65. What is (*i*) period of revolution and (*ii*) sense of rotation of a geostationary satellite ? **[Himachal 06]**

66. The centripetal force on a satellite revolving around the earth is F.

(*i*) What is the gravitational force due to earth on it ?

(*ii*) What is the net force on it ?

67. What is the angular velocity of a geostationary satellite in radian per hour ?

68. A spring balance is suspended inside an artificial satellite revolving around the earth. If a body of mass 1 kg is suspended from it, what would be its reading ?

69. The escape velocity from the earth for a body of 20 g is 11 kms^{-1}. What will be its value for a body of 100 g ?

70. A body lying on the surface of planet Venus has gravitational potential energy equal to -7.5×10^6 J. How much energy will be required for the body to escape from the planet ?

71. Two artificial satellites are revolving around the earth, one closer to its surface and other away from it. Which has larger speed ?

72. Write two conditions for the existence of atmosphere on a planet. **[CBSE(F) 94]**

73. Write the most important application of geostationary satellite. **[Delhi 02]**

74. What would happen to an artificial satellite if its orbital velocity is slightly decreased due to some defects in it ? **[Manipur 97]**

75. What is the frequency of oscillation of a simple pendulum mounted in a cabin that is freely falling under gravity ? **[Delhi 05]**

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76. What is the basis of Kepler's law of areas of planetary motion ?
77. What is the basis of Newton's law of gravitation ?
78. A simple pendulum is mounted inside a spacecraft. What should be its time period of vibration ? [Central Schools 05]
79. The gravitational potential energy of a body at a point in a gravitational field of another body is $-\frac{GMm}{r}$. What does the negative sign show ? [Central Schools 05]
80. What is meant by a geostationary satellite ? [Himachal 05]
81. What is full form of geostationary satellite APPLE ? [Himachal 05]
82. What will be the kinetic energy needed to project a body of mass m from the earth's surface (radius R) to infinity ? [AIEEE 02]
83. What is gravitational force ? [Himachal 07]
84. Define universal gravitatinal constant. [Himachal 07C]
85. What is the weight of a body at the centre of the earth ? [Himachal 05C]
86. Where does the body weigh more – at the surface of the earth or in a mine ? [Himachal 05C]
87. If the change in the value of 'g' at a height 'h' above the surface of earth is same as that at a depth 'x' below it (both x and h being much smaller than radius of earth), then how are x and h related to each other ? [Chandigarh 07]
88. How would the value of 'g' change if the earth were to shrink slightly without any change in mass ? [Central Schools 09]
89. If the radius of the earth shrinks by 1 percent, its mass remaining the same by what percentage will the acceleration due to gravity on its surface change ? [Central Schools 09]
90. The distances of two planets from the sun are 10^{11} m and 10^{10} m respectively. What is the ratio of time period of these two planets ? [Central Schools 09]
91. Name India's first cosmonaut. [Himachal 05]
92. What is weightlessness ? [Himachal 07, 07C]
93. Give two uses of geostationary satellites.
94. Give two uses of polar satellites.
95. What are the time period and height of a geostationary satellite above the surface of the earth ? [Central Schools 12]

Answers

1. Gravitation is the force of attraction between any two bodies in the universe while gravity refers to the force of attraction between any body and the earth.
2. The motion of a body under the influence of gravity alone is called a free fall.
3. Refer to point 2 of Glimpses.
4. No, $F = \frac{G m_1 m_2}{r^2}$ is independent of the presence of other bodies and properties of intervening medium.
5. G is a universal constant, so its value on the moon $= 6.67 \times 10^{-11}\ \text{Nm}^2\ \text{kg}^{-2}$.
6. $[G] = [M^{-1}L^3T^{-2}]$
7. Yes, this acceleration is called acceleration due to gravity.
8. At the centre of the earth g is zero, while $G = 6.67 \times 10^{-11}\ \text{Nm}^2\text{kg}^{-2}$.
9. While the value of g varies from place to place, the value of G remains the same throughout the universe.
10. g is a vector and G is a scalar.
11. Lord Cavendish.
12. Cavendish's torsion balance.
13. Yes, they obey Newton's third law of motion.
14. 0.25 N, because $F \propto 1/r^2$.
15. No, acceleration due to gravity is independent of the mass of the body.
16. $F = G\frac{m_1 m_2}{r^2} = \frac{6.67 \times 10^{-11} \times 1 \times 1}{0.10} = \mathbf{6.67 \times 10^{-10}\ N.}$
17. $\frac{F_{es}}{F_{ps}} = \left(\frac{r_{ps}}{r_{es}}\right)^2 = \left(\frac{40}{1}\right)^2 = 1600 : 1.$
18. $g = \frac{GM_e}{R_e^2}.$
19. $\rho = \frac{3g}{4\pi GR}.$
20. Given $\frac{GM_e m}{(R_e + h)^2} = \frac{1}{3}\frac{GM_e m}{R_e^2}$

$\therefore \quad R_e + h = \sqrt{3}R_e = 1.732\ R_e$

or $\quad h = \mathbf{0.732\ R_e}.$

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21. As $g = \frac{GM}{R^2}$

$$\therefore \quad g' = \frac{G(2M)}{(2R)^2} = \frac{1}{2}\frac{GM}{R^2} = \frac{g}{2}.$$

22. $m = \frac{W}{g} = \frac{1}{9.80} = 0.102$ kg.

23. Zero.

24. The two bodies interact gravitationally due to their masses, so they exert equal forces on each other but in opposite directions.

25. No.

26. Different planets have different escape velocities ($v_e = \sqrt{2GM/R}$) because of their different masses and sizes.

27. The value of g depends on (*i*) the shape of the earth (*ii*) latitude (*iii*) altitude (*iv*) depth.

28. Acceleration due to gravity decreases with altitude.

29. The acceleration due to gravity is minimum at the equator and gradually increases from equator to poles.

30. $d = 2h$.

31. When taken from earth to moon, the weight of a body gradually decreases to zero. It then increases till at the moon's surface, it becomes $mg/6$. However, its inertial and gravitational masses remain unchanged.

32. At the centre of the earth, $g = 0$.

$$\therefore \quad T = 2\pi\sqrt{\frac{l}{g}} = \infty.$$

Hence the pendulum does not oscillate at all.

33. Zero, because the intensity of a gravitational field inside a hollow sphere is zero.

34. The mass of the body will remain 5 kg at the centre of the earth because it remains the same at all places.

35. (*i*) $\frac{g_h}{g} = \frac{R^2}{(R+h)^2} = \frac{R^2}{(R+R)^2} = \frac{1}{4}$

$\therefore \quad mg_h = \frac{1}{4}mg$ *i.e.*, weight becomes one-fourth.

(*ii*) $mg_d = mg\left(1 - \frac{d}{R}\right) = m\left(1 - \frac{R}{R}\right) = 0.$

36. The gravitational attraction of the sun provides the centripetal force to keep the earth in stable orbit around the earth.

37. The weight of the bodies will increase.

38. The value of g does not change.

39. No effect.

40. $-GMm/r$.

41. GMm/r.

42. Nkg^{-1}.

43. Zero.

44. 9.8 N kg^{-1}, zero.

45. Gravitational intensity at a point is equal to the negative of the potential gradient at that point.

46. Because it arises due to attractive forces of gravitation.

47. $U = \frac{GMm}{r} = \left(\frac{GM}{r^2}\right)rm = grm = mg \times r.$

$\therefore$ Weight of the body $= mg = U/r$.

48. The gravitational force of attraction exerted by the planet provides the necessary centripetal force to the satellite.

49. Escape velocity is defined as the minimum velocity with which a body must be thrown vertically upwards in order that it may just escape the gravitational pull of the earth. Its value on the earth's surface is 11.2 km s^{-1}.

50. Orbital velocity is the velocity given to an artificial earth's satellite a few hundred kilometers above the earth's surface so that it may start revolving around the earth.

51. $v_0 = \sqrt{\frac{GM}{R+h}}$.

Clearly, orbital velocity does not depend on mass of a satellite.

52. $v_e = \sqrt{2}\,v_0$.

53. $v_{moon} = 2.4$ kms^{-1} and $v_{sun} = 620$ kms^{-1}.

54. Jupiter.

55. Yes, $v_e \propto \sqrt{\rho}$.

56. This is because the r.m.s. speed of hydrogen molecules is four times that of oxygen molecules.

57. As $v_0 = \sqrt{\frac{GM}{r}}$. For a particular orbit r is constant, so v_0 remains constant.

58. $v_e = \sqrt{2}v_0 = \sqrt{2} \times 8 = \mathbf{11.3\ kms^{-1}}$.

59. Time period of a satellite,

$$T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}.$$

Clearly when h decreases, T also decreases.

60. Yes, when the K.E. of the satellite is doubled, its orbital velocity increases $\sqrt{2}$ times and becomes equal to the escape velocity. So the satellite will escape.

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61. $\frac{v_p}{v_e} = \sqrt{\frac{2GM_p}{R_p}} \times \sqrt{\frac{R_e}{2GM_e}} = \sqrt{\frac{M_p}{M_e} \times \frac{R_e}{R_p}} = \sqrt{8 \times \frac{1}{2}} = 2$

$\therefore \quad v_p = 2v_e = 2 \times 11.2 = 22.4 \text{ kms}^{-1}$.

62. A satellite which revolves around earth with the same angular speed and in the same direction as the earth rotates about its own axis is called geosynchronous satellite.

63. The orbit of a geostationary satellite is called parking orbit. It lies in the equatorial plane at a height of about 36,000 km from the earth's surface.

64. By means of satellites in stationary orbits, television programmes can be transmitted continuously from one part of the world to the other parts.

65. (*i*) 24 hours (*ii*) west to east.

66. (*i*) *F* (*ii*) *F*.

67. $\omega = \frac{2\pi \text{ rad}}{24 \text{ h}} = \frac{\pi}{12} \text{ rad h}^{-1}$.

68. Zero, because the satellite is in a state of free fall.

69. 11.2 kms^{-1}, because the escape velocity does not depend on the mass of the body projected.

70. $+ 7.5 \times 10^6$ J.

71. Orbital speed of a satellite revolving at height h is

$$v_0 = \sqrt{\frac{GM}{R+h}}$$

Clearly smaller the height h, larger is the orbital speed v_0. Hence the satellite revolving closer to the earth's surface has larger speed v_0.

72. The two conditions for the existence of atmosphere on a planet are
   (*a*) High value of acceleration due to gravity on the planet.
   (*b*) Low surface temperature of the planet.

73. A geostationary satellite can be used to communicate radio, T.V. and telephone signals between any two points of the earth.

74. The satellite will fall to the earth.

75. In a freely falling cabin, $g = 0$.

$$\nu = \frac{1}{2\pi}\sqrt{\frac{g}{l}} = 0.$$

Hence the pendulum will not oscillate.

76. Kepler's law of areas is based on the law of conservation of angular momentum.

77. Kepler's laws of planetary motion.

78. A body is in a state of weightlessness inside a spacecraft *i.e.*, $g = 0$

$$\therefore \quad T = 2\pi\sqrt{\frac{l}{g}} = \infty.$$

79. The negative sign shows that the gravitational potential energy of one body is due to the attractive gravitational force exerted by the other body.

80. Refer to point 24 of Glimpses.

81. Ariana Passenger Pay Load Experiment.

82. Required kinetic energy

$$= \frac{1}{2} m \times (\text{escape velocity})^2$$

$$= \frac{1}{2} m (\sqrt{2gR})^2$$

$$= mgR.$$

83. The force of attraction between two bodies by virtue of their masses is called gravitational force.

84. The universal gravitational constant may be defined as the force of attraction between two bodies of unit mass each and placed unit distance apart.

85. Zero, because the value of g is zero at the centre of the earth.

86. At the surface of the earth, because weight decreases with depth.

87. $h = 2x$.

88. The value of 'g' slightly increases.

89. The value of 'g' increases by 2%.

90. $\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2} = \left(\frac{10^{11}}{10^{10}}\right)^{3/2} = 10^{3/2} = 10\sqrt{10}$.

91. Rakesh Sharma.

92. Refer to point 28 of Glimpses.

93. Refer answer to Q.46 on page 8.35.

94. Refer answer to Q.47 on page 8.35.

95. $T = 24$ h and $h = 36{,}000$ km.

Type B : Short Answer Questions — 2 or 3 Marks Each

1. State Newton's law of gravitation. Hence define universal gravitational constant. Give the value and dimensions of G. **[Meghalaya 96, 99 ; Himachal 07]**

2. State the universal law of gravitation. Establish the relationship $M_e = gR_e^2 / G$, where M_e and R_e are the mass and radius of the earth respectively. **[Delhi 02]**

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3. Define acceleration due to gravity. Derive expression for the variation of 'g' with height from the surface of the earth. **[Central Schools 14]**

4. Derive an expression for g (acceleration due to gravity) at a depth d from the surface of earth. Consider the earth as a sphere of uniform mass density. What happens to 'g' at the centre of earth. **[Delhi 13 ; Central Schools 14]**

5. Define acceleration due to gravity. Show that gravity decreases with depth. **[Himachal 05, 06 ; Central Schools 09]**

6. Discuss the variation of 'g' (*i*) with altitude (*ii*) with depth (*iii*) with latitude : Rotation of earth. **[Central Schools 04, 05 ; Delhi 06]**

7. Explain the variation of 'g' with (*i*) shape of earth ; (*ii*) rotation of earth and prove that the weight of body remains unchanged at the poles of earth. **[Chandigarh 03]**

8. Show that the acceleration due to gravity at a height h above the surface of the earth has same value as that at depth $d = 2h$ below the surface of the earth.

9. Write the definitions and expressions of the intensity of gravitational field and gravitational potential. State their SI units. **[Himachal 04]**

10. Define gravitational potential energy. Give its SI unit. **[Delhi 13]**

11. Define the term gravitational field. Show that acceleration due to gravity is equal to the intensity of gravitational field at any point.

12. Define gravitational potential. Give its SI unit. **[Delhi 10, 11]**

13. What do you mean by gravitational potential energy of a body ? Obtain an expression for it for a body of mass m lying at distance r from the centre of the earth. Hence write the expression for gravitational potential. **[Chandigarh 08 ; Delhi 08, 10]**

14. (*i*) What is meant by gravitational field strength ?

(*ii*) Which of the planets of the solar-system has the greatest gravitational field strength ?

(*iii*) What is the gravitational field strength of a planet where the weight of a 60 kg astronaut is 300 N ? **[Delhi 97, 05]**

15. What is the maximum value of potential energy that can be possessed by a heavenly body ? Give the general expression for potential energy of an object near the surface of earth. **[Central Schools 03]**

16. (*i*) Define escape velocity.

(*ii*) Derive expression for the escape velocity of an object from the surface of a planet.

(*iii*) Does it depend on location from where it is projected ? **[Delhi 09]**

17. Define Escape Velocity. Derive an expression for escape velocity of an object from the surface of a planet. **[Delhi 99, 03C, Himachal 02, 03, 05]**

18. Derive an expression for the escape velocity of a satellite projected from the surface of the earth. **[Central Schools 07]**

19. What is escape velocity ? Prove that escape velocity from the surface of earth is 11.2 kms^{-1}. **[Himachal 07C ; Delhi 95]**

20. What happens to a body when it is projected vertically upwards from the surface of the earth with a speed of 11200 m/s, and why ? Compare escpae speeds for two planets of masses M and $4M$ and radii $2R$ and R respectively. **[Delhi 2003]**

21. A black hole is a body from whose surface nothing may ever escape. What is the condition for a uniform spherical mass M to be a black hole ? What should be the radius of such a black hole if its mass is the same as that of the earth ? **[Delhi 03C]**

22. Define the term orbital speed. Establish a relation for orbital speed of a satellite orbiting very close to the surface of the earth. Find the ratio of this orbital speed and escape speed. **[Delhi 05]**

23. What are geostationary satellites ? Calculate the height of the orbit above the surface of the earth in which a satellite, if placed, will appear stationary. **[Delhi 02]**

24. Find the expression of total energy of a satellite revolving around the surface of earth. What is the significance of negative sign in the expression ? **[Chandigarh 07 ; Central Schools 08]**

25. State and explain Kepler's laws of planetary motion. Name the physical quantities which remain constant during the planetary motion. **[Central Schools 01, 07 ; Delhi 11, 12]**

26. State Kepler's law of periods and law of areas regarding planetary motion. **[Central Schools 13]**

27. State and derive Kepler's law of periods (or harmonic law) for circular orbits.

28. (*a*) According to Kepler's second law, the radius vector to a planet from the sun sweeps out equal areas in equal interval of time. The law is consequence of which conservation law ?

(*b*) State Kepler's third law. **[Delhi 09]**

29. (*a*) State Kepler's third law of periods.

(*b*) Two satellites are at different heights (smaller and larger) from the surface of earth. Which would have greater velocity ?

(*c*) What is the formula for escape velocity in terms of g and R ? **[Central Schools 08]**

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30. What is a polar satellite ? Explain how does it scan the entire earth in its each revolution ? Give two important uses of a polar satellite.

31. What do you mean by the term weightlessness ? Explain the state of weightlessness of
    (*i*) a freely falling body
    (*ii*) an astronaut in a satellite orbiting the earth.

32. Distinguish between inertial and gravitational masses, giving an illustration for each. Show that the two types of masses are equivalent.

33. State the conditions necessary for a satellite to appear stationary. **[Himachal 07]**

34. Define orbital velocity. Derive an expression for the orbital velocity of a satellite revolving around a planet. **[Central Schools 14]**

Answers

1. Refer answer to Q. 6 on page 8.3.
2. Refer answer to Q. 6 on page 8.3 & Q. 15 on page 8.9.
3. Refer answer to Q. 18 on page 8.13.
4. Refer answer to Q. 19 on page 8.15.
5. Refer answer to Q.19 on page 8.15.
6. Refer to point 12 of Glimpses.
7. Refer to point 12 of Glimpses.
8. Refer answer to Q. 21 on page 8.16.
9. Refer to points 14 and 15 of Glimpses.
10. Refer to point 16 of Glimpses.
11. Refer answer to Q. 28 and Q. 29 on page 8.20.
12. Refer to point 15 of Glimpses.
13. Refer answer to Q. 32 and Q. 33 on page 8.21.
14. (*i*) Refer to point 14 of Glimpses.
    (*ii*) The gravitational field strength is maximum on Jupiter.
    (*iii*) Gravitational field strength
    $$= \frac{F}{m} = \frac{300}{60}$$
    $$= 5 \text{ Nkg}^{-1}.$$
15. The maximum value of potential energy of a heavenly body is zero. The gravitational P.E. of a body of mass m near the surface of the earth is
    $$U = -\frac{GMm}{R}$$
16. Refer answer to Q.36 on page 8.27. Yes, it depends on height of location from where the satellite is projected.
17. Refer answer to Q. 36 on page 8.27.
18. Refer answer to Q.36 on page 8.27.
19. Refer answer to Q. 36 on page 8.27 and solution of Example 36 on page 8.28.
20. The body will escape the gravitational field of the earth.
    $$\frac{v_e}{v_e'} = \sqrt{\frac{2GM}{2R}} \cdot \sqrt{\frac{R}{2G \times 4M}}$$
    $$= \frac{1}{\sqrt{8}} = 1 : \sqrt{8}.$$
21. Refer to solution of Example 38 on page 8.28.
22. Refer answer to Q. 41 on page 8.31.
23. Refer answer to Q. 43 on page 8.34.
24. Refer answer to Q.48 on page 8.36.
25. Refer answer to Q. 51 on page 8.38. Total mechanical energy and angular momentum are conserved during planetary motion.
26. Refer answer to Q. 51 on page 8.38.
27. Refer answer to Q. 55 on page 8.40.
28. (*a*) Law of conservation of angular momentum.
    (*b*) According to Kepler's third law of periods, the square of the period of revolution of a planet around the sun is proportional to the cube of the semimajor axis of its elliptical orbit.
29. (*a*) Refer answer to part (*b*) of the above question.
    (*b*) Orbital velocity,
    $$v_0 = \sqrt{\frac{GM}{R+h}}$$
    Clearly, the satellite revolving at smaller height h from the surface of the earth will have a greater velocity.
    (*c*) $v_e = \sqrt{2gR}$.
30. Refer answer to Q. 47 on page 8.35.
31. Refer answer to Q. 57 on page 8.43.
32. Refer answer to Q. 58 and Q. 59 on page 8.44.
33. Refer answer to Q. 44 on page 8.35.
34. Refer answer to Q.41 on page 8.31.

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Type C : Long Answer Questions

5 Marks Each

1. Explain how did Newton discover the universal law of gravitation ?
2. What is meant by acceleration due to gravity ? Obtain an expression for it in terms of mass of the earth and gravitational constant. Explain how the mass and the density of the earth can be obtained from the knowledge of G.
3. What do you mean by acceleration due to gravity ? Discuss the variation of g on the surface of the earth due to axial rotation of the earth. Derive the necessary relation. **[Meghalaya 98]**
4. Obtain an expression for the acceleration due to gravity on the surface of the earth in terms of mass of the earth and its radius. Discuss the variation of acceleration due to gravity with altitude, depth and rotation of the earth. **[Kerala 01]**
5. What is escape velocity ? Obtain an expression for the escape velocity on earth. Why is it that there is no atmosphere on the moon ? Explain. **[Chandigarh 06]**
6. (*a*) Define Orbital velocity and establish an expression for it.
   (*b*) Calculate the value of orbital velocity for an artificial satellite of earth orbiting at a height of 1000 km.

   Given : Mass of earth = 6×10^{24} kg

   Radius of earth = 6400 km. **[Delhi 96]**
7. Define orbital velocity and time period of a satellite. Derive expressions for these. **[Himachal 02, 06]**
8. State Kepler's laws of planetary motion. Deduce Newton's law of gravitation from Kepler's law. **[Himachal 02, 05C, 07C ; Chandigarh 04]**

Answers

1. Refer answer to Q. 5 on page 8.2.
2. Refer answer to Q. 15 and 16 on page 8.9.
3. Refer answer to Q. 24 on page 8.18.
4. Refer answer to Q. 15 on page 8.9. Refer to point 12 of Glimpses.
5. Refer answer to Q.36 on page 8.27 and solution of Problem 46 on page 8.49.
6. (*a*) Refer answer to Q. 41 on page 8.31.
   (*b*) Refer to solution of Example 44 on page 8.32.
7. Refer answer to Q. 41 on page 8.31 and Q. 42 on page 8.32.
8. Refer answer to Q. 50 on page 8.38 and Q. 56 on page 8.40.

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Gravitation

GLIMPSES

1. **Gravitation and gravity.** Gravitation is the force of attraction between any two bodies while gravity refers to the force of attraction between any body and the earth.

2. **Newton's law of gravitation.** It states that every body in this universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

   Mathematically, $F = G\frac{m_1 m_2}{r^2}$

   where G is the universal gravitational constant. It was derived by Newton on the basis of Kepler's laws of planetary motion.

3. **Universal gravitational constant (G).** It is equal to the force of attraction between two bodies of unit mass each and separated by unit distance.

   In SI units, $G = 6.67 \times 10^{-11}\ \text{Nm}^2\ \text{kg}^{-2}$

   In CGS system, $G = 6.67 \times 10^{-8}$ dyne $\text{cm}^2\ \text{g}^{-2}$

   Dimensional formula of G is $[M^{-1}L^3T^{-2}]$

4. **Properties of gravitational forces.** The gravitational force between two point masses (*i*) is independent of intervening medium (*ii*) obeys Newton's third law of motion (*iii*) has spherical symmetry (*iv*) is independent of the presence of other bodies (*v*) obeys principle of superposition (*vi*) is conservative and central.

5. **Principle of superposition of gravitational forces.** The gravitational force between two masses acts independently and uninfluenced by the presence of other bodies. Hence the resultant force on a particle due to a number of masses is the vector sum of the gravitational forces exerted by the individual masses on the given particle.

$$\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots\ldots + \vec{F}_n = \sum_{i=1}^{n} \vec{F}_i$$

6. **Shell theorem.** (*i*) The gravitational force on a particle placed anywhere inside a spherical shell is zero (*ii*) If a particle lies outside a spherical shell, the shell attracts the particle as though the mass of the shell were concentrated at the centre of the shell.

7. **Free fall.** The motion of a body under the influence of gravity alone is called a free fall.

8. **Acceleration due to gravity (g).** The acceleration produced in the motion of a body under the effect of gravity is called acceleration due to gravity.

   At the surface of the earth, $g = \frac{GM}{R^2}$

   where M is the mass and R the radius of the earth. It is a vector quantity. Its SI unit is ms^{-2}.

9. **Mass of the earth.** It is given by

$$M = \frac{gR^2}{G}$$

10. **Mean density of the earth.** If the earth is a sphere of mass M and mean density ρ, then its mass would be

$$M = \text{Volume} \times \text{density} = \frac{4}{3}\pi R^3 \rho$$

Also, $M = \frac{gR^2}{G}$

$$\therefore \quad \frac{4}{3}\pi R^3 \rho = \frac{gR^2}{G}$$

or $$\rho = \frac{3g}{4\pi GR}.$$

11. **Weight of a body.** It is the gravitational force with which a body is attracted towards the centre of the earth.

$$\vec{W} = m\vec{g}$$

Weight of a body is a vector quantity. It is measured in newton, kg wt, etc. The weight of a body varies from place to place because of the variation in the value of g.

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12. Variation of acceleration due to gravity.

(i) *Effect of altitude.* At a height h,

$$g_h = g\left(1 - \frac{h}{R}\right)^{-2}$$

and $$g_h = g\left(1 - \frac{2h}{R}\right), \text{ when } h << R$$

Clearly, the value of g decreases with the increase in h.

(ii) *Effect of depth.* At a depth d,

$$g_d = g\left(1 - \frac{d}{R}\right)$$

Clearly, the value of g decreases with the increase in depth d and becomes zero at the centre of the earth.

(iii) *Effect of rotation.* If ω is the angular velocity of the earth about its axis of rotation, then at latitude λ,

$$g_\lambda = g - R\omega^2 \cos^2 \lambda$$

As λ increases, $\cos \lambda$ decreases and g_λ increases. So as we move from equator to pole, the value of g increases.

At equator, $\lambda = 0°$, $\cos \lambda = 1$, hence

$$g_{equa} = g - R\omega^2$$

At poles, $\lambda = 90°$, $\cos \lambda = 0$

hence $$g_{pole} = g - R\omega^2 \times 0 = g$$

Hence g is minimum at equator and maximum at poles.

$$g_{pole} - g_{equa} = g - (g - R\omega^2) = R\omega^2.$$

(iv) *Effect of non-sphericity of the earth.* The earth has an equatorial bulge and is flattened at the poles. As $R_e > R_p$, so the value of g is minimum at the equator and maximum at the poles.

13. **Gravitational field.** It is the space around a material body in which its gravitational pull can be experienced by other bodies.

14. **Intensity of gravitational field.** The intensity or strength of gravitational field at any point in a gravitational field is equal to the force experienced by a unit mass placed at that point. It is a vector quantity directed towards the body producing the gravitational field. It is given by

$$E = \frac{\text{Force}}{\text{Mass}} = \frac{F}{m} = \frac{GM}{r^2}$$

The intensity of gravitational field at a point is equal to the acceleration due to gravity at that point.

SI unit of E is N kg^{-1} and the CGS unit is dyne g^{-1}.

15. **Gravitational potential.** The gravitational potential at a point in the gravitational field of a body is the amount of work done in bringing a body of unit mass from infinity to that point. It is a scalar quantity.

Gravitational potential, $V = \frac{\text{Work done}}{\text{Mass}} = -\frac{GM}{r}$

SI unit of V is J kg^{-1} and its CGS unit is erg g^{-1}.

16. **Gravitational potential energy.** The gravitational potential energy of a body may be defined as the energy associated with it due to its position in the gravitational field of another body and is measured by the amount of work done in bringing a body from infinity to a given point in the gravitational field of the other body.

Gravitational P.E. = Gravitational potential × mass of body

$$U = -\frac{GM}{r} \times m$$

Gravitational intensity (E) and gravitational potential (V) at a point are related as

$$E = -\frac{dV}{dr}$$

17. **Escape velocity (v_e).** It is the minimum velocity with which a body must be projected vertically upwards in order that it may just escape the gravitational field of the earth.

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

For earth, the value of escape velocity is 11.2 kms^{-1}. It is independent of the mass of body projected.

18. **Satellite.** It is a heavenly or an artificial body which is revolving continuously in an orbit around a planet.

19. **Orbital velocity of a satellite (v_0).** It is the velocity required to put a satellite in its orbit around a planet. The orbital velocity of a satellite revolving around the earth at a height h is given by

$$v_0 = \sqrt{\frac{GM}{R+h}} = \sqrt{\frac{gR^2}{R+h}} = R\sqrt{\frac{g}{R+h}}$$

When a satellite revolves close to the surface of the earth, ($h = 0$),

$$v_0 = \sqrt{gR} \quad \text{and} \quad v_e = \sqrt{2}\, v_0.$$

20. **Time period of a satellite (T).** It is the time taken by a satellite to go once around the planet

$$T = \frac{\text{Circumference of the orbit}}{\text{Orbital velocity}}$$

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$$= \frac{2\pi(R+h)}{v_0} = \frac{2\pi(R+h)}{\sqrt{\frac{GM}{R+h}}}$$

$$= 2\pi\sqrt{\frac{(R+h)^3}{GM}} = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}}$$

$$= \sqrt{\frac{3\pi(R+h)^3}{G\rho R^3}}$$

If the satellite revolves just close to the surface of the earth, $h = 0$, then

$$T = 2\pi\sqrt{\frac{R^3}{GM}} = 2\pi\sqrt{\frac{R}{g}} = \sqrt{\frac{3\pi}{G\rho}}$$

21. **Height of a satellite above the earth's surface.** We know that

$$T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}} \quad \text{or} \quad (R+h)^3 = \frac{T^2R^2g}{4\pi^2}$$

or $$h = \left[\frac{T^2R^2g}{4\pi^2}\right]^{1/3} - R$$

22. **Angular momentum of a satellite.** The angular momentum of a satellite of mass m moving with velocity v_0 in an orbit of radius $r\,(= R + h)$ is given by

$$L = m_0vr = m\sqrt{\frac{GM}{r}}.r = \sqrt{GMm^2r}$$

23. **Total energy of a satellite.** For a satellite of mass m moving with velocity v_0 in an orbit of radius r,

Potential energy, $U = -\frac{GMm}{r}$

Kinetic energy, $K = \frac{1}{2}mv_0^2 = \frac{1}{2}m\left(\frac{GM}{r}\right)$

Total energy, $E = K + U$

or $$E = \frac{1}{2}mv_0^2 - \frac{GMm}{r} = -\frac{GMm}{2r}$$

Clearly, $E = -K = U/2$

Negative total energy indicates that the satellite is bound to the earth.

24. **Geostationary or synchronous satellite.** It is satellite which revolves around the earth with the same angular speed and in the same direction as the earth rotates about its own axis. Such a satellite should revolve around the earth from west to east in an orbit concentric and coplanar with the equatorial plane of the earth at a height of 36,000 km.

Three geostationary satellites with a mutual angular separation of 120° can be used to communicate between any of two points of the entire earth.

25. **Polar satellite.** A satellite that revolves in a polar orbit is called a polar satellite. Such a satellite passes once over geographical north and south poles during each round trip. A polar orbit has a smaller radius of 500 – 800 km.

26. **Conservation of quantities in motion under gravitational influence.** During the motion of an object under the gravitational influence of another object the following quantities are conserved : (*a*) Angular momentum (*b*) Total mechanical energy.

Linear momentum is not conserved. Conservation of angular momentum leads to Kepler's second law of planetary motion.

27. **Kepler's laws of planetary motion**

(*i*) ***Law of orbits.*** Every planet moves in an elliptical orbit around the sun, with the sun being at one of the focii.

(*ii*) ***Law of areas.*** The radius vector drawn from the sun to the planet sweeps out equal areas in equal intervals of time *i.e.*, the areal velocity of a planet is constant.

(*iii*) ***Law of periods or Harmonic law.*** The square of the period of revolution (T) of a planet around the sun is proportional to the cube of the semi-major axis r of the ellipse.

$$T^2 \propto r^3.$$

28. **Weightlessness.** A body is said to be in a state of weightlessness when the reaction of the supporting surface is zero or its apparent weight is zero. An astronaut experiences weightlessness in a space satellite. This is not because the gravitational force is small at that location in space. It is because both the astronaut and the satellite are in the state of free fall towards the earth.

29. **Inertial mass.** The mass of a body which measures its inertia and which is given by the ratio of the external force applied on it to the acceleration produced in it along a smooth horizontal surface is called inertial mass.

$$\text{Inertial mass} = \frac{\text{Applied force}}{\text{Acceleration produced}} \quad \text{or} \quad m_i = \frac{F}{a}$$

30. **Gravitational mass.** The mass of a body which determines the gravitational pull due to earth acting upon it is called gravitational mass. On the surface of the earth,

$$F = \frac{GMm_g}{R^2}$$

$\therefore$ Gravitational mass, $m_g = \frac{FR^2}{GM}$.

Inertial mass of a body is equivalent to its gravitational mass.

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IIT Entrance Exam

Multiple Choice Questions with One Correct Answer

1. If the radius of earth were to shrink by one percent (its mass remaining the same), then the acceleration due to gravity on the earth's surface
(a) would decreases (b) would remain unchanged
(c) would increase (d) cannot be predicted [IIT 81]

2. A simple pendulum has a time period T_1 when on the earth's surface, and T_2 when taken to a height R above the earth's surface, where R is the radius of the earth. The value of T_2 / T_1 is
(a) 1 (b) $\sqrt{2}$
(c) 4 (d) 2 [IIT 01]

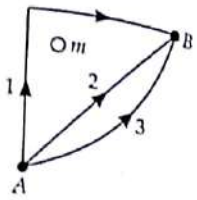
3. If the distance between the earth and the sun were half its present value, the number of days in year would have been
(a) 64.5 (b) 129
(c) 182.5 (d) 730 [IIT 96]

4. A geo-stationary satellite orbits around the earth in a circular orbit of radius 36,000 km. Then, the time period of a spy satellite orbiting a few hundered km above the earth's surface (R_{earth} = 6,400 km) will approximately be
(a) (½) h (b) 1 h
(c) 2 h (d) 4 h [IIT 02]

5. A binary star system consists of two stars A and B which have time periods T_A and T_B, radii R_A and R_B and masses M_A and M_B. Then
(a) if $T_A > T_B$, then $R_A > R_B$
(b) if $T_A > T_B$, then $M_A > M_B$
(c) $\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{R_A}{R_B}\right)^3$ (d) $T_A = T_B$ [IIT 06]

6. A geostationary satellite is orbiting the earth at a height of 6 R above the surface of the earth, R being the radius of the earth. The time period of another satellite at a height of 2.5 R from the surface of earth is
(a) $6\sqrt{2}$ hours (b) 6 hours
(c) $6\sqrt{3}$ hours (d) 10 hours [IIT 87]

7. If W_1, W_2 and W_3 represent the work done in moving a particle from A to B along three different paths 1, 2 and 3 respectively (as shown in the figure) in the gravitational field of a point mass m. Find the correct relation between W_1, W_2 and W_3.



(a) $W_1 > W_2 > W_3$
(b) $W_1 = W_2 = W_3$
(c) $W_1 < W_2 < W_3$
(d) $W_2 > W_1 > W_3$ [IIT 03]

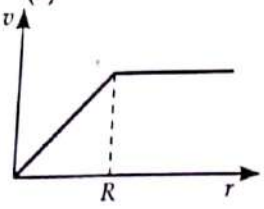
8. If g is the acceleration due to gravity on the earth's surface, the gain in the potential energy of an object of mass m raised from the surface of the earth to a height equal to the radius R of the earth, is
(a) $\frac{1}{2}mgR$ (b) $2mgR$
(c) mgR (d) $\frac{1}{4}mgR$ [IIT 83]

9. An artificial satellite moving in a circular orbit around the earth has total (kinetic + potential) energy E_0. Its potential energy is
(a) $-E_0$ (b) $1.5\,E_0$
(c) $2\,E_0$ (d) E_0 [IIT 97]

10. A spherically symmetric system of particles has a mass density

$$\rho = \begin{cases} \rho_0 & \text{for } r \le R \\ 0 & \text{for } r > R \end{cases}$$

where ρ_0 is a constant. A test mass can undergo circular motion under the influence of the gravitational field of particles. Its speed v as a function of distance r ($0 < r < \infty$) from the centre of the system is represented by

(a)


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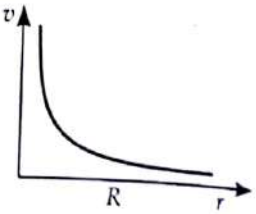
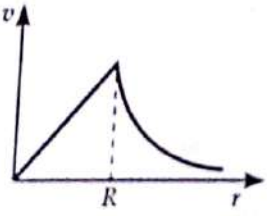
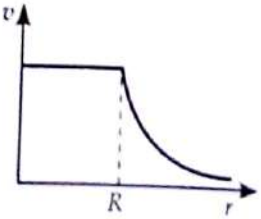


(c)


(d)


[IIT 08]

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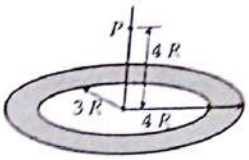
11. A thin uniform annular disc (see figure) of mass M has outer radius $4R$ and inner radius $3R$. The work required to take a unit mass from point P on its axis to infinity is



(a) $\frac{2GM}{7R}(4\sqrt{2}-5)$ (b) $-\frac{2GM}{7R}(4\sqrt{2}-5)$

(c) $\frac{GM}{4R}$ (d) $\frac{2GM}{5R}(\sqrt{2}-1)$ [IIT 2010]

12. A satellite is moving with a constant speed V in a circular orbit about the earth. An object of mass m is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of its ejection, the kinetic energy of the object is

(a) $\frac{1}{2}mV^2$ (b) mV^2

(c) $\frac{3}{2}mV^2$ (d) $2mV^2$ [IIT 2011]

Multiple Choice Questions with One or More than One Correct Answer

13. Imagine a light planet revolving around a very massive star in a circular orbit of radius R with a period of revolution T. If the gravitational force of attraction between the planet and the star is proportional to $R^{-5/2}$, then

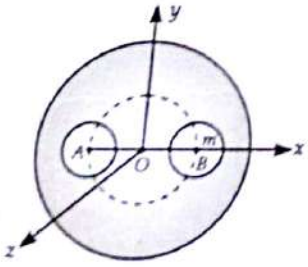
(a) T^2 is proportional to R^3

(b) T^2 is proportional to $R^{7/2}$

(c) T^2 is proportional to $R^{3/2}$

(d) T^2 is proportional to $R^{7/3}$. [IIT 89]

14. A solid sphere of uniform density and radius 4 units is located with its centre at the origin O of coordinates. Two spheres of equal radii 1 unit, with



their centres at $A(-2,0,0)$ and $B(2,0,0)$ respectively, are taken out of the solid leaving behind spherical cavities as shown in the figure. Then

(a) the gravitational force due to this object at the origin is zero

(b) the gravitational force at the point $B(2,0,0)$ is zero

(c) the gravitational potential is the same at all points of circle $y^2+z^2=36$

(d) the gravitational potential is the same at all points of circle $y^2+z^2=4$. [IIT 93]

15. The magnitudes of the gravitational field at distances r_1 and r_2 from the centre of a uniform sphere of radius R and mass m are F_1 and F_2 respectively. Then

(a) $\frac{F_1}{F_2}=\frac{r_1}{r_2}$, if $r_1<R$ and $r_2<R$

(b) $\frac{F_1}{F_2}=\frac{r_2^2}{r_1^2}$, if $r_1>R$ and $r_2>R$

(c) $\frac{F_1}{F_2}=\frac{r_1}{r_2}$, if $r_1>R$ and $r_2>R$

(d) $\frac{F_1}{F_2}=\frac{r_1^2}{r_2^2}$, if $r_1<R$ and $r_2<R$. [IIT 94]

16. A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth.

(a) the acceleration of S is always directed towards the centre of the earth

(b) the angular momentum of S about the centre of the earth changes in direction, but its magnitude remains constant

(c) the total mechanical energy of S varies periodically with time

(d) the linear momentum of S remains constant in magnitude. [IIT 98]

Reasoning Type

Instructions. Each question contains statement – 1 (assertion) and statement – 2 (reason). Of these statements mark correct choice if

(a) statements – 1 and 2 are true and statement – 2 is a correct explanation for statement – 1.

(b) statements – 1 and 2 are true and statement – 2 is not a correct explanation for statement – 1

(c) statement – 1 is true, statement – 2 is false

(d) statement – 1 is false, statement – 2 is true.

17. **Statement 1 :** An astronaut in an orbiting space station above the earth experiences weightlessness.

Statement 2 : An object moving around the earth under the influence of earth's gravitational field is in a state of free-fall. [IIT 08]

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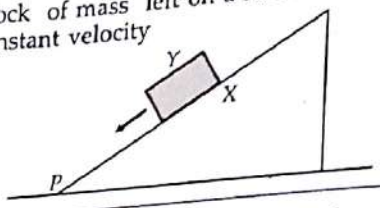
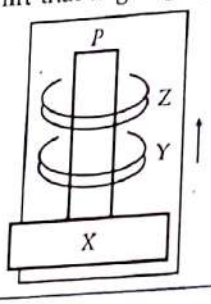
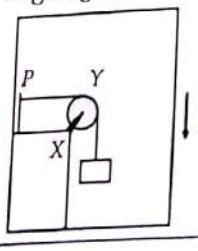
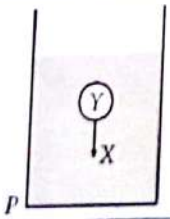
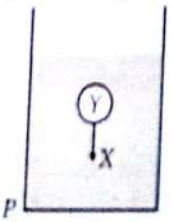
MATCH MATRI TYPE

18. Column II shows five systems in which two objects are labelled as X and Y. Also in each case a point P is shown. Column I gives some statements about X and/or Y. Match these statements to the appropriate system(s) from Column II.

Column I	Column II
(a) The force exerted by on has a magnitude	(p) Block of mass left on a fixed inclined plane , slides on it with a constant velocity 
(b) The gravitational energy of X is continuously increasing	(q) Two ring magnets Y and Z, each of mass M, are kept in frictionless vertical plastic stand so that they repel each other. Y rests on the base X and Z hangs in air in equilibrium. P is the topmost point of the stand on the common axis of the two rings. The whole system is in a lift that is going up with a constant velocity. 
(c) Mechanical energy of the system $X+Y$ is continuously decreasing	(r) A pulley Y of mass m_0 is fixed to a table through a clamp X, a block of mass M hangs from a string that goes over the pulley and is fixed at point P of the table. The whole system down is kept in a lift that is going with a constant velocity 
(d) The torque of the weight of Y about point P is zero	(s) A sphere Y of mass M is put in a nonviscous liquid X kept in a container at rest. The sphere is released and it moves down in the liquid. 
	(t) A sphere Y of mass M is falling with its terminal velocity in a viscous liquid X kept in a container. 

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INTEGER ANSWER TYPE

19. Gravitational acceleration on the surface of a planet is $\frac{\sqrt{6}}{11}g$, where g is the gravitational acceleration on the surface of the earth. The average mass density of the planet is $\frac{2}{3}$ times that of the earth. If the escape speed on the surface of the earth is taken to be 11kms^{-1}, find the escape speed on the surface of the planet in kms^{-1}. [IIT 2010]

Answers and Explanations

1. (c) Acceleration due to gravity on earth's surface

$$g = \frac{GM}{R^2}$$

If R decreases, then g increases.

Hence option (c) is correct.

2. (d) On earth surface, $g_1 = \frac{GM}{R}$

At a height equal to R, $g_2 = \frac{GM}{(2R)^2} = \frac{g_1}{4}$

As $T = 2\pi\sqrt{\frac{l}{g}}$ $\quad\therefore\quad$ $\frac{T_2}{T_1} = \sqrt{\frac{g_1}{g_2}} = \sqrt{\frac{g_1}{g_1/4}} = 2.$

3. (b) According to Kepler's law of periods,

$$T^2 \propto R^3$$

$$\therefore \quad \left(\frac{T_2}{T_1}\right)^2 = \left(\frac{R_2}{R_1}\right)^3 = \left(\frac{R_1/2}{R_1}\right)^3 = \frac{1}{8}$$

$$T_2 = \frac{1}{2\sqrt{2}} \times T_1 = 0.353 \times 365 \text{ days} = \mathbf{129 \text{ days}}.$$

4. (c) For a geostationary satellite,

$$T_1 = 24\,\text{h}, \quad R_1 = 36000 \text{ km}$$

For a satellite on earth's surface,

$$T_2 = ?, \quad R_2 \approx 6400 \text{ km}$$

$$\frac{T_2}{T_1} = \left(\frac{6400}{36000}\right)^3 = \left(\frac{8}{45}\right)^3 = \frac{1}{178}$$

$$T_2 = \frac{T_1}{\sqrt{178}} = \frac{24}{13.34} = 1.8 \text{ h}.$$

For a spying satellite slightly above earth's surface,

$$T_2 = \mathbf{2\ h}.$$

5. (d) The two stars of a binary star system have the same angular velocity.

$$\therefore \quad \omega = \frac{2\pi}{T_A} = \frac{2\pi}{T_B}$$

Hence $T_A = T_B$.

6. (a) For a geostationary satellite,

$$T_1 = 24 \text{ h}, \quad R_1 = 6R + R = 7R$$

For another satellite,

$$T_2 = ?, \quad R_2 = 2.5R + R = 3.5R$$

$$\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{R_2}{R_1}\right)^3 = \left(\frac{3.5R}{7R}\right)^3 = \frac{1}{8}$$

$$T_2 = \frac{1}{2\sqrt{2}} T_1 = \frac{1}{2\sqrt{2}} \times 24$$

$$= \mathbf{6\sqrt{2}\ h}.$$

7. (b) Gravitational field is a conservative field. Work done in taking a body from one position to another along different paths in such a field is same.

8. (a) On earth's surface, $U_1 = -\frac{GMm}{R}$

At a height equal to radius of the earth,

$$U_2 = -\frac{GMm}{R+r} = -\frac{GMm}{2R}$$

$$\Delta U = U_2 - U_1 = -\frac{GMm}{2R} + \frac{GMm}{R}$$

$$= \frac{GMm}{2R}$$

But $\quad g = \frac{GM}{R^2}$

$$\therefore \quad \Delta U = \frac{gR^2 \times m}{2R} = \frac{\boldsymbol{mgR}}{\mathbf{2}}.$$

9. (c) P.E. of a satellite

$$= 2 \times \text{Total energy} = \mathbf{2}E_0.$$

10. (c) For $r < R$,

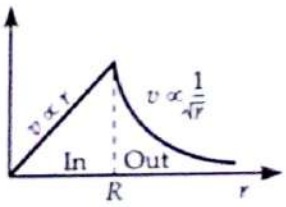
$$F = \frac{GMm}{R^3} r = \frac{mv^2}{r}$$

$$\therefore \quad v^2 \propto r^2 \quad \text{or} \quad v \propto r$$

For $\quad r > R$

$$F = \frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$\therefore \quad v \propto \frac{1}{\sqrt{r}}$$



Hence the correct option is (c).

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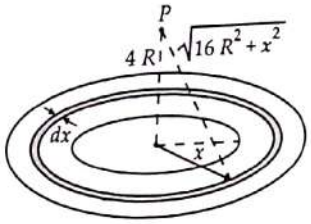
11. (*a*) The mass of annular ring of thickness dx and radius x is

$$dM = \frac{M}{\pi(16R^2 - 9R^2)} \times 2\pi x\, dx$$

$$= \frac{2M}{7R^2} x\, dx$$



Gravitational potential due to the disc at the point P,

$$V_P = \int_{3R}^{4R} \frac{-GdM}{\sqrt{16R^2 + x^2}}$$

$$= -\frac{GM}{7R^2} \int_{3R}^{4R} \frac{2x\, dx}{\sqrt{16R^2 + x^2}}$$

$$= -\frac{GM}{7R^2} \left[\frac{\sqrt{16R^2 + x^2}}{1/2} \right]_{3R}^{4R}$$

$$= -\frac{2GM}{7R^2}(4\sqrt{2}R - 5R)$$

$$= -\frac{2GM}{7R^2}(4\sqrt{2} - 5)$$

Work done in moving a unit mass from P to ∞,

$$W = V_\infty - V_P = 0 - \left(\frac{-2GM}{7R}(4\sqrt{2} - 5) \right)$$

$$= \frac{2GM}{7R}(4\sqrt{2} - 5)$$

12. (*b*) $v_e = \sqrt{2}\, V$

K.E. of the object at the time of ejection,

$$\text{K.E.} = \frac{1}{2} m v_e^2 = \frac{1}{2} m(\sqrt{2}V)^2 = mV^2$$

13. (*b*) For motion of a planet in a circular orbit,

Gravitational force = Centripetal force

$$\frac{GMm}{R^{5/2}} = mR\omega^2 = mR\left(\frac{2\pi}{T}\right)^2$$

or $$T^2 = \frac{4\pi^2}{GM} \cdot R^{7/2}$$

$$\therefore \quad T^2 \propto R^{7/2}.$$

14. (*a*), (*c*), (*d*) The gravitational field is zero at the centre of a solid sphere. The small spheres removed from the big sphere can be considered as negative masses m each located at A and B. The gravitational fields due to these masses at O will be equal and opposite. Thus the resultant force on a mass at O will be zero. Hence option (*a*) is correct.

Option (*b*) is wrong in view of the above discussion.

Now, $y^2 + z^2 = 36$

represents a circle of radius 6 units with centre (0, 0, 0) and the plane of the circle is perpendicular to x-axis. For points outside the sphere, the mass of the sphere can be assumed to be concentrated at the centre. All points of the given circle are equidistant from O, so gravitational potential is same at all points of this circle. Hence option (*c*) is correct.

The above logic also holds for the circle $y^2 + z^2 = 4$ which just touches the sphere. Hence option (*d*) is also correct.

15. (*a*), (*b*) For $r > R$, the gravitational field due to a sphere of radius R,

$$F = \frac{GM}{r^2} \quad i.e., \quad F \propto \frac{1}{r^2}$$

$$\therefore \quad \frac{F_1}{F_2} = \frac{r_2^2}{r_1^2}$$

Hence option (*b*) is correct.

For $r < R$, the gravitational field is

$$F = \frac{GM}{R^3} \times r \quad i.e., \quad F \propto r$$

$$\therefore \quad \frac{F_1}{F_2} = \frac{r_1}{r_2}$$

Hence option (*a*) is correct.

16. (*a*), (*c*) Force on a satellite acts towards the earth, so the acceleration of the satellite S is always directed towards the centre of the earth. Hence option (*a*) is correct.

Net torque due to the gravitational force about centre of the earth is zero. So angular momentum of S remains constant. Hence option (*b*) is incorrect.

As the gravitational force is conservative in nature, total mechanical energy of S remains constant. Speed of S is maximum when it is nearest to earth and minimum when it is farthest. Thus the mechanical energy of S varies periodically with time. Hence option (*c*) is correct.

As the magnitude of velocity of S varies along the elliptical orbit, the linear momentum of S does not remain constant in magnitude. Hence option (*d*) is incorrect.

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17. (*a*) Both statements are correct and statement – 2 is a correct explanation of statement – 1.

18. $a(p, t)$; $b(q, s, t)$; $c(p, r, t)$; $d(q)$

(p) As block Y moves with a constant velocity, force exerted by X may will balance its weight Mg ($p \to a$).

Work is done against the force of friction, mechanical energy of the system decreases ($p \to c$).

(q) As lift is moving upwards, the gravitational P.E. of X is continuously increasing ($q \to b$).

Line of action of Y passes through P, hence torque is zero ($q \to d$).

(r) As the system $X+Y$ comes down with a constant velocity, its P.E. is decreasing but kinetic energy is constant and hence mechanical energy is continuously decreasing ($r \to c$).

(s) As the sphere Y moves down, the centre of mass of liquid X rises up, the gravitational P.E. of X increases ($s \to b$).

(t) As the sphere Y moves down with a constant velocity, the force exerted by the liquid X will balance its weight Mg ($t \to a$). The gravitational P.E. of liquid X rises due to rise of its CM ($t \to b$). The mechanical energy of the system $X+Y$ decreases as work is done against viscous force ($t \to c$).

19.

0	0	0	3

$$g_p = \frac{GM_p}{R_p^2} = \frac{4}{3} G\pi R_p \rho_p$$

$$\Rightarrow \quad \frac{g_p}{g_e} = \frac{R_p \rho_p}{R_e \rho_e}$$

Also, $v_e = \sqrt{2gR}$

$$\Rightarrow \quad \frac{v_p}{v_e} = \sqrt{\frac{g_p R_p}{g_e R_e}} = \left(\frac{g_p}{g_e}\right)\sqrt{\frac{\rho_e}{\rho_p}} = \frac{\sqrt{6}}{11} \times \sqrt{\frac{3}{2}}$$

$$\Rightarrow \quad v_p = \mathbf{3\ km/s}$$

AIEEE

1. If suddenly the gravitational force of attraction between earth and a satellite revolving around it becomes zero, then the satellite will

(a) continue to move in its orbit with same velocity

(b) move tangentially to the original orbit with the same velocity

(c) become stationary in its orbit

(d) move towards the earth. [AIEEE 02]

2. Average density of the earth

(a) does not depend on g

(b) is a complex function of g

(c) is directly proportional to g

(d) is inversely proportional to g [AIEEE 05]

3. The change in the value of g at a height h above the surface of the earth is the same as at a depth d below the surface of earth. When both d and h are much smaller than the radius of earth, then which one of the following is correct ?

(a) $d = \frac{h}{2}$ (b) $d = \frac{3h}{2}$

(c) $d = 2h$ (d) $d = h$ [AIEEE 05]

4. If g is the acceleration due to gravity on the earth's surface, the gain in the potential energy of an object of mass m raised from the surface of the earth to a height equal to the radius R of the earth, is

(a) $\frac{1}{4}mgr$ (b) $\frac{1}{2}mgr$

(c) $2\,mgr$ (d) mgr [AIEEE 04]

5. Energy required to move a body of mass m from an orbit of radius $2R$ to $3R$ is

(a) $GMm/12R^2$ (b) $GMm/3R^2$

(c) $GMm/8R$ (d) $GMm/6R$ [AIEEE 02]

6. A particle of mass 10 g is kept on the surface of a uniform sphere of mass 100 kg and radius 10 cm. Find the work to be done against the gravitational force between them to take the particle far away from the sphere. You may take $G = 6.67 \times 10^{-11}\ \mathrm{Nm^2kg^{-2}}$

(a) 13.34×10^{-10} J (b) 3.33×10^{-10} J

(c) 6.67×10^{-9} J (d) 6.67×10^{-10} J [AIEEE 05]

7. The kinetic energy needed to project a body of mass m from the earth's surface (radius R) to infinity is

(a) $mgR/2$ (b) $2\,mgR$

(c) mgR (d) $mgR/4$ [AIEEE 02]

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8. The escape velocity of a body depends upon mass as

(a) m^0 (b) m

(c) m^2 (d) m^3 [AIEEE 02]

9. The escape velocity for a body projected vertically upwards from the surface of earth is 11 km s^{-1}. If the body is projected at an angle 45° with the vertical, the escape velocity will be

(a) $11/\sqrt{2}$ km s^{-1} (b) 11 km s^{-1}

(c) $11\sqrt{2}$ km s^{-1} (d) 22 km s^{-1} [AIEEE 03]

10. A planet in a distant solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that the escape velocity from the earth is 11 km s^{-1}, the escape velocity from the surface of the planet would be

(a) 0.11 km s^{-1} (b) 1.1 km s^{-1}

(c) 11 km s^{-1} (d) 110 km s^{-1} [AIEEE 08]

11. A satellite of mass m revolves around the earth of radius R at a height x from its surface. If g is the acceleration due to gravity on the surface of the earth, the orbital speed of the satellite is

(a) gx (b) $\frac{gR}{R-x}$

(c) $\frac{gR^2}{R+x}$ (d) $\left(\frac{gR^2}{R+x}\right)^{1/2}$ [AIEEE 04]

12. The time period of an earth satellite in circular orbit is independent of

(a) the mass of the satellite

(b) radius of its orbit

(c) both the mass and radius of the orbit

(d) neither the mass of the satellite nor the radius of its orbit. [AIEEE 04]

13. The time period of a satellite of earth is 5 hours. If the separation between the earth and the satellite is increased to 4 times the previous value, the new time period will become

(a) 10 hours (b) 80 hours

(c) 40 hours (d) 20 hours [AIEEE 03]

14. Suppose that the gravitational force varies inversely as the nth power of distance. Then, the time period of a planet in circular orbit of radius R around the sun will be proportional to

(a) $R^{(n+1)/2}$ (b) $R^{(n-1)/2}$

(c) R^n (d) $R^{(n-2)/2}$ [AIEEE 04]

15. Two spherical bodies of mass M and $5M$ and radii R and $2R$ respectively are released in free space with initial separation between their centres equal to $12R$. If they attract each other due to gravitational force only, then the distance covered by the smaller body just before collision is

(a) 4.5 R (b) 7.5 R

(c) 1.5 R (d) 2.5 R [AIEEE 03]

16. The height at which acceleration due to gravity becomes $g/9$ (where g = the acceleration to gravity on the surface of the earth) in terms of R, the radius of the earth is

(a) $2R$ (b) $\frac{R}{\sqrt{2}}$

(c) $\frac{R}{2}$ (d) $\sqrt{2}R$ [AIEEE 09]

17. Two bodies of masses m and $4m$ are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is

(a) zero (b) $-\frac{4Gm}{r}$

(c) $-\frac{6Gm}{r}$ (d) $-\frac{9Gm}{r}$ [AIEEE 2011]

18. **Directions.** The following question contains :

Statement – 1 and statement – 2. Of the four choices given, choose the one that best describes the two statements.

(a) Statement – 1 is true, statement – 2 is false.

(b) Statement – 1 is false, statement – 2 is true.

(c) Statement – 1 is true, statement – 2 is true. Statement – 2 is a correct explanation for statement – 1.

(d) Statement – 1 is true, statement – 2 is true. Statement – 2 is not a correct explanation for statement – 1.

Statement 1 : For a mass M kept at the centre of a cube of side a, the flux of gravitational field passing through its sides is $4\pi GM$.

Statement 2 : If the direction of a field due to a point source is radial and its dependence on the distance r from the source is given as $1/r^2$, its flux through a closed surface depends only on the strength of the source enclosed by the surface and not on the size or shape of the surface. [AIEEE 08]

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Answers and Explanations

1. (*b*) If the gravitational force suddenly becomes zero, the satellite will move tangentially to the original orbit with the velocity, it was revolving around the earth.

2. (*c*) Acceleration due to gravity on earth's surface

$$g = \frac{GM}{R^2} = \frac{G}{R^2} \times \frac{4}{3}\pi R^3 \rho = \frac{4\pi GR\rho}{3}$$

or $$\rho = \frac{3g}{4\pi GR} \quad \therefore \quad \rho \propto g.$$

3. (*c*) $g_h = g_d$

$$g\left(1-\frac{2h}{R}\right) = g\left(1-\frac{d}{R}\right) \quad \therefore \quad d = 2h.$$

4. (*b*) Refer to the solution of Problem 8 on page 8.75.

5. (*d*) Gravitational P.E. of mass m in an orbit of radius R,

$$U = -\frac{GMm}{R}$$

In an orbit of radius $2R$, $U_i = -\frac{GMm}{2R}$

In an orbit of radius $3R$, $U_f = -\frac{GMm}{3R}$

$$\therefore \quad \Delta U = U_f - U_i = \frac{GMm}{R}\left(\frac{1}{2}-\frac{1}{3}\right) = \frac{GMm}{6R}.$$

6. (*d*) Here, $M = 100$ kg, $m = 10$ g $= 10^{-2}$ kg

$R = 10$ cm $= 0.01$ m

Initial P.E. of the two bodies,

$$U_i = -\frac{GMm}{R} = -\frac{6.67\times10^{-11}\times100\times10^{-2}}{0.01}$$

$$= -6.67\times10^{-9}\ \text{J}$$

When the particle is far away from the sphere, the P.E. of the system will be zero.

$$U_f = 0$$

$$\therefore \quad W = U_f - U_i = +\mathbf{6.67\times10^{-9}\ J}.$$

7. (*c*) As in the above problem,

$$U_i = -\frac{GMm}{R}$$

$$U_f = U_\infty = 0$$

K.E. needed $= U_f - U_i$

$$= \frac{GMm}{R} = \frac{gR^2.m}{R} = \mathbf{mgR}. \qquad [GM = gR^2]$$

8. (*a*) $$v_e = \sqrt{2gR} = \sqrt{\frac{2GM_e}{R}}$$

Clearly, the escape velocity of a body does not depend on its mass m

9. (*b*) The escape velocity of a body does not depend on the angle of projection from the earth's surface. Therefore, $v_e = 11$ km s^{-1}.

10. (*d*) v_e (earth) $= \sqrt{\frac{2GM}{R}} = 11$ km s^{-1}

$$v_e(\text{planet}) = \sqrt{\frac{2G\times10M}{R/10}} = 10\sqrt{\frac{2GM}{R}}$$

$$= 10\times11 = \mathbf{110\ km\ s^{-1}}.$$

11. (*d*) Centripetal force on a satellite

= Gravitational force

$$\frac{mv_0^2}{R+x} = \frac{GMm}{(R+x)^2}$$

$$v_0 = \sqrt{\frac{GM}{R+x}} = \sqrt{\frac{gR^2}{R+x}}.$$

12. (*a*) $$T = \frac{2\pi(R+h)}{v_0} = 2\pi(R+h)\sqrt{\frac{R+h}{GM}} = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$$

Clearly, the time period of a satellite is independent of its mass m.

13. (*c*) According to Kepler's law of periods,

$$T^2 \propto r^3$$

$$\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{r_2}{r_1}\right)^3 = \left(\frac{4}{1}\right)^3 = 64$$

$$T_2 = 8T_1 = 8\times5 = \mathbf{40\ h}.$$

14. (*a*) Refer to the solution of Problem 6 on page 8.53.

15. (*b*) Gravitational force between the two spheres,

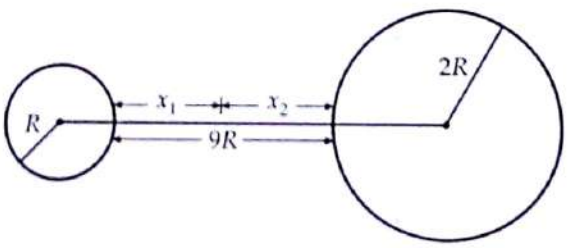
$$F = \frac{GM\times5M}{(12R-x)^2}$$

Acceleration of smaller body,

$$a_1 = \frac{F}{M} = \frac{G\times5M}{(12R-x)^2}$$

Acceleration of larger body,

$$a_2 = \frac{F}{5M} = \frac{GM}{(12R-x)^2}$$



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Distances covered by two bodies in time t,

$$x_1 = \frac{1}{2}a_1 t^2 \quad \text{and} \quad x_2 = \frac{1}{2}a_2 t^2$$

$$\therefore \quad \frac{x_1}{x_2} = \frac{a_1}{a_2} = 5 \quad \Rightarrow \quad x_1 = 5x_2$$

But $x_1 + x_2 = 9R$

$$x_1 + \frac{x_1}{5} = 9R \quad \text{or} \quad x_1 = \frac{45R}{6} = 7.5\ R.$$

16. *(a)* On the earth's surface,

$$g = \frac{GM}{R^2}$$

At height h above earth's surface,

$$g_h = \frac{GM}{(R+h)^2}$$

or $$\frac{g}{9} = \frac{GM}{R^2} \cdot \frac{R^2}{(R+h)^2}$$

or $$\frac{g}{9} = g \cdot \frac{R^2}{(R+h)^2}$$

or $$\frac{R+h}{R} = 3$$

or $$h = 2R.$$

17. *(d)* Position of the null point from mass m,

$$x = \frac{r}{1+\sqrt{\frac{4m}{m}}} = \frac{r}{3}$$

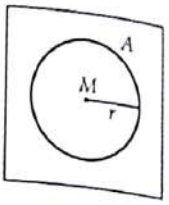
The gravitational potential at the null point,

$$v = -\frac{Gm}{r/3} - \frac{G \cdot 4m}{2r/3} = -\frac{9Gm}{r}$$

18. *(c)* Let A be the Gaussian surface enclosing a point mass M.

Gravitational field, $E = -\dfrac{GM}{r^2}$

Flux, $\phi_g = |E \cdot 4\pi r^2| = 4\pi GM$



Every line passing through A has to pass through B, whether B is a cube or not. It is only for the Gaussian surface, the lines of field should be normal, not necessarily the cube. Hence statement – 1 is correct.

Statement – 2 is also correct when the shape of the earth is spherical, area of the Gaussian surface is $4\pi r^2$. This ensures inverse square law.

DCE and G.G.S. Indraprastha University Engineering Entrance Exam

(Similar Questions)

1. Two balls, each of radius R, equal mass and density are placed in contact, then the force of gravitation between them is proportional to

(a) $F \propto \dfrac{1}{R^2}$ (b) $F \propto R$

(c) $F \propto R^4$ (d) $F \propto \dfrac{1}{R}$ [DCE 07]

2. Acceleration due to gravity g for a body of mass m on earth's surface is proportional (Radius of earth = R, mass of earth = M) to

(a) M/R^2 (b) m

(c) mM (d) $1/R^{3/2}$ [DCE 04]

3. If the density of earth is doubled keeping its radius constant, then acceleration due to gravity g is

(a) 20 m/s^2 (b) 10 m/s^2

(c) 5 m/s^2 (d) 2.5 m/s^2 [IPUEE 04]

4. If the radius of earth decreases by 1% and its mass remains same, then the acceleration due to gravity

(a) increases by 1% (b) decreases by 1%

(c) increases by 2% (d) increases by 2% [DCE 04]

5. If the earth stops rotating, the value of g at the equator

(a) increases (b) decreases

(c) no effect (d) none of these. [DCE 07]

6. Assuming earth to be a sphere of radius R, if $g_{30°}$ is value of acceleration due to gravity at latitude of 30° and g at the equator, the value of $g - g_{30°}$ is

(a) $\frac{1}{4}\omega^2 R$ (b) $\frac{3}{4}\omega^2 R$

(c) $\omega^2 R$ (d) $\frac{1}{2}m\omega R$ [DCE 05]

7. Three particles each of mass m are kept at vertices of an equilateral triangle of side L. The gravitational field at centre due to these particles is

(a) zero (b) $\dfrac{3GM}{L^2}$

(c) $\dfrac{9GM}{L^2}$ (d) $\dfrac{12}{\sqrt{3}}\dfrac{GM}{L^2}$ [DCE 05]

8. When a body is lifted from surface of earth to a height equal to radius of earth, then the change in its potential energy is

(a) mgR (b) $2mgR$

(c) $\frac{1}{2}mgR$ (d) $4mgR$ [DCE 97, 98]

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9. If M is the mass of the earth and R its radius, the ratio of the gravitational acceleration and the gravitational constant is

(a) $\frac{R^2}{M}$ (b) $\frac{M}{R^2}$

(c) MR^2 (d) $\frac{M}{R}$ [IPUEE 06]

10. A particle falls towards earth from infinity. Its velocity on reaching the earth would be

(a) infinity (b) $\sqrt{2gR}$

(c) $2\sqrt{gR}$ (d) zero. [IPUEE 05]

11. The escape velocity for a body of mass 1 kg from the earth's surface is 11.2 kms^{-1}. The escape velocity for a body of mass 100 kg would be

(a) 11.2×10^2 kms^{-1} (b) 112 kms^{-1}

(c) 11.2 kms^{-1} (d) 11.2×10^{-2} kms^{-1} [DCE 03]

12. Escape velocity on earth is 11.2 km/s. What would be the escape velocity on a planet whose mass is 1000 times and radius is 10 times that of earth ?

(a) 112 km/s (b) 11.2 km/s

(c) 1.12 km/s (d) 3.7 km/s [DCE 01]

13. A satellite revolves very near to the earth surface. Its speed should be around

(a) 5 km/s (b) 8 km/s

(c) 2 km/s (d) 11 km/s [IPUEE 04]

14. The value of escape velocity on a certain planet is 2 km/s. Then the value of orbital speed for a satellite orbiting close to its surface is

(a) 12 km/s (b) 1 km/s

(c) $\sqrt{2}$ km/s (d) $2\sqrt{2}$ km/s [DCE 05]

15. The escape velocity from earth is v_{es}. A body is projected with velocity $2v_{es}$. With what constant velocity will it move in the inter planetary space ?

(a) v_{es} (b) $3v_{es}$

(c) $\sqrt{3}v_{es}$ (d) $\sqrt{5}v_{es}$ [DCE 02]

16. What is not conserved in the case of celestial bodies revolving around sun ?

(a) kinetic energy (b) mass

(c) angular momentum

(d) linear momentum. [DCE 97]

17. A body is projected from earth's surface to become its satellite. Its time period of revolution will not depend upon

(a) mass of earth (b) its own mass

(c) gravitational constant

(d) radius of earth. [DCE 97]

18. If gravitational constant is decreasing in time, what will remain unchanged in case of a satellite orbiting around earth ?

(a) time period (b) orbiting radius

(c) areal velocity (d) angular velocity [DCE 99]

19. Satellite is revolving around earth. If its height is increased to four times the height of geostationary satellite, what will become its time period ?

(a) 8 days (b) 4 days

(c) 2 days (d) 16 days [DCE 97]

20. Two satellites A and B have ratio of masses 3 : 1 in circular orbits of radii r and $4r$. The ratio of total mechanical energy of A to B is

(a) 1 : 3 (b) 3 : 1

(c) 3 : 4 (d) 12 : 1 [DCE 02]

21. A body is orbiting around earth at a mean radius which is two times as greater as the parking orbit of a geostationary satellite, the period of body is

(a) 4 days (b) $2\sqrt{2}$ days

(c) 16 days (d) 64 days [DCE 99]

22. A planet moves around the sun. It is closest to sun at a distance d_1 and has velocity v_1 and farthest with distance d_2, its speed at this point will be

(a) $\frac{d_1^2 v_1}{d_2^2}$ (b) $\frac{d_2 v_1}{d_1}$

(c) $\frac{d_1 v_1}{d_2}$ (d) $\frac{d_2^2 v_1}{d_1^2}$ [DCE 02]

23. A satellite moves in elliptical orbit about a planet. The maximum and minimum velocities of satellites are 3×10^4 m/s and 1×10^3 m/s respectively. What is the minimum distance of satellite from planet if maximum distance is 4×10^4 km ?

(a) 4×10^3 km (b) 3×10^3 km

(c) $\frac{4}{3} \times 10^3$ km (d) 1×10^3 km [DCE 06]

24. Kepler discovered

(a) laws of motion (b) laws of rotational motion

(c) laws of planetary motion

(d) laws of curvilinear motion [DCE 2K]

25. Which of the following quantities does not depend upon the orbital radius of the satellite ?

(a) $\frac{T}{R}$ (b) $\frac{T^2}{R}$

(c) $\frac{T^2}{R^2}$ (d) $\frac{T^2}{R^3}$ [DCE 2K]

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26. A satellite is orbiting around the earth with orbital radius R and time period T. The quantity which remains constant is

(a) T/R (b) T^2/R

(c) T^2/R^2 (d) T^2/R^3 [DCE 03]

27. According to Kepler's law, the time period of a satellite varies with its radius as

(a) $T^2 \propto R^3$ (b) $T^3 \propto R^2$

(c) $T^2 \propto (1/R^3)$ (d) $T^3 \propto (1/R^2)$ [IPUEE 05]

28. When the distance between earth and sun is halved, the duration of year will become

(a) more

(b) less

(c) can't be determined

(d) none of the above. [DCE 07]

29. A satellite in a circular orbit of radius R has a period of 4 h. Another satellite with orbital radius $3R$ round the same planet will have a period (in hours)

(a) 16 (b) 4

(c) $4\sqrt{27}$ (d) $4\sqrt{8}$ [IPUEE 07]

30. If a new planet is discovered rotating around the sun with the orbital radius double that of earth, then what will be its time period (in earth's days) ?

(a) 1032 (b) 1023

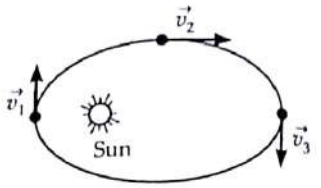
(c) 1024 (d) 1043 [DCE 04]

31. The period of a planet around sun is 27 times that of earth. The ratio of radius of planet's orbit to the radius of earth's orbit is

(a) 4 (b) 9

(c) 64 (d) 27 [DCE 05]

32. Figure shows the velocity of a planet revolving around the sun at three times of a year. Let v be the speed of the planet when its velocity is $\vec{v}$.



Which of the following alternatives is correct ?

(a) $v_2 = 2v_1$ and $v_3 = 3v_1$

(b) $v_3 > v_2 > v_1$

(c) $v_2 = \dfrac{v_1 + v_3}{2}$

(d) $v_1 > v_2 > v_3$ [DCE 08]

33. When a body is taken from poles to equator on the earth, its weight

(a) increases

(b) decreases

(c) remains the same

(d) increases at south pole and decreases at north pole. [DCE 09]

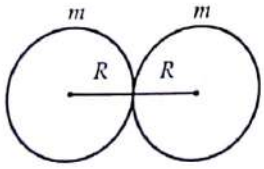
34. The escape velocity of a 10 g body from the earth is 11.2 kms^{-1}. Ignoring air resistances, the escape velocity of 10 kg of the iron ball from the earth will be

(a) 0.0112 kms^{-1} (b) 0.112 kms^{-1}

(c) 11.2 kms^{-1} (d) 0.56 kms^{-1} [DCE 09]

Answers and Explanations

1. (c)



$$F = G\frac{m \times m}{(2R)^2} = G\frac{\left(\frac{4}{3}\pi R^3 \rho\right)^2}{4R^2}$$

$\therefore \quad F \propto R^4$

2. (a) $g = \dfrac{GM}{R^2} \quad \therefore \quad g \propto \dfrac{M}{R^2}$

3. (d) $g = \dfrac{4}{3}\pi G R\rho$ *i.e.*, $g \propto \rho$

If density is doubled, then the value of g also gets doubled.

4. (c) $g = \dfrac{GM}{R^2}$. For constant G and M,

$$\frac{\Delta g}{g} \times 100 = -2\frac{\Delta R}{R} \times 100 = -2(-1)\,\% = +2\%$$

The value of g increases by 2%.

5. (a) At the equator, $g_e = g - R\omega^2$

When $\omega = 0$, $g_e = g$

The value of g increases if the earth stops rotating.

6. (b) $g_{30^\circ} = g - R\omega^2 \cos^2 30^\circ = g - R\omega^2 \times \left(\dfrac{\sqrt{3}}{2}\right)^2$

$\therefore \quad g - g_{30^\circ} = \dfrac{3}{4}\omega^2 R.$

7. (a) Refer to the solution of Example 32 on page 8.24.

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8. (c) At the surface of the earth,

$$U_i = -\frac{GMm}{R}$$

At a height equal to the radius of the earth,

$$U_f = -\frac{GMm}{R+R} = -\frac{GMm}{2R}$$

$$\Delta U = U_f - U_i = \frac{GMm}{R}\left(1-\frac{1}{2}\right)$$

$$= \frac{gR^2M}{R} \times \frac{1}{2} \quad [GM = gR^2] = \frac{1}{2}mgR.$$

9. (b) $g = \frac{GM}{R^2} \quad \therefore \quad \frac{g}{G} = \frac{M}{R^2}$

10. (b) $\frac{1}{2}mv^2 = U_i - U_f = 0 - \left(-\frac{GMm}{R}\right)$

$$\frac{1}{2}mv^2 = \frac{gR^2m}{R} \qquad [GM = gR^2]$$

$$v = \sqrt{2gR}.$$

11. (c) The escape velocity does not depend on the mass of the body projected.

12. (a) $v_e = \sqrt{\frac{2GM}{R}} = 11.2 \text{ km s}^{-1}$

$$v_e' = \sqrt{\frac{2G \times 1000M}{10R}} = 10\sqrt{\frac{2GM}{R}}$$

$$= 10 \times 11.2 = \mathbf{112 \text{ km s}^{-1}}.$$

13. (b) Near earth's surface,

$$v_0 = \sqrt{gR} = 7.2 \text{ km s}^{-1} \approx \mathbf{8 \text{ km s}^{-1}}.$$

14. (c) Near earth's surface,

$$v_0 = \frac{v_e}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \mathbf{\sqrt{2} \text{ km s}^{-1}}.$$

15. (c) $\frac{1}{2}mv^2 = \frac{1}{2}m(2v_{es})^2 - \frac{1}{2}mv_{es}^2$

$$v^2 = 4v_{es}^2 - v_{es}^2 = 3v_{es}^2$$

$$v = \mathbf{\sqrt{3}\, v_{es}}.$$

16. (a) K.E. changes due to the change in the speed of celestial body around the sun.

17. (b) $T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$, Clearly, T does not depend on the mass m of the satellite.

18. (c) The central gravitational force decreases which does not exert any torque. Hence angular momentum and consequently areal velocity remain unchanged.

19. (a) According to Kepler's law of periods,

$$\frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{3/2} = \left(\frac{4}{1}\right)^{3/2} = 8$$

$$\therefore \quad T_2 = 8T_1 = 8 \times 1 \text{ day} = \mathbf{8 \text{ days}}.$$

20. (d) Total energy of a satellite

$$E = -\frac{GMm}{r}$$

$$\frac{E_A}{E_B} = \frac{m_A}{m_B} \cdot \frac{r_B}{r_A} = \frac{3}{1} \times \frac{4r}{r} = 12:1$$

21. (b) $\frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{3/2} = \left(\frac{2}{1}\right)^{3/2} = 2\sqrt{2}$

$$T_2 = 2\sqrt{2}T_1 = 2\sqrt{2} \times 1 \text{ day} = 2\sqrt{2} \text{ days}.$$

22. (c) By conservation of angular momentum,

$$L_1 = L_2 \quad \text{or} \quad mv_1d_1 = m_2v_2d_2 \quad \therefore \quad v_2 = \frac{v_1d_1}{d_2}.$$

23. (c) By conservation of angular momentum

$$mv_{max}d_{min} = Mv_{min}d_{max}$$

$$d_{min} = \frac{v_{min}d_{max}}{v_{max}} = \frac{1 \times 10^3 \times 4 \times 10^4}{3 \times 10^4} = \frac{4}{3} \times 10^3 \text{ km}.$$

24. (c) Kepler discovered the laws of planetary motion.

25. (d) As $T^2 \propto R^3 \quad \therefore \quad \frac{T^2}{R^3} = \text{constant}$

26. (d) Same reason as in the above problem.

27. (a) According to Kepler's law of periods,

$$T^2 \propto R^3.$$

28. (b) By conservation of angular momentum,

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{3/2} = \left(\frac{P_1/2}{P_1}\right)^{3/2} = \frac{1}{2\sqrt{2}}$$

$$T_2 = \frac{1}{2\sqrt{2}}T_1 = \frac{1}{2\sqrt{2}} \times 1 \text{ year} = \frac{1}{2\sqrt{2}} \text{ year}$$

Hence the duration of the year will become less.

29. (c) $\frac{T_2}{T_1} = \left(\frac{3R}{R}\right)^{3/2} = \sqrt{27}$

$$T_2 = \sqrt{27}T_1 = \mathbf{4\sqrt{27} \text{ h}}.$$

30. (a) $\frac{T_2}{T_1} = \left(\frac{2r}{r}\right)^{3/2} = 2\sqrt{2}$

$$T_2 = 2\sqrt{2}T_1 = 2\sqrt{2} \times 365 \text{ days} = \mathbf{1032 \text{ days}}.$$

31. (b) $\frac{r_p}{r_e} = \left(\frac{T_p}{T_e}\right)^{2/3} = \left(\frac{27}{1}\right)^{2/3} = 9.$

32. (d) Linear velocity $\propto \frac{1}{\text{distance from the sun}}$

As $\quad d_1 < d_2 < d_3 \quad \therefore \quad v_1 > v_2 > v_3.$

33. (b) $g_p = g$ and $g_e = g - R\omega^2 < g_p.$

34. (c) Escape velocity is independent of the mass of the body projected provided the air resistance is neglected.

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AIIMS Entrance Exam

1. The force of gravitation is

(*a*) repulsion (*b*) electrostatic

(*c*) conservative (*d*) non-conservative [AIIMS 02]

2. Which of the following is an evidence to show that there must be a force acting on earth and directed toward the sun ?

(*a*) deviation of the falling bodies towards east

(*b*) revolution of the earth round the sun

(*c*) phenomenon of day and night

(*d*) apparent motion of sun round the earth. [AIIMS 80]

3. A man waves his arms, while walking. This is

(*a*) to keep constant velocity

(*b*) to ease the tension

(*c*) to increase the velocity

(*d*) to balance the effect of earth's gravity. [AIIMS 2K]

4. All the known planets move in

(*a*) straight path (*b*) circular path

(*c*) elliptical path (*d*) hyperbolic path [AIIMS 94]

5. Kepler's second law is based on

(*a*) Newton's first law

(*b*) Newton's second law

(*c*) Special theory of relativity

(*d*) conservation of angular momentum. [AIIMS 02]

6. The radius vector, drawn from the sun to a planet sweeps out equal areas in equal times. This is the statement of

(*a*) Kepler's first law (*b*) Kepler's second law

(*c*) Kepler's third law (*d*) Newton's third law [AIIMS 95]

7. The orbital speed of Jupiter is

(*a*) greater than the orbital speed of earth

(*b*) less than the orbital speed of earth

(*c*) equal to the orbital speed of earth

(*d*) proportional to distance from the earth. [AIIMS 94]

8. For a planet moving around the sun in an elliptical orbit of semimajor and semiminor axes a and b respectively and period T,

(*a*) the torque acting on the planet about the sun is non-zero

(*b*) the angular momentum of the planet about the sun is constant

(*c*) the planet moves with a constant speed around the sun

(*d*) the areal velocity is $\pi ab/T$. [AIIMS 2K]

9. If mass of a body is M on the earth surface, then the mass of the same body on the moon surface is

(*a*) $M/6$ (*b*) zero

(*c*) M (*d*) none of these. [AIIMS 97]

10. Two spheres of same size, one of mass 2 kg and another of mass 4 kg are dropped simultaneously from the top of Qutab Minar (height = 72 km). When they are 1 m above the ground, the two spheres have the same

(*a*) momentum (*b*) kinetic energy

(*c*) potential energy (*d*) acceleration [AIIMS 06]

11. Two planets of radii r_1 and r_2 are made from the same material. The ratio of the acceleration of gravity g_1/g_2 at the surfaces of the planets is

(*a*) r_1/r_2 (*b*) r_2/r_1

(*c*) $(r_1/r_2)^2$ (*d*) $(r_2/r_1)^2$ [AIIMS 85]

12. If the radius of earth shrinks by one percent and its mass remaining the same, then acceleration due to gravity on the earth's surface will

(*a*) decrease (*b*) increase

(*c*) remain constant (*d*) either (*a*) or (*c*). [AIIMS 94]

13. At what depth below the surface of the earth, is the value of g same as that of a height of 5 km ?

(*a*) 10 km (*b*) 7.5 km

(*c*) 5 km (*d*) 2.5 km [AIIMS 2K]

14. A body weighed 250 N on the surface. Assuming the earth to be a sphere of uniform mass density, how much would it weigh half way down to the centre of earth ?

(*a*) 240 N (*b*) 210 N

(*c*) 195 N (*d*) 125 N [AIIMS 95]

15. Knowing that mass of moon is $M/81$ (where M is the mass of earth), find the distance of the point, where gravitational field due to earth and moon cancel each other. Given that the distance between the earth and moon is $60\,R$, where R is the radius of earth.

(*a*) $2\,R$ (*b*) $4\,R$

(*c*) $6\,R$ (*d*) $8\,R$ [AIIMS 2K]

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16. The velocity with which a projectile, must be fired so that it escapes earth's gravitation, does not depend on
(a) mass of the earth (b) mass of the projectile
(c) radius of the projectile's orbit
(d) gravitational constant. [AIIMS 03]

17. The angular velocity of rotation of a star (of mass M and radius R) at which the matter starts to escape from its equator, is
(a) $\sqrt{2GM^2/R}$ (b) $\sqrt{2GM/R^3}$
(c) $\sqrt{2GM/R}$ (d) $\sqrt{2GR/M}$ [AIIMS 95]

18. In what manner, does the escape velocity of a particle depend upon its mass ?
(a) m^2 (b) m
(c) m^0 (d) m^{-1} [AIIMS 94]

19. Escape velocity of a body, when projected from the earth's surface is 11.2 km s^{-1}. If it is projected at an angle of 60° with the horizontal, then escape velocity will be
(a) 11.2 km s^{-1} (b) 11.6 km s^{-1}
(c) 12.8 km s^{-1} (d) 16.2 km s^{-1} [AIIMS 99]

20. The mass of moon is 1/81 of earth's mass and its radius ¼ of that of earth. If the escape velocity from the earth's surface is 11.2 km s^{-1}, its value for the moon is
(a) 0.14 km s^{-1} (b) 0.76 km s^{-1}
(c) 2.45 km s^{-1} (d) 5.28 km s^{-1} [AIIMS 2K]

21. The escape velocity from the earth is 11.2 km s^{-1}. The escape velocity from a planet having twice the radius and the same mean density as the earth is
(a) 22.4 km s^{-1} (b) 11.2 km s^{-1}
(c) 5.5 km s^{-1} (d) 15.5 km s^{-1} [AIIMS 01]

22. There is no atmosphere on the moon, because
(a) it is closer to the earth and also it has the inactive inert gases in it.
(b) it is too far from the sun and has very low pressure in its outer surface.
(c) escape velocity of gas molecules is greater than their root mean square velocity.
(d) escape velocity of gas molecules is less than their root mean square velocity. [AIIMS 94]

23. A missile is launched with a velocity less than escape velocity. The sum of its kinetic and potential energies is
(a) zero (b) negative
(c) positive (d) first (a) then (b) [AIIMS 94]

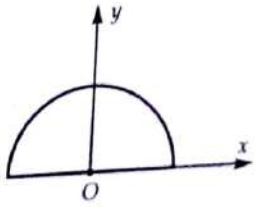
24. A satellite of the earth is revolving in a circular orbit with a uniform speed v. If the gravitational force suddenly disappears, the satellite will
(a) continue to move with velocity v along the original orbit
(b) move with a velocity v tangentially to the original orbit
(c) fall down with increasing velocity
(d) ultimately come to rest, somewhere on the original orbit. [AIIMS 82]

25. Two satellites of masses m_1 and m_2 ($m_1 > m_2$) are going around the earth in orbits of radii r_1 and r_2 ($r_1 > r_2$). Which statement about their velocities is correct ?
(a) $v_1 = v_2$ (b) $v_1 / r_1 = v_2 / r_2$
(c) $v_1 > v_2$ (d) $v_1 < v_2$ [AIIMS 94]

26. If v be the orbital velocity of a satellite in a circular orbit close to the earth's surface and v_e is the escape velocity from the earth, then relation between the two is
(a) $v_e = v$ (b) $v_e = \sqrt{2}v$
(c) $v = \sqrt{3}v_e$ (d) $v_e = 2v$ [AIIMS 02]

27. A satellite is in an orbit around the earth. If its kinetic energy is doubled, then
(a) it will maintain its path
(b) it will fall on the earth
(c) it will rotate with a great speed
(d) it will escape out of earth's gravitational field. [AIIMS 2K]

28. A wire of length l and mass m is bent in the form of a semicircle. The gravitational field intensity at the centre of semicircle is



(a) $\frac{Gm}{\pi l}$ along x-axis (b) $\frac{Gm}{\pi l}$ along y-axis
(c) $\frac{2\pi Gm}{l^2}$ along y-axis (d) $\frac{2\pi Gm}{l^2}$ along x-axis [AIIMS 2009]

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ASSERTIONS AND REASONS

Directions. In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as

(*a*) If both assertion and reason are true and reason is the correct explanation of the assertion.

(*b*) If both assertion and reason are true but reason is not correct explanation of the assertion.

(*c*) If assertion is true, but reason is false.

(*d*) If both assertion and reason are false.

29. **Assertion.** The stars twinkle, while the planets do not.

Reason. The stars are much bigger in size than the planets. [AIIMS 03]

30. **Assertion.** A planet is a heavenly body revolving round the sun.

Reason. Star is a self-luminous body made of gaseous material. [AIIMS 02]

31. **Assertion.** The comets do not obey Kepler's laws of planetary motion.

Reason. The comets do not have elliptical orbits. [AIIMS 95]

32. **Assertion.** The square of the period of revolution of a planet is proportional to the cube of its distance from the sun.

Reason. Sun's gravitational field is inversely proportional to the square of its distance from the planet. [AIIMS 96]

33. **Assertion.** The earth is slowing down and as a result, the moon is coming nearer to it.

Reason. The angular momentum of the earth-moon system is not conserved. [AIIMS 05]

34. **Assertion.** The length of the day is slowly increasing.

Reason. The dominant effect causing a slow down in the rotation of the earth is the gravitational pull of other planets in the solar system. [AIIMS 03]

35. **Assertion.** The earth without its atmosphere would be inhospitably cold.

Reason. All heat would escape in the absence of atmosphere. [AIIMS 02]

36. **Assertion.** The time-period of pendulum on a satellite orbiting the earth is infinity.

Reason. Time-period of a pendulum is inversely proportional to $\sqrt{g}$. [AIIMS 02]

37. **Assertion.** If a pendulum falls freely, then its time period is infinite.

Reason. Free falling body has acceleration equal to g. [AIIMS 97]

38. **Assertion.** An astronaut experiences weightlessness in a space satellite.

Reason. When a body falls freely, it does not experience gravity. [AIIMS 07]

39. **Assertion.** The difference in the value of acceleration due to gravity at pole and equator is proportional to square of angular velocity of earth.

Reason. The value of acceleration due to gravity is minimum at the equator and maximum at the pole. [AIIMS 2010]

Answers and Explanations

1. (*c*) The force of gravitation is a conservative force.

2. (*b*) The revolution of the earth around the sun is due to the centripetal force provided by the gravitational attraction of the sun on the earth.

3. (*d*) The waving of arms helps in keeping the centre of gravity at a suitable position. This helps the man to walk comfortably.

4. (*c*) All the known planets move in elliptical orbits.

5. (*d*) Kepler's second law states that the radius vector drawn from the sun to a planet sweeps out equal areas in equal intervals of time. It is based on the law of conservation of angular momentum.

6. (*b*) Refer answer to the above problem.

7. (*b*) $v_0 = \sqrt{\frac{GM_{sun}}{r}}$

or $v_0 \propto \frac{1}{\sqrt{r}}$

As the Jupiter is at a larger distance from the sun than the earth, so the orbital speed of Jupiter is less than that of the earth.

8. (*b*) For a planet moving around the sun, the angular momentum of the planet is conserved.

9. (*c*) The mass of a body remains constant everywhere.

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10. (*d*) Acceleration due to gravity is independent of the mass of the body.

11. (*a*) $g_1 = \frac{GM_1}{r_1^2} = \frac{G}{r_1^2} \times \frac{4}{3} r_1^3 \rho = \frac{4}{3} \pi G r_1 \rho$

$$g_2 = \frac{4}{3} \pi G r_2 \rho$$

$$\therefore \quad \frac{g_1}{g_2} = \frac{r_1}{r_2}.$$

12. (*b*) $g = \frac{GM}{R^3}$

As the radius of the earth decreases, the value of g increases.

13. (*a*) $d = 2h = 2 \times 5 = 10$ km.

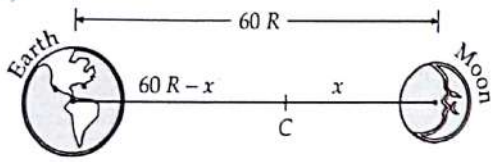
14. (*d*) At depth,

$$d = \frac{R}{2}$$

$$g_d = g\left(1 - \frac{d}{R}\right) = g\left(1 - \frac{R/2}{R}\right) = \frac{g}{2}$$

New weight $= \frac{mg}{2} = \frac{250}{2} = 125$ N.

15. (*c*)



Gravitational field due to earth at C

= Gravitational field due to moon at C

$$\frac{GM}{(60R - x)^2} = \frac{G(M/81)}{x^2}$$

or $\quad 81x^2 = (60R - x)^2$

or $\quad 9x = 60R - x$

or $\quad x = 6R$

16. (*b*) The escape velocity does not depend on the mass of the projectile.

17. (*b*) $v_e = \sqrt{\frac{2GM}{R}}$

$$\omega_e = \frac{v_e}{R} = \sqrt{\frac{2GM}{R^3}}.$$

18. (*c*) The escape velocity of a particle does not depend on its mass.

19. (*a*) The escape velocity of a body does not depend on its angle of projection from the earth's surface.

So, $\quad v_e = 11.2$ km s^{-1}.

20. (*a*) $v_e(\text{earth}) = \sqrt{\frac{2GM}{R}} = 11.2$ km s^{-1}

$$v_e(\text{moon}) = \sqrt{\frac{2G(M/81)}{R/4}}$$

$$= \frac{2}{9}\sqrt{\frac{2GM}{R}} = \frac{2}{9} \times 11.2 \text{ ms}^{-1}$$

$$= 2.45 \text{ km s}^{-1}.$$

21. (*a*) $v_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G}{R} \cdot \frac{4}{3} \pi R^3 \rho} = \sqrt{\frac{8\pi}{3} G\rho \times R}$

$$\therefore \quad v_e \propto R$$

$$\frac{v_e(\text{planet})}{v_e(\text{earth})} = \frac{2R}{R} = 2$$

$$v_e(\text{planet}) = 2 \times 11.2$$

$$= 22.4 \text{ km s}^{-1}.$$

22. (*d*) The required escape velocity of gas molecules is less than their root mean square velocity.

23. (*c*) For velocity less than escape velocity, the missile is bound due to the gravitational field of the earth. Hence its total energy is negative.

24. (*b*) In the absence of the gravitational force, the satellite will move with velocity v tangentially to the original orbit.

25. (*d*) For a satellite, $\frac{GM_e m}{r^2} = \frac{mv^2}{r}$

or $\quad v = \sqrt{\frac{GM_e}{r}} \quad$ *i.e.*, $\quad v \propto \frac{1}{\sqrt{r}}$

$$\therefore \quad \frac{v_1}{v_2} = \sqrt{\frac{r_2}{r_1}}$$

As $r_1 > r_2$, so, $v_1 < v_2$.

26. (*b*) Near the earth's surface,

$$v_e = \sqrt{2} v_0$$

27. (*d*) Kinetic energy,

$$K = \frac{1}{2} m v_0^2 \quad \text{or} \quad v_0 = \sqrt{\frac{2K}{m}}$$

When kinetic energy is doubled,

$$v_0' = \sqrt{\frac{2 \times 2K}{m}} = \sqrt{2}\sqrt{\frac{2K}{m}} = \sqrt{2} v_0$$

But $\quad \sqrt{2} v_0 = v_e$ (escape velocity)

Hence satellite will escape out of earth's gravitational field.

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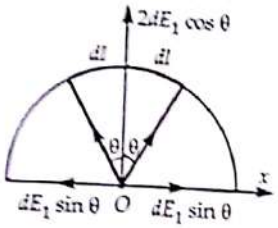
28. (c) Consider two symmetrically located elements, $dl = r\,d\theta$. Here $\pi r = l$



Net field due to the two elements is along y-axis.

$$dE = 2\,dE_1\cos\theta = \frac{2G(m/l)r\,d\theta}{r^2}\cos\theta$$

$$E = \int dE = \frac{2GM}{lr}\int_0^{90^\circ}\cos\theta\,d\theta = \frac{2Gm}{l^2}[\sin\theta]_0^{90^\circ}$$

$$= \frac{2Gm}{l(l/\pi)}(1-0) = \frac{2\pi Gm}{l^2}, \text{ along } y\text{-axis}$$

29. (b) Both the assertion and reason are true but the reason is not a correct explanation of the assertion. Planets are much closer to the earth than the stars. Light coming from planet does not show intensity variations as does the light from stars.

30. (b) Both the assertion and reason are true but the reason is not a correct explanation of the assertion.

31. (b) Both the reason and assertion are true but the reason is not a correct explanation of the assertion. Some of the comets are non-periodic and move along hyperbolic or parabolic paths. They do not obey Kepler's laws of planetary motion.

32. (b) Both the assertion and reason are true but the reason is not correct explanation of the assertion.

$$\frac{GMm}{r^2} = \frac{mv^2}{r} = \frac{m}{r}\times\left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 mr}{T^2}$$

or $$T^2 = \left(\frac{4\pi^2}{GM}\right)r^3 \quad \therefore \quad T^2 \propto r^3.$$

33. (d) Both the assertion and reason are false. The earth is not slowing down. The speed of rotation of the earth about its axis remains constant. Further, the angular momentum of the earth-moon system is conserved.

34. (d) Both the assertion and reason are false. The length of the day depends on the speed of rotation of the earth about its axis. As it remains constant, the length of the day is not changing. Further, the earth is not slowing down due to the gravitational pull of other planets in the solar system.

35. (a) Both the assertion and reason are true. The atmosphere reflects back the heat radiated by the earth.

36. (a) Both the assertion and reason are true. The time period of a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

In a satellite, the bodies are in the state of weightlessness *i.e.*, $g = 0$.

$$\therefore \quad T = 2\pi\sqrt{\frac{l}{0}} = \infty$$

37. (b) Both the assertion and reason are true, but the reason is not correct explanation of the assertion. For correct reasoning, refer to the solution of the above problem.

38. (b) Both the assertion and reason are true but the reason is not a correct explanation of the assertion. The astronaut feels weightlessness because both the astronaut and the satellite are in a state of free fall towards the earth.

39. (b) $g_p - g_e = R_e\omega^2$. Both assertion and reason are true but the reason is not the correct explanation of the assertion.

CBSE PMT Prelims and Final Exams

1. Two spheres of masses m and M are situated in air and the gravitational force between them is F. The space around the masses is now filled with a liquid of specific gravity 3. The gravitational force will now be

(a) $3F$ (b) F

(c) $F/3$ (d) $F/9$ [CBSE PMT 03]

2. The earth (mass $= 6\times10^{24}$ kg) revolves around the sun with an angular velocity of 2×10^{-7} rad/s in a circular orbit of radius 1.5×10^{8} km. The force exerted by the sun on the earth, in newton, is

(a) 36×10^{21} (b) 27×10^{39}

(c) zero (d) 18×10^{25} [CBSE PMT 03]

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3. Two particles of equal mass go around a circle of radius R under the action of their mutual gravitational attraction. The speed v of each particle is

(a) $\frac{1}{2}\sqrt{\frac{Gm}{R}}$ (b) $\sqrt{\frac{4Gm}{R}}$

(c) $\frac{1}{2R}\sqrt{\frac{1}{Gm}}$ (d) $\sqrt{\frac{Gm}{R}}$ [CBSE PMT 95]

4. If the gravitational force between two objects were proportional to $1/R$ (and not as $1/R^2$), where R is the distance between them, then a particle in a circular path (under such a force) would have its orbital speed v, proportional to

(a) R (b) R^0 (independent of R)

(c) $1/R^2$ (d) $1/R$ [CBSE PMT 89, 94]

5. Gravitational force is required for

(a) stirring of liquid (b) convection

(c) conduction (d) radiation [CBSE PMT 06]

6. What will be the formula of mass of the earth in terms of g, R and G ?

(a) $G\frac{R}{g}$ (b) $g\frac{R^2}{G}$

(c) $g^2\frac{R}{G}$ (d) $G\frac{g}{R}$ [CBSE PMT 96]

7. The acceleration due to gravity g and mean density of the earth ρ are related by which of the following relations ? (where G is the gravitational constant and R is the radius of the earth)

(a) $\rho = \frac{3g}{4\pi GR}$ (b) $\rho = \frac{3g}{4\pi GR^3}$

(c) $\rho = \frac{4\pi gR^2}{3G}$ (d) $\rho = \frac{4\pi gR^3}{3G}$ [CBSE PMT 95]

8. The density of a newly discovered planet is twice that of earth. The acceleration due to gravity at the surface of the planet is equal to that at the surface of the earth. If the radius of the earth is R, the radius of the planet would be

(a) $2R$ (b) $4R$

(c) $\frac{1}{4}R$ (d) $\frac{1}{2}R$ [CBSE PMT 04]

9. Imagine a new planet having the same density as that of earth but it is 3 times bigger than the earth in size. If the acceleration due to gravity on the surface of earth is g and that on the surface of the new planet is g', then

(a) $g' = g/9$ (b) $g' = 27g$

(c) $g' = 9g$ (d) $g' = 3g$ [CBSE PMT 05]

10. The acceleration due to gravity on the planet A is 9 times the acceleration due to gravity on planet B. A man jumps to a height of 2 m on the surface of A. What is the height of jump by the same person on the planet B ?

(a) (2/9) m (b) 18 m

(c) 6 m (d) (2/3) m [CBSE PMT 03]

11. A body of weight 72 N moves from the surface of earth at a height half of the radius of earth, then gravitational force exerted on it will be

(a) 36 N (b) 32 N

(c) 144 N (d) 50 N [CBSE PMT 2K]

12. The radius of earth is about 6400 km and that of mars is 3200 km. The mass of the earth is about 10 times mass of mars. An object weighs 200 N on the surface of earth. Its weight on the surface of mars will be

(a) 20 N (b) 8 N

(c) 80 N (d) 40 N [CBSE PMT 94]

13. A ball is dropped from a spacecraft revolving around the earth at a height of 120 km. What will happen to the ball ?

(a) it will fall down to the earth gradually

(b) it will go very far in the space

(c) it will continue to move with the same speed along the original orbit of spacecraft

(d) it will move with the same speed, tangentially to the spacecraft. [CBSE PMT 96]

14. The largest and the shortest distances of the earth from the sun are r_1 and r_2. Its distance from the sun when it is at perpendicular to the major-axis of the orbit drawn from the sun, is

(a) $\frac{r_1 + r_2}{4}$ (b) $\frac{r_1 + r_2}{r_1 - r_2}$

(c) $\frac{2r_1r_2}{r_1 + r_2}$ (d) $\frac{r_1 + r_2}{3}$ [CBSE PMT 88]

15. The period of revolution of planet A around the sun is 8 times that of B. The distance of A from the sun is how many times greater than that of B from the sun ?

(a) 4 (b) 5

(c) 2 (d) 3 [CBSE PMT 97]

16. The distances of two planets from the sun are 10^{13} m and 10^{12} m respectively. The ratio of time periods of the planets is

(a) $\sqrt{10}$ (b) $10\sqrt{10}$

(c) 10 (d) $1/\sqrt{10}$ [CBSE PMT 88, 94]

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17. A body of mass m is placed on earth surface which is taken from earth surface to a height of $h = 3R$. Then change in gravitational potential energy is

(a) $\frac{mgR}{4}$ (b) $\frac{2}{3}mgR$

(c) $\frac{3}{4}mgR$ (d) $\frac{mgR}{2}$ [CBSE PMT 02]

18. The escape velocity of a sphere of mass m is given by (G = universal gravitational constant ; M_e = Mass of the earth and R_e = Radius of the earth)

(a) $\sqrt{\frac{2GM_em}{R_e}}$ (b) $\sqrt{\frac{2GM_e}{R_e}}$

(c) $\sqrt{\frac{GM_e}{R_e}}$ (d) $\sqrt{\frac{2GM_e + R_e}{R_e}}$ [CBSE PMT 99]

19. For a satellite escape velocity is 11 km/s. If the satellite is launched at an angle of 60° with the vertical, then escape velocity will be

(a) 11 km/s (b) $11\sqrt{3}$ km/s

(c) $\frac{11}{\sqrt{3}}$ km/s (d) 33 km/s [CBSE PMT 89]

20. The escape velocity from earth is 11.2 km/s. If a body is to be projected in a direction making an angle 45° to the vertical, then the escape velocity is

(a) 11.2×2 km/s (b) 11.2 km/s

(c) $11.2/\sqrt{2}$ km/s (d) $11.2\sqrt{2}$ km/s [CBSE PMT 93]

21. The escape velocity of a body on the surface of the earth is 11.2 km/s. If the earth's mass increases to twice its present value and radius of the earth becomes half, the escape velocity becomes

(a) 22.4 km/s (b) 44.8 km/s

(c) 5.6 km/s (d) 11.2 km/s [CBSE PMT 97]

22. For a planet having mass equal to mass of the earth, the radius is one fourth of radius of the earth. Then escape velocity for this planet will be

(a) 11.2 km/sec (b) 22.4 km/sec

(c) 5.6 km/sec (d) 44.8 km/sec. [CBSE PMT 2K]

23. With what velocity should a particle be projected so that its height becomes equal to radius of earth ?

(a) $\left(\frac{GM}{R}\right)^{1/2}$ (b) $\left(\frac{8GM}{R}\right)^{1/2}$

(c) $\left(\frac{2GM}{R}\right)^{1/2}$ (d) $\left(\frac{4GM}{R}\right)^{1/2}$ [CBSE PMT 01]

24. The earth is assumed to be a sphere of radius R. A plateform is arranged at a height R from the surface of the earth. The escape velocity of a body from this platform is fv, where v is its escape velocity from the surface of the earth. The value of f is

(a) 1/2 (b) $\sqrt{2}$

(c) $1/\sqrt{2}$ (d) 1/3 [CBSE PMT 96]

25. A satellite A of mass m is at a distance of r from the surface of the earth. Another satellite B of mass $2m$ is at a distance of $2r$ from the earth's surface. Their time periods are in the ratio of

(a) 1 : 2 (b) 1 : 16

(c) 1 : 32 (d) $1 : 2\sqrt{2}$ [CBSE PMT 95]

26. The mean radius of earth is R, its angular speed on its own axis is ω and the acceleration due to gravity at earth's surface is g. What will be the radius of the orbit of a geostationary satellite ?

(a) $(R^2g/\omega^2)^{1/3}$ (b) $(Rg/\omega^2)^{1/3}$

(c) $(R^2\omega^2/g)^{1/3}$ (d) $(R^2g/\omega)^{1/3}$ [CBSE PMT 92]

27. For a satellite moving in an orbit around the earth, the ratio of kinetic energy to potential energy is

(a) 1/2 (b) $1/\sqrt{2}$

(c) 2 (d) $\sqrt{2}$ [CBSE PMT 95]

28. The satellite of mass m is orbiting around the earth in a circular orbit with a velocity v. What will be its total energy ?

(a) $(3/4)mv^2$ (b) $(1/2)mv^2$

(c) mv^2 (d) $-(1/2)mv^2$ [CBSE PMT 91]

29. Two satellites of earth, S_1 and S_2 are moving in the same orbit. The mass of S_1 is four times the mass of S_2. Which one of the following statements is true ?

(a) The potential energies of earth and satellite in the two cases are equal

(b) S_1 and S_2 are moving with the same speed

(c) The kinetic energies of the two satellites are equal

(d) The time period of S_1 is four times that of S_2

[CBSE PMT 97]

30. A planet is moving in an elliptical orbit around the sun. If T, V, E and L stand respectively for its kinetic energy, gravitational potential energy, total energy and magnitude of angular momentum about the centre of force, which of the following is correct ?

(a) T is conserved

(b) V is always positive

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(c) E is always negative

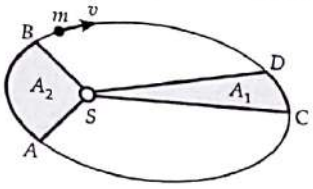
(d) L is conserved but direction of vector L changes continuously [CBSE PMT 90]

31. A satellite in force free space sweeps stationary interplanetary dust at a rate of $dM/dt = \alpha v$, where M is mass and v is the speed of satellite and α is a constant. The acceleration of satellite is

(a) $\frac{-\alpha v^2}{2M}$ (b) $-\alpha v^2$

(c) $\frac{-2\alpha v^2}{M}$ (d) $\frac{-\alpha v^2}{M}$ [CBSE PMT 94]

32. The figure shows elliptical orbit of a planet m about the sun S. The shaded area of SCD is twice the



shaded area SAB. If t_1 is the time for the planet to move from C to D and t_2 is the time to move from A to B, then

(a) $t_1 = 4t_2$ (b) $t_1 = 2t_2$

(c) $t_1 = t_2$ (d) $t_1 > t_2$ [CBSE PMT 09]

33. A particle of mass M is situated at the centre of a spherical shell of same mass and radius a. The gravitational potential at a point situated at $a/2$ distance from the centre, will be

(a) $-\frac{3GM}{a}$ (b) $-\frac{2GM}{a}$

(c) $-\frac{GM}{a}$ (d) $-\frac{4GM}{a}$

[CBSE Pre 2010, Final 2011]

34. A particle of mass m is thrown upwards from the surface of the earth, with a velocity u. The mass and the radius of the earth are, respectively, M and R. G is gravitational constant and g is acceleration due to gravity on the surface of the earth. The minimum value of u so that the particle does not return to earth, is

(a) $\sqrt{2gR^2}$ (b) $\sqrt{\frac{2GM}{R^2}}$

(c) $\sqrt{\frac{2GM}{R}}$ (d) $\sqrt{\frac{2gM}{R^2}}$ [CBSE Pre 2010]

35. The radii of circular orbits of two satellites A and B of the earth are $4R$ and R, respectively. If the speed of satellite A is $3V$, the the speed of satellite B will be

(a) $3V/4$ (b) $6V$

(c) $12V$ (d) $3V/2$ [CBSE Pre 2010]

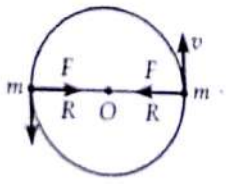
Answers and Explanations

1. (b) The gravitational force between two bodies is independent of the nature of the medium between them.

2. (a) Force exerted by the sun = Centripetal force

$$F = Mr\omega^2$$
$$= 6\times10^{24}\times1.5\times10^{11}\times(2\times10^{-7})^2$$
$$= 36\times10^{21}\ \text{N}.$$

3. (a) The two masses, separated by a distance $2R$, revolve about the common centre of mass O.



Centripetal force = Mutual gravitational attraction

$$\frac{mv^2}{R} = \frac{Gm\times m}{(2R)^2} \quad \text{or} \quad v^2 = \frac{GM}{4R}$$

$$\therefore \quad v = \frac{1}{2}\sqrt{\frac{GM}{R}}.$$

4. (b) Centripetal force = Gravitational force

$$\frac{mv^2}{R} = \frac{GMm}{R} \quad [\text{Here } R^2 \to R]$$

$$\therefore \quad v = \sqrt{GM}$$

Hence v is independent of R.

5. (b) Gravitational force is needed for setting up convection currents.

6. (b) As $g = \frac{GM}{R^2}$

$$\therefore \quad M = g\frac{R^2}{G}$$

7. (a) $g = \frac{GM}{R^2} = \frac{G}{R^2}\times\frac{4}{3}\pi R^3\rho$

$$\therefore \quad \rho = \frac{3g}{4\pi GR}.$$

8. (d) $g_e = \frac{4}{3}\pi G R_e \rho_e$

$$g_p = \frac{4}{3}\pi G R_p \rho_p$$

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As $g_p = g_e$

$\therefore \quad R_p\rho_p = R_e\rho_e$

or $R_p \times 2\rho_e = R \times \rho_e$

or $R_p = \dfrac{R}{2}$.

9. *(d)* $g = \dfrac{4}{3}\pi G R\rho$

$$g' = \frac{4}{3}\pi G R_p \rho = \frac{4}{3}\pi G \times 3R \times \rho$$

$\therefore \quad g' = 3g.$

10. *(b)* For the same gain of P.E. in both cases

$$mg_A h_A = mg_B h_B$$
$$9g_B \times 2 = g_B \times h_B$$
$$h_B = 18 \text{ m}.$$

11. *(b)* $\dfrac{F'}{F} = \dfrac{R^2}{(R+h)^2} = \dfrac{R^2}{\left(R+\dfrac{R}{2}\right)^2} = \dfrac{4}{9}$.

$$F' = \frac{4}{9}F = \frac{4}{9} \times 72 = \mathbf{32\ N}.$$

12. *(c)* $\dfrac{F_m}{F_e} = \dfrac{M_m}{M_e}\left(\dfrac{R_e}{R_m}\right)^2$

$$= \frac{M_m}{10\,M_m}\left(\frac{6400}{3200}\right)^2 = \frac{4}{10} = \frac{2}{5}$$

$$F_m = \frac{2}{5}F_e = \frac{2}{5} \times 200 = \mathbf{80\ N}.$$

13. *(c)* As no external torque is being applied, by law of conservation of angular momentum, the ball will continue to move with the same angular velocity along the original orbit of spacecraft.

14. *(c)* Refer to the solution of Problem 64 on page 8.50.

15. *(a)* According to Kepler's law of periods,

$$T^2 \propto r^3$$

$$\therefore \quad \left(\frac{r_A}{r_B}\right)^3 = \left(\frac{T_A}{T_B}\right)^2 = \left(\frac{8T_B}{T_B}\right)^2 = 64$$

$$\therefore \quad \frac{r_A}{r_B} = 4$$

or $r_A = 4r_B$.

16. *(b)* $\dfrac{T_1}{T_2} = \left(\dfrac{r_1}{r_2}\right)^{3/2} = \left(\dfrac{10^{13}}{10^{12}}\right)^{3/2}$

$$= 10^{3/2} = \mathbf{10\sqrt{10}}.$$

17. *(c)* $U_i = -\dfrac{GMm}{R}$

$$U_f = -\frac{GMm}{R+h}$$

$$= -\frac{GMm}{R+3R} = -\frac{GMm}{4R}$$

$$\Delta U = U_f - U_i = \frac{GMm}{R}\left(1 - \frac{1}{4}\right)$$

$$= \frac{gR^2m}{R} \times \frac{3}{4} = \frac{3}{4}mgR \qquad [GM = gR^2]$$

18. *(b)* $v_e = \sqrt{\dfrac{2GM_e}{R_e}}$

19. *(a)* Escape velocity does not depend on the angle of projection of the body from the earth's surface.

$\therefore \quad v_e = 11 \text{ km s}^{-1}$.

20. *(b)* $v_e = 11.2 \text{ km s}^{-1}$.

Same reason as in the above problem.

21. *(a)* $v_e = \sqrt{\dfrac{2GM}{R}} = 11.2 \text{ km s}^{-1}$.

$$v_e' = \sqrt{\frac{2G \times 2M}{R/2}} = 2\sqrt{\frac{2GM}{R}}$$

$$= 2 \times 11.2 = \mathbf{22.4\ km\ s^{-1}}.$$

22. *(b)* $v_e = \sqrt{\dfrac{2GM}{R}} = 11.2 \text{ km s}^{-1}$

$$v_e' = \sqrt{\frac{2GM}{R/4}} = 2\sqrt{\frac{2GM}{R}}$$

$$= 2 \times 11.2 = \mathbf{22.4\ km\ s^{-1}}.$$

23. *(a)* Initial K.E. = Final P.E. − Initial P.E.

$$\frac{1}{2}mv^2 = -\frac{GMm}{R+R} - \left(-\frac{GMm}{R}\right)$$

$$= \frac{GMm}{R}\left(1 - \frac{1}{2}\right) = \frac{GMm}{2R}$$

$\therefore \quad v = \sqrt{\dfrac{GM}{R}}$.

24. *(c)* Escape velocity from earth's surface,

$$v_e = \sqrt{2gR}$$

Let v_e' be the escape velocity from the platform at height R. As total energy of body at infinity is zero,

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$\frac{1}{2}mv_e^2$ + P.E. of the body on the platform = 0

$$\frac{1}{2}mv_e^2 + \left(-\frac{GMm}{R+R}\right) = 0$$

$$\frac{1}{2}mv_e^2 = \frac{GMm}{2R}$$

$$v_e = \sqrt{\frac{GM}{R}} = \sqrt{gR}$$

But $v_e = fv$

$$\sqrt{gR} = f \times \sqrt{2gR}$$

or $$f = \frac{1}{\sqrt{2}}.$$

25. *(d)* Time period does not depend on mass of the satellite.

$$\frac{T_A}{T_B} = \left(\frac{r_A}{r_B}\right)^{3/2} = \left(\frac{r}{2r}\right)^{3/2} = \frac{1}{2\sqrt{2}} = 1 : 2\sqrt{2}.$$

26. *(a)* $\frac{GMm}{r^2} = mr\omega^2$

or $$r^3 = \frac{GM}{\omega^2} = \frac{gR^2}{\omega^2}$$

$$\therefore \quad r = \left(\frac{gR^2}{\omega^2}\right)^{1/3}.$$

27. *(a)* $$\text{P.E.} = -\frac{GMm}{r}$$

Now $$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$\therefore \quad \text{K.E.} = \frac{1}{2}mv^2 = \frac{GMm}{2r}$$

$$\frac{\text{K.E.}}{\text{P.E.}} = \frac{GMm}{2r} \times \frac{r}{GMm} = \frac{1}{2}.$$

28. *(d)* As $$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$\therefore \quad \text{K.E.} = \frac{1}{2}mv^2 = \frac{GMm}{2r}$$

$$\text{P.E.} = -\frac{GMm}{r} = -mv^2$$

Total energy = K.E. + P.E.

$$= \frac{1}{2}mv^2 - mv^2 = -\frac{1}{2}mv^2.$$

29. *(b)* For a satellite of mass m moving in an orbit of radius r,

$$\text{K.E.} = \frac{GMm}{2r}, \quad \text{P.E.} = -\frac{GMm}{r}$$

$$\text{Orbital speed} = \sqrt{\frac{GM}{r}}$$

$$\text{Time period} = \sqrt{\frac{4\pi^2 r^3}{GM}}$$

As S_1 and S_2 have different m, their potential energies are also different. Hence option *(a)* is false.

S_1 and S_2 are moving with the same orbital speed as it is independent of m. Hence option *(b)* is true.

Because of different m, the kinetic energies of S_1 and S_2 are different. Hence option *(c)* is false.

S_1 and S_2 have same time period because it is independent of m. Hence option *(d)* is false.

30. *(c)* The total energy E is negative because planet is bound to the sun. Because of attractive field, V is negative. Kinetic energy T changes with distance from the sun. When the motion is in a plane, the direction of $\vec{L}$ does not change.

31. *(d)* Retarding force,

F = Velocity × Rate of change of mass

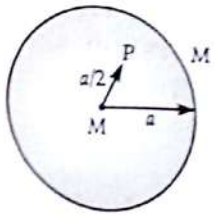
$$= -v\frac{dM}{dt} = -v \times \omega v = -\omega v^2$$

$$a = \frac{F}{M} = -\frac{\omega v^2}{M}.$$

32. *(b)* $\frac{A_1}{t_1} = \frac{A_2}{t_2}$ or $\frac{A_1}{A_2} = \frac{t_1}{t_2} = 2$

$$\therefore \quad t_1 = 2t_2$$

33. *(a)* $V_P = -\frac{GM}{a} - \frac{GM}{a/2} = -\frac{3GM}{a}$



34. *(c)* For the particle to escape the gravitational field of the earth,

$$\frac{1}{2}mu^2 = \frac{GMm}{R}$$

or $$u = \sqrt{\frac{2GM}{R}}$$

35. *(b)* $v_0 = \sqrt{\frac{GM}{r}}$ i.e., $v_0 \propto \frac{1}{\sqrt{r}}$

$$\frac{v_B}{v_A} = \sqrt{\frac{r_A}{r_B}} = \sqrt{\frac{4R}{R}} = 2$$

$$v_B = 2v_A = 2 \times 3\text{V} = 6\text{ V}.$$

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Delhi PMT and VMMC Entrance Exam

1. The mass of moon is 1% of mass of earth. The ratio of gravitational pull of earth on moon and that of moon on earth will be

(a) 1 : 1 (b) 1 : 10
(c) 1 : 100 (d) 2 : 1 [VMMC 05]

2. The value of acceleration due to gravity g at distance r from earth's centre such that $r < R$ depends on r according to relation (R = radius of earth)

(a) $g \propto \frac{1}{r^2}$ (b) $g \propto \frac{1}{r}$
(c) $g \propto r$ (d) $g \propto r^2$ [VMMC 07]

3. If R is the radius of a planet, g is the acceleration due to gravity, then the mean density of the planet is given by

(a) $\frac{3gG}{4\pi R}$ (b) $\frac{4\pi gR}{3G}$
(c) $\frac{4\pi GR}{3g}$ (d) $\frac{3g}{4\pi GR}$ [DPMT 93]

4. Acceleration due to gravity
(a) decreases from equator to poles
(b) decreases from poles to equator
(c) is maximum at the centre of the earth
(d) is maximum at the equator. [DPMT 02, 05]

5. If the mass of earth is 80 times of that of moon and its diameter is double that of moon and g on earth is 9.8 m / sec^2, then the value of g on moon is

(a) 0.98 m / s^2 (b) 0.49 m / s^2
(c) 9.8 m / s^2 (d) 4.9 m / s^2 [DPMT]

6. The acceleration due to gravity at a depth d is g_d and at height h above the surface of earth is g_h. If $d = 2h$, then the ratio g_d / g_h is equal to

(a) 1 (b) 1/2
(c) 2/1 (d) 1/4 [DPMT 07]

7. Assuming earth to be a sphere of uniform density, what is the value of acceleration due to gravity at a point 100 km below the earth's surface ?

(Given $R = 6380 \times 10^3$ m)

(a) 9.66 m / s^2 (b) 5.06 m / s^2
(c) 7.64 m / s^2 (d) 3.10 m / s^2 [DPMT 92]

(Similar Questions)

8. If the earth loses its gravity, then for a body
(a) weight becomes zero, but not the mass
(b) mass becomes zero but not weight
(c) neither mass nor weight is zero
(d) both mass and weight are zero

[DPMT 96 ; VMMC 04]

9. Three equal masses, m each are placed at the three corners of an equilateral triangle of side length l. The gravitational field at centre of triangle is

(a) zero (b) $9Gm/l^2$
(c) $3Gm/l^2$ (d) Gm/l^2 [DPMT 02]

10. A ball is dropped from a spacecraft revolving around the earth at a height of 120 km. What will happen to the ball ?

(a) it will go very far in the space
(b) it will move with the same speed tangentially to the spacecraft
(c) it will fall down to the earth gradually
(d) it will continue to move with the same speed along the original orbit of spacecraft [DPMT 91]

11. A thin uniform rod of mass m and length l is hinged at the lower end to a level floor and stands vertically. It is now allowed to fall, then its upper end will strike the floor with the velocity

(a) $\sqrt{2gl}$ (b) $\sqrt{5gl}$
(c) $\sqrt{3gl}$ (d) $\sqrt{mgl}$ [VMMC 05]

12. The velocity with which a projectile must be fired so that it escapes earth's gravitational field (escape velocity) doesn't depend on

(a) mass of the earth (b) mass of the projectile
(c) radius of orbit
(d) universal gravitational constant G. [DPMT 2K]

13. The escape velocity of a projectile, from the earth is approximately

(a) 7 km/sec (b) 11.2 km/sec
(c) 112 km/sec (d) 1.1 km/sec [DPMT 93]

14. The escape velocity for a body projected vertically upwards from the surface of earth is 11 kms^{-1}. If the body is projected at an angle 45° with the vertically, the escape velocity will be

(a) 22 kms^{-1} (b) 11 kms^{-1}
(c) $11\sqrt{2}$ kms^{-1} (d) $\frac{11}{\sqrt{2}}$ kms^{-1} [DPMT 06, 08]

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15. Escape velocity from a planet is v_e. If its mass is increased to 8 times and its radius is increased to 2 times, then the new escape velocity would be

(a) v_e (b) $\sqrt{2}v_e$

(c) $2v_e$ (d) $2\sqrt{2}v_e$ [DPMT 05]

16. The escape velocity from earth is 11.2 kms^{-1}. Another planet is having mass 1000 times and radius 10 times that of earth, then escape velocity at that planet will be

(a) 11.2 km/s (b) 112 km/s

(c) 1.12 km/s (d) 1120 km/s [DPMT 04]

17. Mass of moon is 1/81 time that of earth and its radius is 1/4 the earth's radius. If escape velocity at surface of earth is 11.2 km/s, then its value at surface of moon is

(a) 0.14 km/s (b) 2.5 km/s

(c) 0.5 km/s (d) 5 km/s [DPMT 95]

18. The minimum energy required to launch a m kg satellite from earth's surface in a circular orbit at an altitude of $5R$, R is the radius of earth, will be

(a) $3mgR$ (b) $\frac{5}{6}mgR$

(c) $2mgR$ (d) $\frac{1}{5}mgR$ [VMMC 06]

19. Two satellites of masses m_1 and m_2 (where m_1 is greater than m_2) are going around the earth in orbits of radii r_1 and r_2 ($r_1 > r_2$). Which statement about velocities is correct ?

(a) $v_1 > v_2$ (b) $v_1 < v_2$

(c) $v_1 = v_2$ (d) $v_1 / r_1 > v_2 / r_2$ [DPMT 94]

20. Black hole is

(a) super surface of atmosphere

(b) ozone layer

(c) super dense planetary material

(d) none of these [VMMC 04]

21. The time period of a satellite is related to the density of earth (ρ) as

(a) $\rho^{1/2}$ (b) ρ

(c) $\rho^{-3/2}$ (d) $\rho^{-1/2}$ [DPMT 05]

22. Potential energy of a satellite having mass 'm' and rotating at a height of 6.4×10^6 m from the earth centre is

(a) $-mgR_e$ (b) $-0.5mgR_e$

(c) $-0.67mgR_e$ (d) $-0.33mgR_e$ [DPMT 95]

23. The total energy of a satellite is E. What is its P.E. ?

(a) $2E$ (b) $-2E$

(c) E (d) $-E$ [DPMT 08]

24. According to Kepler's laws, which of the following is correct ?

(a) $T \propto R^{3/2}$ (b) $T \propto R^3$

(c) $T \propto R^{2/3}$ (d) $T \propto R^2$

(where T is the time period of rotation around sun and R is the average distance between planet and sun.) [DPMT 2K]

25. A satellite is moving on a circular path of radius r around the earth has a time period T. If its radius slightly increases by Δr, the change in its time period is

(a) $\frac{3}{2}\left(\frac{T}{r}\right)\Delta r$ (b) $\left(\frac{T}{r}\right)\Delta r$

(c) $\frac{3}{2}\left(\frac{T^2}{r^2}\right)\Delta r$ (d) none of these [VMMC 06]

26. If the radius of earth's orbit is made one-fourth, then duration of one year will become

(a) 8 times (b) 1/4 times

(c) 4 times (d) 1/8 times [DPMT 99]

27. If the orbital radius of the earth is made 4 times, then find the duration of the year if earlier it was T.

(a) T (b) $2T$

(c) $4T$ (d) $8T$ [DPMT 07]

28. The nearest and farthest planets from Sun are

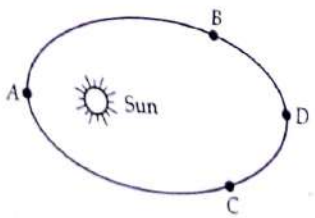
(a) Mars and Venus

(b) Mercury and Pluto

(c) Venus and Pluto

(d) Earth and Neptune. [DPMT 05]

29. The maximum rotational kinetic energy of a planet moving around the sun is at position



(a) A (b) B

(c) D (d) C [DPMT 07]

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30. If radius of the earth is reduced, then
(a) time duration is reduced
(b) earth rotates slower
(c) time period of earth decreases
(d) duration of day increases. [DPMT 08]

31. Two stars of masses m_1 and m_2 are parts of a binary star system. The radii of their orbits are r_1 and r_2 respectively, measured from the CM of the system. The magnitude of the gravitational force m_1 exerts on m_2 is

(a) $\frac{m_1 m_2 G}{(r_1 + r_2)^2}$ (b) $\frac{m_1 G}{(r_1 + r_2)^2}$

(c) $\frac{m_2 G}{(r_1 + r_2)^2}$ (d) $\frac{(m_1 + m_2)}{(r_1 + r_2)^2}$ [DPMT 09]

32. A point mass m is placed inside a spherical shell of radius R and mass M at a distance $R/2$ from the centre of the shell. The gravitational force exerted by the shell on the point mass is

(a) $\frac{GMm}{R^2}$ (b) $-\frac{GMm}{R^2}$

(c) zero (d) $4\frac{GMm}{R^2}$ [DPMT 09]

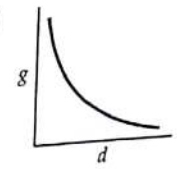
33. A man is standing on an international space station, which is orbiting earth at an altitude 520 km with a constant speed 7.6 km/s. If the man's weight is 50 kg, his acceleration is

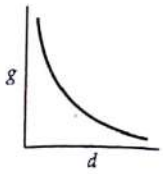
(a) 7.6 km/s^2 (b) 7.6 m/s^2
(c) 8.4 m/s^2 (d) 10 m/s^2. [DPMT 09]

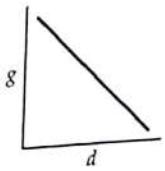
34. A synchronous satellite goes round the earth once in every 24 h. What is the orbit of the synchronous satellite in terms of the earth's radius ? Given mass of the earth, $m_e = 5.98 \times 10^{24}$ kg, radius of the earth, $r_e = 6.37 \times 10^6$ m, Universal gravitational constant, $G = 6.67 \times 10^{-11}$ Nm^2/kg^2.

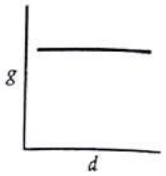
(a) 2.4 r_e (b) 3.6 r_e
(c) 4.8 r_e (d) 6.6 r_e [DPMT 09]

35. Which of the following graphs shows the variation of acceleration due to gravity g with depth d from the surface of the earth ?

(a) 

(b) 

(c) 

(d) 

[DPMT 2011]

36. At what altitude (h) above the earth's surface would the acceleration due to gravity be one-fourth of its value at the earth's surface ?

(a) $h = R$ (b) $h = 4R$
(c) $h = 2R$ (d) $h = 16R$ [DPMT 2011]

Answers and Explanations

1. (a) Gravitational forces are mutually equal and opposite.

2. (c) For $r < R$,

$$g = \frac{GM}{R^3} r \quad \therefore \quad g \propto r$$

3. (d) $g = \frac{GM}{R^2}$

$$= \frac{G}{R^2} \times \frac{4}{3}\pi R^3 \rho = \frac{4}{3}\pi G R \rho$$

$$\therefore \quad \rho = \frac{3g}{4\pi G R}.$$

4. (b) Acceleration due to gravity decreases from poles to equator.

5. (b) For earth,

$$g = \frac{GM}{R^2} = 9.8 \text{ ms}^{-2}$$

For moon,

$$g' = \frac{G(M/80)}{(R/2)^2} = \frac{1}{20}\frac{GM}{R^2}$$

$$= \frac{1}{20} \times 9.8 = \mathbf{0.49 \text{ ms}^{-2}}.$$

6. (a) For $d = 2h$, $g_d = g_h$

7. (a) For $r < R$, $g \propto r$,

$$\frac{g'}{g} = \frac{6280}{6380}$$

$$g' = \frac{628}{638} \times 9.8 = \mathbf{9.66 \text{ ms}^{-2}}.$$

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8. (*a*) Weight = mg

As g becomes zero, weight is zero. Mass is the quantity of matter which is independent of g.

9. (*a*) Refer to the solution of Example 32 on page 8.24.

10. (*d*) In the absence of any external torque, the ball will continue to move with the same speed along the original orbit of space craft.

11. (*c*) $v = \sqrt{3gl}$

Refer to the solution of Example 38 on page 7.36.

12. (*a*) Escape velocity does not depend on the mass of the projectile.

13. (*b*) From the earth's surface,

$$v_e = \sqrt{2gR} = 11.2 \text{ kms}^{-1}.$$

14. (*b*) Escape velocity does not depend on the angle of projection of the projectile.

15. (*c*) $v_e = \sqrt{\dfrac{2GM}{R}}$

$$v_e' = \sqrt{\frac{2G\times 8M}{2R}} = 2\sqrt{\frac{2GM}{R}} = 2v_e.$$

16. (*b*) $v_e = \sqrt{\dfrac{2GM}{R}} = 11.2 \text{ kms}^{-1}$

$$v_e' = \sqrt{\frac{2G\times 100M}{10R}} = 10\sqrt{\frac{2GM}{R}}$$

$$= 10 \times 11.2 = 112 \text{ kms}^{-1}.$$

17. (*b*) $v_e = \sqrt{\dfrac{2GM}{R}}$

$$v_e' = \sqrt{\frac{2G(M/81)}{R/4}} = \frac{2}{9}\sqrt{\frac{2GM}{R}}$$

$$= \frac{2}{9} \times 11.2 \simeq 2.5 \text{ kms}^{-1}.$$

18. (*b*) On earth's surface,

$$U_i = -\frac{GMm}{R}$$

At an altitude of $5R$,

$$U_f = -\frac{GMm}{R+5R} = -\frac{GMm}{6R}$$

Required energy = $U_f - U_i$

$$= \frac{GMm}{R}\left(1 - \frac{1}{6}\right) = \frac{5}{6}\frac{GMm}{R} = \frac{5}{6}\frac{gR^2m}{R} = \frac{5}{6}mgR.$$

19. (*b*) $v_0 \propto \dfrac{1}{\sqrt{r}}$ $[r_1 > r_2]$

$$\therefore \quad \frac{v_1}{v_2} = \sqrt{\frac{r_2}{r_1}} < 1$$

or $v_1 < v_2$.

20. (*c*) Black hole is a super dense planetary material formed due to the continued compression of the core of a star during supernova explosion.

21. (*d*) $T = \sqrt{\dfrac{3\pi(R+h)^3}{G\rho R^3}}$

$$\therefore \quad T \propto \frac{1}{\sqrt{\rho}}$$

22. (*b*) Height $h = 6.4\times10^6$ m is the radius R_e of the earth.

$$\therefore \quad U = -\frac{GMm}{R_e + R_e}$$

$$= -\frac{gR_e^2 m}{2R_e} = -0.5\,mg\,R_e.$$

23. (*a*) P.E. = 2 × Total energy = **2 E**.

24. (*a*) According to Kepler's law of periods,

$$T^2 \propto R^3$$

i.e., $T \propto R^{3/2}$.

25. (*a*) $T^2 \propto r^3$; $T \propto r^{3/2}$

or $T \propto r^{3/2}$

or $T = kr^{3/2}$

$$\therefore \quad \frac{dT}{dr} = \frac{3}{2}kr^{1/2}$$

$$= \frac{3}{2}\left(\frac{T}{r^{3/2}}\right)r^{1/2} = \frac{3}{2}\left(\frac{T}{r}\right)$$

$$\therefore \quad \Delta T = \frac{3}{2}\left(\frac{T}{r}\right)\Delta r.$$

26. (*d*) $\dfrac{T_2}{T_1} = \left(\dfrac{r/4}{r}\right)^{3/2} = \left(\dfrac{1}{4}\right)^{3/2} = \dfrac{1}{8}$

$$T_2 = \frac{1}{8}T_1 = \frac{1}{8} \times 1 \text{ year} = \frac{1}{8} \text{ year}.$$

27. (*d*) $\dfrac{T_2}{T_1} = \left(\dfrac{4r}{r}\right)^{3/2} = (4)^{3/2} = 8$

$$T_2 = 8T_1 = 8T \qquad [T_1 = T]$$

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28. (c) The nearest planet is Mercury and farthest is Pluto.

29. (a) When the planet is at point A, its distance from the sun is minimum and its velocity is maximum. Hence rotational K.E. is maximum at A.

30. (c) $I = \frac{2}{5}MR^2$

When R decreases (for constant M), the M.I. decreases.

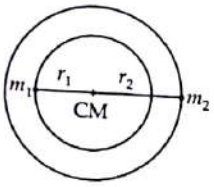
But $I\omega = \text{constant}$

$\therefore$ $\omega = \frac{2\pi}{T}$ increases

Hence period of rotation T decreases.

31. (a) Separation between given stars

$= r_1 + r_2$



$\therefore$ $F_1 = G\frac{m_1 m_2}{(r_1 + r_2)^2}$.

32. (c) At inner position of the shell, the gravitational field intensity (E) is zero.

$\therefore$ $F = mE = 0$.

33. (c) $g_h = \frac{R^2}{(R+h)^2}g$

$= \frac{(6400 \times 10^3)^2}{[(6400 + 520) \times 10^3]^2} \times 10 = 8.4 \text{ m/s}^2$.

34. (d) $T^2 = \frac{4\pi^2 r^3}{GM}$

or $r = \left[\frac{T^2 GM}{4\pi^2}\right]^{1/3}$

$= \left[\frac{(24 \times 3600)^2 \times 6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{4 \times 9.87}\right]^{1/3}$

$= 4.225 \times 10^7 \text{ m} = 42250 \text{ km} = 6.6\ r_e$.

35. (b) $g_d \propto \frac{1}{r}$ for $r < R$

36. (a) $\frac{1}{4}g = g\left(\frac{R}{R+h}\right)^2 \Rightarrow h = R$

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SAMPLE VALUE BASED QUESTIONS

Chapter 2 : UNITS AND MEASUREMENTS

Question 1

Ramesh was travelling alongwith his father in a train. When the train was moving fast, Ramesh looked out of the window of the train. To his surprise, he found that the nearby trees, houses, lamp-posts, etc. seem to move rapidly in a direction opposite to the train's motion while the distant objects such as hill tops, the Moon, the stars, etc. seem to be stationary. Out of shear enthusiasm, he asked his father, who was a science teacher, to explain him the reason behind this observation. He gave the explanation of the phenomenon in detail.

Answer the following questions based on the above information :

(*a*) *What are the values being displayed by Ramesh here ?*

(*b*) *Name the phenomenon involved in the above observation.*

(*b*) *Explain the above observation clearly.*

Answer

(*a*) Keen observer and curiosity.

(*b*) Parallax.

(*c*) The line joining the object and the eye is called the line of sight. The direction of the line of sight of the nearby objects changes very rapidly due to the fast motion of the train and so they appear to move in a direction opposite to the train's motion. But the direction of line of sight of a distant object almost does not change due to large distance from the eye, so the distant objects appear stationary.

Question 2

One day a small piece of hair fell into the left eye of Kavita. It caused severe itching in her eye and she was restless. On seeing this, her elder brother Sunil took her to the eye specialist. The doctor removed the hair from her eye. She felt a sigh of relief and expressed gratitude to both the doctor and her elder brother.

(*a*) *What were the values displayed by Sunil ?*

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(b) *A student measures the thickness of a human hair by looking at it through a microscope of magnification 100. He makes 20 observations and finds that the average width of the hair in the field of view of the microscope is 3.5 mm. What is the estimate on the thickness of hair ?*

Answer

(a) Presence of mind, caring attitude and concern for his sister.

(b) Average thickness of hair as observed through the microscope = 3.5 mm

Magnification produced by the microscope = 100

$$\text{Actual thickness of hair} = \frac{\text{Observed thickness}}{\text{Magnification}}$$

$$= \frac{3.5}{100} = \mathbf{0.035\ mm.}$$

Chapter 3 : MOTION IN A STRAIGHT LINE

Question 3

An old lady, holding a purse in her hand, was crossing the road. She was feeling difficulty in walking. A pickpocket snached the purse from her and started running away. On seeing this incident, Suresh decided to help that old lady. He informed the police inspector who was standing nearby. The inspector chased the pickpocket and caught hold of him. He collected the purse from the pickpocket and gave back the purse to the old lady.

(a) *What were the values displayed by Suresh ?*

(b) *A police jeep is chasing with velocity 45 km/h. A thief in another jeep is moving with 155 km/h. Police fires a bullet with a muzzle velocity of 180 m/s. With what velocity does the bullet strike the jeep of the thief ?*

Answer

(a) Presence of mind, helping nature and sense of social responsibility.

(b) Relative speed of bullet with respect to thief's jeep

$$= (v_b + v_p) - v_t$$

$$= 180\,\text{ms}^{-1} + 45\,\text{kmh}^{-1} - 155\,\text{kmh}^{-1}$$

$$= 180\,\text{ms}^{-1} - 110 \times \frac{5}{18}\,\text{ms}^{-1}$$

$$= 180\,\text{ms}^{-1} - 30.5\,\text{ms}^{-1}$$

$$= \mathbf{149.5\ ms^{-1}.}$$

Question 4

Mohan drived a car at a speed of 70 km/h along a straight road for 8.4 km. Then the car suddenly ran out of petrol. Mohan did not lose his cool. Instead he walked for 30 min to reach a petrol pump at a distance of 2 km.

(a) *What were the values displayed by Mohan ?*

(b) *What was the average velocity from the beginning of his drive till he reached the petrol pump ?*

Answer

(a) Patience, determination and courage.

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(*b*) Average speed $= \frac{s_1 + s_2}{t_1 + t_2} = \frac{8.4 \text{ km} + 2 \text{ km}}{\frac{8.4}{70} h + \frac{30}{60} h}$

$= \frac{10.4}{0.12 + 0.5} = \frac{10.4}{0.62}$

$= 16.8 \text{ km / h}.$

Chapter 4 : MOTION IN A PLANE

Question 5

Sachin was going to a nearby shopping complex alongwith his father who was a Physics teacher. Suddenly it started raining heavily. Sachin's father advised him to hold umbrella slightly inclined with the vertical in the direction of motion. Out of his curiosity, Sachin asked his father why should I hold umbrella slightly inclined even when the rain drops are falling vertically downwards. His father explained him the reason behind it satisfactorily.

(*a*) *What are the values being displayed by Sachin ?*

(*b*) *What are the values displayed by Sachin's father ?*

(*c*) *Give reason for holding the umbrella slightly inclined with the vertical.*

Answer

(*a*) Quest for knowledge.

(*b*) Concern for the child and scientific thinking.

(*c*) The child experiences the velocity of the rain relative to himself. To protect himself from the rain, the child should hold umbrella in the direction of relative velocity of rain w.r.t. the child.

Question 6

Three athletes A, B and C throw their javelins from the similar positions with the same initial speeds making angles 30°, 45° and 60° respectively with the horizontal. Each one tries to throw his javelin to a maximum distance.

(*a*) *Out of three athletes, which one is wiser than the other two ? What value has he displayed ?*

(*b*) *Give a condition for maximum horizontal range of a projectile.*

Answer

(*a*) *B* is wiser than *A* and C. He displays scientific thinking.

(*b*) The angle of projection with the horizontal must be 45°.

Chapter 5 : LAWS OF MOTION

Question 7

Raju saw her grandmother trying to clean a carpet. She was feeling difficulty in lifting the carpet. Raju helped his grandmother in cleaning the carpet by beating it with a stick.

(*a*) *What are the values displayed by Raju ?*

(*b*) *Name the scientific principle involved in Raju's action.*

(*c*) *Give one such more example.*

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Answer

(a) Empathy, helping and caring.

(b) Inertia of rest.

(c) When we shake the branch of a tree, its fruits and dry leaves fall down due to inertia of rest.

Question 8

Ravi went to the market alongwith his grandfather and purchased a weighing machine. They came back to their society complex and entered the lift. Ravi placed the weighing machine on the floor of the lift and asked his grandfather to stand on it. The reading on the machine was 50 kg. When the lift started moving up, the reading increased to 75 kg and when the lift stopped at 10th floor, the reading again dropped to 50 kg. Then both came out of the lift and went to their apartment. His grandfather again checked his weight and found it to be 50 kg. Ravi's grandfather was confused and insisted that the machine is faulty and we must get it changed from the shopkeeper. Ravi told to grandfather that the machine is not faulty and explained why it showed different readings in the lift.

(a) What are the values exhibited by Ravi ?

(b) Why did the machine show increased reading during upward motion ?

Answer

(a) General awareness and scientific thinking.

(b) When the lift accelerates upwards, the apparent weight increases.

Apparent weight $= m(g + a)$.

Question 9

Read the following passage carefully :

The Aristotelian law of motion can be stated as, "An external force is required to keep a body in motion". But there is a serious flaw in Aristotelian law of motion. From our common experience, we can agree with the law of motion as proposed by Aristotle. Let us take an example of a small child playing with a simple (non-electric) toy car on a floor. Even the child also knows intuitively that it needs to constantly drag the string attached to the toy car with some force to keep it going. If child releases the string, the toy car would come to rest because the external force of friction on the toy car by the floor opposes its motion. The child has to apply an external force on the toy car in the direction of motion ; to counter the opposing force of friction. The force applied by the child cancels the force of friction and thus the toy car is kept in motion. Had there been no friction, the child would not be required to apply any force to keep the toy car in uniform motion. The opposing forces such as friction (solids) and viscous forces (for fluids) are always present in the natural world. **[Central Schools 13]**

Answer the following on the basis of above passage :

(a) What is the net external force acting on a body in uniform motion ? What a force can do ?

(b) Name the opposing force in case of motion on the surface of solids and give its one disadvantage.

(c) Name the opposing force in case of motion in fluids and give its one advantage.

Answer

(a) Net external force on a body in uniform motion is zero. A force can change state of rest, the speed or direction of motion of an object.

(b) **Friction.** Friction causes wear and tear of machinery.

(c) **Viscosity.** The knowledge of viscosity helps to select a suitable lubricant for a given machine in different seasons.

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Chapter 6 : WORK, ENERGY AND POWER

Question 10

One day Arvind went to Super Bazar to purchase some groceries. There he saw an old lady struggling with her shopping. He immediately showed her the lift and explained to her how she can carry her goods from one floor to the other. Even then the old lady showed hesitation to use the lift. On seeing this, Arvind took the lady into the lift and showed her how to operate the lift. The old lady was very happy and easily finished her shopping.

(*a*) *What are the values shown by Arvind ?*

(*b*) *An elevator which can carry a maximum load of 1800 kg is moving up with a constant speed of* $2\ ms^{-1}$. *The frictional force opposing the motion is 4000 N. Determine the maximum power delivered by the motor to the elevator in horse power.*

Answer

(*a*) Empathy, helping nature, social responsibility.

(*b*) The downward force on the elevator is

$$F = mg + F_f$$

$$= 1800 \times 10 + 4000 = 22000\ \text{N}$$

The motor must supply enough power to balance this force.

$$\therefore \quad P = Fv$$

$$= 22000 \times 2\ \text{W} = \frac{44000}{746}\ \text{hp} \simeq \mathbf{59\ hp.}$$

Question 11

Puneet decides to use his bath tub water to generate electric power to run a 40 W bulb. The bath tub is located at a height of 10 m from the ground and it holds 200 litres of water. He instals a water driven wheel generator on the ground. Initially, he slowly drains water from the bath but the bulb does not glow. He gradually increases the rate of water drainage and finally succeeds in making the bulb glow.

(*a*) *What are the values being displayed by Puneet ?*

(*b*) *What should be the minimum rate of water drainage from the bath tub for making the bulb glow ? Given efficiency of generator = 90% and* $g = 98\ ms^{-2}$.

Answer

(*a*) Scientific thinking, determination and curiosity.

(*b*) Let V be the minimum volume of water flowing out per second from the tub kept at height h which makes the bulb glow.

Then,

$$\text{Input power} = (V\rho)gh \qquad [\because V\rho = \text{mass of water flowing per second}]$$

$$\text{Output power} = 40\ \text{W}$$

$$\text{Efficiency,} \quad \eta = \frac{\text{Output power}}{\text{Input power}}$$

$$\frac{90}{100} = \frac{40}{V\rho gh}$$

$$\therefore \quad V = \frac{40 \times 100}{90\rho gh} = \frac{40 \times 100}{90 \times 1000 \times 9.8 \times 10} = 0.453 \times 10^{-3}\ \text{m}^3/\text{s} = \mathbf{0.453\ litre/s.}$$

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Question 12

Ravi came to stay in a multistoried building. He noticed that motor supplying water to the second floor has power rating X kW while of that supplying water to 8th floor has Y kW. He asked his father the reason behind the difference of the power ratings. His father explained him the reason. **[Delhi 14]**

(*a*) What values does Ravi posses ?

(*b*) Which power rating is more *X* or *Y* ?

(*c*) A motor pumps up 1000 kg of water through length of 10 m in 5 s. If the efficiency of the motor is 60%, calculate the power of the motor in kilowatt.

Answer

(*a*) Keen observer, curiosity and quest for knowledge.

(*b*) *Y* kW power rating is more than *X* kW power rating.

(*c*) Output power

$$= \frac{mgh}{t} = \frac{1000 \times 9.8 \times 10}{5} = 1960 \text{ W}$$

As $\eta = \dfrac{\text{Output power}}{\text{Input power}}$ $\therefore$ $\dfrac{60}{100} = \dfrac{1960}{\text{Input power}}$

$$\text{Input power} = \frac{1960 \times 100}{60} = 3600 \text{ W} = 3.6 \text{ kW}$$

Hence, the power of the motor is 3.6 kW.

Chapter 7 : SYSTEMS OF PARTICLES & ROTATIONAL MOTION

Question 13

A Physics teacher performs a demonstration in front of her students. She sits on a stool which is free to rotate about a vertical axis almost without any frictional opposition. She holds two heavy two loads in her outstretched hands and asks a student to rotate her stool slowly. Now she pulls her arms closer to her body, her angular speed atonce increases. When she again outstretches her hands, her angular speed decreases. Now she calls one of her students Kavita and asks her to explain this observation. Kavita explains the reasoning in a clear cut manner.

(*a*) *What are the values displayed by Kavita ?*

(*b*) *Which conservation law is involved in this demonstration ? Explain clearly.*

Answer

(*a*) Keen observer, intelligent, critical thinking.

(*b*) Law of conservation of angular momentum is involved here. In the absence of any external torque, angular momentum remains constant.

$$L = I_1\omega_1 = I_2\omega_2$$

When the arms are brought in or out, *I* changes and so ω changes.

Question 14

Kushagra noticed that a wheel was getting detached from his uncle's car. He took the car to a workshop and got it repaired. He informed his uncle about this matter who expressed gratitude to Kushagra.

(*a*) *What according to you are the values displayed by Kushagra ?*

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(b) *A flywheel is revolving with a constant angular velocity. A chip of its rim breaks and flies away. What will be the effect on its angular velocity ?*

Answer

(a) Concern for elders, resourceful.

(b) Due to the decrease in its mass, the moment of inertia of the flywheel will decrease. To conserve angular momentum, the angular velocity of the flywheel will increase.

Chapter 8 : GRAVITATION

Question 15

Mythili was a student of Class IX. She was sitting in a garden alongwith her grandmother, who was a retired Physics teacher. Suddenly she saw an orange falling from the tree. Immediately she asked her grandmother that both the orange and earth experience equal and opposite forces of gravitation, then why it is the orange that falls towards the earth and not the earth towards the orange. Her grandmother explained her the reason in a simple way.

(a) *What are the values being displayed by Mythili ?*

(b) *What in your opinion may be the reason for this observation ?*

Answer

(a) Power of observation and curiosity.

(b) According to the law of gravitation, the orange and earth exert equal and opposite forces on each other. But the mass of the earth is extremely large as compared to that of orange, so the acceleration of the earth is very small and is not noticeable.

Question 16

Deepa and Shilpa were two students of Class XI. Once they were discussing the law of gravitation with each other. Deepa asked Shilpa that a person sitting in a satellite feels weightlessness but a person standing on the moon has weight though moon is also a satellite of the earth. Both were not able to find a suitable answer for this problem. They went to their Physics teacher and sought answer for this question. The teacher explained the reasoning behind this nicely.

(a) *What are the values being displayed by Deepa and Shilpa ?*

(b) *What can be the possible answer for their problem ?*

Answer

(a) Curiosity and critical thinking.

(b) In the artificial satellite, the gravitational attraction due to the earth is used up in providing the necessary centripetal force. Net force on the person is zero, he feels weightlessness. For the person standing on the moon, the gravitational attraction of the moon is left unbalanced, which provides some weight to that person.

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